# Speculation in commodity futures markets: A simple equilibrium model*

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April 30, 2015

**Abstract**

We propose a comprehensive equilibrium model of the interaction between the physical and the derivative markets of a commodity. There are four types of traders, each corresponding to a basic economic function: inventory storers and industrial processors of the commodity, both of which operate on all markets; speculators who operate on the futures market only; and spot traders. The model exhibits a surprising variety of behaviors at equilibrium, which we can use to analyze price relations in any market, according to the characteristics of the commodity under consideration. In particular, it integrates the normal backwardation theory and the storage theory. The paper addresses the political economy of regulatory issues like speculators' presence in the market and their influence on prices: we identifiy precisely the losers and winners of the financialization of commodity markets. This is made possible by the clarification of the comparative statics in terms of the basic effects and their interactions. On the theory side, we give the necessary and sufficient conditions on the fundamentals of this economy for a rational expectations equilibrium to exist, and we show that it is unique.

JEL Codes: D40; D81; D84; G13; Q00.

## 1 Introduction

In the field of commodity derivatives markets, some questions are as old as the markets themselves, and they remain open today. Speculation is a good example. In his famous article about speculation

*The authors acknowledge conversations with Bruno Biais, Eugenio Bobenrieth, Alexander Guembel, Roger Guesnerie, Larry Karp, Sophie Moinas, Marion Oury, and Jean-Charles Rochet; remarks from Gabrielle Demange and Sven Rady; and comments from audiences in Paris (FIME Research Initiative, Chair Finance and Sustainable Development, ACPR, CEMAP, ENSTA and Mines ParisTech), ETH Zurich, Montreal (IAES), Toulouse School of Economics, Santiago de Chile (CMM), Lyon (AFFI and ISFA), Bonn (Haussdorf Center for Mathematics), Toronto (Fields Institute), Milan (Bocconi University), and Cachan (École Normale Supérieure). This article is based on work supported by the Chair Finance and Sustainable Development and the FIME Research Initiative.

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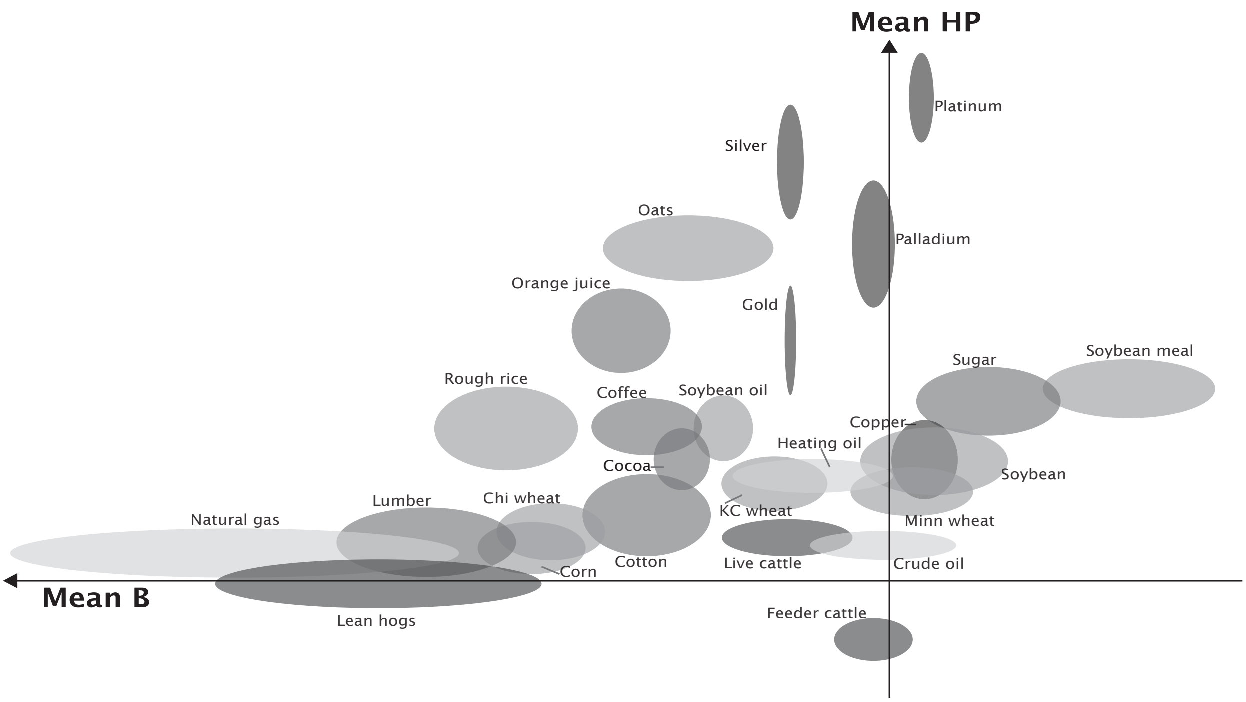


Figure 1: Map of commodity markets. After tables in Kang et al. (2014).

and economic activity, Kaldor (1939) wrote: "Does speculation exert a price-stabilizing influence, or the opposite? The most likely answer is that it is neither, or rather that it is both simultaneously." More than 70 years later, in June 2011, the report of the G20 (FAO et al. (2011)) states: "The debate on whether speculation stabilizes or destabilizes prices resumes with renewed interest and urgency during high price episodes. [...] More research is needed to clarify these questions and in so doing to assist regulators in their reflections about whether regulatory responses are needed and the nature and scale of those responses."

Our model of commodity trading provides insights into this question. We propose a new analysis of the relation between the spot and the futures prices that explains why speculation could be accused of destabilizing markets, and gives a general theoretical framework. In the literature this analysis is usually split into two strands: the storage theory and the normal backwardation theory (also named the hedging pressure theory after De Roon et al. (2000)). The former focuses on the cost of the storage of the underlying asset; the latter on the risk premium. Although they are complementary, to the best of our knowledge these two strands have remained apart up to now.

Figure 1 maps 26 commodity markets along two dimensions: the mean basis B (i.e. the difference between the futures price and the spot price) in the horizontal axis, the mean hedging pressure HP on the vertical one. The data are taken from tables in Kang et al. (2014), a study from 1996 to 2012. These authors examine five categories of commodity futures contracts traded on American markets:

energy, metals, grains, soft commodities and live stocks[1]. The ellipses are drawn according to the standard deviations of the mean basis and hedging pressure. They show that whereas the positions of certain commodities are quite stable (this is the case for example, for the cocoa and the copper), others are more volatile (see the energy markets).

We unify the four regions of this map through a single theory. The regions integrate the storage theory, which explains the implications of a contango (the "current basis", i.e. the difference between the futures price and the *current* spot price, is positive) or a backwardation (the current basis is negative) on the futures market. The regions also integrate the normal backwardation theory that analyzes the "expected basis", that is, the difference between the futures price and the expected spot price. The sign and the magnitude of the bias depend directly on which regime prevails. For example, the futures price can be predicted to be lower (resp. higher) than the expected spot price if there are relatively more (resp. less) storers compared to processors, or if they are relatively more (resp. less) risk averse. The precise thresholds depend on the number of speculators and their risk aversion. So the model depicts the way futures are used to reallocate risk between agents and the price to pay for such a transfer, and it provides insights into the main economic function of derivative markets: hedging.[2]

We show why, in the presence of storage, speculation is both stabilizing and destabilizing, depending on the commodity and on the period. This dependency is made completely explicit. The results make a case for prudence before "evidence" of malfunctioning futures markets. Volatilities and their evolution are the object of much discussion, and we think that a model is useful when it comes to interpreting these changes normatively. To synthesize a number of descriptive results, analyzing the distribution of the impact of speculation across different types of agents, we found simple reasons why agents take pro or con positions as to whether speculation should be encouraged or restricted. The economic determinant of political positions appears to be the weight the agents have in the commodity under consideration and not their specific economic role. For example, an industrialist can be for speculation in a market where hedging is costly to him, and against in an other where hedging is profitable.

The model is built such as to allow for a wide variety of commodities. The financial market interacts with the physical market through two time periods, a single commodity, and a numéraire. There is a spot market at times $t = 1$ and $t = 2$, and a futures market in which contracts are traded at $t = 1$ and settled at $t = 2$. The spot market is physical because of a nonnegativity constraint on inventories, while the futures market is financial because shorting is allowed. There are four types of traders, each corresponding to a basic economic function: spot traders motivated by immediate needs and sensitive to the current price; inventory storers and industrial processors of the commodity, both of which operate on all markets; and speculators who operate on the futures market only. All of them are utility maximizers and have mean-variance utilities. Uncertainty

[1] Table 4 in Appendix D reproduces the values obtained by the authors for these two measures, as well as their standard deviation.

[2] Our model also operates if we assume that all operators (or even a single one) are risk-neutral. The model is still valid and gives the four regimes.

originates in the amount of the commodity produced and in the demand of the spot traders at $t = 2$. The solution is a rational expectation equilibrium with incomplete markets. Our focus is on the basic diversity of objectives among the players in the field. We leave aside in particular the vast and important literature on informational asymmetries,[3] where different behaviors come from different information, not different goals. Such a question is investigated in the context of commodity markets by Leclercq and Praz (2013), among others. We also leave aside the herding effects that might affect the price dynamics of any asset.

Because the model is tractable, our main contributions are: we prove the existence and uniqueness of the equilibrium, we exhibit extended comparative statistics, and we develop a political-economic approach to the regulation of commodity futures markets.

We give necessary and sufficient conditions on the fundamentals of this economy for a rational expectations equilibrium to exist, and we show that it is unique. The equilibrium is characterized by a system of equations that is piecewise linear; the four different regimes coming from the nonnegativity constraints on inventories and committed inputs., Uniqueness is proved by showing that these four regimes do not overlap. Existence on the other hand is based on the nonnegativity conditions on the deterministic and random prices, which yields restrictions on the distribution of the exogenous random factors in the model.

Our model allows for new types of comparative statics. We focus the analysis on a major concept: speculation. Increasing the number of speculators is a metaphor for diverse exogenous events that change the importance of speculation in the futures market. The increase could plainly refer to more speculators in search of profit opportunities or diversification. Or, the increase could refer to a decrease in the apparent (as for the futures market) risk aversion of speculators due, for example, to a smaller correlation between the commodity and other (unmodeled) investment vehicles. Another scenario might be a relaxation of access restrictions on the futures markets.

We show that having futures markets rather than not improves the existence conditions, but increasing the number of speculators when there is a futures market can degrade the existence conditions in a situation where storage is already large. Concerning prices, speculators facilitate hedging and therefore increased speculation increases storage. In the period when storers buy and store, this stabilizes prices (volatility diminishes) and all the more so as speculators are more active. In the period when the storers release their inventories, these bigger quantities destabilize prices.

Beyond these descriptive predictions, we use our model to perform a welfare analysis and to draw regulatory implications. This question, again, is as old as the derivatives markets. Newbery (2008) summarizes well the usual yet ambivalent appreciation of the impact of derivatives markets on welfare. The author makes a difference between what he calls the "layman" and "the body of informed opinion." He explains that to the first, "the association of speculative activity with volatile markets is often taken as proof that speculators are the cause of the instability;" whereas to the second, "volatility creates a demand for hedging or insurance." Our analysis demands a basic distinction: profits from speculation and benefits from hedging are impacted differently by

[3] On this topic, see e.g., Vives (2008).

regulation. As far as speculation only is concerned, all incumbent agents prefer that the number of speculators does not increase. With the rare exception where the physical and financial positions of storers and processors match exactly, they have opposite views on the desirability of speculators. Indeed, the agent that needs more hedging than what the other agent can supply wants more speculators. This is because speculation reduces the costs of hedging for those most in need of it. The other type wants less speculators because they undermine a privileged position. For example, storers want more speculation if they weigh more than processors; and they are against if they weigh less, while processors take the opposite position. To the best of our knowledge, such a political-economic tension has not been clarified before.

The remainder of the paper is as follows: Section 2 is the literature review. Section 3 presents the model. In Section 4, we solve individual programs. Section 5 has the definition of the equilibrium and shows existence and uniqueness results. Section 6 provides a detailed analysis of the possible regimes. In Section 7 we divulge the comparative statistics with respect to speculation, and we divulge the political-economic propositions. Most of the proofs are relegated to the Appendix.

## 2 Literature review

The questions we address have been investigated before; but contrary to what is done in this paper, the literature on commodity prices separates two questions. The first question is the links between the spot and the futures prices. It is usually associated to the theory of storage initiated by Kaldor (1940), Working (1949) and Brennan (1958). The second question is the bias in the futures price (as compared with the expected future spot price). It was investigated first by Keynes (1930) through the theory of normal backwardation. The same separation is true for the equilibrium models developed so far.

A series of equilibrium models for commodity prices focus on the bias in the futures price and the risk transfer function of the derivative market. This is the case, for example, in Anderson and Danthine (1983a), Anderson and Danthine (1983b), Hirshleifer (1988), Hirshleifer (1989), Guesnerie and Rochet (1993), and Acharya et al. (2013). The study by Anderson and Danthine (1983a) is an important source of issues and modeling ideas. The models developed by Hirshleifer (1988) and Hirshleifer (1989) are also inspired by Anderson and Danthine (1983a). In these papers, Hirshleifer analyzes the coexistence of futures and forward markets. Hirshleifer (1989) also asks whether or not vertical integration and futures trading can be a substitute means of diversifying risk: in a context of incomplete markets, integration creates insurance opportunities that are not available otherwise.

Contrary to Anderson and Danthine (1983b), Hirshleifer (1989) and Routledge et al. (2000), our time horizon is limited to two periods in which our focus is on risk management and speculation via storage and futures. Anderson and Danthine (1983b) is the "inter-temporal" extension of Anderson and Danthine (1983a): they allow the futures position to be revised once within the cash market's holding period. To obtain results while keeping tractable equations, the authors however must simplify their model so that only one category of hedger remains in the new version. This is a clear limitation.

Routledge et al. (2000) give another interesting example of inter-temporal analysis. It is related to the literature on equilibrium models that focuses on the *current* spot price and the role of inventories in the behavior of commodity prices, as in Deaton and Laroque (1992), and in Chambers and Bailey (1996). In these models, markets are complete and there is in fact a single type of (representative) agent. Futures prices can be derived from the equilibrium prices; these models are not fit for the political economy of regulatory changes because all problems are already solved.

Beyond the question of the risk premium, equilibrium models have also been used to examine the possible destabilizing effect of the presence of a futures market and to analyze welfare issues. This is the case for Hart and Kreps (1986), Newbery (1987), Gregoir and Salanié (1991), Guesnerie and Rochet (1993), and, more recently, Baker and Routledge (2012), who focus on Pareto optimal risk allocations. As the model proposed by Guesnerie and Rochet (1993) is devoted to the analysis of mental ("eductive") coordination strategies, it is more stripped down than ours. As in Newbery (1987), our explicit formulas for equilibrium prices allow for interesting comparisons based on the presence or absence of a futures market. We complete these by the analysis of increased speculation, and by the description of the incidence of structural changes on the different types of agents: they appear to be very contrasted, which gives importance to political-economic considerations.

Apart from the specific behavior of prices, the nonnegativity constraint on inventories raises another issue. Empirical facts indeed testify that there is more than a nonnegativity constraint in commodity markets: the level of inventories never falls to zero, thus leaving unexploited some supposedly profitable arbitrage opportunities. The concept of a convenience yield associated with inventories, initially developed by Kaldor (1940) and Brennan (1958) is generally used to explain such a phenomenon, which has been regularly confirmed by an empirical point of view, since Working (1949).[4] In their model, Routledge et al. (2000) introduce a convenience yield in the form of an embedded timing option associated with physical stocks, yet the model predicts zero inventories with positive probability. Contrary to these authors, we do not take into account the presence of a convenience yield in our analysis. While this approach might be an interesting improvement to our work, it is not compatible with a two-period model.

Recent attempts to test equilibrium models must also be mentioned, as they are rare. The tests undertaken by Acharya et al. (2013) could be used as a fruitful source of inspiration for further developments. As far the analysis of the risk premium is concerned, the empirical tests performed by Hamilton and Wu (2014) and Szymanowska et al. (2014), as well as the simulations proposed by Bessembinder and Lemmon (2002) are other possible directions.

# 3 The model

The model is based on two time periods. There is one commodity, a numéraire, and two markets: the spot market at times $t = 1$ and $t = 2$ and a futures market in which contracts are traded at

[4]For a recent and exhaustive study on this question, see for example Symeonidis et al. (2012)

$t = 1$ and settled at $t = 2$. The model allows for short positions on the futures market. When an agent sells (resp. buys) futures contracts, his or her position is short (resp. long), and the amount of $f$ he or she holds is negative (resp. positive). On the spot market, short positions are not allowed. In other words, the futures market is financial, while the spot market is physical.

There are three types of inter-temporal traders.

- *Processors (P)*, or industrial users, use the commodity to produce other goods that they sell to consumers. Because of the inertia of their own production process and/or because all of their production is sold forward, they decide at $t = 1$ how much to produce at $t = 2$. They cannot store the commodity, so they have to buy all of their input on the spot market at $t = 2$. They also trade on the futures market.

- *Storers (I for inventory)* have storage capacity and can use this capacity to buy the commodity at $t = 1$ and release it at $t = 2$. They trade on the spot market at $t = 1$ and at $t = 2$. We thus separate the roles of the processor and the storer, although in reality, processors can also hold inventory. The storers also operate on the futures market.

- *Speculators (S)*, or money managers, use the commodity price as a source of risk to make a profit out of their positions in futures contracts. They do not trade on the spot market.

Further, the futures and spot markets operate in a sort of partial equilibrium framework: in the background, there are other sellers of the commodity, and processors as well. These additional agents are referred to as *spot traders*, and their global effect can be described by a demand function. At time $t = 1$, the (net) demand is $\mu_1 - mP_1$, and it is $\tilde{\mu}_2 - m\tilde{P}_2$ at time $t = 2$. The $P_t$ is the spot price at time $t$ and the (net) demand can be either positive or negative; the superscript $\sim$ indicates a random variable.

All traders make their decisions at time $t = 1$, conditionally on the information available for $t = 2$. The timing is as follows:

- For $t = 1$, the commodity is in total supply $\omega_1$, and the spot and the futures markets are open. On the spot market, there are spot traders and storers on the demand side, and the price is $P_1$. On the futures market, the processors, the storers, and the speculators all initiate a position, and the price is $F$. However, the storers have to decide simultaneously how much to buy on the spot market and what position to take on the futures market.

- For $t = 2$, the commodity is in total supply $\tilde{\omega}_2$, to which one has to add the inventory carried by the storers from $t = 1$, and the spot market is open. The processors and the spot traders are on the demand side, and the price is $\tilde{P}_2$. The futures contracts are then settled. We assume that there is a perfect convergence of the basis at the expiration of the futures contract. Thus, at time $t = 2$, the position on the futures market is settled at price $\tilde{P}_2$ that is prevailing on the spot market.

There are $n_P$ processors, $n_S$ speculators, and $n_I$ storage companies. We assume that all agents (except the spot traders) are risk averse, inter-temporal utility maximizers. To make their decisions at time $t = 1$, they need to know the distribution of the spot price $\tilde{P}_2$ at $t = 2$.

Uncertainty is modeled by a probability space $(\Omega, \mathcal{A}, P)$. The $\tilde{\omega}_2$, $\tilde{\mu}_2$, and $\tilde{P}_2$ are random variables on $(\Omega, \mathcal{A}, P)$. At time $t = 1$, their realizations are unknown, but their distributions are common knowledge.

Before we proceed, some clarifications are in order.

- Production of the commodity can be seen as inelastic: the quantities $\omega_1$ and $\tilde{\omega}_2$ that reach the spot market at times $t = 1$ and $t = 2$ are exogenous to the model. Traders know $\omega_1$ and $\mu_1$, and share the same priors about $\tilde{\omega}_2$ and $\tilde{\mu}_2$.
- Alternatively, spot demand can be seen as net demand, meaning that there can be some spot supplies. This view gives meaning to *negative* spot demand. The same remark applies to time $t = 2$.
- We set the risk-free interest rate at 0.

# 4 Optimal positions and market clearing

## 4.1 Utilities

All agents have mean-variance utilities. For all of them, a profit $\tilde{\pi}$ brings the utility:

$$E[\tilde{\pi}] - \frac{1}{2}\alpha_i \mathrm{Var}[\tilde{\pi}] \tag{1}$$

where $\alpha_i$ is the risk aversion parameter of a type $i$ individual. This choice has a long tradition and has been largely discussed, see e.g. Hirshleifer (1988).

## 4.2 Profit maximization

For the speculator, the profit resulting from a position in the futures market $f_S$ is the r.v.:

$$\tilde{\pi}_S(f_S) = f_S\,(\tilde{P}_2 - F),$$

and the optimal position is

$$f_S^\star = \frac{\mathrm{E}[\tilde{P}_2] - F}{\alpha_S \mathrm{Var}[\tilde{P}_2]}. \tag{2}$$

This position is purely speculative. It depends mainly on the level and on the sign of the bias in the futures price. The speculator goes long whenever he or she thinks that the expected spot price is higher than the futures price. Otherwise, he or she goes short. Further, he or she is all the more inclined to take a position if the risk aversion and the volatility of the underlying asset are low.

The storer can hold any nonnegative inventory. However, storage is costly: holding a quantity $X$ between $t = 1$ and $t = 2$ costs $\frac{1}{2}CX^2$. The parameters $C$ (cost of storage) and $\alpha_I$ (risk aversion) characterize the storer. The storer has to decide how much inventory to buy at $t = 1$, if any, and what position to take in the futures market, if any. If the storer buys $X \geq 0$ on the spot market at $t = 1$, resells it on the spot market at $t = 2$, and takes a position $f_I$ on the futures market; then the resulting profit is the r.v.:

$$\tilde{\pi}_I(X, f_I) = X\,(\tilde{P}_2 - P_1) + f_I\,(\tilde{P}_2 - F) - \frac{1}{2}CX^2.$$

The optimal position on the physical market is:

$$X^\star = \frac{1}{C}\max\{F - P_1, 0\}. \tag{3}$$

The storer holds inventories if the futures price is higher than the current spot price. This position is the only one in the model that directly links the spot and the futures prices. This is consistent with the theory of storage and, more precisely, its analysis of contango and the informational role of futures prices. The optimal position on the futures market is:

$$f_I^\star = -X^\star + \frac{\mathrm{E}[\tilde{P}_2] - F}{\alpha_I \mathrm{Var}[\tilde{P}_2]}. \tag{4}$$

This position can be decomposed into two elements. First, a negative position $-X^\star$ that hedges the physical position: the storer sells futures contracts in order to protect himself against a decrease in the spot price. Second, a speculative position, structurally identical to that of the speculator, that reflects the storer's risk aversion and his or her expectations about the relative level of the futures and the expected spot prices.

The processor decides at time $t = 1$ how much input $Y$ to buy at $t = 2$, and which position $f_P$ to take on the futures market. The revenue from sales at date $t = 2$ is $(Y - \frac{\beta}{2}Y^2)\,Z$, where $Z$ is our convention for the forward price of the output, and the other factor reflects decreasing marginal revenue. Due to these forward sales of the production, this revenue is known at time $t = 1$. The resulting profit is the r.v.:

$$\tilde{\pi}_P(Y, f_P) = \left(Y - \frac{\beta}{2}Y^2\right) Z - Y\tilde{P}_2 + f_P(\tilde{P}_2 - F).$$

Therefore, the processor's optimal decisions are:

$$Y^\star = \frac{1}{\beta Z}\max\{Z - F, 0\}, \tag{5}$$

$$f_P^\star = Y^\star + \frac{\mathrm{E}[\tilde{P}_2] - F}{\alpha_P \mathrm{Var}[\tilde{P}_2]}. \tag{6}$$

The futures market is also used by the processor to plan his or her production, particularly if the price of his or her input $F$ is below that of his or her output $Z$. The position on the futures market can be decomposed into two elements: a hedge position $Y^\star$ (the processor goes long on futures contracts in order to protect himself against an increase in the spot price) and a speculative position.

In this framework, all agents have the possibility to undertake speculative operations in the futures market. After having hedged 100 percent of their physical positions, they adjust this position according to their expectations. This result is consistent with the volatile hedging positions recently documented, on a large number of commodity markets, by Cheng and Xiong (2014) and Kang et al. (2014). The separation of the physical and the futures decisions was derived from Danthine (1978). As shown by Anderson and Danthine (1983a), this property does not hold if the final good price ($Z$ here) is endogenous and stochastic unless a second futures market for the final good is introduced. Further, in the absence of a futures market (a scenario analyzed in Appendix F), the quantities held on the physical market necessarily have a speculative dimension.

### 4.3 Market clearing

Appendix A shows why we can take $C = 1$ and $\beta Z = 1$ to simplify the equations without loss of generality.

**The spot market at time 1.** On the supply side, we have the harvest $\omega_1$. On the demand side, we have the inventory $n_I X^\star$ bought by the storers, and the demand of the spot traders. Market clearing requires

$$\omega_1 = n_I X^\star + \mu_1 - mP_1;$$

hence,

$$P_1 = \frac{1}{m}\left(\mu_1 - \omega_1 + n_I X^\star\right). \tag{7}$$

**The spot market at time 2.** We have, on the supply side, the harvest $\tilde{\omega}_2$, and the inventory $n_I X^\star$ sold by the storers; on the demand side, we have the input $n_P Y^\star$ bought by the processors and the demand of the spot traders. The market clearing condition is:

$$\tilde{\omega}_2 + n_I X^\star = n_P Y^\star + \tilde{\mu}_2 - m\tilde{P}_2,$$

with $X^\star$ and $Y^\star$ as above. We get:

$$\tilde{P}_2 = \frac{1}{m}\left(\tilde{\mu}_2 - \tilde{\omega}_2 - n_I X^\star + n_P Y^\star\right). \tag{8}$$

**The futures market.** Market clearing requires:

$$n_S f_S^\star + n_P f_P^\star + n_I f_I^\star = 0.$$

Replacing the $f_i^\star$ by their values, we get:

$$F = \mathrm{E}[\tilde{P}_2] - \frac{\mathrm{Var}[\tilde{P}_2]}{\frac{n_P}{\alpha_P} + \frac{n_I}{\alpha_I} + \frac{n_S}{\alpha_S}}(n_I X^\star - n_P Y^\star). \tag{9}$$

Remark that if different agents of the same type $K$ $(K = P, I, S)$ have different risk aversions $\alpha_{Kj}$ (for $j = 1, \ldots, N_K$), then we have $\sum_j 1/\alpha_{Kj}$ instead of $N_K/\alpha_K$ in Equation (9). This is an illustration of a more general fact: we sum up the inverse of the risk aversions (a.k.a. risk tolerances) of all agents to represent the inverse of the overall (or market) risk aversion.

### 4.4 Summary and definition

Let's set

$$\begin{aligned} \xi_1 &:= \mu_1 - \omega_1, \\ \tilde{\xi}_2 &:= \tilde{\mu}_2 - \tilde{\omega}_2, \\ \xi_2 &:= \mathrm{E}[\tilde{\mu}_2 - \tilde{\omega}_2]. \end{aligned}$$

The $\xi_1$, $\tilde{\xi}_2$, and $\xi_2$ represent scarcity. The distribution of $\tilde{\xi}_2$ is common knowledge. We also assume that $\mathrm{Var}[\tilde{\xi}_2] > 0$. Thus, $\tilde{\xi}_2$ is the only source of uncertainty in the model.

The equations characterizing the equilibrium result from the optimal choices on the physical market (Equations 3 and 5), the clearing of the spot market at dates 1 and 2 (Equations 7 and 8), as well as the clearing of the futures market (9):

$$\begin{cases} X^\star &= \max\{F - P_1, 0\} & (3) \\ Y^\star &= \max\{Z - F, 0\} & (5) \\ P_1 &= \frac{1}{m}(\xi_1 + n_I X^\star) & (7) \\ \tilde{P}_2 &= \frac{1}{m}(\tilde{\xi}_2 - n_I X^\star + n_P Y^\star) & (8) \\ F &= \mathrm{E}[\tilde{P}_2] - \frac{\mathrm{Var}[\tilde{P}_2]}{\frac{n_P}{\alpha_P} + \frac{n_I}{\alpha_I} + \frac{n_S}{\alpha_S}}\mathrm{HP} & (9) \end{cases}$$

HP (Equation (9)) represents the Hedging Pressure or the unbalance of hedging positions:

$$\mathrm{HP} := n_I X^\star - n_P Y^\star. \tag{10}$$

Equation (9) gives a formal expression for the bias in the futures price, which confirms and refines the findings of Anderson and Danthine (1983a). The equation shows that the bias depends primarily on the fundamental economic structures (storage and production costs embedded in the hedging pressure and the number of operators), secondarily on the subjective parameters (agents' risk aversions), and thirdly on the volatility of the underlying asset. Further, the sign of the bias depends only on the sign of the hedging pressure, which is endogenous. Because the risk aversion of the operators only influences the speculative part of the futures position, it does not impact the sign of the bias, at least in this partial equilibrium equation. Furthermore, when HP $= 0$, there is no bias in the futures price, and the risk transfer function of the market is entirely undertaken by the hedgers because their positions on the futures market are opposite and match exactly. Thus, the absence of bias is not exclusively the consequence of risk neutrality but might have other structural causes.

**Definition 1.** An *equilibrium* is a family $(X^\star, Y^\star, P_1, F, \tilde{P}_2)$ such that processors, storers, and speculators act as price-takers; all markets clear; and all prices are nonnegative:

$$X^\star \geq 0, \quad Y^\star \geq 0, \tag{11}$$

$$P_1 \geq 0, \quad F \geq 0, \tag{12}$$

$$\tilde{P}_2 \geq 0 \qquad \text{a.s.} \tag{13}$$

## 5 Existence and uniqueness of the equilibrium

Because of (8), we can derive the expectation and the variance of $\tilde{P}_2$:

$$\mathrm{E}[\tilde{P}_2] = \frac{1}{m}(\xi_2 - n_I X^\star + n_P Y^\star), \tag{8E}$$

$$\mathrm{Var}[\tilde{P}_2] = \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2}. \tag{8V}$$

We also introduce the following notation where $m$ is the price sensitivity of the demand:

$$\gamma := 1 + \frac{1}{m} \frac{\mathrm{Var}[\tilde{\xi}_2]}{\frac{n_P}{\alpha_P} + \frac{n_I}{\alpha_I} + \frac{n_S}{\alpha_S}}$$

This parameter encodes a lot of information about the structure of the market. We have $1 \leq \gamma \leq +\infty$. If one of the agents is risk-neutral (for instance, the processor: $\alpha_P = 0$), then $\gamma = 1$. If all of the agents are pure arbitragers, so that $\alpha_K = +\infty$ for all $K$, then $\gamma = +\infty$. All of these particular cases are integrated into the general calculations, provided some elementary precautions are taken.

**Theorem 1.** *An equilibrium exists if and only if $\tilde{\xi}_2 \geq B(\xi_1)$ a.s., where $B$ is a piecewise-linear continuous decreasing function. And, the equilibrium is then unique.*

The exact expression of $B$ is in Appendix B.3. Figure 3 shows how it looks, the hatched region corresponding to nonexistence (i.e. negative prices with positive probability).

To prove this theorem, we insert Equation (8E) into Equation (9). We get:

$$mF - \gamma(n_P Y^\star - n_I X^\star) = \xi_2. \tag{14}$$

We now have two equations, (7) and (14) for $P_1$ and $F$. Replacing $X^\star$ and $Y^\star$ with their values given by (3) and (5), we get a system of two nonlinear equations for two variables:

$$\begin{cases} mP_1 - n_I \max\{F - P_1, 0\} = \xi_1, & (15) \\ mF + \gamma\left(n_I \max\{F - P_1, 0\} - n_P \max\{Z - F, 0\}\right) = \xi_2. & (16) \end{cases}$$

If we can solve this system with $P_1 > 0$ and $F > 0$, then we get $\tilde{P}_2$ from (8). So the problem is reduced to solving (16) and (15). Consider the mapping $\varphi : \mathbb{R}_+^2 \to \mathbb{R}^2$ defined by:

$$\varphi(P_1, F) = \begin{pmatrix} mP_1 - n_I \max\{F - P_1, 0\} \\ mF + \gamma\left(n_I \max\{F - P_1, 0\} - n_P \max\{Z - F, 0\}\right) \end{pmatrix}.$$

In $\mathbb{R}_+^2$, take $P_1$ as the horizontal coordinate and $F$ as the vertical one as depicted in Figure 2. We denote by $O$ the origin in $\mathbb{R}_+^2$, by $A$ the point $(0, Z)$, and by $M$ the point $(Z, Z)$.

There are four regions, separated by the straight lines $F = P_1$ and $F = Z$:

- Region 1, where $F > P_1$ and $F < Z$, and both $X^\star$ and $Y^\star$ are positive
- Region 2, where $F > P_1$ and $F > Z$, and $X^\star > 0$ and $Y^\star = 0$
- Region 3, where $F < P_1$ and $F > Z$, and $X^\star = 0$ and $Y^\star = 0$
- Region 4, where $F < P_1$ and $F < Z$, and $X^\star = 0$ and $Y^\star > 0$

Moreover, in the regions where $X^\star > 0$, we have $X^\star = F - P_1$; and in the regions where $Y^\star > 0$, we have $Y^\star = Z - F$. So, in each region, the mapping is linear and continuous across the boundaries.

To conclude, we need to show that the system (16) and (15) have a unique solution. It can be rewritten as:

$$\varphi(P_1, F) = \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix},$$

and it has a unique solution if and only if the right-hand side belongs in the image of $F$, which is depicted by Figure 3 and developed in Appendix B.1.

A complementary question exists for a given $(\xi_1, \xi_2)$. Is there a distribution of $\tilde{\xi}_2$ such that an equilibrium exists? This gives the whole set of constraints pictured in Figure 4. There are two characteristic points that are denoted $\varphi'(O)$ and $\varphi'(A)$. For a low $\xi_1$, the constraint $P_1 \geq 0$ matters more because excessive abundance at $t = 1$ must be avoided; but for a low $\xi_2$, the constraint $\tilde{P}_2 \geq 0$ is the most restrictive one because excessive abundance at $t = 2$ should be avoided. This is developed in Appendix B.3, Proposition 2.

Theorem 2 and Figure 9 in Appendix F show that in the absence of a futures market, the existence conditions for an equilibrium are restricted. Whatever the effects on the price levels and volatility, the futures market is unambiguously stabilizing, as long as only existence is considered.

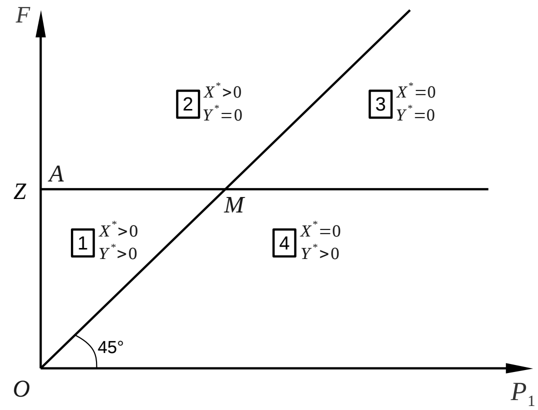


Figure 2: Physical and financial decisions in space $(P_1, F)$: the four regions.

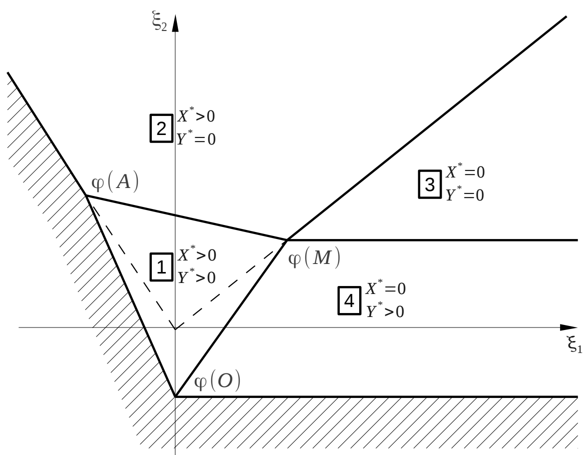


Figure 3: Physical and financial decisions in space $(\xi_1, \xi_2)$: the four regions.

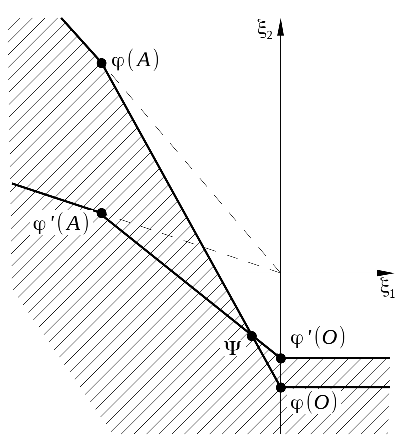


Figure 4: Proposition 2's existence conditions in space $(\xi_1, \xi_2)$: zoom on Region 1.

# 6 Equilibrium analysis

In this section, we analyze the equilibrium in two steps. First, we examine the four regions depicted in Figure 2. They correspond to very different types of interactions between the physical and the financial markets. Second, we turn to Figure 3 that directly shows the impact of "initial net scarcity" ($\xi_1$) and "expected net scarcity" ($\xi_2$).

## 6.1 Prices and physical and financial positions

A first general comment on Figure 2 is that in Regions 1 and 2 where $X^\star > 0$, the futures market is in contango: $F > P_1$. Inventories are positive, and they can be used for inter-temporal arbitrages. In Regions 3 and 4, there is no inventory ($X^\star = 0$), and the market is in backwardation: $F < P_1$. These configurations are fully consistent with the theory of storage.

The other meaningful comparison concerns $F$ and $E[\tilde{P}_2]$. From Equation (9), the hedging pressure HP gives the sign and magnitude of $E[\tilde{P}_2] - F$, that is, the price paid by the hedgers to transfer their risk in the futures market. The analysis of the four possible regions, but with a focus on Region 1 (it is the only one where all operators are active, and it has two important subcases), discloses the reasons for the classical conjecture that there is backwardation on the expected basis, that is, $F < E[\tilde{P}_2]$. More interestingly, we show why the reverse inequality is also plausible, as mentioned by several empirical studies.[5]

The equation HP $= 0$ cuts Region 1 into two parts, 1U and 1L. It passes through $M$ as can be seen in Figure 5. This frontier can be rewritten as:

$$\Delta: \quad n_I(F - P_1) - n_P(Z - F) = 0. \tag{17}$$

- Along the line $\Delta$, there is no bias in the futures price, and the risk is exchanged between hedgers: storers and processors have perfectly matching positions, they hedge each other.
- Above $\Delta$, HP $> 0$ and $F < E[\tilde{P}_2]$. These conditions concern the upper part of Region 1 (Subregion 1U) and Region 2. The net hedging position is short, and speculators in long position are indispensable to the clearing of the futures market. In order to induce their participation, there must be a profitable bias between the futures price and the expected spot price. This backwardation on the expected basis corresponds to the situation depicted by Keynes (1930) as the normal backwardation theory.
- Below $\Delta$, HP $< 0$ and $F > E[\tilde{P}_2]$. These conditions apply to the lower part of Region 1 (Subregion 1L) and Region 4. The net hedging position is long and the speculators must be short, which requires that the expected spot price is lower than the futures price.

Table 1 summarizes for each Region the relations between the prices and the physical and financial positions. The table shows very different regimes. For example, in Region 2, we simultaneously

[5] For extensive analyses of the bias in a large number of commodity markets, see e.g., Fama and French (1987), Kat and Oomen (2007) and Gorton et al. (2013).

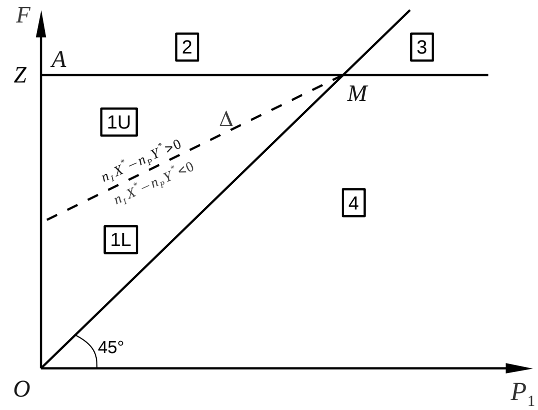


Figure 5: Physical and financial decisions in space $(P_1, F)$ (zoom on Region 1).

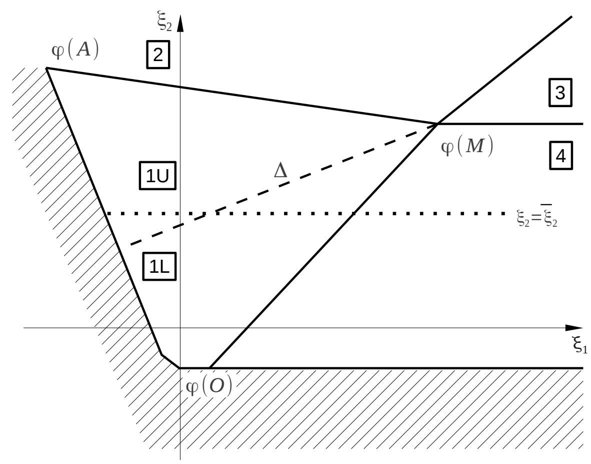


Figure 6: Physical and financial decisions in space $(\xi_1, \xi_2)$ (zoom on Region 1).

| | | | |
|---|---|---|---|
| $\boxed{2}$ | $P_1 < F$ | $F < E[\tilde{P}_2]$ | $F > Z$ |
| | $X^\star > 0$ | $f_S > 0$ | $Y^\star = 0$ |
| $\boxed{1U}$ | $P_1 < F$ | $F < E[\tilde{P}_2]$ | $F < Z$ |
| | $X^\star > 0$ | $f_S > 0$ | $Y^\star > 0$ |
| $\boxed{\Delta}$ | $P_1 < F$ | $F = E[\tilde{P}_2]$ | $F < Z$ |
| | $X^\star > 0$ | $f_S = 0$ | $Y^\star > 0$ |
| $\boxed{1L}$ | $P_1 < F$ | $F > E[\tilde{P}_2]$ | $F < Z$ |
| | $X^\star > 0$ | $f_S < 0$ | $Y^\star > 0$ |
| $\boxed{4}$ | $P_1 > F$ | $F > E[\tilde{P}_2]$ | $F < Z$ |
| | $X^\star = 0$ | $f_S < 0$ | $Y^\star > 0$ |
| $\boxed{3}$ | $P_1 > F$ | $F = E[\tilde{P}_2]$ | $F > Z$ |
| | $X^\star = 0$ | $f_S = 0$ | $Y^\star = 0$ |

Table 1: Relations between prices and physical and financial positions. In reference to figures, regions are listed counter-clockwise.

have a contango on the current basis and a backwardation on the expected basis (or a positive bias). In short, $P_1 < F < E[\tilde{P}_2]$. In Region 3, in the absence of hedging of any sort, the futures market is dormant; and there is no bias on the expected basis. Region 4 is the opposite of Region 2: the market is in backwardation; and, as $X^\star = 0$, the net hedging position is long, the net speculative position is short, and the bias is negative. In short, $P_1 > F > E[\tilde{P}_2]$.

### 6.2 Supply shocks

Let's examine Figure 6. Assuming that no market is open before $\xi_1$ and that $\xi_1$ brings no news about $\xi_2$, we can take $\xi_2$ as fixed; and see what happens to the equilibrium variables, depending on the value of $\xi_1$. To fix ideas, suppose that we expect a moderate scarcity at date 2 ($\xi_2 = \overline{\xi}_2$ in Figure 6).

If period 1 experiences abundance (Subregion 1U), there is massive storage. This storage exists because the current price is low and the expected profits are attractive. It also exists because future scarcity is expected, and storers need more hedging than processors. Thus, there is a positive bias in the futures price, and speculators have long positions. For a less marked abundance (Subregion 1L), storage is more limited. The storers' hedging needs diminish while that of the processors increases. So the net hedging position is long, the bias in the futures price becomes negative, and the speculators have short positions. If the commodity is even scarcer (Region 4), then there is no storage. With no storage, only the processors are active, and they hedge their positions. This example illustrates why, when there is a contango on the current basis, there is either an expected backwardation or an expected contango.

Different categories of commodities can be mapped in Figure 6 according to their characteristics. Due to their fundamental economic structures, some of them will be, more than the others, rooted in a specific region. For example if depreciation is very quick, or the storage costs are very high, the commodity should be situated in Region 4, where $X^\star = 0$ and prices are in backwardation. For the majority of precious metals on the contrary, Region 2, characterized by a high level of stocks ($X^\star > 0$ and $Y^\star = 0$) and prices in contango, is more likely. In these markets, inventories are especially abundant: they are value reserves rather than for use. Region 3, where $X^\star = Y^\star = 0$, is for commodities whose production is almost immediately consumed by the spot traders. As Region 1 is the richest one, it should gather the majority of commodities. Lastly, a market doesn't have to be in the same Region all the time, because its fundamental economic structures change. This is not a paradox, but the recognition that markets normally experience changing supply and demand conditions. Moreover, risk aversions also depend on changing factors, exogenous to the market. The same can be said of the demand by spot traders.

### 6.3 Empirical illustration

Data in Kang et al. (2014), a study of 26 commodity markets from 1994 to 2012, allow for an empirical illustration of our equilibrium analysis. Among their statistics, two are of particular interest for us: the mean basis and the mean hedging pressure for each commodity, both computed over the entire period. Because these measures are proxies of notions of our equilibrium analysis, an analogy can be made between the regions Figure 6 and Figure 1 presented in the introduction.

Of course, the link between the two figures it is not perfect. The values computed by Kang et al. (2014) are means over a long period and cannot be considered independently of their standard deviations. Moreover, the authors do not provide information about the level of physical positions. In addition, Kang et al. (2014) use the data publicly provided by the Commodity Futures Trading Commission (CFTC). To obtain the hedging pressure, they take the difference between the short and long positions, aggregated over all maturities, of the commercial traders. This category however includes swaps dealers like the Commodity Index Traders (CIT), who do not operate on the physical market and might exert a buying hedging pressure on some markets (see for example Hamilton and Wu (2015)).[6] Moreover, the data on commercial positions do not distinguish the hedging from the speculative component of these positions. We thus suppose, at least for sake of this empirical illustration, that speculative components are marginal.

When the mean basis is positive, on the left-hand side of Figure 7, the market is in contango. This corresponds to Regions 1 and 2 in our equilibrium analysis. When the mean basis is negative, in the right-hand side of Figure 7, markets are in backwardation, which characterizes Regions 3 and 4. The left-hand side part of the abscissa axis in Figure 7, where the mean hedging pressure is equal to zero, corresponds to the line $\Delta$ in Figure 6. The positive mean hedging pressure, on the upper left-hand side of Figure 7, is a situation where short hedging positions dominate and it corresponds to Regions 2 and 1U. Conversely, a negative hedging pressure characterizes Regions 1L and 4. Region 3 can be identified to the right-hand side of the horizontal axis in Figure 7. Finally, the upper right-hand side of Figure 7, where the hedging pressure is positive and markets are in backwardation, is not possible in our equilibrium analysis.

A large majority of commodities are positioned in Regions 1 and 2. As expected, Region 1 is the richest part of the map. However, markets are not equally distributed among its sub-regions: Region 1U is more populated.

Given their fundamental economic structures, the markets are also quite well positioned. Precious metals (silver, platinum, palladium and gold) are close to each other and characterized by a high short hedging pressure. Lean hogs and feeder cattle stand in Region 1L where the long hedging pressure dominates. Although situated in Region 1U, the market for live cattle is not far. This result is also quite intuitive: the stocks of animals are characterized by very quick changes and high depreciation, so inventories are presumed to be low in such markets. Overall, energy markets as well as seasonal agricultural products (with the exception of oats, an illiquid market) are situated around the line $\Delta$, where hedging pressure is low.

A market is not necessarily anchored in a specific region. This is all the more true that it is situated near a frontier (as is the case for crude oil, lean hogs and Minneapolis wheat, among others) and that their mean basis or hedging pressure is volatile. The percentage of positive or negative mean hedging pressure recorded over the whole period, provided by Table 4 (Appendix D) also

[6] This is the reason why the CFTC proposes since 2006 a more precise report that identifies clearly the swaps dealers. This report has been however available over a rather short period.

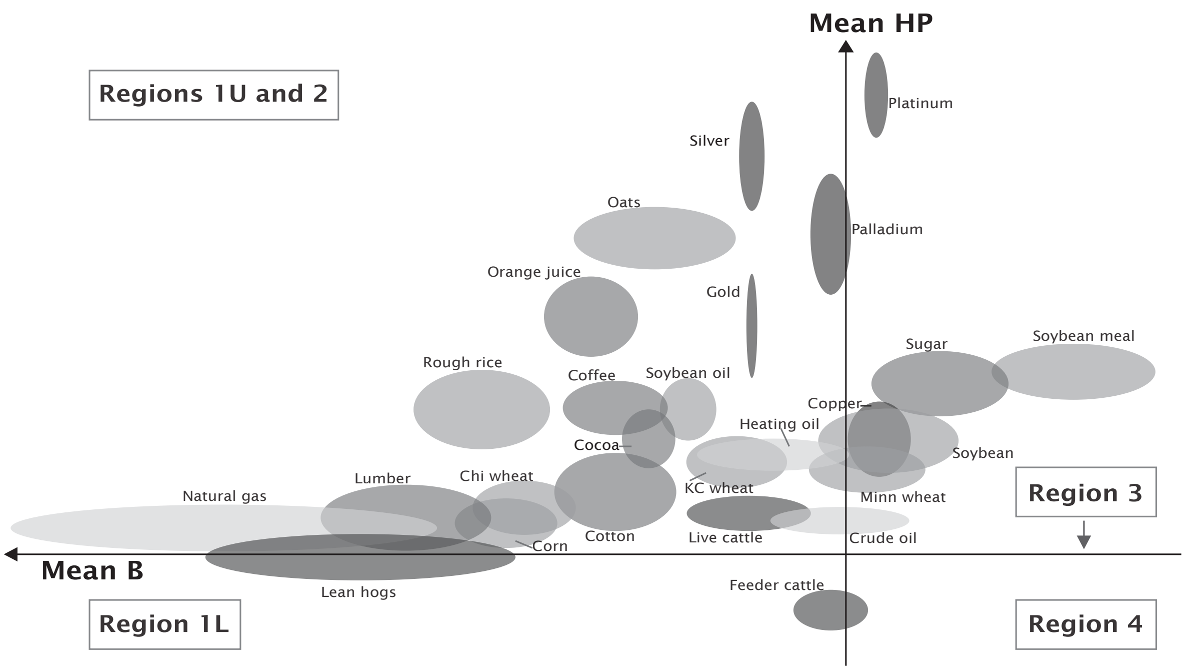


Figure 7: Map of commodity markets, according to Kang et al. (2014) and to our analysis.

gives precious indications. For example, the level of the average short hedging pressure is 1.8% for natural gas. However, this measure is positive at 58.8%. So in more than 40% of the cases, natural gas will be in the Subregion 1L.

Finally, according to the measures obtained by Kang et al. (2014), six markets can be found in the upper right-hand side of Figure 7, a region that does not exist in our setting. Several reasons can explain this result. First, as the link between Figures 6 and 1 is not perfect, the positions of *all* markets must be considered more as an empirical illustration than as a proof. Second, the characteristics of an individual market can play a role in its positioning. This is the case for the high standard deviations recorded on the basis of the soybean and the wheat traded in Minneapolis. Third, this finding could be due to some simplifications voluntary made in the model, like the absence of a seasonality in the prices (this could play a role for the agricultural markets) or the assumption that there is only one futures contract, worldwide, for each commodity.

# 7 The impact of speculation

The impact of speculation can be studied in two ways: first, by examining the difference between having and not having speculators; and second, by examining the effect of “increasing speculation.”

The first approach in particular is taken by Newbery (1987). He shows, in a model without storage, that speculators facilitate hedging by producers by encouraging them to take more risk and thus having a more volatile production. This situation can increase the price variance. We propose related results in Appendix F (No Futures), but storage brings a different perspective. Speculation decreases the price variance when storers store (period 1 in this model): storage stabilize prices particularly as hedging is made cheaper by the speculators. In contrast, speculation can increase the price variance when storers release their inventories (period 2 in Region 2 for this model). The releases are shocks on the spot market that increase volatility. We have already mentioned in Section 5 the result in Appendix F: Theorem 2 and Figure 9 prove that in the absence of a futures market, the existence conditions for an equilibrium are restricted. In this qualitative sense, speculation is stabilizing.

We use the second approach in this section. Increased speculation comes from an easier access to the futures markets (extensive effect), or from a sudden rise in risk appetite (intensive effect). We translate these changes as an increase in the number of speculators $n_S$, or as a decrease of the risk aversion $\alpha_i$. All these possible causes impact the synthetic index $\gamma$ in the same way: it decreases.

### 7.1 Speculators' impact on prices and quantities

Increased speculation has *equilibrium* effects, so that the causal relations between speculation, prices, and quantities must be used with care. However, we propose a sequence of concomitant theoretical facts. The proofs are in the Appendix E.1

The equilibrium analysis assumes that $\xi_1$ is known when the markets open, namely at date 1. But for the observer, $\xi_1$, or rather $\tilde{\xi}_1$, can be seen as random at a previous stage: date 0. In order to analyze the variance, we consider that the prices are functions of two random factors: $\tilde{\xi}_1$ and $\tilde{\xi}_2$, with $\xi_i = \mathrm{E}[\tilde{\xi}_i]$ $(i = 1, 2)$. We assume that the two factors are independent. We focus in particular on $\mathrm{Var}_0[\cdot]$ instead of $\mathrm{Var}_1[\cdot|\ \xi_1]$, as was done implicitly up to now. (The subscript gives the date at which the statistics are calculated; given the absence of ambiguity in the sequel, the subscript will be dropped.)

We first study Region 2 and Subregion 1U where $\mathrm{E}[\tilde{P}_2] > \tilde{F}$, and where the physical agents are sellers in aggregate in period 2 (they prefer the sure — as of date 1 — $\tilde{F}$ to the random $\tilde{P}_2$). References below are to the columns of Table 2. In the table, the absolute value $|\mathrm{E}[\tilde{P}_2] - \tilde{F}|$ gives the (equilibrium) cost of risk coverage for physical agents. This cost is the starting point of our economic analysis.

First are the prices and quantities in levels.

- Increasing speculation increases the overall capacity to absorb risk. In our competitive setting, this increase means that hedging becomes cheaper: the expected margin $\mathrm{E}[\tilde{P}_2] - \tilde{F} > 0$ decreases (see column $|\mathrm{E}[\tilde{P}_2] - \tilde{F}|$).

- As risk management becomes cheaper for storers, they increase their inventories whatever the shock observed in period 1 (see column $\tilde{X}^{\star}$).

- For the processors, hedging is a double win: it reduces risk and is profitable. The rent (or subsidy) they receive is diminished by increased speculation; thus they reduce what they take (see column $\tilde{Y}^\star$).

- Increased inventories mean an increased demand in period 1, thus a price increase (see columns $\tilde{P}_1$). Logically, the effect is a lower price in period 2 due to the extra units drawn from the inventories (see column $\tilde{P}_2$).

Next are the variances.

- The decrease in the hedging cost enables storers to be more reactive to first-period prices, so that overall, their opportunistic purchases attenuate even more production or demand shocks on prices: the covariance of the inventories and the price is negative, and it increases in absolute value with the speculation. This increase explains the lower variance in $\tilde{P}_1$ (see column $\mathrm{Var}[\tilde{P}_1]$).

- The consequence of the previous effect is that there is more variance in the quantity of the commodity delivered in period 2. This increased variance adds noise to the current shocks and thus the variance in $\tilde{P}_2$ increases (see column $\mathrm{Var}[\tilde{P}_2]$.)

- The $\tilde{F}$ and $\tilde{P}_2$ get closer as the speculation increases (see columns $\tilde{F}$ and $\tilde{P}_2$). This convergence means that their variances have the same type of variation with respect to speculation (see columns $\mathrm{Var}[\tilde{F}]$ and $\mathrm{Var}[\tilde{P}_2]$).

Whenever $\mathrm{E}[\tilde{P}_2] < \tilde{F}$, for Region 4 and Subregion 1L, the effects are similar but reverse. Processors are the agents who need more speculation, and they increase their position as the speculation increases. Storers in contrast lose the rent they draw from being structural contrarians.

| | $\lvert\mathrm{E}[\tilde{P}_2] - \tilde{F}\rvert$ | $\tilde{F}$ | $\tilde{X}^\star$ | $\tilde{Y}^\star$ | $\tilde{P}_1$ | $\tilde{P}_2$ | $\mathrm{Var}[\tilde{F}]$ | $\mathrm{Var}[\tilde{P}_1]$ | $\mathrm{Var}[\tilde{P}_2]$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | $\searrow$ | $\nearrow$ | $\nearrow$ | 0 | $\nearrow$ | $\searrow$ | $\searrow$ | $\searrow$ | $\nearrow$ | $\mathrm{E}[\tilde{P}_2] - \tilde{F} > 0$ |
| 1U | $\searrow$ | $\nearrow$ | $\nearrow$ | $\searrow$ | $\nearrow$ | $\searrow$ | $\searrow$ | $\searrow$ | $\nearrow$ | |
| 1L | $\searrow$ | $\searrow$ | $\searrow$ | $\nearrow$ | $\searrow$ | $\nearrow$ | $\searrow$ | $\searrow$ | $\nearrow$ | $\tilde{F} - \mathrm{E}[\tilde{P}_2] > 0$ |
| 4 | $\searrow$ | $\searrow$ | 0 | $\nearrow$ | $\longleftrightarrow$ | $\nearrow$ | $\longleftrightarrow$ | $\longleftrightarrow$ | $\longleftrightarrow$ | |
| 3 | $\longleftrightarrow$ | $\longleftrightarrow$ | 0 | 0 | $\longleftrightarrow$ | $\longleftrightarrow$ | $\longleftrightarrow$ | $\longleftrightarrow$ | $\longleftrightarrow$ | $\tilde{F} = \mathrm{E}[\tilde{P}_2]$ |

Table 2: Impact of speculators on prices and quantities. Legend: $\nearrow$ variable increases; $\searrow$ variable decreases; 0 variable is null; $\longleftrightarrow$ no impact on variable.

**Speculation, prices, and quantities in summary.** Table 2 shows that Regions 2 and 4 can be viewed as subcases of subregions 1U and 1L. For example, $\tilde{P}_1$ decreases in Subregion 1L, whereas it is constant in Region 4. This is due to the fact that the storers are active in Subregion 1L but not in Region 4. Inventories indeed appear as the transmission channel for shocks in "space" (between the financial and the physical markets) and in time (between dates 1 and 2). Thus, a shock appearing in the financial market (i.e. the rise of speculation) impacts the level and variances of the physical quantities and the prices.

As far as the level of the different variables is concerned, our model shows that the impact of an increase in speculation depends, in the end, on which side of the hedging demand dominates. The physical quantities, for example, increase for the agents benefiting from lower hedging costs whereas they decrease for the others. These changes amplify the difference in the positions of the operators and consequently their market impact.

The analysis of the variances is less straightforward. The most simple effect is the impact on $\mathrm{Var}[\tilde{F}]$, which always diminishes under the pressure of a more intense speculative activity (provided that there are stocks in the economy). Concerning the spot prices, an increase in speculation has a stabilizing effect at time 1 and a destabilizing one at time 2. However, the latter result might be modified in a three-period model where the quantities at time 2 are influenced by the futures price of a contract expiring at time 3. It could also be changed if the price of the output, $Z$, could be adjusted as an answer to the shock. Up to now, there is nothing in the model that could absorb a shock at time 2. This version of the model illustrates the fact that financial markets might "destabilize" the underlying markets, though the term is inappropriate since it only refers to a statistical property. Of course a higher price volatility does not mean lower welfare, quite the contrary, more volatility means that prices are more effective/informative signals. The impact of markets on price volatilities is often a naïve aspect of welfare analysis.

### 7.2 Speculators' impact on utilities

In this subsection, we express the equilibrium's indirect utilities of the various types of agents, and we compute their sensitivities with respect to the parameters, in particular the number of speculators. We proceed in two steps. First, we compute the indirect utilities as functions of the equilibrium prices $P_1$ and $F$. Second, we compute the elasticities of $P_1$ and $F$ to deduce the elasticities of the indirect utilities. We restrict ourselves to the richer case, that is, Region 1 where all agents are active. Therefore, we have $F < Z$ and $P_1 < F$. For the sake of simplicity, we return to an analysis where $\xi_1$ is known.

The speculators' indirect utility is given by:

$$U_S = f_S^\star(\mathrm{E}[\tilde{P}_2] - F) - \frac{1}{2}\alpha_S f_S^{\star 2}\mathrm{Var}[\tilde{P}_2],$$

where we have to substitute the value of $f_S^\star$ and and $\mathrm{Var}[\tilde{P}_2]$, which leads to:

$$U_S = \frac{\left(\mathrm{E}[\tilde{P}_2] - F\right)^2}{2\alpha_S \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2}}. \tag{18}$$

The storers' indirect utility is given by:

$$U_I = (X^\star + f_I^\star)\mathrm{E}[\tilde{P}_2] - X^\star P_1 - f_I^\star F - \frac{1}{2}X^{\star 2} - \frac{1}{2}\alpha_I(X^\star + f_I^\star)^2\mathrm{Var}[\tilde{P}_2],$$

where we substitute the values of $f_I^\star$, $X^\star$ and $\mathrm{Var}[\tilde{P}_2]$:

$$U_I = \frac{\left(\mathrm{E}[\tilde{P}_2] - F\right)^2}{2\alpha_I \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2}} + \frac{X^{\star 2}}{2}. \tag{19}$$

Similarly, the processors' indirect utility is:

$$U_P = \frac{\left(\mathrm{E}[\tilde{P}_2] - F\right)^2}{2\alpha_P \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2}} + \frac{Y^{\star 2}}{2}. \tag{20}$$

For all agents, we see a clear separation between the two components of the indirect utilities. The speculative component is associated with the level of the expected basis. The hedging component changes with the category of agent considered. For the storers, it is positively related to the current basis $F - P_1$, and for the processors, it rises with the margin on the processing activity $Z - F$.

We can use directly Table 2 to produce Table 3 in which there are relatively few ambiguities left.

| | $U_S$ | $U_I$ | $U_P$ | |
|---|---|---|---|---|
| $\boxed{2}$ | $\searrow$ | See App. | $\searrow$ | $\mathrm{E}[\tilde{P}_2] - \tilde{F} > 0$ |
| $\boxed{\text{1U}}$ | $\searrow$ | See App. | $\searrow$ | |
| $\boxed{\text{1U near } \Delta}$ | $\searrow$ | $\nearrow$ | $\searrow$ | |
| $\boxed{\text{1L near } \Delta}$ | $\searrow$ | $\searrow$ | $\nearrow$ | $\tilde{F} - \mathrm{E}[\tilde{P}_2] > 0$ |
| $\boxed{\text{1L}}$ | $\searrow$ | $\searrow$ | See App. | |
| $\boxed{4}$ | $\searrow$ | $\searrow$ | See App. | |
| $\boxed{3}$ | $\longleftrightarrow$ | $\longleftrightarrow$ | $\longleftrightarrow$ | $\tilde{F} = \mathrm{E}[\tilde{P}_2]$ |

Table 3: Impact of increasing speculation on welfare. Effects of 1U and 1L are expressed close to line $\Delta$. *See App.* = See Appendix E.2 for complete results.

**Speculation and welfare in summary.** To conclude, all agents are speculators in some way. The speculative component is all the more important as the futures market is biased, whatever the sign of the bias. All agents lose on this part of their utilities from having more speculators. Consequently, the pure speculator always loses from more competition.

As far as storers and processors are concerned, their second (hedging-related) utility components go in strictly opposite directions in Region 1, and weakly opposite directions in Regions 2 and 4. Agents lose (gain) on the hedging components from having more speculators when they have the smaller (bigger) physical position; namely, storers in Regions 4 and 1L,and processors in Regions 2 and 1U.

One case is particularly worthy of mention. Near line $\Delta$ that separates subregions 1U and 1L, speculation is small because the physical positions of the storers and the processors almost match.[7] In that case, agents with the smaller physical position see their privileged position (they play the best part in the hedging market) erode from an increased number of speculators, hence the difference between 1U and 1L near $\Delta$.

In terms of political economy (in the sense that economic interests may determine political positions), we can state the message as follows: we cannot systematically assign a pro or con position to storers nor to processors. The prediction is that they have opposite interests.

## 8 Conclusion

Our model shows the interaction between spot and futures markets and exhibits a surprising variety of behaviors. In equilibrium, there might be a contango or a backwardation, the futures price might be higher or lower than the expected spot price, inventory holders might or might not hold inventory, industrial processors might or might not sell forward, adding speculators might increase or decrease the hedging benefits of inventory holders and of industrial processors. All of these behaviors depend on the market fundamentals and the realization of shocks in the physical market. This rich variety of behaviors can also be found in commodities markets.

The diversity of objectives between agents, in particular if we consider the pair storers/processors, has not received sufficient attention. The clear opposition of their views, like for the interest of developing speculation, sheds light on apparently confused and contradictory debates. Economic interest explains the diversity of political postures between agents and across time.

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[7] In the neighborhood of the frontier with $\Delta$ cutting Region 1 into two parts, the speculation component is of second order with respect to the hedging component.

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# A Simplification without loss of generality

Although we assume that all individuals are identical in each category of agents, more subtle assumptions could be retained without much complication. For example, if the storers have different technologies, say, storer $i$ (with $i = 1, \ldots, n_I$) has technology $C_i$, then instead of $\frac{n_I}{C}\max\{F - P_1, 0\}$, the total inventories are $\left(\sum_i 1/C_i\right)\max\{F - P_1, 0\}$. In other words, storers can be aggregated so that only their number matter. It suffices to redefine variables as follows:

$$
\begin{aligned}
n_I &:= \begin{cases} n_I/C & \text{if storers are identical,} \\ \sum_i 1/C_i & \text{otherwise,} \end{cases} \\
X^\star &:= \max\{F - P_1, 0\}.
\end{aligned}
$$

Similarly, if the processors have different technologies, say, processor $i$ (with $i = 1, \ldots, n_P$) has technology $\beta_i$, then the total input demand is $(\sum_i 1/(\beta_i Z)) \max\{Z - F, 0\}$ instead of $\frac{n_P}{\beta Z} \max\{Z - F, 0\}$. It suffices to redefine variables as follows:

$$
\begin{aligned}
n_P &:= \begin{cases} \frac{n_P}{\beta Z} & \text{if processors are identical,} \\ \frac{1}{Z} \sum_i \frac{1}{\beta_i} & \text{otherwise,} \end{cases} \\
Y^\star &:= \max\{Z - F, 0\}.
\end{aligned}
$$

# B Proofs of existence and uniqueness of the equilibrium

## B.1 Quasi-equilibrium

We need an intermediary concept in which the positivity of $\tilde{P}_2$ is not required:

**Definition 2.** A *quasi-equilibrium* is a family $(X^\star, Y^\star, P_1, F, \tilde{P}_2)$ such that all prices, except possibly $\tilde{P}_2$, are nonnegative; processors, storers, and speculators act as price-takers; and all markets clear.

Technically speaking, a quasi-equilibrium is a family

$$(X^\star, Y^\star, P_1, F, \tilde{P}_2) \in \mathbb{R}_+^4 \times L^0(\Omega, \mathcal{A}, P)$$

such that Equations (3), (5), (7), (8), and (9) are satisfied.

**Lemma 1.** *There is a quasi-equilibrium if and only if* $(\xi_1, \xi_2)$ *verifies:*

$$
\begin{aligned}
&\xi_2 \geq -n_P \gamma Z && \text{if } \xi_1 \geq 0, && (21) \\
&\xi_2 \geq -n_P \gamma Z - \frac{m + (n_I + n_P)\gamma}{n_I} \xi_1 && \text{if } -n_I Z \leq \xi_1 \leq 0, && (22) \\
&\xi_2 \geq -\frac{m + n_I \gamma}{n_I} \xi_1 && \text{if } \xi_1 \leq -n_I Z, && (23)
\end{aligned}
$$

*and then it is unique.*

We examine the images by $\varphi$ of Regions 1 to 4, depicted by Figure 3.

In Region 1 (triangle $OAM$), we have:

$$\varphi(P_1, F) = \begin{pmatrix} m P_1 - n_I (F - P_1) \\ mF + \gamma(n_I(F - P_1) - n_P(Z - F)) \end{pmatrix}.$$

The images $\varphi(O)$, $\varphi(A)$ and $\varphi(M)$ are easily computed:

$$
\begin{aligned}
\varphi(O) &= (0, -\gamma n_P Z), \\
\varphi(A) &= Z\,(-n_I, m + \gamma n_I), \\
\varphi(M) &= m\,Z\,(1, 1).
\end{aligned}
$$

From this, one can find the images of all four regions (see Figure 3).

The image of Region 1 is the triangle $\varphi(O)\varphi(A)\varphi(M)$.

The image of Region 2 is bounded by the segment $\varphi(A)\varphi(M)$ and by two infinite half-lines: one of which is the image of $\{P_1 = 0, F \geq Z\}$, the other being the image of $\{P_1 = F, F \geq Z\}$. In Region 2, we have:

$$\varphi(P_1, F) = \begin{pmatrix} mP_1 - n_I(F - P_1) \\ mF + \gamma n_I(F - P_1) \end{pmatrix}.$$

The first half-line emanates from $\varphi(A)$ and is carried by the vector $(-n_I,\, m + \gamma n_I)$. The second half-line emanates from $\varphi(M)$ and is carried by the vector $(1, 1)$. Both of them (if lengthened) go through the origin.

The image of Region 4 is bounded by the segment $\varphi(O)\varphi(M)$ and by two infinite half-lines, one of which is the image of $\{F = 0\}$, the other being the image of $\{P_1 \geq Z, F = Z\}$. In Region 4, we have:

$$\varphi(P_1, F) = \begin{pmatrix} mP_1 \\ mF - \gamma n_P\,(Z - F) \end{pmatrix},$$

so the first half-line emanates from $\varphi(O)$ and is horizontal with vertical coordinate $-\gamma n_P Z$, and the second emanates from $\varphi(M)$ and is horizontal.

The image of Region 3 is entirely contained in $\mathbb{R}^2_+$ where it is the remainder of the three images we described.

## B.2 Prices and quantities: explicit expressions

The region is determined by $(\xi_1, \xi_2)$, and the final expressions of equilibrium prices are as follows. A remark for all subsequent calculations. Starting from Region 1, setting $n_P = 0$ gives expressions for Region 2; setting $n_I = 0$ gives expressions for Region 4.

**Region 1.**

$$P_1 = \frac{m(m + (n_I + n_P)\gamma)\frac{\xi_1}{m} + mn_I\frac{\xi_2}{m} + n_I n_P \gamma Z}{m(m + (n_I + n_P)\gamma) + mn_I + n_I n_P \gamma}, \tag{24}$$

$$F = \frac{mn_I\gamma\frac{\xi_1}{m} + m(m + n_I)\frac{\xi_2}{m} + (m + n_I)n_P\gamma Z}{mn_I\gamma + m(m + n_I) + (m + n_I)n_P\gamma}, \tag{25}$$

$$\tilde{P}_2 = \frac{\tilde{\xi}_2}{m} + \frac{mn_I\frac{\xi_1}{m} - ((m + n_I)n_P + mn_I)\frac{\xi_2}{m} + (m + n_I)n_P Z}{mn_I\gamma + m(m + n_I) + (m + n_I)n_P\gamma}, \tag{26}$$

All denominators are equal. They are written in different ways only to show that $P_1$ and $F$ are convex combinations of $\frac{\xi_1}{m}$, $\frac{\xi_2}{m}$, and $Z$. Thus:

$$\mathrm{E}[\tilde{P}_2] - F = (\gamma - 1)\frac{mn_I\left(\frac{\xi_2}{m} - \frac{\xi_1}{m}\right) - n_P(m + n_I)\left(Z - \frac{\xi_2}{m}\right)}{m(m + n_I + (n_I + n_P)\gamma) + n_I n_P \gamma}. \tag{27}$$

Quantities:

$$X^\star = \frac{m(m + n_P\gamma)\left(\frac{\xi_2}{m} - \frac{\xi_1}{m}\right) + mn_P\gamma\left(Z - \frac{\xi_2}{m}\right)}{m(m + n_I + (n_I + n_P)\gamma) + n_I n_P \gamma}, \tag{28}$$

$$Y^\star = \frac{mn_I\gamma\left(\frac{\xi_2}{m} - \frac{\xi_1}{m}\right) + m(m + n_I(1+\gamma))\left(Z - \frac{\xi_2}{m}\right)}{m(m + n_I + (n_I + n_P)\gamma) + n_I n_P \gamma}. \tag{29}$$

**Region 2.**

$$P_1 = \frac{(m + n_I\gamma)\frac{\xi_1}{m} + n_I\frac{\xi_2}{m}}{m + n_I(1+\gamma)}\ ;\ F = \frac{n_I\gamma\frac{\xi_1}{m} + (m + n_I)\frac{\xi_2}{m}}{m + n_I(1+\gamma)}\ ;\ \tilde{P}_2 = \frac{\tilde{\xi}_2}{m} + \frac{n_I\left(\frac{\xi_1}{m} - \frac{\xi_2}{m}\right)}{m + n_I(1+\gamma)}\ ;$$
$$X^\star = \frac{m\left(\frac{\xi_2}{m} - \frac{\xi_1}{m}\right)}{m + n_I(1+\gamma)}\ ;\ Y^\star = 0.$$

Therefore:

$$\mathrm{E}[\tilde{P}_2] - \tilde{F} = (\gamma - 1)\frac{n_I\left(\frac{\xi_2}{m} - \frac{\tilde{\xi}_1}{m}\right)}{m + n_I(1+\gamma)} > 0. \tag{30}$$

**Region 3.**

$$P_1 = \frac{\xi_1}{m}\ ;\ F = \frac{\xi_2}{m}\ ;\ \tilde{P}_2 = \frac{\tilde{\xi}_2}{m}\ ;\ X^\star = 0\ ;\ Y^\star = 0\ ;\ \mathrm{E}[\tilde{P}_2] - \tilde{F} = 0.$$

**Region 4.**

$$P_1 = \frac{\xi_1}{m}\ ;\ F = \frac{m\frac{\xi_2}{m} + n_P\gamma Z}{m + n_P\gamma}\ ;\ \tilde{P}_2 = \frac{\tilde{\xi}_2}{m} + \frac{n_P\left(Z - \frac{\xi_2}{m}\right)}{m + n_P\gamma}\ ;\ X^\star = 0\ ;\ Y^\star = \frac{m\left(Z - \frac{\xi_2}{m}\right)}{m + n_P\gamma}.$$

Therefore:

$$\tilde{F} - \mathrm{E}[\tilde{P}_2] = (\gamma - 1)\frac{n_P\left(Z - \frac{\xi_2}{m}\right)}{m + n_P\gamma} > 0. \tag{31}$$

## B.3 Equilibrium

A quasi-equilibrium is an equilibrium if $\tilde{P}_2$ is almost surely positive. If some (probable) realizations of $\tilde{\xi}_2$ are sufficiently low, then there is no equilibrium: states of extreme abundance are inconsistent with positive prices. By Equation (8), the exact condition is:

$$\tilde{\xi}_2 \geq n_I X^\star - n_P Y^\star \qquad \text{a.s.}$$

**Proposition 1.** *An equilibrium exists if and only if* $(\xi_1, \xi_2)$ *satisfies (21), (22), and (23); plus the additional condition of*

$$\begin{aligned}
&\textit{In Region 1,} && \tilde{\xi}_2 \geq m \frac{m n_I \left(\frac{\xi_2}{m} - \frac{\xi_1}{m}\right) - n_P (m + n_I) \left(Z - \frac{\xi_2}{m}\right)}{m(m + n_I + (n_I + n_P)\gamma) + n_I n_P \gamma} && \textit{a.s.} \\
&\textit{In Region 2,} && \tilde{\xi}_2 \geq \frac{m n_I \left(\frac{\xi_2}{m} - \frac{\xi_1}{m}\right)}{m + n_I(1+\gamma)} && \textit{a.s.} \\
&\textit{In Region 3,} && \tilde{\xi}_2 \geq 0 && \textit{a.s.} \\
&\textit{In Region 4,} && \tilde{\xi}_2 \geq -\frac{m n_P (Z - \frac{\xi_2}{m})}{m + n_P \gamma} && \textit{a.s.}
\end{aligned}$$

*And, the equilibrium is then unique.*

*Proof.* In Region 1, given Eq. (26) in Appendix B.2, $\tilde{P}_2 \geq 0$ a.s. is equivalent to:

$$0 \leq \frac{\tilde{\xi}_2}{m} + \frac{m n_I \frac{\xi_1}{m} - ((m + n_I) n_P + m n_I) \frac{\xi_2}{m} + (m + n_I) n_P Z}{m n_I \gamma + m(m + n_I) + (m + n_I) n_P \gamma},$$

which gives the expression of the Theorem after rearrangement.

Starting from the condition for Region 1, taking $n_P = 0$ yields the condition for Region 2, taking $n_I = 0$ yields the condition for Region 4, and taking $n_I = n_P = 0$ yields the condition for Region 3. $\square$

A complementary question exists for a given $(\xi_1, \xi_2)$. Is there a distribution of $\tilde{\xi}_2$ such that an equilibrium exists?

**Proposition 2.** *If* $(\xi_1, \xi_2)$ *supports a unique quasi-equilibrium under the terms of Lemma 1, then there is a distribution of* $\tilde{\xi}_2$ *that supports an equilibrium if and only if*

$$\begin{aligned}
&\frac{\xi_2}{m} \geq -\frac{n_I \xi_1 + (m + n_I) n_P Z}{m(m + (\gamma - 1) n_P) + n_I (m\gamma + (\gamma - 1) n_P)} && \textit{in Region 1,} \\
&\frac{\xi_2}{m} \geq -\frac{n_I}{m + n_I \gamma} \frac{\xi_1}{m} && \textit{in Region 2,} \\
&\frac{\xi_2}{m} \geq 0 && \textit{in Region 3,} \\
&\frac{\xi_2}{m} \geq -\frac{n_P Z}{m + (\gamma - 1) n_P} && \textit{in Region 4.}
\end{aligned}$$

*Proof.* Starting from Proposition 1 and because the limit case allowing us to draw the frontier occurs when $\operatorname{ess\,inf}\{\tilde{\xi}_2\} = \xi_2$, we find the conditions above after rearrangement.

Some precisions about the constraints added by this proposition are depicted by Figure 4. The characteristic points of the boundary are:

$$\varphi'(O) = \left(0, -\frac{mn_P}{m + n_P(\gamma - 1)} Z\right);$$
$$\varphi'(A) = \left(-n_I Z, \frac{n_I^2}{m + n_I \gamma} Z\right).$$

Remark that $\varphi'(O)$ is above $\varphi(O)$ (both have the same negative abscissa), and that $\varphi'(A)$ is below $\varphi(A)$ (both have the same positive abscissa). The intersection point $\Psi$ of the two sets of constraints is in Region 1, with:

$$\Psi = \left(-\frac{n_I n_P(\gamma - 1)Z}{m + (n_I + n_P)(\gamma - 1)}, -\frac{mn_P Z}{m + (n_I + n_P)(\gamma - 1)}\right).$$

□

# C Comparative statics on the existence of the equilibrium

This appendix deals with the existence of an equilibrium when $\gamma$ increases by examining the changes in the boundaries.

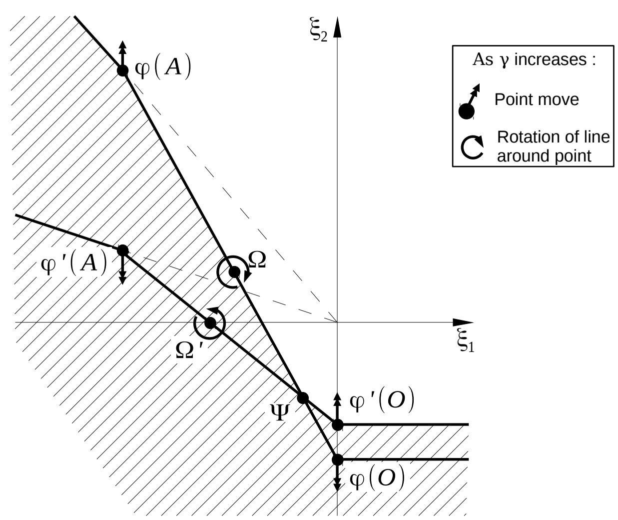


Figure 8: Existence conditions of the equilibrium: comparative statics on $\gamma$.

See Figure 4 for a starting point. Figure 8 illustrates how the characteristic points move as $\gamma$ increases. Point $\varphi(M)$ remains fixed, while $\varphi(O)$, $\varphi'(O)$, $\varphi(A)$ and $\varphi'(A)$ move vertically, as the calculations show. The relevant points, namely $\varphi'(O)$ and $\varphi(A)$, move vertically upwards. This movement implies that the existence conditions are restrained in Regions 2 and 4.

The effects on Region 1 are mixed. The segment $\varphi(A)\varphi(O)$ rotates clockwise around the fixed point (i.e., whose value does not depend on $\gamma$):

$$\Omega = \left(-\frac{n_I n_P Z}{n_I + n_P}, \frac{m n_P Z}{n_I + n_P}\right)$$

that is in the NW quadrant (calculations available on request). The segment $\varphi'(A)\varphi'(O)$ rotates anticlockwise around the fixed point (i.e. whose value does not depend on $\gamma$):

$$\Omega' = \left(-\frac{(m + n_I)n_P Z}{n_I}, 0\right)$$

that is on the horizontal axis (calculations available on request). This rotation proves that Region 1 enlarges on one side, as far as existence is considered.

Overall, speculation facilitates existence of an equilibrium in space $(\xi_1, \xi_2)$, except for one side of Region 1, namely $(\Omega, \Psi)$.

The comparative statics on $\gamma$ are now clear: the more frictious the markets (the higher $\gamma$), the tighter the existence conditions in Region 2 and Region 4 where only one type of actor actually has a physical position. In Region 1, where physical positions of storers and processors compensate (more or less) for each other, the conclusion is not clear-cut.

# D Empirical illustration of the equilibrium analysis: the data

This appendix reproduces the data used in Subsection 6.3. They are extracted from two Tables in Kang et al. (2014), which provide for summary statistics of commodity futures basis and hedging pressure, for 26 commodity markets, from 1994 to 2012.

Table 4 exhibits the values of two quantities which are of particular interest in our setting. First, the mean log basis $B_{i,t}$ recorded on the period for commodity $i$:

$$B_{i,t} = \frac{\log F_i(t, T_2) - \log F_i(t, T_1)}{T_2 - T_1},$$

where $F_i(t, T_n)$ is the futures price recorded at week $t$ for the maturity $n$, and $T_1$ and $T_2$ correspond to the maturities of the closest to maturity and the next to maturity contracts. Second, the average hedging pressure (HP) over the period. The authors define it as the net short (short minus long) positions of hedgers in commodity $i$, summed over all maturities, and divided by the total open interest of that commodity. They also give the percentage of weeks when there is a short hedging pressure (SHP). This definition of the hedging pressure is very close to ours, except for the division by the open interest, which allows essentially for comparison between commodity markets.

(RMK: Plus other differences mention in the text – see Subsection 6.3)

| Commodity | Mean B | (Std dev) | Mean HP | (Std dev) | Percent. of SHP |
|---|---|---|---|---|---|
| Crude oil | 0.2% | (22.9%) | 3% | (7.7%) | 68.0% |
| Heating oil | 2.4% | (24.2%) | 10.3% | (8.7%) | 87.2% |
| Natural gas | 20.5% | (61.7%) | 1.8% | (11.5%) | 58.8% |
| Platinum | −1.0% | (3.9%) | 49.1% | (23.9%) | 95.0% |
| Palladium | 0.5% | (6.7%) | 32.5% | (33.9%) | 76.5% |
| Silver | 3.1% | (2.1%) | 43.1% | (15.3%) | 100.0% |
| Copper | −1.1% | (10.4%) | 10.8% | (21.1%) | 68.0% |
| Gold | 3.1% | (1.8%) | 22.7% | (29.2%) | 74.0% |
| Chi Wheat | 10.6% | (17.0%) | 3.7% | (15.2%) | 52.8% |
| KC Wheat | 3.6% | (16.6%) | 8.9% | (14.5%) | 74.1% |
| Minn Wheat | −0.7% | (19.2%) | 8.2% | (13.0%) | 73.0% |
| Corn | 11.2% | (16.9%) | 2.1% | (13.9%) | 56.7% |
| Oats | 6.3% | (26.7%) | 33.7% | (17.5%) | 95.0% |
| Soybean | −1.4% | (22.3%) | 11.0% | (17.4%) | 73.0% |
| Soybean Oil | 5.2% | (9.3%) | 14.5% | (17.5%) | 75.0% |
| Soybean Meal | −7.5% | (27.0%) | 18.9% | (15.7%) | 83.8% |
| Rough Rice | 12.0% | (22.5%) | 14.0% | (22.3%) | 72.3% |
| Cotton | 7.6% | (20.1%) | 4.8% | (21.9%) | 60.2% |
| Orange Juice | 8.4% | (15.5%) | 24.4% | (22.5%) | 84.4% |
| Lumber | 14.5% | (28.1%) | 10.1% | (18.5%) | 65.7% |
| Cocoa | 6.5% | (8.8%) | 11.3% | (16.5%) | 72.7% |
| Sugar | −3.1% | (22.6%) | 17.3% | (18.3%) | 79.1% |
| Coffee | 7.6% | (17.3%) | 14.9% | (15.2%) | 78.7% |
| Lean Hogs | 16.0% | (51.1%) | −0.4% | (13.0%) | 52.2% |
| Live Cattle | 3.2% | (20.5%) | 3.6% | (9.9%) | 61.0% |
| Feeder Cattle | 0.5% | (12.3%) | −7.4% | (11.4%) | 26.7% |

Table 4: Summary statistics of commodity futures basis and hedging pressure, 1994 − 2012.

# E Prices' and utilities' sensitivities in Section 7

This appendix depicts in detail the impact of speculators, first on prices and quantities and second on utilities. The analysis of an increase in speculation is studied as a decrease in $\gamma$, as explained at the beginning of Section 7.

## E.1 Speculators' impact on prices and quantities

To perform the comparative statics, we focus first on Regions 2 and 4 in order to examine the mechanisms, then discuss Region 1 in which the previous effects are mixed in interesting ways. The equilibrium prices are drawn from Appendix B.2.

**Region 2.** The facts that $\tilde{P}_1$ and $\tilde{F}$ are weighted averages of $\tilde{\xi}_1/m$ and $\xi_2/m$ and that $\frac{\xi_2}{m} \geq \frac{\tilde{\xi}_1}{m}$ in Region 2 determine immediately the variations of the three prices given in Table 2.

Moreover,

$$\mathrm{Var}[\tilde{P}_1] = \left(\frac{m + n_I\gamma}{m + n_I(1+\gamma)}\right)^2 \frac{\mathrm{Var}[\tilde{\xi}_1]}{m^2}, \qquad \mathrm{Var}[\tilde{F}] = \left(\frac{n_I\gamma}{m + n_I(1+\gamma)}\right)^2 \frac{\mathrm{Var}[\tilde{\xi}_1]}{m^2},$$
$$\mathrm{Var}[\tilde{P}_2] = \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2} + \left(\frac{n_I}{m + n_I(1+\gamma)}\right)^2 \frac{\mathrm{Var}[\tilde{\xi}_1]}{m^2}, \qquad \mathrm{Var}[\tilde{X}^\star] = \left(\frac{-1}{m + n_I(1+\gamma)}\right)^2 \mathrm{Var}[\tilde{\xi}_1],$$

with obvious comparative statics.

Thus, we can conclude directly from Eq.(30) that $\mathrm{E}[\tilde{P}_2] - \tilde{F}$ decreases as speculation increases.

**Region 4.** Again, $\tilde{P}_1$ and $\tilde{F}$ are weighted averages of $\tilde{\xi}_1/m$ and $\xi_2/m$. These averages, in addition to the fact that $\frac{\xi_2}{m} \leq Z$ in Region 4, determine the comparative statics on the three prices.

Therefore, we can conclude directly from Eq.(31) that $\tilde{F} - \mathrm{E}[\tilde{P}_2]$ decreases as speculation increases.

**Region 1.** The two subregions 1U and 1L are separated by the line $\Delta$ already encountered and defined by $n_I\tilde{X}^\star - n_P\tilde{Y}^\star = 0 \Leftrightarrow \mathrm{E}[\tilde{P}_2] - F = 0$ (see Eq.27).

Taking into account that prices and quantities have the form $\frac{A+B\gamma}{C+D\gamma}$, with positive numerators and denominators, and that such expressions increase with respect to $\gamma$ if $BC - DA \geq 0$, 1U and 1L are the relevant subregions, the former resembling Region 2 and the latter Region 4. This is true for the levels of $\tilde{P}_1$, $\tilde{F}$, $\tilde{P}_2$, $\tilde{X}^\star$, and $\tilde{Y}^\star$, whose variations are summarized in Table 2.

Thus,

$$\mathrm{Var}[\tilde{P}_1] = \left(\frac{m(m + (n_I + n_P)\gamma)}{m(m + n_I + (n_I + n_P)\gamma) + n_I n_P \gamma}\right)^2 \frac{\mathrm{Var}[\tilde{\xi}_1]}{m^2},$$
$$\mathrm{Var}[\tilde{P}_2] = \left(\frac{m n_I}{m(m + n_I + (n_I + n_P)\gamma) + n_I n_P \gamma}\right)^2 \frac{\mathrm{Var}[\tilde{\xi}_1]}{m^2} + \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2},$$
$$\mathrm{Var}[\tilde{F}] = \left(\frac{m n_I \gamma}{m(m + n_I + (n_I + n_P)\gamma) + n_I n_P \gamma}\right)^2 \frac{\mathrm{Var}[\tilde{\xi}_1]}{m^2}.$$

The position with regard to $\Delta$ is not relevant for the variances, which are monotonic in the same way whatever subcase is used. See Table 2 in subsection 7.1.

## E.2 Utilities

In what follows, we explain the results gathered in Table 3.

First, we particularize the indirect utilities (formulas 18, 19, and 20) to the case when the markets are in equilibrium. In that case, $\tilde{P}_2$ becomes a function of $(P_1, F)$, and the formulas become:

$$U_S = \frac{\text{Var}[\tilde{\xi}_2]}{2m^2\alpha_S\left(\sum \frac{n_i}{\alpha_i}\right)^2}\left(n_I\left(F-P_1\right)-n_P\left(Z-F\right)\right)^2; \tag{32}$$

$$U_I = \frac{\text{Var}[\tilde{\xi}_2]}{2m^2\alpha_I\left(\sum \frac{n_i}{\alpha_i}\right)^2}\left(n_I\left(F-P_1\right)-n_P\left(Z-F\right)\right)^2 + \frac{(F-P_1)^2}{2}; \tag{33}$$

$$U_P = \frac{\text{Var}[\tilde{\xi}_2]}{2m^2\alpha_P\left(\sum \frac{n_i}{\alpha_i}\right)^2}\left(n_I\left(F-P_1\right)-n_P\left(Z-F\right)\right)^2 + \frac{(Z-F)^2}{2}. \tag{34}$$

Formulas (32), (33), and (34) give us the indirect utilities of the agents at equilibrium in terms of the equilibrium prices $P_1$ and $F$. These can in turn be expressed in terms of the fundamentals of the economy, namely $\xi_1$ and $\tilde{\xi}_2$ (see Appendix B.2): substituting formulas (24), (25), and (26), we get new expressions that can be differentiated to give the sensitivities of the indirect utilities with respect to the parameters in the model.

To investigate whether an increase in the number of speculators increases or decreases the welfare of speculators, of inventory holders, and of industry processors, we work directly with formulas (32), (33), and (34) and take the sensitivities of $P_1$ and $F$ with respect to parameter $n_S$. In contrast, the complete substitution of equilibrium values seems unworkable.

**Sensitivity of $U_S$.** Differentiating formula (32) yields:

$$\begin{aligned}
\frac{dU_S}{dn_S} =& \frac{\text{Var}[\tilde{\xi}_2]}{m^2\alpha_S\left(\sum \frac{n_i}{\alpha_i}\right)^2}\left(n_I(F-P_1)-n_P(Z-F)\right)\left(m+n_P\left(1+\frac{m}{n_I}\right)\right)\frac{dP_1}{dn_S} \\
& - \frac{\text{Var}[\tilde{\xi}_2]}{m^2\alpha_S^2\left(\sum \frac{n_i}{\alpha_i}\right)^3}\left(n_I(F-P_1)-n_P(Z-F)\right)^2 \\
=& - \frac{\text{Var}[\tilde{\xi}_2]}{m^2\alpha_S^2\left(\sum \frac{n_i}{\alpha_i}\right)^3}\left(1-\frac{\text{Var}[\tilde{\xi}_2]}{m\sum\frac{n_i}{\alpha_i}}\frac{\left(m+n_P\left(1+\frac{m}{n_I}\right)\right)}{\left(\frac{m}{n_I}+1\right)(m+\gamma n_P)+\gamma m}\right) \\
& \times \left(n_I(F-P_1)-n_P(Z-F)\right)^2. \\
=& - \frac{\text{Var}[\tilde{\xi}_2]}{m^2\alpha_S^2\left(\sum \frac{n_i}{\alpha_i}\right)^3}\,\frac{m^2+n_In_P+m(2n_I+n_P)}{m^2+mn_I+\gamma(n_In_P+m(n_I+n_P))} \\
& \times \left(n_I(F-P_1)-n_P(Z-F)\right)^2.
\end{aligned} \tag{35}$$

The sign of $\frac{dU_S}{dn_S}$ is constant in Region 1: it is negative. Adding speculators decreases the remuneration associated to risk bearing.

**Sensitivity of $U_I$.** Differentiating formula (33) yields:

$$
\begin{aligned}
\frac{dU_I}{dn_S} &= -\frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2\alpha_S\alpha_I\left(\sum\frac{n_i}{\alpha_i}\right)^3}\,\frac{m^2+n_In_P+m(2n_I+n_P)}{m^2+mn_I+\gamma(n_In_P+m(n_I+n_P))}\\
&\qquad\times(n_I(F-P_1)-n_P(Z-F))^2+(F-P_1)\left(\frac{dF}{dn_S}-\frac{dP_1}{dn_S}\right)\\
&= -\frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2\alpha_S\alpha_I\left(\sum\frac{n_i}{\alpha_i}\right)^3}\,\frac{m^2+n_In_P+m(2n_I+n_P)}{m^2+mn_I+\gamma(n_In_P+m(n_I+n_P))}\\
&\qquad\times(n_I(F-P_1)-n_P(Z-F))^2\\
&\quad+\frac{F-P_1}{n_I\alpha_S}\frac{\mathrm{Var}[\tilde{\xi}_2]}{\left(\sum\frac{n_i}{\alpha_i}\right)^2}\frac{n_I(F-P_1)-n_P(Z-F)}{\left(\frac{m}{n_I}+1\right)(m+\gamma n_P)+\gamma m}.
\end{aligned}
\tag{36}
$$

As mentioned before, the utility due to speculative activities decreases when $n_S$ increases. As far as the utility of hedging is concerned, the effect depends on the sign of $n_I(F-P_1)-n_P(Z-F)$. This line has already been encountered; it separates Region 1 into two subcases. In Subregion 1U, the utility of hedging increases for the storers because they need more hedging than processors. The opposite conclusion arises in Subregion 1L.

As far as the total utility is concerned, we will not pursue the calculations further, noting simply that $n_I(F-P_1)-n_P(Z-F)$ factors, so that the result is of the form:

$$\frac{dU_I}{dn_S}=A(n_I(F-P_1)-n_P(Z-F))\times(K_1(F-P_1)+K_2(Z-F)),$$

for suitable constants $A$, $K_1$, and $K_2$. This means that the sign changes across

- the line $\Delta$, already encountered, defined by $n_I\,(F-P_1)+n_P\,(Z-F)=0$
- the line $D$, defined by the equation $K_1(F-P_1)+K_2(Z-F)=0$

Both $\Delta$ and $D$ go through the point $M$ where $P_1=F=Z$. If $K_2/K_1<0$, the line $D$ enters Region 1, if $K_2/K_1>0$, it does not. So, if $K_2/K_1<0$, Region 1 is divided into three subregions by the lines $D$ and $\Delta$, and the sign changes when one crosses from one to the other. If $K_2/K_1>0$, Region 1 is divided in two subregions by the line $\Delta$, and the sign changes across $\Delta$. In all cases, the response of inventory holders to an increase in the number of speculators depends on the equilibrium.

**Sensitivity of $U_P$.** Differentiating formula (34) yields:

$$
\begin{aligned}
\frac{dU_P}{dn_S} = & -\frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2\alpha_S\alpha_P\left(\sum\frac{n_i}{\alpha_i}\right)^3}\frac{m^2+n_In_P+m(2n_I+n_P)}{m^2+mn_I+\gamma(n_In_P+m(n_I+n_P))} \\
& \times(n_I(F-P_1)-n_P(Z-F))^2+(F-Z)\frac{dF}{dn_S} \\
= & -\frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2\alpha_S\alpha_P\left(\sum\frac{n_i}{\alpha_i}\right)^3}\frac{m^2+n_In_P+m(2n_I+n_P)}{m^2+mn_I+\gamma(n_In_P+m(n_I+n_P))} \\
& \times(n_I(F-P_1)-n_P(Z-F))^2 \\
& +(F-Z)\left(\frac{m}{n_I}+1\right)\frac{1}{m\alpha_S}\frac{\mathrm{Var}[\tilde{\xi}_2]}{\left(\sum\frac{n_i}{\alpha_i}\right)^2}\frac{n_I(F-P_1)-n_P(Z-F)}{\left(\frac{m}{n_I}+1\right)(m+\gamma n_P)+\gamma m}.
\end{aligned}
\tag{37}
$$

Again, the utility due to speculation decreases, and the utility linked with hedging depends on the sign of: $n_I(F-P_1)-n_P(Z-F)$. In Subregion 1U, the utility of hedging decreases for the processors, and it increases in Subregion 1L.

We will not pursue the calculations further, noting simply that $n_I(F-P_1)-n_P(Z-F)$ factors again, so that:

$$
\frac{dU_P}{dn_S} = A^\star(n_I(F-P_1)-n_P(Z-F))\times(K_1^\star(F-P_1)+K_2^\star(Z-F))
$$

As in the preceding case, there is a line $D^\star$ (different from $D$) that enters Region 1 if $K_1^\star/K_2^\star<0$ and does not if $K_1^\star/K_2^\star>0$. In the first case, Region 1 is divided into three subregions by $D$ and $\Delta^\star$, in the second it is divided into two subregions by $\Delta$, and the sign of $\frac{dU_I}{dn_S}$ changes when one crosses the frontiers.

# F Comparison with the no-futures scenario (NF)

This appendix is devoted to the case where there is no futures market (scenario NF). In this case the speculators are inactive, and there are three kinds of operators: storers, processors, and spot traders.

**Optimal positions.** The optimal position of the storer becomes:

$$
X^\star_{\mathrm{NF}} = \frac{1}{C+\alpha_I\frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2}}\max\{\mathrm{E}[\tilde{P}_2]-P_1,0\}. \tag{38}
$$

The storer holds inventory if the expected price is higher than the current spot price. The processor's activity depends on the fact that the forward price of the output is higher than the expected spot price of the commodity:

$$Y^{\star}_{\text{NF}} = \frac{1}{\beta Z + \alpha_P \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2}} \max\{Z - \mathrm{E}[\tilde{P}_2], 0\}. \tag{39}$$

When there is no futures market, uncertainty on the future spot price determines the decisions undertaken in the physical market: the separation between physical decision, hedging, and speculation is no longer true.

**Equilibrium: existence conditions and regions.**

**Theorem 2.** *Existence conditions on $(\xi_1, \xi_2)$ are stricter in the scenario NF than in the basic case.*

*Proof.* To prove this theorem, we begin by taking Eq.(8) that depicts the expected equilibrium at date 2:

$$\mathrm{E}[\tilde{P}_2] = \frac{1}{m}\left(\xi_2 - N_I X^{\star}_{\text{NF}} + N_P Y^{\star}_{\text{NF}}\right).$$

Hence $\mathrm{Var}[\tilde{P}_2]$ is a constant $\frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2}$. This constant allows us to follow exactly the same line of reasoning as in the basic scenario, with the modified mapping $\varphi_{\text{NF}} : \mathbb{R}^2_+ \to \mathbb{R}^2$ defined by:

$$\varphi_{\text{NF}}(P_1, \mathrm{E}[\tilde{P}_2]) = \begin{pmatrix} mP_1 - \frac{n_I C}{C + \alpha_I \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2}} \max\{\mathrm{E}[\tilde{P}_2] - P_1, 0\} \\ m\mathrm{E}[\tilde{P}_2] + \frac{n_I C}{C + \alpha_I \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2}} \max\{\mathrm{E}[\tilde{P}_2] - P_1, 0\} - \frac{n_P \beta Z}{\beta Z + \alpha_P \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2}} \max\{Z - \mathrm{E}[\tilde{P}_2], 0\} \end{pmatrix}.$$

Formally, the analysis is identical to the one done in the basic case. We reuse previous calculations by applying the variables and parameters in the transposition given in Table 5.

| | | | | |
|---|---|---|---|---|
| Basic scenario: | $F$ | $\gamma$ | $n_I$ | $n_P$ |
| | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ |
| NF scenario: | $\mathrm{E}[\tilde{P}_2]$ | 1 | $\overbrace{n_I^{\text{NF}} = n_I \times \frac{C}{C + \alpha_I \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2}}}$ | $\overbrace{n_P^{\text{NF}} = n_P \times \frac{\beta Z}{\beta Z + \alpha_P \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2}}}$ |

Table 5: Variables and parameters transposition.

We can now turn to existence conditions, the proof being in three steps. Calculations are available on request.

We use the complete conditions of existence from Proposition 2:

1. The $\varphi'_{\text{NF}}(O)$ is above $\varphi'(O)$, where we use the convention that $\varphi'_{\text{NF}}(O)$ is the equivalent in the NF scenario of $\varphi'(O)$ in the basic model. We use similar conventions in the following.

2. Note that the slope of the left frontier of Region 2 is steeper (in absolute value) in the case of NF than in the basic model. This increased steepness also restricts existence possibilities in case NF.

3. $\varphi_{\text{NF}}(A)$ has a higher abscissa and a lower ordinate than $\varphi(A)$, leading also to more restrictive conditions in scenario NF.

The three properties are pictured in Figure 9. □

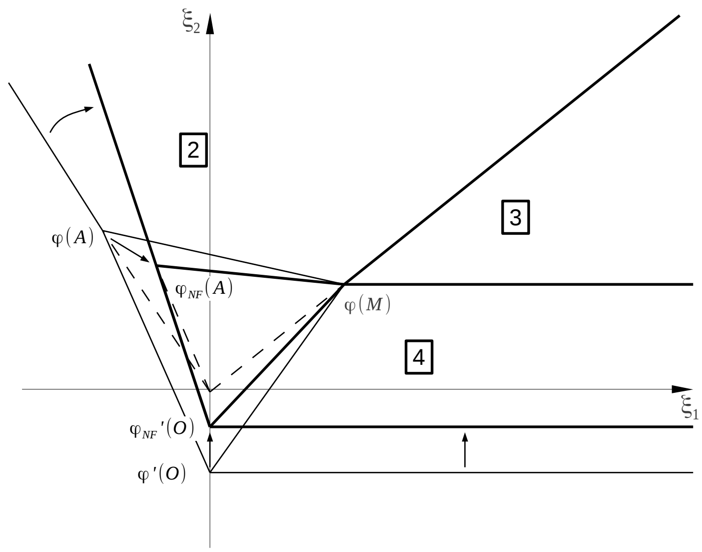


Figure 9: Existence conditions: comparison between with and without futures market.

The four regions of the NF scenario are included in those of the basic scenario. Region 1 diminishes. Region 2 grows at the expense of basic Region 1, and it is cut on its left border. Region 3 does not change. Region 4 grows at the expense of basic Region 1, and it is cut on its bottom border. These results are consistent with the previous conclusion about the optimal positions on the physical market.

**Prices and volatility.** The absence of a futures market also impacts price levels and volatilities. For instance, Equations (7) and (8) suggest that lower values for inventories and production lead to lower levels of the spot price at date 1 and also possibly at date 2.

In order to analyze the variances, we consider $\xi_1$ as random, as we did in Section 7. All things being equal, having futures or not can change the region where the equilibrium is. Yet, for simplicity we compare the variances region by region, as if the equilibrium was in the same region whatever the scenario.

Prices in the NF scenario can be retrieved directly, or with Table 5 and the equations of Appendix

B.2. In Region 1 (other regimes can be retrieved with the same trick as for previous calculations):

$$\tilde{P}_1^{\mathrm{NF}} = \frac{m(m + n_I^{\mathrm{NF}} + n_P^{\mathrm{NF}})\frac{\tilde{\xi}_1}{m} + mn_I^{\mathrm{NF}}\frac{\xi_2}{m} + n_I^{\mathrm{NF}}n_P^{\mathrm{NF}}Z}{m(m + n_I^{\mathrm{NF}} + n_P^{\mathrm{NF}}) + mn_I^{\mathrm{NF}} + n_I^{\mathrm{NF}}n_P^{\mathrm{NF}}}, \tag{40}$$

$$\mathrm{E}[\tilde{P}_2^{\mathrm{NF}}] = \frac{mn_I^{\mathrm{NF}}\frac{\tilde{\xi}_1}{m} + m(m + n_I^{\mathrm{NF}})\frac{\xi_2}{m} + (m + n_I^{\mathrm{NF}})n_P^{\mathrm{NF}}Z}{mn_I^{\mathrm{NF}} + m(m + n_I^{\mathrm{NF}}) + (m + n_I^{\mathrm{NF}})n_P^{\mathrm{NF}}}, \tag{41}$$

$$\tilde{P}_2^{\mathrm{NF}} = \frac{\tilde{\xi}_2}{m} + \frac{mn_I^{\mathrm{NF}}\frac{\tilde{\xi}_1}{m} - ((m + n_I^{\mathrm{NF}})n_P^{\mathrm{NF}} + mn_I^{\mathrm{NF}})\frac{\xi_2}{m} + (m + n_I^{\mathrm{NF}})n_P^{\mathrm{NF}}Z}{mn_I^{\mathrm{NF}} + m(m + n_I^{\mathrm{NF}}) + (m + n_I^{\mathrm{NF}})n_P^{\mathrm{NF}}}. \tag{42}$$

Let us compare the variance of $\tilde{P}_1$ in Region 1 in the two scenarios:

$$\mathrm{Var}[\tilde{P}_1] = \left(\frac{m(m + (n_I + n_P)\gamma)}{m(m + n_I + (n_I + n_P)\gamma) + n_I n_P \gamma}\right)^2 \frac{\mathrm{Var}[\tilde{\xi}_1]}{m^2}.$$

with

$$\mathrm{Var}[\tilde{P}_1^{\mathrm{NF}}] = \left(\frac{m(m + n_I^{\mathrm{NF}} + n_P^{\mathrm{NF}})}{m(m + 2\underline{n}_I + n_P^{\mathrm{NF}}) + n_I^{\mathrm{NF}}n_P^{\mathrm{NF}}}\right)^2 \frac{\mathrm{Var}[\tilde{\xi}_1]}{m^2}.$$

The latter is unambiguously bigger than the former.[8] Markets are stabilizing the price in period 1, as we saw in Subsection 7.1, because purchases are countercyclical. In the absence of a futures market, the stabilizing effect is attenuated.

Concerning period 2, we have to compare:

$$\mathrm{Var}[\tilde{P}_2] = \left(\frac{mn_I}{m(m + n_I + (n_I + n_P)\gamma) + n_I n_P \gamma}\right)^2 \frac{\mathrm{Var}[\tilde{\xi}_1]}{m^2} + \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2}.$$

with

$$\mathrm{Var}[\tilde{P}_2^{\mathrm{NF}}] = \left(\frac{mn_I^{\mathrm{NF}}}{m(m + 2n_I^{\mathrm{NF}} + n_P^{\mathrm{NF}}) + n_I^{\mathrm{NF}}n_P^{\mathrm{NF}}}\right)^2 \frac{\mathrm{Var}[\tilde{\xi}_1]}{m^2} + \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2}.$$

In Region 4, they are identical: due to the absence of storage, the absence of futures leaves the two periods independent statistically. Yet, quantities are higher if there are futures.

In Region 2, the variance is bigger in the base scenario. This is the effect underlined in Newbery (1987): the facilitation of storage transports shocks from the first period to the second one.

The comparison is ambiguous in Region 1, and our attempts to factorize the difference has not produced particularly interesting conditions. One reason is that passing from one scenario to another is a qualitative step that does not have smooth effects on mathematical expression.

[8] We analyzed the numerator after reduction of the difference to the same denominator. All terms have the same sign. Calculations are available upon request.