Energy & Commodity Markets I

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Overview

1. Traditional commodity markets issues and models. Electricity price models.
2. Energy production from exhaustible resources and renewables: game threoretic models.
3. Financialization of commodity markets.

Commodity Forward Curves

- By commodities, we refer to
  - Agriculturals: (soybeans, corn, coffee, sugar, ...)
  - Metals (copper, aluminum, gold, silver, ...)
  - Energy (oil, natural gas, coal, power, carbon, ...)
  - Livestock (live cattle, frozen pork bellies, ...),
  - Shipping / Freight, Weather (temperature, rainfall)
- They are widely traded on standardised exchanges such as the Chicago Board of Trade (CBOT), the London Metal Exchange (LME), New York Mercantile Exchange (NYMEX).
- Idealized notion: spot price (S_t), akin to stock price. But storability is a major distinction.
- The basic financial instrument is the forward, with the forward price $F(t,T)$ being the price agree today (time t) for delivery on maturity date T, and so

$$\lim_{t \to T} F(t,T) = S_T.$$

Forward Prices when no Storage Cost

- Portfolio A: Long forward at time t
  - Time t: costs nothing to enter forward contract;
  - Time T: portfolio realizes $S_T - F(t,T)$.
- Portfolio B: Buy spot, borrow $\$x$
  - Time t: costs $S_t - x$;
  - Time T: portfolio realizes $S_T - xe^{r(T-t)}$.
- Choose $x = Fe^{-r(T-t)}$, then portfolios A and B have identical payoffs.
- No arbitrage implies the must cost the same to enter, so

$$0 = S_t - x \qquad \Rightarrow \qquad F(t,T) = S_t e^{r(T-t)}.$$

- But the 'buy spot' side of the arbitrage cannot easily be executed when the commodity has to be stored. Leads to modifications described as cost of carry and convenience yield.

Backwardation & Contango

- Upward sloping forward curve: contango .
- Downward sloping: backwardation .
- Historically, forward curves have been in *backwardation* about 75% of the time. But in contango more often more recently.
- Keynes' theory of backwardation (1930s): $F(t, T)$ should be a downward biased estimate of S_T, *i.e.* spot should be greater than forward prices.
- Explained by the hedging pressure on commodity producers:
  - Producers : long commodity, **sell** forwards to hedge;
  - Consumers : short commodity, **buy** forwards to hedge.
- Keynes: producers dominate due to fragmentation of consumers. They pay a premium to lock in prices.
- More recent contango (oil): pushed up by speculators? Financialization.

Convenience Yield

- Typical modification to forward prices:

$$F(t,T) = S_t e^{(r+c-\delta)(T-t)},$$

  where the (vague) variables are:
  - c is cost of carry: rent for storage;
  - δ is convenience yield: benefits holder of spot, but not the holder of the forward, *inversely* related to inventory levels.
- Large δ: backwardation.
- Small δ, large c: contango.
- Making c, δ time-varying and random: can switch between backwardation and contango.

Spot Price Models

- Under a risk-neutral measure Q, spot is GBM:

$$\frac{dS_t}{S_t} = (r - \delta)\, dt + \sigma\, dW_t^Q.$$

- No arbitrage arguments dictate that

$$F(t,T) = \boldsymbol{E}^Q\{S_T \mid S_t\} = e^{(r-\delta)(T-t)} S_t.$$

- Leads to

$$\frac{dF(t,T)}{F(t,T)} = \sigma\, dW_t^Q.$$

  Volatility of forward is constant and independent of T.
- Samuelson effect: volatility is greater for shorter maturity forwards.
- Captured by mean-reverting spot price models.

Schwartz One-Factor Model (1997)

- Spot price is expOU. $S_t = e^{Y_t}$, where

$$dY_t = \alpha(m - Y_t)\,dt + \sigma\,dW_t^Q.$$

- Leads to

$$\frac{dS_t}{S_t} = \alpha\left(m + \frac{\sigma^2}{2\alpha} - \log S_t\right)\,dt + \sigma\,dW_t^Q.$$

- Forward prices

$$F(t,T) = \exp\left(e^{-\alpha(T-t)}\log S_t + m(1 - e^{-\alpha(T-t)}) + \frac{\sigma^2}{4\alpha}(1 - e^{-2\alpha(T-t)}\right.$$

and consequently

$$\frac{dF(t,T)}{F(t,T)} = \sigma\,e^{-\alpha(T-t)}\,dW_t^Q,$$

which captures the Samuelson effect, but suggests volatility goes to zero at long maturities T.

Other Limitations

- Limited range of forward curve shapes.
- With one α, misprice either the long or the short end.
- Perfect correlation between front and back end of forward curve. Both must move in sync, but not observed to do so empirically.
- Leads naturally to multi-factor models, *e.g.* Schwartz two-factor model:

$$\begin{aligned} \frac{dS_t}{S_t} &= (r - \delta_t)\, dt + \sigma_1\, dW_t^Q \\ d\delta_t &= \alpha(\mu - \delta_t)\, dt + \sigma_2\, dB_t^Q. \end{aligned}$$

- Also Schwartz & Smith (2000); Schwartz 3-factor which has stochastic (Vasicek) interest rate.

Derivatives I: Options on Forwards

- For example a call option to go long a forward with expiration T' on date $T < T'$, with strike K. Payoff $(F(T, T') - K)^+$.
- For the GBM model, have Black's formula

$$C_t = e^{-r(T-t)} \left[F(t, T')N(d_1) - KN(d_2) \right],$$

where $N(\cdot)$ is the standard normal cdf, and

$$d_{1,2} = \frac{\log\left(\frac{F(t,T')}{K}\right) \pm \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}.$$

- Similar calculations can be made in the Schwartz models.

Derivatives II: Spread Options

- For example, calendar spreads with payoff :

$$(F(T_1, T_2) - F(T_1, T_3 - K))^+, \qquad \text{where} \qquad T_1 < T_2 < T_3.$$

- For $K = 0$, there is (in the GBM model) Margrabe's formula.
- Valuation of such products motivates Gaussian Exponential Factor Models (GEM):

$$\frac{dF(t,T)}{F(t,T)} = \sum_{i=1}^{N} \sigma_i(t,T)\, dW_t^{Q,i},$$

which are flexible since don't have to be derived from a spot model. See book of G. Swindle.

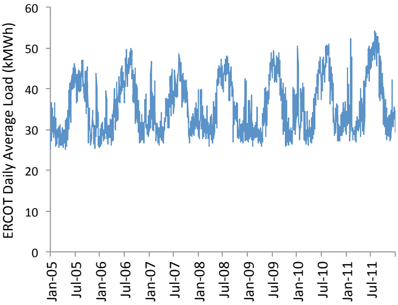
- Using such products for hedging, motivates *stretching* Black-Scholes-type models as much as possible, even for spiky electricity markets.

A Model for Hedging Load and Price Risk in the Texas Electricity Market (Coulon, Powell, Sircar 2013)

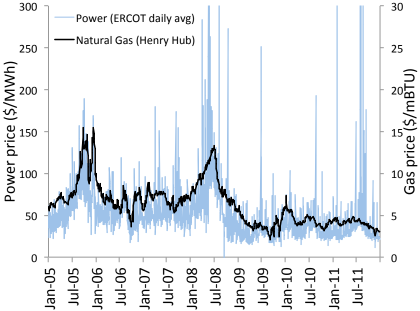
- Use of financial products, *e.g.* futures and options, by retail suppliers to hedge electricity price and demand spikes has grown recently.
- As well as spikes, power prices have strong seasonal components, are linked to fuel prices (esp. natural gas), driven by demand (or load), and depend on transmission capacity.
- Typical in financial options problems to use continuous time stochastic models built on Brownian motion. Here we want to try and capture the above features, *even spikes*, with diffusions.
- Motivated by convenient pricing formulas which can reduce the simulation burden on an optimization program for hedging risk.

Texas Data & Related Models

- US electricity market in Texas: ERCOT (Electric Reliability Council of Texas) serve 20m+ customers.
- Power prices volatile and weather extreme! Heat waves – e.g. Aug '11, total load 68.4 Gwh, day ahead prices $\$3000/MWh$.
- Modelling prices directly (reduced form):
  - Jump diffusions: Cartea & Figueroa '05; Geman & Roncoroni '06; Hambly, Howison & Kluge '09.
  - Lévy processes: Benth *et al.* '08; Veraart & Veraart '12.
- Structural models:
  - Barlow '02 (price driven purely by demand);
  - Pirrong & Jermakyan '08 (include fuel prices);
  - Cartea & Villaplana '05 (include capacity changes); Coulon & Howison '09; Aid *et al.* '12 (both).



(a) Historical daily average ERCOT loads



(b) Historical daily average electricity and gas prices

Figure: *ERCOT load and electricity prices over 2005-11, and natural gas prices. Seasonality in loads and spikes in prices are clear.*

Building the Model I

- First de-seasonalize the ERCOT load L_t:

$$L_t = S(t) + \bar{L}_t,$$

where the seasonal component (estimated using hourly data) is given by

$$S(t) = a_1 + a_2 \cos(2\pi t + a_3) + a_4 \cos(4\pi t + a_5) + a_6 t + a_7 \mathbb{1}_{we}.$$

Here $a_2(h)$ to $a_5(h)$ are the seasonal components, $a_6(h)$ picks up the upward trend , and $a_7(h)$ captures the drop in demand on weekends, and h is the hour;

- Then fit the residual load $\bar{L}_t$ to an Ornstein-Uhlenbeck (OU) model:

$$d\bar{L}_t = -\kappa_L \bar{L}_t \, dt + \eta_L \, dW_t^{(L)}.$$

- Relation to natural gas priceG_t, model as expOU:

$$d \log G_t = \kappa_G (m_G - \log G_t) \, dt + \eta_G \, dW_t^{(G)},$$

where $W^{(L)}$ and $W^{(G)}$ are independent.

Building the Model II

- An additional factor X proxies for the effect of capacity outages and grid congestion:

$$X_t = S_X(t) + \bar{X}_t,$$

with seasonal component treated similarly to that of load:

$$S_X(t) = b_1 + b_2 \cos(2\pi t + b_3) + b_4 \cos(4\pi t + b_5).$$

- The process $\bar{X}_t$ follows

$$d\bar{X}_t = -\kappa_X \bar{X}_t \, dt + \eta_X \, dW_t^{(X)},$$

where the Brownian motions $W^{(X)}$ and $W^{(L)}$ are correlated with parameter ν.

Spike Mechanism

- The price P_t can spike in any hour as follows:

$$P_t = G_t \exp(\alpha_{m_k} + \beta_{m_k} L_t + \gamma_{m_k} X_t) \quad \text{for } t_k \leq t < t_{k+1}, k \in \mathbb{N}.$$

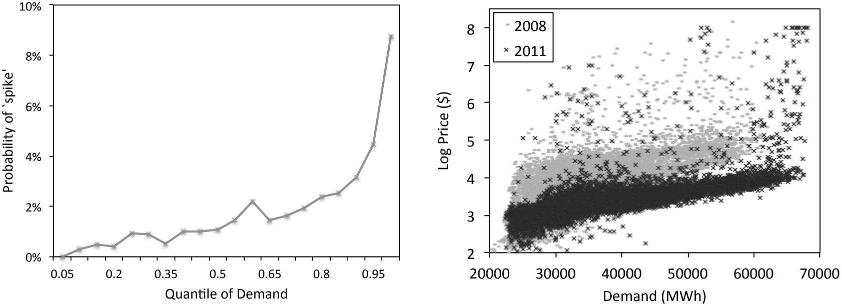
- At each hour t_k, the value of $m_k \in \{1, 2\}$ is determined by an independent coin flip whose probabilities depend on the current load $\bar{L}_{t_k}$:

$$m_k = \begin{cases} 1 & \text{with probability } 1 - p_s \Phi\left(\frac{\bar{L}_{t_k} - \mu_s}{\sigma_s}\right) \\ 2 & \text{with probability } p_s \Phi\left(\frac{\bar{L}_{t_k} - \mu_s}{\sigma_s}\right), \end{cases}$$

where $\Phi(\cdot)$ is the standard normal cdf.

- The choice of $\alpha_2, \beta_2, \gamma_2$ (the spike regime) allows for a steeper and potentially more volatile price to load relationship.
- Model for price is a mixture of lognormals: convenient for the pricing of forwards and options.

Data & Fits



(a) Probability of spike vs load (b) Hourly price vs load for 2008 and 2011

Figure: Relationship between power price and load (right), and between spike probability and load (left).

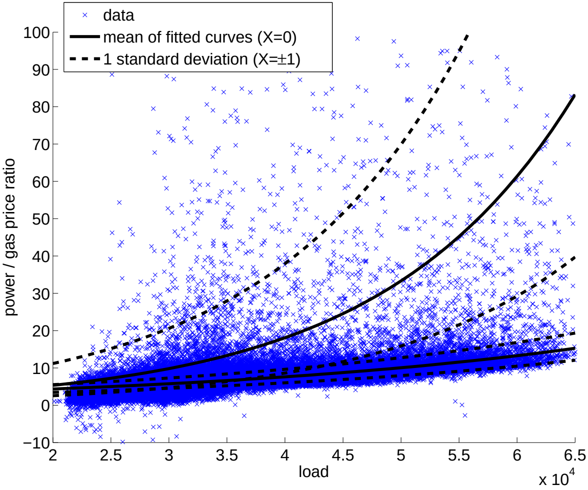


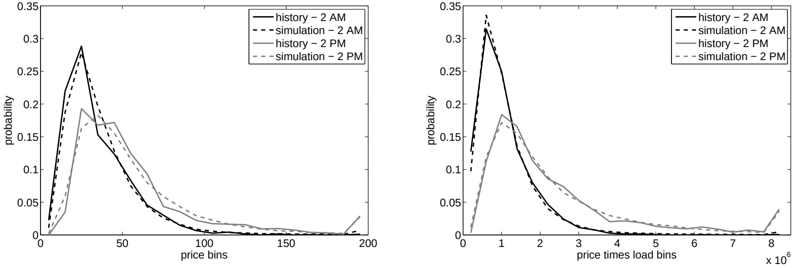
Figure: *Fitting result - solid lines illustrate exponential price-load relationship when $X = 0$ (at the mean), while dotted lines represent one standard deviation bands ($X = \pm 1$).*

Fitting to Data

- Primarily, use Maximum Likelihood Estimation after deseasonalization.
- Find that the factors move on very different time scales: gas very slowly mean-reverting over months ($\kappa_G = 1.07$); load mean reverting over several days ($\kappa_L = 92.6$) and X_t mean reverting on an intra-day time scale ($\kappa_X = 1517$).
- Find that $p_s = 0.129$ implies that we visit the spike regime approximately 6.5% of the time on average
- $6.11 \times 10^{-5} = \beta_2 > \beta_1 = 2.79 \times 10^{-5}$ so that in the spike regime the exponential relationship between price and load is significantly steeper, as expected in order to produce extreme spikes.
- Moreover, $0.741 = \gamma_2 > \gamma_1 = 0.237$, as the spike regime is also more volatile than the normal price regime.

(In-sample) Model Performance

Simulate 100 paths of seven years of price and load dynamics, while always using the historical gas prices from 2005-11:



(a) Price

(b) Price times Load

Figure: *Histograms for P_t (left) and $P_t \times L_t$ (right): model simulation vs. historical data*

A little known option pricing result from the 70's

$$V_0 = S_0 \Phi_2\left(h + \sigma\sqrt{T_1}, k + \sigma\sqrt{T_2}; \sqrt{\frac{T_1}{T_2}}\right) - K_2 \mathrm{e}^{-rT_2} \Phi_2\left(h, k; \sqrt{\frac{T_1}{T_2}}\right) - K\mathrm{e}^{-rT_1}\Phi(h),$$

where $h = \frac{\log(S_0/S^*)+(r-\frac{1}{2}\sigma^2)T_1}{\sigma\sqrt{T_1}}$, $k = \frac{\log(S_0/K)+(r-\frac{1}{2}\sigma^2)T_2}{\sigma\sqrt{T_2}}$ and $S^* = \ldots$

- That was Geske's 1977 result for compound options (a call on a call):

$$V_0 = \mathrm{e}^{-rT_1}\mathbb{E}^{\mathbb{Q}}\left[\left(C^{BS}_{T_1}(T_2, K_2) - K_1\right)^+\right]$$

- It follows from the following very useful result:

$$\int_{-\infty}^{h} e^{cx}\Phi\left(\frac{a+bx}{d}\right)\frac{\mathrm{e}^{-\frac{1}{2}x^2}}{\sqrt{2\pi}}dx = \mathrm{e}^{\frac{1}{2}c^2}\Phi_2\left(h-c, \frac{a+bc}{\sqrt{b^2+d^2}}; \frac{-b}{\sqrt{b^2+d^2}}\right)$$

  where a, b, c, d, h are constants, and $\Phi(\cdot)$, $\Phi_2(\cdot,\cdot;\rho)$ the Gaussian cdf's.
- Interestingly, this same result proves valuable for energy derivatives...

Derivative Contracts

- Explicit pricing of forward contracts on power. Allows to calibrate dynamics under (risk-neutral) pricing measure $\mathbb{Q}$ under certain assumptions.
- Options prices are also computable in closed-form:

$$\begin{aligned} V_t^p &= \mathrm{e}^{-r(T-t)}\mathbb{E}_t^{\mathbb{Q}}\left[(P_T-K)^+\right] \\ &= \mathrm{e}^{-r(T-t)}\Big\{F^g(t,T)c_1\Phi\left(d_1^+\right)-K\Phi\left(d_1^-\right) \\ &\quad +p_s\sum_{i=1}^{2}(-1)^i\Big[F^g(t,T)c_i\Phi_2\big(d_i^+,g_i^+;\lambda_i\big) \\ &\quad -K\Phi_2\big(d_i^-,g_i^-;\lambda_i\big)\Big]\Big\} \end{aligned}$$

- Also: spread options on power and gas.
- No closed form for options on elec. forwards, but simulation fast due to forwards explicit formula.

Revenue Hedging

- Consider here the case of a retail power utility company which faces the choice of waiting and buying the required amount of electricity each hour from the spot market, or hedging its obligations in advance through the purchase of forwards, options or some combination of the two.
- Simplified: we make a single hedging decision today. This is a *static* hedge, so we do not (yet) consider dynamically re-hedging through time.
- Setting $\tau = 1/365$ and p^{fixed} to be the price it charges its customers per MWh, the firm's revenue R_T over the one-day period $[T, T+\tau]$ in the future is given by

$$R_T = \sum_{j=1}^{24} L_{T_j} \left(p^{\mathit{fixed}} - P_{T_j} \right), \quad \text{where } T_j = T + \frac{j}{24}\tau.$$

Hedging Instruments

We allow trading in the following contracts in order to hedge the firm's risk:

- Forward contracts with delivery on a particular hour j, with price $F^p(t, T_j)$
- Call options on these forwards, with payoff at $T_j - \tau$ of $\tilde{V}^p_{T_j-\tau} = (F^p(T_j - \tau, T_j) - K(j))^+$
- Spark spread options on forwards, with payoff $\tilde{V}^{p,g}_{T_j-\tau} = (F^p(T_j - \tau, T_j) - hF^g(T_j - \tau, T_j))^+$
- Call options on the hourly spot power price, with payoff $V^p_{T_j} = \left(P_{T_j} - K(j)\right)^+$
- Spark spread options on spot energy prices, with payoff $V^{p,g}_{T_j} = \left(P_{T_j} - hG_{T_j}\right)^+$,

where h and $K(j)$ are the heat rates and strikes specified in the option contracts. We allow the hourly strikes to vary due to the large price variation through the day, and in the base case consider only at-the-money (ATM) options for all hours.

Variance Reduction by Hedging

- Minimize

$$\mathrm{Var}\left(R_T - \sum_{n=1}^{N} \theta_n U_T^{(n)}\right)$$

where the $\theta_1, \ldots, \theta_N$ represent the quantities purchased of N available hedging products with payoffs $U_T^{(1)}, \ldots, U_T^{(N)}$.

- First choose $N = 1$ to compare products one by one. See the substantial benefit obtained from using derivative products to hedge risk. Can clearly observe an optimal hedge quantity .
- When taken alone forward contracts are the most effective at variance reduction. The next most effective hedge is options on spot power.
- Spread options on spot power and gas provide slightly less variance reduction, since they essentially isolate and hedge only the risk of L_t and X_t, not the natural gas risk.

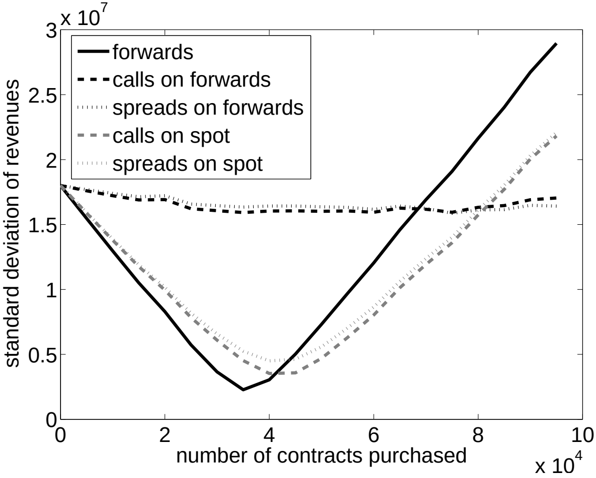
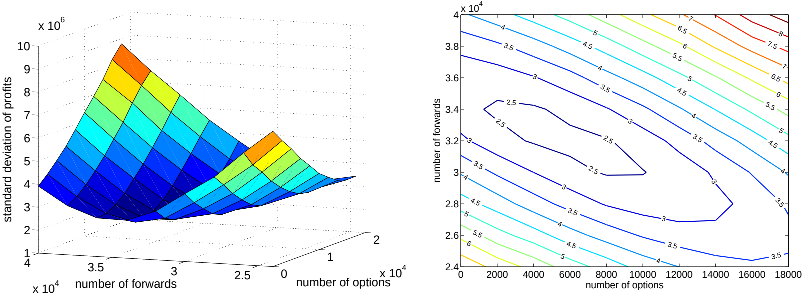


Figure: *Comparison of variance reduction from different hedging strategies (forwards vs. various options).*

Several Hedging Instruments

- By combining several different products, such as forwards and options, even greater variance reduction may be possible.
- Now allow purchases of both forwards and spot spread options.
- We can observe that a minimum in the variance surface appears somewhere in the middle, implying that simultaneously trading in the two products can provide further risk reduction than trading in either product alone.
- However, even in the static case, finding an optimal hedging portfolio across many maturities and many product types may require a sophisticated optimization algorithm ... to do!



(a) Surface plot (b) Contour plot (all contour labels $\times 10^6$)

Figure: Variance reduction when trading in both options and forwards.