Energy mix models for emission market optimization

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Optimization of emission market

Under **linear utility**, is possible

- *Market designs for emissions trading schemes.*
  SIAM Review SIAM Review, 52(3):403-452, (2010).

since it connects to optimal control

- *Optimal stochastic control and carbon price formation.*
  SIAM Journal on Control and Optimization 48:2168-2190, (2009)

and suggest how to find best market design.

for **risk averse** agents, does not work in the same way.

In this talk, we show how to address the optimal market design in risk-averse settings.

Advantage: Clear arguments and proofs.

Explicit and simple in this talk: One period model, in the so-called *energy mix* formulation

A generalization is straight-forward

Extended emission trading systems

- administrator sets emissions cap and allocates permits
- a penalty applies for each unit of pollutant not covered
- taxation/subsidy of production in monetary units and emission certificates, in technology-sensitive way

Equilibrium modeling

Question: How to find the best *cap-and-trade scheme*?

Answer: For Linear utility, equilibrium modeling $\Longrightarrow$ calculations using aggregated market quantities

Equilibrium-like modeling

Non-linear utility: Equilibrium modeling $\Longrightarrow$ one needs individual quantities, thus calculation impossible. But there is a risk neutral measure. With respect to risk neutral measure the equilibrium looks like under linear utility.

Observation: Risk-neutral measure carries essential information. Given risk-neutral measure, equilibrium can be (re-)constructed

Suggestion: Describe the market directly under risk neutral measure to obtain

- realistic,
- flexible,
- numerically tractable

equilibrium-like models to optimize market design

One-period equilibrium model

- compliance date T
- action times $t = 0$ and T
- finite number of agents $i \in I$ follow polluting (energy) production
- revenue of agent i for strategy $(\vartheta^i, \xi_0^i) = (\vartheta_0^i, \vartheta_T^i, \xi_0^i)$, given allowance price $A = (A_0, A_T)$ and energy prices $P = (P_0)$ is

$$L^{A,P,i}(\vartheta^i, \xi_0^i) = \underbrace{-\vartheta_0^i A_0 - \vartheta_T^i A_T}_{\text{trading}} - \underbrace{C^i(\xi_0^i)}_{\text{costs}} + \underbrace{V^i(\xi_0^i)}_{\text{volume}} P_0 - \underbrace{\pi}_{\text{penalty}} (\underbrace{E^i(\xi_0^i)}_{\text{emission}} + \underbrace{N^i}_{\substack{\text{surprise}\\ \text{emission}}} - \vartheta_0^i - \vartheta_T^i - \gamma_0^i)^+$$

Model ingredients

For the agent $i \in I$, we have

- production plans $\xi_0^i \in \Xi^i$
- costs, volume and emission of production plan ξ_0^i, which are given by functions

$$V^i, C^i, E^i : \Xi^i \to [0, \infty[$$

- ϑ_t^i change of allowance number by trade at time $t = 0, T$
- $\gamma_0^i \in [0, \infty[$ initial allocation
- unpredictable emission N^i

Model ingredients

Risk aversion of agent $i \in I$

is described by agent-specific utility function U^i

Rational behavior

Given prices $A = (A_0, A_T)$, $P = (P_0)$ each agent $i \in I$ maximizes

$$\mathcal{U}^i \to \mathbb{R}, \qquad (\vartheta^i, \xi_0^i) = (\vartheta_0^{*i}, \vartheta_T^{*i}, \xi_0^{*i}) \mapsto \mathbb{E}\left(U^i(L^{A,P,i}(\vartheta^i, \xi_0^i))\right)$$

over all feasible policies $\mathcal{U}^i$ available to the agent i

Energy demand

$D_0 \in \mathbb{R}$ is exogenously specified, it can and must be met

$$D_0 = \sum_{i \in I} V^i(\xi_0^i)$$

Definition

Given energy demand $D_0 \in \mathbb{R}_+$, the prices $(A^*, P^*) = (A_0^*, A_T^*, P_0^*)$ are called equilibrium prices, if, for each agent $i \in I$, there exists a strategy $(\vartheta^{i*}, \xi_0^{i*}) = (\vartheta_0^{*i}, \vartheta_T^{*i}, \xi_0^{*i})$ with

(i) the energy demand is covered:

$$\sum_{i \in I} V^i(\xi_0^{i*}) = D_0,$$

(ii) the certificates are in zero net supply:

$$\sum_{i \in I} \vartheta_t^{*i} = 0 \quad \text{for } t = 0 \text{ and } t = T,$$

(iii) each agent $i \in I$ is satisfied by the own policy, thus

$$\mathbb{E}(U^i(L^{A^*,P^*,i}(\vartheta^{*i}, \xi_0^{*i}))) \geq \mathbb{E}(U^i(L^{A^*,P^*,i}(\vartheta^i, \xi_0^i))$$

holds for any alternative strategy (ϑ^i, ξ_0^i).

In equilibrium, only aggregated quantities do matter

Define aggregated quantities as functions on $\times_{i\in I}\Xi^i$ by

$$V(\xi_0)=\sum_{i\in I}V^i(\xi_0^i)\quad C(\xi_0)=\sum_{i\in I}C^i(\xi_0^i),\quad E(\xi_0)=\sum_{i\in I}E^i(\xi_0^i)$$

They describe the overall production volume, costs, and emissions of the market production plan $\xi_0=(\xi_0^i)\in\times_{i\in I}\Xi^i$

Knowing them and the unpredictable emission

$$\sum_{i\in I}N^i=N$$

under the risk-neutral measure $\mathbb{Q}$ the equilibrium production schedule $\xi_0^*=(\xi_0^{*i})_{i\in I}$ can be recovered, in some sense

aggregated quantities do matter

Given energy demand D_0, let $(A^*, P^*) = (A_0^*, A_T^*, P_0^*)$ be the equilibrium prices with strategies $(\vartheta^{i*}, \xi_0^{i*})$ $i \in I$.

(i) For each overall production schedule $\xi_0 = (\xi_0^i)_{i \in I} \in \times_{i \in I} \Xi^i$ which covers the demand $V(\xi_0) \geq V(\xi_0^*) = D_0$ it holds that

$$C(\xi_0^*) + A_0^* E(\xi_0^*) \quad \leq \quad C(\xi_0) + A_0^* E(\xi_0).$$

(ii) There exists a risk-neutral measure $\mathbb{Q}$ such that

$$A_0^* = \mathbb{E}^{\mathbb{Q}}(A_T^*).$$

iii) If $N = \sum_{i \in I} N^i$ has no point masses, then the terminal allowance price is digital

$$A_T^* = \pi 1_{\{E(\xi_0^*) + N - \gamma_0 \geq 0\}}.$$

where $\gamma_0 = \sum_{i \in I} \gamma_0^i$ is the total allocation

Invisible hand of Adam Smith

A proxy for the the impact of production ξ_0, is the *social burden*

$$B(\xi_0) = C(\xi_0) + \pi(E(\xi_0) + N - \gamma_0)^+$$

caused by the overall production plan $\xi_0 \in \times_{i \in I} \Xi^i$.

It turns out that the equilibrium strategy minimizes the social burden among all production strategies which cover a given demand.

Social optimality of equilibrium

Given energy demand D_0, let $(A^*, P^*) = (A_0^*, A_T^*, P_0^*)$ be the equilibrium prices with the corresponding strategies $(\vartheta^{i*}, \xi_0^{i*})$ $i \in I$. Let $\mathbb{Q}$ be a risk neutral measure from the equilibrium, then

$$E^{\mathbb{Q}}(B(\xi_0^*)) \leq E^{\mathbb{Q}}(B(\xi_0))$$

holds for each $\xi_0 = (\xi_0^i)_{i \in I} \in \times_{i \in I} \Xi^i$ with the same production volume $V(\xi_0) \geq V(\xi_0^*) = D_0$.

Risk-neutral measure inherits crucial information about equilibrium.

Under mild additional assumptions, we obtain equilibrium production schedule ξ_0^* from aggregated quantities γ_0, V, C, E, N and a risk neutral measure $\mathbb{Q}$ as follows:

0) take a risk neutral measure $\mathbb{Q}$ from the equilibrium

1) the equilibrium production schedule ξ_0^* must be a solution to the the *deterministic problem*

$$\text{minimize } C(\xi_0) + A_0 E(\xi_0)$$
$$\text{subject to } V(\xi_0) \geq D_0, \text{ over } \xi_0 \in \times_{i \in I} \Xi^i.$$

where A_0 equals to the equilibrium allowance allowance price A_0^*, which will be determined in the next step

2) assume that

for each $A_0 \in [0, \pi]$, there exists unique
solution $\xi_0^*(D_0, A_0)$ to deterministic problem

and define for $A_0 \in [0, \pi]$:

$$C^*(D_0, A_0) = C(\xi_0^*(D_0, A_0)), \quad E^*(D_0, A_0) = E(\xi_0^*(D_0, A_0)),$$

3) the price A_0^* must be a solution to fixed point problem

$$A_0 = \pi \mathbb{E}^{\mathbb{Q}}(1_{\{E^*(D_0, A_0) + N - \gamma_0 \geq 0\}})$$
subject to $A_0 \in [0, \pi]$,

suppose that this problem admits a unique solution
$A_0^* = A_0^*(D_0)$ calculate $\xi_0^* = \xi_0^*(D_0, A_0^*(D_0))$.

4) Having obtained ξ_0^*, determine $C(\xi_0^*)$, $E(\xi_0^*)$ to assess the performance of the market. (Since the demand is random, consider the averaged performance)

To carry out this program, one needs to specify only the fluctuations N of the non-predictable emissions under a risk neutral measure!

Technology space: is represented by $\mathbb{R}^2_+$ such that $(c, e) \in \mathbb{R}^2_+$ represents a technology with production costs c (Euro per MWh) and emission rate e (tonnes of CO_2 per MWh).

Capacity allocation: is represented by Borel measure on $\mathbb{R}^2_+$

$$q : \mathcal{B}(\mathbb{R}^2_+) \to \mathbb{R}_+.$$

Aggregated production schedule: is represented by a set $R \subset \mathbb{R}^2_+$ with aggregated volume, costs, emissions

$$\int_R q(\mathrm{d}c, \mathrm{d}e), \quad \int_R c \cdot q(\mathrm{d}c, \mathrm{d}e), \quad \int_R e \cdot q(\mathrm{d}c, \mathrm{d}e)$$

Equilibrium-like production schedules

$$R(P_0, A_0) = \{(c, e) \in \mathbb{R}^2_+ \; : \; c + A_0 e \leq P_0\}$$

equilibrium-like production schedules are

$$R(P_0, A_0) = \{(c, e) \in \mathbb{R}^2_+ \; : \; c + A_0 e \leq P_0\}$$

thus we define equilibrium-like volume

$$V(P_0, A_0) \;\; = \;\; \int_{R(P_0,A_0)} q(\mathrm{d}c, \mathrm{d}e),$$

equilibrium-like costs,

$$C(P_0, A_0) \;\; = \;\; \int_{R(P_0,A_0)} c \cdot q(\mathrm{d}c, \mathrm{d}e)$$

equilibrium-like emissions

$$E(P_0, A_0) \;\; = \;\; \int_{R(P_0,A_0)} e \cdot q(\mathrm{d}c, \mathrm{d}e)$$

In the equilibrium, energy demand $D_0 \in [0, \infty[$ must be satisfied, thus we define at demand D_0 and emission price A_0 the energy price

$$P^*(D_0, A_0) \quad = \quad \inf\{P_0 \in [0, \infty[: \; V(P_0, A_0) \geq D_0\}$$

the emission

$$E^*(D_0, A_0) \quad = \quad E(P^*(D_0, A_0), A_0)$$

the production costs

$$C^*(D_0, A_0) \quad = \quad C(P^*(D_0, A_0), A_0)$$

the penalty payment

$$\Pi^*(D_0, A_0) \quad = \quad \pi \mathbb{E}^{\mathbb{Q}}(1_{\{E^*(D_0, A_0) + N - \gamma_0 \geq 0\}})$$

For the expected penalty payment

$$\Pi^*(D_0, A_0) = \pi \mathbb{E}^{\mathbb{Q}}(1_{\{E^*(D_0,A_0)+N-\gamma_0 \geq 0\}}) = \pi \int_{\mathbb{R}} 1_{\{E^*(D_0,A_0)+n-\gamma_0 \geq 0\}} \nu(\mathrm{d}n)$$

we have supposed that

$$\left. \begin{array}{c} \text{with respect to the risk-neutral measure } \mathbb{Q}, \\ \text{the uncontrollable emission } N \text{ follows a} \\ \text{distribution } \nu \text{ on } \mathbb{R} \text{ without point masses.} \end{array} \right\}$$

The equilibrium allowance price $A_0^*(D_0)$ at demand D_0 is determined as the solution to the fixed point problem

$$A_0 = \Pi^*(D_0, A_0), \qquad A_0 \in [0, \pi].$$

The market performance can be assessed in terms of the

consumer's burden

$$B^{*c}(D_0) \;=\; D_0 P^*(D_0, A^*(D_0)),$$

producer's burden

$$B^{*p}(D_0) \;=\; -D_0 P^*(D_0, A_0^*(D_0)) + C^*(D_0, A^*(D_0)) + \Pi^*(D_0, A_0^*(D_0)),$$

environmental burden

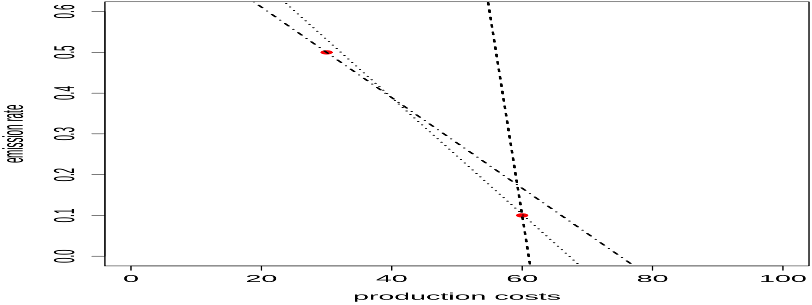
$$B^{*e}(D_0) \;=\; E^*(D_0, A_0^*(D_0)).$$

which depend on energy demand D_0.

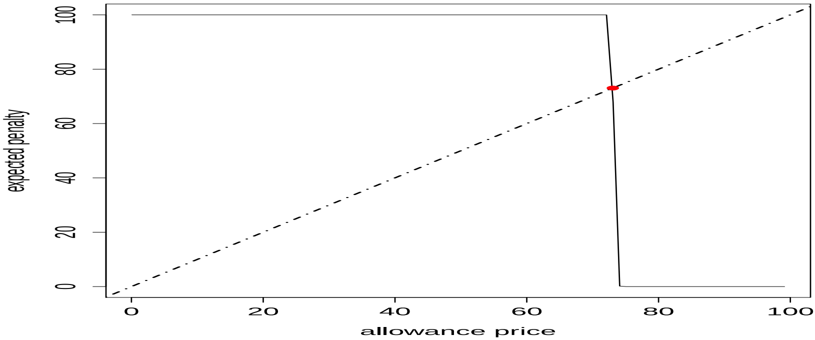
Example

The energy mix model provides a minimalistsic framework to obtain a picture of the energy market response

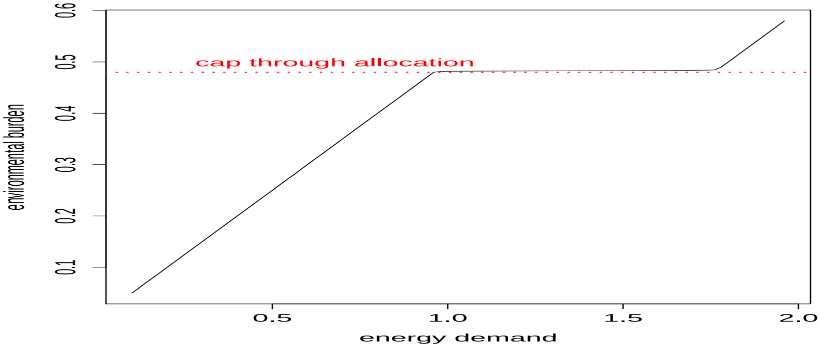
Consider a two-technology market as previously



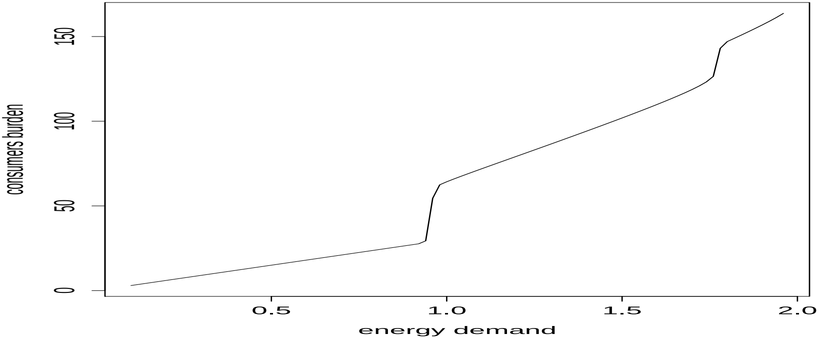
Solution to the fixed point problem is



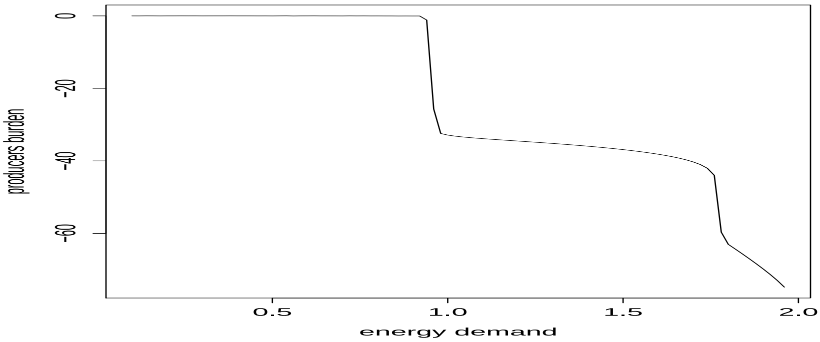
Environmental burden $d_0 \mapsto B^{*e}(d_0)$



Consumers burden $d_0 \mapsto B^{*c}(d_0)$



Producers burden $d_0 \mapsto B^{*p}(d_0)$



Optimization of emission market

The demand is random, thus we quantify this

with respect to the objective measure,
the energy demand follows
a distribution $\rho : \mathcal{B}([0, \infty[) \to \mathbb{R}_+$.

with this, introduce the expectation of the burden variables

$$\bar{B}^{*c} = \int_{\mathbb{R}} B^{*c}(d_0)\rho(\mathrm{d}d_0), \ \ \bar{B}^{*p} = \int_{\mathbb{R}} B^{*p}(d_0)\rho(\mathrm{d}d_0), \ \ \bar{B}^{*e} = \int_{\mathbb{R}} B^{*e}(d_0)\rho(\mathrm{d}d_0).$$

Optimizing market architecture, one determines a reasonable relation between them by adjusting γ_0 and π.

Optimizing extended scheme within energy mix model

Remember that the a regulator can, in a technology-sensitive way

i) tax/subsidise production in monetary units, and

ii) tax/subsidise production in emission allowance units.

Such intervention is given by

$$\theta : \mathbb{R}^2_+ \to \mathbb{R}^2, \qquad (c, e) \mapsto (\theta_1(c, e), \theta_2(c, e))$$

Such intervention is given by

$$\theta : \mathbb{R}_+^2 \to \mathbb{R}^2, \qquad (c, e) \mapsto (\theta_1(c, e), \theta_2(c, e))$$

where for each unit of energy produced by the technology with production costs $c \in \mathbb{R}_+$ and specific emission rate $e \in \mathbb{R}_+$:

- $\theta_1(c, e)$ is taxation (if $\theta_1(c, e) > 0$) or subsidy (if $\theta_1(c, e) < 0$) of $|\theta_1(c, e)|$ $\mathrm{EURO/MWh}$
- $\theta_2(c, e)$ is the taxation (if $\theta_2(c, e) > 0$) or subsidy (if $\theta_2(c, e) < 0$) of $|\theta_2(c, e)|$ tonnes of $\mathrm{CO_2}$ certificates

The effective production costs and emission rates are

$$\Theta : \mathbb{R}^2_+ \to \mathbb{R}^2_+, \quad (c, e) \mapsto (c, e) + \theta(c, e).$$

which induces an image measure

$$q^\Theta : \mathcal{B}(\mathbb{R}^2_+) \to \mathbb{R}_+, \quad M \mapsto q(\Theta^{-1}(M)).$$

Replacing q by q^Θ, the same considerations apply.

More precisely, one can investigate market equilibrium with intervention θ to see that it is related to equilibrium without intervention but other capacity allocation, q^Θ instead of q.

However, the true consumers burden must be re-defined as

$$B^{*c}(D_0) = D_0 P^*(D_0, A^*(D_0)) - K(D_0, A^*(D_0)).$$

with correction $K^{*c}(D_0, A^*(D_0))$ due to the policy financing

$$K(d,a) = \int_{\Theta^{-1}(R(P^*(d,a),a))} \theta_1(c,e) q(\mathrm{d}c, \mathrm{d}e).$$

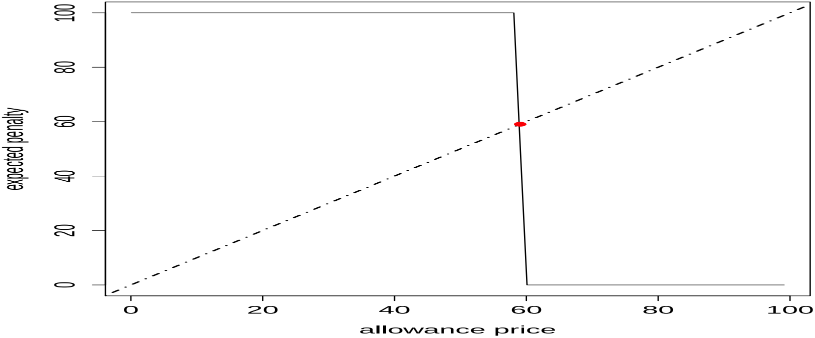
Also, the environmental burden B^{*e} must be re-defined as

$$B^{*e}(d) = \int_{\Theta^{-1}(R(P^*(d,a),a))} e q(\mathrm{d}c, \mathrm{d}e).$$

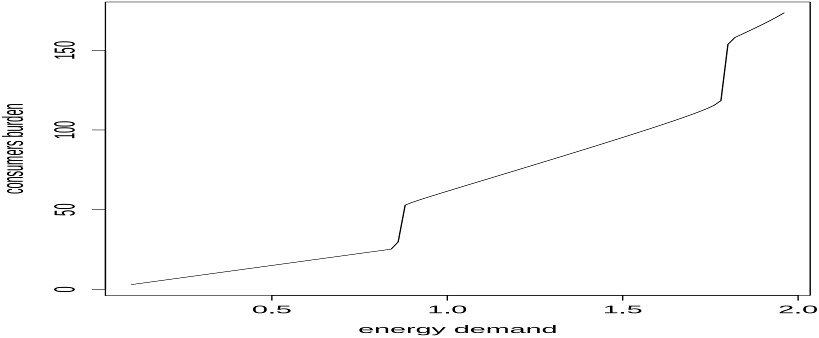
The taxation/subsidy, described by θ provides a tremendous degree of freedom and improves optimization

Example

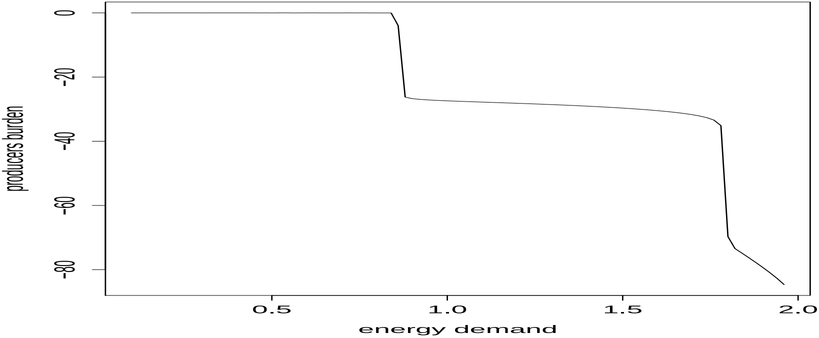
Subsidy in terms of emission certificates lowers the certificate price



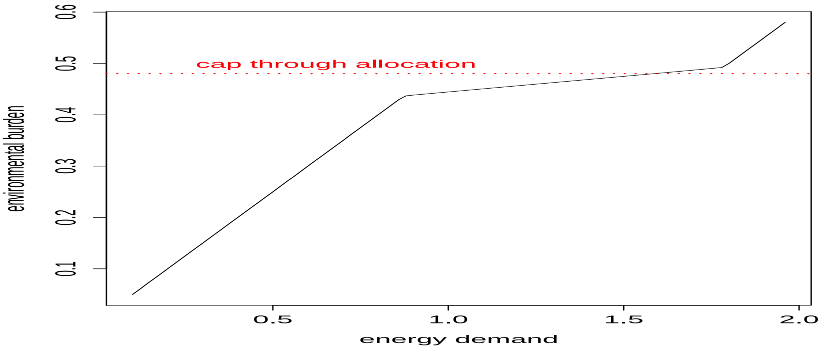
and changes the consumers burden



and changes the producers burden



and makes the environmental burden more uncertain



Also social optimality

can be used to re-construct production schedule ξ_0^* from the risk neutral measure $\mathbb{Q}$ as solution of the *control problem*

$$\text{minimize } C(\xi_0) + \pi \mathbb{E}^{\mathbb{Q}}((E(\xi_0) + N - \gamma_0)^+)$$
$$\text{subject to } V(\xi_0) \geq D_0, \text{ over } \xi_0 \in \times_{i \in I} \Xi^i.$$

Namely, under above assumptions the solution A_0^* to

$$\text{minimize } C^*(D_0, A_0) + \pi\mathbb{E}^{\mathbb{Q}}((E^*(D_0, A_0) + N - \gamma_0)^+) \\ \text{over } A_0 \in [0, \pi].$$

yields everything

$$V(\xi_0^*) = V^*(D_0, A_0^*), \quad E(\xi_0^*) = E(D_0, A_0^*) \\ C(\xi_0^*) = C^*(D_0, A_0^*), \quad P_0^* = P^*(D_0, A_0^*)$$

we need to calculate the burden variables.

In multi-period setting, social optimality provides a convenient way to study equilibrium via control theory.

Equilibrium model in multi-period setting

- compliance date T
- action times $t = 0, \ldots, T$
- all processes on $(\Omega, \mathcal{F}, P, (\mathcal{F}_t)_{t=0}^{T})$ are adapted
- finite number of agents $i \in I$

focus on energy production as main source of pollution

Revenue of agent i for strategy (ξ^i, ϑ^i), given allowance price $A = (A_t)_{t=0}^{T}$ and energy prices $P = (P_t)_{t=0}^{T-1}$

$$L^{A,P,i}(\vartheta^i, \xi^i) = \sum_{t=0}^{T-1}(-\vartheta^i_t A_t - \underbrace{C^i(\xi^i_t)}_{\text{costs}} + \underbrace{V^i(\xi^i_t)}_{\text{volume}} P_t) - \vartheta^i_T A_T$$

$$- \underbrace{\pi}_{\text{penalty}} (\sum_{t=0}^{T-1}(\underbrace{E^i(\xi^i_t)}_{\text{emission}} - \vartheta^i_t) - \vartheta^i_T + N^i - \gamma^i_0)^+$$

Definition of equilibrium is basically the same

$A^* = (A_t^*)_{t=0}^T, P^* = (P_t^*)_{t=0}^{T-1}$ are equilibrium prices, there exist strategies $(\vartheta^{*i}, \xi^{*i}) \in \mathcal{U}^i$, $i \in I$ such that:

(i) Each agent $i \in I$ is satisfied by the own policy

$$(\vartheta^{*i}, \xi^{*i}) \text{ is maximizer to } (\vartheta^i, \xi^i) \mapsto \mathbb{E}(U^i(L^{A^*,P^*,i}(\vartheta^i, \xi^i)))$$

(ii) Changes in allowance positions are in zero net supply

$$\sum_{i \in I} \vartheta_t^{*i} = 0, \text{ for all } t = 0, \dots, T.$$

(ii) The demand is covered

$$D_t = \sum_{i \in I} V_t^i(\xi_t^{*i}), \qquad t = 0, \dots, T-1$$

Invisible hand (of Adam Smith)

The social burden is

$$B(\xi) = \sum_{t=0}^{T-1} C(\xi_t) + \pi(\sum_{t=0}^{T-1} E(\xi_t) + N - \gamma_0)^+$$

In any equilibrium

$$(A_t^*)_{t=0}^T, \ (P_t^*)_{t=0}^{T-1}, \ (\xi_t^{*i})_{t=0}^{T-1}, \ (\vartheta_t^{*i})_{t=0}^T, \ i \in I \text{ with } \mathbb{Q}$$

the production $\xi^* = ((\xi_t^{*i})_{i \in I})_{t=0}^{T-1}$ satisfies

$$\mathbb{E}^{\mathbb{Q}}(B(\xi^*)) \leq \mathbb{E}^{\mathbb{Q}}(B(\xi))$$

for each production strategy ξ which meets the demand

$$\sum_{i \in I} V_t^i(\xi_t^i) = D_t, \qquad t = 0, \dots, T-1.$$

Equivalently, under above assumptions, a solution $(A_t^*)_{t=0}^T$ to the stochastic control problem

$$\text{minimize} \quad \mathbb{E}\left(\sum_{t=0}^{T-1} C^*(D_t, A_t) + \pi\left(\sum_{t=0}^{T-1} E^*(D_t, A_t) + N - \gamma_0\right)^+\right)$$
$$\text{over all adapted processes } (A_t)_{t=0}^T$$

yields everything we need

$$V(\xi_t^*) = V^*(D_t, A_t^*), \qquad E(\xi_t^*) = E^*(D_t, A_t^*)$$
$$C(\xi_t^*) = C^*(D_t, A_t^*), \qquad P_t^* = P^*(D_t, A_t^*), \quad t = 0, \ldots, T-1$$

to calculate the burden variables in order to asses the performance of the scheme.

Alternatively, under above assumptions solution $(A_t^*)_{t=0}^T$ to the fixed point problem

$$A_T = 1_{\{\sum_{t=0}^{T-1} E(D_t, A_t) + N - \gamma_0\}}, \qquad A_t = \mathbb{E}_t^{\mathbb{Q}}(A_{t+1}), \quad t = 0, \ldots T-1$$

yields everything we need

$$\begin{aligned} V(\xi_t^*) &= V^*(D_t, A_t^*), & E(\xi_t^*) &= E^*(D_t, A_t^*) \\ C(\xi_t^*) &= C^*(D_t, A_t^*), & P_t^* &= P^*(D_t, A_t^*), \quad t = 0, \ldots, T-1 \end{aligned}$$

to calculate the burden variables in order to asses the performance of the scheme.
Existence and uniqueness of fixed-point problem

- On fair pricing of emission-related derivatives. Bernoulli 16(4):1240-1261, (2010).

Conclusion

Optimization of environmental market is possible since equilibrium production can be deduced from aggregated market quantities using risk-neutral measure

market architecture optimization boils down to a realistic choice of risk neutral measure plus some numerical investigations

One-period models are simple and relevant, helping better understand the working principles of environmental protection by financial instruments

Multi-period models require solutions to high-dimensional optimal control problems, which is interesting...