Order Flows and Execution Costs

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joint w/Kyle Bechler, Sebastian Jaimungal

Optimal Execution Modeling

Typical OE models are "reduced-form"

- Ingredients include
  - *reference price* (e.g. Brownian motion)
  - realized price (e.g. reference + offset) ;
  - price impact specification (e.g. transient linear impact w/exponential resilience);
  - liquidity state, etc.,
- Trading is continuous: diffusion setting
- Scheduling time-scale: 5–10 minutes

Data

- Actual data is based on the Limit Order Book
- Full information about all ticker events and snapshots of the book
- Mathematically represented in terms of point processes or queues
- Time-scale of $\ll 1$ second

How to reconcile/connect these frameworks?

OE vs LOB

- Existing OE models are **oblivious** to LOB states

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- Typical OE algorithms are "open-loop" – calibrate and forget
- Wish to come up with a higher-level picture of LOB at the time-scale of the scheduler to build a "closed-loop" system:
- LOB $\rightarrow$ Liquidity/Market State $\rightarrow$ OE algorithm

What is Liquidity?

Challenges:

- How to measure liquidity of the LOB?
- How to measure price impact via the LOB?
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- Static snapshots of the LOB have been extensively studied
- For example short-term effects from book imbalance and “micro price” which in turn drives mid-price movements
- Liquidity can be statically measured in terms of
  - Bid-Ask Spread
  - Depth
  - Execution Cost
- But these features are too detailed/fleeting at the OE scale
- Dynamic view is provided by the order flows (time-scale of minutes)

Outline

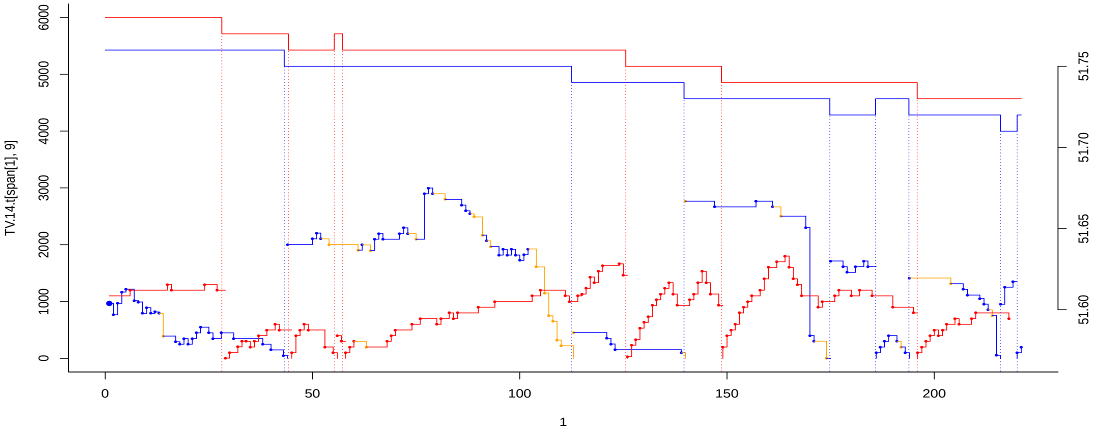
- Aim: include LOB info into OE schedulers
- **PART I:** Initial empirical analysis with a view to explain what is (statistically) important for order execution
- **PART II:** A simple model to incorporate order flow into OE
- For another take, see *Incorporating Order-Flow into Optimal Execution* by Cartea and Jaimungal (on SSRN, 2015)

Order Flow

- Need to aggregate LOB data to understand the more persistent features relevant for OE
- Moving from microstructure to the meso-scale (complements line of research that aggregates from point processes to diffusions)
- Aggregation naturally leads to consideration of Order Flows
- Related to metrics such as *Market Toxicity* or *Liquidity State*
- There is a lot of evidence that flows are persistent (hours or even days)

$\Rightarrow$ Trading shows up as a disturbance of order flow – links to price impact

LOB Evolution



Tick-by-tick evolution of LOB queues at the touch. Market orders are in orange. Mid price is plotted at the top (TEVA, May 3, 2011)

Main Aspects of Order Flow

- Must separately treat Limit Orders and Market Orders
- Consequently, must also separately treat each side of the book (cross-effects)
- Aggregating by time-slicing or volume-slicing?
- Order Flow has intrinsic links to price behavior: mechanical (slippage) and informational (price discovery)
- Economically, OF is tied to informational costs:
  - Participants fear that other traders are better informed
  - Noise traders trade in all directions; informed traders are deliberate
  - $\Rightarrow$ When order flow is toxic market makers provide less liquidity
  - Example of a metric: VPIN by Easley, Lopez de Prado, O'Hara (2012abcd) – ties toxicity to flow imbalance = ratio of noise/informed traders

Empirical Order Flow

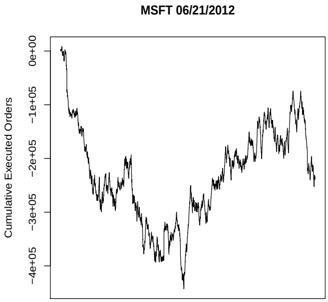


Figure: Cumulative net volume of MSFT executed trades over June 21, 2012

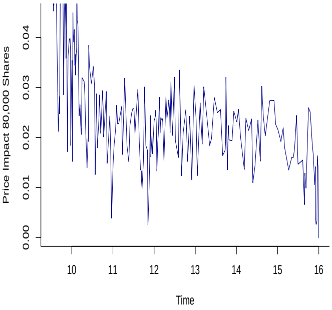


Figure: Price Impact for MSFT over same day

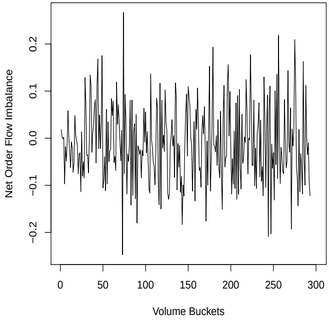


Figure: MSFT Net Order Flow over buckets of $1,000,000$

Notation

- LOB consists of pairs (p_t^j, v_t^j) indexed from the top queue.
- Mid-price: $P^M(t) := (p_1^A(t) + p_1^B(t))/2$
- Market orders (MO) and Limit orders (LO) form asynchronous time-series: time-stamp T_i, signed volume O_i, limit price S_i
- To synchronize & aggregate, construct **buckets**
- Slice the ticker according to intervals defined by (τ_k) (e.g. equal volume)
- Market buys: $VM_k^A = \sum_{i:\tau_k \le T_i^M \le \tau_{k+1}} O_i^M 1_{\{O_i^M > 0\}}$.
- $MI_k = VM_k^A - VM_k^B$
- Contemporaneous ask limit orders at the touch:

$$VL_k^A = \sum_{i:\tau_k \le T_i^L \le \tau_{k+1}} O_i^L 1_{\{S_i^L = p_1^B(T_i^L)\}}$$

How to Model Price Impact

- Permanent Price Impact
- Transient Price Impact
  1. Large orders walk the book (depends on the LOB shape)
  2. Liquidity providers replenish the queues and the book bounces back (resiliency)

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  1. Large orders walk the book (depends on the LOB shape)
  2. Liquidity providers replenish the queues and the book bounces back (resiliency)
- Obizhaeva/Wang: Realized execution price: $P_t = \tilde{P}_t + D_t$
- Unaffected $\tilde{P}$ is arithmetic BM
- Resilience D decays exponentially to zero
- Each term is linearly impacted by trading:

$$\tilde{P}_t = \tilde{P}_0 + \lambda MI_t + \sigma W_t, \qquad D_t = e^{-\rho t} D_0 + \nu MI_t.$$

- Assuming Gaussian D_0, gives a conditional Gaussian distribution for ΔP, linear in MI_t

LOB Perspective

- Immediate effect of a trade is deterministic given the LOB state. Cannot distinguish various sub-effects
- Difficult to measure long-term effects
- Have to describe the dynamic impact on LOB – eg arrival/cancellation rates

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Large Tick Assets

- Concentrate on liquid US equities
- Tick size is \$0.01; asset price is \$20-\$50
- Spread is almost always 1 tick
- The two queues at the touch are deep ($> 10AES$)
- Orders rarely “walk the book” and there is no *shape* to the LOB
- Typical immediate price impact is zero
- examples: MSFT, INTC, TEVA

Price Impact of Market Flow

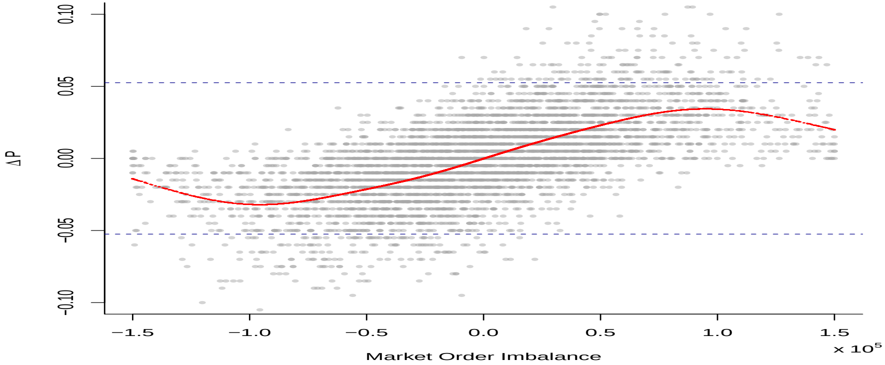


Figure: Relationship between mid-price change ΔP and net market order flow MI across **volume-aggregated** slices of 150,000 shares (1/80 of ADV) for MSFT during Jan-Mar 2011. The average price change was computed using Loess regression.

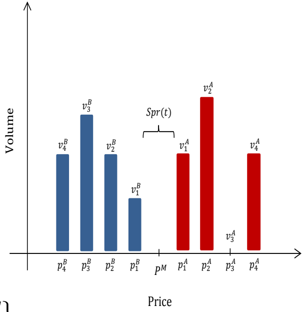
Explaining Price Movement based on Flows

- One sided flow consumes liquidity asymmetrically so the price moves
- Lots of buying $\rightarrow$ price goes up
- (Extreme MI typically anticipates low ΔP – observed S-shape)
- BUT: Even under balanced flow the price can make significant moves
- Good state: $MI = 50,000$ and $\Delta P = 0$.
- Bad state: $MI = 20,000$ and $\Delta P = 0.04$ large price move based on small net volume.
- Potential causes of such weak LOB resilience: are:
  1. Illiquid LOB
  2. Cancellations of limit orders (vanishing liquidity)

Static Liquidity of the LOB

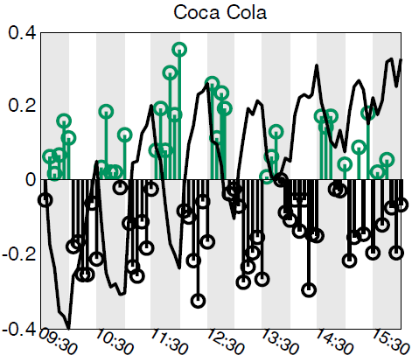
- Spread: $p_1^A - p_1^B$. Always 1-2 ticks – not indicative
- Static volume imbalance: $|v_1^A(t) - v_1^B(t)|$ - jumps each time the mid-price moves
- Depth: $\sum_i v_i^j$ (e.g. top 2 levels of the book)
- Price Impact: total effect relative to touch if instantaneously sell M shares:

$$PI_i := M^{-1} \sum_{i=1}^{i^M} v_i (S_i - p_1) + \bar{v_{i^M}} (S_{i^M} - p_1), \; i^M = \min\{i : \sum_{j<i} v_j > M\}$$



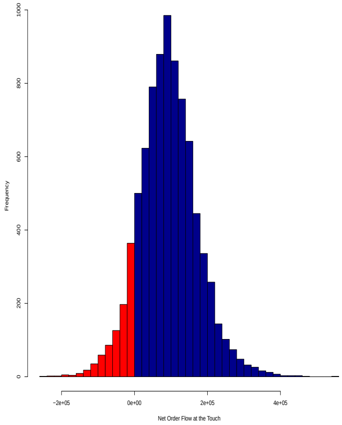
Vanishing Liquidity

- Static depth can also be misleading
- One-sided market flows can induce wild swings in additions/cancellations and generate a lot of mid-price volatility
- This depends on the correlation between VL amd MO and further latent regimes
- Picture from Lehalle et al (2012) – July 19, 2012



Limit Order Flows

- Limit order flow should generally be positive (more additions than cancellations) – counteracts market orders
- Instances where VL is negative are highly indicative of major price moves
- This is not surprising from the LOB mechanics
- But shows that time series of limit flows is crucial to identify reduced liquidity/toxicity



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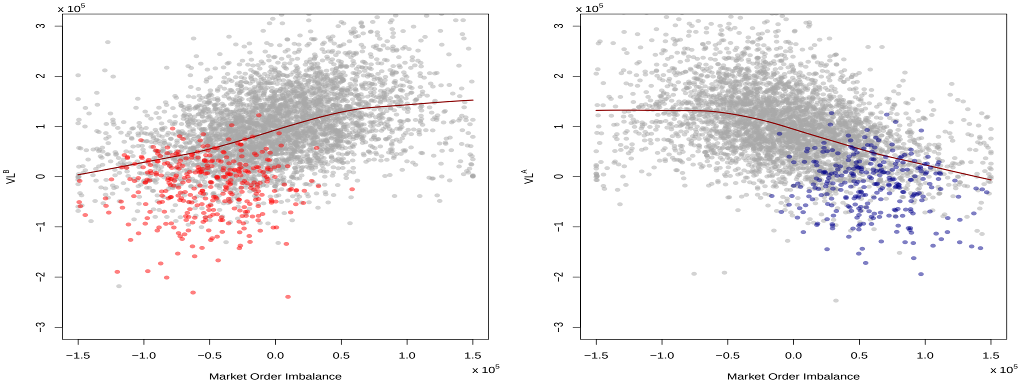
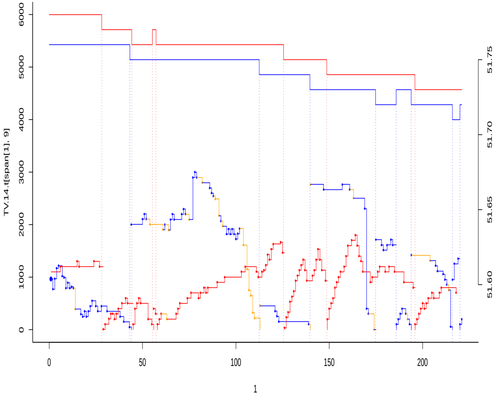


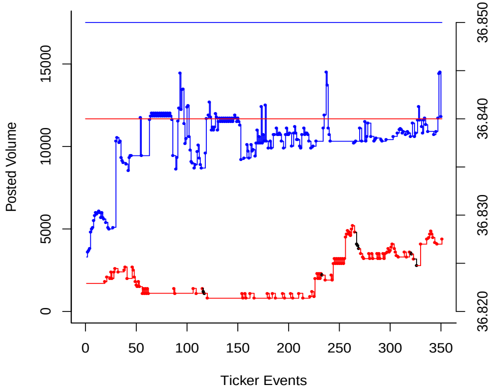
Figure: Relationship between net market order flow *MI* and concurrent top-level limit flow VL^j for MSFT during Jan-Mar 2011. Left: Bid, right: Ask. Highlighted points indicate buckets where the price change *exceeded 5 ticks*.

Can We Predict Scarce Liquidity?

- Generalized additive model to explain ΔP
- (GAM) Logistic regression to predict large price moves aka scarce liquidity
- Most important predictor is MO flow
- In both cases LO have much more explanatory power than price impact or book imbalance.
- This is not actionable (LO is contemporaneous)
- There is also some memory of scarce liquidity (based on ACF for volume-sliced time series)

Dependence Between Limit & Market Orders



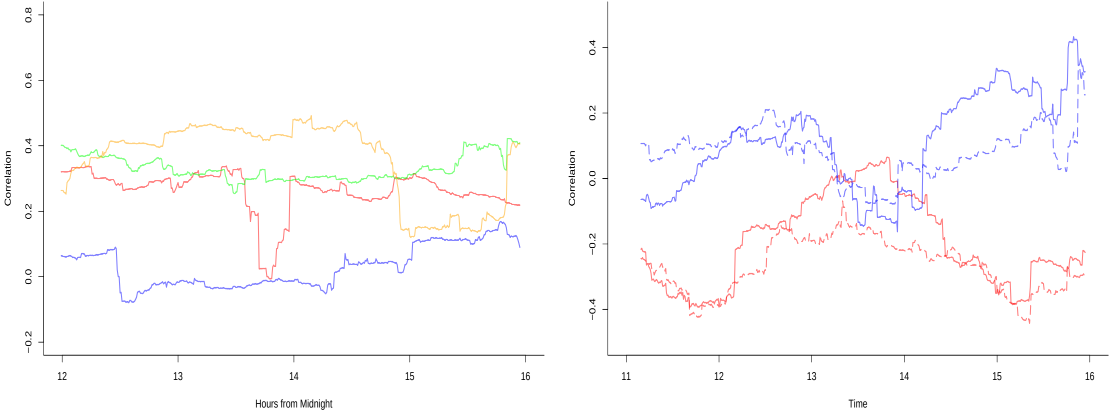


$-$*ve* Corr (TEVA) $+$*ve* Corr (ORCL)

Dependence Between Limit & Market Orders

- Could represent via regimes: resilient (*VL* and *MI* positive correlation)
- Scarce: negative correlation (price fade/"front running")
- Limit order flow is a key factor to explain price move conditional on MO
- **Open Problem:** How to define model-free dependence of 4 asynchronous marked point processes?

Correlation Between Limit & Market Orders



Smoothed contemporaneous correlation between *VL* and *MI* over the past 2.5 hours using 30-second buckets (4 days in 2011). Left: TEVA. Right: MSFT. Solid: bid-side. Dashed: ask-side

Another View of Persistency in Liquidity

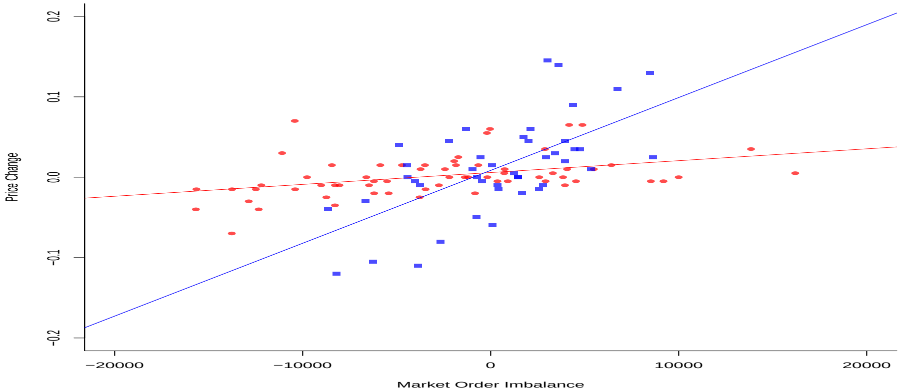


Figure: ΔP against MI and accompanying linear best fit lines for Feb 10, 2011 and March 9, 2011 in TEVA. Higher correlation implies stronger MI impact (also see Cartea & Jaimungal, 2015).

Current Work

- Quantify observed stylized features using statistical models (nonparametric regression)
- Suggest some ways of measuring the temporal correlation between limit and market order flows
- Investigate existence of statistical regimes (hidden Markov factors??)

Part II: Incorporating OF into OE

Two main concerns when executing a large order:

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Two main concerns when executing a large order:

1. Price Impact
   - Spatial effect: consume liquidity in terms of standing limit orders
2. Information Leakage
   - Temporal effect: executed trade appears on the ticker tape
   - Other traders react and adjust their behavior (eg “front-running”) – will receive worse price in the future

AIM: integrate these ideas into a dynamic optimal execution framework

Treat order flow as a (controlled) state variable

Literature Landscape

- Work with continuous trading rates
- Assume a parametric form for inventory risk (Gatheral-Schied)
- Risk neutral agents (Gueant & Lehalle, ...)
- No fill risk (in contrast to Cartea & Jaimungal, Gueant et al, Bayraktar-L, ...)
- Treat only market orders (in the future: hybrid models like in Guilbaud & Pham, Carmona & Webster, ...)
- Postulate one-sided trading (in the future: allow two-sided to consider potential for market manipulation)
- Flat (constant depth) LOB $\rightarrow$ linear price impact
- Flat LOB $\rightarrow$ quadratic instantaneous execution cost
- (Order book resilience (Obizhaeva & Wang, Alfonsi et al))
- (Empirical evidence: Farmer, Bouchaud, ...)

Contributions

- Introduce order flow imbalance as a state variable
- Suggest a simple mechanism for informational costs (inspired by ELO12) – temporal impact beyond usual spatial impact
- Will also endogenize the execution horizon T – can trade faster under favorable conditions
- Derive closed-form approximate strategies for the resulting optimization problem

Execution Model

- Inventory x_t: liquidate x_0
- $t \mapsto x_t$ is absolutely continuous; trading rate is α_t
- $dx_t = -\alpha_t dt$
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- Order flow imbalance Y_t:
  - Mean-reverting to zero
  - Stationary in long-run
  - Affected by execution algorithms
- $Y_t > 0$: buyers-market; $Y_t < 0$: sellers market

Order Flow Dynamics

- Unaffected order flow: $dY_t^0 = -\beta Y_t^0 dt + \sigma\, dW_t$
- With execution:
$$dY_t = (-\beta Y_t - \phi(\alpha_t))\, dt + \sigma\, dW_t$$
- i.e. $Y_t = Y_t^0 + \int_0^t e^{-\beta(t-s)} \alpha_s ds$
- Typical cases:
  - $\phi(\alpha) = \phi_t$ (deterministic information cost)
  - $\phi(\alpha) = \eta\alpha$ (linear in trading rate)
- Assume that flow is independent of price (empirical relationship is not clear) - more on this later

Optimization Problem

- Objective: $v(x,y) := \inf_{\alpha\in\mathcal{A}} \mathbb{E}_{x,y}\left[\int_0^{T_0} g(\alpha_s) + \lambda(x_s) + \kappa Y_s^2 ds\right]$
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- κY^2 : information cost
- Unbalanced order flow: Higher liquidity costs

HJB Equation

- $0 = -\beta Y v_Y + \frac{1}{2}\sigma^2 v_{YY} + \inf_{\alpha \geq 0}\{g(\alpha) - \alpha v_x - \phi(\alpha) v_Y\} + \kappa Y^2 + \lambda(x)$
- Finite-fuel boundary condition: $v(0, y) = 0$ for all y
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- Hard to understand the structure
- Positivity constraint on α is challenging
- To gain insights: build approximating problems by
  - (i) solving the fixed-horizon problem
  - (ii) optimizing over T

Fixed Horizon Problem

$$u(T,x,y) = \inf_{(\alpha_t)\in\mathcal{A}(T,x)} \mathbb{E}_{x,y}\left[\int_0^T \alpha_s^2 + \lambda(x_s^\alpha) + \kappa Y_s^2\, ds\right]$$

HJB equation becomes

$$u_T = \frac{1}{2}\sigma^2 u_{yy} + \kappa y^2 + \lambda(x) - \beta y u_y + \inf_\alpha \left\{\alpha^2 - \alpha u_x - \eta\alpha u_y\right\} \tag{1}$$

Singular boundary condition: $\lim_{T\downarrow 0} u(T,x,y) = \infty$ if $x \neq 0$

Proposition

The solution of (1) *has the form*

$$u(T,x,y) = x^2 A(T) + y^2 B(T) + xyC(T) + D(T), \tag{2}$$

where A, B, C, D solve a matrix Riccati ordinary differential equation.

Note: Riccati equations parameterized in terms of time-to-maturity τ

Execution Speed

- The corresponding optimal rate of liquidation is

$$\alpha_t^D = \frac{x_t(2A(\tau) + \eta C(\tau)) + Y_t(C(\tau) + 2\eta B(\tau))}{2}.$$

- Execution rate is linear in x_t and in Y_t (generalizes Almgren-Chriss)
- The Proposition only treats the unconstrained case $\alpha \in \mathbb{R}$: if T is large relative to x_0 or Y_t is negative enough then $\alpha^D < 0$
- As $t \to T$, the dynamic trading rate stabilizes, resembling a VWAP strategy.

Myopic Strategies $(\phi(\alpha) = \phi_t)$

- Suppose agent myopically optimizes only against price impact:
  $u^M(T,x,y) := \inf_{(x_t)} \left(\int_0^T \dot{x}_s^2 + \lambda(x_s) ds \right) + \int_0^T \kappa \mathbb{E}_y[Y_s^2] ds =: \mathcal{I} + \mathcal{O}$
- If $\lambda(x) = cx^2$ solution is $\left\{ \begin{aligned} x_t^{MH} &= \frac{x \sinh(\sqrt{c}(T-t))}{\sinh(\sqrt{c}T)} \\ \alpha_t^{MH} &= \frac{\sqrt{c}x \cosh(\sqrt{c}(T-t))}{\sinh(\sqrt{c}T)} \end{aligned} \right\}$
- Now $\phi_t = \eta \alpha_t^{MH}$ is the above deterministic function of $t \to Y_t$ is Gaussian with known moments
- $\mathcal{I}^{MH}(T,x) = \sqrt{c}x^2 \coth(\sqrt{c}T)$
  $\mathcal{O}^{MH}(T,x,y) = \kappa \int_0^T \left(ye^{-\beta t} - \int_0^t e^{-\beta(t-s)} \eta \alpha_s^{MH} ds \right)^2 + \frac{\sigma^2}{2\beta}(1 - e^{-2\beta t})\, dt$

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- Similarly have closed-form expressions for cases $\lambda(x) = cx$ (Quadratic) and $\lambda(x) = 0$ (VWAP)
- Next step: Take existing closed-form expressions u^M and optimize over T

Optimizing the Horizon

- $T^* = \arg\min_T \; u(T, x, y)$
- Lemma: $T^* \in (0, \infty)$ (closed-form for $T \mapsto u(T, x, y)$)
- Open-loop (static): find T^* at the outset and implement $\alpha_t(T^*(x, y), x_t, Y_t)$

Optimizing the Horizon

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- Open-loop (static): find T^* at the outset and implement $\alpha_t(T^*(x, y), x_t, Y_t)$
- Closed-loop (dynamic): continuously recompute T^*:

$$\tilde{\alpha}_t^M(x, y) := \alpha^M(T^*(x_t, Y_t), x_t, Y_t)$$

- Realized horizon $T_0(x, y)$ becomes random
- Dynamically recomputing T^* - adapt to changing Y_t without the indefinite horizon finite-fuel problem

Next up: we show these are in fact good approximations!

Comparison of costs

	Optimal Execution Cost				
	v	$\tilde{u}^D$	$\tilde{u}^{ML}$	u^D	u^{ML}
$\mathbb{E}[J(\alpha)]$	**4.257**	4.264	4.317	4.483	4.547
$SD(J(\alpha))$	1.50	1.45	1.39	1.77	1.84
$\mathbb{E}[T_0]$	3.87	3.70	3.48	3.43	3.43

Legend:

- v: directly from the HJB **pde** (fully numerical)
- $\tilde{u}(x, y)$: **closed-loop** optimization of T^*; T_0 is random
- $u(T^*, x, y)$: **static** optimization $T_0 = T^*$
- u^{ML}: **VWAP** on $[0, T^*]$
- ^{D} superscript refers to dynamic strategies based on Proposition 1 (**Riccati** eqns)

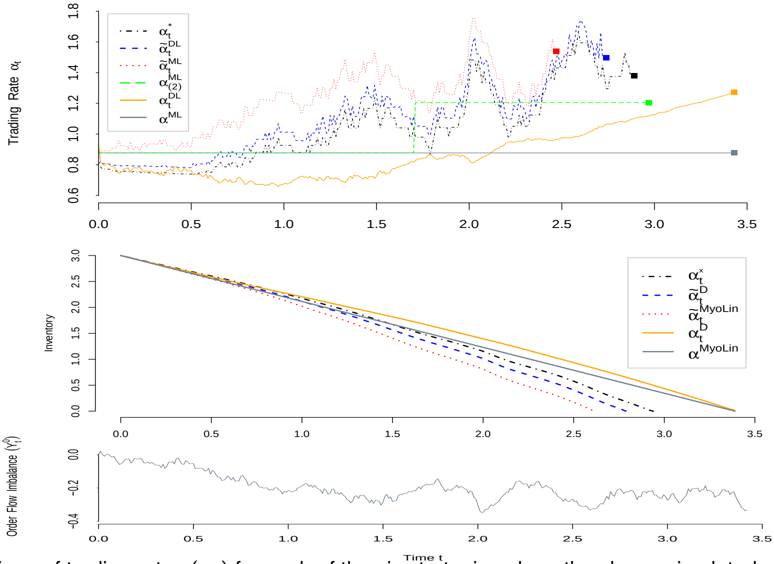

Figure: Comparison of trading rates (α_t) for each of the six strategies along the shown simulated path of (Y_t^0) (The realized (Y_t) depends on the strategy chosen).

Execution Paths

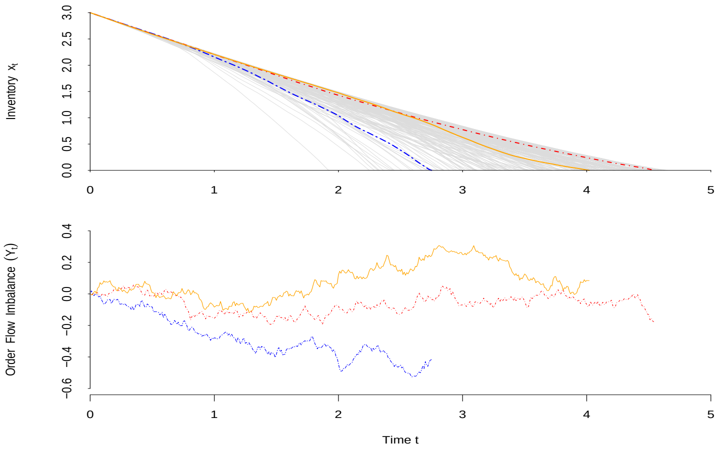

Figure: Top: 200 simulated trajectories from strategy $\tilde{\alpha}_t^D$. Highlighted are three trajectories resulting from different Y_t-paths. Bottom: Corresponding realizations of $t \mapsto Y_t$.

Extensions

- How to estimate/filter/infer Y_t from LOB data?
- information leakage $\phi(\alpha)$ depends on Y_t?
- Correlated S_t and Y_t: dependence between price movement and order flow

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Thank You!

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