Energy & Commodity Markets II

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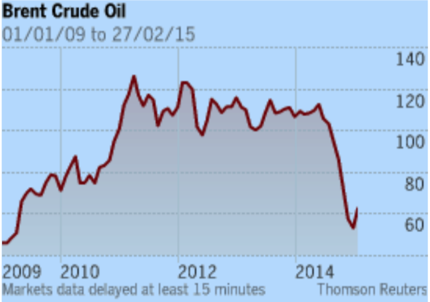
Overview

1. Traditional commodity markets issues and models. Electricity price models.
2. Energy production from exhaustible resources and renewables: game threoretic models.
3. Financialization of commodity markets.

Game Theoretic Models of Energy Production

- Recent decline in oil prices (around \$100 per barrel in June 2014 to around \$50 in January 2015) illustrates evolution of energy production as a result of competition between different sources.



- Drop was prompted in large part by OPEC's strategic decision not to decrease its oil output in the face of increased production of *shale oil* in the US, itself arising from new technologies (fracking), spurred by investment in exploration and research in times of higher oil prices.

Energy Production

- These complex interactions are in addition to long-running concerns about dwindling fossil fuel reserves (‘peak oil’), as well as climate change transitioning to sustainable energy sources.
- Build models successively incorporating various of these features starting from a competitive oligopolistic view of an idealized global energy market, in which *game theory* describes the outcome of competition.
- Oligopoly is in a *Cournot framework*: players choose *quantities* of production and prices are determined by total supply.
- Reasonable for energy production: major players determine their output relative to their production costs, as in the expected scenario that OPEC will cut production in order to increase the market price of oil.

Game Changers

- Start with static, or one-period games to see for instance the non-competitiveness of producing from a relatively expensive renewable source, such as wind, against a cheap fossil fuel.
- The nature of the complexities calls for a dynamic model in which there are (cliché) *game changers* over time, *e.g.*
  - dwindling reserves of oil or coal, ramping up their scarcity value;
  - discoveries of new oil reserves (over 30 major finds in 2009);
  - technological innovation such as *fracking*, led to extraction of shale oil and gas;
  - government subsidies for renewables such as solar and wind;
  - varying costs of production, *e.g.* cheaper solar due to falling silicon prices and improved solar cell efficiency (Solyndra).
- Many if not all these phenomena are unpredictable and dramatic: motivate the development of *stochastic models*, with potentially significant 'jumps' (for instance in costs or reserves).

Static Cournot Games

- $N \geq 1$ profit-maximizing producers (or players), with per-unit (constant) cost of production $s_i \geq 0$, which will be different, reflecting the costs of heterogeneous energy sources.
- Market is specified by a decreasing inverse demand curve $P(\cdot)$. Given total production level $Q = q_1 + \ldots + q_N$, the market clearing price is $P(Q)$.
- Example: linear inverse demand, $P(Q) = 1 - Q$.
- Profit of player i is the quantity produced times price minus cost:

$$\pi(q_i, Q_{-i}, s_i) = \begin{cases} q_i \left(P(Q_{-i} + q_i) - s_i\right) & \text{if } q_i > 0, \\ 0 & \text{if } q_i = 0, \end{cases}$$

where $Q_{-i} = \sum_{j \neq i} q_j$.

Nash Equilibrium

- A Nash equilibrium is a vector $\vec{q}^* = (q_1^*, q_2^*, \ldots, q_N^*) \in [0, \infty)^N$ such that, for all i,

$$\pi(q_i^*, Q_{-i}^*, s_i) = \max_{q_i \in [0,\infty)} \pi(q_i, Q_{-i}^*, s_i), \tag{1}$$

where $Q_{-i}^* = \sum_{j \neq i} q_j^*$.

- Assume: price function P is twice continuously differentiable, with $P' < 0$ everywhere on $(0, \infty)$; and there exists $\eta \in (0, \infty)$ such that $P(\eta) = 0$. And order the firms by their costs:

$$0 \leq s_1 \leq s_2 \leq \ldots \leq s_N < P(0^+).$$

- Define *relative prudence* of P:

$$\rho(Q) = -\frac{Q\,P''(Q)}{P'(Q)}, \qquad \overline{\rho} = \sup_{Q \in (0,\infty)} \rho(Q).$$

Theorem

Suppose that $\overline{\rho} < 2$. *Then there is a unique Nash equilibrium which can be constructed as follows. Let* $\bar{Q}^* = \max\{Q_n^* \mid 1 \leq n \leq N\}$, *where* Q_n^* *is the unique non-negative solution to the scalar equation*

$$QP'(Q) + nP(Q) = \sum_{j=1}^{n} s_j.$$

The unique Nash equilibrium production quantities are given by

$$q_i^*(\vec{s}) = \max\left\{\frac{P\left(\bar{Q}^*\right) - s_i}{-P'\left(\bar{Q}^*\right)}, 0\right\}, \quad 1 \leq i \leq N,$$

and the corresponding profits are

$$G_i(\vec{s}) = q_i^*(\vec{s})(P(\bar{Q}^*) - s_i), \quad 1 \leq i \leq N.$$

In particular, q_i^* *and* G_i *are Lipschitz continuous, and the number of active players (that is, players with* $q_i^* > 0$*), in the unique equilibrium is* $m = \min\left\{n \mid Q_n^* = \bar{Q}^*\right\}$.

Blockading

- The constraint $q_i \geq 0$ endogenizes the market structure in terms of the cost profile $\vec{s}$.
- Oligopolies with symmetric costs generate a trivial market structure: either all firms active or all firms inactive.
- When firms are asymmetric, some firms may be inactive in equilibrium. In dynamic models, asymmetric costs induce different entry times into the market.
- Especially pertinent to energy markets, where producers using different fuels and technologies have widely different costs of production. For example, oil and coal sources are much cheaper than renewables, such as solar or wind.

Heterogeneous Costs: just oil sources

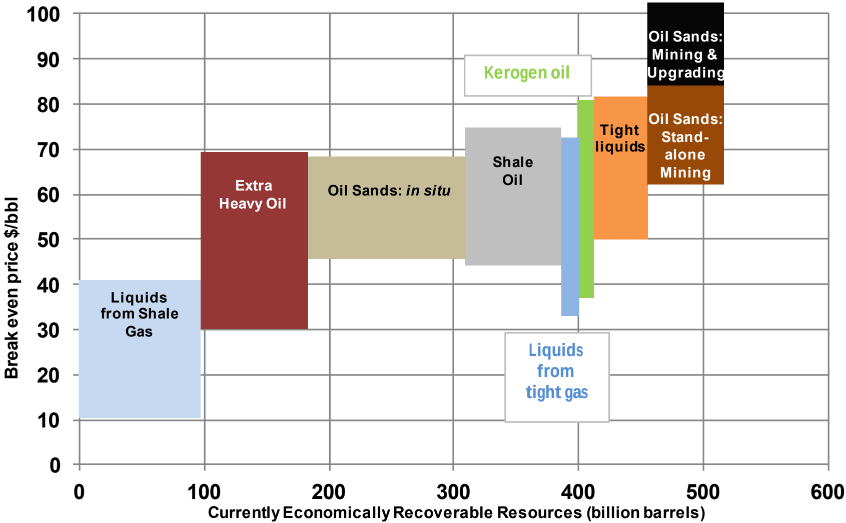
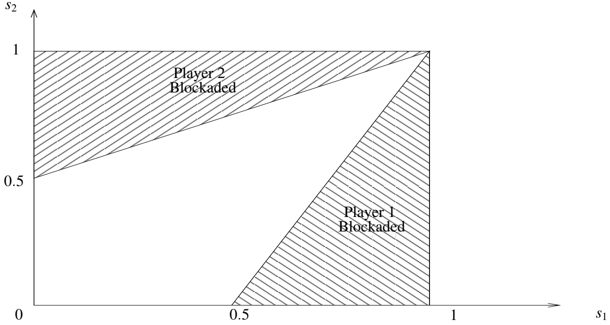


Figure: *Estimated oil extraction costs from varying sources, 2012.*

Duopoly: $N = 2$ with linear demand $P(Q) = 1 - Q$



Current example: OPEC holding back on cuts in production to drive shale oil producers out of the market and into bankruptcy.

Monopoly Exhaustible Resources

- Hotelling (1931) : a single producer who has reserves $x(t)$ at time t, with:

$$\frac{dx}{dt} = -q(x(t))\mathbb{1}_{\{x(t)>0\}}, \tag{2}$$

  where $q(x(t))$ is his production (or extraction) rate. When his reserves run out, he no longer participates in the market.
- The producer extracts to maximize lifetime discounted profit:

$$v(x) = \sup_{q \geq 0} \int_0^{\tau_x} e^{-rt} q(x(t)) P(q(x(t))\, dt,$$

  where τ_x is the exhaustion time starting at $x(0) = x$:

$$\tau_x = \inf\{t > 0 \mid x(t) = 0\}.$$

- Hotelling's rule (for a monopolist with exhaustible resources):

$$\frac{d}{dt} v'(x(t)) = rv'(x(t)), \tag{3}$$

  or $v'(x(t)) = v'(x(0))e^{rt}$.

- Incorporating other players leads to nonzero-sum differential games and PDEs.
- Existence and regularity theory is scarce (outside of the case of linear-quadratic (LQ) games).
- To illustrate the complexity in the duopoly case, let $x_i(t)$ be the reserves of each player, depleted at their extraction rates q_i:

$$\frac{dx_i}{dt} = -q_i(\vec{x}(t)), \quad i = 1,2 \quad \text{where} \quad \vec{x}(t) = (x_1(t), x_2(t)).$$

- A Nash (or Markov perfect) equilibrium (q_1^*, q_2^*):

$$v_i(\vec{x}) = \sup_{q_i \geq 0} \int_0^{\tau_{x_i}} e^{-rt} q_i(\vec{x}(t)) P\left(q_i(\vec{x}(t)) + q_j^*(\vec{x}(t))\right) \, dt, \; i = 1,2; j \neq i,$$

where $\tau_{x_i} = \inf\{t > 0 \mid x_i(t) = 0\}$ are the exhaustion times starting at $x_i = x_i(0)$.

- PDEs for the value functions in $x_1, x_2 > 0$:

$$rv_i = \sup_{q_i \geq 0} \pi \left(q_i, Q^*_{-i}, \frac{\partial v_i}{\partial x_i} \right) - q^*_j \frac{\partial v_i}{\partial x_j}.$$

- This identifies the infinitesimal problem in the dynamic programming equation as the static Nash equilibrium problem with scarcity costs $s_i = \frac{\partial v_i}{\partial x_i}$.
- As players deplete their reserves, their marginal costs may rise sufficiently to make further production uneconomical and causing them to drop out of competition.

N-player dynamic game

- Using the notation of the static Theorem the dynamic game PDEs are:

$$rv_i = G_i(\mathcal{D}v) - q_j^*(\mathcal{D}v)\frac{\partial v_i}{\partial x_j}, \quad i = 1,2; j \neq i, \quad \mathcal{D}v = \left(\frac{\partial v_i}{\partial x_i}, \frac{\partial v_j}{\partial x_j}\right).$$

- In a model of *only* exhaustible resources, when player i runs out, player j has a monopoly until he also exhausts, which lead to boundary conditions on $x_i = 0$ and at $(0,0)$ respectively.
- A more nuanced (and optimistic) view allows that when an oil producer exits, he is replaced by an inexhaustible producer (such as from solar).
- Alternatively, one can consider models with a single exhaustible producer, and hence a single state variable, along with $N-1$ renewable producers.
- This maintains game effects but minimizes mathematical complexity, and allows to study the effect of blockading.

Dynamic Cournot Model for Energy Production

- The oil producer (Player 0) has reserves $x(t)$ at time t, and chooses his production rate $\bar{q}_0(x(t))$, depleting reserves as

$$\frac{dx}{dt} = -\bar{q}_0(x(t))\mathbb{1}_{\{x(t)>0\}}.$$

Others produce energy at rates $\bar{q}_i(x(t))$, $i = 1, \ldots, N-1$.

- Price given by linear inverse demand function:

$$P(t) = 1 - \bar{q}_0(x(t)) - \sum_{j=1}^{N-1} \bar{q}_j(x(t)).$$

Note maximum (choke) price is 1.

- Players maximize discounted lifetime profit. Player 0's value function:

$$v_0(x) = \sup_{\bar{q}_0} \int_0^\infty e^{-rt}\bar{q}_0(x(t))P(t)\mathbb{1}_{\{x(t)>0\}}dt.$$

Aside: Static Cournot Game

- In a *static* Cournot game between N players with ordered costs $(s_0, s_1, \cdots, s_{N-1})$, the number of active players in equilibrium depends on the distribution of the costs. Let

$$G_i(s_0, s) = \max_{q_i \geq 0} q_i \left(1 - Q - s_i\right), \qquad Q = \sum_{j=0}^{N-1} q_j.$$

- Let $S^{(n)} = \sum_{j=0}^{n-1} s_j$. If $n \leq N-1$ players participate, the equilibrium total supply is: $Q^{\star,n} = \frac{n - S^{(n)}}{(n+1)}$.

Proposition

Let $\bar{Q}^\star = \max\left\{Q^{\star,n} | 0 \leq n \leq N-1\right\}$*. Then the unique Nash equilibrium quantities are given by*

$$q_i^\star(s_0, s) = \max\left\{1 - \bar{Q}^\star - s_i, 0\right\}, \quad G_i = \left(q_i^\star\right)^2, \quad 0 \leq i \leq N-1.$$

The number of active players in the unique equilibrium is $m = \min\left\{n \mid Q^{\star,n} = \bar{Q}^\star\right\}$*. (The others are blockaded).*

Value Functions and Feedback Strategies

We look for a *Markov Perfect* Nash equilibrium. Player 0's value function:

$$v_0(x) = \sup_{\bar{q}_0} \int_0^\infty e^{-rt} \bar{q}_0(x(t)) P(t) \mathbb{1}_{\{x(t)>0\}} dt.$$

When oil runs out, the remaining firms ($i = 1, \ldots, N-1$) with their inexhaustible resources repeatedly play a static game with profit flow $G_i(1, s)$:

$$w_i(x) = \sup_{\bar{q}_i} \int_0^\infty e^{-rt} \bar{q}_i(x(t)) \left(P(t) - s_i\right) \mathbb{1}_{\{x(t)>0\}} dt + \frac{1}{r} G_i(1, s).$$

The HJB equation is $rv_0 = G_0(v_0', s)$ with $v_0(0) = 0$, and the equilibrium production rates are:

$$\bar{q}_i^\star(x(t)) = q_i^\star(v_0'(x(t)), s), \qquad i = 0, \ldots, N-1.$$

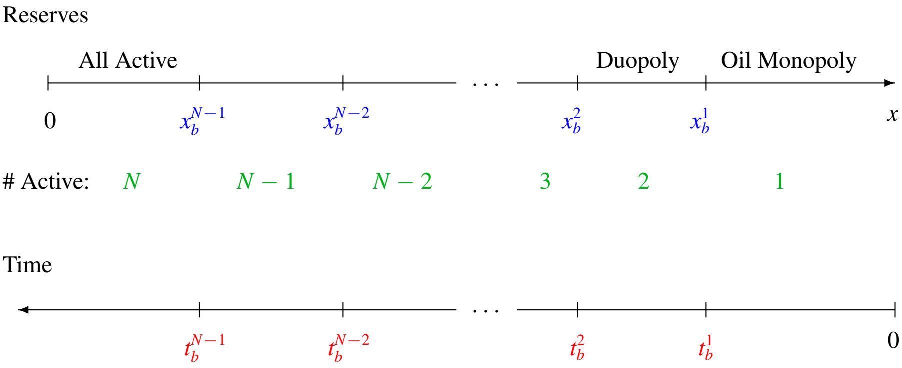
Oil producer's scarcity value (shadow cost) is encoded in $v_0'(x)$.

Blockading Points

For $n = 0, \ldots, N-1$, let

$$x_b^n = \inf\{x \geq 0 : \bar{q}_n^\star(x) = 0\}, \quad t_b^n = \inf\{t \geq 0 : \bar{q}_n^\star(x(t)) > 0\}.$$

Let $S^{(k)} = \sum_{j=1}^{k} s_j$ and assume s is s.t. $s_{N-1} < \frac{1+S^{(N-2)}}{N-1}$: guarantees everyone else participates when oils runs out.



Low Oil Reserves: Value Function

Proposition

For $x \in (0, x_b^{N-1})$, *Player* 0*'s value function is given by*

$$v^{(N)}(x) = \frac{1}{r}\left(\frac{1+S^{(N-1)}}{N+1}\right)^2 \left(1+\mathbf{W}\left(\theta(x)\right)\right)^2,$$

with $\theta(x) = -e^{-\mu_N x - 1}$, *and,* $\mu_N = \frac{r(N+1)^2}{2N(1+S^{(N-1)})}$, *and where* $\mathbf{W}\left(\cdot\right)$ *is the Lambert-*$\mathbf{W}$ *function.*

$$\begin{aligned}
\bar{q}_0^\star(x(t)) &= \frac{1}{(N+1)}\left(1 - N v^{(N)'}(x(t)) + S^{(N-1)}\right), \\
\bar{q}_i^\star(x(t)) &= \frac{1}{(N+1)}\left(1 - (N+1)s_i + v^{(N)'}(x(t)) + S^{(N-1)}\right),
\end{aligned}$$

where $v^{(N)'}(x) = -(1+S^{(N-1)})\mathbf{W}\left(\theta(x)\right)/N.$

Blockading Point

Let $\alpha_n = (n+1)s_n - (1 + S^{(n-1)})$.

Proposition

The last blockading point is given by:

$$x_b^{N-1} = \frac{1}{\mu_N}\left[-1 + \frac{N\alpha_{N-1}}{1+S^{(N-1)}} - \log\left(\frac{N\alpha_{N-1}}{1+S^{(N-1)}}\right)\right],$$

provided $\alpha_{N-1} > 0$*, otherwise* $x_b^{N-1} = \infty$*. Suppose that for* $n \in \{2, \ldots, N-1\}$, $x_b^n < \infty$*. If* $\alpha_{n-1} > 0$*, then*

$$x_b^{n-1} = x_b^n + \frac{1}{\mu_n}\left[-\frac{n(n+1)}{1+S^{(n-1)}}\left(s_n - s_{n-1}\right) - \log\left(\frac{\alpha_{n-1}}{\alpha_n}\right)\right],$$

otherwise $x_b^{n-1} = \infty$.

Assume hereon s such that all $\alpha_n > 0 \Rightarrow x_b^n < \infty$.

Hotelling's Rule

- A modified version of Hotelling's rule for exhaustible resources holds:

Proposition

For $n \in \{1, \ldots, N\}$*, for* $x \in (x_b^n, x_b^{n-1})$*, (we identify* $x_b^N = 0$ *and* $x_b^0 = \infty$*),*

$$\frac{d}{dt} v^{(n)\prime}(x(t) - x_b^n) = \left(\frac{1}{2} + \frac{1}{2n}\right) r\, v^{(n)\prime}(x(t) - x_b^n).$$

- Coincides with the classical Hotelling rule (1931) for $n = 1$: the marginal value grows (exponentially) at the discount rate.

Market Price

- It can be shown that $P^{(n)}(x_b^{n-1} - x_b^n) = s_{n-1}$, *i.e.* the blockading point x_b^{n-1} is exactly the point at which the market price equals the cost of Firm $n-1$.
- Turns out there is an autonomous linear ODE for the price:

$$\frac{d}{dt}P(t) = \left(\frac{1}{2} + \frac{1}{2n}\right) r \left(P(t) - \frac{1 + S^{(n-1)}}{n+1}\right).$$

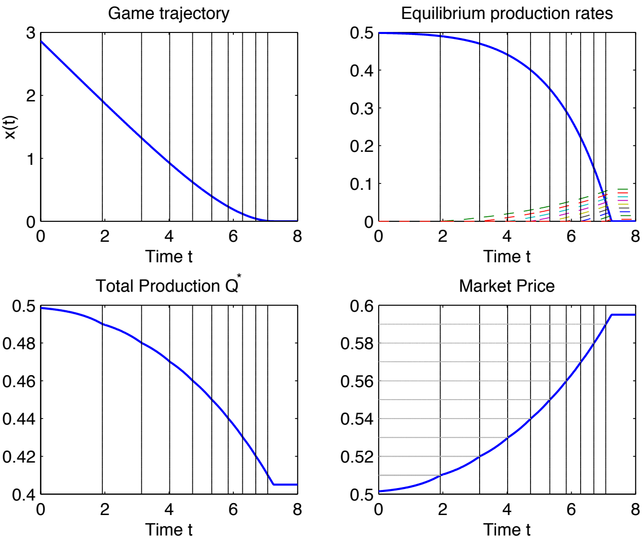
- For $n \in \{2, \ldots, N-1\}$, the time at which Firm n enters the game is

$$t_b^n = t_b^{n-1} + \frac{2n}{(n+1)r} \log\left(\frac{\alpha_n}{\alpha_{n-1}}\right),$$

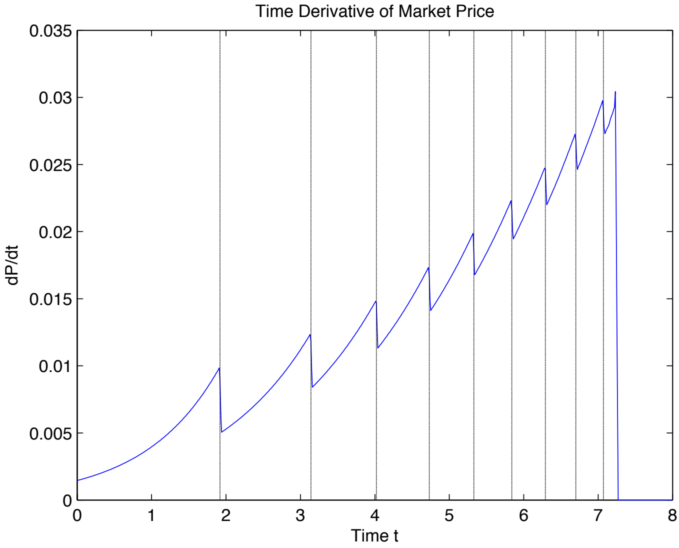
and for $n = 1$ by

$$t_b^1 = \frac{1}{r} \log\left(\frac{s_1 - \frac{1}{2}}{P(0) - \frac{1}{2}}\right).$$

Example: $N = 10$, $s = (0.51, 0.52, \ldots, 0.59)$



dP/dt


Time Derivative of Market Price
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
dP/dt
0
1
2
3
4
5
6
7
8
Time t

Summary

- Exhaustibility wins over increased competition: oil runs low, competing energy sources enter the market, but price rises. However, exponential rate of price increase decreases like $(\frac{1}{2} + \frac{1}{2n})r$.
- Remains to understand the blockading issue with multiple exhaustible suppliers: involves strongly coupled systems of nonlinear PDEs with nonsmooth coefficients.
- Those PDEs require subtle regularization in the form of trembling: bounding below $\bar{q}_i \geq \varepsilon$ and passing $\varepsilon \downarrow 0$.
- Next: incorporate exploration.

Exploration and Random Discoveries

- So far: exhaustibility or scarcity leads to price increases/shocks.
- However there were over 30 new discoveries in 2009. Proved reserves of crude oil rose 13% to 25.2 billion barrels in 2010, the largest annual increase since 1977, and the highest total level since 1991.
- We analyze effect of exploration and random discoveries in a dynamic Cournot game . This was studied in the monopoly context: Pindyck '78, Arrow & Chang '82, Deshmukh & Pliska '80-'85, Soner '85, Hagan *et al.* '94.
- Concentrate on two-player game: player 2 is clean (solar) with fixed cost $c > 0$; player 1 produces oil at zero cost, but can explore for new reserves.

Axis Game with Exploration

The remaining reserves X of Player 1 follows

$$dX_t = -q_1(X_t)\,\mathbb{1}_{\{X_t>0\}}\,dt + \delta\,dN_t,$$

where (N_t) is a controlled point process with intensity λa_t, penalized by cost $\mathcal{C}(a_t)$. Market price: $P(t) = \big(1 - q_1(X_t) - q_2(X_t)\big)$.
Value functions of each player:

$$\begin{aligned}
v(x) &= \sup_{q_1,a} \mathbb{E}\left[\int_0^\infty e^{-rt}\left(q_1(X_t)P(t) - \mathcal{C}(a_t)\right)\,dt \mid X_0 = x\right], \\
w(x) &= \sup_{q_2\geq 0} \mathbb{E}\left[\int_0^\infty e^{-rt} q_2(X_t)\big(P(t) - c\big)\,\mathbb{1}_{\{X_t>0\}}\,dt \right. \\
&\qquad\qquad \left. + \int_0^\infty e^{-rt}\frac{1}{4}(1-c)^2\,\mathbb{1}_{\{X_t=0\}}\,dt \mid X_0 = x\right].
\end{aligned}$$

Axis Game HJB System

The ODEs for v and w are

$$\begin{aligned}
\sup_{q_1,a}\left\{(1-q_1-q_2^*)q_1 - q_1 v'(x) - \mathcal{C}(a) + a\lambda\Delta v(x)\right\} - rv(x) &= 0,\\
\sup_{q_2\geq 0}\left\{(1-q_1^*-q_2-c)q_2\right\} - q_1^* w'(x) + a^*(x)\lambda\Delta w(x) - rw(x) &= 0,
\end{aligned}$$

where $\Delta v(x) = v(x+\delta) - v(x)$ is the non-local or jump term, and

$$a^*(x) = \operatorname*{argsup}_{a\geq 0}\{-\mathcal{C}(a) + a\lambda\Delta v(x)\}$$

is the optimal exploration effort.
Boundary conditions:

$$v(0) = \sup_a \frac{a\lambda v(\delta) - \mathcal{C}(a)}{\lambda a + r}, \qquad w(0) = \frac{(1-c)^2/4 + \lambda a^*(0) w(\delta)}{\lambda a^*(0) + r}.$$

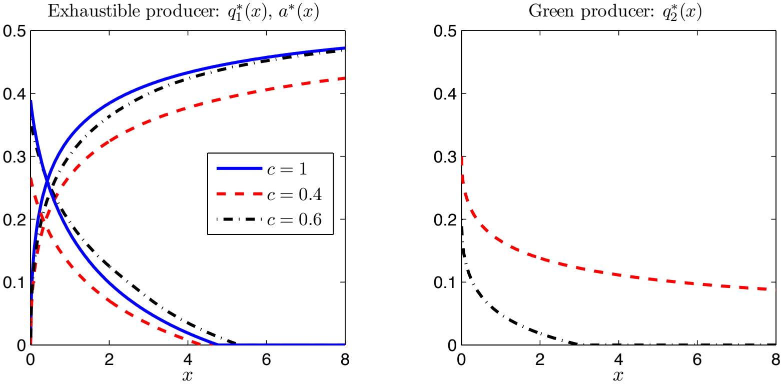
Power Function Costs

- If $a^* > 0$ for all x then X^* is recurrent on its full state space. Therefore $\sup_t X_t^* = +\infty$ and reserves will become arbitrarily large infinitely often.
- Unrealistic for describing non-renewable resources, and suggests that we should take $\mathcal{C}'(0) > 0$.
- Then there exists a *saturation level* x_{sat} such that $a^*(x) = 0$ for $x > x_{\text{sat}}$ and X^* would be positive recurrent on $[0, x_{\text{sat}} + \delta)$ only.
- Take $\mathcal{C}(a) = \frac{1}{\beta} a^\beta + \kappa a$, with $\beta > 1, \kappa \geq 0$. Note that $\mathcal{C}'(0) = \kappa$.

  Then $a^*(x) = \left[(\lambda \Delta v(x) - \kappa)^+ \right]^{\gamma - 1}$, where $\beta^{-1} + \gamma^{-1} = 1$, and

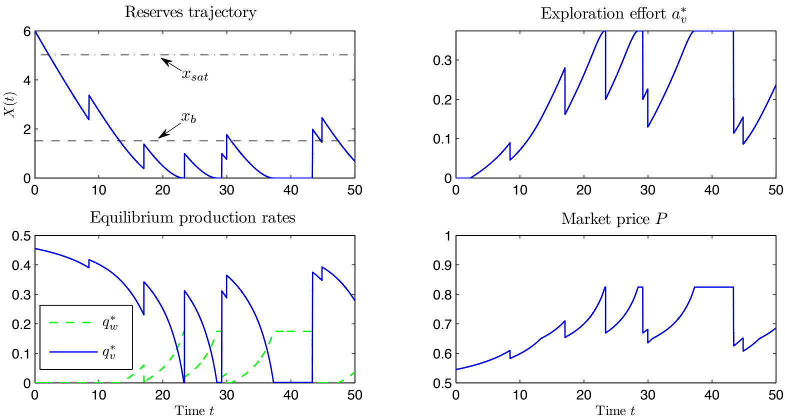
$$\frac{1}{9}(1 - 2v' + c)^2 + \frac{1}{\gamma} \left[(\lambda \Delta v(x) - \kappa)^+ \right]^\gamma - rv = 0.$$

Effect of Competition on Exploration Effort



The parameters are $\delta = 1$, $\lambda = 1$, $r = 0.1$, $\mathcal{C}(a) = 0.1a + a^2/2$.

Sample Game Dynamics



Hotelling's Rule Updated

Monopoly exhaustible resources, Hotelling 1931:

$$\frac{d}{dt}v'(X_t^*) = rv'(X_t^*).$$

Here we have

$$\frac{d}{dt}v'(X_t^*)\mid_{X_t^*=x} = \mathcal{D}v'(x) = \lambda a^*(x)\Delta v'(x) - q_1^*(x)v''(x),$$

and we find:

$$\mathcal{D}v'(x) = \begin{cases} rv'(x) + q_1^*(x)\dfrac{\partial}{\partial x}q_2^*(x) & \text{if} \quad x < x_{\mathrm{b}} \wedge x_{\mathrm{sat}} \\ \dfrac{3}{4}rv'(x) & \quad x_{\mathrm{sat}} < x < x_{\mathrm{b}} \\ rv'(x) & \quad x > x_{\mathrm{b}}. \end{cases}$$

With competition, shadow prices grow *slower* than r.

Continuum Mean Field Game Approximations

- With a finite number of exhaustible players, the HJB system of PDEs needs numerical resolution. Even in the two-player case, these equations are hard to handle.
- Instead, one may study the market dynamics when the number of firms tends to infinity by using the concept of a mean field game (MFG), proposed by Lasry & Lions and Caines *et al.*: continuum limit of an infinity of small players.
- The interaction is modeled by assuming that each player only sees and reacts to the other players throught their average action.
- Optimization against the distribution of other players leads to a backward (in time) HJB equation; their actions determine the evolution of the state distribution, encoded by a forward Kolmogorov equation.
- In examples, these equations are numerically tractable and provide good qualitative approximation for the small # player problem.

Some References

	# Players	Type	Demand	Randomness	Replenish
Hotelling (1931)	1	–	linear	Determ.	No
Dasgupta and Stiglitz (1981)	N	Cournot	constant	single-shock	No
Deshmukh and Pliska (1983)	1	–	regimes	Poisson	Yes
Benchekroun (2008)	N	Cournot	linear	Determ.	Yes
Benchekroun et al (2009)	N	Cournot	linear	Determ.	Yes
Harris et al (2010)	1+g	Cournot	linear	Brownian	No
Ludkovski and Sircar (2011)	1+g	Cournot	linear	Poisson	Yes
Ledvina and Sircar (2012)	1+N	Bertrand	linear	Determ.	No
Ludkovski and Yang (2014)	1+g	Cournot	linear	Poisson	Yes
Colombo and Labrecciosa (2013)	N	Cournot	linear	Determ.	Yes
Dasarathy and Sircar (2014)	1+N	Cournot	linear	Poisson	Yes
Guéant et al (2010)	∞	Cournot	CES	Determ.	No
Chan and Sircar (2014)	∞	Bertrand	linear	Brownian	No