Survival of Hedge Funds: Frailty vs Contagion

Darolles, S., Gagliardini, P., Gourieroux, C.

February, 2015
1. Economic motivation
Financial entities exposed to liquidity risk(s) ....

- on the asset component of the balance sheet (market liquidity)

- on the liability component of the balance sheet (funding liquidity)

... reinforced by *spirals* between funding liquidity and market liquidity (Brunnermeier, Pedersen (2009))
Why Hedge Funds?

- Large sample of different hedge fund strategies
- Different exposures to market liquidity (market portfolios)
- Different exposures to funding liquidity (leverage, redemption policies)

Liquidity risk(s) directly observed through Hedge Fund liquidations (and not returns)
This paper introduces new models to analyze liquidation risk dependence across individual Hedge Funds (HF).

The model specification focuses on disentangling:

1) **Dynamic frailty**: the effect of unobserved common exogenous shocks (a.k.a. systematic risk factors)

2) **Contagion effects**: an endogenous shock on one fund has an impact on the other funds.
i) Examples of common exogenous shocks

- Large cash withdrawals of direct investors during **funding liquidity crisis**
- Deleveraging imposed by prime brokers
- Outflow of funds of funds investors

Effect by means of the **liability component of the balance sheet**
ii) Example of endogenous shock

**Fund 1** is liquidated and sells illiquid asset A
→ “market price" of asset A decreases
→ NAV of **Fund 2** invested in asset A decreases
→ likelihood of liquidation of **Fund 2** increases

Effect by means of the **asset component of the balance sheet**
Main empirical findings

- The introduction of the systematic frailty factor diminishes the importance of contagion phenomena.
- The largest contribution to the variance-covariance matrix of the liquidation count comes from the frailty process, through a direct effect (64%) or an indirect one, via the contagion network (24%).
- A large fraction of the frailty factor (70%) is explained by proxies for funding liquidity.
Liquidation risk is especially important for 3 types of market participants:

- Institutional investors are interested in the low correlation of Hedge Funds returns with traditional asset classes but want to avoid short liquidity issues.

- Regulators want to monitor both the market and the funding liquidity risks.

- Funds of funds are sensitive to liquidation risks dependencies between individual funds.
1 Economic motivation
2 Hedge funds data
3 Contagion modeling
4 Empirical analysis
5 The funding liquidity factor
6 Stress-tests
7 Concluding remarks
2. Hedge funds data
Hedge funds data

- Lipper Tass Database, 6406 individual funds from February 1977 to May 2009
- **Active funds** are in "Live" database
- **Liquidated funds** are in the "Graveyard" database with status code "Liquidated", as well as in the "Live" database if they no longer report performance during a sufficiently long time
- Funds of funds are eliminated
- We focus on **9 management styles** with the larger sizes
Frequency Counts of Liquidated HF

- Convertible Arbitrage
- Emerging Markets
- Equity Market Neutral
- Event Driven
- Fixed Income Arbitrage
- Global Macro
- Long/Short Equity Hedge
- Managed Futures
- Multi-Strategy

Darolles, S., Gagliardini, P., Gourieroux, C. Survival of Hedge Funds: Frailty vs Contagion
Lexis diagram for Emerging Markets

The horizontal axis represents calendar time and the vertical axis represents age in months
We focus in the following on the modeling of the time concentration effects (clustering) in Hedge Funds liquidation.
3. Contagion modeling
Microscopic analysis

By means of liquidation models at the level of individual hedge funds (to capture the fund specific component of liquidation risk)

Macroscopic analysis

The data are aggregated by management styles (to eliminate by aggregation the idiosyncratic effects)
Macroscopic analysis

Since the number of funds in a given management style is sufficiently large,

- we consider the liquidation counts
- we apply a Poisson approximation conditional on common factors and lagged counts

This facilitates the analysis of the liquidation dynamic by leading to affine models
The model

At a given date $t$, the HF are classified:

by type $k = 1, \ldots, K$ (management style, domicile country, ...)

by age $h = 1, \ldots, H$

Number of HF at the beginning of the period: $n_{k,h,t}$

Number of liquidated HF during the period: $Y_{k,h,t}$

Aggregated over age as $Y_{k,t} = \sum_h Y_{k,h,t}$

Underlying exogenous common shock (frailty): $F_t$
A Poisson regression model with lagged liquidation counts and unobserved frailty as regressors

\[ Y_{k,t} \sim \mathcal{P} \left( \left( \frac{n_{k,t}}{n_{k,t_0}} \right) (a_k + b_k F_t + \sum_{k'=1}^{K} c_{k,k'} Y^*_{k',t-1}) \right) \]

with two adjustments for cohort sizes and \( Y^*_{k,t} \equiv Y_{k,t} / n_{k,t} \)

Parameters: intercepts \( a_k \), factor sensitivities \( b_k \), contagion matrix \( c_{k,k'} \)

Inspired by the literature in epidemiology on contagion (Anderson, Britt (2000))
• The standardization of \( Y^* \) captures a part of the competitive pressure

• The lagged counts are useful to fit the liquidation clustering and its diffusion between the management styles

• A single factor for tractability, but also for the compatibility with the literature

• A large number of parameters because of the cross-effects:
  management style \( \times \) factor
  management style \( \times \) lagged liquidation counts
Frailty dynamic

AutoRegressive Gamma (ARG) process

The transition of Markov process \((F_t)\) is a noncentral gamma distribution \(\gamma(\delta, \eta F_{t-1}, \nu)\)

\(\delta\): degrees of freedom

\(\nu\): scale parameter

\(\rho = \nu \eta < 1\): the serial correlation
The model is flexible enough to test for frailty and/or contagion effects by analyzing the significance of the regressors in the liquidation intensity:

\[ \iff \text{considering if } b_k = 0, \forall k, \text{ or } c_{k,k'} = 0, \forall k, k' \]
Literature on financial contagion

- No real consensus (Forbes, Rigobon (2002))
- Mostly focused on contagion between asset returns
- Difficult to rely on the interpretation suggested by epidemiological models
- Definition of "risk infected" or "sick" assets (Boyson, Stahel, Stulz (2010))
Literature on financial contagion

- Contagion models for asset returns also differ on the explanatory variables introduced in the equations.
- Presence or not of common exogenous factors capturing the dependence across assets ...
- Observability of these common factors ...
- Presence or not of (lag) asset returns among the explanatory variables ...
Litterature on financial contagion

Our paper:

- consider both frailty and contagion in a dynamic framework
- develop statistical inference

Thus the constrained specifications (common factors only or asset returns only) are special cases and can be easily tested.

Empirically, these constrained specifications appear to be misspecified, which may imply significant biases in contagion analysis.
4. Empirical analysis
i) Model with pure contagion

Estimated by maximum likelihood on the Poisson regression model with lagged counts

ii) Model with contagion and frailty

- The likelihood function involves large-dimensional integrals
- Estimated by an appropriate GMM approach
Static moment restrictions based on the conditional Laplace transform:

\[
E[\exp(-u_k Y_{k,t}) | F_t, Y_{t-1}] = \exp\{-\gamma_{k,t}(a_k + b_k F_t + c'_k Y_{t-1}^*)(1 - e^{-u_k})\}
\]

\[
\downarrow
\]

\[
E[\exp\{-u_k Y_{k,t} + \gamma_{k,t}(a_k + c'_k Y_{t-1}^*)(1 - e^{-u_k})\} | F_t, Y_{t-1}]
= \exp\{-\gamma_{k,t}b_k(1 - e^{-u_k})F_t\}
\]

\[
\downarrow
\]

By integrating out the unobservable frailty, we get

\[
E[\exp\{\log(1 - v/\gamma_{k,t}) Y_{k,t} + v(a_k + c'_k Y_{t-1}^*)\}] = \frac{1}{(1+vb_k/\delta)^{\delta}}, \forall v \in \mathcal{V}
\]
Dynamic moment restrictions based on the conditional Laplace transform

By considering the joint Laplace transform of $Y_{k,t}$, $Y_{l,t-1}$:

$$E[\exp\{-u_{k,t} Y_{k,t} - \tilde{u}_{l,t-1} Y_{l,t-1} + \nu (a_k + c_k' Y_{t-1}^*) + \tilde{\nu} (a_l + c_l' Y_{t-2}^*)\}]$$

$$= E[\exp(-v b_k F_t - \tilde{\nu} b_l F_{t-1})]$$

Dynamic moment restrictions are required to identify the factor dynamic
### Table: Estimated contagion parameters $c_{k,k'}$ in pure contagion model

<table>
<thead>
<tr>
<th></th>
<th>CON</th>
<th>EM</th>
<th>EMN</th>
<th>ED</th>
<th>FIA</th>
<th>GM</th>
<th>LSE</th>
<th>MF</th>
<th>MS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CON</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EM</td>
<td>0.19*** (0.07)</td>
<td>-0.13** (0.06)</td>
<td>0.11** (0.05)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EMN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.17*** (0.05)</td>
</tr>
<tr>
<td>ED</td>
<td></td>
<td>0.35*** (0.11)</td>
<td>0.16*** (0.07)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FIA</td>
<td></td>
<td></td>
<td>0.08*** (0.04)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GM</td>
<td></td>
<td></td>
<td></td>
<td>0.36*** (0.08)</td>
<td>0.20*** (0.07)</td>
<td></td>
<td></td>
<td>0.08*** (0.03)</td>
<td></td>
</tr>
<tr>
<td>LSE</td>
<td>0.42** (0.17)</td>
<td>0.42** (0.20)</td>
<td></td>
<td>0.78*** (0.23)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MF</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.27** (0.11)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MS</td>
<td>-0.16*** (0.04)</td>
<td></td>
<td>0.15*** (0.06)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.07* (0.03)</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**
- ****: Significant at the 10% level.
- ****: Significant at the 5% level.
- ****: Significant at the 1% level.

---

Darolles, S., Gagliardini, P., Gourieroux, C. *Survival of Hedge Funds: Frailty vs Contagion*
Contagion scheme for the pure contagion model

Darolles, S., Gagliardini, P., Gourieroux, C.

Survival of Hedge Funds: Frailty vs Contagion
<table>
<thead>
<tr>
<th></th>
<th>$a_k$</th>
<th>$b_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CON</td>
<td>0.00</td>
<td>1.08**</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(0.55)</td>
</tr>
<tr>
<td>EM</td>
<td>0.10</td>
<td>0.69**</td>
</tr>
<tr>
<td></td>
<td>(0.23)</td>
<td>(0.27)</td>
</tr>
<tr>
<td>EMN</td>
<td>0.27</td>
<td>0.84**</td>
</tr>
<tr>
<td></td>
<td>(0.30)</td>
<td>(0.40)</td>
</tr>
<tr>
<td>ED</td>
<td>0.00</td>
<td>1.39**</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(0.70)</td>
</tr>
<tr>
<td>FIA</td>
<td>0.00</td>
<td>0.31**</td>
</tr>
<tr>
<td></td>
<td>(0.20)</td>
<td>(0.13)</td>
</tr>
<tr>
<td>GM</td>
<td>0.76***</td>
<td>0.33***</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>LSE</td>
<td>2.98***</td>
<td>4.55**</td>
</tr>
<tr>
<td></td>
<td>(1.10)</td>
<td>(1.92)</td>
</tr>
<tr>
<td>MF</td>
<td>1.18</td>
<td>0.63**</td>
</tr>
<tr>
<td></td>
<td>(0.75)</td>
<td>(0.25)</td>
</tr>
<tr>
<td>MS</td>
<td>0.00</td>
<td>0.89**</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.41)</td>
</tr>
</tbody>
</table>

**Table:** Estimated contagion parameters $a_k$ and $b_k$ in the model with contagion and frailty
<table>
<thead>
<tr>
<th></th>
<th>CON</th>
<th>EM</th>
<th>EMN</th>
<th>ED</th>
<th>FIA</th>
<th>GM</th>
<th>LSE</th>
<th>MF</th>
<th>MS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CON</td>
<td></td>
<td></td>
<td>0.17***</td>
<td></td>
<td></td>
<td>0.15**</td>
<td></td>
<td>0.20***</td>
<td></td>
</tr>
<tr>
<td>EM</td>
<td>0.17***</td>
<td></td>
<td>0.21**</td>
<td></td>
<td>0.10*</td>
<td>0.35***</td>
<td></td>
<td>0.22***</td>
<td>0.10**</td>
</tr>
<tr>
<td>EMN</td>
<td></td>
<td>0.21**</td>
<td></td>
<td></td>
<td>0.10*</td>
<td>0.09**</td>
<td></td>
<td>0.20*</td>
<td></td>
</tr>
<tr>
<td>ED</td>
<td>0.35***</td>
<td>0.10*</td>
<td>0.09**</td>
<td>0.23**</td>
<td></td>
<td>0.39**</td>
<td></td>
<td>0.20*</td>
<td></td>
</tr>
<tr>
<td>FIA</td>
<td></td>
<td></td>
<td>0.22***</td>
<td></td>
<td>0.07</td>
<td>0.20*</td>
<td></td>
<td>0.27**</td>
<td></td>
</tr>
<tr>
<td>GM</td>
<td></td>
<td></td>
<td></td>
<td>0.20*</td>
<td></td>
<td>0.11</td>
<td></td>
<td>0.27**</td>
<td></td>
</tr>
<tr>
<td>LSE</td>
<td></td>
<td></td>
<td>0.39**</td>
<td>0.23**</td>
<td></td>
<td></td>
<td>0.29***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MF</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.27**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.29***</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table:** Estimated contagion parameters $c_{k,k'}$ in the model with contagion and frailty
The matrix of estimated contagion coefficients provides the structure of the network between the HF strategies.

The number of contagion channels is clearly diminished when frailty is taken into account.
Contagion scheme for the model with contagion and frailty

Darolles, S., Gagliardini, P., Gourieroux, C.  
Survival of Hedge Funds: Frailty vs Contagion
The relative importance of frailty and contagion can be measured by considering the decomposition of the variance of liquidation counts:

\[ V(Y_t) = \text{diag}[E(Y_t)] \quad \text{(standard Poisson)} \]

\[ + CV(Y_t)C' \quad \text{(contagion)} \]

\[ + \sigma^2 bb' + \sigma^2 \rho C(Id - \rho C)^{-1}bb' + \sigma^2 \rho bb'(Id - \rho C')^{-1}C' \quad \text{(frailty)} \]

\[ \text{(direct)} \quad \text{(indirect)} \]

with \( \rho = \eta \nu \) and \( \sigma^2 = 1/\delta \)
We get the following decomposition of variance:

<table>
<thead>
<tr>
<th></th>
<th>Poisson</th>
<th>Contagion</th>
<th>Frailty (direct)</th>
<th>Frailty (indirect)</th>
</tr>
</thead>
<tbody>
<tr>
<td>percentage</td>
<td>6, 54%</td>
<td>5, 10%</td>
<td>64, 30%</td>
<td>24, 06%</td>
</tr>
</tbody>
</table>

Darolles, S., Gagliardini, P., Gourieroux, C.  
Survival of Hedge Funds: Frailty vs Contagion
5. The funding liquidity factor
To reinforce the funding liquidity risk interpretation of the frailty factor,

- we filter the unobserved factor path
- we investigate how this factor is related with other funding liquidity proxies introduced in the literature
Filtering of the factor
The factor interpretation

TED spread

VIX

Credit spread

Darolles, S., Gagliardini, P., Gourieroux, C.  
Survival of Hedge Funds: Frailty vs Contagion
We estimate the regression:

$$\hat{F}_t = I(VIX_t < c)(\beta_1 + \beta_2 TED_t + \beta_3 TEDL_t + \beta_4 VIX_t + \beta_4 VIXL_t + \beta_5 SPR_t)$$
$$+ I(VIX_t > c)(\gamma_1 + \gamma_2 TED_t + \gamma_3 TEDL_t + \gamma_4 VIX_t + \gamma_4 VIXL_t + \gamma_5 SPR_t) + e_t$$

where $\hat{F}_t$ is the filtered value of the frailty, and the 2 regimes corresponds to "good equilibrium" (HF provide liquidity) and "bad equilibrium" (HF are liquidity demanders).

The estimated value of $c$ is 25.
A large fraction (70%) of the common factor is explained by the proxies for funding liquidity.

The coefficient of TED is statistically significant and larger in the "bad equilibrium".

The effect of the volatility passes through the lagged value VIXL, which has a negative impact (more pronounced in bad equilibrium).
6. Stress-tests
The estimated model can be used to perform stress-tests on a portfolio of individual hedge funds

i) We may stress the current value of the common factor $F_t$ by replacing this value with an extreme quantile of its distribution

ii) We may also stress the parameters by changing the form of the contagion matrix, for instance by increasing the contagion
For a given stress scenario, we can compute the term structure of expected liquidation counts:

\[ E_{\theta^s}(Y_{k,t+\tau} | F_t = q_{\alpha^s}, Y_t) \]

and the term structure of overdispersion of these counts:

\[ V_{\theta^s}[Y_{k,t+\tau} | F_t = q_{\alpha^s}, Y_t] / E_{\theta^s}[Y_{k,t+\tau} | F_t = q_{\alpha^s}, Y_t] \]

where \( q_{\alpha^s} \) is the quantile of \( F_t \), and \( \theta^s \) is the model parameter, in the stress scenario.
Shock on the factor value : median $\rightarrow$ extreme quantile
Shock on the contagion matrix: $C \rightarrow 2C$
● The effects of the shocks depend on the environment through the lagged liquidation counts

● This is a dynamic stress-test, which accounts for both lagged liquidation counts and frailty dynamic

● Our analysis is very different from that in models with observable factors, assuming a crystallized scenario for the future evolution of the factor. These scenarios neglect the risk of liquidation correlation
7. Concluding remarks
The aim of this paper is to disentangle the two sources of dependence between HF liquidation risks:

- exogenous systematic factors, described by the unobserved frailty,
- endogenous contagion effects, corresponding to the impact of lagged liquidation counts

Such an analysis is a preliminary step before measuring and managing systemic risk in the hedge fund industry.
The analysis completes the literature

• on contagion between the (individual) hedge fund returns

• on dependence between the funding liquidity and market liquidity

The causal contagion scheme captures a part of the spiral effect highlighted in Brunnermeier, Pedersen (2009)