# Systemic risk in the repo market.

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#### **Alexander Shkolnik**

UC Berkeley ads2@berkeley.edu

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#### The repurchase agreement (repo)



- A spot sale of a security and a simultaneous forward agreement to repurchase at a later date (*R* repo rate, *h* haircut).
- Securities used: Treasuries, Bonds, MBS, ABS, other (AAA-rated).

### **Repo market**

- Hedge funds a typical borrower.
- Money market funds a typical cash investor.
- Dealer banks (Goldman, Citigroup, Merrill Lynch, Barclays, PNP Paribas, etc.) are the intermediaries.
- The Federal reserve uses repos to implement monetary policy.

#### **Primary dealers' net repo financing**



Figure 10: Source: FR 2004C. Primary Dealers' Net Repo Financing. The figure shows the value of the repos minus reverse repos of the primary dealers.

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### **Related literature**

- Financial networks: Allen & Gale (2000), Eisenberg & Noe (2001), Acemoglu, Ozdaglar & Tahbaz-Selehi (2013), Glasserman & Young (2014) and many others.
- Secured lending: Dang, Gorton & Holmstrom (2013), Zhang (2013), Lee (2013), Eren (2014), Martin, Skeie & von Thadden (2014).
- Instability of credit: Hawtrey (1923), Hawtrey (1934), Schumpeter (1934), Minsky (1957), Minsky (1967) and others.
  - One bank's cash outflow is another's inflow.
  - Spending of one bank induces spending at another, and so on ...

## **Modeling & analysis overview**

- A dynamic model of the repo market.
  - Perspective: modeler has the same information set as the market,
  - $\mathbb{F} = \{\mathcal{F}_s\}_{s \ge 0}$  information filtration observed by the market.
- Write SDEs to model repo events occuring on a network.

- Analysis: derive spectra of ODE system Jacobian:
  - out-of-equilibrium system trajectories,
  - equilibria: stability, chaotic behaviour, etc.

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### Model construction (purchase only)

• Network of  $n \ge 1$  dealer banks. At time  $s \ge 0$ ,

 $N_s^j$  - units of collateral posted by *j* in repo,  $I_s(j)$  - name of *j*'s counterparty,  $\alpha_s^i$  - size of *i*'s collateral pool.

• SDE model takes the form

$$\Delta \alpha_s^i = -\Delta N_s^i + \sum_{j=1}^n \mathbf{1}_{\{I_s(j)=i\}} \Delta N_s^j \tag{1}$$

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• Take expectation to obtain ODEs.

### **Probabilistic modeling assumptions**

(Intuitive general principles which are empirically<sup>a</sup> supported.)

• Borrow in proportion to collateral pool size.

$$N_{\cdot}^{i} - \int_{0}^{\cdot} \alpha_{s}^{i} \, ds \quad is \ a \ martingale. \tag{2}$$

• Lend in proportion to cash available. On  $\{\Delta N_s^j = 1\}$ ,

$$P(I_s(j) = i | \mathcal{F}_{s-}) \propto \zeta_{s-}^i v_{ij}$$
(3)

where  $\zeta_s^i$  is the cash balance of dealer *i* at time  $s \ge 0$ .

 $(v_{ij}$  - probability both parties agree to contact.)

<sup>&</sup>lt;sup>a</sup>(Kirk, McAndrews, Sastry & Weed 2014)

### **Deterministic laws**

• Integrate SDE, take expectation, apply assumptions and differentiate:

$$\begin{bmatrix} \dot{a}_i \\ \dot{c}_i \end{bmatrix} = \begin{bmatrix} -1 & A/C \\ 1 & -A/C \end{bmatrix} \begin{bmatrix} a_i \\ c_i \end{bmatrix}$$
(4)

for i = 1, ..., n where at time  $s \ge 0$ 

$$a_i(s) = \mathbf{E}[\alpha_s^i]$$

$$c_i(s) = \mathbf{E}[\varsigma_s^i]$$

$$A = \sum_{i=1}^n \alpha^i \text{ (total system assets)}$$

$$C = \sum_{i=1}^n \varsigma^i \text{ (total system cash)}$$

#### **Purchase market stability**

C - total cash, A - total assets.

**Theorem.** Trajectories  $\{(a_i(s), c_i(s))\}_{s \ge 0}$  converge to a globally stable equilibrium  $(a_i^*, c_i^*) = \lim_{s \to \infty} (a_i(s), c_i(s))$  where

$$a_i^* = \frac{A/C}{1 + A/C} a_i(0)$$
(5)

$$c_i^* = (C/A) a_i^*$$
 (6)

on  $(\mathbb{R}_+, \mathbb{R}_+) \setminus \{(0, 0), (A, C)\}.$ 

## **Repurchase & Rehypothecation**

- Rehypothecation is the **re-use** of collateral.
- Unlimited rehypothecation implies (*no repurchase*) system debt

 $D(s) \ge As > C$  (system cash) (7)

(equals when R = 0) where A is the total system assets.

• Sum over all dealer banks to get system rates.



#### **Rehypothecation by primary dealers**



Figure 11: The total value of collateral that dealer banks were permitted to repledge (Dashed Lines) and the total value of collateral that dealer banks repledged. (Solid Lines) The data is obtained from the 10Q and 10K filings of the dealer banks.

#### **Model assumptions (repurchase)**

• Dealers repurchase only when cash is available, i.e.

$$K^{i}_{\cdot} - \int_{0}^{\cdot} \zeta^{i}_{s} \mathbf{1}_{\{\zeta^{i} > 0, L^{i}_{s} > 0\}} ds \quad is \ a \ martingale.$$
(8)

where  $L^{i} = N^{i} - K^{i}$  has not been repurchased and

 $K_s^i$  - units of security repurchased by *i*,

 $N_s^i$  - units of collateral posted by *i* in repo.

 $\zeta_s^i$  - cash holdings of dealer *i*.

• Also assume repo interest is paid daily.

### **Deterministic laws**

• Integrate SDE, take expectation, apply assumptions and differentiate:

$$\begin{bmatrix} \dot{a}_i \\ \dot{c}_i \end{bmatrix} = \begin{bmatrix} -1 & 1 + A/C \\ 1 + h + R & -(1 + R) - (1 - h)A/C \end{bmatrix} \begin{bmatrix} a_i \\ c_i \end{bmatrix} + \dots$$

for i = 1, ..., n where at time  $s \ge 0$ 

 $a_i(s) = \mathbf{E}[\alpha_s^i]$   $c_i(s) = \mathbf{E}[\zeta_s^i]$   $A = \sum_{i=1}^n \alpha^i \text{ (total system assets)}$   $C = \sum_{i=1}^n \zeta^i \text{ (total system cash)}$  R = repo rate h = haircut

#### **Repo market stability**

*C* - total cash, *A* - total assets,  $\lambda_i \in (0, 1)$  - demand to collect repo interest, *h* - haircut, *R* - repo rate.

**Theorem.** Trajectories  $\{(a_i(s), c_i(s))\}_{s \ge 0}$  converge to a globally stable equilibrium  $(a_i^*, c_i^*) = \lim_{s \to \infty} (a_i(s), c_i(s))$  where

$$a_{i}^{*} = (1 + A/C)c_{i}^{*} - \lambda_{i}C$$
(9)

$$c_i^* = \frac{(h/R)\lambda_i C - a_i(0)}{h/R - A/C}$$
(10)

on  $(\mathbb{R}_+, \mathbb{R}_+)$  if and only if

$$1 \le A/C < h/R$$
 (bounded leverage) (11)

or A/C < 1 and  $R \leq 0$ .

### Infinite-debt equilibrium

C - total cash, A - total assets, h - haircut, R - repo rate.

• Bounded leverage condition

 $1 \le A/C < h/R$  (bounded leverage) (12)

- Netted inter-dealer debt is finite but system debt  $\uparrow \infty$ .
- Sum over all dealer banks to obtain system rates.



### **Finite-debt equilibria (intuition)**

- Suppose there is only a single security in the market.
- The security performs a random walk over the network moving from dealer j to dealer i at event time  $T_k$  with probability

$$\frac{\zeta_{T_k}^i}{C} \tag{13}$$

- On each move it leaves a loan on dealer *i*'s balance sheet.
- Repurchase at rate proportional the number of loans.

#### **Model assumptions (repurchase)**

• Dealers repay in proportion to loans held, i.e.

$$K^{i}_{\cdot} - \int_{0}^{\cdot} L^{i}_{s} ds \quad is \ a \ martingale.$$
 (14)

where  $L^{i} = N^{i} - K^{i}$  has not been repurchased and

 $K_s^i$  - units of security repurchased by *i*,

 $N_s^i$  - units of collateral posted by *i* in repo.

### **Deterministic laws**

• Integrate SDE, take expectation, apply assumptions and differentiate:

$$\begin{bmatrix} \dot{a}_i \\ \dot{c}_i \end{bmatrix} = \begin{bmatrix} -1-z & A/C \\ 1+z(1+R) & -A/C \end{bmatrix} \begin{bmatrix} a_i \\ c_i \end{bmatrix} - za_i(0) \begin{bmatrix} -1 \\ (1+R) \end{bmatrix}$$

for i = 1, ..., n where at time  $s \ge 0$ 

- $a_{i}(s) = \mathbf{E}[\alpha_{s}^{i}]$   $c_{i}(s) = \mathbf{E}[\zeta_{s}^{i}]$   $A = \sum_{i=1}^{n} \alpha^{i} \text{ (total system assets)}$   $C = \sum_{i=1}^{n} \zeta^{i} \text{ (total system cash)}$  R = repo rate
  - z = fraction in repo

#### **Repo market instability**

C - total cash, A - total assets,  $z \in (0, 1]$  - fraction in repo, R - repo rate.

**Theorem.** Trajectories  $\{(a_i(s), c_i(s))\}_{s \ge 0}$  converge to a globally stable equilibrium  $(a_i^*, c_i^*) = \lim_{s \to \infty} (a_i(s), c_i(s))$  where

$$a_i^* = a_i(0)$$
 (15)

$$c_i^* = (C/A) a_i^*$$
 (16)

on  $(\mathbb{R}_+, \mathbb{R}_+)$  only if  $R \leq 0$ . If R > 0, equilibrium (15)-(16) is unstable.

### **Remedies for instability**

- Solution 1: Allow for (Fed) open market operations.
  - Sell assets to soak up excess reserves in parts of network.
  - Purchase assets to inject liquidity in rest of network.
- Solution 2: Ensure return r > 0 on portfolio with s = r R satisfies

$$\frac{z}{1-z} < 1 + \frac{s}{1+R}.$$
 (17)

• The equilibrium associated with (17) is a **finite-debt** equilibrium.

### Nonlinear phenomena

• Suppose only one dealer demands a haircut *h*. For constant z > 0.

$$\dot{a} = -a + \frac{A}{C}c - c + \lambda C \tag{18}$$

$$\dot{c} = a - (1 - h)\frac{A}{C}c + ha\frac{c}{C} - (1 + R)(e - a - c) - \lambda C$$
(19)

$$\dot{h} = z - ac \tag{20}$$

• Two (non-trivial) equilibria corresponding to low and high regimes

$$\pm(\sqrt{zA/C},\sqrt{zC/A},0) \tag{21}$$

• Model exhibits complex and chaotic behavior.

### Stable orbit about high regime



#### **Stable orbit converges to high regime**



### **Stable orbit leaves high regime**



## **Oscillations between the two regimes**



### **Extensions**

- Variance and asymptotic analysis,
- Dealer defaults,
- Restricted network topology (e.g. central clearing),
- Heterogeneous repo contracts,
- Some simple emperical evaluation.

### **Some conclusions**

- Model of cash and collateral flows with survival contraints.
- The framework provides an elegant way analyze the behaviour of a very complex system.
- The model is highly sensitive without amplification through shocks.
- Aggregate variables (e.g. A/C) directly related to market instability.

# Questions

Systemic risk in the repo market. March 25, 2015. 28-1 **References** 

Acemoglu, Daron, Asuman Ozdaglar & Alireza Tahbaz-Selehi (2013), Systemic risk and stability in financial networks. NBER Working Paper No. 18727.

- Allen, Franklin & Douglas Gale (2000), 'Financial contagion', *Journal of Political Economy* **108**(1), 1--33.
- Bottazzi, Jean-Marc, Jaime Luque & Mario R. Pascoa (2012), `Securities market theory: posession, repo and rehypothecation', *Journal of Economic Theory* **147**, 477--500.
- Dang, Tri Vi, Gary Gorton & Bengt Holmstrom (2013), Haircuts and repo chains. Working Paper.
- Eisenberg, Larry & Thomas Noe (2001), 'Systemic risk in financial systems', *Management Science* **47**(2), 236--249.

Systemic risk in the repo market. March 25, 2015. 28-2 Glasserman, Paul & Peyton Young (2014), How likely is contagion in financial networks. Working Paper.

- Hawtrey, Ralph (1923), *Currency and Credit*, Longmans, Green & Co.
- Hawtrey, Ralph (1934), *The Art of Central Banking*, Longmans, Green & Co.
- Kirk, Adam, James McAndrews, Parinitha Sastry & Phillip Weed (2014), `Matching collateral supply and financing demands in dealer banks', *Economic Policy Review of the Federal Reserve Bank of New York (forthcoming)* **20**(2), 00--00.
- Lee, Jeongmin (2013), Collateral circulation and repo spreads. Job Market Paper.
- Martin, Antoine, David Skeie & Ernst-Ludwig von Thadden (2014), Repo runs. Working Paper.
- Minsky, Hyman (1957), Central Banking and Money Market Changes, Quarterly Journal of Eco-

Systemic risk in the repo market. March 25, 2015. 28-3 Schumpeter, Joseph (1934), *The theory of economic development: an inquiry into profits, capital, credit, interest and the business cycle.*, Harvard University Press.

Zhang, Shengxing (2013), Collateral risk, reportlover and shadow banking. Job Market Paper.