Hidden Illiquidity With Multiple Central Counterparties

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OTC vs CCP

Over-the-counter market

Centrally cleared market
Key Idea of the Paper

• Margin requirements need to reflect the price impact/liquidation cost/concentration risk of large illiquid positions at default
  – Need to grow superlinearly with position size

• This creates an incentive for clearing members to split their positions across CCPs

• So the CCPs need to charge more than the “right” amount of margin because of what they don’t see

• This may not work if different CCPs have different views on the “right” amount of margin, creating a race to the bottom

• Counteracting this effect requires some coordination or information sharing between CCPs and/or common members
Netting Reduces Total Counterparty Risk

Over-the-counter market

Centrally cleared market

Bilateral netting

The CCP always has a matched book and zero net exposure, in theory
But What Happens If A Clearing Member Fails?

• If a clearing member fails, the CCP needs to restore a matched book but may incur a loss in doing so.

• The failure of a CCP could cascade to failures of other clearing members.

• CCPs are a potential source of systemic risk.
• CCP holds margin from each clearing member to absorb potential losses over a liquidation period of 5-10 days

• This is “initial” margin as opposed to variation margin

• Clearing members also contribute to a default fund
Consider Margin Proportional to Standard Deviation (Market Risk)

$n$ types of swaps cleared by $K$ CCPs

Dealer wants to clear swaps of size $x = (x^1, x^2, ..., x^n)$

Allocation:
- $x_1$ types to CCP 1
- $x_2$ types to CCP 2
- ... $x_K$ types to CCP $K$

Margin:
- $a(x_1' \Sigma x_1)^{1/2}$ for CCP 1
- $a(x_2' \Sigma x_2)^{1/2}$ for CCP 2
- ... $a(x_K' \Sigma x_K)^{1/2}$ for CCP $K$

$\Sigma =$ covariance matrix of 10-day price changes

$x_1 + x_2 + ... + x_K = x$

$x = (x^1, x^2, ..., x^n)$
Consider Margin Proportional to Standard Deviation (Market Risk)

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Dealer wants to clear swaps of size $x = (x^1, x^2, ..., x^n)$

Allocation: $x_1$, $x_2$, ..., $x_K$

Margin: $a(x_1'\Sigma x_1)^{1/2}$, $a(x_2'\Sigma x_2)^{1/2}$, ..., $a(x_K'\Sigma x_K)^{1/2}$

$\Sigma =$ covariance matrix of 10-day price changes

$x_1 + x_2 + ... + x_K = x$

How should the dealer allocate the position to minimize total margin?
Incorporating Market Impact

- Standard deviation is positively homogeneous: doubling the size of the swap doubles the margin requirement
  \[ \left( \lambda x^\top \Sigma \lambda x \right)^{1/2} = \lambda \left( x^\top \Sigma x \right)^{1/2}, \quad \lambda \geq 0 \]

- But liquidating or replacing a large position will produce a more-than-proportional increase in the loss because of market impact

- Margin should be superlinear in position size; e.g.,
  \[ f(x) = (x^\top \Sigma x)^{\alpha/2}, \quad \alpha > 1 \]

  Then \[ f(\lambda x) = \lambda^\alpha f(x), \quad \lambda > 0 \]
Superlinear Margin

$n$ types of swaps cleared by $K$ CCPs

Dealer wants to clear swaps of size $x = (x^1, x^2, ..., x^n)$

Allocation: $x_1$, $x_2$, ..., $x_K$

How should the dealer allocate the position to minimize total margin?
The Dealer’s Margin Minimization Problem

- Suppose all CCPs apply margin function $f$
- Dealer’s problem:

\[
\min_{x_1, x_2, \ldots, x_K} \sum_{i=1}^{K} f(x_i) \quad \text{subject to } x_1 + x_2 + \cdots + x_K = x
\]
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**Proposition:** (a) If $f$ is

(i) Subadditive: $f(x + y) \leq f(x) + f(y)$

(ii) Positively homogeneous: $f(\lambda x) = \lambda f(x), \forall \lambda \geq 0$

(as in the case of standard deviation) then clearing everything through one CCP is optimal, as is any allocation of the form $x_i = k_i x$, $k_i \geq 0$, $k_1 + k_2 + \cdots + k_K = 1$.

(b) If $f$ is strictly convex, then the unique optimum is an equal allocation

$$x_i = x/K, \quad i = 1, \ldots, K.$$
Margin Requirement Through Price Impact

• Consider a scalar position of size $x$ cleared in a market with $K$ CCPs
• Suppose the margin function is given by

$$f(x) = xF(x)$$

- Size of position
- Price impact of liquidation

• We will assume $F(0)=0$ and $f$ increasing and strictly convex
Why The Right Model Yields The Wrong Margin

- The dealer optimally sends \( \frac{x}{K} \) to each CCP
- Each CCP collects margin equal to 
  \[ f\left(\frac{x}{K}\right) = \left(\frac{x}{K}\right)F\left(\frac{x}{K}\right) \]
- But the total market impact if the dealer fails will be \( F(x) \) so each CCP should collect margin equal to 
  \( \left(\frac{x}{K}\right)F(x) \)
- In other words, each CCP needs to replace the “true” margin function \( f \) with the “wrong” margin function 
  \[ g(x) = xF(Kx) \]

In order to end up with the right level of margin
Is Liquidity An Issue?

Q1 2013

5Y CDS, 2013

1Y CDS, 2013
CDS Margin Methodology: Liquidity Charges

- **ICE Clear Credit:**
  
  - “Positions that exceed selected thresholds are subject to additional, exponentially increasing, initial margin requirements.”

- **CME Group:**
  
  - “The liquidity risk requirement is designed to capture the liquidity and concentration premium during liquidation of the credit portfolio of a defaulted member.
  
  - For large positions, this loss scales super-linearly by the number of days liquidation will take at a constant unwinding rate, therefore by the position size”

- **LCH.Clearnet**
  
  - “Liquidity charge: In order to take into account the actual cost of liquidating a portfolio, bid-ask spreads need to be covered. Therefore, a specific charge is added, to model the cost of transaction, which increases for positions in excess of a given size.”
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• Full disclosure: I serve on the risk committee of a swaps CCP
What If The CCPs Have Different Models?

- We simplify to two CCPs
- We allow vector positions
- CCP $i$ believes the true price impact for vector position $x$ is $G_i(x)$
- CCP $i$ charges margin as if the price impact were $F_i(x)$
- In other words, it charges $x^T F_i(x)$
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- A dealer trading $x$ minimizes margin by solving

$$\min_{x_1, x_2} x_1^T F_1(x_1) + x_2^T F_2(x_2) \quad \text{subject to } x_1 + x_2 = x$$

- CCPs want to set margin charges to end up with enough margin after the dealer optimizes
Equilibrium

Given price impact beliefs $G_1, G_2$ for the two CCPs, an equilibrium is defined by

- Allocation functions $x_i : \mathbb{R}^n \rightarrow \mathbb{R}^n$, $i = 1, 2$

- Price impact functions $F_i : \mathbb{R}^n \rightarrow \mathbb{R}^n$, $i = 1, 2$,
  \[ F_i(0) = 0 \text{ and } x \mapsto x^\top F_i(x) \text{ strictly convex} \]

satisfying

- $(x_1(x), x_2(x))$ solves the dealer’s allocation problem for all $x$

- $x_i^\top F_i(x_i) \geq x_i^\top G_i(x)$ (Sufficient margin condition)
Equilibrium

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- Allocation functions $x_i : \mathbb{R}^n \to \mathbb{R}^n$, $i = 1, 2$
- Price impact functions $F_i : \mathbb{R}^n \to \mathbb{R}^n$, $i = 1, 2$, with $F_i(0) = 0$ and $x \mapsto x^\top F_i(x)$ strictly convex

satisfying

- $(x_1(x), x_2(x))$ solves the dealer’s allocation problem for all $x$
- $x_i^\top F_i(x_i) = x_i^\top G_i(x)$ (Sufficient margin condition)

We assume a competitive market in which CCPs cannot collect excess margin
Linear Price Impact

- Specialize to the case of linear price impact

\[ G_i(x) = G_ix, \quad G_i \in \mathbb{R}^{n \times n} \]

- Further suppose that

\[ F_i(x) = F_ix, \quad F_i \in \mathbb{R}^{n \times n} \]

- In other words, CCP margin charges are quadratic,

\[ x \mapsto x^T F_ix \]

- We assume that the matrices \( G_i \) and \( F_i \) are symmetric and positive definite
Digression on Linear Price Impact

• This is a multivariate Kyle (1985) model
  – In the usual, scalar Kyle model, price impact is linear, transaction cost is quadratic
• Do price impacts across different swaps make sense?
  • Yes
    – CDS for firms in the same sector
    – 1-year and 5-year CDS for the same firm
    – Different series of the same index (the London Whale trade)
    – Also for interest rate swaps

• Cross-asset impacts are very difficult to estimate. Could be based on correlations in returns, but we are interested in impact at dealer’s default
Equilibrium With Linear Price Impact

**Theorem.** A necessary and sufficient condition for an equilibrium is that the CCPs have common beliefs on market impact, meaning $G_1 = G_2 \equiv G$.

In this case, all equilibria are determined by matrices $F_1, F_2$ satisfying

$$G^{-1} = F_1^{-1} + F_2^{-1}$$

CCPs need to agree on “true” price impact but not on the margin they charge.
Discussion

\[ G^{-1} = F_1^{-1} + F_2^{-1} \]

- Special case: \( F_i = 2G \), charge twice your belief and get half the volume

- More generally, we can have

\[ F_1 = \frac{G}{\alpha}, \quad F_2 = \frac{G}{1 - \alpha}, \quad \alpha \in (0, 1). \]

The CCP that sets the margin lower gets more of the volume and needs to correct less for hidden illiquidity
**Parallel Sum of Matrices**

- The operation

\[(F_1^{-1} + F_2^{-1})^{-1}\]

is called the *parallel sum* of matrices (Anderson and Duffin 1969)

- It yields the *effective margin* in the market, so our condition states that the effective margin needs to equal the CCPs’ share view on the margin required

Margin requirements combine like resistors connected in parallel:

- resistance ~ price impact per unit traded
- current ~ size of trade
- voltage ~ price impact of trade
If They Disagree: A Race to the Bottom

- Consider the scalar case with price impact views $G_1 < G_2$
- Suppose, initially, they charge according to their views, $F_i = G_i$.
- A dealer trading $x$ minimizes margin by setting
  \[ x_1 = \frac{F_2}{F_1 + F_2} x, \quad x_2 = \frac{F_1}{F_1 + F_2} x \]
- CCPs update their charges to have enough margin:
  \[ \hat{F}_1 x_1^2 = x_1(G_1 x), \quad \hat{F}_2 x_2^2 = x_2(G_2 x) \]
- This yields
  \[ \frac{\hat{F}_2}{\hat{F}_1} = \left( \frac{G_2}{G_1} \right) \left( \frac{F_2}{F_1} \right) \to \infty, \quad x_1 \to x, \quad x_2 \to 0 \]
- The CCP that estimates a higher liquidation cost gets driven out
Equilibrium With Non-Participation

• We expand the strategy space for each CCP, allowing it to decide whether to clear certain types of swaps (as opposed to just setting margin levels)

• This partitions the set of swap types into three groups:
  – Cleared only by CCP 1
  – Cleared by both
  – Cleared only by CCP2

• We partition vectors and matrices in accordance with this decomposition

• We remove any swap types not cleared by either CCP
Equilibrium With Non-Participation

**Theorem.** An equilibrium exists if and only if the CCPs’ price impact views have a common block diagonal structure

\[
G_i = \begin{pmatrix}
G_i(1,1) & & \\
& G_i(2,2) & \\
& & G_i(3,3)
\end{pmatrix}, \quad i = 1, 2,
\]

with \(G_1(2,2) = G_2(2,2) \equiv G(2,2)\). In this case, all equilibria are determined by matrices \(F_1, F_2\),

\[
F_1 = \begin{pmatrix}
G_1(1,1) & \\
& F_1(2,2)
\end{pmatrix}, \quad F_2 = \begin{pmatrix}
F_2(2,2) & \\
& G_2(3,3)
\end{pmatrix},
\]

satisfying

\[
G(2,2)^{-1} = F_1(2,2)^{-1} + F_2(2,2)^{-1}
\]

CCPs need to

- agree on “true” price impact for swaps they both clear
- clear anything that impacts anything they clear
Adding Uncertainty

Previously we had

- \((x_1(x), x_2(x))\) solves the dealer’s allocation problem for all \(x\)
- \(x_i^\top F_i x_i = x_i^\top G_i x\) (Sufficient margin condition)

Now we add (uncorrelated, zero mean) uncertainty to

- Each CCP’s inference about total position size: \(x + \epsilon_i\)
- Each CCP’s views on price impact: \(G_i\) stochastic, uncorrelated with \(\epsilon_i\)

Equilibrium condition becomes

\[
x_i^\top F_i x_i = x_i^\top \mathbb{E}[G_i(x + \epsilon)] = x_i^\top \mathbb{E}[G_i] x
\]

and results go through replacing \(G_i\) with \(\mathbb{E}[G_i]\)
What Can We Say With Nonlinear Price Impact?

• For the scalar case, we have a general characterization of equilibrium, but it is difficult to apply.

• Example:

  If common view of price impact is

  \[ G(x) = cx^\beta, \quad \beta > 0, \]

  then we get an equilibrium with \( F_i(x) = b_i x^\beta, \ i = 1, 2, \) for any \( b_1, b_2 \) satisfying

  \[ b_1^{-1/\beta} + b_2^{-1/\beta} = c^{-1/\beta} \]

• Similarity with linear case is not accidental. Both are consequences of *effective margin*.
Effective Margin

The effective margin requirement for the market is the inf-convolution of the individual margin requirements:

\[
f_{\text{eff}}(x) = \min_{x_1} \{ f_1(x_1) + f_2(x - x_1) \}
\]

\[
= (f_1 \Box f_2)(x).
\]
Effective Margin

The effective margin requirement for the market is the inf-convolution of the individual margin requirements:

\[ f_{\text{eff}}(x) = \min_{x_1} \{ f_1(x_1) + f_2(x - x_1) \} \]
\[ = (f_1 \Box f_2)(x). \]

For proper convex functions (Rockafellar 1973, Thm. 16.4)

\[(f_1 \Box f_2)^* = (f_1^* + f_2^*)\]

where \( f^* \) is the conjugate of \( f \), \( f^*(y) = \sup_x \{ x^\top y - f(x) \} \). In the strictly convex quadratic case

\[ f(x) = x^\top F x, \quad f^*(x) = x^\top F^{-1} x \]

and the effective margin is given by

\[ x^\top (F_1^{-1} + F_2^{-1})^{-1} x \]
Equilibrium With Nonlinear Price Impact

**Theorem:** [Scalar case, nonlinear impact]

(i) If the CCPs have common beliefs $G_1 = G_2 = G$, then an equilibrium exists. All equilibria result in proportional allocations $x_1 = \alpha x$ and $x_2 = (1 - \alpha)x$, for some $\alpha \in (0, 1)$.

(ii) If an equilibrium with proportional allocations exists, then the CCPs have common beliefs $G_1 = G_2 = G$.

(iii) In any equilibrium with common beliefs, $f_{\text{eff}} = g$, meaning that the effective margin equals the shared view on required margin and

$$g = (f_1^* + f_2^*)^*$$

where $g(x) = xG(x)$. 

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Back to the Real World: Implications

- CCPs need to consider liquidation cost/price impact in setting margin
  - This requires superlinear margin

- Because superlinear margin creates an incentive for dealers to spread positions, CCPs need to account for what they don’t see in setting margin
  - Margin needs to be higher than what the “right” model says
  - Good backtesting is bad
  - CCPs and/or dealers need to share information about trades at other CCPs

- To avoid a race to the bottom, CCPs need shared information about “true” liquidation cost. Potential solutions:
  - Firm commitments to buy (short puts) from dealers as part of their guarantee fund contributions
  - Fed and CFTC recently called for standard stress tests for CCPs. Add impact of other CCPs to these stress tests
Thank You