Order Flows and Execution Costs

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Optimal Execution Modeling

Typical OE models are “reduced-form”

- Ingredients include
  - reference price (e.g. Brownian motion)
  - realized price (e.g. reference + offset)
  - price impact specification (e.g. transient linear impact w/exponential resilience)
  - liquidity state, etc.,

- Trading is continuous: diffusion setting
- **Scheduling** time-scale: 5–10 minutes
Data

- Actual data is based on the Limit Order Book
- Full information about all ticker events and snapshots of the book
- Mathematically represented in terms of point processes or queues
- Time-scale of $\ll 1$ second

How to reconcile/connect these frameworks?
OE vs LOB

- Existing OE models are **oblivious** to LOB states
  
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- Typical OE algorithms are “open-loop” – calibrate and forget

- Wish to come up with a higher-level picture of LOB at the time-scale of the scheduler to build a “closed-loop” system:

- LOB $\rightarrow$ Liquidity/Market State $\rightarrow$ OE algorithm
What is Liquidity?

Challenges:

- How to measure liquidity of the LOB?
- How to measure price impact via the LOB?
- How to reconcile Market Orders and Limit Orders in LOB?
What is Liquidity?

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- Static snapshots of the LOB have been extensively studied
- For example short-term effects from book imbalance and “micro price” which in turn drives mid-price movements
- Liquidity can be statically measured in terms of
  - Bid-Ask Spread
  - Depth
  - Execution Cost
- But these features are too detailed/fleeting at the OE scale
- Dynamic view is provided by the order flows (time-scale of minutes)
Outline

- **Aim**: include LOB info into OE schedulers
- **PART I**: Initial empirical analysis with a view to explain what is (statistically) important for order execution
- **PART II**: A simple model to incorporate order flow into OE
- For another take, see *Incorporating Order-Flow into Optimal Execution* by Cartea and Jaimungal (on SSRN, 2015)
Order Flow

- Need to aggregate LOB data to understand the more persistent features relevant for OE
- Moving from microstructure to the meso-scale (complements line of research that aggregates from point processes to diffusions)
- Aggregation naturally leads to consideration of Order Flows
- Related to metrics such as Market Toxicity or Liquidity State
- There is a lot of evidence that flows are persistent (hours or even days)

⇒ Trading shows up as a disturbance of order flow – links to price impact
LOB Evolution

Tick-by-tick evolution of LOB queues at the touch. Market orders are in orange. Mid price is plotted at the top (TEVA, May 3, 2011)
Main Aspects of Order Flow

- Must separately treat Limit Orders and Market Orders
- Consequently, must also separately treat each side of the book (cross-effects)
- Aggregating by time-slicing or volume-slicing?
- Order Flow has intrinsic links to price behavior: mechanical (slippage) and informational (price discovery)
- Economically, OF is tied to informational costs:
  - Participants fear that other traders are better informed
  - Noise traders trade in all directions; informed traders are deliberate
  - When order flow is toxic market makers provide less liquidity
  - Example of a metric: VPIN by Easley, Lopez de Prado, O’Hara (2012abcd) – ties toxicity to flow imbalance = ratio of noise/informed traders
Empirical Order Flow

**Figure:** Cumulative net volume of MSFT executed trades over June 21, 2012

**Figure:** Price Impact for MSFT over same day

**Figure:** MSFT Net Order Flow over buckets of 1,000,000
Notation

- LOB consists of pairs \((p^j_t, v^j_t)\) indexed from the top queue.
- Mid-price: \(P^M(t) := (p^A_1(t) + p^B_1(t))/2\)
- Market orders (MO) and Limit orders (LO) form asynchronous time-series: time-stamp \(T_i\), signed volume \(O_i\), limit price \(S_i\)
- To synchronize & aggregate, construct **buckets**
- Slice the ticker according to intervals defined by \((\tau_k)\) (e.g. equal volume)
- Market buys: \(VM^A_k = \sum i : \tau_k \leq T^M_i \leq \tau_{k+1} O^M_i \{O^M_i > 0\}\).
- \(MI_k = VM^A_k - VM^B_k\)
- Contemporaneous ask limit orders at the touch:

\[
VL^A_k = \sum_{i : \tau_k \leq T^L_i \leq \tau_{k+1}} O^L_i \{S_i = p^B_1(T^L_i)\}
\]
How to Model Price Impact

- **Permanent** Price Impact
- **Transient** Price Impact

1. Large orders walk the book (depends on the LOB shape)
2. Liquidity providers replenish the queues and the book bounces back (resiliency)
How to Model Price Impact

- **Permanent** Price Impact
- **Transient** Price Impact
  1. Large orders walk the book (depends on the LOB shape)
  2. Liquidity providers replenish the queues and the book bounces back (resiliency)
- Obizhaeva/Wang: Realized execution price: $P_t = \tilde{P}_t + D_t$
- Unaffected $\tilde{P}$ is arithmetic BM
- Resilience $D$ decays exponentially to zero
- Each term is linearly impacted by trading:
  \[ \tilde{P}_t = \tilde{P}_0 + \lambda M_t + \sigma W_t, \quad D_t = e^{-\rho t} D_0 + \nu M_t. \]
- Assuming Gaussian $D_0$, gives a conditional Gaussian distribution for $\Delta P$, linear in $M_t$
LOB Perspective

- Immediate effect of a trade is deterministic given the LOB state. Cannot distinguish various sub-effects
- Difficult to measure long-term effects
- Have to describe the dynamic impact on LOB – eg arrival/cancellation rates

Large Tick Assets

Concentrate on liquid US equities

Tick size is $0.01; asset price is $20-$50

Spread is almost always 1 tick

The two queues at the touch are deep (>10 AES)

Orders rarely “walk the book” and there is no shape to the LOB

Typical immediate price impact is zero examples: MSFT, INTC, TEVA
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- examples: MSFT, INTC, TEVA
Price Impact of Market Flow

Figure: Relationship between mid-price change $\Delta P$ and net market order flow $M_l$ across volume-aggregated slices of 150,000 shares (1/80 of ADV) for MSFT during Jan-Mar 2011. The average price change was computed using Loess regression.
Explaining Price Movement based on Flows

- One sided flow consumes liquidity asymmetrically so the price moves
- Lots of buying → price goes up
- (Extreme MI typically anticipates low $\Delta P$ – observed S-shape)
- BUT: Even under balanced flow the price can make significant moves
- Good state: $MI = 50,000$ and $\Delta P = 0$.
- Bad state: $MI = 20,000$ and $\Delta P = 0.04$ large price move based on small net volume.
- Potential causes of such weak LOB resilience: are:
  1. Illiquid LOB
  2. Cancellations of limit orders (vanishing liquidity)
Static Liquidity of the LOB

- Spread: $p_A^1 - p_B^1$. Always 1-2 ticks – not indicative
- Static volume imbalance: $|v_A^1(t) - v_B^1(t)|$ - jumps each time the mid-price moves
- Depth: $\sum_i v_i^j$ (e.g. top 2 levels of the book)
- Price Impact: total effect relative to touch if instantaneously sell $M$ shares:

$$Pl_i := M^{-1} \sum_{i=1}^{i^M} v_i (S_i - p_1) + \tilde{v}_i^M (S_i^M - p_1), i^M = \min\{i : \sum_{j<i} v_j > M\}$$
Vanishing Liquidity

- Static depth can also be misleading
- One-sided market flows can induce wild swings in additions/cancellations and generate a lot of mid-price volatility
- This depends on the correlation between VL and MO and further latent regimes
Limit Order Flows

- Limit order flow should generally be **positive** (more additions than cancellations) – counteracts market orders
- Instances where VL is negative are **highly indicative** of major price moves
- This is not surprising from the LOB mechanics
- But shows that time series of limit flows is crucial to identify reduced liquidity/toxicity
**Limit Order Flows**

**Figure:** Relationship between net market order flow $MI$ and concurrent top-level limit flow $VL_j$ for MSFT during Jan-Mar 2011. Left: Bid, right: Ask. Highlighted points indicate buckets where the price change exceeded 5 ticks.
Can We Predict Scarce Liquidity?

- Generalized additive model to explain $\Delta P$
- (GAM) Logistic regression to predict large price moves aka scarce liquidity
- Most important predictor is MO flow
- In both cases LO have much *more explanatory power* than price impact or book imbalance.
- This is not actionable (LO is contemporaneous)
- There is also some memory of scarce liquidity (based on ACF for volume-sliced time series)
Dependence Between Limit & Market Orders

−ve Corr (TEVA)

+ve Corr (ORCL)
Dependence Between Limit & Market Orders

- Could represent via regimes: **resilient** ($VL$ and $MI$ positive correlation)
- **Scarce**: negative correlation (price fade/"front running")
- Limit order flow is a key factor to explain price move conditional on MO
- **Open Problem**: How to define model-free dependence of 4 asynchronous marked point processes?
Correlation Between Limit & Market Orders

Another View of Persistency in Liquidity

Figure: $\Delta P$ against $MI$ and accompanying linear best fit lines for Feb 10, 2011 and March 9, 2011 in TEVA. Higher correlation implies stronger MI impact (also see Cartea & Jaimungal, 2015).
Current Work

- Quantify observed stylized features using statistical models (nonparametric regression)
- Suggest some ways of measuring the temporal correlation between limit and market order flows
- Investigate existence of statistical regimes (hidden Markov factors??)
Part II: Incorporating OF into OE

Two main concerns when executing a large order:

1. Price Impact
   - Spatial effect: consume liquidity in terms of standing limit orders
Part II: Incorporating OF into OE

Two main concerns when executing a large order:

1. Price Impact
   - **Spatial** effect: consume liquidity in terms of standing limit orders
2. Information Leakage
   - **Temporal** effect: executed trade appears on the ticker tape
   - Other traders react and adjust their behavior (eg “front-running”) – will receive worse price in the future

**AIM:** integrate these ideas into a dynamic optimal execution framework

Treat order flow as a (controlled) state variable
Literature Landscape

- Work with continuous trading rates
- Assume a parametric form for inventory risk (Gatheral-Schied)
- Risk neutral agents (Gueant & Lehalle, ...)
- No fill risk (in contrast to Cartea & Jaimungal, Gueant et al, Bayraktar-L, ...)
- Treat only market orders (in the future: hybrid models like in Guilbaud & Pham, Carmona & Webster, ...)
- Postulate one-sided trading (in the future: allow two-sided to consider potential for market manipulation)
- Flat (constant depth) LOB $\rightarrow$ linear price impact
- Flat LOB $\rightarrow$ quadratic instantaneous execution cost
- (Order book resilience (Obizhaeva & Wang, Alfonsi et al))
- (Empirical evidence: Farmer, Bouchaud, ... )
Contributions

- Introduce order flow imbalance as a state variable
- Suggest a simple mechanism for informational costs (inspired by ELO12) – temporal impact beyond usual spatial impact
- Will also endogenize the execution horizon $T$ – can trade faster under favorable conditions
- Derive closed-form approximate strategies for the resulting optimization problem
Execution Model

- Inventory $x_t$: liquidate $x_0$
- $t \mapsto x_t$ is absolutely continuous; trading rate is $\alpha_t$
- $dx_t = -\alpha_t dt$
- Unaffected price $S_t$: martingale
Execution Model

- Inventory $x_t$: liquidate $x_0$
- $t \mapsto x_t$ is absolutely continuous; trading rate is $\alpha_t$
- $dx_t = -\alpha_t dt$
- Unaffected price $S_t$: martingale
- Order flow imbalance $Y_t$:
  - Mean-reverting to zero
  - Stationary in long-run
  - Affected by execution algorithms
- $Y_t > 0$: buyers-market; $Y_t < 0$: sellers market
Order Flow Dynamics

- **Unaffected** order flow: \( dY_t^0 = -\beta Y_t^0 dt + \sigma dW_t \)
- With execution:
  \[
  dY_t = (-\beta Y_t - \phi(\alpha_t)) dt + \sigma dW_t
  \]
- i.e. \( Y_t = Y_t^0 + \int_0^t e^{-\beta(t-s)} \alpha_s ds \)
- Typical cases:
  - \( \phi(\alpha) = \phi_t \) (deterministic information cost)
  - \( \phi(\alpha) = \eta \alpha \) (linear in trading rate)
- Assume that flow is independent of price (empirical relationship is not clear) - more on this later
Optimization Problem

- **Objective:** \( v(x, y) := \inf_{\alpha \in A} E_{x,y} \left[ \int_{0}^{T_0} g(\alpha_s) + \lambda(x_s) + \kappa Y_s^2 ds \right] \)
- **Realized horizon** \( T_0 := \inf \{ t : x_t^\alpha = 0 \} \) – endogenous to the strategy \( \alpha \)
Optimization Problem

- **Objective:** $\nu(x, y) := \inf_{\alpha \in A} \mathbb{E}_{x, y} \left[ \int_0^{T_0} g(\alpha_s) + \lambda(x_s) + \kappa Y_s^2 ds \right]$
- **Realized horizon** $T_0 := \inf\{t : x_t^\alpha = 0\}$ – endogenous to the strategy $\alpha$
- **$g(\alpha)$:** price impact
- **$g(\alpha) = \alpha^2$** (constant-depth LOB)
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- \( g(\alpha) = \alpha^2 \) (constant-depth LOB)
- \( \lambda(x) \): inventory risk
- \( \lambda(x) = cx^2 \) (Almgren-Chriss criterion) / \( \lambda(x) = cx \) (similar to Gatheral-Schied)

\(\text{Unbalanced order flow: Higher liquidity costs}\)
Optimization Problem

- Objective: $v(x, y) := \inf_{\alpha \in \mathcal{A}} \mathbb{E}_{x,y} \left[ \int_{T_0}^T g(\alpha_s) + \lambda(x_s) + \kappa Y_s^2 ds \right]$
- Realized horizon $T_0 := \inf\{t : x_t^\alpha = 0\}$ – endogenous to the strategy $\alpha$
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- $g(\alpha) = \alpha^2$ (constant-depth LOB)
- $\lambda(x)$: inventory risk
- $\lambda(x) = cx^2$ (Almgren-Chriss criterion) / $\lambda(x) = cx$ (similar to Gatheral-Schied)
- $\kappa Y^2$ : information cost
- Unbalanced order flow: Higher liquidity costs
HJB Equation

- $0 = -\beta Yv_Y + \frac{1}{2} \sigma^2 v_{YY} + \inf_{\alpha \geq 0} \{g(\alpha) - \alpha v_x - \phi(\alpha)v_Y\} + \kappa Y^2 + \lambda(x)$
- Finite-fuel boundary condition: $v(0, y) = 0$ for all $y$
- Nonlinear parabolic PDE
- Hard to understand the structure
- Positivity constraint on $\alpha$ is challenging
HJB Equation

\[ 0 = -\beta Y v_Y + \frac{1}{2} \sigma^2 v_{YY} + \inf_{\alpha \geq 0} \{ g(\alpha) - \alpha v_x - \phi(\alpha) v_Y \} + \kappa Y^2 + \lambda(x) \]

- Finite-fuel boundary condition: \( v(0, y) = 0 \) for all \( y \)
- Nonlinear parabolic PDE
- Hard to understand the structure
- Positivity constraint on \( \alpha \) is challenging
- To gain insights: build approximating problems by
  (i) solving the fixed-horizon problem
  (ii) optimizing over \( T \)
Fixed Horizon Problem

\[ u(T, x, y) = \inf_{(\alpha_t) \in A(T,x)} \mathbb{E}_{x,y} \left[ \int_0^T \alpha_s^2 + \lambda(x_s^{\alpha}) + \kappa Y_s^2 \, ds \right] \]

HJB equation becomes

\[ u_T = \frac{1}{2} \sigma^2 u_{yy} + \kappa y^2 + \lambda(x) - \beta y u_y + \inf_{\alpha} \left\{ \alpha^2 - \alpha u_x - \eta \alpha u_y \right\} \quad (1) \]

Singular boundary condition: \( \lim_{T \downarrow 0} u(T, x, y) = \infty \) if \( x \neq 0 \)

Proposition

The solution of (1) has the form

\[ u(T, x, y) = x^2 A(T) + y^2 B(T) + xyC(T) + D(T), \quad (2) \]

where \( A, B, C, D \) solve a matrix Riccati ordinary differential equation.

Note: Riccati equations parameterized in terms of time-to-maturity \( \tau \)
Execution Speed

- The corresponding optimal rate of liquidation is
  \[ \alpha^D_t = \frac{x_t(2A(\tau) + \eta C(\tau)) + Y_t(C(\tau) + 2\eta B(\tau)))}{2}. \]

- Execution rate is linear in \( x_t \) and in \( Y_t \) (generalizes Almgren-Chriss)

- The Proposition only treats the unconstrained case \( \alpha \in \mathbb{R} \): if \( T \) is large relative to \( x_0 \) or \( Y_t \) is negative enough then \( \alpha^D < 0 \)

- As \( t \to T \), the dynamic trading rate stabilizes, resembling a VWAP strategy.
**Myopic Strategies** \( (\phi(\alpha) = \phi_t) \)

- Suppose agent myopically optimizes only against price impact:
  \[
  u^M(T, x, y) := \inf_{(x_t)} \left( \int_0^T \dot{x}_s^2 + \lambda(x_s) ds \right) + \int_0^T \kappa \mathbb{E}_Y [Y_s^2] ds =: I + O
  \]

- If \( \lambda(x) = cx^2 \) solution is
  \[
  \begin{align*}
  x_{t}^{MH} &= \frac{x \sinh(\sqrt{c}(T - t))}{\sinh(\sqrt{c}T)} \\
  \alpha_{t}^{MH} &= \frac{\sqrt{c}x \cosh(\sqrt{c}(T - t))}{\sinh(\sqrt{c}T)}
  \end{align*}
  \]

- Now \( \phi_t = \eta \alpha_{t}^{MH} \) is the above deterministic function of \( t \to Y_t \) is Gaussian with known moments

- \( I^{MH}(T, x) = \sqrt{c}x^2 \coth(\sqrt{c}T) \)

- \( O^{MH}(T, x, y) = \kappa \int_0^T \left( ye^{-\beta t} - \int_0^t e^{-\beta(t-s)} \eta \alpha_{s}^{MH} ds \right)^2 + \frac{\sigma^2}{2\beta} (1 - e^{-2\beta t}) dt \)
Myopic Strategies \((\phi(\alpha) = \phi_t)\)

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\[
\mathcal{I}^{MH}(T, x) = \sqrt{cx^2} \coth(\sqrt{c}T)
\]

\[
\mathcal{O}^{MH}(T, x, y) = \kappa \int_0^T \left( ye^{-\beta t} - \int_0^t e^{-\beta(t-s)} \eta \alpha_{s}^{MH} ds \right)^2 + \frac{\sigma^2}{2\beta} (1 - e^{-2\beta t}) dt
\]

- Similarly have closed-form expressions for cases \(\lambda(x) = cx\) (Quadratic) and \(\lambda(x) = 0\) (VWAP)

- Next step: Take existing closed-form expressions \(u^M\) and optimize over \(T\)
Optimizing the Horizon

- \( T^* = \arg \min_T \ u(T, x, y) \)
- Lemma: \( T^* \in (0, \infty) \) (closed-form for \( T \mapsto u(T, x, y) \))
- Open-loop (static): find \( T^* \) at the outset and implement \( \alpha_t(T^*(x, y), x_t, Y_t) \)

Dynamically recomputing \( T^* \) - adapt to changing \( Y_t \) without the indefinite horizon

Finite-fuel problem

Next up: we show these are in fact good approximations!
### Optimizing the Horizon

- \( T^* = \arg \min_T u(T, x, y) \)
- **Lemma:** \( T^* \in (0, \infty) \) (closed-form for \( T \mapsto u(T, x, y) \))
- Open-loop (static): find \( T^* \) at the outset and implement \( \alpha_t(T^*(x, y), x_t, Y_t) \)
- Closed-loop (dynamic): continuously recompute \( T^* \):
  \[
  \tilde{\alpha}_t^M(x, y) := \alpha^M(T^*(x_t, Y_t), x_t, Y_t)
  \]
- Realized horizon \( T_0(x, y) \) becomes random
- Dynamically recomputing \( T^* \) - adapt to changing \( Y_t \) without the indefinite horizon finite-fuel problem

Next up: we show these are in fact **good approximations**!
## Comparison of costs

<table>
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<th></th>
<th>$\nu$</th>
<th>$\tilde{\nu}^D$</th>
<th>$\tilde{\nu}^{ML}$</th>
<th>$u^D$</th>
<th>$u^{ML}$</th>
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<tbody>
<tr>
<td>$\mathbb{E}[J(\alpha)]$</td>
<td>4.257</td>
<td>4.264</td>
<td>4.317</td>
<td>4.483</td>
<td>4.547</td>
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<tr>
<td>$SD(J(\alpha))$</td>
<td>1.50</td>
<td>1.45</td>
<td>1.39</td>
<td>1.77</td>
<td>1.84</td>
</tr>
<tr>
<td>$\mathbb{E}[T_0]$</td>
<td>3.87</td>
<td>3.70</td>
<td>3.48</td>
<td>3.43</td>
<td>3.43</td>
</tr>
</tbody>
</table>

**Legend:**
- $\nu$: directly from the HJB pde (fully numerical)
- $\tilde{\nu}(x, y)$: closed-loop optimization of $T^*$; $T_0$ is random
- $u(T^*, x, y)$: static optimization $T_0 = T^*$
- $u^{ML}$: VWAP on $[0, T^*]$
- $^D$ superscript refers to dynamic strategies based on Proposition 1 (Riccati eqns)
Figure: Comparison of trading rates ($\alpha_t$) for each of the six strategies along the shown simulated path of ($Y_t^0$) (The realized ($Y_t$) depends on the strategy chosen).
Execution Paths

**Figure:** Top: 200 simulated trajectories from strategy $\hat{\alpha}_t^D$. Highlighted are three trajectories resulting from different $Y_t$-paths. Bottom: Corresponding realizations of $t \mapsto Y_t$. 
Extensions

- How to estimate/filter/infer $Y_t$ from LOB data?
- Information leakage $\phi(\alpha)$ depends on $Y_t$?
- **Correlated** $S_t$ and $Y_t$: dependence between price movement and order flow.
Extensions

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Thank You!
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