Too Fast or Too Slow? Determining the Optimal Speed of Financial Markets

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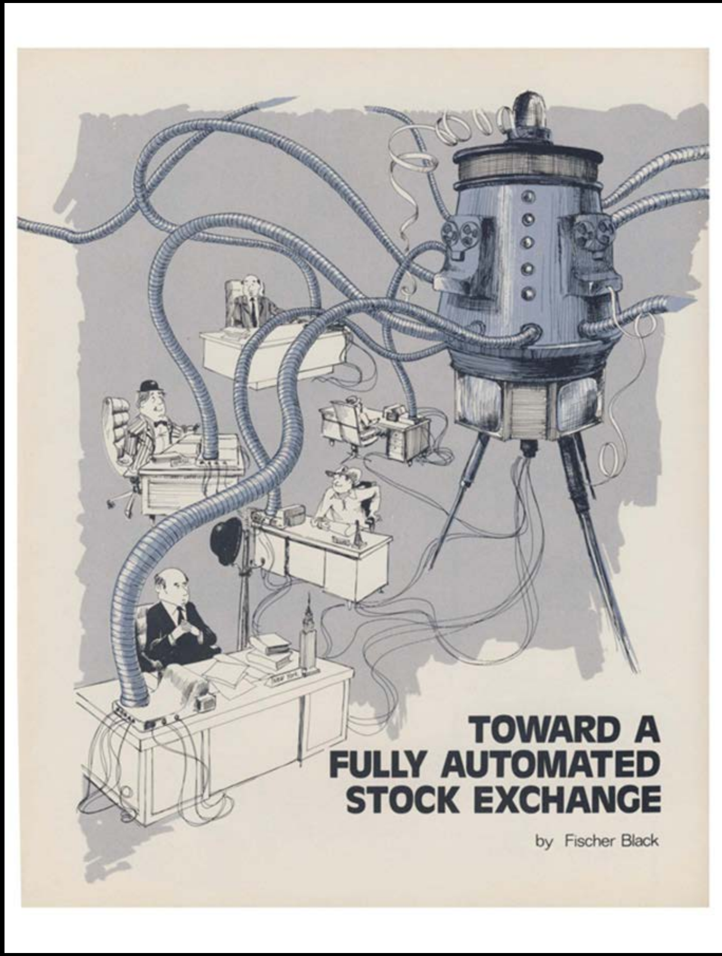
Motivation

- Modern financial markets are *really* fast
- Current latency:
  - intramarket: **1-100** microseconds
  - intermarket: **0.1 – 100** milliseconds
- Are these speeds really necessary?
- Is there an optimal speed of trading, and if so, what determines this speed?

Results

- We model the trading of a security via periodic batch auctions
- Liquidity is maximized at intermediate speeds:
  - if too slow, orders "sit" too long before transacting
  - if too fast, not enough orders are "mixed"
- Three factors determine the optimal speed of a security:
  - volatility of the security
  - intensity of trading in the security
  - correlation of security with the market
- Using rough estimates of these factors, the optimal clearing speed for a typical U.S. stock:
  - 0.2 to 0.9 seconds

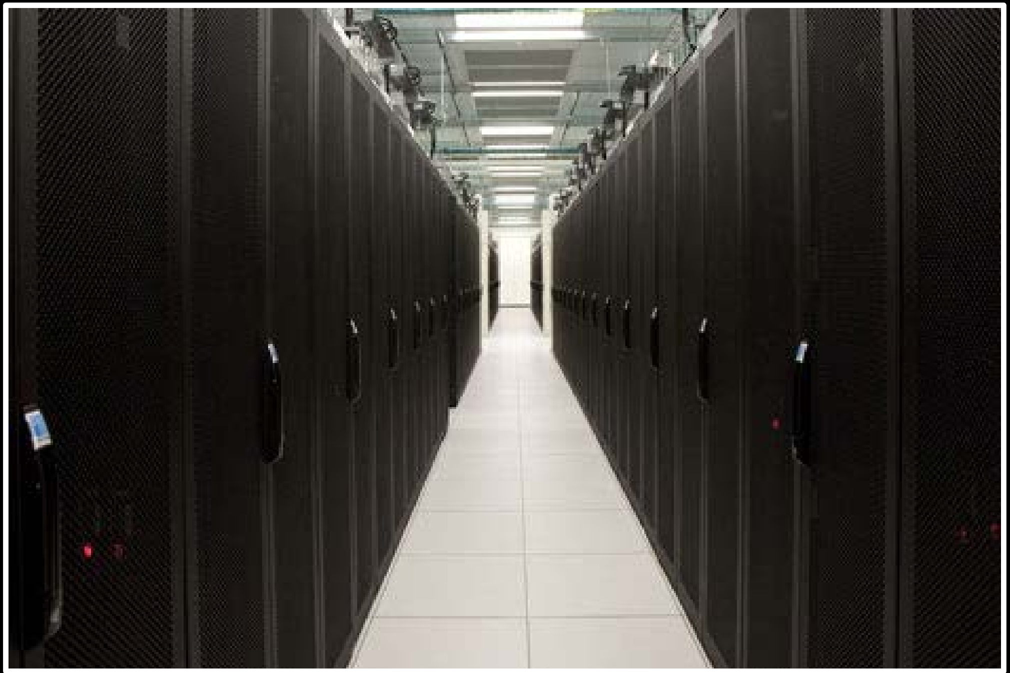
The Automation of Financial Markets



In 1971, Fischer Black predicted that most activity on financial exchanges could (and would) be automated.

Also: Demsetz (1968), Black (1971), Fama (1970 pg. 399), Garbade and Silber (1978), Hakansson, Beja, and Kale (1985), Amihud and Mendelson (1988)

NYSE today



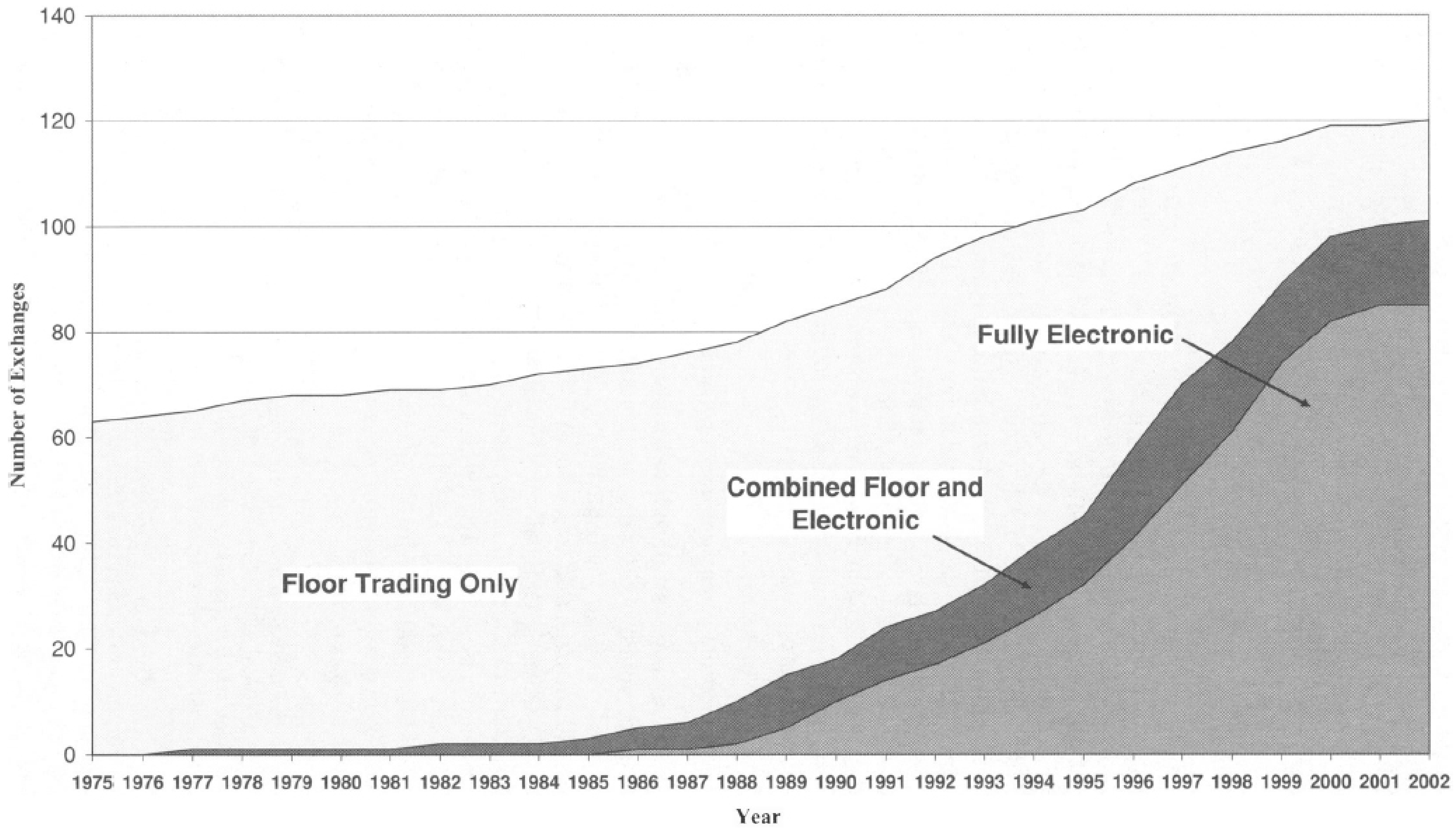
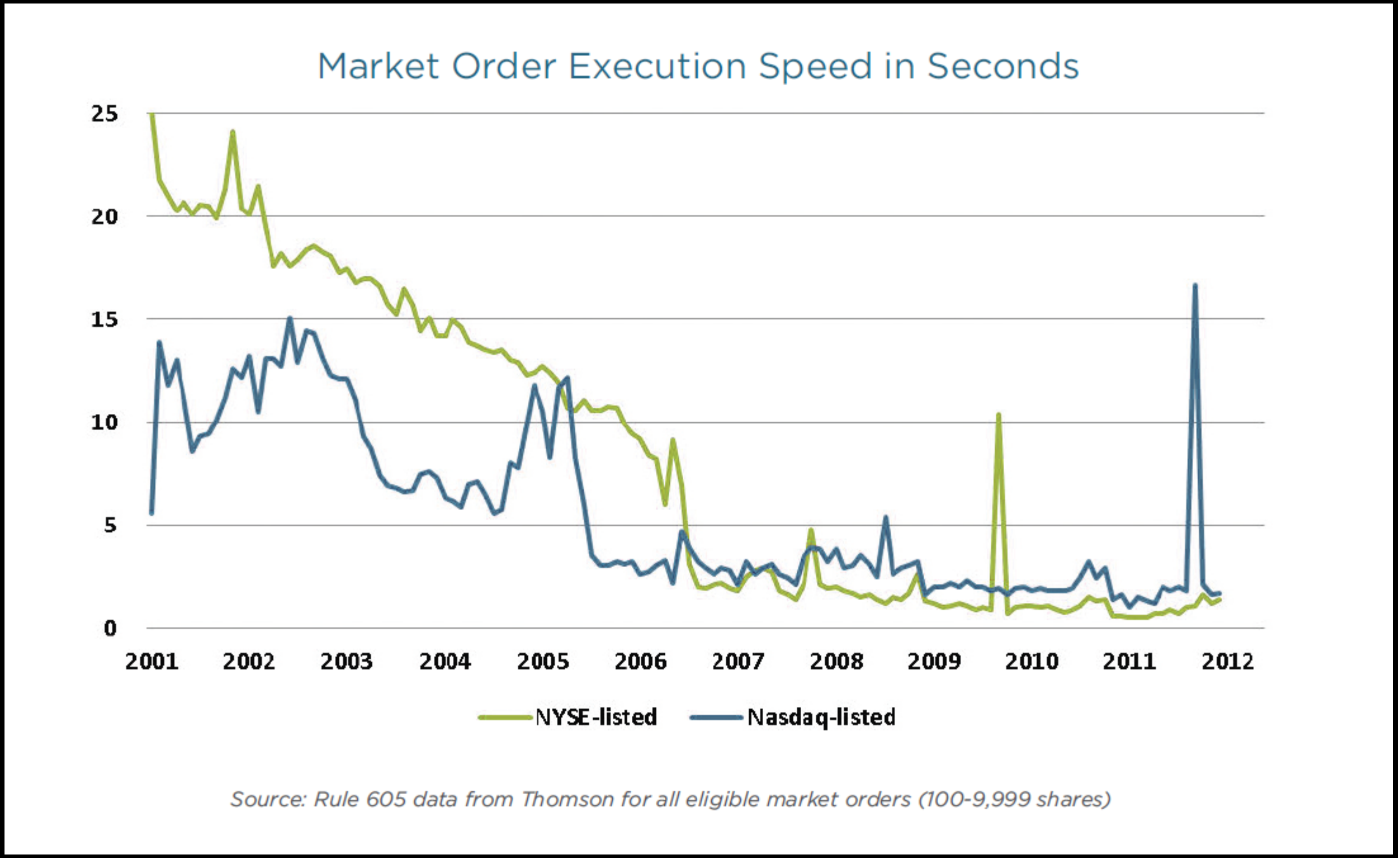


Figure 1. The global shift from floor trading to electronic trading. Based on automation of the leading stock exchange in 120 countries from 1975 to 2002.

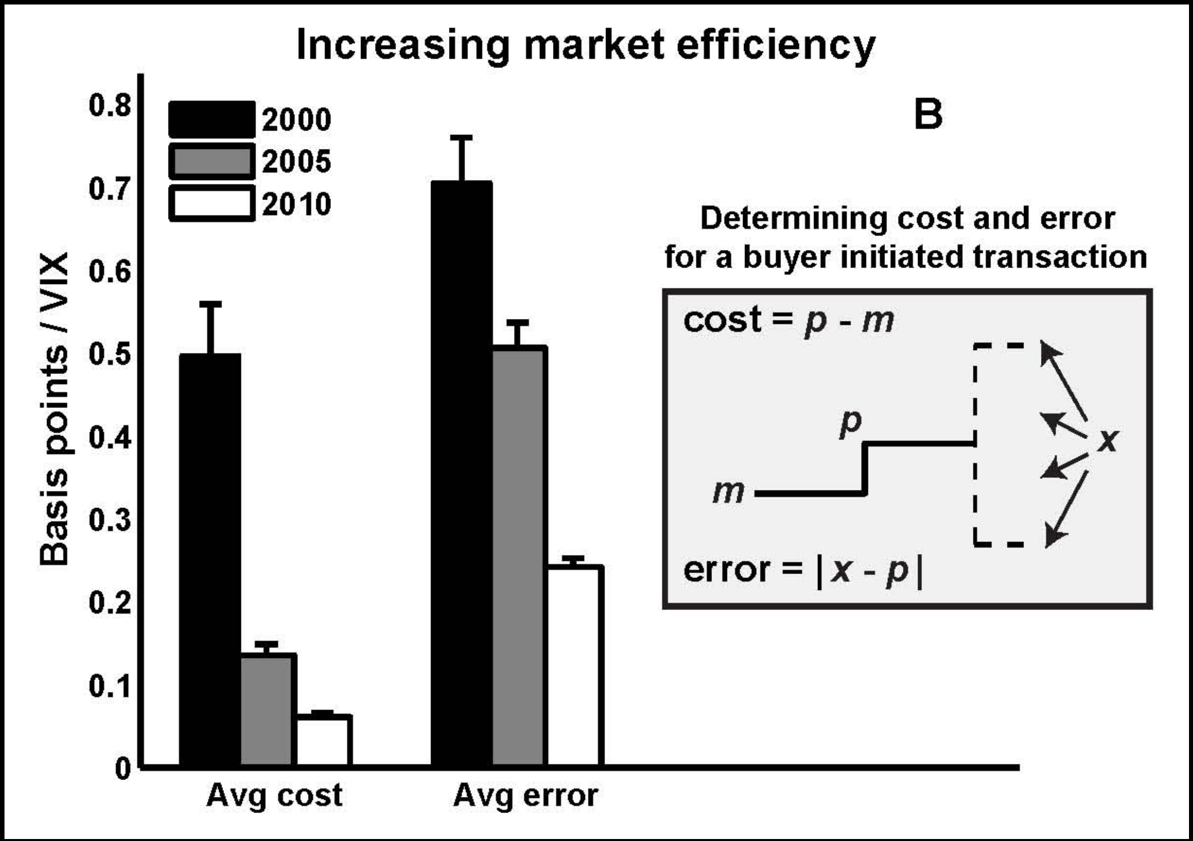
Global shift to electronic trading from 1975 to 2002 (from Jain (2005)).

With automation came speed



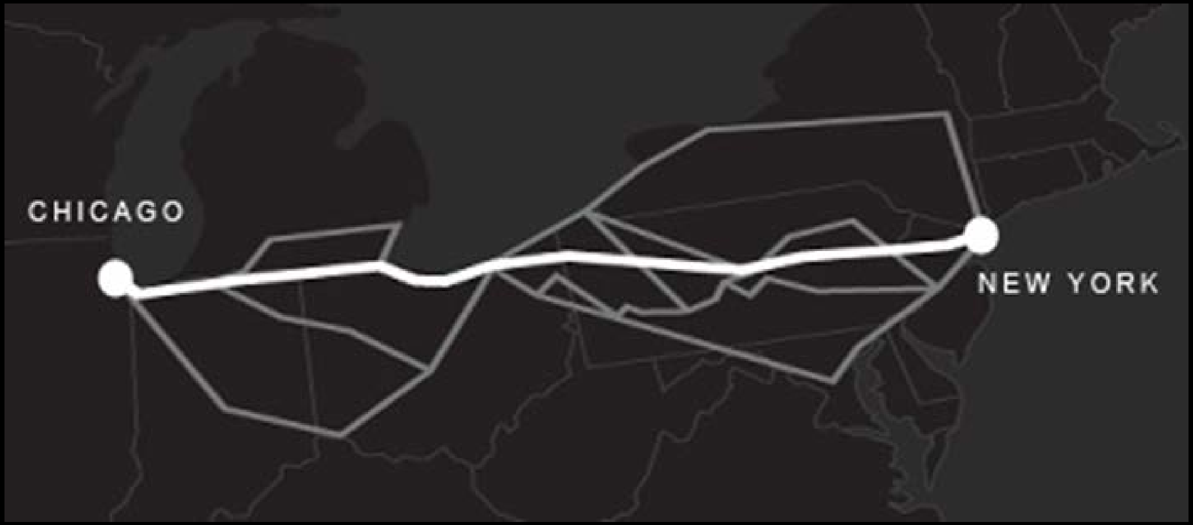
(Angel, Harris, and Spatt, 2013).

… as well as lower costs and increased efficiency



Data from 35 large-cap US stocks during the last full week of February in 2000, 2005, and 2010. Error bars report the standard error of the mean across the 35 stocks (Gerig, 2013).

but are current speeds necessary?



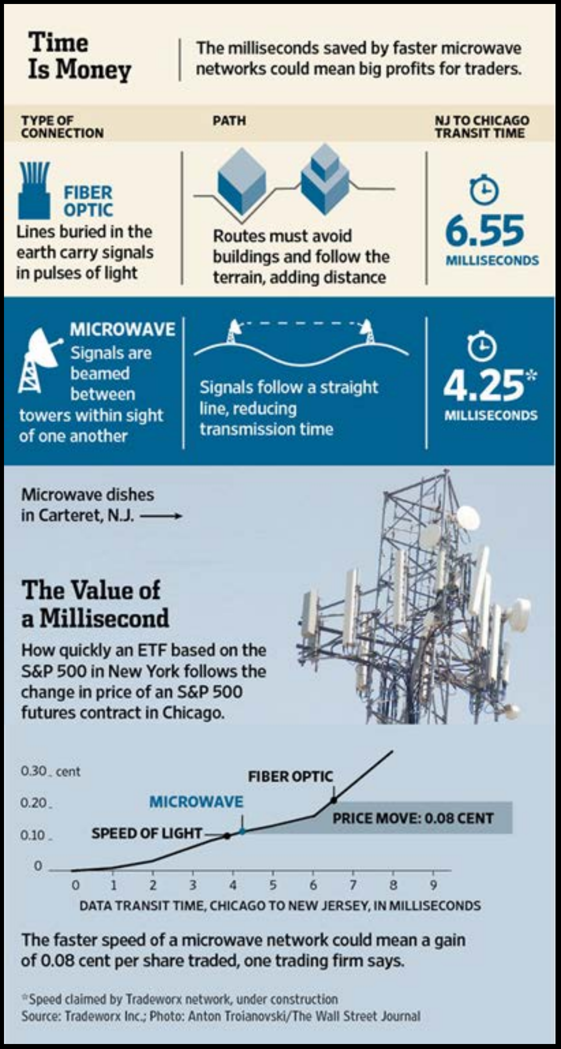
U.S. Equities Exchanges

BATS Making Markets Better®

BATS System Performance

World-class, Sustained Low Latency

Order Latency	10 Gig Connection		1 Gig Connection	
	Binary	FIX	Binary	FIX
Average	82 microseconds	95 microseconds	104 microseconds	126 microseconds
80% of orders within	107 microseconds	107 microseconds	135 microseconds	135 microseconds
99% of orders within	137 microseconds	127 microseconds	135 microseconds	155 microseconds
99.9% of orders within	307 microseconds	257 microseconds	235 microseconds	255 microseconds



… probably not necessary for price discovery

- "… price discovery at the nano-second interval cannot possibly give a significant allocative efficiency benefit over price discovery on a second-by-second basis."
  - Adair Turner, FSA Chairman (at the time)

What about liquidity/transaction costs?

- There is no need for markets to clear faster than the rate of order arrival
- … but, the current rate of order arrival is *very* high
  - on average, there is more than one U.S. equity trade *per millisecond* during the trading day (see www.utpplan.com)
  - milliseconds might be important!!!

How is liquidity affected by market speed?

How do you model the relationship?

Our Analysis

- We extend the model of Garbade and Silber (1979).
- Batch auction model where τ is the time between clearings.

  $\rightarrow$ Smaller τ = faster markets. ($1/\tau$ = clearing frequency)
- Our liquidity measure (really an inverse liquidity measure):

  $\rightarrow$ **Liquidity Risk**, notated V,

  > *. . . the variance of the difference between the equilibrium value of an asset at the time a market participant decides to trade and the transaction price ultimately realized.*

  $\rightarrow$ V = variance of implementation shortfall

Original Model

Garbade and Silber (1979) – JoF paper, but most of you are probably unaware of the model!

- Unobservable equilibrium price evolves as driftless brownian motion.
- Investors arrive sequentially to market and submit linear excess demand schedules with different reservation prices.
- Reservation prices are normally distributed around the unobserved equilibrium price.
- Markets clear at regular intervals, i.e., it is a batch auction market.

Optimal τ

Intermediate τ minimizes liquidity risk (maximizes liquidity).

- If markets are too fast (τ very small), the book will be sparse and clearing prices will be noisy.
- If markets are too slow (τ very large), equilibrium prices are likely to change significantly by the time an order clears.

In either case (τ too large or too small), V is large.

What Determines the Optimal τ?

Which factors determine τ^*?

In Garbade and Silber (1979), τ^* is related to:

1. **Price volatility**. The more volatile the security, the quicker it should trade.
2. **Intensity of trade**. The more investors who trade the security, the quicker it should trade.

Our contribution:

3. **Cross-correlation with other securities!** The more correlated the security is to others, the quicker it should trade.

Results

We **update** the model of **Garbade and Silber (1979)** to include **multiple securities**, and we **estimate** the current optimal speed of trading for a typical U.S. stock.

Main findings:

1. Higher correlations increase liquidity and allow markets to speed up.
2. For sufficiently large correlations, continuous trading is optimal.
3. Rough estimates indicate that the optimal speed of trading for a typical U.S. stock is approximately 0.2 to 0.9 seconds.

$\Rightarrow$ For many securities, current financial markets are perhaps unnecessarily fast.

Three model treatments:

1. Public Market
2. Public Market + Liquidity Provider
3. Public Market + Liquidity Provider + Market Security

Public Market

Public Market

Single security traded by **public investors**.

- $\tau =$ time interval between (periodic) clearings,
- $\omega =$ arrival rate of investors between clearings,
- Number of active investors per clearing: $K = \omega\tau$.
- Investors submit **excess demand schedules** to the market:

$$D(p) = a(r_i - p),$$

with $a > 0$, r_i is the **reservation price** of i, and p is the **clearing price** in the public market

$$p = \sum_i^K r_i / K.$$

Public Market

- Let r_i be normally distributed around the **unobservable equilibrium price**, m_i, at all times:

$$\begin{aligned} r_i &= m_i + g_i, \\ g_i &\sim N(0, \sigma^2), \end{aligned}$$

where g_i is uncorrelated across investors.

- **Average reservation price** at market clearing t

$$\begin{aligned} \bar{r}_t &= \sum_i^K (m_i + g_i)/K = \bar{m}_t + f_t, \\ f_t &\sim N\left(0, \frac{\sigma^2}{\omega\tau}\right). \end{aligned}$$

Public Market

- We assume m is a **driftless Brownian motion** such that the **average equilibrium price** at time t is

$$\begin{aligned} \bar{m}_t &= \bar{m}_{t-\tau} + e_t, \\ e_t &\sim N(0, \tau\psi^2), \end{aligned}$$

with e_t serially uncorrelated, $e_t \perp g_i$, and $e_t \perp f_t$.

Results: Public Market

Liquidity risk, V_P:

- variance of difference between **equilibrium price at trading decision** and **clearing price**

$$\begin{aligned} V_P &= \mathrm{Var}[\bar{r}_t - m_i], \\ &= \mathrm{Var}[\bar{r}_t - \bar{m}_t] + \mathrm{Var}[\bar{m}_t - m_i] \\ &= \frac{\sigma^2}{\omega\tau} + \frac{\tau\psi^2}{4}. \end{aligned}$$

- **Note:** V_P increases with volatility, ψ, and decreases with intensity of trade, ω.

$$\Rightarrow \boxed{\tau_P^* = \frac{2\sigma}{\psi\sqrt{\omega}}} \text{ and } \boxed{V_P^* = \frac{\sigma\psi}{\sqrt{\omega}}}$$

- **Also Note:** The optimal clearing interval, τ^* decreases with both volatility and intensity of trade.

Results: Public Market

$$V_P = \frac{\sigma^2}{\omega\tau} + \frac{\tau\psi^2}{4}$$

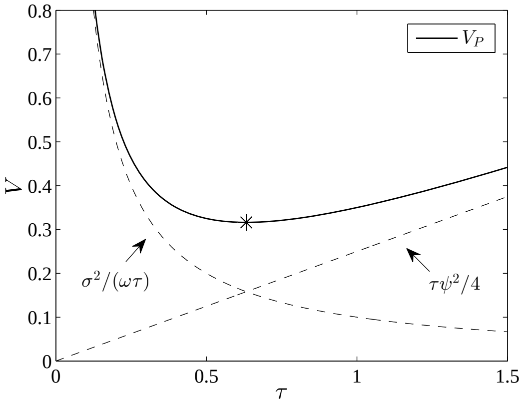


Figure: Liquidity risk, V_P, versus the time between market clearings, τ, without liquidity provider. Parameters: $\psi = 1$, $\sigma = 1$, and $\omega = 10$.

Market Efficiency in Public Market

Problem: mean-reversion in the clearing prices.

Solution: introduce liquidity provider or short-term "speculator" (who would naturally want to step in).

Result: security is more liquid and can clear faster.

Public Market + **Liquidity Provider**

Liquidity Provider

- Include a **single, competitive, risk-neutral liquidity provider**:
  - observes the **average reservation price** ($\bar{r}_t$) directly before the clearing,
  - pushes the clearing price towards her estimate of the equilibrium price ($\hat{m}_t$) $\rightarrow$ **Kalman filter**.

- **Liquidity risk**:

$$V_L = Var[\hat{\mathbf{m}}_{\mathbf{t}} - m_i] \leq V_P.$$

- **Main results**:

$$\boxed{\tau_L^* = \frac{\tau_P^*}{\sqrt{3}}} \text{ and } \boxed{V_L^* < V_P^*}$$

Results: Liquidity Provider

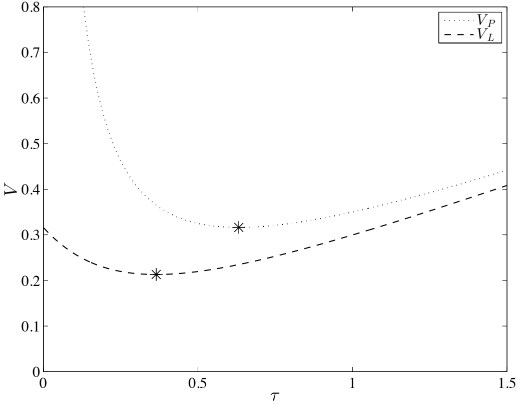


Figure: Liquidity risk in public market (V_P) and in the market with liquidity provider (V_L). Parameters: $\psi = 1$, $\sigma = 1$, and $\omega = 10$.

Public Market + Liquidity Provider + **Multiple Securities**

Multiple Securities

General setup with N securities:

- average reservation prices (observation)

$$\begin{aligned} \bar{\mathbf{r}}_t &= \bar{\mathbf{m}}_t + \mathbf{f}_t, \\ \mathbf{f}_t &\sim N(0, \boldsymbol{\Sigma}). \end{aligned}$$

- average equilibrium prices (system)

$$\begin{aligned} \bar{\mathbf{m}}_t &= \bar{\mathbf{m}}_{t-\tau} + \mathbf{e}_t, \\ \mathbf{e}_t &\sim N(0, \boldsymbol{\Psi}), \end{aligned}$$

where $\bar{\mathbf{r}}$, $\bar{\mathbf{m}}$, $\bar{\mathbf{f}}$, and $\bar{\mathbf{e}}$ are $(N \times 1)$ vectors and $\boldsymbol{\Sigma}$ and $\boldsymbol{\Psi}$ are $(N \times N)$ matrices.

Multiple Securities (Kalman Filter)

- Let $\hat{\mathbf{m}}_t$ be the estimate of $\bar{\mathbf{m}}_t$ based on $\{\bar{\mathbf{r}}_t, \bar{\mathbf{r}}_{t-1}, \bar{\mathbf{r}}_{t-2}, \dots\}$:

$$\begin{aligned} P(\bar{\mathbf{m}}_t | \bar{\mathbf{r}}_t, \bar{\mathbf{r}}_{t-1}, \dots) &\sim N(\hat{\mathbf{m}}_{t-1} + \mathbf{G}_t[\bar{\mathbf{r}}_t - \hat{\mathbf{m}}_{t-1}],\ \mathbf{S}_t), \\ P(\bar{\mathbf{m}}_{t+1} | \bar{\mathbf{r}}_t, \bar{\mathbf{r}}_{t-1}, \dots) &\sim N(\hat{\mathbf{m}}_t, \mathbf{R}_{t+1}), \end{aligned}$$

  where

$$\begin{aligned} \mathbf{G}_t &= \mathbf{R}_t(\mathbf{R}_t + \mathbf{\Sigma})^{-1} \text{ [Kalman gain]}, \\ \mathbf{R}_{t+1} &= \mathbf{S}_t + \mathbf{\Psi}, \\ \mathbf{S}_t &= \mathbf{R}_t - \mathbf{G}_t \mathbf{R}_t. \end{aligned}$$

- Best estimate of $\bar{\mathbf{m}}_t$ is

$$\hat{\mathbf{m}}_t = \hat{\mathbf{m}}_{t-1} + \mathbf{G}_t(\bar{\mathbf{r}}_t - \hat{\mathbf{m}}_{t-1}).$$

- Solution Method: search for convergence of estimation variance to limiting value, i.e., $\mathbf{R}_{t+1} = \mathbf{R}_t$. The result is the Riccati equation:

$$\mathbf{R}(\mathbf{R} + \mathbf{\Sigma})^{-1}\mathbf{R} - \mathbf{\Psi} = 0.$$

Market Security

In order to present analytic results, we treat the multiple security model as a special case of a two security market where the second security is the "market security".

- Idealized assumptions:
  1. **Two securities**, one of them being the **"market security"**,
  2. The market security is **perfectly liquid**,
  3. ρ = **correlation** of equililibrium price changes.
     - $\omega_M \gg 1$,
     - $\mathbf{\Sigma} \approx \begin{pmatrix} \sigma^2/(\omega\tau) & 0 \\ 0 & 0 \end{pmatrix}$.
- **Liquidity risk**:

$$V_M = Var[\hat{\mathbf{m}}_{\mathbf{t}}(\rho) - m_i] \leq V_L.$$

$\Rightarrow$ Market information helps reduce liquidity risk further!

Market Security and the Importance of ρ

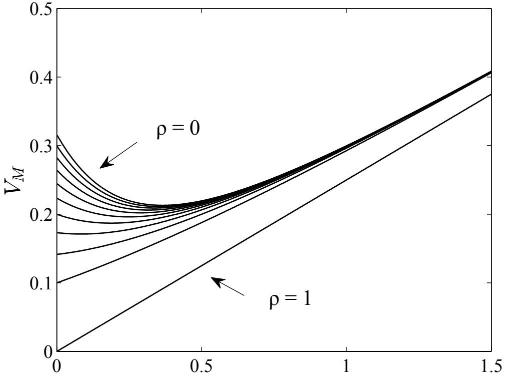


Figure: Liquidity risk, V_M, versus the time between market clearings, τ, with liquidity provider and market information. Parameters: $\psi = 1$, $\sigma = 1$, $\omega = 10$. Values of ρ are between 0 and 1. Critical value: $|\rho^c| = \sqrt{3/4}$.

Results: Comparing Liquidity Risk (V^*)

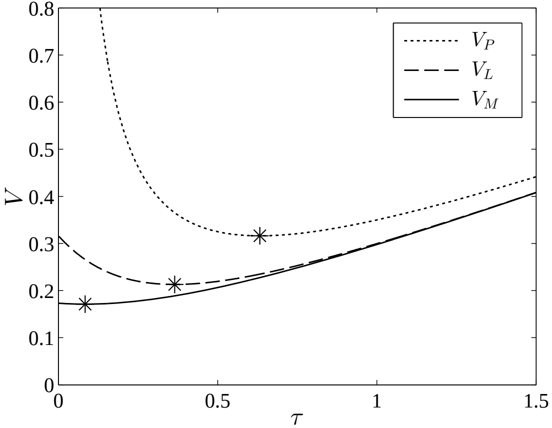


Figure: Comparison of liquidity risk, V, for the different models. Parameters: $\psi = 1$, $\sigma = 1$, $\omega = 10$, and $\rho = 0.84$. *Note*: $\rho < \rho^c$.

Market Security and the Importance of ρ

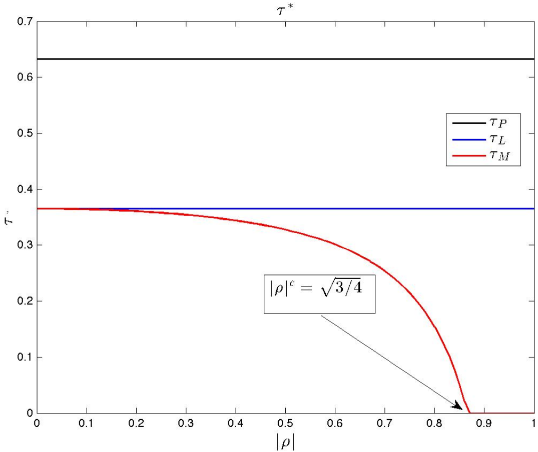


Figure: Speed versus correlation. Here we compare τ^* for the different models. Parameters: $\psi = 1$, $\sigma = 1$, $\omega = 10$. Note: $\tau_M^* = h(\rho)\tau_L^*$.

Estimating Optimal Speed

Estimating τ_M^* (in seconds) for a typical U.S. stock:

Para.	Value	Remarks
ψ	0.0001	Corresponds to annualized volatility of 25%
σ	0.0003	Corresponds to $0.01 quoted spread for a $33 stock
ω	5	Average number of quote changes per second per Tape A/B security (utpplan.com).
ω^{peak}	100	... during peak times.
ρ	0.75	Note: $\rho < \rho^c$

$\Rightarrow \tau^* \approx 0.9$ seconds, $\tau^*_{\text{peak}} \approx 0.2$ seconds.

Conclusions

- Market quality has improved considerably as a result of speed and automation ... but is there a limit to the benefits of speed?
- We model the trading of a security via periodic batch auctions and study how market quality is affected as the clearing frequency is changed.
- The optimal clearing frequency of a security depends on three factors
  $\rightarrow$ volatility
  $\rightarrow$ intensity of trading
  $\rightarrow$ correlation with the market
- At a critical correlation threshold $\rho^c \approx 0.87$, it becomes optimal for a security to continuously clear.
- Rough estimates suggest that a typical U.S. stock should trade at intervals between 0.2 to 0.9 seconds.

Thank you for your attention!

Appendix

$$
\begin{aligned}
V_L &= Var[\hat{\mathbf{m}}_{\mathbf{t}} - m_i], \\
&= \frac{\frac{\tau\psi^2}{2}\left(1+\sqrt{1+\frac{4\sigma^2/\omega}{\tau^2\psi^2}}\right)+4\sigma^2/(\omega\tau)}{2\left(1+\sqrt{1+\frac{4\sigma^2/\omega}{\tau^2\psi^2}}\right)}.
\end{aligned}
$$

$$
\begin{aligned}
V_M &= Var[\hat{\mathbf{m}}_{\mathbf{t}}(\rho) - m_i], \\
&= \frac{(1/2+\Theta)\,\tau\psi^2+(1/2-\Theta)\tau\psi^2\sqrt{1+\frac{4\sigma^2/\omega}{\Theta\tau^2\psi^2}}+4\sigma^2/(\omega\tau)}{2\left(1+\sqrt{1+\frac{4\sigma^2/\omega}{\Theta\tau^2\psi^2}}\right)},
\end{aligned}
$$

with $\Theta = 1 - \rho^2$.