What to Model and What For?

Market Microstructure in Practice 2/3

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Outline

Goals of This Session

The previous sessions presented different market features, now we will see how use markets for trading and what they look at. We will go through:

- **macroscopic statistics** , including seasonalities
- **orderbook dynamics** : statistics more than models, since models will be proposed during other talks
- metaorder dynamics, including **market impact** (statistics more than models).

Next session is about optimal trading.

Outline

Market Participants

Each participant has a different viewpoint, depending on his time scale, and his utility function. For the purpose of modelling and understanding market dynamics for trading, it is useful to split participants across:

- **The Buy Side** : Investors / Asset Managers. They issue **metaorders**;
- **The Sell Side - Agency** : Intermediaries ; excluding market markers or risk takers. They trade metaorders of their clients (including internal clients).
- **Market operators and vendors** : exchanges, trading facilities, providers of trading frameworks, etc. They sell technology and services around trading (especially analytics).
- **Market makers and Prop. Shops** (including HFT firms): their core business is trading.
- **Regulators** : they target to monitor a smooth and orderly functioning market.

Specialization Inside Participants (1/2)

On the Buy Side, small asset managers are not experts in microstructure, but have to guarantee *best execution* to their clients, hence they more and more have to deal with reporting on their trading flows (100% done by their broker on their behalf).
Largest asset managers structured their trading around one internal **Dealing Desk**. It centralizes all the metaorder of all the portfolio managers, and reroute to different brokers. They often have **DMA** (Direct Market Access) to specific venues (Dark Pools for instance). They monitor the efficiency of the trading methodologies of their brokers, conducting **TCA** (Transaction Cost Analysis).

The Sell Side is the main algo trading provider. Of course they provide voice trading too, but even *care orders* are now traded internally using algos. They are members of the exchanges / trading facilities, and operate direct market access for dealing desks of their clients. They provide access to their **Broker Crossing Network**. They offer *Execution Services*, and performance analysis.

Market Operators and Vendors are on the technology side, they are of paramount importance in terms of knowledge dissemination. For instance AlgoBoxes provider (like Fidessa or Flextrade), give away *templates* of trading algorithms. The best choice for them is it implement simple academic models. They offer *Analytical Engines*, and again they have to implement meaningful computations to sell their products.

Specialization Inside Participants (1/2)

Market Makers and Prop. Shops are 100% focussed on trading, with an intensive on opportunist trading. Market makers have agreements with market operators (rebates, special order types, etc). They fear adverse selection (i.e. trading against information they do not have in their databases). They often make the difference between tactics based on pure latency, and more model based ones (they have quants).

Regulators are monitoring all this. They have more information on the data they access, but are in the process of addressing (or not) the needed Terabytes of hard drives needed (and the means to process them). In the US the Flash Crash put pressure in a data crunching direction (see the **MIDAS** web site of the SEC for instance). In Europe it is difficult since the Authority of one Member State does not acces to the data of trading venues regulated by another Member... Nevertheless the ESMA does it best to issue studies, like [Bouveret et al., 2014].

Algorithmic trading

- In most brokerage firms, Algorithmic trading executes more than 80% of the orders (in value). It is thus used internally (to execute care or corporate orders) and by (low touch) clients.
- Algo users pay fees (2 to 8 bps) and have no guarantee on the execution price.
- Buy sides would like to have Algos the most similar as possible from one provider to another, since integration in the trading tools is a never-ending challenge. But Sell sides try to differentiate, leading to Algos with fancy names and “new” features.

Algo types:

- **Long duration Algos** are controlled via a *Benchmark*, giving an indication on the criterion to minimize (reflecting user’s belief in good proxies for liquidity and risk), and having different parameters.
- **Short duration Algos** (or Robots) are synthesizing smart features of orders (icebergs, on the close, stop orders, snipers, etc) or seek liquidity on more than one platform (SOR, liquidity seekers). Their parameters make the balance between speed and price.
- Algos can be **customized**: benchmarked algos can send robots, and the type of an algo can **change on market conditions**, like: “*start with a VWAP and switch to PoV if the market volume is more than 2 times the ADV*”.
- **Program trading Algos** are dedicated to portfolio trading, with synchronization features (in cash or market exposure).

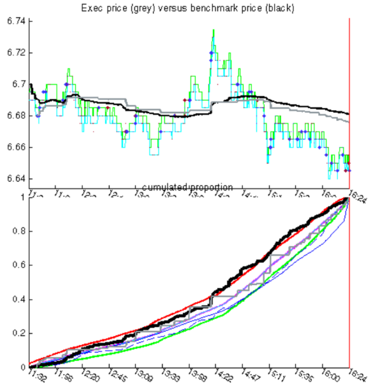
Trading algorithms: Typical uses

Benchmark	Region of preference	Order characteristics	Market context	Type of hedged risk
PoV	Asia	Large order size (more than 10% of ADV: Average daily consolidated volume)	Possible negative news	Do not miss the rapid propagation of an unexpected news event (especially if I have the information)
VWAP / TWAP	Asia and Europe	Medium size (from 5 to 15% of ADV)	Any “unusual” volume is negligible	Do not miss the slow propagation of information in the market
Implementation Shortfall (IS)	Europe and US	Small size (0 to 6% of ADV)	Possible price opportunities	Do not miss an unexpected price move in the stock
Liquidity Seeker	US (Europe)	Any size	The stock is expected to “oscillate” around its “fair value”	Do not miss a liquidity burst or a relative price move on the stock

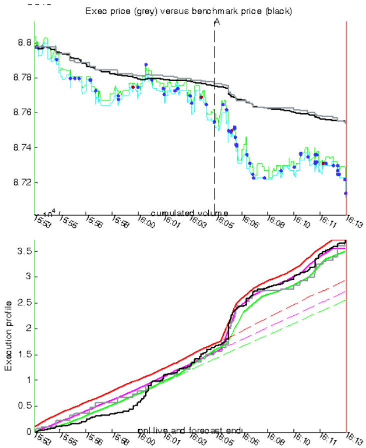
Trading algorithms: Typical features

Benchmark	Type of stock	Type of trade	Main feature
PoV	Medium to large market depth	(1) Long duration position	(1) Follows current market flow, (2) Very reactive, can be very aggressive, (3) More price opportunity driven if the range between the max percent and min percent is large
VWAP / TWAP	Any market depth	(1) Hedging order, (2) Long duration position, (3) Unwind tracking error (delta hedging of a fast evolving inventory)	(1) Follows the “usual” market flow, (2) Good if market moves with unexpected volumes in the same direction as the order (up for a buy order), (3) Can be passive
Implementation Shortfall (IS)	Medium liquidity depth	(1) Alpha extraction, (2) Hedge of a non-linear position (typically Gamma hedging), (3) Inventory-driven trade	(1) Will finish very fast if the price is good and enough liquidity is available, (2) Will “cut losses” if the price goes too far away
Liquidity Seeker	Poor a fragmented market depth	(1) Alpha extraction, (2) Opportunistic position mounting, (3) Already split / scheduled order	(1) Relative price oriented (from one liquidity pool to another, or from one security to another), (2) Capture liquidity everywhere, (3) Stealth (minimum information leakage using fragmentation)

Typical Algo Life



A Real VWAP Exec



A Combination of Algos

Pre and Post trade analysis, TCA consultancy

With electronic trading, a full range of services is offered:

- Buy sides are performing **TCA (Transaction Cost Analysis)** to compare their brokers on a monthly basis;
- **Pre trade analysis** provides automated assistance to Algo parameters tuning (taking market impact / market risk into account), it computes betas, exposures, etc of a list of algos sent by a Dealing Desk.
- **Post trade analysis** : at the end of the day, the generation of a report is automated, to allow to understand fast what happened during the day for each Dealing desk. Performances are compared with unexpected and expected new arrivals, and with usual performances and performances of other clients.
- **Monitoring** is conducted during the life of algos, analytics are grouping trading lines according to market conditions to dynamically explain the potential causes of performance degradation, so that sales-traders can adjust some parameters or call the Buy side Dealing Desk.
- **Execution Consultancy** is available on demand too. Experts discuss with clients to help them to tune parameters for their own use, or to design - customize new Algos. Backtest can be done, models or parameters can be sent to clients by FIX/FTP.

Outline

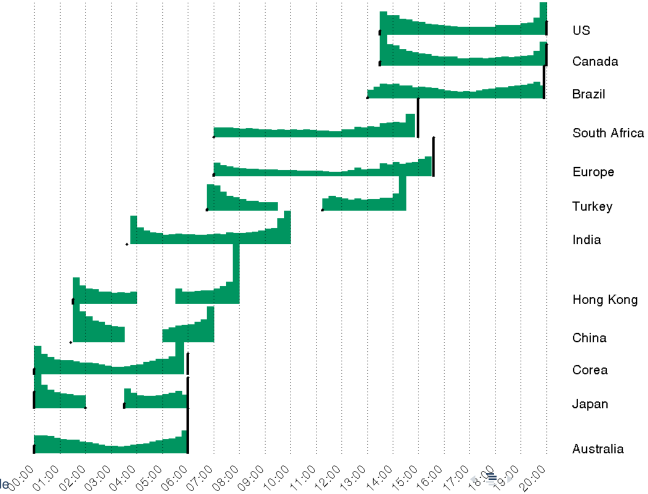
Observing the Market Short Term Dynamics

First of all, short term dynamics have to be considered with respect to some microstructural characteristics:

- The **Matching Rules** : pro-rata rules vs. Time-priority (FIFO) ones; Fixing Auction vs. Continuous Auctions (simultaneously or not); micro-auctions like in Korea or a computed (and not traded) close price exists like in India. Bilateral trading (for instance with a *last look*) can be implemented vs. multilateral anonymous trading.
- **Order types** are part of the rules but are less influencing dynamics compared to the previous list. Nevertheless hidden / Iceberg orders, conditional orders (book or cancel, fill or kill, etc), pegged orders, are of importance.
- The **fragmentation** /multi-listing, with associated **latency** issues are important too.
- The **tick size** is of paramount importance.

Descriptive Statistics: Volume Curves

The first important feature of intraday dynamics is the **seasonality** : morning, afternoon and evening are not comparable. It is important to take it into account to *stationarize* a dataset.



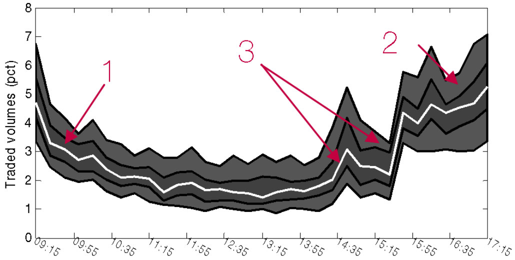
Dark bars are **Fixing auctions**, green ones for continuous trading. In Europe Fixing auctions are 2 times more liquid than in the US. But there is a pause between the continuous phase and the prefixing.

All the volume curves are more or less **U-shaped** , the influence of a re-open with or without fixing is noticeable.

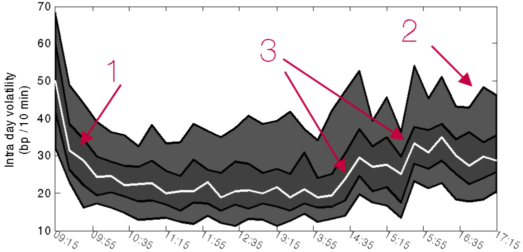
The influence of the opening of US markets on Brasil and Europe can be easily seen.

Usual phases (in Europe):

- Open: uncertainty on prices and unwind of the overnight positions;
- Macro economic news + NY opens;
- Close: need to hedge the positions.



Volumes



Volatility

Measure or estimate: the case of Intra day volatility

First of all, note Log Volume are more iid than volatility.

Moreover, concerning the **volatility** :

- Proportion of volatility seems to be more stable by volatility itself
- How could we use proportion of volatility?
- But measuring volatility itself is difficult

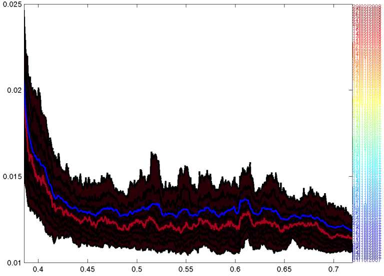
Our prices are discretized (rounded?) on a price grid

A lot of interesting papers have been written on this subject, see [Aït-Sahalia and Jacod, 2007], [Zhang et al., 2005], [Hayashi and Yoshida, 2005], [Robert and Rosenbaum, 2010]

Bid-ask spread : when microstructure limits the use of diffusion models

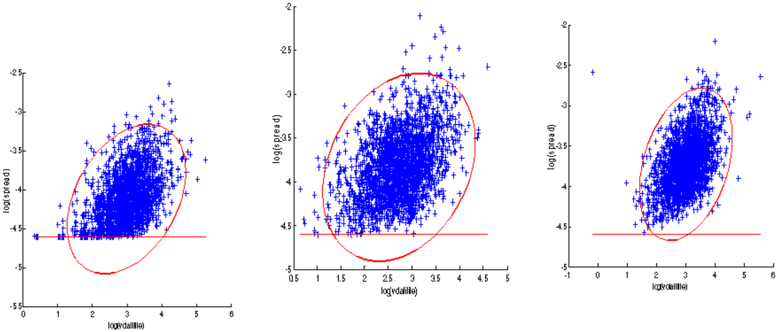
If only one microstructure effect should be kept, it is the Bid-Ask spread:

- it is the difference between the buy and sell prices
- it should be influenced by the volatility
- and by the volume of transactions.

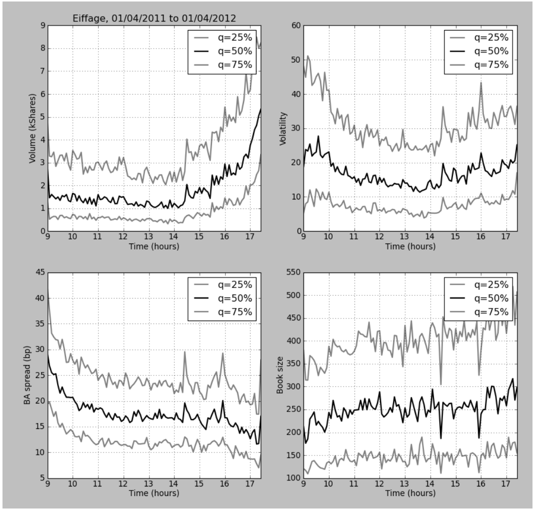


Bid Ask Spread on a typical French Stock

"*Spread effect*" on three stocks

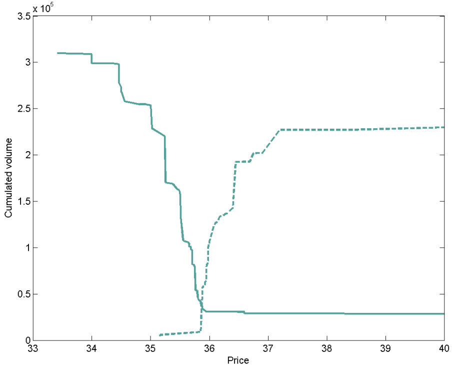


A fourth curve? (liquidity)



- Each one of the previous curves (volume, volatility, bid-ask spread) is influence by slightly different factors.
- Since the Bid-Ask spread is (badly) influenced by the tick size (remember in June 2009, when the tick size has been used in Europe as a competition weapon between trading venues), we can add the quantity at the first limit. It capture some effects the Bid-Ask can't when it is constrained by the tick (I took a large tick French stock on the left).
- We can see the *expected volatility-spread relation* is not verified during the end of the day. It is due to a competition for the rebate between traders (you need to hedge or close your position by the end of day, it is better to pay less than the spread + fees but half spread - rebate).

During the fixing auction, the two sides of the orderbook can be filled without generating any transactions. A theoretical price, volume and imbalance are published by the exchange (or not... depending on the exchange).



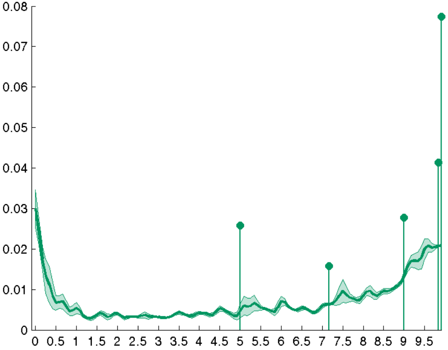
A simple calculation shows at the first order, the equilibrium price moves by

$$\delta s = \frac{-\delta V}{f^-_{\mathrm{LOB}}(S^*) + f^+_{\mathrm{LOB}}(S^*)} + o(\delta V),$$

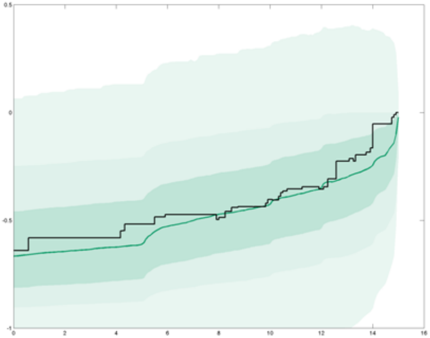
when a participant insert a marketable volume δV in the book.

The Open Fixing: Real Data

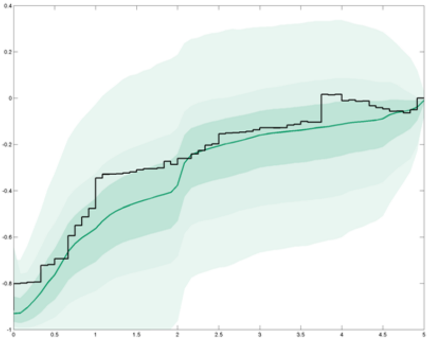
Decomposition of the Open of Crédit Agricole



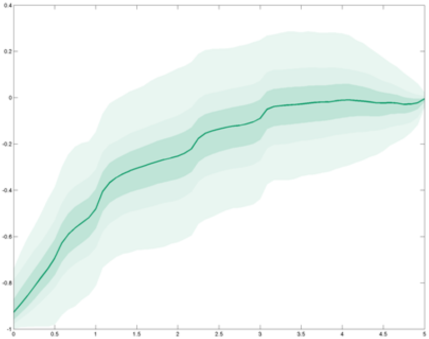
Cumulative matched volumes



The Close of Crédit Agricole



The close on a market with less transparency



Outline

Towards orderbook modelling

Different viewpoints on orderbook modelling:

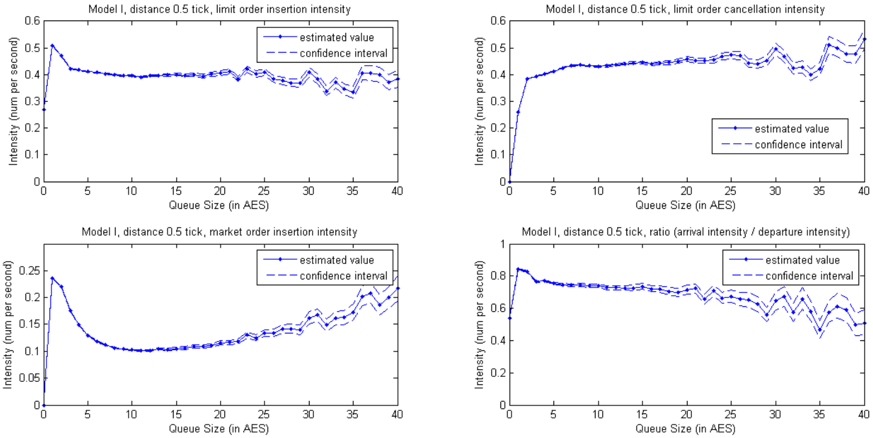
- Descriptive model, reproducing stylized facts (like Hawkes [Bacry et al., 2013], or [Robert and Rosenbaum, 2011] and [Fodra and Pham, 2013]);
- Structural models, typically flow-driven models (see [Cont and De Larrard, 2013], [Huang et al., 2013] or [Smith et al., 2003]);
- Theoretical models, like [Roşu, 2009], [Bayraktar et al., 2007], [Abergel and Jedidi, 2011] or [Lachapelle et al., 2013].

Each angle focus on one aspect of the dynamics:

- reproduce,
- understand,
- explain.

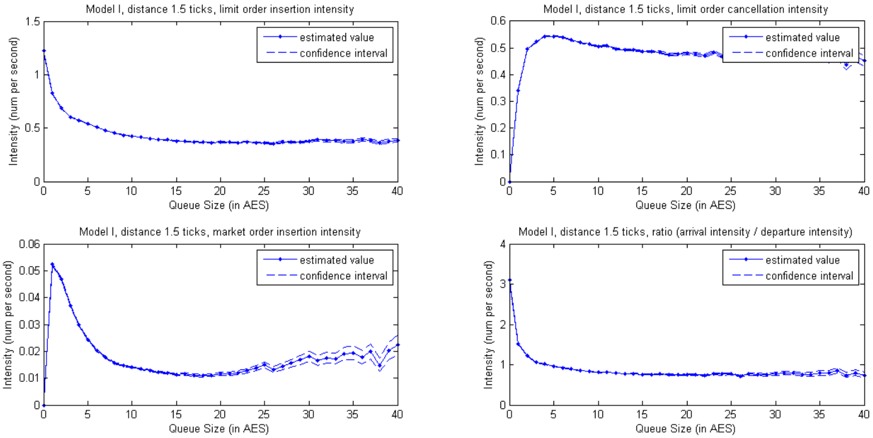
Some Descriptive Statistics on OB Dynamics: No Conditioning

Just estimate the intensity of the processes providing or consuming liquidity, unconditionally:



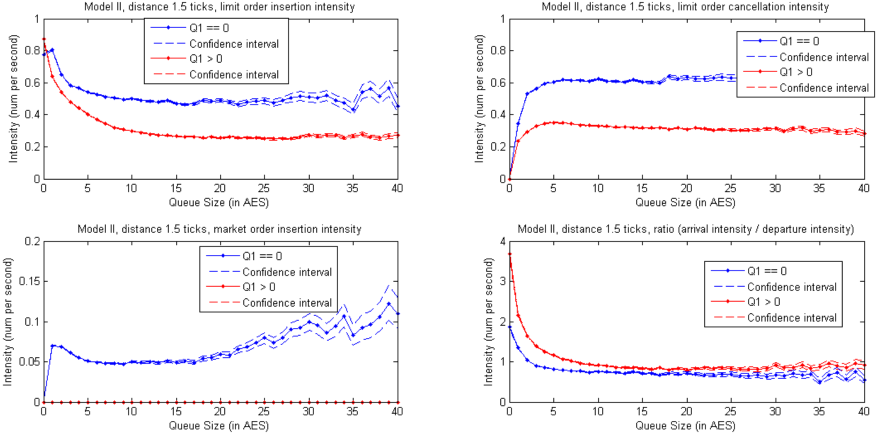
First Limit

Just estimate the intensity of the processes providing or consuming liquidity, unconditionally:



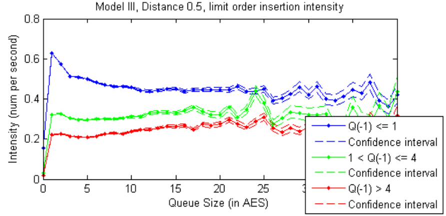
Second Limit

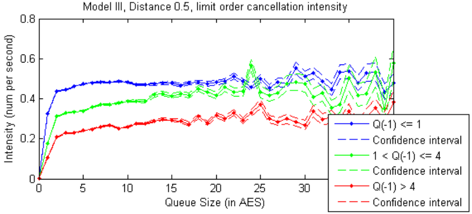
Just estimate the intensity of the processes providing or consuming liquidity, condition queue 2 by the quantity on queue 1:

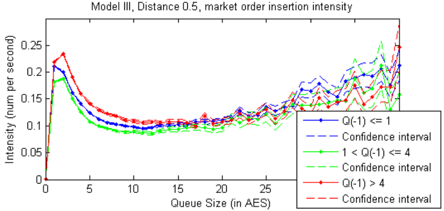


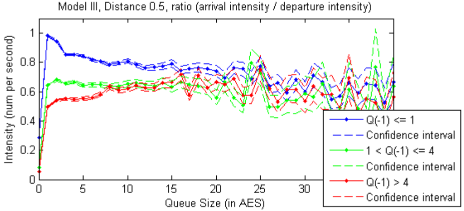
Some Descriptive Statistics on OB Dynamics: Full Conditioning

Just estimate the intensity of the processes providing or consuming liquidity, now condition queue 1 by the quantity on queue -1 (i.e. the opposite one):









In [Huang et al., 2013], we produce distribution laws for such conditioning, and show that even the “full conditioning” does provide **a good description at very small time scale**, but not at large ones.

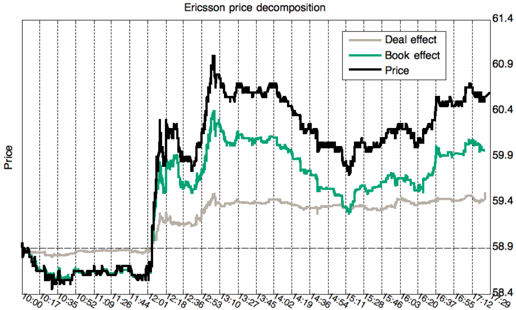
When you zoom out,**the obtained price dynamics are too mean reverting**. It surprised us but we understood it comes from the fact that in such a model, conditionally to the fact a first limit is fully consumed, the next one (i.e. 2nd becoming 1st), is very large and cannot easily be consumed too.

Going back to [Cont and De Larrard, 2013], it is not hard to identify the difference lays between the law to choose the distribution of the 2nd limit conditionally to the fact it just become a first limit. **If we add a parameter to renew or not the limit, the price become diffusive enough at large scales**. This parameter can be seen as the probability to observe a mechanical or an informational consumption of the first limit.

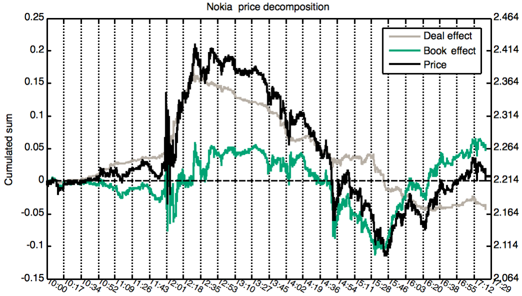
The Big Question: Mechanical or Informational Moves?

I have make an experiment with Paul Besson (see the working paper on SSRN [Besson and Lehalle, 2014]) to try to split price moves caused by trades or just “quote shifts”, and look at it for different types of events: Examples (18 oct 2012):

easy to process news



difficult to process news



Going Further: Modelling Each Participant in the Book

In order to provide an instance of anticipation behaviors, we model in [Lachapelle et al., 2013] a specific configuration where the service rate is known to increase (improved service) as soon as the queue reaches a certain level. We can for instance think of the starting up of a new server.
The arrival rate of players is continuous and stochastic. As usual, they arrive following a Poisson process with intensity λ.
The service rate is also Poisson and has the specificity to take two values:

- a low value μ_1 below a certain queue size threshold S,
- a higher value μ_2 ($\mu_2 > \mu_1$), above the threshold S.

Consequently the service rate is a function depending upon the queue size variable x:

$$\mu(x) = \mu_1 \mathbf{1}_{x<S} + \mu_2 \mathbf{1}_{x\geq S}, \ 0 \leq \mu_1 < \mu_2.$$

The unit size of a task in the queue is q. The queuing discipline is a process sharing one (with no priority), i.e. individual service in a queue of size x is worth q/x.
The pay-off gained by a player per unit os task is a nonnegative constant P. On the other hand, there is a cost c of waiting in the queue.

Now, as usual in game theory, there is a value function u for any player. The value function depends upon the queue size x. It is her expected Profit & Loss (P&L) of a player entering the queue. Note that we assume that agents are risk neutral and that their reservation utility is set to 0, which means that an agent decide to enter the queue as soon as the value function is positive: $u(x) > 0$.

Equilibrium : the value function equation I

In this paragraph we introduce the equation verified by the value function at the equilibrium. Below we detail the equilibrium equation for each probability event.

$$
\begin{aligned}
(1) \qquad u(x) = \; & (1 - \lambda \mathbf{1}_{u(x)>0} dt - \mu(x) dt) \cdot u(x) && \leftarrow \text{nothing happens} \\
& + \lambda \mathbf{1}_{\{u(x)>0\}} dt \cdot u(x+q) && \leftarrow \text{new queue entrance} \\
& + \mu(x) dt \cdot \left(\frac{q}{x} P + (1 - \frac{q}{x}) u(x-q) \right) && \leftarrow \text{service} \\
& - cqdt && \leftarrow \text{waiting cost}
\end{aligned}
$$

We can perform a Taylor expansion for small q in the discrete equation above. In this way we derive the following differential equation:

$$
\begin{aligned}
0 = \frac{\mu(x)}{x}(P - u) - c + (\lambda \mathbf{1}_{\{u>0\}} - \mu(x)) u' + \\
q\left(\frac{1}{2} (\lambda \mathbf{1}_{\{u>0\}} - \mu(x)) u'' + \frac{\mu(x)}{x} u' \right),
\end{aligned}
$$

where the second order term is the last one (blue term).

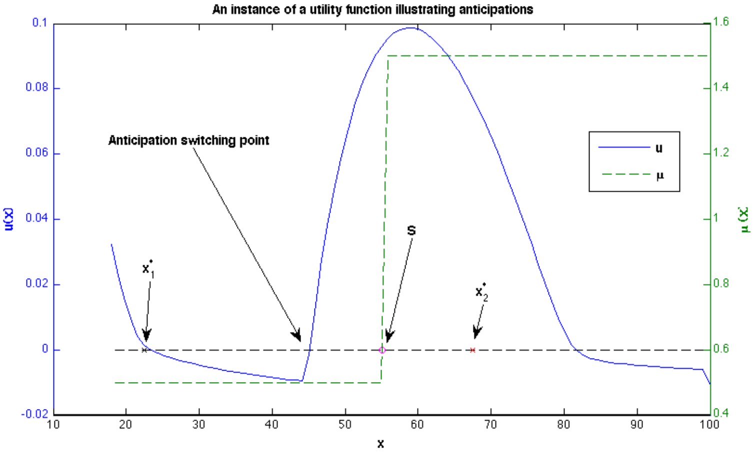
First order analysis

Before approximating numerically the solution to 1, we propose to get some insights on the shape of the solution by doing a first order analysis. More precisely, the solution of the queuing system describe above is characterized by the sign of the value function u. Consequently we are interested in finding potential sign switching points of u. The core modeling ingredient is the value of the Poisson arrival rate λ relative to $\mu(x)$.
For the first order analysis we look at the first order equation:

$$0 = \frac{\mu(x)}{x}(P - u(x)) - c + (\lambda \mathbf{1}_{\{u(x)>0\}} - \mu(x))u'(x). \tag{2}$$

Let us remark that equation (2) corresponds to a trivial shared risk Mean Field Game monotone system with $N = 1$, as described in the previous section. Note that in the framework of this model, the mean field aspect does not come from the continuum of agents (for every instant, the number of players is finite), but rather to the stochastic continuous structure of entries and exits of players.
Now we look at the the case where $\lambda > \mu(x)\ \forall x$. Then the term multiplying the derivative in (2) changes sign as u does. It is a simple matter to see that this happens at least twice: $x_1^* = \mu_1 P/c$ and $x_2^* = \mu_2 P/c$.

An anticipation behavior I



Two queues I

The *market price* will be centered on a constant P. The *market depth* is δ, meaning that no transaction will take place at a price lower than $P - \delta$ or higher than $P + \delta$. The (time varying) size of the bid queue (waiting buy orders) is Q_t^b and the size of the ask one (waiting sell orders) is Q_t^a.
When a market (buying) order hits the ask queue, the transaction price is p^{buy} and when the bid queue is lifted by a market (selling) order, the transaction price is p^{sell}. The price takes into account instantaneous queue size adjustments depending upon the order size q.

$$p_q^{\mathrm{buy}}(Q_t^a) := P + \frac{\delta q}{Q_t^a - q}, \quad p_q^{\mathrm{sell}}(Q_t^b) := P - \frac{\delta q}{Q_t^b - q} \tag{3}$$

The decision process I

When a buy (resp. sell) order arrives, its owner has to make a *routing decision*:

- if $v(Q^a_t, Q^b_t + q) < p^{\rm buy}(Q^a_t)$ (resp. $u(Q^a_t + q, Q^b_t) > p^{\rm sell}(Q^b_t)$) it is more valuable to route the order to the bid (resp. ask) queue (i.e. sending a limit order). In such a case the order will be a *Liquidity Provider (LP)*. We define symmetrically *Liquidity Consumer (LC)* orders. This decision is formalized in the model by setting the variable $R^{\oplus}_{\rm buy}(v, Q^a_t, Q^b_t + q)$ to 1 when $v(Q^a_t, Q^b_t + q) < p^{\rm buy}(Q^a_t)$, and to zero otherwise:

$$\begin{aligned} R^{\oplus}_{\rm buy}(v, Q^a_t, Q^b_t + q) &:= \mathbf{1}_{v(Q^a_t, Q^b_t + q) < p^{\rm buy}(Q^a_t)}, \quad \text{LP buy order} \\ R^{\oplus}_{\rm sell}(u, Q^a_t + q, Q^b_t) &:= \mathbf{1}_{u(Q^a_t + q, Q^b_t) > p^{\rm sell}(Q^b_t)}, \quad \text{LP sell order.} \end{aligned} \tag{4}$$

- otherwise the order goes Liquidity Consumerly to the ask (resp. bid) queue to obtain a trade. It will be a liquidity remover in this case:

$$\begin{aligned} R^{\ominus}_{\rm buy}(Q^a_t, Q^b_t) &:= 1 - R^{\oplus}_{\rm buy}(Q^a_t, Q^b_t), \quad \text{LC buy order} \\ R^{\ominus}_{\rm sell}(Q^a_t, Q^b_t) &:= 1 - R^{\oplus}_{\rm sell}(Q^a_t, Q^b_t), \quad \text{LC sell order.} \end{aligned}$$

  The price of such a transaction is $p^{\rm buy}$ (resp. $p^{\rm sell}$) as defined by equality (3). Note that we omit the dependence on u, v when it is unnecessary for the understanding of the equations.

The dynamics I

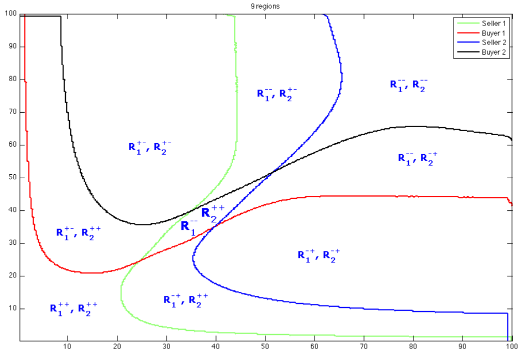
$$
\begin{aligned}
(5)\qquad & u(Q_t^a, Q_t^b) = \\
& (1 - \lambda_{\rm buy}\, dt - \lambda_{\rm sell}\, dt - 2\lambda^- dt)\ u(Q_t^a, Q_t^b) && \leftarrow \text{nothing} \\
& + (\lambda_{\rm sell} R_{\rm sell}^{\ominus}(u, Q_t^a + q, Q_t^b) + \lambda^-) dt\ u(Q_t^a, Q_t^b - q) && \leftarrow \text{sell order, LC} \\
& +\ \lambda_{\rm sell} R_{\rm sell}^{\oplus}(u, Q_t^a + q, Q_t^b) dt\ u(Q_t^a + q, Q_t^b) && \leftarrow \text{sell order, LP} \\
& + (\lambda_{\rm buy} R_{\rm buy}^{\ominus}(v, Q_t^a, Q_t^b + q) + \lambda^-) dt \cdot \Big[&& \leftarrow \text{buy order, LC} \\
& \qquad \underbrace{\frac{q}{Q_t^a} p^{\rm buy}(Q_t^a)}_{\text{trade part (ask)}} \ + \underbrace{(1 - \frac{q}{Q_t^a})\, u(Q_t^a - q, Q_t^b)\Big]}_{\text{removing (ask)}} \\
& +\ \lambda_{\rm buy}\ R_{\rm buy}^{\oplus}(v, Q_t^a, Q_t^b + q) dt\ u(Q_t^a, Q_t^b + q) && \leftarrow \text{buy order, LP} \\
& -\ c_a q\, dt. && \leftarrow \text{cost to maintain inventory}
\end{aligned}
$$

$$\text{(Ask)}\quad 0=[(\lambda_b R_b^{\ominus}+\lambda^-)\frac{1}{x}(p^b(x)-u)-c_a] + [\lambda_s R_s^{\oplus}-\lambda_b R_b^{\ominus}-\lambda^-]\cdot\partial_x u+[\lambda_b R_b^{\oplus}-\lambda_s R_s^{\ominus}-\lambda^-]\cdot\partial_y u,$$

$$\text{(Bid)}\quad 0=[(\lambda_s R_s^{\ominus}+\lambda^-)\frac{1}{y}(p^s(y)-v)+c_b] + [\lambda_s R_s^{\oplus}-\lambda_b R_b^{\ominus}-\lambda^-]\cdot\partial_x v+[\lambda_b R_b^{\oplus}-\lambda_s R_s^{\ominus}-\lambda^-]\cdot\partial_y v.$$

The system has to be understood locally in the four regions

$$R^{++}=\{(x,y), R_s^{\oplus}(x,y)=R_b^{\oplus}(x,y)=1\},$$
$$R^{--}=\{(x,y), R_s^{\ominus}(x,y)=R_b^{\ominus}(x,y)=1\},$$
$$R^{+-}=\{(x,y), R_s^{\oplus}(x,y)=R_b^{\ominus}(x,y)=1\},$$
$$R^{-+}=\{(x,y), R_s^{\ominus}(x,y)=R_b^{\oplus}(x,y)=1\}$$



Decision curves

Conclusion on Orderbook Dynamics

We can go as far as we want in the sophistication of models. We can use them:

- **reproduce**,
- **understand**,
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Conclusion on Orderbook Dynamics

We can go as far as we want in the sophistication of models. We can use them:

- **reproduce** , useful to simulate or test a trading logic (for Sell side and Market makers and prop. shops).
- **understand** , good to build estimators of what you can expect from the market state in the next few minutes (for Market makers, prop. shops and buy sides).
- **explain** . for regulators and policy makers.

Outline

What is Market Impact?

The easy answer is "**the price response to a volume pressure**". But

- a lot of participants a jointly buying and selling (volume pressure of who?), is there a different market impact for each participant?
- at which time scale (millisecond or week?), child orders or metaorders?

Is it different from the **price formation process**? In [Kyle, 1985], using a game theoretic approach between an informed trader, a market maker and noise traders:

> *[...] modeling price innovations as functions of quantities traded is not inconsistent with modelling price innovations as the consequence of new information.*

Why bother with Market Impact?

Allocation I buy and the price goes up: my investment choice is good. **Not that true in presence of impact**.

Hedging and risk management my optimal strategy protects me against a risks. But **my impact can add costs** (and a very bad way if I am in a one way market).

Execution my trading profile has to take my impact into account. when do I start and stop to measure my **slippage**?

Market making and arbitrage I should act as Kyle's market maker to earn money "arbitraging" the impact (i.e. providing liquidity) but I would like to detect if I am in presence of **adverse selection**.

For all this we need market impact models, at different time scales, but consistent from one scale to the other.

Our approach

A lot of papers have been written about market impact.

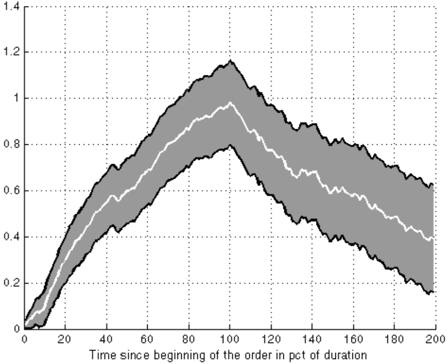
- About **metaorders** , after theoretical papers, more practical ones appeared targeting parameters estimation for optimal trading or just TCA (see [Almgren et al., 2005] or [Engle et al., 2012]), and then others using accurate databases of metaorders ([Bershova and Rakhlin, 2012], [Moro et al., 2009] or [Waelbroeck and Gomes, 2013]). More recently [Brokmann et al., 2014] and [Zarinelli et al., 2014] focussed on daily market impact.
  The latter are very informative but their methodology is not that clear. We would like to establish a clear methodology to asses market impact on databases of metaorders.
- On another side, models have been proposed to explain impact of **atomic trades** (i.e. child orders), like [Eisler et al., 2010].
  We aim to reconcile small and large time scales, using Hawkes processes.

Our Database

We have a database of metaorders electronically traded by a pure agency intermediary in 2009. We selected some of them, long enough, similar enough, traded a steady-state and homogenous way.

	Filters	Nbe of metaorders
Original database Ω	-	**398,812**
Intraday database $\Omega^{(in)}$	smooth execution condition	**299,824**
Database for temporary $\Omega^{(te)}$	10 child orders minimum	191,324
	consistency with HF market data	**84,794**
Database for transient $\Omega^{(tr)}$	$V_D > 500,000$	56,047
	$T > 3$ minutes	51,218
	trading rate $\in [3\%, 40\%]$	33,518
	daily part. $\in [0.1\%, 15\%]$	**32,756**
Daily data-base for decay $\Omega^{(de)}$	$t_0(\omega) + 2T(\omega)$	**20,486**

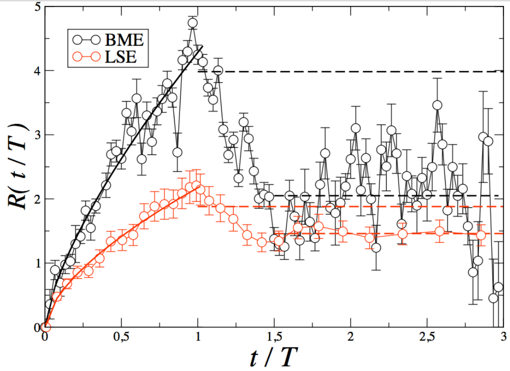
in [Lehalle et al., 2013]



Market Impact is a process in several phases:

- the **transient impact**, concave in time,
- reaches its maximim, the **temporary impact**, at the end of the metaorder,
- then a **decay**,
- until a stationary level; the price moved by a **permanent** shift.

in [Moro et al., 2009]



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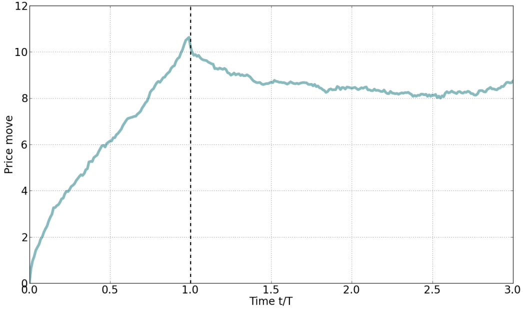
On our database



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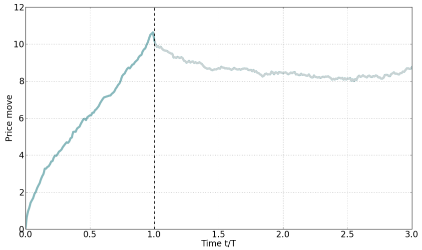
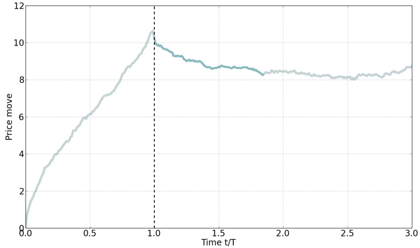


- The meta order has a concave transient impact;
- the largest past price move, the less future move (for the same size);
- could be memory, i.e. a **latent orderbook effect**: other market participants react actively to the price change (investors) or volume pressure (market makers),
- could be just decay of the price impact of each atomic / child order.

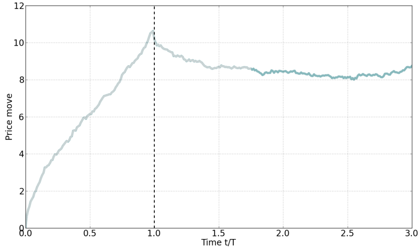
At the end of this phase, the price reaches the **temporary market impact** of the metaorder.

Impact Decay



- Once the trading process of the metaorder stops, there is a decay;
- could be a microscopic effect: a **zero intelligence** (or more sophisticated, like [Huang et al., 2013] or [Mastromatteo et al., 2013]) model in front of an imbalanced orderbook generates decay;
- or could be a **fair pricing condition**: the market makers asked me a premium during my trading (i.e. if the trading rate is constant the decay should take place to a 2/3 of the temorary impact).

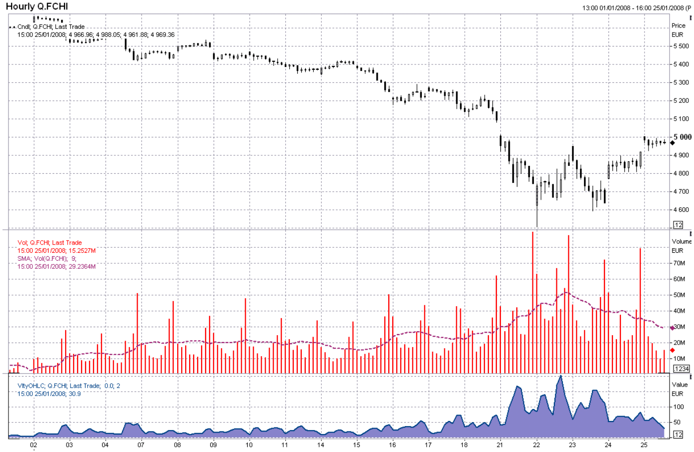
Permanent Impact



- the permanent impact can be seen as a **mechanical effect** of the trading process giving birth to the price formation: a large buyer moved the price up. It can be how the offer and demand diffuse information in price (it is a forward / transport like process).
- It can be seen as a **price discovery process**: the "fair price" is higher because of an information, the more investors trade this information, the faster the price join this pre determined level (it is a backward / future expected valuation process).

A good question: how metaorders of all investors mix to generate the price formation, what are the permanent impacts of two investors trading the same information?

Sometimes the Market Impact is Visible



Market Impact During The unwinding of J Kerviel’s position by Société Générale.

Daily Market Impact: Mechanical or Informational Effect?

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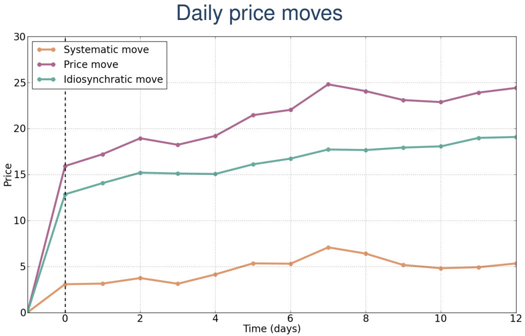
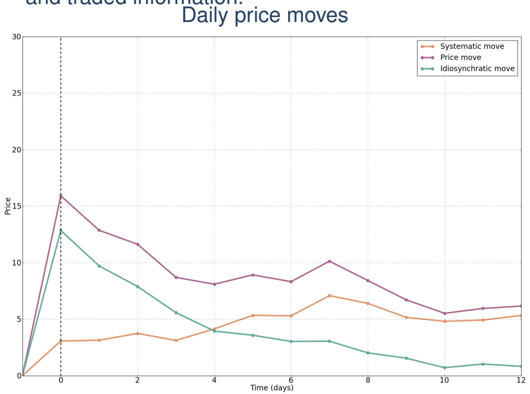


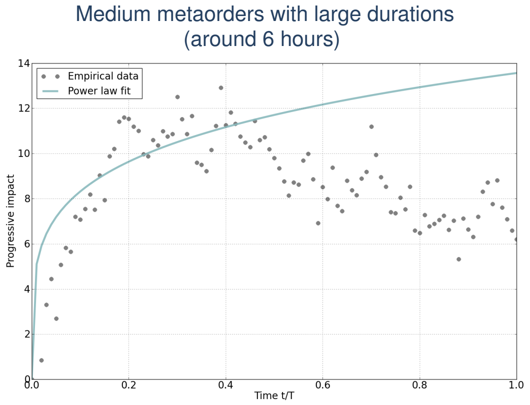
- If you plot the long term price moves (x-axis in days), you see an regular increase;
- But the same stock is traded today, tomorrow, the day after, etc.

We have enough data to investigate long term impact, exploring the relationships between permanent impact and traded information.



- If you plot the long term price moves (x-axis in days), you see an regular increase;
- But the same stock is traded today, tomorrow, the day after, etc.
- Once you remove the market impact of "future" trades (similarly to [Waelbroeck and Gomes, 2013]), you obtain a different shape.
- If you look each curve: the yellow one contains the CAPM β (the metaorders are trading market-wide moves), the green curve contains the idiosyncratic moves, this shape can be read as **the daily decay of metaorders impact**.

Conclusion on Impact

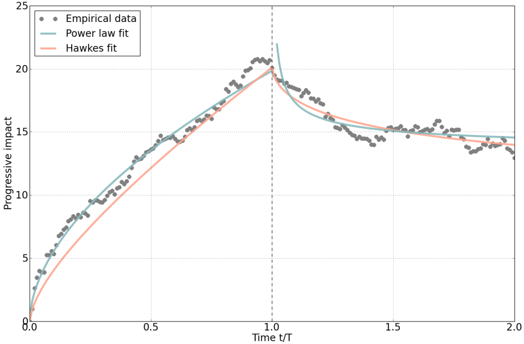


In [Bacry et al., 2014]:

- We notice a **dependence of the market impact to the duration** of the metaorder,
- but it can be discussed... This underlines the need for a shared methodology among researchers.

Conclusion on Impact

Medium durations (around 2 hours)



$\beta^{(tr)} = 0.55, \beta^{(d)} = -0.48$

In [Bacry et al., 2014]:

- We notice a **dependence of the market impact to the duration** of the metaorder,
- but it can be discussed... This underlines the need for a shared methodology among researchers.
- We introduce **exogenous metaorders in a microstructural Hawkes model**, building a structural link between the transient and decay shapes (for steadily traded metaorders).
- This link is validated on the data, showing this model seems to capture market impact a parsimonious way across intraday time scales (from trade by trade to metaorders).

Outline

Conclusion

We have seen each market participant has a specific focus. Nevertheless they all have to cope with

- **Seasonalities** (long term –i.e. morning / mid day / evening– and short ones –i.e. during pre fixing of auction calls–);
- The **liquidity flows** (consuming or adding flows), and their (potential) relation to price moves;
- The **market impact** of their large orders.

Moreover, being able to disentangle **mechanical vs. information parts of price moves** is of paramount importance in the understanding of price formation.

For Regulators, understanding the effect of a change of some rules like the tick size, taker/maker fees, minimum resting time on the efficiency of the price formation, is really important. Nevertheless the efficiency is not that well defined.

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