Systemic Risk and Central Counterparty Clearing

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Systemic Risk and Financial Networks
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Outline

Financial Network

Central Counterparty Clearing

Comparative statics

Does a CCP reduce systemic risk?

Pareto optimality analysis

Outline

Setup

- Two periods $t = 0, 1, 2$
- Values at $t = 1, 2$ are random variables on $(\Omega, \mathcal{F})$
- m interlinked banks $[m] := \{1, 2, \ldots, m\}$

Instruments

Bank i holds

- Cash γ_i: zero return
- External asset (e.g. long-term investment maturing at $t = 2$):
  - fundamental value Q_i at $t = 1, 2$
  - liquidation value $P_i < Q_i$ at $t = 1$
- Interbank liabilities:
  - formation at $t = 0$
  - realization/expiration at $t = 1$: L_{ij}
- No external debt

Interbank liabilities realize at $t = 1$

- $L_{ij}(\omega)$ cash-amount bank i owes bank j
- $L_i = \sum_{j \in [m]} L_{ij}$ total nominal liabilities of bank i
- $\sum_{j \in [m]} L_{ji}$ total nominal receivables from other banks

Bank i's nominal balance sheet at $t = 1$

- Assets

$$\gamma_i + \sum_{j \in [m]} L_{ji} + Q_i$$

- Liabilities

$$L_i + \text{nominal net worth}$$

- Nominal cash balance

$$\gamma_i + \sum_{j \in [m]} L_{ji} - L_i$$

Liquidation of external asset at $t = 1$

- If bank i's cash balance is negative,

$$\gamma_i + \sum_{j \in [m]} L_{ji} < L_i,$$

  it sells external assets at liquidation price $P_i < Q_i$

- Bank i is bankrupt if

$$\underbrace{\gamma_i + \sum_{j \in [m]} L_{ji} + P_i}_{\text{liquidation value of assets}} < L_i,$$

  and then bank j receives a part of liquidation value of bank i's assets

Interbank liability clearing equilibrium

Interbank liability clearing equilibrium defined as (L^*_{ij}) satisfying

1. Fair allocation:
$$0 \leq L^*_{ij} \leq L_{ij}$$
2. Clearing: $L^*_i = \sum_{j \in [m]} L^*_{ij}$ satisfies
$$L^*_i = L_i \wedge \left(\gamma_i + \sum_{j \in [m]} L^*_{ji} + P_i \right), \ i \in [m]$$

Assumption 1.
Let (L^*_{ij}) be any interbank liability clearing equilibrium

Example of interbank clearing equilibrium

Eisenberg and Noe (2001): proportionality rule $\Pi_{ij} = L_{ij}/L_i$ and

$$L^*_{ij} = \Pi_{ij} L^*_i$$

with clearing vector $\mathbf{L}^* = (L^*_1, \dots, L^*_m)$ determined as fixed point

$$\Phi(\mathbf{L}^*) = \mathbf{L}^*$$

where $\Phi : [0, \mathbf{L}] \to [0, \mathbf{L}]$ is given by

$$\Phi_i(\ell) = L_i \wedge \left(\gamma_i + \sum_{j \in [m]} \ell_j \Pi_{ji} + P_i\right), \; i \in [m]$$

Theorem 1.1 (Eisenberg and Noe (2001)).

If $\gamma_i + P_i > 0$ for all i then there exists a unique interbank clearing equilibrium.

Bank i's terminal net worth at $t = 2$

- Fraction of liquidated external asset

$$Z_i = \frac{\left(L_i - \gamma_i - \sum_{j\in[m]} L^*_{ji}\right)^+}{P_i} \wedge 1$$

- Assets

$$A_i = \gamma_i + \sum_{j\in[m]} L^*_{ji} + Z_i P_i + (1 - Z_i) Q_i$$

- Net worth

$$C_i = A_i - L_i$$

Bankruptcy characterization

- Shortfall of bank i equals

$$C_i^- = L_i - L_i^*$$

- Bank i is bankrupt if and only if

$$C_i < 0 \quad (\text{or } L_i^* < L_i)$$

- If bank i is bankrupt then all its external assets are liquidated

$$Z_i = 1$$

Aggregate surplus identify

Lemma 1.2.
The aggregate surplus satisfies

$$\sum_{i\in[m]} C_i^+ = \sum_{i\in[m]} \gamma_i + \sum_{i\in[m]} Q_i - \sum_{i\in[m]} Z_i(Q_i - P_i).$$

Outline

Central Clearing Counterparty (CCP)

- We label the CCP as $i = 0$
- All liabilities are cleared through the CCP

$\rightarrow$ star shaped network

- Proportionality rule: CCP liabilities have equal seniority

$\rightarrow$ interbank clearing equilibrium is trivial (no fixed point problem)

Capital structure of CCP

- The CCP is endowed with
  - external equity capital γ_0
  - **guarantee fund**

    $$\sum_{i=1}^{m} \mathbf{g_i}$$

    where $\mathbf{g_i} \leq \gamma_i$ is received from bank i at time $t = 0$
- Guarantee fund is hybrid, junior to CCP equity capital
- Banks' shares in the guarantee fund have equal seniority

Liabilities

- Bank i's net exposure to CCP

$$\Lambda_i = \sum_{j=1}^{m} L_{ji} - \sum_{j=1}^{m} L_{ij}$$

- Bank i's nominal liability to the CCP (**netting**)

$$\widehat{L}_{i0} = \left(\Lambda_i^- - \mathbf{g_i}\right)^+$$

- CCP's nominal liability to bank i

$$\widehat{L}_{0i} = (1 - f)\Lambda_i^+$$

→ CCP charges a volume based fee f on bank i's receivables

$$f \times \Lambda_i^+$$

Nominal guarantee fund

- Bank i's nominal share in the guarantee fund:

$$G_i = (\Lambda_i + g_i)^+ - \Lambda_i^+$$

- Linking facts:

$$G_i - \widehat{L}_{i0} = g_i - \Lambda_i^-, \quad G_i \times \widehat{L}_{i0} = 0$$

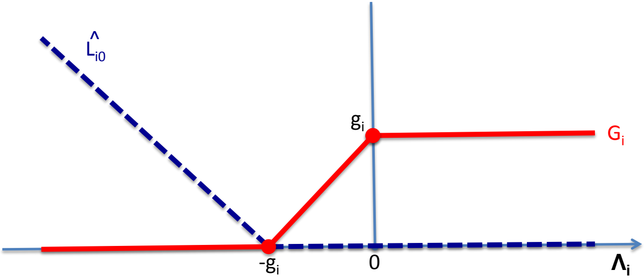

Figure: G_i and $\widehat{L}_{i0}$ as functions of Λ_i

CCP's nominal balance sheet at $t = 1$

Denote $G_{\text{tot}} = \sum_{i=1}^{m} G_i$ total nominal value of guarantee fund

- Assets: $\gamma_0 + \sum_{i=1}^{m} g_i + \sum_{i=1}^{m} \widehat{L}_{i0}$,
- Liabilities: $\widehat{L}_0 + G_{\text{tot}} +$ nominal net worth $\big(\gamma_0 + \sum_{i=1}^{m} f\Lambda_i^+\big)$.

Liability clearing equilibrium

- Fraction of external assets liquidated ($\widehat{L}_{i0} \times \widehat{L}_{0i} = 0$)

$$\widehat{Z}_i = \frac{\left(\gamma_i - g_i - \widehat{L}_{i0}\right)^-}{P_i} \wedge 1$$

- Clearing payment of bank i to CCP

$$\widehat{L}_i^* = \widehat{L}_{i0} \wedge (\gamma_i - g_i + P_i)$$

- Value of CCP's total assets become

$$\widehat{A}_0 = \gamma_0 + \sum_{i=1}^m g_i + \sum_{i=1}^m \widehat{L}_i^*$$

- Clearing payment of CCP

$$\widehat{L}_0^* = \widehat{L}_0 \wedge \widehat{A}_0$$

- Bank i receives (proportionality rule)

$$\widehat{L}_{0i}^* = \frac{\widehat{L}_{0i}}{\widehat{L}_0} \times \widehat{L}_0^*$$

Liquidation of the guarantee fund at $t = 2$

- Guarantee fund = first layer, prior to nominal net worth

$$G^*_{\mathrm{tot}} = G_{\mathrm{tot}} \wedge \left(\widehat{A}_0 - \widehat{L}^*_0 - \gamma_0 - \sum_{i=1}^{m} f \Lambda_i^+ \right)^+$$

- Bank i receives (proportionality rule)

$$G^*_i = \frac{G_i}{G_{\mathrm{tot}}} \times G^*_{\mathrm{tot}}$$

Terminal net worth

- CCP

$$\widehat{C}_0 = \widehat{A}_0 - \widehat{L}_0 - G^*_{\mathrm{tot}}$$

- Bank i's assets

$$\widehat{A}_i = \gamma_i + \widehat{Z}_i P_i + (1 - \widehat{Z}_i) Q_i + \frac{\widehat{L}_{0i}}{\widehat{L}_0} \times \widehat{L}^*_0 + G^*_i - g_i$$

- Bank i's net worth

$$\widehat{C}_i = \widehat{A}_i - \widehat{L}_{i0}$$

- Shortfall of CCP and banks becomes

$$\widehat{C}^-_i = \widehat{L}_i - \widehat{L}^*_i$$

Aggregate surplus identity with CCP

Lemma 2.1.
The aggregate surplus satisfies

$$\sum_{i=0}^{m} \widehat{C}_i^{+} = \sum_{i=0}^{m} \gamma_i + \sum_{i\in[m]} Q_i - \sum_{i\in[m]} \widehat{Z}_i(Q_i - P_i).$$

Outline

Independence from fee and guarantee fund policy

Write $\mathbf{g} = (g_1, \ldots, g_m)$

Lemma 3.1.

- *Number of liquidated assets $\widehat{Z}_i$ does not depend on $(f, \mathbf{g})$*
- *Shortfall of bank i does not depend on $(f, \mathbf{g})$*

$$\widehat{C}_i^- = (\Lambda_i + y_i P + \gamma_i)^-$$

- *Aggregate surplus dos not depend on $(f, \mathbf{g})$*

Sensitivity results

- CCP:

$$\frac{\partial \widehat{C}_0}{\partial f} \geq 0, \quad \frac{\partial \widehat{C}_0}{\partial g_i} \geq 0$$

- Bank i:

$$\frac{\partial \widehat{C}_i}{\partial f} = \frac{\partial \widehat{C}_i^+}{\partial f} \leq 0$$
$$\frac{\partial \widehat{C}_i}{\partial g_j} = \frac{\partial \widehat{C}_i^+}{\partial g_j} \begin{cases} \geq 0 & \text{if} \quad i \neq j \\ \leq 0 & \text{if} \quad i = j \end{cases}$$

Aggregate sensitivity results

- Aggregate net worth of financial system is non-decreasing in $\mathbf{g}$

$$\frac{\partial \sum_{k=0}^{m} \widehat{C}_k}{\partial g_i} = -\frac{\partial \widehat{C}_0^-}{\partial g_i} \geq 0, \quad \text{for all } i \in [m]$$

- Aggregate net worth of the banks is non-increasing in $\mathbf{g}$

$$\frac{\partial \sum_{k=1}^{m} \widehat{C}_k}{\partial g_i} = -\frac{\partial \widehat{C}_0^+}{\partial g_i} \leq 0, \quad \text{for all } i \in [m]$$

- Same for f

Impact of CCP on net worth of banks

- Compare financial network with and without CCP
- **Convention:** For comparison we set

$$C_0 = \gamma_0$$

CCP state-wise impact

- CCP always reduces
  - liquidation losses
    $$\widehat{Z}_i \leq Z_i$$
  - bank shortfalls (bankruptcy cost)
    $$\widehat{C}_i^- \leq C_i^-$$
- CCP always improves
  - aggregate terminal bank net worth
    $$\sum_{i=1}^m \widehat{C}_i \geq \sum_{i=1}^m C_i$$
  - aggregate surplus
    $$\sum_{i=0}^m \widehat{C}_i^+ = \sum_{i=0}^m C_i^+ + \underbrace{(Q_i - P_i) \sum_{i=1}^m (Z_i - \widehat{Z}_i)}_{\geq 0}$$
- CCP imposes shortfall risk $\widehat{C}_0^- \geq 0$

CCP capital impact decomposition

Lemma 3.2.
Difference in capital of bank $i \in [m]$ is given by

$$\widehat{C}_i - C_i = T_1 + T_2 + T_3$$

where . . .

…difference in capital due to

- counterparty default:

$$T_1 = -\frac{\Lambda_i^+}{\sum_{i=1}^m \Lambda_i^+}\widehat{C}_0^- + \sum_{j=1}^m (L_{ji} - L_{ji}^*)$$

- liquidation loss:

$$T_2 = (Z_i - \widehat{Z}_i)(Q_i - P_i) \geq 0$$

- fees and losses in guarantee fund:

$$T_3 = -f\Lambda_i^+ - \frac{G_i}{G_{\mathrm{tot}}}\left(G_{\mathrm{tot}} - G_{\mathrm{tot}}^*\right) \leq 0$$

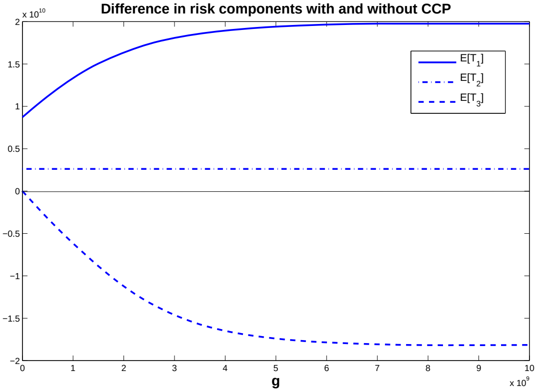

Figure: Expected capital difference components, for $f = 0$

Outline

Systemic risk measure as in Chen et al. (2013)

- Write $\mathbf{C} = (C_0, \ldots, C_m)$, and similarly $\widehat{\mathbf{C}}$
- Generic coherent risk measure $\rho(X)$
- Aggregation function, $\alpha \in [1/2, 1]$,

$$A_\alpha(\mathbf{C}) = \underbrace{\alpha \sum_{i=0}^{m} C_i^-}_{\text{bankruptcy cost}} - \underbrace{(1-\alpha) \sum_{i=0}^{m} C_i^+}_{\text{tax benefits}}$$

- Systemic risk measure

$$\rho_\alpha(\mathbf{C}) = \rho\left(A_\alpha(\mathbf{C})\right)$$

Impact on aggregation function

Lemma 4.1.

$$A_\alpha(\widehat{\mathbf{C}}) - A_\alpha(\mathbf{C}) = \alpha \widehat{C}_0^- - \Delta_\alpha$$

where

$$\Delta_\alpha = \alpha \sum_{i\in[m]} \left(C_i^- - \widehat{C}_i^- \right) + (1-\alpha)(Q-P) \sum_{i\in[m]} \left(Z_i - \widehat{Z}_i \right)$$

is nonnegative, $\Delta_\alpha \geq 0$*, and does not depend on* $(f, \mathbf{g})$*. Hence*

$$\rho_\alpha(\widehat{\mathbf{C}}) - \rho_\alpha(\mathbf{C}) = \rho\left(A_\alpha(\widehat{\mathbf{C}})\right) - \rho\left(A_\alpha(\mathbf{C})\right) \leq \rho\left(A_\alpha(\widehat{\mathbf{C}}) - A_\alpha(\mathbf{C})\right)$$
$$\leq \alpha\rho\left(\widehat{C}_0^-\right) + \rho(-\Delta_\alpha)$$

with equlity if $\rho(X) = \mathbb{E}[X]$

Impact on systemic risk measure

Theorem 4.2.
The CCP reduces systemic risk if (and only if)

$$\alpha\rho\left(\widehat{C}_0^-\right) < -\rho\left(-\Delta_\alpha\right)$$

(for $\rho(X) = \mathbb{E}[X]$*). The RHS does not depend on* $(f, \mathbf{g})$*.*

Outline

CCP and banks' utility function

- CCP and banks are risk neutral
- Utility function = expected surplus

$$u_i(f, \mathbf{g}) = \mathbb{E}\left[\widehat{C}_i^+\right]$$

- Participation constraints: $(f, \mathbf{g})$ is feasible if

$$\begin{aligned} &u_0(f, \mathbf{g}) \geq \gamma_0 \quad \text{competitive case} \\ &u_i(f, \mathbf{g}) \geq \mathbb{E}\left[C_i^+\right], \quad i \in [m], \quad \text{monopolistic case} \end{aligned}$$

Symmetric case

- $\gamma_i \equiv \gamma$, $g_i \equiv g$, and

$$(Q_i, P_i, \{L_{ij}\}_{j=1\ldots m}, \{L_{ji}\}_{j=1\ldots m}), \quad i \in [m]$$

  is exchangeable
- Consequence:

$$u_0(f, g) + mu_1(f, g) = \gamma_0 + \mathbb{E}\left[\geq 0\right] \equiv \text{constant}$$

- Consequence: every feasible (f, g) is Pareto optimal

Numerical result: parameters

- Complete inter dealer network based on BIS 2010 data
- $m = 14$ banks
- $\gamma_0 = \$5bn$
- $\gamma = \$10bn$
- total notional $\$16tn$

Numerical result: Pareto optimal policies

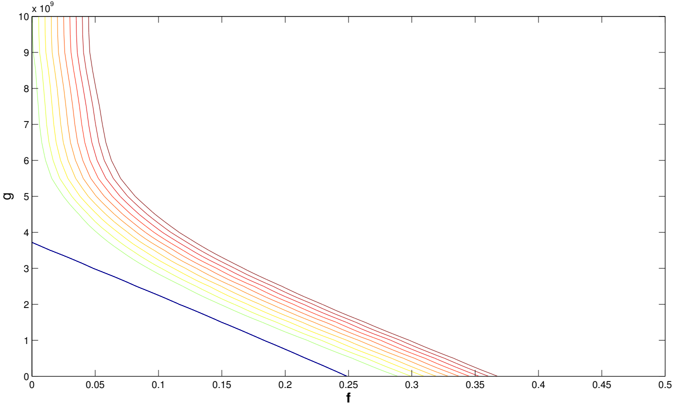

Figure: Feasible Pareto optimal policies, and systemic risk zero line

Conclusion

- Simple general financial network setup with and without CCP
- CCP always improves aggregate surplus through lower forced liquidation losses
- CCP always reduces banks' bankruptcy cost
- CCP introduces tail risk, and may increase systemic risk
- Find sufficient (and necessary) condition for systemic risk reduction
- Numerical example shows that CCP reduces systemic risk for feasible fee and guarantee fund policies (open question: does this hold in general?)