Agent-based Networks of Corporate Lending

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Parts based on research with U. Kochańska (ECB) and Ch. Kok (ECB)

Motivation

- Recent financial crisis: loss of trust on the interbank market; concerns about failure of one of the key players spreading contagion; small shocks with detrimental effects
- A response from regulators: measures to mitigate the risk $\Rightarrow$ higher capital standards + reducing bilateral exposures
  - Large Exposure limits;
  - Credit Valuation Adjustment to unlock the risk in OTC exposures and immediately reflect it in the capital
  - Standard settlement practices (CCP framework)
  - ...but usually only interbank market modelled $\rightarrow$ a large part of the network is neglected
- Our aim:
  - fill the gap in the literature to improve understanding of:
    - linkages (and emergence of links) between banks and the real economy (non-bank corporate sector)
    - risk stemming from interconnectedness
- Approach: modelling of banks' reactions to these measures and to the changing macroeconomic environment with links to corp sector (combining risk/return trade-offs, funding conditions...)

Outline

Modeling framework – agent-based interbank+corporate networks

- Four round model – endogenous formation
- Interbank augmented by non-bank corporate sector (called: firms)
  1. offers of interbank placements based on individual optimisation of interbank asset structures
  2. funding diversification
  3. negotiation phase: matching offers and preferred funding structure in a bargaining game
  4. price (i.e. interest rate) adjustment (if demand $\neq$ supply)

Scope for application

- stress tests and dynamic balance sheet tool
- assessing network effects of credit provision to the real economy (shocks from corporate sector)
- parametrisation of LE and concentration limits (so far only for interbank) and sector RWs

Literature – towards network formation

- Networks in other research areas: game theory of Jackson and Wolinsky (1996)
- Extensions in finance – exogenous networks: game theory – optimal responses of banks to shocks to incentives to lend Cohen-Cole (2011); Bluhm et al. (2013). Acemoglu et al. (2013): dealing with social inefficiency of financial networks; Georg (2011) models interbank exposures as residuals of banks' investment activities (but networks simply drawn from a distribution)
- Jackson and Watts (2002) combine stochastic games and matching problems to study general principles of network formation in economics; Acemoglu et al. (2014) create the interbank structure based on equilibrium of lending contracts and repayments
- Agent-based approach to address overly complex equilibria – Markose (2012); Grasselli (2013)
- Matching (Chen, 2013); (Duffie and Sun 2012) and price formation (Eisenschmidt, 2009) $\Rightarrow$ mechanisms important for us

Formation of the lending network – Endogenous networks

The aim of the project is to:

1. understand foundations of the topology of lending networks in the economy
2. capture sensitivity of the interbank network structures to the heterogeneity of banks (in terms of size of balance sheet, capital position, general profitability, counterparty credit risk) and the changes of market and bank specific risk parameters
3. provide a framework to assess effectiveness of rule designed to mitigate systemic risk on the interbank system (esp. pertaining to capital requirements, size and diversity of interbank exposures)

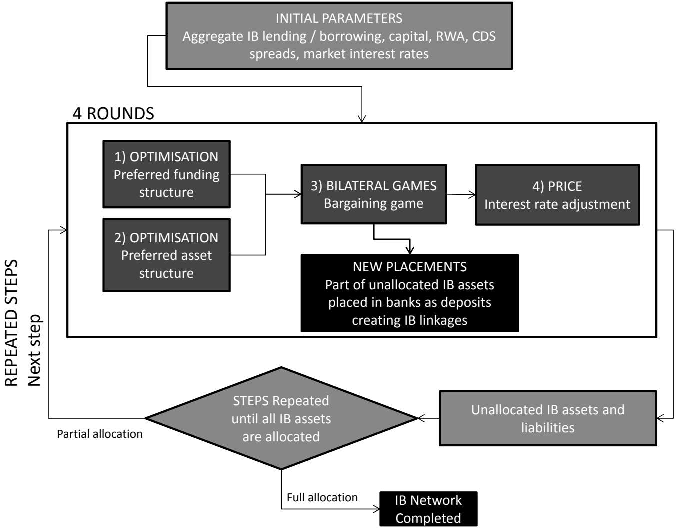
4 round model – outline

The following 4 rounds are repeated until $\simeq$all interbank assets of a predefined volume are invested (separate for interbank and bank-firm network)

1. Firms make loan offers to other banks and firms which are drawn from a probability map: offers based on optimisation of their interbank asset structures and corporate lending portfolio
2. Firms formulate their preferred structure of interbank (banks) and bank (firms) funding from banks drawn in round 1: based on the diversification of the funding (rollover) risk
3. Firms enter negotiation phase: bargaining game in order to try to match the preferred allocation of the assets and the preferred structure of interbank (bank) funding
4. Firms reconsider their pricing offers: firms with open funding gap incrementally adjust their offers of interest payments on new loan (optional feature, not used so far in the exposition)

At each step, assets are "matched" with liabilities incrementally

Figure 1: The sequential four round procedure of the interbank formation (**formation of bank-firm links separate** but analogous)



Prerequisites

- (nodes) N banks and M non-bank firms: capital and bank borrowing + out-degree distribution within (NACE) sectors
- (exposures) Let L_{ij} denotes the interbank (bank) placement (loan) of bank j in bank (firm) i.
- (capital position – constraint for risk-taking) total capital e and capital $e^I \leq e$ allocated to the interbank assets, $e^C \leq e$ allocated to non-bank firms; risk weights ω of exposures.
- (probability map P) of interbank and bank-firm connections drawn from P allowing for capturing possible customer relationship between banks and firms. Each bank j draws its counterparties $B_j^k \subset \mathbb{N}/\{j\}$, enlarging the set at each step k: $\bar{B}_j^{k+1} = \bar{B}_j^k \cup B_j^{k+1}$;
  In addition, firms choose max number (m_j) of banks granting loans based on out-degree distribution, i.e. $\#B_j^{k+1} \leq m_j$
- (matching) at step k incremental matching of assets and liabilities: $\bar{a}_j^k = \bar{a}_j^{k-1} - \sum_i L_{ij}^k$, where L^k is a matrix of placements at step k

1^{st} round – Criteria for investment of interbank assets

General idea of banks' optimising behaviour

Assumption (i): each bank maximises return from loan portfolio adjusted by risk related to interest rates and counterparts' defaults (with a predefined risk aversion parameter) and taking into account customer relationship, i.e. a drawn sample of banks and firms
Assumption (ii): optimisation of interbank portfolio separate from optimisation of non-bank corporate loan portfolio

Each bank maximises the following function of its interbank exposure breakdown:

$$J(L_{1j}, \ldots, L_{Nj}) = \sum_{i \in \bar{B}_j^k} r_i L_{ij} - \kappa_j (\sigma * L_{\cdot j}^{\top})^{\top} Q(\sigma * L_{\cdot j}) \tag{1}$$

Outcome: a matrix of exposures $L^{I,k}$, whereby optimisation subject to constraints...

...Constraints of the admissible set of strategies

The maximisation is subject to some feasibility and capital constraints.

1. budget constraint: $\sum_{j|j\neq i} L_{ij} = \bar{a}_j^k$ and $L_{jj} = 0$, for $\bar{a}_j^0 = \bar{a}_j$ being exogenously determined;
2. counterpart's size constraint: $L_{ij} \leq \bar{l}_i^k$;
3. capital constraint: $\sum_{i|i\neq j} \omega_i(L_{ij}^k + L_{ij}) \leq e_j^I - \gamma^\top(\bar{L}_{\cdot j} + L_{\cdot j})$;
4. large exposure limit constraint: $L_{ij} \leq \chi e_j$.

What if the constraints are too stringent for a bank j? $\Rightarrow$ bank j reduces its interbank lending and (technically) the optimisation is solved for $\bar{a}_j^k$ replaced by $\bar{a}_i^k - 2\Delta\bar{a}_i^k$, $\bar{a}_i^k - 3\Delta\bar{a}_i^k$,... until $\bar{a}_i^k - k_i\Delta\bar{a}_i^k$ gives a feasible set of constraints

2^{nd} round – funding diversification

Diversification risk gauged by default risk

$$X_j := \begin{cases} 0 & \text{with probability } p_j \\ 1 & \text{with probability } 1 - p_j \end{cases} \tag{2}$$

Assumption: p_js are risky (variance based on time series of CDS spreads)
For a covariance matrix $\bar{D}_X^2$ of X, the optimised funding risk is measured

$$F(L_{i1}^k, \ldots, L_{iN}^k) = \kappa^F [L_{i1}^k \ \ldots \ L_{iN}^k] \bar{D}_X^2 [L_{i1}^k \ \ldots \ L_{iN}^k]^\top \tag{3}$$

Outcome: a matrix of interbank deposits $L^{F,k}$, whereby optimisation on the admissible set:
$\mathcal{A}_i^F := \{y \in \mathbb{R}_+^N | j \in \bar{B}_j^k \Rightarrow y_j \leq \bar{a}_j^k \text{ and } j \notin \bar{B}_j^k \Rightarrow y_j = 0\}$.

REMARK: inclusion of non-bank corporate sector implies that (3) is also solved by non-bank firms ($\Rightarrow L^{F,k}$ is $(N+M) \times (N+M)$ matrix)

3^{rd} round – the game

Assumption: banks negotiate loans in pairs simultaneously (pair (i', j) knows the outcome of (i'', j) after both games are completed). Case $L_{ij}^{I,k} > L_{ij}^{F,k}$

$$G_{ij}^{k}(x) = \left[U_{ij}^{I,k*} - s_{ij}^{I,k} \cdot (x - L_{ij}^{F,k})\right]\left[U_{ij}^{a,k*} - s_{ij}^{a,k} \cdot (L_{ij}^{I,k} - x)\right] \tag{4}$$

where $s_{ij}^{I,k}$ is a measure of how much bank i is willing to deviate from his optimal funding strategy, i.e.

$$s_{ij}^{I,k} = \max\left(\frac{U_{ij}^{I,k}(L_{ij}^{F,k}) - U_{ij}^{I,k}(L_{ij}^{I,k})}{|L_{ij}^{I,k} - L_{ij}^{F,k}|}, 0\right),$$

where $U_{ij}^{I,k}(x) = -F(L_{i1}^{F,k}, \ldots, L_{ij-1}^{F,k}, x, L_{ij+1}^{F,k}, \ldots, L_{iN}^{F,k})$
(for $s_{ij}^{a,k}$ analogously,... and for $L_{ij}^{I,k} < L_{ij}^{F,k}$ similar)
Goal of the game: maximisation of G_{ij}^{k}; outcome $\rightarrow \tilde{L}^{G,I,k}$ and $\tilde{L}^{G,F,k}$

3^{rd} round – correction to the game

Observation: the outcome of the game may not be consistent with the budget constraint of the given size of banks' aggregate interbank portfolios (i.e. $\bar{a}^k$ and $\bar{l}^k$)

Correction

- if $\sum_i \tilde{L}_{ij}^{G,I,k} > \bar{a}_j^k$ then $\tilde{L}_{ij}^{G,I,k} := \tilde{L}_{ij}^{G,I,k} * \frac{\bar{a}_j^k}{\sum_i \tilde{L}_{ij}^{G,I,k}}$
- if $\sum_j \tilde{L}_{ij}^{G,F,k} > \bar{l}_i^k$ then $\tilde{L}_{ij}^{G,F,k} := \tilde{L}_{ij}^{G,F,k} * \frac{\bar{l}_i^k}{\sum_j \tilde{L}_{ij}^{G,F,k}}$
- $L^{G,k} = \min(\tilde{L}^{G,I,k}, \tilde{L}^{G,F,k})$ (element-wise)

Aggregation of the step k with outcomes of steps $1, \ldots, k-1$

- $\bar{L}^{k+1} = \bar{L}^k + L^{G,k}$

Update: $\bar{e}^{I,k+1}$, $\bar{a}^{k+1}$ and $\bar{l}^{k+1}$

4^{th} round – price adjustment [optional]

- After the first 3 rounds of a step k some banks may still have a gap in the interbank funding $\Rightarrow$ adjustment to the offered interest rate on new interbank deposits to increase a chance to obtain funding in step $k+1$
- If at the step $k+1$ the gap amounts to $g_i^{k+1} := l_i - \sum_j \bar{L}_{ij}^{k+1}$ then the adjusted offered rate satisfies $r_i^{k+1} = r_i^k \exp(\alpha g_i^{k+1} / l_i)$.

REMARKS

- in the baseline case we assume $\alpha = 0.25$
- no clearing mechanism implemented – would be good to have it since price adjustment mechanism does not prove to be fully effective (a sequential 4-round model with only one step and price adjustment does not lead to full allocation of assets / liabilities)

Sensitivity analysis based on a stylised setup

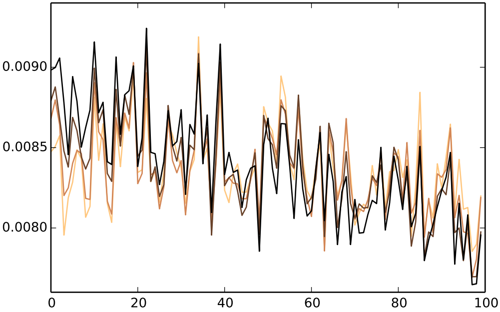
Assumptions

- 100 banks ($bank \in \{1, \dots, 100\}$)
- uniform probability map (5% probability of connection for all pairs of nodes)
- uniform return (0.05) and investment risk (0.1) and funding risk (0.1) parameters
- no correlation

Scenario 1: Risk of investment on the interbank become heterogenous (part 1)

- x-axis: banks
- y-axis: betweenness centrality
- light copper lines correspond to the baseline case; the darker the color the higher heterogeneity of investment risk ($\sigma_{bank}^{line} = \sigma_{bank}^{base} + line * \mathrm{d}\sigma * \frac{bank}{100}$)
- Centrality of more 'risky' nodes decreases and the (relatively) less 'risky' increases

Figure 2: Betweenness centrality vs changing risk of the return from interbank investment

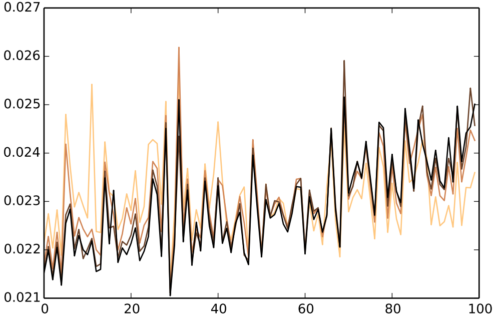


Source: own calculations

Scenario 1: Risk of investment on the interbank become heterogenous (part 2)

- x-axis: banks
- y-axis: Marginal (of last step tranche) price
- light copper lines correspond to the baseline case; the darker the color the higher heterogeneity of investment risk ($\sigma_{bank}^{line} = \sigma_{bank}^{base} + line * \mathrm{d}\sigma * \frac{bank}{100}$)
- (Unsurprisingly) price paid by banks that have more investment risk increases since they must search longer for funding.

Figure 3: Marginal price vs changing risk of return from interbank investment

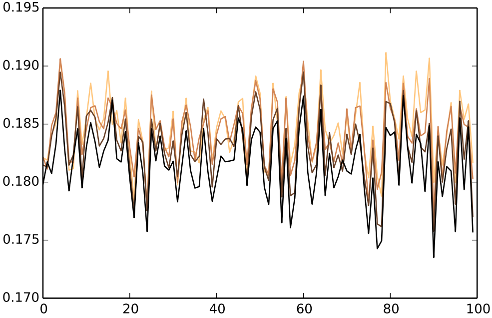


Source: own calculations

Scenario 1: Risk of investment on the interbank become heterogenous (part 3)

- x-axis: banks
- y-axis: clustering coefficient
- light copper lines correspond to the baseline case; the darker the color the higher heterogeneity of investment risk ($\sigma_{bank}^{line} = \sigma_{bank}^{base} + line * \mathrm{d}\sigma * \frac{bank}{100}$)
- Heterogeneity of risk translates into uniform shift of number of triangles associated with nodes.

Figure 4: Clustering coeff. vs changing risk of investment into interbank assets

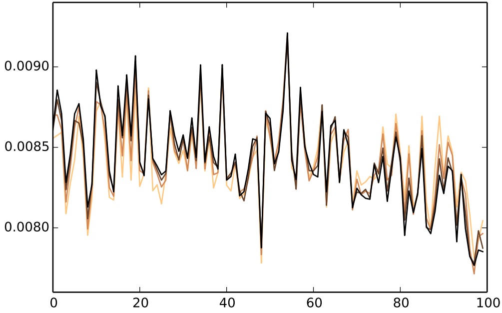


Source: own calculations

Scenario 2: Risk of investment on the interbank become heterogenous

- x-axis: banks
- y-axis: Betweenness centrality
- light copper lines correspond to the baseline case; the darker the color the higher heterogeneity of funding risk ($\sigma_{bank}^{line} = \sigma_{bank}^{base} + line * \mathrm{d}\sigma * \frac{bank}{100}$)
- Centrality of more 'risky' nodes decreases and the (relatively) less 'risky' increases

Figure 5: Betweenness centrality vs changing risk of the funding (roll-over) risk



Source: own calculations

Table 1: Overview of data inputs

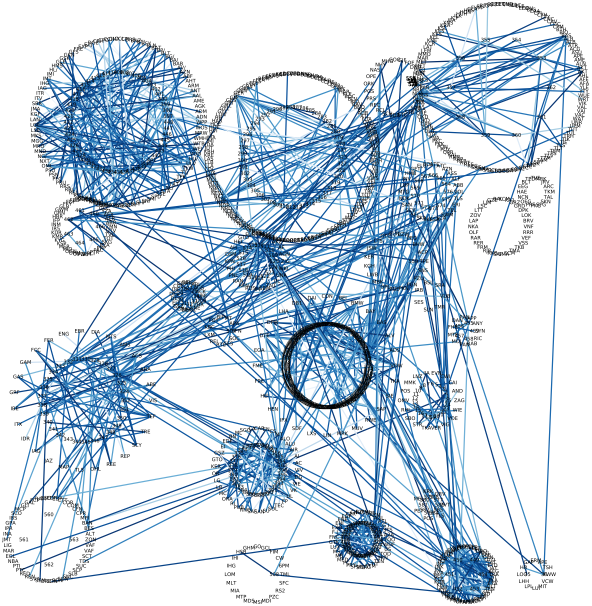
Item	Description	Sources
Coverage		
Banks	As identified in 2011 EBA Disclosures; 80 banks from EU countries. + 500 randomly generated banks based on TA	EBA, Halaj and Kok (2014) + Bankscope
Non-financial corporations	Members of the benchmark equity indices in the countries covered by EBA Disclosures and Halaj and Kok (2014); total 700 firms	Bloomberg and ECB
Attributes		
Banks	Total assets, IB assets, securities, securities MtM, equity, CT1 capital, IB liabilities	EBA
Banks	Loans to non-fin. corporations: calculated by using avg. country ratio of such loans to TA based on the ECB (MFI) balance sheet dataset	ECB calculations
Banks	Economic activity code (NACE), CDS of senior debt with 5 maturity, and long-term issuer ratings by Moody's, Fitch and S&P.	Bloomberg
Non-financial corporations	Total assets, total equity, total liabilities, NACE code, CDS spreads of senior debt with 5 maturity, and long term ratings by Moody's, Fitch and S&P.	Bloomberg
Non-financial corporations	Loans from banks: calculated by using the average country ratio of loans to total assets of NFCs based on the ECB EA Accounts dataset.	ECB calculations
Lending relations and other supportive variables		
Lending relationship	Defined as the number of loans with different banks; average figures by country and NACE sector were applied based on the data provided through the Working Group on Credit Registers	ECB calculations
Interest rates on loans by size and country	Avg. interest rates on loans by size of loan and by country based on the ESCB MIR data; categories of loans as follows: (below 0.25 EUR mn), (equal or above 0.25-1 EUR mn), and (over 1 EUR mn).	ECB calculations
Expected default frequencies	Avg. of expected default frequencies for non-financial corp. by country and NACE.	Moody's KMV and ECB

Sampling of the network

- Observed nodes (banks + non-bank corporate firms) and +500 generated banks
  - generated banks: based on the total assets and proportional allocation of other attributes
- Lending relationship:
  - {bank}–{firm}: based on aggregate Credit Register data
    - ★ → out-degree distribution (for each NACE sector) → the cardinal number of set B_j^k of firms k to which a bank j grants loans is constrained by a number m_j drawn from the out-degree distribution, i.e. $\#B_j^k \leq m_j$

    +

    - ★ → probability that a bank in a given country lends to a firm from a given country and a given (NACE) sector
  - {EBA sample bank}–{EBA sample bank}: EBA disclosures
  - {small bank}–{EBA sample bank}: arbitrary [small] probability of connection ($= 0.01$)



Applications – policy implications

Event-driven contagion (realised)

Deterioration of credit quality in a given sector (NACE) – corporate loan losses trigger contagion

Risk weights – policy tool to limit exposures to risky sectors (realised)

Specific sectors can be targeted to force banks to use more capital for more risky sectors

Large Exposure limits – compactness of the networks (realised)

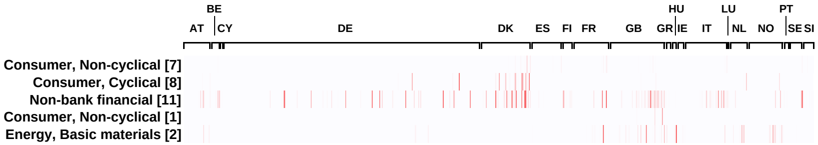
lower bilateral exposures allowed $\Rightarrow$ more connections

Network reactions to adverse market conditions (planned)

passing macro scenarios via dynamic BS model (Hałaj, 2013):

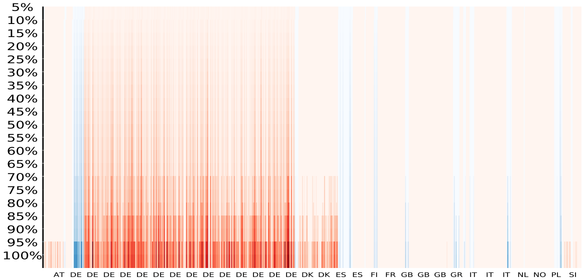
baseline macro scenario $\Rightarrow$ optimising behaviour of banks $\Rightarrow$ change in banks' preferred aggregate interbank lending and borrowing $\Rightarrow$ endogenous formation of the interbank under specified regulatory regime $\Rightarrow$ adverse macro shock $\Rightarrow$ banks defaults $\Rightarrow$ contagion

Figure 7: Contagion simulation



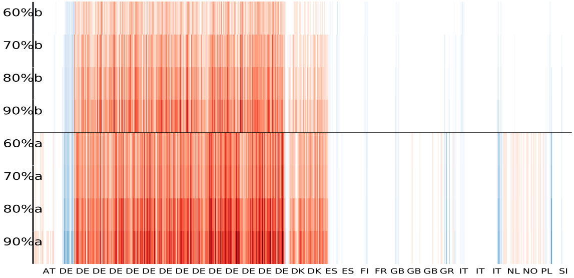
- Contagion mechanism – cascade triggered by a deterioration of credit quality of loan portfolios to companies in a given NACE sector (manufacturing in DE) imposing 5% PD and 50% LGD
- "Spectral" graph shows impact of the contagion losses of 500+ banks (the darker the bar, the higher the fraction of capital wiped out by contagion)

Figure 8: Contagion simulation for different deterioration of credit quality



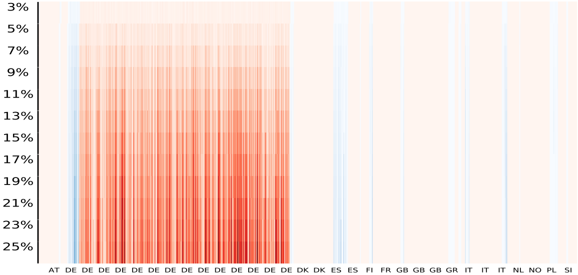
- Contagion mechanism – cascade triggered by a deterioration of credit quality of loan portfolios to companies in a given NACE sector for (y-axis) $\mathrm{PD} \in \{5\%, 10\%, \ldots, 100\%\}$ and 50% LGD
- "Spectral" graph of contagion losses of 500+ banks (the darker the bar, the higher the fraction of capital wiped out by contagion)

Figure 9: Risk-weight policy impact on contagion triggered by deterioration credit quality



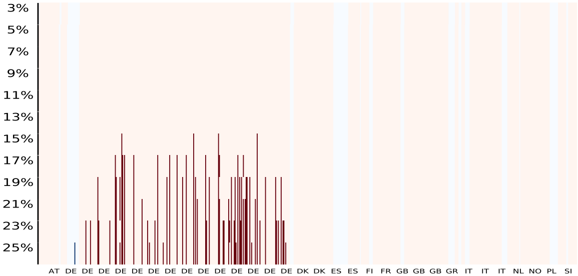
- "Spectral" graph of contagion losses of 500+ banks (the darker the bar, the higher the fraction of capital wiped out by contagion)
  - a) baseline case (PD$\in \{60\%, \ldots, 90\%\}$)
  - b) risk weights increased by a factor of 1.5 for all exposures to the manufacturing sector in Germany

Figure 10: Defaults of banks in the cascade of contagion spreading triggered by losses in the portfolio of loans to the manufacturing sector in DE



- Defaults of banks triggered by banks failing to pay back their obligations as a result of losses related to decreasing credit quality of manufacturing loan portfolio in (counterfactual example!) Germany
- Each bar indicates a defaulting bank under LE$\in \{3\%, 5\%, \ldots, 25\%\}$

Figure 11: Second round defaults of banks in the cascade of contagion spreading triggered by losses in the portfolio of loans to the manufacturing sector in DE



- Defaults of banks triggered by banks failing to pay back their obligations as a result of losses related to decreasing credit quality of manufacturing loan portfolio in Germany
- Each bar indicates a defaulting bank only because their debtors defaulted under LE$\in \{3\%, 5\%, \ldots, 25\%\}$

Where may the predefined volumes of interbank assets and liabilities come from? Endogenous balance sheet

Conclusions

- Endogenous interbank networks give an important insight into the role of banks' investment and funding strategies in shaping the interbank market and non-bank firms' funding channels. The simple, mechanistic cascade models are too simplistic in assuming that banks do not react to actions of other interbank participants and market conditions.
- It is easier to introduce heterogeneity of agents if the network approach is taken rather than macroeconomic (e.g. general equilibrium) framework.
- In the proposed framework, we are able to analyse different policy measures addressing the systemic risk – their ultimate impact on the market structure and efficiency in reducing the contagion risk.
- Still, a more consistent clearing mechanism would be warranted.
- The model needs to be calibrated to the observed interbank / lending networks. How far are we from the truth? (Target 2 data to be used)