Systemic Risk and Central Counterparty Clearing

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Systemic Risk and Financial Networks
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Outline

Financial Network

Central Counterparty Clearing

Comparative statics

Does a CCP reduce systemic risk?

Pareto optimality analysis
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Pareto optimality analysis
Setup

- Two periods $t = 0, 1, 2$
- Values at $t = 1, 2$ are random variables on $(\Omega, \mathcal{F})$
- $m$ interlinked banks $[m] := \{1, 2, \ldots, m\}$
Instruments

Bank \( i \) holds

- **Cash** \( \gamma_i \): zero return
- **External asset** (e.g. long-term investment maturing at \( t = 2 \)):
  - fundamental value \( Q_i \) at \( t = 1, 2 \)
  - liquidation value \( P_i < Q_i \) at \( t = 1 \)
- **Interbank liabilities**:
  - formation at \( t = 0 \)
  - realization/expiration at \( t = 1 \): \( L_{ij} \)
- **No external debt**
Interbank liabilities realize at $t = 1$

- $L_{ij}(\omega)$ cash-amount bank $i$ owes bank $j$
- $L_i = \sum_{j \in [m]} L_{ij}$ total nominal liabilities of bank $i$
- $\sum_{j \in [m]} L_{ji}$ total nominal receivables from other banks
Bank $i$’s nominal balance sheet at $t = 1$

- **Assets**
  \[ \gamma_i + \sum_{j \in [m]} L_{ji} + Q_i \]

- **Liabilities**
  \[ L_i + \text{nominal net worth} \]

- **Nominal cash balance**
  \[ \gamma_i + \sum_{j \in [m]} L_{ji} - L_i \]
Liquidation of external asset at $t = 1$

- If bank $i$’s cash balance is negative,

\[
\gamma_i + \sum_{j \in [m]} L_{ji} < L_i,
\]

it sells external assets at liquidation price $P_i < Q_i$

- Bank $i$ is bankrupt if

\[
\underbrace{\gamma_i + \sum_{j \in [m]} L_{ji} + P_i}_{\text{liquidation value of assets}} < L_i,
\]

and then bank $j$ receives a part of liquidation value of bank $i$’s assets
Interbank liability clearing equilibrium

Interbank liability clearing equilibrium defined as \((L^*_{ij})\) satisfying

1. Fair allocation:
   \[
   0 \leq L^*_{ij} \leq L_{ij}
   \]

2. Clearing: \(L^*_i = \sum_{j \in [m]} L^*_{ij}\) satisfies
   \[
   L^*_i = L_i \wedge \left( \gamma_i + \sum_{j \in [m]} L^*_j + P_i \right), \ i \in [m]
   \]

**Assumption 1.**

Let \((L^*_{ij})\) be any interbank liability clearing equilibrium
Example of interbank clearing equilibrium

Eisenberg and Noe (2001): proportionality rule \( \Pi_{ij} = \frac{L_{ij}}{L_i} \) and

\[
L_{ij}^* = \Pi_{ij} \frac{L_i^*}{L_i}
\]

with clearing vector \( \mathbf{L}^* = (L_1^*, \ldots, L_m^*) \) determined as fixed point

\[
\Phi(\mathbf{L}^*) = \mathbf{L}^*
\]

where \( \Phi : [0, \mathbf{L}] \rightarrow [0, \mathbf{L}] \) is given by

\[
\Phi_i(\ell) = L_i \land \left( \gamma_i + \sum_{j \in [m]} \ell_j \Pi_{ji} + P_i \right), \quad i \in [m]
\]

**Theorem 1.1 (Eisenberg and Noe (2001)).**

If \( \gamma_i + P_i > 0 \) for all \( i \) then there exists a unique interbank clearing equilibrium.
Bank $i$’s terminal net worth at $t = 2$

- Fraction of liquidated external asset
  \[ Z_i = \frac{(L_i - \gamma_i - \sum_{j \in [m]} L_{ji}^*)^+}{P_i} \wedge 1 \]

- Assets
  \[ A_i = \gamma_i + \sum_{j \in [m]} L_{ji}^* + Z_i P_i + (1 - Z_i) Q_i \]

- Net worth
  \[ C_i = A_i - L_i \]
Bankruptcy characterization

- Shortfall of bank $i$ equals

\[ C_i^- = L_i - L_i^* \]

- Bank $i$ is bankrupt if and only if

\[ C_i < 0 \quad \text{or} \quad L_i^* < L_i \]

- If bank $i$ is bankrupt then all its external assets are liquidated

\[ Z_i = 1 \]
Lemma 1.2.
The aggregate surplus satisfies

$$\sum_{i \in [m]} C_i^+ = \sum_{i \in [m]} \gamma_i + \sum_{i \in [m]} Q_i - \sum_{i \in [m]} Z_i(Q_i - P_i).$$
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Does a CCP reduce systemic risk?

Pareto optimality analysis
Central Clearing Counterparty (CCP)

- We label the CCP as $i = 0$
- All liabilities are cleared through the CCP
  - star shaped network
- Proportionality rule: CCP liabilities have equal seniority
  - interbank clearing equilibrium is trivial (no fixed point problem)
Capital structure of CCP

- The CCP is endowed with
  - external equity capital $\gamma_0$
  - guarantee fund $\sum_{i=1}^{m} g_i$

  where $g_i \leq \gamma_i$ is received from bank $i$ at time $t = 0$

- Guarantee fund is hybrid, junior to CCP equity capital
- Banks’ shares in the guarantee fund have equal seniority
Liabilities

- Bank $i$’s net exposure to CCP
  \[ \Lambda_i = \sum_{j=1}^{m} L_{ji} - \sum_{j=1}^{m} L_{ij} \]

- Bank $i$’s nominal liability to the CCP (netting)
  \[ \hat{L}_{i0} = (\Lambda_i - g_i)^+ \]

- CCP’s nominal liability to bank $i$
  \[ \hat{L}_{0i} = (1 - f)\Lambda_i^+ \]

→ CCP charges a volume based fee $f$ on bank $i$’s receivables
  \[ f \times \Lambda_i^+ \]
Nominal guarantee fund

- Bank $i$’s nominal share in the guarantee fund:

$$G_i = (\Lambda_i + g_i)^+ - \Lambda_i^+$$

- Linking facts:

$$G_i - \hat{L}_{i0} = g_i - \Lambda_i^-,$$
$$G_i \times \hat{L}_{i0} = 0$$

Figure: $G_i$ and $\hat{L}_{i0}$ as functions of $\Lambda_i$
CCP’s nominal balance sheet at $t = 1$

Denote $G_{\text{tot}} = \sum_{i=1}^{m} G_i$ total nominal value of guarantee fund

- **Assets:** $\gamma_0 + \sum_{i=1}^{m} g_i + \sum_{i=1}^{m} \hat{L}_i$,  
- **Liabilities:** $\hat{L}_0 + G_{\text{tot}} + \text{nominal net worth} \left( \gamma_0 + \sum_{i=1}^{m} f\Lambda_i^+ \right)$. 

Central Counterparty Clearing
Liability clearing equilibrium

- Fraction of external assets liquidated \((\hat{L}_{i0} \times \hat{L}_{0i} = 0)\)

\[
\hat{Z}_i = \frac{\left(\gamma_i - g_i - \hat{L}_{i0}\right)^-}{P_i} \wedge 1
\]

- Clearing payment of bank \(i\) to CCP

\[
\hat{L}^*_i = \hat{L}_{i0} \wedge (\gamma_i - g_i + P_i)
\]

- Value of CCP’s total assets become

\[
\hat{A}_0 = \gamma_0 + \sum_{i=1}^m g_i + \sum_{i=1}^m \hat{L}^*_i
\]

- Clearing payment of CCP

\[
\hat{L}^*_0 = \hat{L}_0 \wedge \hat{A}_0
\]

- Bank \(i\) receives (proportionality rule)

\[
\hat{L}^*_{0i} = \frac{\hat{L}_{0i}}{\hat{L}_0} \times \hat{L}^*_0
\]
Liquidation of the guarantee fund at $t = 2$

- Guarantee fund = first layer, prior to nominal net worth

$$G^*_{\text{tot}} = G_{\text{tot}} \wedge \left( \hat{A}_0 - \hat{L}^*_0 - \gamma_0 - \sum_{i=1}^{m} f \wedge_i^+ \right)^+$$

- Bank $i$ receives (proportionality rule)

$$G^*_i = \frac{G_i}{G_{\text{tot}}} \times G^*_{\text{tot}}$$
Terminal net worth

- **CCP**
  \[
  \hat{C}_0 = \hat{A}_0 - \hat{L}_0 - G^*_\text{tot}
  \]

- **Bank \(i\)’s assets**
  \[
  \hat{A}_i = \gamma_i + \hat{Z}_i P_i + (1 - \hat{Z}_i) Q_i + \frac{\hat{L}_{0i}}{\hat{L}_0} \times \hat{L}^*_0 + G^*_i - g_i
  \]

- **Bank \(i\)’s net worth**
  \[
  \hat{C}_i = \hat{A}_i - \hat{L}_{i0}
  \]

- **Shortfall of CCP and banks becomes**
  \[
  \hat{C}^-_i = \hat{L}_i - \hat{L}^*_i
  \]
Lemma 2.1.

The aggregate surplus satisfies

$$\sum_{i=0}^{m} \widehat{C}_i^+ = \sum_{i=0}^{m} \gamma_i + \sum_{i \in [m]} Q_i - \sum_{i \in [m]} \widehat{Z}_i (Q_i - P_i).$$
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Pareto optimality analysis
Independence from fee and guarantee fund policy

Write \( \mathbf{g} = (g_1, \ldots, g_m) \)

**Lemma 3.1.**
- Number of liquidated assets \( \hat{Z}_i \) does not depend on \((f, \mathbf{g})\)
- Shortfall of bank \( i \) does not depend on \((f, \mathbf{g})\)

\[
\hat{C}_i^- = (\Lambda_i + y_i P + \gamma_i)^-
\]
- Aggregate surplus does not depend on \((f, \mathbf{g})\)
Sensitivity results

- **CCP:**

\[ \frac{\partial \hat{C}_0}{\partial f} \geq 0, \quad \frac{\partial \hat{C}_0}{\partial g_i} \geq 0 \]

- **Bank \(i\):**

\[ \frac{\partial \hat{C}_i}{\partial f} = \frac{\partial \hat{C}_i^+}{\partial f} \leq 0 \]

\[ \frac{\partial \hat{C}_i}{\partial g_j} = \frac{\partial \hat{C}_i^+}{\partial g_j} \begin{cases} \geq 0 & \text{if } i \neq j \\ \leq 0 & \text{if } i = j \end{cases} \]
Aggregate sensitivity results

- Aggregate net worth of financial system is non-decreasing in $g$
  \[
  \frac{\partial \sum_{k=0}^{m} \hat{C}_k}{\partial g_i} = - \frac{\partial \hat{C}_0^-}{\partial g_i} \geq 0, \quad \text{for all } i \in [m]
  \]

- Aggregate net worth of the banks is non-increasing in $g$
  \[
  \frac{\partial \sum_{k=1}^{m} \hat{C}_k}{\partial g_i} = - \frac{\partial \hat{C}_0^+}{\partial g_i} \leq 0, \quad \text{for all } i \in [m]
  \]

- Same for $f$
Impact of CCP on net worth of banks

- Compare financial network with and without CCP
- **Convention:** For comparison we set

  \[ C_0 = \gamma_0 \]
CCP state-wise impact

- CCP always reduces
  - liquidation losses
    \[ \hat{Z}_i \leq Z_i \]
  - bank shortfalls (bankruptcy cost)
    \[ \hat{C}_i^- \leq C_i^- \]

- CCP always improves
  - aggregate terminal bank net worth
    \[ \sum_{i=1}^{m} \hat{C}_i \geq \sum_{i=1}^{m} C_i \]
  - aggregate surplus
    \[ \sum_{i=0}^{m} \hat{C}_i^+ = \sum_{i=0}^{m} C_i^+ + (Q_i - P_i) \sum_{i=1}^{m} (Z_i - \hat{Z}_i) \geq 0 \]

- CCP imposes shortfall risk \( \hat{C}_0^- \geq 0 \)
CCP capital impact decomposition

**Lemma 3.2.**

*Difference in capital of bank $i \in [m]$ is given by*

$$\hat{C}_i - C_i = T_1 + T_2 + T_3$$

*where ...*
...difference in capital due to

- counterparty default:

\[ T_1 = -\frac{\Lambda_i^+}{\sum_{i=1}^{m} \Lambda_i^+} \hat{C}_0^- + \sum_{j=1}^{m} (L_{ji} - L_{ji}^*) \]

- liquidation loss:

\[ T_2 = (Z_i - \hat{Z}_i)(Q_i - P_i) \geq 0 \]

- fees and losses in guarantee fund:

\[ T_3 = -f \Lambda_i^+ - \frac{G_i}{G_{tot}} (G_{tot} - G_{tot}^*) \leq 0 \]
Figure: Expected capital difference components, for $f = 0$
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Systemic risk measure as in Chen et al. (2013)

- Write \( \mathbf{C} = (C_0, \ldots, C_m) \), and similarly \( \hat{\mathbf{C}} \)
- Generic coherent risk measure \( \rho(X) \)
- Aggregation function, \( \alpha \in [1/2, 1] \),

\[
A_\alpha(\mathbf{C}) = \alpha \sum_{i=0}^{m} C_i^- - (1 - \alpha) \sum_{i=0}^{m} C_i^+
\]

- Systemic risk measure

\[
\rho_\alpha(\mathbf{C}) = \rho(A_\alpha(\mathbf{C}))
\]
Lemma 4.1.

\[ A_\alpha(\hat{C}) - A_\alpha(C) = \alpha \hat{C}_0^- - \Delta_\alpha \]

where

\[ \Delta_\alpha = \alpha \sum_{i \in [m]} \left( C_i^- - \hat{C}_i^- \right) + (1 - \alpha)(Q - P) \sum_{i \in [m]} \left( Z_i - \hat{Z}_i \right) \]

is nonnegative, \( \Delta_\alpha \geq 0 \), and does not depend on \((f, g)\). Hence

\[ \rho_\alpha(\hat{C}) - \rho_\alpha(C) = \rho \left( A_\alpha(\hat{C}) \right) - \rho \left( A_\alpha(C) \right) \leq \rho \left( A_\alpha(\hat{C}) - A_\alpha(C) \right) \]

\[ \leq \alpha \rho \left( \hat{C}_0^- \right) + \rho(-\Delta_\alpha) \]

with equality if \( \rho(X) = \mathbb{E}[X] \)
Impact on systemic risk measure

**Theorem 4.2.**
The CCP reduces systemic risk if (and only if)

\[ \alpha \rho \left( \hat{C}_0^- \right) < -\rho \left( -\Delta_\alpha \right) \]

(for \( \rho(X) = \mathbb{E}[X] \)). The RHS does not depend on \((f, g)\).
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Pareto optimality analysis
CCP and banks’ utility function

- CCP and banks are risk neutral
- Utility function = expected surplus
  \[ u_i(f, g) = \mathbb{E}\left[\hat{C}_i^+\right] \]
- Participation constraints: \((f, g)\) is feasible if
  \[ u_0(f, g) \geq \gamma_0 \quad \text{competitive case} \]
  \[ u_i(f, g) \geq \mathbb{E}\left[C_i^+\right], \quad i \in [m], \quad \text{monopolistic case} \]
Symmetric case

- $\gamma_i \equiv \gamma, g_i \equiv g$, and

$$ (Q_i, P_i, \{L_{ij}\}_{j=1}^{m}, \{L_{ji}\}_{j=1}^{m}), \quad i \in [m] $$

is exchangeable

- Consequence:

$$ u_0(f, g) + mu_1(f, g) = \gamma_0 + \mathbb{E}[\geq 0] \equiv \text{constant} $$

- Consequence: every feasible $(f, g)$ is Pareto optimal
Numerical result: parameters

- Complete inter dealer network based on BIS 2010 data
- \( m = 14 \) banks
- \( \gamma_0 = \$5bn \)
- \( \gamma = \$10bn \)
- total notional \( \$16tn \)
Numerical result: Pareto optimal policies

Figure: Feasible Pareto optimal policies, and systemic risk zero line
Conclusion

- Simple general financial network setup with and without CCP
- CCP always improves aggregate surplus through lower forced liquidation losses
- CCP always reduces banks’ bankruptcy cost
- CCP introduces tail risk, and may increase systemic risk
- Find sufficient (and necessary) condition for systemic risk reduction
- Numerical example shows that CCP reduces systemic risk for feasible fee and guarantee fund policies (open question: does this hold in general?)