Static Models of Central Counterparty Risk

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1The opinions expressed in this presentation are my own and do not necessarily reflect the views of the Board of Governors or its staff.
In response to the 2007-2009 financial crisis, the Group of Twenty (G-20) initiated a reform program in 2009 to reduce the systemic risk from OTC derivatives. The G-20’s reform program includes the following elements:

- All standardised OTC derivatives should be cleared through central counterparties (CCPs).
In response to the 2007-2009 financial crisis, the Group of Twenty (G-20) initiated a reform program in 2009 to reduce the systemic risk from OTC derivatives. The G-20’s reform program includes the following elements:

▶ All standardised OTC derivatives should be cleared through central counterparties (CCPs).

▶ Non-centrally-cleared derivatives should be subject to higher capital requirements. In 2011, the G-20 agreed to add margin requirements on non-centrally cleared derivatives to the reform program.
The Clearing Mandate

- Title VII of the Dodd-Frank Act and the European Market Infrastructure Regulation implement the G-20 clearing mandate in the US and Europe.

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- 42% of the global IRS market ($380 trillion at February 2013), 14% of the global CDS market ($22 trillion), about 12% of the commodity swaps ($2.6 trillion), 2% of the equity swaps ($6.3 trillion), and a de minimis share of FX swaps ($67.4 trillion) were being cleared in February 2013.²

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The potential loss exceeding these layers of the default waterfall resources is to be absorbed by the survivors’ unfunded default funds.
The state of research on CCPs is not conclusive:

**Bilateral vs Multilateral Netting: Would CCPs increase net exposure?**

Cont and Kokholm [2012]: “The impact of central clearing on total expected exposure is highly sensitive to assumptions on heterogeneity across asset classes in terms of riskyness and correlation of exposures across asset classes.” (Compare with the results of Duffie and Zhu [2011].)
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▶ **Bilateral vs Multilateral Netting: Would CCPs increase net exposure?**

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▶ **Are CCPs “good” for the financial system?**

Pirrong [2013]: “Greater netting and collateral redistributes risk in the system. CCPs transform counterparty risk into liquidity risk, which can be more systemically destabilizing. Making one part of the financial system invulnerable does not make the system as a whole safer.”
CCPs collectively take a mathematical-model-based approach on margin requirements. The remaining layers of the default waterfall are specified broadly. Post financial crisis, this fragmented CCP risk measurement approach has been mainly shaped by international standard setting bodies (SSBs) responsible for financial market infrastructures (FMIs).³

³The committee on payments and market infrastructures (CPMI) and the technical committee of the international organization of securities commissions (IOSCO)
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Post clearing mandate, SSBs have devised formulaic methods based on which clearing members are to hold capital against their exposures to CCPs. The CCP risk capital depends on all layers of the default waterfall resources; particularly prefunded and unfunded default funds.

³The committee on payments and market infrastructures (CPMI) and the technical committee of the international organization of securities commissions (IOSCO)
In the absence of a coherent CCP risk measurement framework:

- CCPs may engage in non-unifiable risk management practices.
- There will be inconsistencies in the CCP risk capital calculations; it would be impossible to develop a risk sensitive method.
- The central clearing of OTC derivatives may be disincentivized.\(^4\)

\(^4\) Regulatory reform of OTC derivatives: an assessment of incentives to clear centrally, BIS[2014].
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We introduce a model-based framework for the default waterfall of typical derivatives CCPs:

- It can be viewed as a common ground for CCPs, their direct clearing members, bank regulators, and CCP regulators.
- It can be used for a risk sensitive definition of the CCP risk capital based on which non-model-based methods can be evaluated.

\(^4\) *Regulatory reform of OTC derivatives: an assessment of incentives to clear centrally, BIS[2014].*
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How Should Banks be Capitalized Against Their Exposures to CCPs?

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- **It is impossible to define the second class of losses in a conceptually sound way in the absence of a unifying default waterfall model for CCPs.**
Consider a derivatives CCP with \( n \) direct clearing members; \( CM_i \) defaults with probability \( p_i \) based on a fixed time \( T > 0 \), \( i = 1, ..., n \). Default indicators \( Y_i \)'s are dependent through \( t \)-copula threshold models:

\[
Y_i = 1\{X_i > x_i\} \quad \text{and} \quad X_i = \frac{a_i Z + \sqrt{1 - a_i^2} \xi_i}{\lambda},
\]

where \( Z \) and \( \xi_i \)'s are independent standard normals, \( \lambda \equiv \sqrt{K/\nu} \) with \( K \sim \chi^2_{\nu} \) being independent of \( Z \) and \( \xi_i \)'s, and \( x_i \)'s are chosen such that \( E[Y_i] = p_i \).
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where $Z$ and $\xi_i$’s are independent standard normals, $\lambda \equiv \sqrt{\frac{K}{\nu}}$ with $K \sim \chi^2_v$ being independent of $Z$ and $\xi_i$’s, and $x_i$’s are chosen such that $E[Y_i] = p_i$.

Portfolio credit loss distributions are skewed with a relatively heavy upper tail. These empirical properties can not be captured by normal copula models; they assign very small probabilities to simultaneous defaults, (McNeil et al. [2005]).
Let $C_i$ denote the CCP’s collateralized credit exposure to $CM_i$ at its default in the presence of the $CM_i$’s VM and IM. We use *dynamic* counterparty credit risk measures, e.g., EPE, to define $C_i$’s,

$$C_i \equiv (1 - \delta_i) \int_0^T E[e_i(t)] dt,$$

where $\delta_i$ is the $CM_i$’s recovery rate and,
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where $\delta_i$ is the $CM_i$’s recovery rate and,

$e_i(t)$ denotes the CCP’s collateralized exposure to $CM_i$ at time $t$,

$$e_i(t) \equiv \max\{V_i^+(t + \Delta) - VM_i(t) - IM_i(t), 0\},$$

where $V_i^+(t) \equiv \max\{V_i(t), 0\}$, $VM_i(t) \equiv V_i^+(t - \hat{\Delta})$, and $IM_i(t)$ is the VaR or ES associated with $V_i^+(t + \Delta) - V_i^+(t - \hat{\Delta})$. 
Let $U_i = (C_i - DF_i)^+$ denote the CCP’s exposure to $CM_i$ in the presence of $VM_i$, $IM_i$, and $DF_i$. 
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The CCP’s counterparty credit loss in the presence of the defaulter-pay resources is, $L^{(1)} = \sum_{i=1}^{n} U_i Y_i$. 

Surviving members’ prefunded default funds reduce $L^{(1)}$ to $L^{(2)} = (\sum_{i=1}^{n} U_i Y_i - E - DF_s) +$, where, $DF_s \equiv DF - \sum_{i=1}^{n} DF_i Y_i$.

$L^{(2)}$ or part of it is allocated to the surviving members in the form of unfunded default funds.
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CCPs often specify their $DF$ based on the so called Cover 1/Cover 2 principle of PFMI:\(^5\)

CCPs should maintain financial resources to cover the default of one or two participants that would potentially cause the largest aggregate credit exposures for the CCP in extreme but plausible market conditions.

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*CCPs should maintain financial resources to cover the default of one or two participants that would potentially cause the largest aggregate credit exposures for the CCP in extreme but plausible market conditions.*

Compare the PFMI driven-DF for CCPs with large $n$ and homogeneous portfolios and CCPs with smaller $n$ and heterogenous portfolios.
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- Credit quality of the clearing members and their correlation do not play any role in specifying DF.
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- Compare the PFMI driven-DF for CCPs with large $n$ and homogeneous portfolios and CCPs with smaller $n$ and heterogenous portfolios.

- Credit quality of the clearing members and their correlation do not play any role in specifying DF.

- The allocation of DF to clearing members remains a subjective matter.

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View CCPs as financial institutions exposed to their \textit{portfolio counterparty credit risk}; the portfolio constituents are the members’ margined portfolios. The static model is a one-period approximation of the portfolio counterparty credit risk.

\footnote{See Tasche [1999], Tasche [2008], Denault [2001], and Kalkbrener [2005].}
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Define the CCPs’ total **DF based on a particular risk measure** associated with $L = \sum_{i=1}^{n} C_i Y_i$,

$$DF \equiv ES_\alpha(L) = E[L|L \geq \text{VaR}_\alpha(L)].$$

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Allocate DF to clearing members based on the so called Euler capital allocation principle,\(^6\)

\[ DF_i = C_i E[Y_i|L \geq VaR_\alpha(L)]. \]

\(^6\)See Tasche [1999], Tasche [2008], Denault [2001], and Kalkbrener [2005].
The CCPs’ loss in the absence of unfunded default funds becomes,

\[ L^{(2)} = \left( \sum_{i=1}^{n} U_i Y_i - E - DF_s \right)^+ = \left( \sum_{i=1}^{n} C_i Y_i - E - DF \right)^+, \]

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The \( DF \)-based or \( C \)-based loss allocation rules: consider the CCP’s unfunded default fund capital call on \( CM_i \),

\[ \frac{DF_i \bar{Y}_i}{\sum_{j=1}^{n} DF_j \bar{Y}_j} L^{(2)}, \quad \text{and} \quad \frac{C_i \bar{Y}_i}{\sum_{j=1}^{n} C_j \bar{Y}_j} L^{(2)}, \]

where \( \bar{Y}_j = 1 - Y_j, \ j = 1, ..., n \), the former has more desirable risk management justifications.
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**The uncapped case:**

$$\tilde{DF}_{i}^{uc} \equiv L_{i}^{uc} = \frac{DF_i \bar{Y}_i}{DF_s} L^{(2)} \geq 0,$$

where $DF_s = \sum_{j=1}^{n} DF_j \bar{Y}_j$. 
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▶ The capped case:

$$\tilde{DF}_i \equiv L_i = \min\{L^{uc}_i, \beta DF_i \bar{Y}_i\},$$

where $\beta > 0$. 

The Unfunded Default Funds
Define the $CM_i$’s unanticipated loss assuming its time-$T$ survival,

$$\tilde{DF}_{i}^{uc,s} \equiv L_{i}^{uc,s} = \frac{DF_i}{DF_{s,i}} \left( \sum_{j \neq i} C_j Y_j - E - DF \right)^+,$$

where $DF_{s,i} \equiv DF - \sum_{j \neq i} DF_j Y_j$. 
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The capped case from the $CM_i$’s perspective becomes,

$$\tilde{D}F_{i}^{s} \equiv L_{i}^{s} = \min\{L_{i}^{uc,s}, \beta DF_i\},$$

where $\beta > 0$. 

The uncapped case:

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L_{i}^{t,uc,s} = L_{i}^{uc,s} = \frac{DF_{i}}{DF_{s,i}} \left( \sum_{j \neq i} C_{j}Y_{j} - E - DF \right)^{+}
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Total Losses to $CM_i$

- **The uncapped case:**

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- **The capped case:** let $\tilde{Y}_{i}$ denote the CCP’s default indicator from $CM_i$’s perspective assuming it’s survival at time $T$. The $CM_i$’s total potential loss becomes,

  $$L_{i}^{t,s} = L_{i}^{s} + \tilde{U}_{i}\tilde{Y}_{i},$$

where $L_{i}^{s} = \min\{L_{i}^{uc,s}, \beta DF_{i}\}$, and $\tilde{U}_{i}$ denotes the $CM_i$’s loan-equivalent exposure to the CCP at its default.
Total Losses to $\text{CM}_i$

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where $L_{i}^{s} = \min\{L_{i}^{uc,s}, \beta DF_i\}$, and $\tilde{U}_i$ denotes the $\text{CM}_i$’s loan-equivalent exposure to the CCP at its default.

- And,

\[
E[\tilde{Y}_i] = P_{ccp,i} = P \left( \sum_{j \neq i} C_j Y_j > E + DF + \tilde{D}F_{i}^s + \sum_{j \neq i} \tilde{D}F_{j} \right) .
\]
The uncapped case:

\[ L_{i}^{t,uc,s} = \min \left\{ DF_{i}, \frac{DF_{i}}{DF_{s,i}} L_{i}^{(1)} \right\} + L_{i}^{uc,s}, \]

where \( L_{i}^{(1)} = \left( \sum_{j \neq i} (C_{j} - DF_{j})Y_{j} - E \right)^{+} \) represents the CCP’s loss in the presence of defaulter-pay resources and conditional on \( CM_{i} \)'s time-\( T \) survival.
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The capped case:

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where \( L_{i}^{s} = \min\{L_{i}^{uc,s}, \beta DF_{i}\}. \)
The CCP risk capital based on *expected losses*:

\[
E[L_{i}^{t,s}] = E\left[\min\left\{ DF_{i}, \frac{DF_{i}}{DF_{s,i}} L_{i}^{(1)}\right\}\right] + E[L_{i}^{s}] + \tilde{U}_{i} P_{ccp,i},
\]

where

\[
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The CCP Risk Capital

- The CCP risk capital based on expected losses:

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\]

- The CCP risk capital based on unexpected losses:

\[
\text{VaR}_{\alpha}(L_{i,t,s}^t) - E[L_{i,t,s}^t].
\]
The CCP risk capital based on expected losses:

\[ E[L_{t,s}^i] = E \left[ \min \left\{ DF_i, \frac{DF_i}{DF_{s,i}} L_i^{(1)} \right\} \right] + E[L_i^s] + \tilde{U}_i P_{ccp,i}, \]

where

\[ L_i^s = \min \left\{ \frac{DF_i}{DF_{s,i}} \left( \sum_{j \neq i} C_j Y_j - E - DF \right)^+ , \beta DF_i \right\}. \]

The CCP risk capital based on unexpected losses:

\[ \text{VaR}_\alpha (L_{t,s}^i) - E[L_{t,s}^i]. \]

Basel II’s credit risk capital is based on unexpected loss. Basel III’s counterparty credit risk capital does not accept a well-defined expected or unexpected loss characterization.
The BCBS has often developed both non-model-based and model-based methods for risk capital requirements. For instance: The Basel II’s non-model-based *Standardized* approach and the model-based *Foundation Internal Ratings Based (IRB)/Advanced IRB* approaches for credit risk capital.\(^7\)

\(^7\)See Gordy [2003] for model-theoretic aspects of Basel II’s credit risk framework.
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The regulatory CCP risk capital is not model-based. Since the CCP risk capital depends on the default waterfall in a non-straightforward way, non-model based methods will not be conceptually sound and logically consistent.

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The interim framework for the CCP risk capital was first published by BCBS in 2012. A consultative document modified the interim framework in 2013. BCBS-CPMI-IOSCO published their finalized CCP risk capital requirements in April 2014.

\textsuperscript{7}See Gordy [2003] for model-theoretic aspects of Basel II’s credit risk framework.
The Regulatory CCP Risk Capital

- The $CM_i$’s CCP risk capital:

  \[
  \text{Default fund capital charges} \, + \, \text{Trade exposure capital charges}
  \]
The regulatory CCP risk capital:

*The $CM_i$’s CCP risk capital:*

*Default fund capital charges + Trade exposure capital charges*

*The one-period-model’s average-based CCP risk capital:*

$$
E[L_{i}^{t,s}] = E \left[ \min \left\{ DF_{i}, \frac{DF_{i}}{DF_{s,i}} L_{i}^{(1)} \right\} \right] + E[L_{i}^{s}] + \tilde{U}_{i} P_{ccp,i},
$$

where

$$
L_{i}^{s} = \min \left\{ \frac{DF_{i}}{DF_{s,i}} \left( \sum_{j \neq i} C_{j} Y_{j} - E - DF \right)^{+}, \beta DF_{i} \right\}.
$$
The Regulatory CCP Risk Capital

- The $CM_i$’s CCP risk capital:

  \[ \text{Default fund capital charges} + \text{Trade exposure capital charges} \]

- The one-period-model’s average-based CCP risk capital:

  \[
  E[L_{t=s}^i] = E \left[ \min \left\{ DF_i, \frac{DF_i}{DF_{s,i}} L_{i}^{(1)} \right\} \right] + E[L_i^s] + \tilde{U}_i P_{ccp,i},
  \]

  where

  \[
  L_i^s = \min \left\{ \frac{DF_i}{DF_{s,i}} \left( \sum_{j \neq i} C_j Y_j - E - DF \right)^+ , \beta DF_i \right\}.
  \]

- What are the regulatory “Default fund capital charges” supposed to represent?
The BCBS-CPMI-IOSCO’s Finalized Rule

Default Fund-Exposure Capital Charges

The CCP’s hypothetical capital requirements:

\[ K_{ccp} = (\text{capital ratio}) \times RW \times \left( \sum_{i=1}^{n} (C_i - DF_i)^+ \right), \]

the risk weight is 20\%, and the minimum capital ratio has been historically set equal to 8\%. 

Ghamami

Static Models of Central Counterparty Risk
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- \( K_{cm_i} \) can not be rationalized.
During times of financial stress, the market volatility increases. This in turn increases the margin requirements. High margin requirements affect the funding and market liquidity; liquidity dry-ups further increase the market volatility. 

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8Brunnermeier and Pedersen [2009].
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Consider the CCPs’ expected losses in the presence of all layers of the default waterfall,

\[ E[L^{(3)}] = E[(\sum_{i=1}^{n} C_i Y_i - E - DF_\alpha - \tilde{DF})^+] \].

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\(^8\)Brunnermeier and Pedersen [2009].
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The mix of VM, IM, E, and DF can be chosen such that margins’ procyclicality be reduced while the CCPs maintain the same level of financial resiliency.

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CCPs are supposed to bring transparency to the OTC derivatives market. International regulatory risk management standards influence and shape the dynamics of financial markets.
Conclusion

- CCPs are supposed to bring transparency to the OTC derivatives market. International regulatory risk management standards influence and shape the dynamics of financial markets.

- The current central clearing environment is fragmented: The mathematical-model-based treatment of margin requirements; The PFMI’s principle-based treatment of the guarantee fund; The formulaic CCP risk capital rules.
CCPs are supposed to bring transparency to the OTC derivatives market. International regulatory risk management standards influence and shape the dynamics of financial markets.

The current central clearing environment is fragmented: The mathematical-model-based treatment of margin requirements; The PFMI’s principle-based treatment of the guarantee fund; The formulaic CCP risk capital rules.

The proposed CCP risk measurement framework, which models the CCPs’ default waterfall coherently, can improve the status quo. It can be used for a risk sensitive and conceptually sound definition of the CCP risk capital.
Appendix: The Legacy Ratio-Tranches Methods

- The CCP’s hypothetical capital requirements:

\[ K_{ccp} = \text{(capital ratio)} \times RW \times \left( \sum_{i=1}^{n} C_i \right), \]

the risk weight is to represent the credit quality of the clearing members, and the minimum capital ratio has been historically set equal to 8%. 
Appendix: The Legacy Ratio-Tranches Methods

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- The Ratio approach:

\[ K_{cm_i} = \frac{DF_i}{DF} K_{ccp}. \]

- The Tranches approach:

\[ K_{cm_i} = \begin{cases} K_{ccp} & \text{if } DF < K_{ccp} \\ K_{ccp} + \frac{16K_{ccp}(DF-K_{ccp})}{DF} & \text{if } K_{ccp} < DF \end{cases} \]
Consider the Ratio Approach:

\[ K_{cmi} = 0.08 \times RW \times \frac{DF_i}{DF} \left( \sum_{i=1}^{n} C_i \right) \]
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Recall:

\[ L_{i}^{uc,s} = \frac{DF_i}{DF_{s,i}} \left( \sum_{j \neq i} C_j Y_j - DF \right)^+ \]
Consider the Ratio Approach:

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Recall:

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The normal-copula-based approximation of \( K_{cm_i} \) in the exchangeable case for large \( n \):

\[ E[L_{i}^{uc,s}] \approx \frac{DF_i}{DF(1 - p) + pDF_i} \left( \theta_1 \sum_{i=1}^{n} C_i + \theta_2 \left( \sum_{i=1}^{n} C_i^2 \right)^{\frac{1}{2}} \right), \]

where \( \theta_1 \) and \( \theta_2 \) depend on \( p \), the correlation between normals underlying the copula model, and the confidence level associated with \( DF \). For our numerical examples the order of \( \theta_1 \) varies from \( 10^{-2} \) to \( 10^{-4} \), and the order of \( \theta_2 \) varies from \( 10^{-3} \) to \( 10^{-5} \).
Estimate the CCP’s credit-equivalent exposures to the clearing members:

\[ C_i \equiv (1 - \delta_i) \int_0^T E[e_i(t)] dt, \]

where\( e_i(t) \equiv \max\{V_i^+(t + \Delta) - VM_i(t) - IM_i(t), 0\}, \)
\( VM_i(t) \equiv V_i^+(t - \hat\Delta), \) and \( IM_i(t) \) is the VaR or ES associated with \( V_i^+(t + \Delta) - V_i^+(t - \hat\Delta); i = 1, \ldots, n. \)

\[ ^9 \text{See Ghamami and Zhang [2014] for efficient Monte Carlo CCR in the absence of IM.} \]
Appendix: Monte Carlo CCP Risk Measurement

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- This will be computationally intensive; particularly for high confidence level-IM’s. It would be challenging to develop efficient Monte Carlo schemes for estimation of \( C_i \)'s as \( V_i \) is the value of the \( CM_i \)'s portfolio consisting of possibly thousands of derivatives transactions.

See Ghamami and Zhang [2014] for efficient Monte Carlo CCR in the absence of IM.
Use Monte Carlo to estimate $q \equiv \text{VaR}_\alpha (L)$, $L = \sum_{i=1}^{n} C_i Y_i$. 

For efficient estimation of $DF$ and $DF_i$'s, recall, $Y_i = 1\{X_i > x_i\}$, $X_i = a_i Z + \sqrt{1-a_i^2} \xi \lambda$, $E[L | L > q] = E[L 1\{L > q\}]P(L > q)$. And use the importance sampling algorithm of Bassamboo et al. [2008].
Appendix: Monte Carlo CCP Risk Measurement

- Use Monte Carlo to estimate $q \equiv \text{VaR}_\alpha(L), \ L = \sum_{i=1}^{n} C_i Y_i$.

- Use the Monte Carlo estimate of $q$ in place of the true value of $q$ to estimate $DF = E[L|L > q]$ and $DF_i = C_i E[Y_i|L > q], \ i = 1, \ldots, n$. 

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$$X_i = \frac{a_i Z + \sqrt{1 - a_i^2 \xi_i}}{\lambda}, \ E[L|L > q] = \frac{E[L1\{L > q\}]}{P(L > q)}$$

and use the importance sampling algorithm of Bassamboo et al. [2008].