A closed-form execution strategy to target VWAP
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VWAP

- Executions are delegated to brokers
- How can investors measure the performance of brokers?
  - Obtaining what the market has borne is sometimes enough,
  - A popular benchmark is

\[
\text{VWAP} = \frac{\int_0^T S_u \mu_u \, du}{\int_0^T \mu_u \, du},
\]

where \( S_t \) is the midprice and \( \mu_t \) is the volume traded by the entire market.
siblings:
vwap and other volume-based algorithms
Targeting VWAP via POV or POCV

- Brokers may guarantee VWAP and charge a fee
  - Incentive is to outperform VWAP

- Account for price impact:
  - slice parent order over a time window,
  - do ‘bits’ in a volume-based fashion to target VWAP.

- Formulation: set up a performance criteria where the investor/broker
  - seeks to execute a large order over a trading horizon $T$,
  - maximises sales proceeds when liquidating shares,
  - accounts for permanent and temporary impact of trades,
  - tracks (penalises deviations from) POV or POCV,
  - ensures that by the end of trading horizon target quantity is achieved.
Volume patterns

Figure: ORCL traded volume for orders sent to NASDAQ in all of 2013 using 5 minute buckets – heat-map of the data and 25%, 50% and 75% quantiles.
Price Impact of Market Orders
Permanent Impact

We assume a linear relationship between net order-flow and changes in the midprice, thus for every trading day we perform the regression

$$\Delta S_n = b \mu_n + \varepsilon_n,$$

(2)

where

- $\Delta S_n = S_{n\tau} - S_{(n-1)\tau}$ is the change in the midprice,
- $\mu_n$ is net order-flow defined as the difference between the volume of buy and sell MOs during the time interval $[(n-1)\tau, n\tau]$, and
- $\varepsilon_n$ is the error term (assumed normal). In the empirical analysis we choose $\tau = 5$ min.
Figure: Order-Flow and effect on the drift of midprice of INTC.
Temporary Impact

- Assume that temporary price impact is linear in the rate of trading so the difference between the execution price that the investor receives and the midprice is $k \nu$,
- To do this we take a snapshot of the LOB each second,
- Determine the price per share for various volumes (by walking through the LOB),
- Compute the difference between the price per share and the best quote at that time,
- Perform a linear regression.
Figure: An illustration of how the temporary impact may be estimated from snapshots of the LOB using INTC on Nov 1, 2013. The first panel is at 11:00am, the second from 11:00am to 11:01am and the third contains the entire day.
Parameters

Table: Permanent and temporary price impact parameters for Nasdaq stocks, ADV of MOs, average midprice, $\sigma$ volatility (hourly) of arithmetic price changes, mean arrival (hourly) of MOs $\lambda^\pm$, and average volume of MOs $\mathbb{E}[^\eta^\pm]$. Data are from Nasdaq 2013.

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Figure: Price Impact INTC using daily observations for 2013.
Targeting VWAP via POV
The Model

- The investor searches for an optimal speed to liquidate $N$ shares over a trading horizon $T$.
- Inventory
  \[ dQ^\nu_t = -\nu_t \, dt, \quad Q^\nu_0 = N, \quad (3) \]
  where $\nu_t$ is the speed of liquidation.
- The rest of the market trades at speed $\mu^+$ for buy side and $\mu^-$ for sell side.
- And the idea is to target, at every instant in time, a percentage $\rho \in [0, 1]$ of the overall traded volume.
- The overall traded volume, at every instant in time, is
  \[ \mu^+_t + \mu^-_t + \nu_t. \]
The Model, price impact

- Execution price

\[ \hat{S}_t^\nu = S_t^\nu - k \nu_t , \]  \hspace{1cm} (4)

\( k > 0 \) is the temporary impact parameter.

- The midprice satisfies

\[ dS_t^\nu = b \left( \mu_t^+ - (\nu_t + \mu_t^-) \right) dt + dM_t , \hspace{1cm} S_0^\nu = S , \]  \hspace{1cm} (5)

where \( b \geq 0 \) is the permanent impact parameter, and \( M = \{ M_t \}_{0 \leq t \leq T} \) is a martingale (independent of all other processes).

- Cash

\[ dX_t^\nu = \hat{S}_t^\nu \nu_t dt , \hspace{1cm} X_0^\nu = X_0 . \]  \hspace{1cm} (6)
The Model (cont)

Performance criteria is

\[ H^{\nu}(t, x, S, \mu, q) = \mathbb{E}_{t,x,S,\mu,q} \left[ X^{\nu}_{T^{\nu}} + Q^{\nu}_{T^{\nu}} (S^{\nu}_{T^{\nu}} - \alpha Q^{\nu}_{T^{\nu}}) - \varphi \int_{t}^{T^{\nu}} (\nu_u - \chi_u^{\nu})^2 \, du \right] , \]

where \( \mu = \{\mu^+, \mu^-\} \),

\[ \chi^{\nu}_t := \tilde{\rho} \times (\mu^+_t + \mu^-_t + \nu_t) , \]

\[ T^{\nu} = T \land \inf \{ t : Q^{\nu}_t = 0 \} , \]

and her value function

\[ H(t, x, S, \mu, q) = \sup_{\nu \in A} H^{\nu}(t, x, S, \mu, q) , \]

where \( A \) is the set of admissible strategies consisting of \( F \)-predictable processes such that \( \int_0^T |\nu_u| \, du < +\infty, \mathbb{P}\)-a.s.
The value function satisfies
\[ 0 = \left( \partial_t + \mathcal{L}^M + \mathcal{L}^\mu \right) H \]
\[ + \sup_\nu \left\{ (S - k\nu) \nu \partial_x H - \nu \partial_q H + b((\mu^+ - \mu^-) - \nu) \partial_S H - \varphi (\nu - \rho \mu)^2 \right\}, \]
(10)

for \( q > 0 \), with terminal condition
\[ H(t, x, S, \mu, q) = x + q(S - \alpha q), \]
(11)

where \( \mu = \mu^+ + \mu^- \), \( \mathcal{L}^\mu \) represents the infinitesimal generator of \( \mu \), \( \mathcal{L}^M \) the infinitesimal generator of \( M \), and we have introduced the re-scaled parameters
\[ \varphi = \bar{\varphi}(1 - \bar{\rho})^2 \quad \text{and} \quad \rho = \bar{\rho}/(1 - \bar{\rho}). \]
Solving the DPE

The DPE (10) admits the solution

\[ H(t, x, S, \mu, q) = x + qS + h_0(t, \mu) + h_1(t, \mu)q + h_2(t)q^2, \quad q > 0 \]

where

\[ h_2(t) = -\left( \frac{T - t}{k + \varphi} + \frac{1}{\alpha - \frac{1}{2}b} \right)^{-1} - \frac{1}{2}b, \quad (12a) \]

\[ h_1(t, \mu) = \frac{\varphi \rho}{(T - t) + \zeta} \int_t^T \mathbb{E}_{t, \mu} \left[ \mu_s^+ + \mu_s^- \right] ds \]

\[ + \frac{b}{(T - t) + \zeta} \int_t^T \left( (T - s) + \zeta \right) \mathbb{E}_{t, \mu} \left[ \mu_s^+ - \mu_s^- \right] ds, \quad (12b) \]

\[ h_0(t, \mu) = \int_t^T \mathbb{E}_{t, \mu} \left[ \frac{(h_1(t, \mu_s) - 2\varphi \rho (\mu_s^+ + \mu_s^-))^2}{4(k + \varphi)} - \varphi \rho^2 (\mu_s^+ + \mu_s^-)^2 \right] ds, \quad (12c) \]

and the constant

\[ \zeta = \frac{k + \varphi}{\alpha - \frac{1}{2}b}. \]
Optimal liquidation speed (Target POV!)

\[ \nu_t^* = \frac{1}{(T-t) + \zeta} Q_t^{\nu^*} \]  

\[ + \frac{\varphi}{k + \varphi} \rho \left\{ (\mu_t^+ + \mu_t^-) - \frac{1}{(T-t) + \zeta} \int_t^T \mathbb{E}[ (\mu_s^+ + \mu_s^-) | F_t^\mu ] \, ds \right\} \]  

\[ - \frac{b}{k + \varphi} \int_t^T \frac{(T-s) + \zeta}{(T-t) + \zeta} \mathbb{E} [ (\mu_s^+ - \mu_s^-) | F_t^\mu ] \, ds , \quad \forall t < \tau^{\nu^*} \]  

where \( F_t^\mu \) denotes the natural filtration generated by \( \mu \), and \( \nu_t^* = 0 \) for \( t \geq \tau \), is the admissible optimal control we seek.
Optimal liquidation speed (targeting POV)

- The first component is TWAP-like.

- The second component consists of targeting instantaneous total order-flow plus a weighted average of future expected total order-flow. Although the POV target is $\rho \mu_t$, the strategy targets a lower amount since $\frac{\varphi \rho}{k+\varphi} \leq \rho$, where equality is achieved if the costs of missing the target are $\varphi \to \infty$ and $k$ remains finite, or there is no temporary impact $k \downarrow 0$.

- The last component acts to correct her trading based on her expectations of the net order-flow from that point in time until the end of the trading horizon. When there is a current surplus of buy trades she slows down her trading rate to allow the midprice to appreciate before liquidating the rest of her order.
From POV to VWAP

Investor must liquidate all shares. Do this by letting $\alpha \rightarrow \infty$

$$\lim_{\alpha \rightarrow \infty} \nu_t^* = \frac{1}{T-t} Q_t^\nu$$

$$+ \frac{\varphi}{k + \varphi} \left( \rho \mu_t - \frac{1}{T-t} \int_t^T \mathbb{E}[\mu_s^+ + \mu_s^- | \mathcal{F}_t^\mu] \, ds \right)$$

$$- \frac{b}{k + \varphi} \int_t^T \frac{(T-s)}{T-t} \mathbb{E}[\mu_s^+ - \mu_s^- | \mathcal{F}_t^\mu] \, ds,$$

and follow this by taking the limit $\varphi \rightarrow \infty$, resulting in

$$\lim_{\varphi \rightarrow \infty} \lim_{\alpha \rightarrow \infty} \nu_t^* = \frac{1}{T-t} Q_t^\nu + \rho \left( (\mu_t^+ + \mu_t^-) \right)$$

$$- \frac{1}{T-t} \int_t^T \mathbb{E} \left[ (\mu_s^+ + \mu_s^-) \mid \mathcal{F}_t^\mu \right] \, ds.$$
Targeting VWAP via POCV
Accumulated Volume

- Accumulated volume $V$ of orders, excluding the agent’s, is
  \[ V_t = \int_0^t (\mu_u^+ + \mu_u^-) \, du. \]

- The investor’s performance criteria is now modified to
  \[ H^\nu(t, x, S, \mu, V, q) = \mathbb{E}_{t,x,S,\mu,V,q} \left[ X_{T_{\nu}}^\nu + Q_{T_{\nu}}^\nu (S_{T_{\nu}}^\nu - \alpha Q_{T_{\nu}}^\nu) \right. \]
  \[ \left. - \bar{\varphi} \int_t^{T_{\nu}} \left( (\sqrt{\gamma} - Q_u^\nu) - \bar{\rho} (V_u + (\sqrt{\gamma} - Q_u^\nu)) \right)^2 du \right] \]
  and $T_{\nu} = T \land \inf\{ t : Q_t^\nu = 0 \}$.

- The investor’s value function is
  \[ H(t, x, S, \mu, V, q) = \sup_{\nu \in \mathcal{A}} H^\nu(t, x, S, \mu, V, q). \]
The value function should satisfy the DPE

\[ 0 = \left( \partial_t + \mathcal{L}^M + \mathcal{L}^{\mu, V} \right) H - \varphi \left( (\mathcal{M} - q) - \rho V \right)^2 \]
\[ + \sup_{\nu} \left\{ b ((\mu^+ - \mu^-) - \nu) \partial_s H + (S - k \nu) \nu \partial_x H - \nu \partial_q H \right\}, \] (18)

for \( q > 0 \), subject to \( H(T, x, S, y, q) = x + q (S - \alpha q) \) and

\[ \varphi = \tilde{\varphi} (1 - \tilde{\rho})^2 \quad \text{and} \quad \rho = \tilde{\rho} / (1 - \tilde{\rho}). \]

Here, \( \mathcal{L}^{\mu, V} \) denotes the infinitesimal generator of the joint process \((\mu_t, V_t)_{0 \leq t \leq T}\).
The DPE (18) admits the solution

\[ H(t, x, S, \mu, V, q) = x + q S + h_0(t, \mu, V) + h_1(t, \mu, V) q + h_2(t) q^2, \quad q > 0 \quad (19) \]

where

\[ h_2(t) = -\sqrt{k \varphi} \frac{\gamma e^{\xi (T-t)} + e^{-\xi (T-t)}}{\gamma e^{\xi (T-t)} - e^{-\xi (T-t)}} - \frac{1}{2} b, \quad (20a) \]

\[ h_1(t, \mu, V) = 2 \varphi \int_t^T \ell(u, t) (\mathfrak{M} - \rho \mathbb{E}_{t, \mu, V} [V_u]) \, du \]

\[ + b \int_t^T \ell(u, t) \mathbb{E}_{t, \mu, V} [(\mu^+_u - \mu^-_u)] \, du, \quad (20b) \]

\[ h_0(t, \mu, V) = \int_t^T \mathbb{E}_{t, \mu} \left[ \frac{1}{4k} (h_1(u, \mu_u, V_u))^2 - \varphi (\mathfrak{M} - \rho V_u)^2 \right] \, du, \quad (20c) \]

the function

\[ \ell(u, t) := \frac{\gamma e^{\xi (T-u)} - e^{-\xi (T-u)}}{\gamma e^{\xi (T-t)} - e^{-\xi (T-t)}}, \quad (20d) \]

and the constants

\[ \xi = \sqrt{\frac{\varphi}{k}}, \quad \text{and} \quad \gamma = \frac{\alpha - \frac{1}{2} b + \sqrt{k \varphi}}{\alpha - \frac{1}{2} b - \sqrt{k \varphi}}. \]
Moreover, the trading speed, for \( q > 0 \), is given by

\[
\nu_t^* = \xi \frac{1 + \gamma}{\gamma e^{\xi(T-t)} - e^{-\xi(T-t)}} Q_t^{\nu^*}
\]

\[
- \xi^2 \int_t^T \ell(u, t) \left\{ \left( \mathfrak{N} - Q_t^{\nu^*} \right) - \rho \mathbb{E} \left[ V_u \mid \mathcal{F}_t^{\mu, V} \right] \right\} \, du \quad \text{(21b)}
\]

\[
- \frac{b}{2k} \int_t^T \ell(u, t) \mathbb{E} \left[ (\mu_u^+ - \mu_u^-) \mid \mathcal{F}_t^{\mu, V} \right] \, du, \quad \forall t < \tau^{\nu^*}. \quad \text{(21c)}
\]

And the optimal inventory path

\[
Q_t^{\nu^*} = \ell(t, 0) \mathfrak{N}
+ \int_t^T \int_u^T \ell(u, s) \left\{ \frac{\varphi}{k} \left( \mathfrak{N} - \rho \mathbb{E} \left[ V_s \mid \mathcal{F}_u^{\mu, V} \right] \right) + \frac{b}{2k} \mathbb{E} \left[ (\mu_s^+ - \mu_s^-) \mid \mathcal{F}_u^{\mu, V} \right] \right\} \, ds \, du. \quad \text{(22)}
\]
Interpreting liquidation speed

- (21a) is an Almgren-Chriss like strategy which approaches TWAP near maturity.

- (21b) corrects for the weighted average of the difference between what the investor has liquidated so far \((N - Q_{t^*})\) and her future targeted volume \(\rho \mathbb{E} \left[ V_u \mid \mathcal{F}_t^{\mu, V} \right] \).

- (21c) contributes a weighted average of future net order-flow and accounts for the permanent impact on the midprice. If future net order-flow is expected to be buy heavy, then investor slows down.

- Finally, as \(t \to T\) the second and third terms become negligible and the investor ignores order-flow entirely and instead focuses on completing her trades.
Targeting VWAP

Let $\alpha \to \infty$, so that the investor ensures that she completely liquidates her position by the terminal time, $\gamma \to 1$, therefore $\ell(u, t) \to \frac{\sinh(\xi (T-u))}{\sinh(\xi (T-t))}$ and so we have

$$\lim_{\alpha \to \infty} \nu^*_t = \xi \frac{Q_t^\nu^*}{\sinh(\xi (T-t))}$$

$$- \xi^2 \int_t^T \frac{\sinh(\xi (T-u))}{\sinh(\xi (T-t))} \left\{ (\mathcal{N} - Q_t^\nu^*) - \rho \mathbb{E} \left[ V_u \mid \mathcal{F}_t^{\mu, V} \right] \right\} du$$

$$- \frac{b}{2k} \int_t^T \frac{\sinh(\xi (T-u))}{\sinh(\xi (T-t))} \mathbb{E} \left[ (\mu_u^+ - \mu_u^-) \mid \mathcal{F}_t^{\mu, V} \right] du.$$
Strategy Performance

Assume that other market participants trades follow

\[ d\mu^+_t = -\kappa^+ \mu^+_t \, dt + \eta^+_{1+N^+_t} \, dN^+_t, \quad (23a) \]

\[ d\mu^-_t = -\kappa^- \mu^-_t \, dt + \eta^-_{1+N^-_t} \, dN^-_t, \quad (23b) \]

where

- \( \kappa^\pm \geq 0 \) are the mean-reversion rates,
- \( N^+_t \) and \( N^-_t \) are independent homogeneous Poisson processes with intensities \( \lambda^+ \) and \( \lambda^- \), respectively,
- \( \{\eta^+_1, \eta^+_2, \ldots\} \) are non-negative i.i.d. random variables with distribution function \( F \), with finite first moment, independent from all processes.
- In addition, we require \( \kappa^\pm > \lambda^\pm \mathbb{E}[\eta^\pm_1] \) to ensure that \( \mu^\pm \) remain bounded \( \mathbb{P}\)-a.s..
VWAP as target

Compute VWAP as

\[
\text{VWAP} = \frac{\int_0^T S_u^\nu \left( \mu_u^+ + \mu_u^- + \nu_u \right) du}{\int_0^T \left( \mu_u^+ + \mu_u^- + \nu_u \right) du}, \tag{24a}
\]

while the execution price is computed as

\[
\text{Exec. Price} = \frac{X_T^\nu}{\mathcal{N}} = \frac{\int_0^T \hat{S}_u^\nu \nu_u du}{\mathcal{N}}. \tag{24b}
\]
Table: The statistics of the execution price, VWAP, and relative error (computed as (Exec. Price - VWAP)/VWAP for each simulation) and reported in basis points (i.e., $\times 10^4$).

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POCV-VWAP performance

Table: The statistics of the execution price, VWAP, and relative error (computed as \((\text{Exec. Price} - \text{VWAP})/\text{VWAP}\) for each simulation) and reported in basis points (i.e., \(\times 10^4\)). **For POCV we set** \(\varphi = 10^5 \times k\)

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<td>$40.61</td>
<td>11.0</td>
<td>$37.92</td>
</tr>
<tr>
<td>95%</td>
<td>$40.73</td>
<td>31.9</td>
<td>$37.96</td>
</tr>
<tr>
<td>%(t: v_t^* &lt; 0)</td>
<td>0.83%</td>
<td>0.013%</td>
<td>0.55%</td>
</tr>
</tbody>
</table>