Too Fast or Too Slow?
Determining the Optimal Speed of Financial Markets

Austin Gerig
US Securities and Exchange Commission
Co-author: Daniel Fricke

The Securities and Exchange Commission, as a matter of policy, disclaims responsibility for any private publication or statement by any of its employees. The views expressed herein are those of the author and do not necessarily reflect the views of the Commission or of the author’s colleagues upon the staff of the Commission.
Motivation

• Modern financial markets are *really* fast
• Current latency:
  – intramarket: 1-100 microseconds
  – intermarket: 0.1 – 100 milliseconds
• Are these speeds really necessary?
• Is there an optimal speed of trading, and if so, what determines this speed?
Results

• We model the trading of a security via periodic batch auctions

• Liquidity is maximized at intermediate speeds:
  – if too slow, orders “sit” too long before transacting
  – if too fast, not enough orders are “mixed”

• Three factors determine the optimal speed of a security:
  – volatility of the security
  – intensity of trading in the security
  – correlation of security with the market

• Using rough estimates of these factors, the optimal clearing speed for a typical U.S. stock:
  – 0.2 to 0.9 seconds
The Automation of Financial Markets

In 1971, Fischer Black predicted that most activity on financial exchanges could (and would) be automated.

NYSE today
Global shift to electronic trading from 1975 to 2002 (from Jain (2005)).
With automation came speed.

Market Order Execution Speed in Seconds

Source: Rule 605 data from Thomson for all eligible market orders (100-9,999 shares)

(Angel, Harris, and Spatt, 2013).
... as well as lower costs and increased efficiency

Data from 35 large-cap US stocks during the last full week of February in 2000, 2005, and 2010. Error bars report the standard error of the mean across the 35 stocks (Gerig, 2013).
but are current speeds necessary?
… probably not necessary for price discovery

- “… price discovery at the nano-second interval cannot possibly give a significant allocative efficiency benefit over price discovery on a second-by-second basis.”

– Adair Turner, FSA Chairman (at the time)
What about liquidity/transaction costs?

• There is no need for markets to clear faster than the rate of order arrival
• … but, the current rate of order arrival is very high
  • on average, there is more than one U.S. equity trade *per millisecond* during the trading day (see www.utpplan.com)
  • milliseconds might be important!!!
How is liquidity affected by market speed?

How do you model the relationship?
• We extend the model of Garbade and Silber (1979).

• Batch auction model where $\tau$ is the time between clearings.
  \[ \rightarrow \text{Smaller } \tau = \text{faster markets. (} 1/\tau = \text{clearing frequency)} \]

• Our liquidity measure (really an inverse liquidity measure):
  \[ \rightarrow \text{Liquidity Risk, notated } V, \]
  \[ \ldots \text{the variance of the difference between the equilibrium value of an asset at the time a market participant decides to trade and the transaction price ultimately realized.} \]
  \[ \rightarrow V = \text{variance of implementation shortfall} \]
Garbade and Silber (1979) – JoF paper, but most of you are probably unaware of the model!

- Unobservable equilibrium price evolves as driftless brownian motion.
- Investors arrive sequentially to market and submit linear excess demand schedules with different reservation prices.
- Reservation prices are normally distributed around the unobserved equilibrium price.
- Markets clear at regular intervals, i.e., it is a batch auction market.
Intermediate $\tau$ minimizes liquidity risk (maximizes liquidity).

- If markets are too fast ($\tau$ very small), the book will be sparse and clearing prices will be noisy.
- If markets are too slow ($\tau$ very large), equilibrium prices are likely to change significantly by the time an order clears.

In either case ($\tau$ too large or too small), $V$ is large.
What Determines the Optimal $\tau$?

Which factors determine $\tau^*$?

In Garbade and Silber (1979), $\tau^*$ is related to:

1. **Price volatility.** The more volatile the security, the quicker it should trade.
2. **Intensity of trade.** The more investors who trade the security, the quicker it should trade.

Our contribution:

3. **Cross-correlation with other securities!** The more correlated the security is to others, the quicker it should trade.
We update the model of Garbade and Silber (1979) to include multiple securities, and we estimate the current optimal speed of trading for a typical U.S. stock.

Main findings:

1. Higher correlations increase liquidity and allow markets to speed up.
2. For sufficiently large correlations, continuous trading is optimal.
3. Rough estimates indicate that the optimal speed of trading for a typical U.S. stock is approximately 0.2 to 0.9 seconds.

⇒ For many securities, current financial markets are perhaps unnecessarily fast.
Three model treatments:

① Public Market
② Public Market + Liquidity Provider
③ Public Market + Liquidity Provider + Market Security
Public Market
Public Market

Single security traded by public investors.

- \( \tau \) = time interval between (periodic) clearings,
- \( \omega \) = arrival rate of investors between clearings,
- Number of active investors per clearing: \( K = \omega \tau \).
- Investors submit excess demand schedules to the market:

\[
D(p) = a (r_i - p),
\]

with \( a > 0 \), \( r_i \) is the reservation price of \( i \), and \( p \) is the clearing price in the public market

\[
p = \frac{1}{K} \sum_{i} r_i.
\]
Public Market

- Let \( r_i \) be normally distributed around the unobservable equilibrium price, \( m_i \), at all times:

\[
\begin{align*}
    r_i &= m_i + g_i, \\
    g_i &\sim N(0, \sigma^2),
\end{align*}
\]

where \( g_i \) is uncorrelated across investors.

- Average reservation price at market clearing \( t \)

\[
\bar{r}_t = \frac{K}{i} (m_i + g_i) / K = \bar{m}_t + f_t,
\]

\[
f_t \sim N \left( 0, \frac{\sigma^2}{\omega_T} \right).
\]
• We assume $m$ is a **driftless Brownian motion** such that the average equilibrium price at time $t$ is

$$\bar{m}_t = \bar{m}_{t-\tau} + e_t,$$

$$e_t \sim N(0, \tau \psi^2),$$

with $e_t$ serially uncorrelated, $e_t \perp g_i$, and $e_t \perp f_t$. 
Liquidity risk, $V_P$:
- variance of difference between equilibrium price at trading decision and clearing price

\[ V_P = \text{Var}[\bar{r}_t - m_i], \]
\[ = \text{Var}[\bar{r}_t - \bar{m}_t] + \text{Var}[\bar{m}_t - m_i] \]
\[ = \frac{\sigma^2}{\omega \tau} + \frac{\tau \psi^2}{4}. \]

- **Note:** $V_P$ increases with volatility, $\psi$, and decreases with intensity of trade, $\omega$.

\[ \Rightarrow \tau_P^* = \frac{2\sigma}{\psi \sqrt{\omega}} \quad \text{and} \quad V_P^* = \frac{\sigma \psi}{\sqrt{\omega}}. \]

- **Also Note:** The optimal clearing interval, $\tau^*$ decreases with both volatility and intensity of trade.
Results: Public Market

\[ V_P = \frac{\sigma^2}{\omega \tau} + \frac{\tau \psi^2}{4} \]

Figure: Liquidity risk, \( V_P \), versus the time between market clearings, \( \tau \), without liquidity provider. Parameters: \( \psi = 1 \), \( \sigma = 1 \), and \( \omega = 10 \).
Market Efficiency in Public Market

**Problem:** mean-reversion in the clearing prices.

**Solution:** introduce liquidity provider or short-term “speculator” (who would naturally want to step in).

**Result:** security is more liquid and can clear faster.
Public Market + Liquidity Provider
• Include a **single, competitive, risk-neutral liquidity provider**:
  - observes the average reservation price ($\bar{r}_t$) directly before the clearing,
  - pushes the clearing price towards her estimate of the equilibrium price ($\hat{m}_t$) → **Kalman filter**.

• **Liquidity risk**:

\[
V_L = \text{Var}[\hat{m}_t - m_i] \leq V_P.
\]

• **Main results**:

\[
\tau^*_L = \frac{\tau^*_P}{\sqrt{3}} \quad \text{and} \quad V^*_L < V^*_P
\]
Figure: Liquidity risk in public market ($V_P$) and in the market with liquidity provider ($V_L$). Parameters: $\psi = 1$, $\sigma = 1$, and $\omega = 10$. 
Public Market + Liquidity Provider + Multiple Securities
General setup with $N$ securities:

- average reservation prices (observation)

$$\bar{r}_t = \bar{m}_t + f_t,$$
$$f_t \sim N(0, \Sigma).$$

- average equilibrium prices (system)

$$\bar{m}_t = \bar{m}_{t-\tau} + e_t,$$
$$e_t \sim N(0, \Psi),$$

where $\bar{r}$, $\bar{m}$, $\bar{f}$, and $\bar{e}$ are $(N \times 1)$ vectors and $\Sigma$ and $\Psi$ are $(N \times N)$ matrices.
Multiple Securities (Kalman Filter)

- Let $\hat{m}_t$ be the estimate of $\bar{m}_t$ based on $\{\bar{r}_t, \bar{r}_{t-1}, \bar{r}_{t-2}, \ldots\}$:

$$
P(\bar{m}_t | \bar{r}_t, \bar{r}_{t-1}, \ldots) \sim N(\hat{m}_{t-1} + G_t [\bar{r}_t - \hat{m}_{t-1}], S_t),$$
$$
P(\bar{m}_{t+1} | \bar{r}_t, \bar{r}_{t-1}, \ldots) \sim N(\hat{m}_t, R_{t+1}),$$

where

$$
G_t = R_t (R_t + \Sigma)^{-1} \text{ [Kalman gain]},
$$
$$
R_{t+1} = S_t + \Psi,
$$
$$
S_t = R_t - G_t R_t.
$$

- Best estimate of $\bar{m}_t$ is

$$
\hat{m}_t = \hat{m}_{t-1} + G_t (\bar{r}_t - \hat{m}_{t-1}).
$$

- Solution Method: search for convergence of estimation variance to limiting value, i.e., $R_{t+1} = R_t$. The result is the Riccati equation:

$$
R (R + \Sigma)^{-1} R - \Psi = 0.
$$
Market Security

In order to present analytic results, we treat the multiple security model as a special case of a two security market where the second security is the “market security”.

- Idealized assumptions:
  1. Two securities, one of them being the “market security”,
  2. The market security is perfectly liquid,
  3. \( \rho = \text{correlation} \) of equililibrium price changes.
    - \( \omega_M \gg 1 \),
    - \( \Sigma \approx \begin{pmatrix} \sigma^2/(\omega \tau) & 0 \\ 0 & 0 \end{pmatrix} \).

- Liquidity risk:

  \[ V_M = \text{Var}[\hat{m}_t(\rho) - m_i] \leq V_L. \]

  \( \Rightarrow \) Market information helps reduce liquidity risk further!
Market Security and the Importance of $\rho$

**Figure:** Liquidity risk, $V_M$, versus the time between market clearings, $\tau$, with liquidity provider and market information. Parameters: $\psi = 1$, $\sigma = 1$, $\omega = 10$. Values of $\rho$ are between 0 and 1. Critical value: $|\rho^c| = \sqrt{3/4}$. 
Results: Comparing Liquidity Risk ($V^*$)

Figure: Comparison of liquidity risk, $V$, for the different models. Parameters: $\psi = 1$, $\sigma = 1$, $\omega = 10$, and $\rho = 0.84$. Note: $\rho < \rho^c$. 
Market Security and the Importance of $\rho$

Figure: Speed versus correlation. Here we compare $\tau^*$ for the different models. Parameters: $\psi = 1$, $\sigma = 1$, $\omega = 10$. Note: $\tau^*_M = h(\rho)\tau^*_L$. 

$|\rho|^c = \sqrt{3/4}$
Estimating Optimal Speed

Estimating $\tau^*_M$ (in seconds) for a typical U.S. stock:

<table>
<thead>
<tr>
<th>Para.</th>
<th>Value</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td>0.0001</td>
<td>Corresponds to annualized volatility of 25%</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.0003</td>
<td>Corresponds to $0.01 quoted spread for a $33 stock</td>
</tr>
<tr>
<td>$\omega$</td>
<td>5</td>
<td>Average number of quote changes per second per Tape A/B security (utpplan.com).</td>
</tr>
<tr>
<td>$\omega^{peak}$</td>
<td>100</td>
<td>… during peak times.</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.75</td>
<td>Note: $\rho &lt; \rho^c$</td>
</tr>
</tbody>
</table>

$\Rightarrow \tau^* \approx 0.9$ seconds, $\tau^{peak}_* \approx 0.2$ seconds.
Conclusions

• Market quality has improved considerably as a result of speed and automation ... but is there a limit to the benefits of speed?

• We model the trading of a security via periodic batch auctions and study how market quality is affected as the clearing frequency is changed.

• The optimal clearing frequency of a security depends on three factors
  → volatility
  → intensity of trading
  → correlation with the market

• At a critical correlation threshold $\rho_c \approx 0.87$, it becomes optimal for a security to continuously clear.

• Rough estimates suggest that a typical U.S. stock should trade at intervals between 0.2 to 0.9 seconds.
Thank you for your attention!
Appendix

\[ V_L = \text{Var}[^{\hat{m}}_t - m_i], \]
\[ = \frac{\tau \psi^2}{2} \left( 1 + \sqrt{1 + \frac{4\sigma^2/\omega}{\tau^2 \psi^2}} \right) + \frac{4\sigma^2}{\omega \tau}, \]
\[ = \frac{2 \left( 1 + \sqrt{1 + \frac{4\sigma^2/\omega}{\tau^2 \psi^2}} \right)}{2 \left( 1 + \sqrt{1 + \frac{4\sigma^2/\omega}{\tau^2 \psi^2}} \right)}. \]

\[ V_M = \text{Var}[^{\hat{m}}_t(\rho) - m_i], \]
\[ = (1/2 + \Theta) \tau \psi^2 + (1/2 - \Theta) \tau \psi^2 \sqrt{1 + \frac{4\sigma^2/\omega}{\Theta \tau^2 \psi^2}} + \frac{4\sigma^2}{\omega \tau}, \]
\[ = \frac{2 \left( 1 + \sqrt{1 + \frac{4\sigma^2/\omega}{\Theta \tau^2 \psi^2}} \right)}{2 \left( 1 + \sqrt{1 + \frac{4\sigma^2/\omega}{\Theta \tau^2 \psi^2}} \right)}, \]
with \( \Theta = 1 - \rho^2. \)