Static Models of Central Counterparty Risk

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Systemic Risk and Financial Networks, IPAM

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[1]The opinions expressed in this presentation are my own and do not necessarily reflect the views of the Board of Governors or its staff.

In response to the 2007-2009 financial crisis, the Group of Twenty (G-20) initiated a reform program in 2009 to reduce the systemic risk from OTC derivatives. The G-20's reform program includes the following elements:

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The 2009 G-20 Clearing Mandate

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- All standardised OTC derivatives should be cleared through central counterparties (CCPs).
- Non-centrally-cleared derivatives should be subject to higher capital requirements. In 2011, the G-20 agreed to add margin requirements on non-centrally cleared derivatives to the reform program.

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- 42% of the global IRS market ($\$380$ trillion at February 2013), 14% of the global CDS market ($\$22$ trillion), about 12% of the commodity swaps ($\$2.6$ trillion), 2% of the equity swaps ($\$6.3$ trillion), and a de minimis share of FX swaps ($\$67.4$ trillion) were being cleared in February 2013.[2]

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- The potential loss exceeding the defaulter-pay resources are to be absorbed by the **CCP's equity contributions** and the survivor-pay **prefunded default funds**.
- The potential loss exceeding these layers of the default waterfall resources is to be absorbed by the survivors' **unfunded default funds**.

The state of research on CCPs is not conclusive:

- **Bilateral vs Multilateral Netting: Would CCPs increase net exposure?**
  Cont and Kokholm [2012]: "The impact of central clearing on total expected exposure is highly sensitive to assumptions on heterogeneity across asset classes in terms of *riskyness* and correlation of exposures across asset classes." (Compare with the results of Duffie and Zhu [2011].)

CCPs, Systemic Risk, and the Financial System

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- **Are CCPs "good" for the financial system?**
  Pirrong [2013]: "Greater netting and collateral redistributes risk in the system. CCPs transform counterparty risk into liquidity risk, which can be more systemically destabilizing. Making one part of the financial system invulnerable does not make the system as a whole safer."

CCP Risk Management and the CCP Risk Capital

- CCPs collectively take a mathematical-model-based approach on margin requirements. The remaining layers of the default waterfall are specified broadly. Post financial crisis, this fragmented CCP risk measurement approach has been mainly shaped by international standard setting bodies (SSBs) responsible for financial market infrastructures (FMIs).[3]

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- Post clearing mandate, SSBs have devised formulaic methods based on which clearing members are to hold capital against their exposures to CCPs. The CCP risk capital depends on all layers of the default waterfall resources; particularly prefunded and unfunded default funds.

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- In the absence of a coherent CCP risk measurement framework:
  - CCPs may engage in non-unifiable risk management practices.
  - There will be inconsistencies in the CCP risk capital calculations; it would be impossible to develop a risk sensitive method.
  - The central clearing of OTC derivatives may be disincentivized.[4]

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  - The central clearing of OTC derivatives may be disincentivized.[4]
- We introduce a model-based framework for the default waterfall of typical derivatives CCPs:
  - It can be viewed as a common ground for CCPs, their direct clearing members, bank regulators, and CCP regulators.
  - It can be used for a risk sensitive definition of the CCP risk capital based on which non-model-based methods can be evaluated.

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How Should Banks be Capitalized Against Their Exposures to CCPs?

- **Losses to a direct clearing member due to the CCP's default**: The replacement cost of the derivatives portfolio it has cleared with the CCP; "losing" the IM posted if not held in a bankruptcy remote manner; "losing" the prefunded default fund (?)

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- **Losses to a direct clearing member due to the default of other clearing members**: "losing" all or part of the prefunded default fund contributions (?) potential losses due to the CCP's unfunded default fund capital calls.
- **It is impossible to define the second class of losses in a conceptually sound way in the absence of a unifying default waterfall model for CCPs.**

- Consider a derivatives CCP with n direct clearing members; CM_i defaults with probability p_i based on a fixed time $T > 0$, $i = 1, ..., n$. Default indicators Y_i's are dependent through t-copula threshold models:

$$Y_i = 1\{X_i > x_i\} \text{ and } X_i = \frac{a_i Z + \sqrt{1 - a_i^2}\xi_i}{\lambda},$$

where Z and ξ_i's are independent standard normals, $\lambda \equiv \sqrt{\frac{K}{v}}$ with $K \sim \chi^2_\nu$ being independent of Z and ξ_i's, and x_i's are chosen such that $E[Y_i] = p_i$.

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- Portfolio credit loss distributions are skewed with a relatively heavy upper tail. These empirical properties can not be captured by normal copula models; they assign very small probabilities to simultaneous defaults, (McNeil et al. [2005]).

- Let C_i denote the CCP's collateralized credit exposure to CM_i at its default in the presence of the CM_i's VM and IM. We use *dynamic* counterparty credit risk measures, e.g., EPE, to define C_i's,

$$C_i \equiv (1-\delta_i)\int_0^T E[e_i(t)]dt,$$

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- $e_i(t)$ denotes the CCP's collateralized exposure to CM_i at time t,

$$e_i(t) \equiv \max\{V_i^+(t+\Delta) - VM_i(t) - IM_i(t), 0\},$$

where $V_i^+(t) \equiv \max\{V_i(t), 0\}$, $VM_i(t) \equiv V_i^+(t-\hat{\Delta})$, and $IM_i(t)$ is the VaR or ES associated with $V_i^+(t+\Delta) - V_i^+(t-\hat{\Delta})$.

CCP Losses in the One-Period Model

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- Surviving members' prefunded default funds reduce $L^{(1)}$ to

$$L^{(2)} = \left(\sum_{i=1}^{n} U_i Y_i - E - DF_s \right)^+,$$

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- $L^{(2)}$ or part of it is allocated to the surviving members in the form of unfunded default funds.

- CCPs often specify their DF based on the so called *Cover 1/Cover 2* principle of PFMI: [5]

  CCPs should maintain financial resources to cover the default of one or two participants that would potentially cause the largest aggregate credit exposures for the CCP in extreme but plausible market conditions.

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The PFMI and Prefunded Default Funds

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- Credit quality of the clearing members and their correlation do not play any role in specifying DF.
- The allocation of DF to clearing members remains a subjective matter.

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The Prefunded Default Funds in the One-Period Model

- View CCPs as financial institutions exposed to their *portfolio counterparty credit risk*; the portfolio constituents are the members' margined portfolios. The static model is a one-period approximation of the portfolio counterparty credit risk.

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- Define the CCPs' total **DF based on a particular risk measure** associated with $L = \sum_{i=1}^{n} C_i Y_i$,

$$DF \equiv ES_\alpha(L) = E[L | L \geq VaR_\alpha(L)].$$

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- Allocate DF to clearing members based on the so called **Euler capital allocation principle**,[6]

$$DF_i = C_i E[Y_i|L \geq VaR_\alpha(L)].$$

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- The CCPs' loss in the absence of unfunded default funds becomes,

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- **The DF-based or C-based loss allocation rules**: consider the CCP's unfunded default fund capital call on CM_i,

$$\frac{DF_i \bar{Y}_i}{\sum_{j=1}^{n} DF_j \bar{Y}_j} L^{(2)}, \quad \text{and} \quad \frac{C_i \bar{Y}_i}{\sum_{j=1}^{n} C_j \bar{Y}_j} L^{(2)},$$

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- Consider the DF-based allocation of,

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- **The uncapped case**:

$$\tilde{DF}_i^{uc} \equiv L_i^{uc} = \frac{DF_i \bar{Y}_i}{DF_s} L^{(2)} \geq 0,$$

where $DF_s = \sum_{j=1}^{n} DF_j \bar{Y}_j$.

The Unfunded Default Funds

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- **The capped case**:

$$\tilde{DF}_i \equiv L_i = \min\{L_i^{uc}, \beta DF_i \bar{Y}_i\},$$

where $\beta > 0$.

The Unfunded Default Funds: *'The Conditional Case'*

- Define the CM_i's unanticipated loss assuming its time-T survival,

$$\tilde{DF}_i^{uc,s} \equiv L_i^{uc,s} = \frac{DF_i}{DF_{s,i}} \left(\sum_{j \neq i} C_j Y_j - E - DF \right)^+ ,$$

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- The capped case from the CM_i's perspective becomes,

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where $\beta > 0$.

Total Losses to CM$_i$

- **The uncapped case**:

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- **The capped case**: let $\tilde{Y}_i$ denote the CCP's default indicator from CM_i's perspective assuming it's survival at time T. The CM_i's total potential loss becomes,

$$L_i^{t,s} = L_i^s + \tilde{U}_i \tilde{Y}_i,$$

where $L_i^s = \min\{L_i^{uc,s}, \beta DF_i\}$, and $\tilde{U}_i$ denotes the CM_i's loan-equivalent exposure to the CCP at its default.

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- And,

$$E[\tilde{Y}_i] = P_{ccp,i} = P\left(\sum_{j \neq i} C_j Y_j > E + DF + \tilde{DF}_i^s + \sum_{j \neq i} \tilde{DF}_j \right).$$

- **The uncapped case**:

$$L_i^{t,uc,s} = \min\left\{DF_i, \frac{DF_i}{DF_{s,i}} L_i^{(1)}\right\} + L_i^{uc,s},$$

where $L_i^{(1)} = \left(\sum_{j \neq i}(C_j - DF_j)Y_j - E\right)^+$ represents the CCP's loss in the presence of defaulter-pay resources and conditional on CM_i's time-T survival.

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where $L_i^s = \min\{L_i^{uc,s}, \beta DF_i\}$.

- The CCP risk capital based on *expected losses*:

$$E[L_i^{t,s}] = E\left[\min\left\{DF_i, \frac{DF_i}{DF_{s,i}} L_i^{(1)}\right\}\right] + E[L_i^s] + \tilde{U}_i P_{ccp,i},$$

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- The CCP risk capital based on *unexpected losses*:

$$VaR_\alpha(L_i^{t,s}) - E[L_i^{t,s}].$$

- Basel II's credit risk capital is based on unexpected loss. Basel III's counterparty credit risk capital does not accept a well-defined expected or unexpected loss characterization.

- The BCBS has often developed both non-model-based and model-based methods for risk capital requirements.
  For instance: The Basel II's non-model-based *Standardized* approach and the model-based *Foundation Internal Ratings Based* (IRB)/*Advanced IRB* approaches for credit risk capital.[7]

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The Regulatory CCP Risk Capital

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- The regulatory CCP risk capital is not model-based. Since the CCP risk capital depends on the default waterfall in a non-straightforward way, non-model based methods will not be conceptually sound and logically consistent.

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- The interim framework for the CCP risk capital was first published by BCBS in 2012. A consultative document modified the interim framework in 2013. BCBS-CPMI-IOSCO published their finalized CCP risk capital requirements in April 2014.

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  Default fund capital charges + *Trade exposure capital charges*

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  Default fund capital charges + *Trade exposure capital charges*

- The one-period-model's average-based CCP risk capital:

$$E[L_i^{t,s}] = E\left[\min\left\{DF_i, \frac{DF_i}{DF_{s,i}} L_i^{(1)}\right\}\right] + E[L_i^s] + \tilde{U}_i P_{ccp,i},$$

where

$$L_i^s = \min\left\{\frac{DF_i}{DF_{s,i}}\left(\sum_{j\neq i} C_j Y_j - E - DF\right)^+, \beta DF_i\right\}.$$

- **What are the regulatory "Default fund capital charges" supposed to represent?**

Default Fund-Exposure Capital Charges

- *The CCP's hypothetical capital requirements*:

$$K_{ccp} = (\text{capital ratio}) \times RW \times \left(\sum_{i=1}^{n} (C_i - DF_i)^+ \right),$$

the *risk weight* is 20%, and the minimum capital ratio has been historically set equal to 8%.

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- K_{cm_i} **can not be rationalized.**

Margins' Procyclicality and the One-Period Model

- During times of financial stress, the **market volatility** increases. This in turn increases the **margin requirements**. High margin requirements affect the **funding and market liquidity**; liquidity dry-ups further increase the market volatility. [8]

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- Consider the CCPs' expected losses in the presence of all layers of the default waterfall,

$$E[L^{(3)}] = E[(\sum_{i=1}^{n} C_i Y_i - E - DF_\alpha - \tilde{DF})^+].$$

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- The mix of VM, IM, E, and DF can be chosen such that margins' procyclicality be reduced while the CCPs maintain the same level of financial resiliency.

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Conclusion

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- The current central clearing environment is fragmented: The mathematical-model-based treatment of margin requirements; The PFMI's principle-based treatment of the *guarantee fund*; The formulaic CCP risk capital rules.
- The proposed CCP risk measurement framework, which models the CCPs' default waterfall coherently, can improve the status quo. It can be used for a risk sensitive and conceptually sound definition of the CCP risk capital.

Appendix: The Legacy Ratio-Tranches Methods

- *The CCP's hypothetical capital requirements*:

$$K_{ccp} = (\text{capital ratio}) \times RW \times (\sum_{i=1}^{n} C_i),$$

the *risk weight* is to represent the credit quality of the clearing members, and the minimum capital ratio has been historically set equal to 8%.

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- **The Ratio approach**: $K_{cm_i} = \frac{DF_i}{DF} K_{ccp}$.
- **The Tranches approach**:

$$K_{cm_i} = \frac{DF_i}{DF} \begin{cases} K_{ccp} & \text{if } \ DF < K_{ccp} \\ K_{ccp} + \frac{.16 K_{ccp}(DF - K_{ccp})}{DF} & \text{if } \ K_{ccp} < DF \end{cases}$$

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- Recall: $L_i^{uc,s} = \frac{DF_i}{DF_{s,i}} \left(\sum_{j \neq i} C_j Y_j - DF\right)^+$
- The normal-copula-based approximation of K_{cm_i} in the *exchangeable case* for large n:

$$E[L_i^{uc,s}] \approx \frac{DF_i}{DF(1-p) + pDF_i} \left(\theta_1 \sum_{i=1}^{n} C_i + \theta_2 (\sum_{i=1}^{n} C_i^2)^{\frac{1}{2}}\right),$$

where θ_1 and θ_2 depend on p, the correlation between normals underlying the copula model, and the confidence level associated with DF. For our numerical examples the order of θ_1 varies from 10^{-2} to 10^{-4}, and the order of θ_2 varies from 10^{-3} to 10^{-5}.

Appendix: Monte Carlo CCP Risk Measurement

- Estimate the CCP's credit-equivalent exposures to the clearing members:

$$C_i \equiv (1-\delta_i)\int_0^T E[e_i(t)]dt,$$

where $e_i(t) \equiv \max\{V_i^+(t+\Delta) - VM_i(t) - IM_i(t), 0\}$, $VM_i(t) \equiv V_i^+(t-\hat{\Delta})$, and $IM_i(t)$ is the VaR or ES associated with $V_i^+(t+\Delta) - V_i^+(t-\hat{\Delta})$; $i = 1, ..., n$.[9]

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- This will be computationally intensive; particularly for high confidence level-IM's. It would be challenging to develop efficient Monte Carlo schemes for estimation of C_i's as V_i is the value of the CM_i's portfolio consisting of possibly thousands of derivatives transactions.

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- For efficient estimation of DF and DF_i's, recall, $Y_i = 1\{X_i > x_i\}$,

$$X_i = \frac{a_i Z + \sqrt{1 - a_i^2}\xi_i}{\lambda}, \; E[L|L > q] = \frac{E[L1\{L > q\}]}{P(L > q)}$$

  and use the importance sampling algorithm of Bassamboo et al. [2008].