Three approaches to a fair operation, access, and design of transportation systems

Ralf Borndörfer

joint work with Thomas Breugem, Martin Grötschel, Nam Dũng Hoàng, Anke Reuter, Markus Reuther, Thomas Schlechte, Christoph Schulz

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Three approaches to …

- … a fair **operation** of transportation systems:
  employee preferences (equitable rosters)
  with Thomas Breugem, Christof Schulz, Thomas Schlechte

- … a fair **access** to transportation systems:
  market access for **operators** (track auctions)
  with Thomas Schlechte and Anke Reuter

- … a fair **design** of transportation systems:
  cost sharing for **users** (ticket prices)
  with Martin Grötschel and Nam Dũng Hoàng

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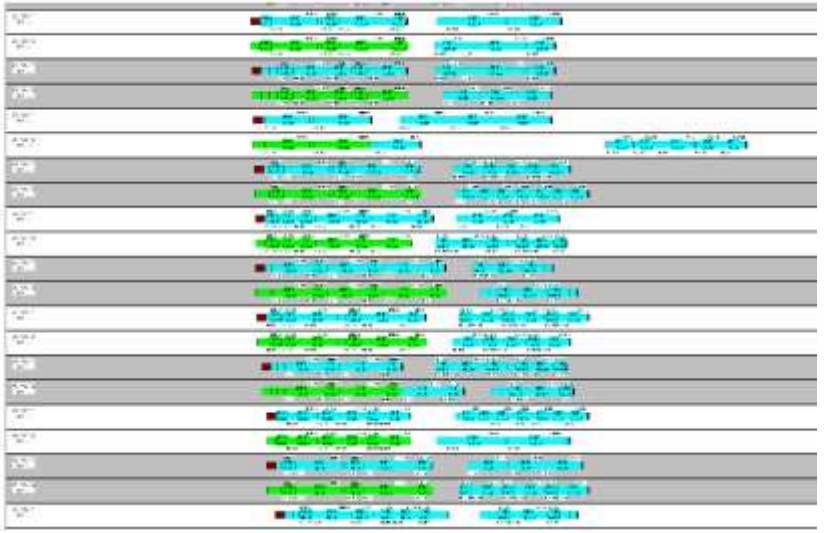
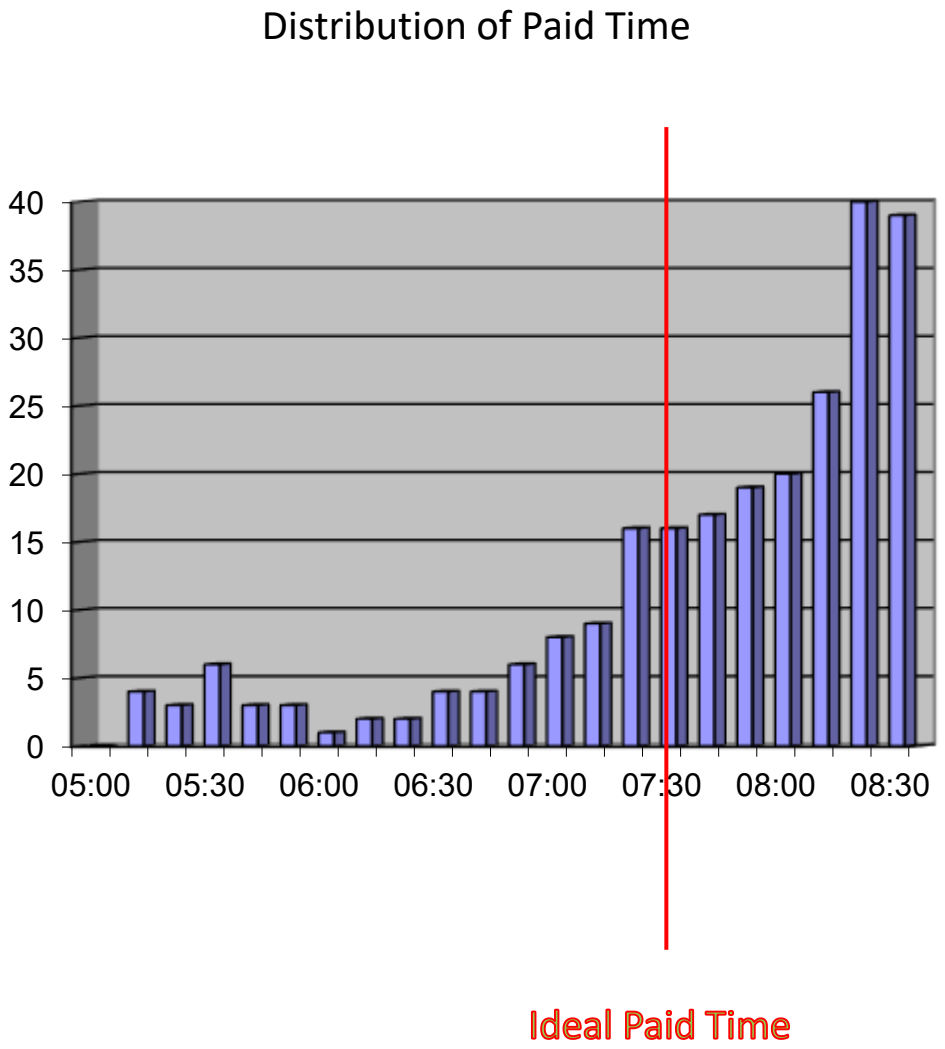
Duty Scheduling at BVG

- Duty optimization since 2010
- 6 bus depots (# duties Mo-Fr 1500, Sa 1100, Su 900)
- ~90 changes with 3956 timetables / year
- ~110 versions with 612 rosters / year
- 4 tram depots (# duties Mo-Fr 580, Sa 440, Su 390 duties)
- 10 subway groups (# duties Mo-Fr 250, Sa 210, Su 190)
- indicators
  - average paid time of all duties
  - # of split-duties / types of duties
  - vehicle changes / duty
  - costs of transit times, breaks
  - duration of individual duties

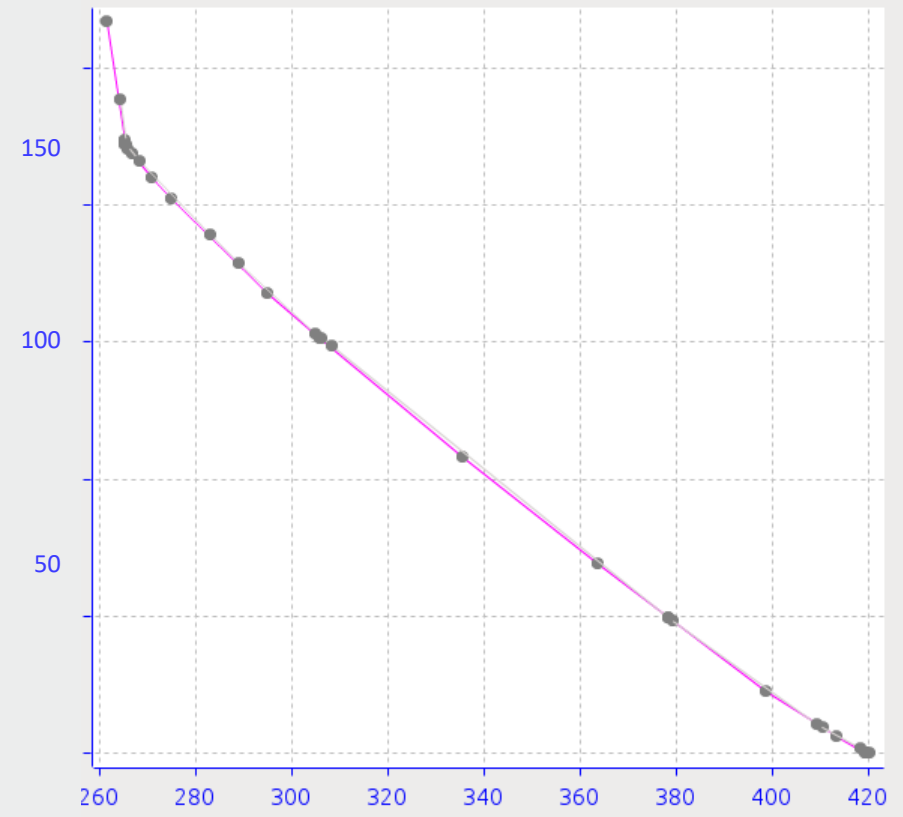


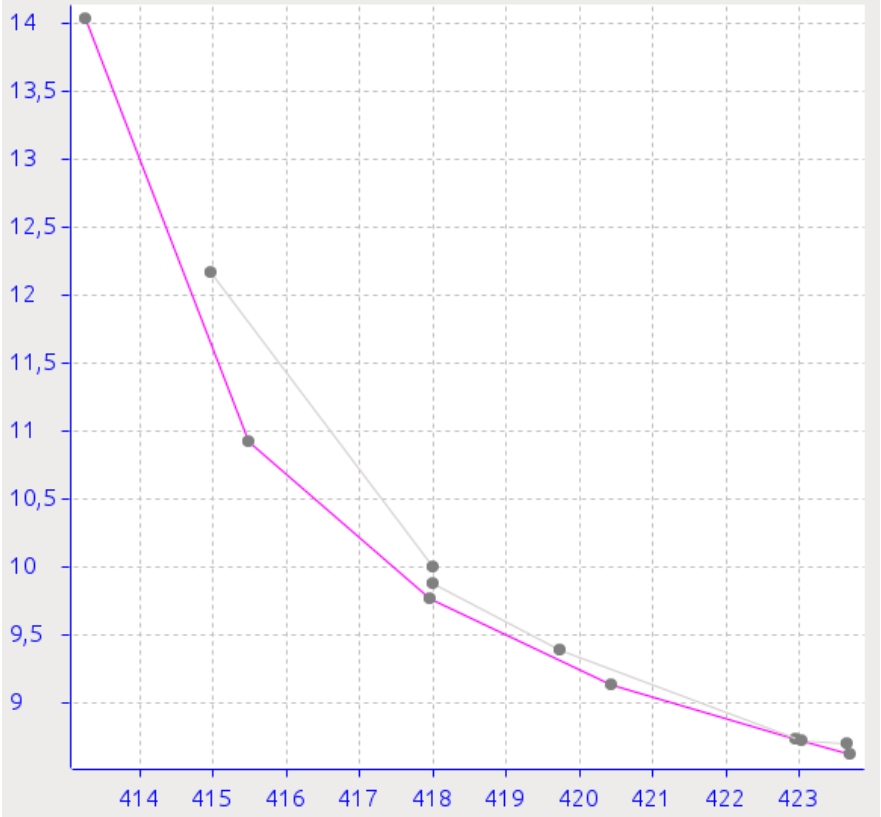
Bus Depot Cicerostraße

- 247 duties
- 1878:16 paid time
- 1472:33 driving time
- 7:36 average duty time



Pareto Optimization

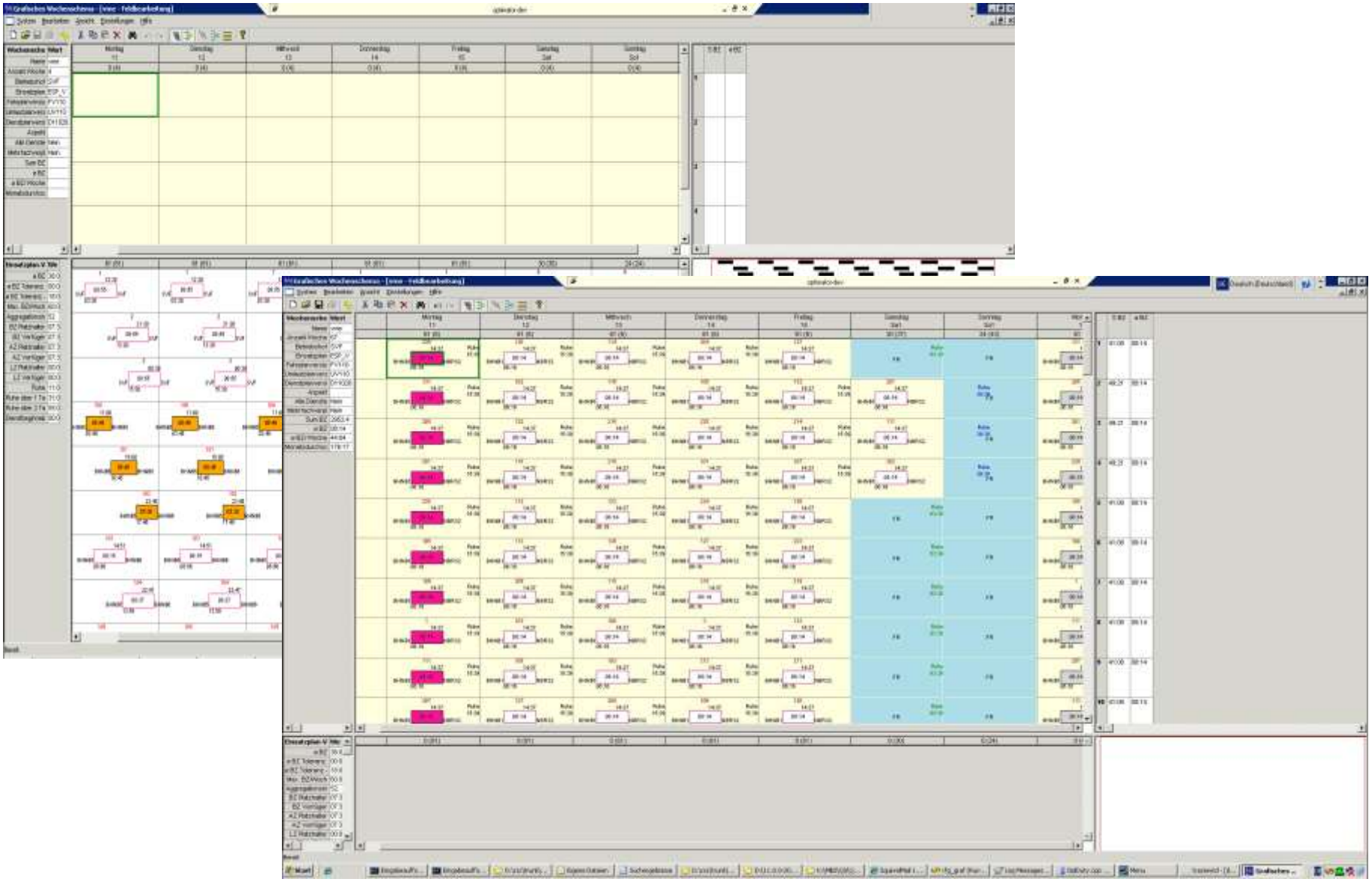




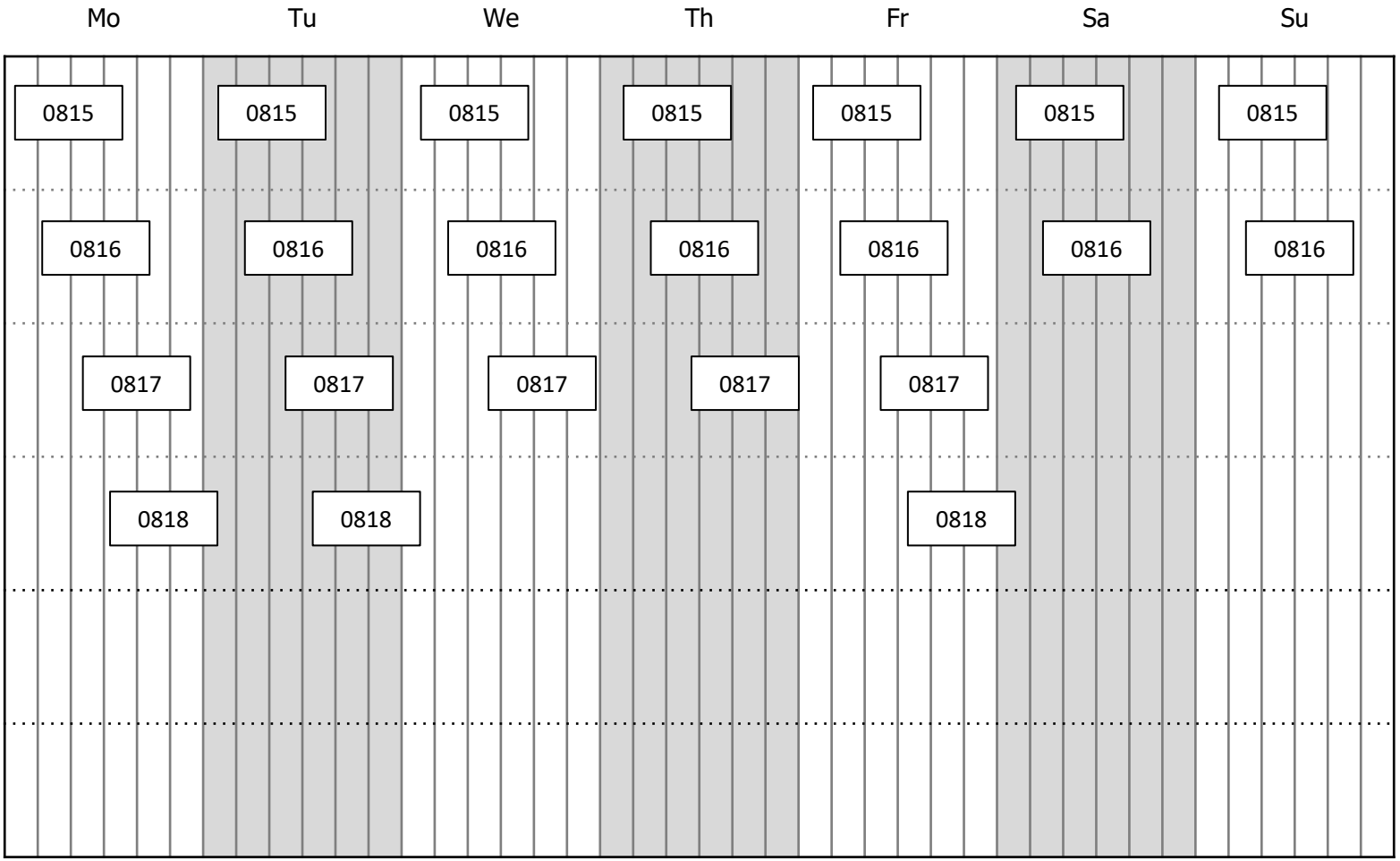
split duties vs. cost index

deviation from paid time penalty vs. cost index

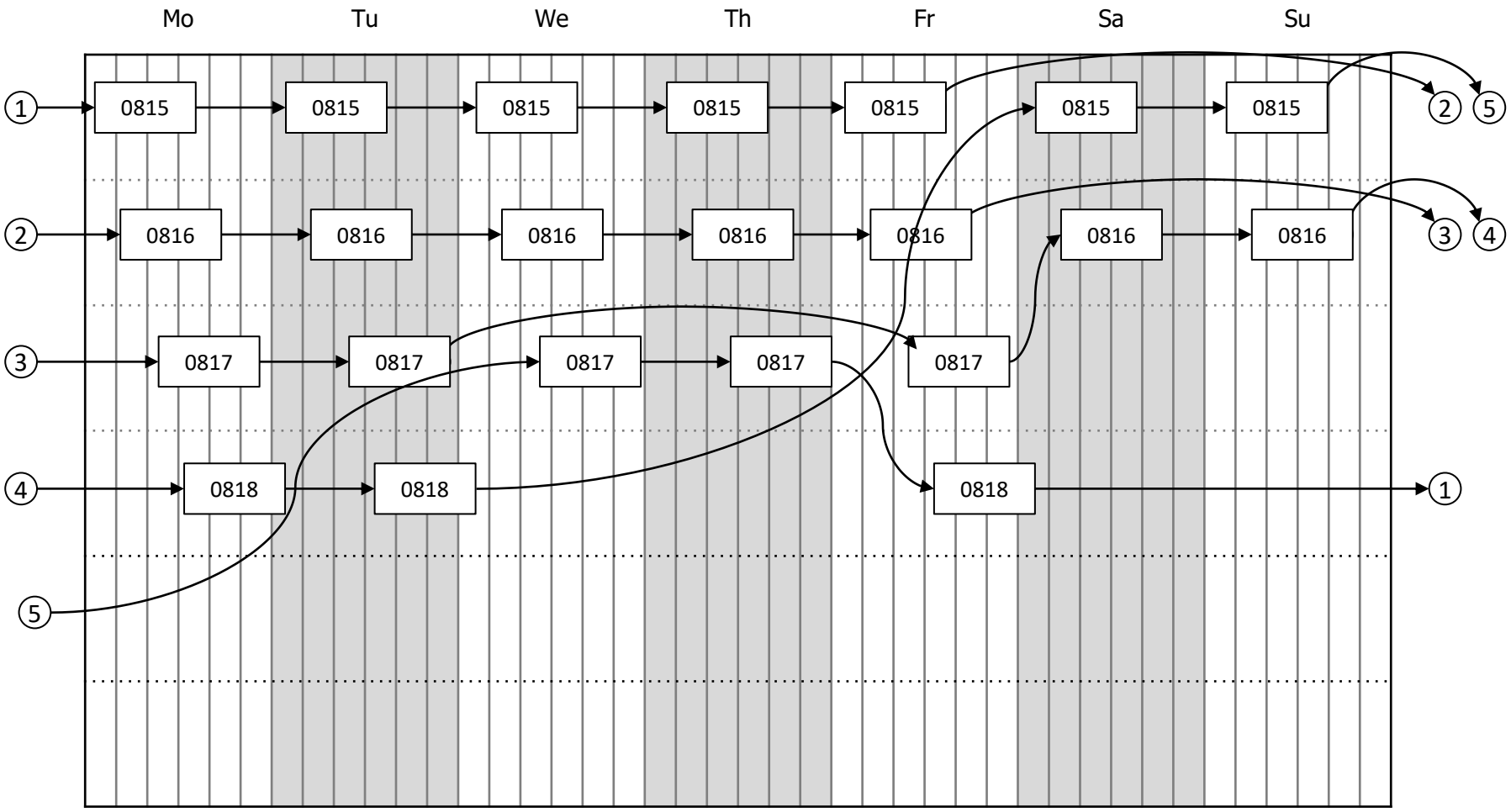
Duties



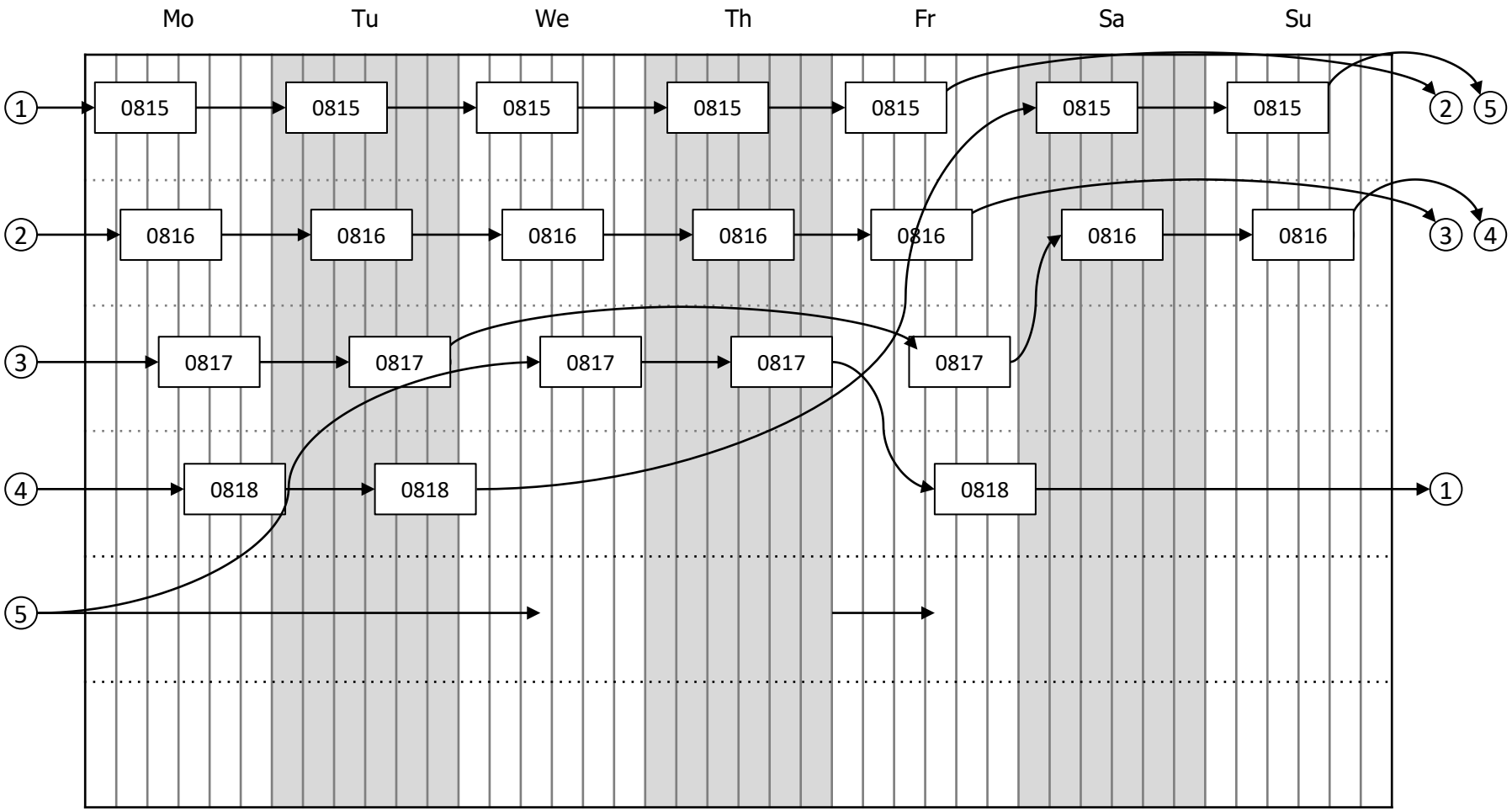
Duties



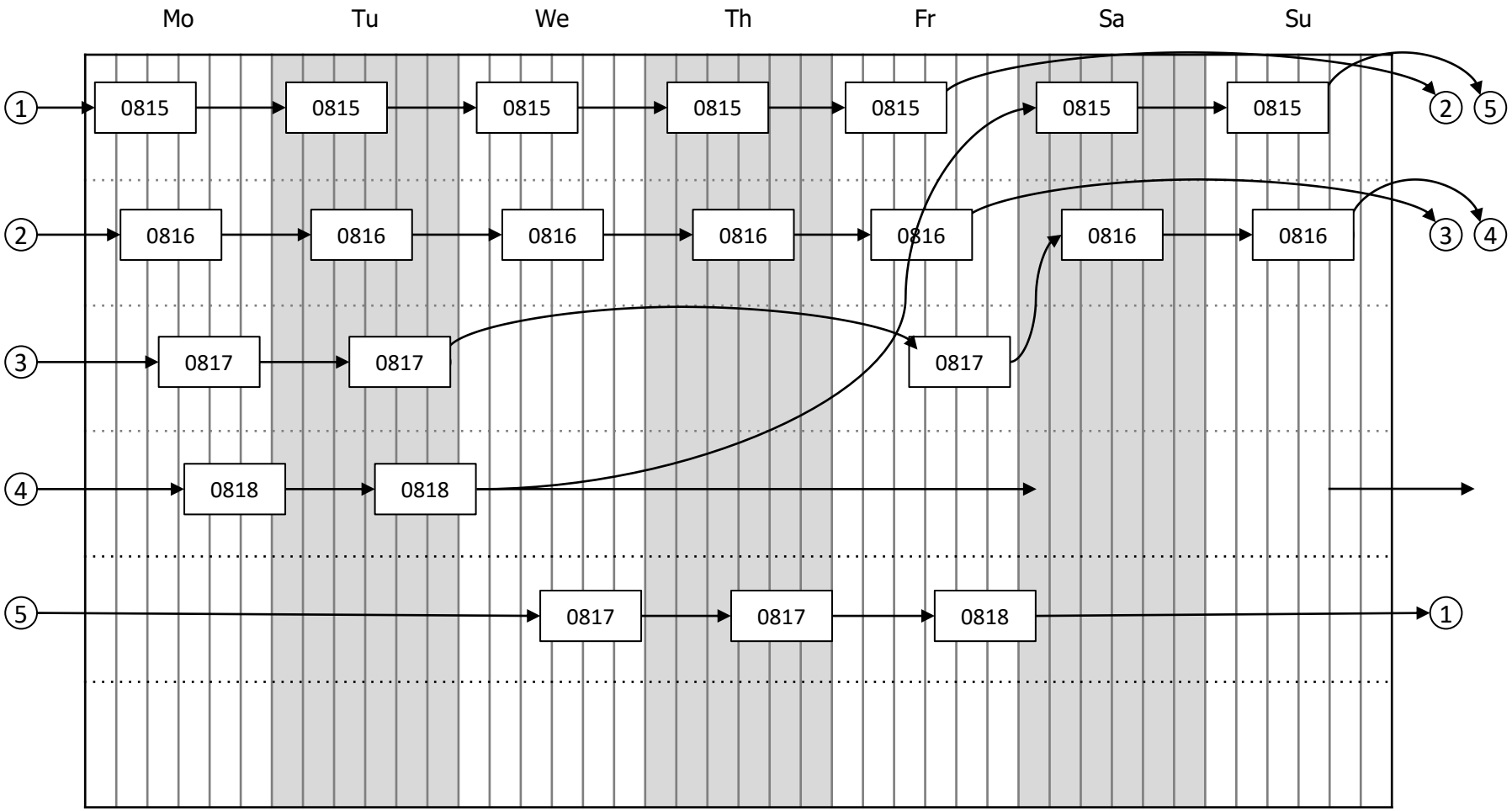
Weekly Roster Scheme



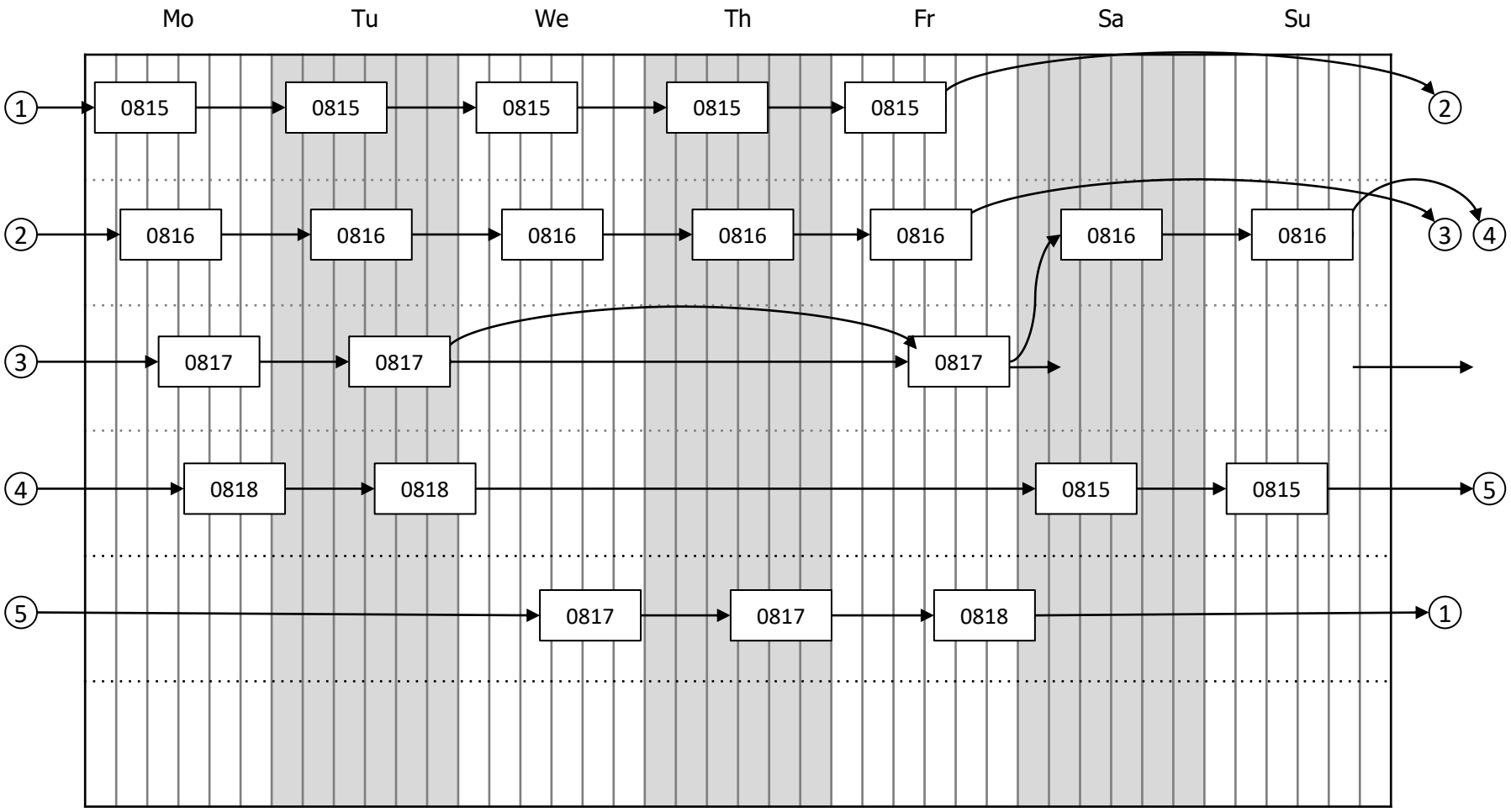
Weekly Roster Scheme



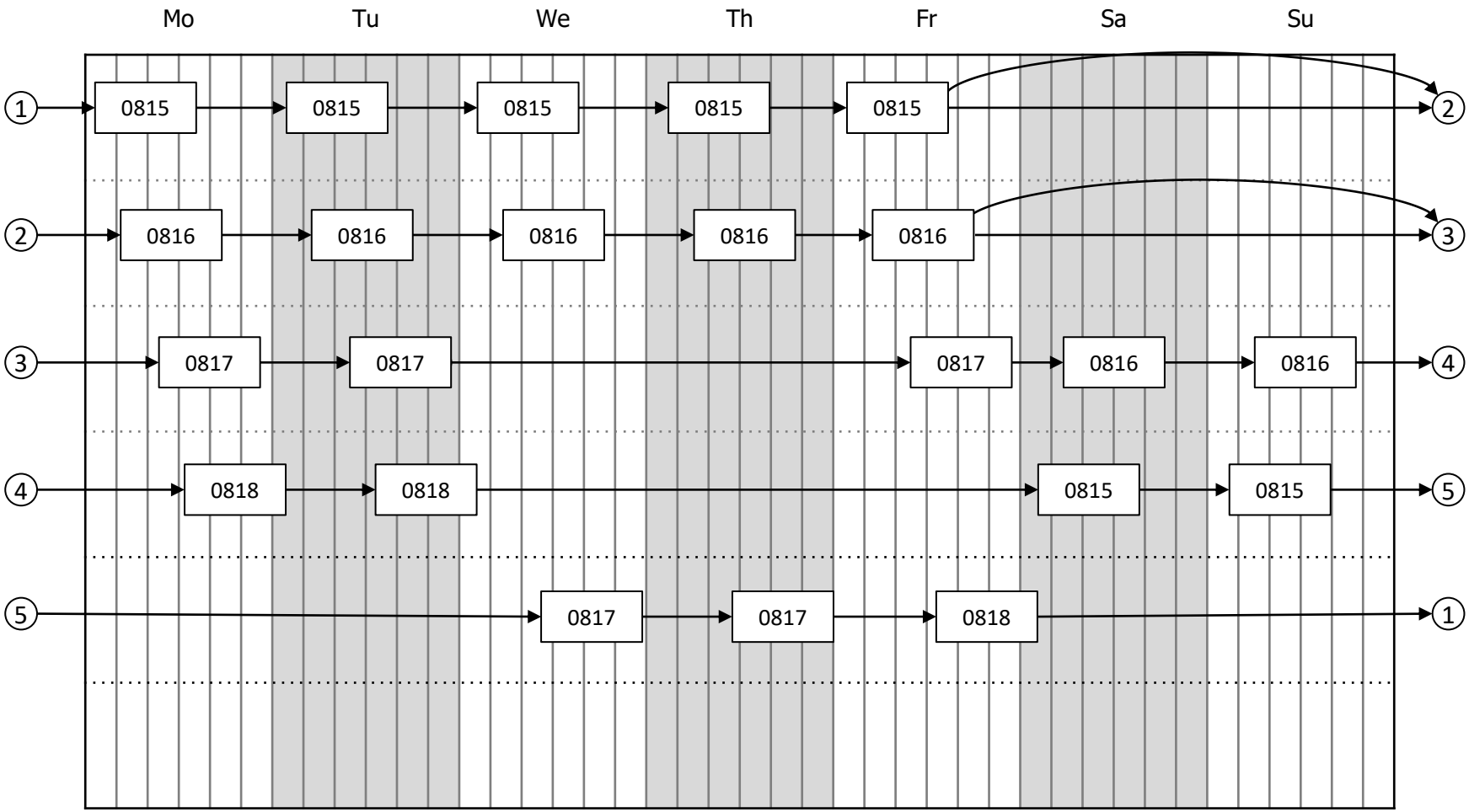
Weekly Roster Scheme



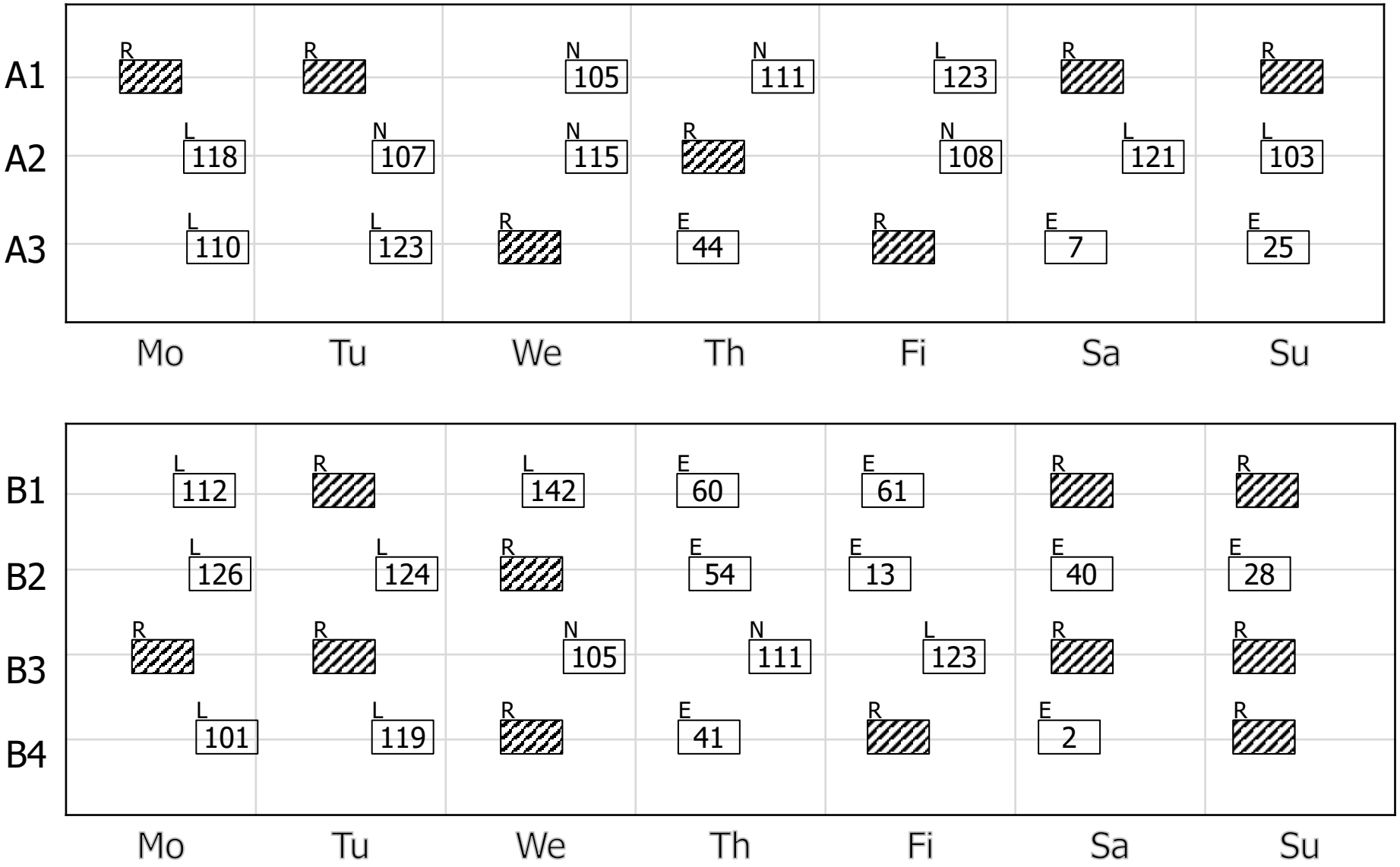
Weekly Roster Scheme



Weekly Roster Scheme



Roster Groups



Fairness vs. Attractiveness

- Perceived Fairness

value of attribute a for duty d

$$\sum_{k \in K_r} \sum_{s \in S_k} \sum_{(t,d) \in s} g_{ad} x_s^k \leq n_r z_a \qquad \forall a \in A, r \in R$$

duties in roster scheme r

$$\sum_{k \in K_r} \sum_{s \in S_k} \sum_{(t,d) \in s} g_{ad} x_s^k \geq n_r v_a \qquad \forall a \in A, r \in R$$

roster dequence s is assigned to row k

$$\sum_{a \in A} w_a (z_a - v_a)$$

- Perceived Attractiveness

Value of assigning duty d to cell t for constraint p

$$\sum_{k \in K_r} \sum_{s \in S_k} \sum_{(t,d) \in s} f_{td}^p x_s^k \leq b_p + \delta_p$$

threshold | violation

Rostering Model (Fairness Level ζ)

$$\min \sum_{k \in K} \sum_{s \in S_k} c_s^k x_s^k + \sum_{p \in P \setminus P_K} c_p \delta_p$$

$$\text{s.t.} \sum_{a \in A} w_a (z_a - v_a) \leq \zeta$$

$$\sum_{s \in S_k} x_s^k = 1 \qquad \forall k \in K$$

$$\sum_{k \in K} \sum_{s \in S_k} h_{ds}^k x_s^k = 1 \qquad \forall d \in D$$

$$\sum_{k \in K_r} \sum_{s \in S_k} \sum_{(t,d) \in s} f_{td}^p x_s^k \leq b_p + \delta_p \qquad \forall p \in P \setminus P_K$$

$$\sum_{k \in K_r} \sum_{s \in S_k} \sum_{(t,d) \in s} g_{ad} x_s^k \leq n_r z_a \qquad \forall a \in A, r \in R$$

$$\sum_{k \in K_r} \sum_{s \in S_k} \sum_{(t,d) \in s} g_{ad} x_s^k \geq n_r v_a \qquad \forall a \in A, r \in R$$

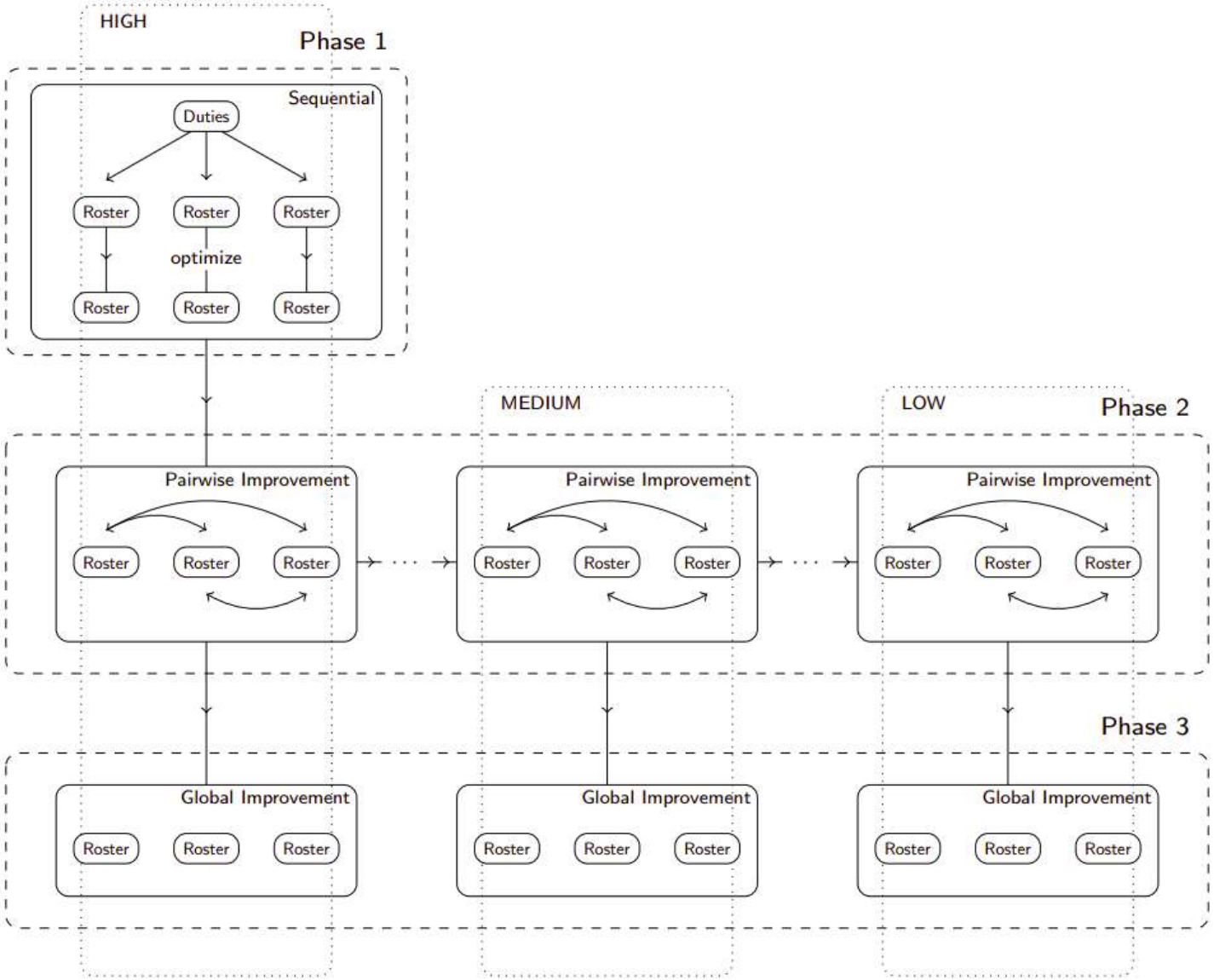
$$\ell_a \leq v_a, z_a \leq u_a \qquad \forall a \in A$$

$$x_s^k \in \{0,1\} \qquad \forall k\, K, s \in S$$

$$\delta_p \in \Delta_p \qquad \forall p \in P \setminus P_K$$

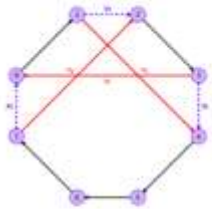
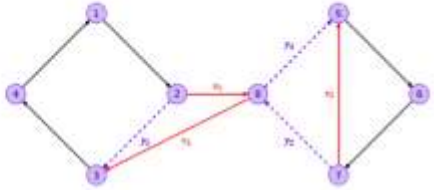
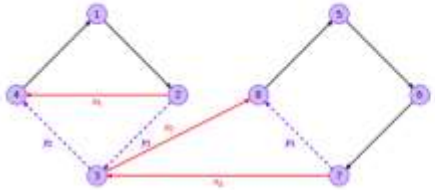
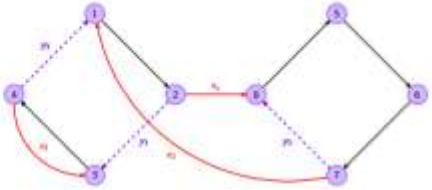
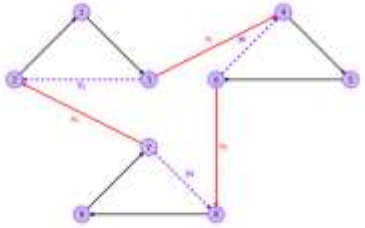
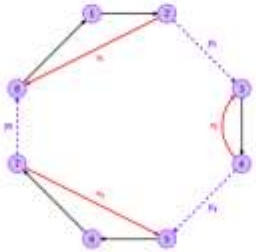
$$v_a, z_a \in \mathbb{R} \qquad \forall\, a \in A$$

Three-Phase Heuristic

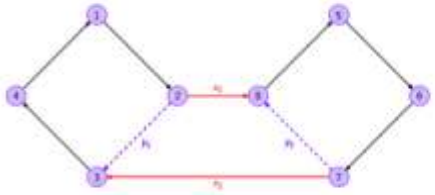
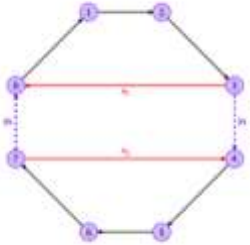


Variable Depth Exchange Heuristic (DEX)

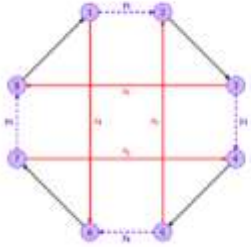
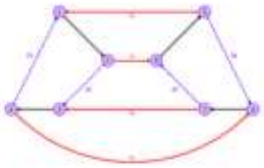
- 3-edge exchanges

- 2-edge exchanges

- 4-edge exchanges

NS Depot Utrecht

Freie Universität Berlin ZIB

	Duties				Employees			
	E	L	N	∑	E	LN	M	# gr.
1	55	29	29	113	14	12	4	3
2	58	33	24	115	12	12	4	3
3	38	22	11	71	6	6	6	3
4	37	17	15	69	6	8	6	3
5	74	62	55	191	12	12/12	12	4
6	88	36	31	155	14	8	12/6	4
7	86	36	37	159	12/6	12	12	4
8	74	30	16	120	6/6	8	12	4
9	126	70	54	250	14/6	12/8	12/12	6
10	142	63	61	266	12/12	12/8	12/12	6

Small Test Instances

Freie Universität Berlin ZIB

	Fairness	Phase 1	Phase 2	Phase 3	Optimal	Root	Gain/%	Gap/%
1	Extreme	1175.8	1175.8	1175.8	1175.8	1155.6	0.0	0.0
	High		1172.6	1172.6	1169.2	1133.4	0.3	0.0
	Moderate		1172.3	1172.3	1164.2	1128.5	0.3	0.7
	Low		1172.3	1165.8	1133.8	1115.4	0.8	2.7
	Poor		1143.1	1142.7	1120.8	1104.8	2.8	1.9
2	Extreme	1288.0	1288.0	1288.0	1288.0	1172.3	0.0	0.0
	High		1288.0	1248.6	1216.4	1158.3	3.1	2.6
	Moderate		1257.3	1226.3	1192.2	1149.3	4.8	2.8
	Low		1257.3	1210.2	1186.1	1135.8	6.0	2.0
	Poor		1204.5	1196.7	1177.8	1126.6	7.1	1.6
3	Extreme	979.3	979.3	979.3	979.3	800.9	0.0	0.0
	High		979.3	979.3	853.8	793.4	0.0	12.8
	Moderate		859.0	859.0	830.3	788.4	12.3	3.3
	Low		802.4	802.4	794.0	781.5	18.1	1.0
	Poor		791.3	787.5	787.5	777.6	19.6	0.0
4	Extreme	910.5	910.5	910.5	910.5	776.7	0.0	0.0
	High		872.1	872.1	847.2	767.3	4.2	2.9
	Moderate		872.1	872.1	827.7	761.3	4.2	5.1
	Low		808.4	808.4	788.0	746.8	11.2	2.5
	Poor		774.2	774.2	774.2	742.7	15.0	0.0

Large Test Instances

Freie Universität Berlin ZIB

	Fairness	Phase 1	Phase 2	Phase 3	Root	Gain/%	Gap/%
5	Extreme	2041.5	2041.5	2039.5	1542.5	0.1	24.4
	High		1780.2	1778.2	1529.0	12.9	14.0
	Moderate		1668.0	1630.5	1514.5	20.1	7.1
	Low		1546.2	1544.1	1491.8	24.4	3.4
6	Extreme	1811.7	1651.6	1650.4	1296.4	8.9	21.4
	High		1513.2	1512.0	1291.5	16.5	14.6
	Moderate		1412.4	1411.1	1282.3	22.1	9.1
	Low		1358.2	1356.9	1265.7	25.1	6.7
7	Extreme	1956.3	1559.0	1557.8	1328.4	20.4	14.7
	High		1510.4	1503.7	1318.0	23.1	12.3
	Moderate		1469.5	1465.1	1299.2	25.1	11.3
	Low		1412.1	1410.7	1262.5	27.9	10.5
8	Moderate	1502.0	1397.4	1396.9	1120.4	7.0	19.8
	Low		1292.7	1207.9	1097.8	19.6	9.1
9	Moderate	2877.0	2252.1	2249.2	1774.0	21.8	21.1
	Low		2194.0	2182.3	1768.2	24.1	19.0
10	Moderate	3009.0	1863.7	1859.2	1526.0	38.2	17.9
	Low		1759.6	1759.6	1486.5	41.5	15.5

Time limit 30 minutes

Three approaches to …

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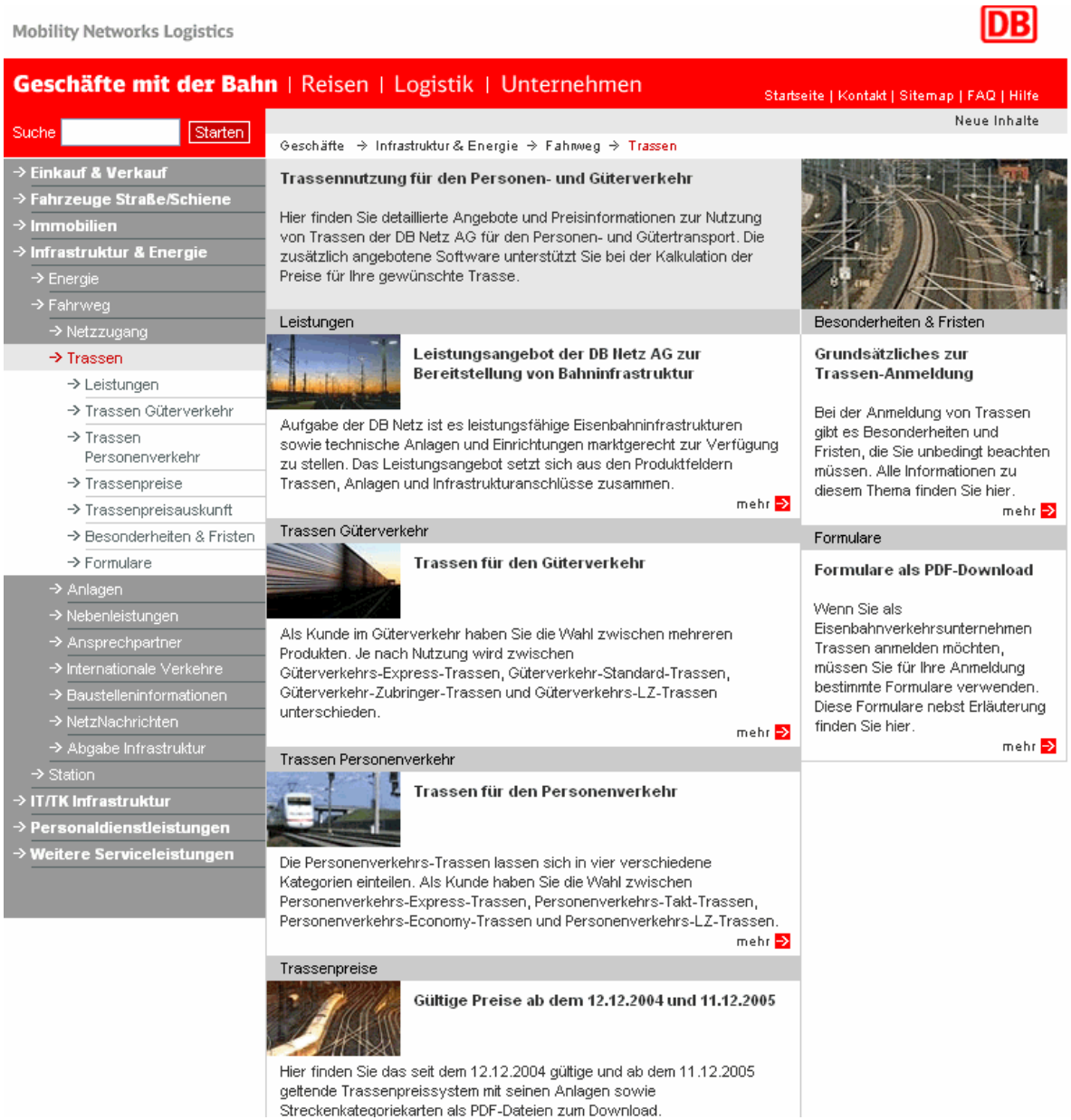
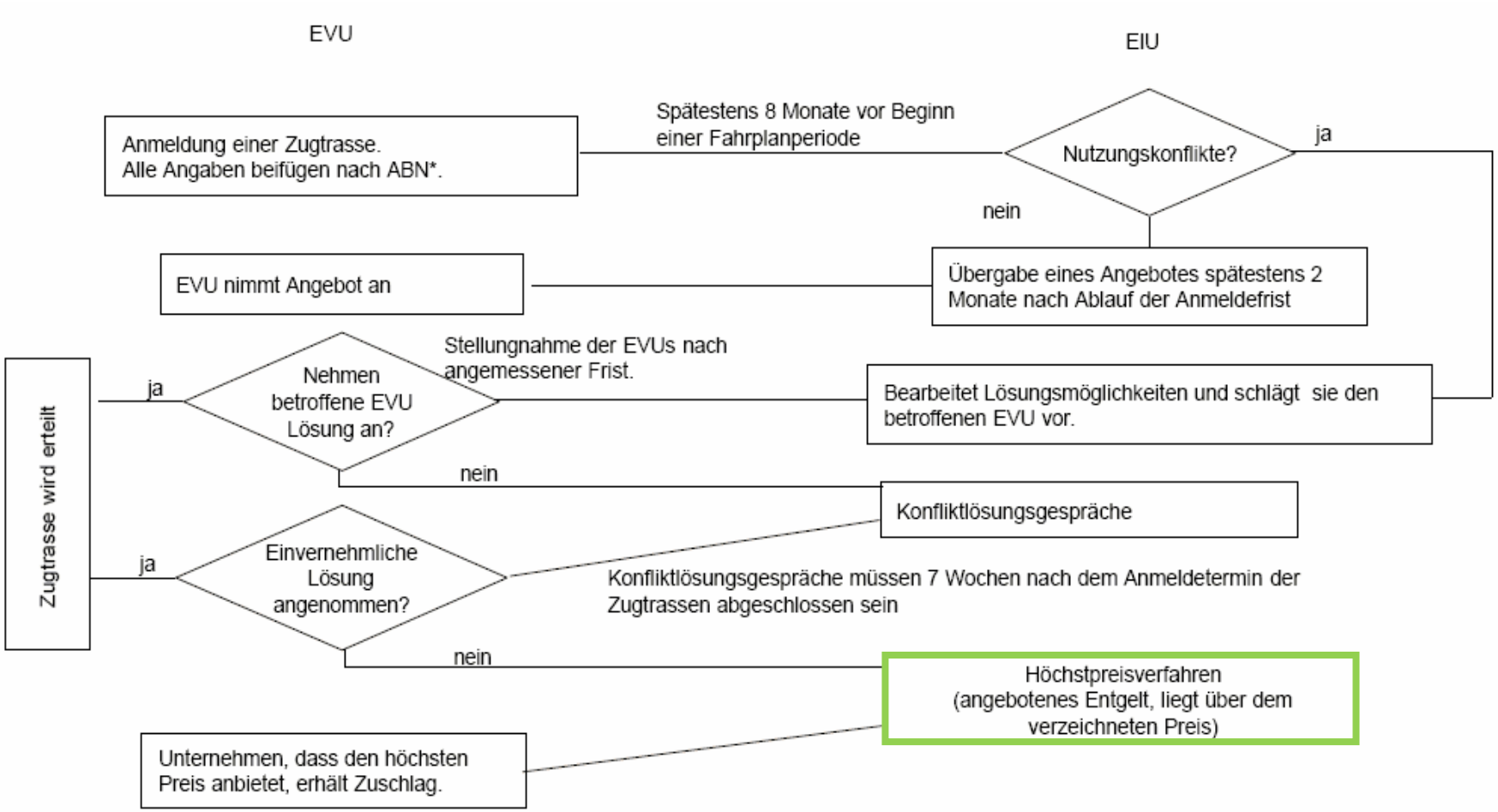
Ordering a Train Route

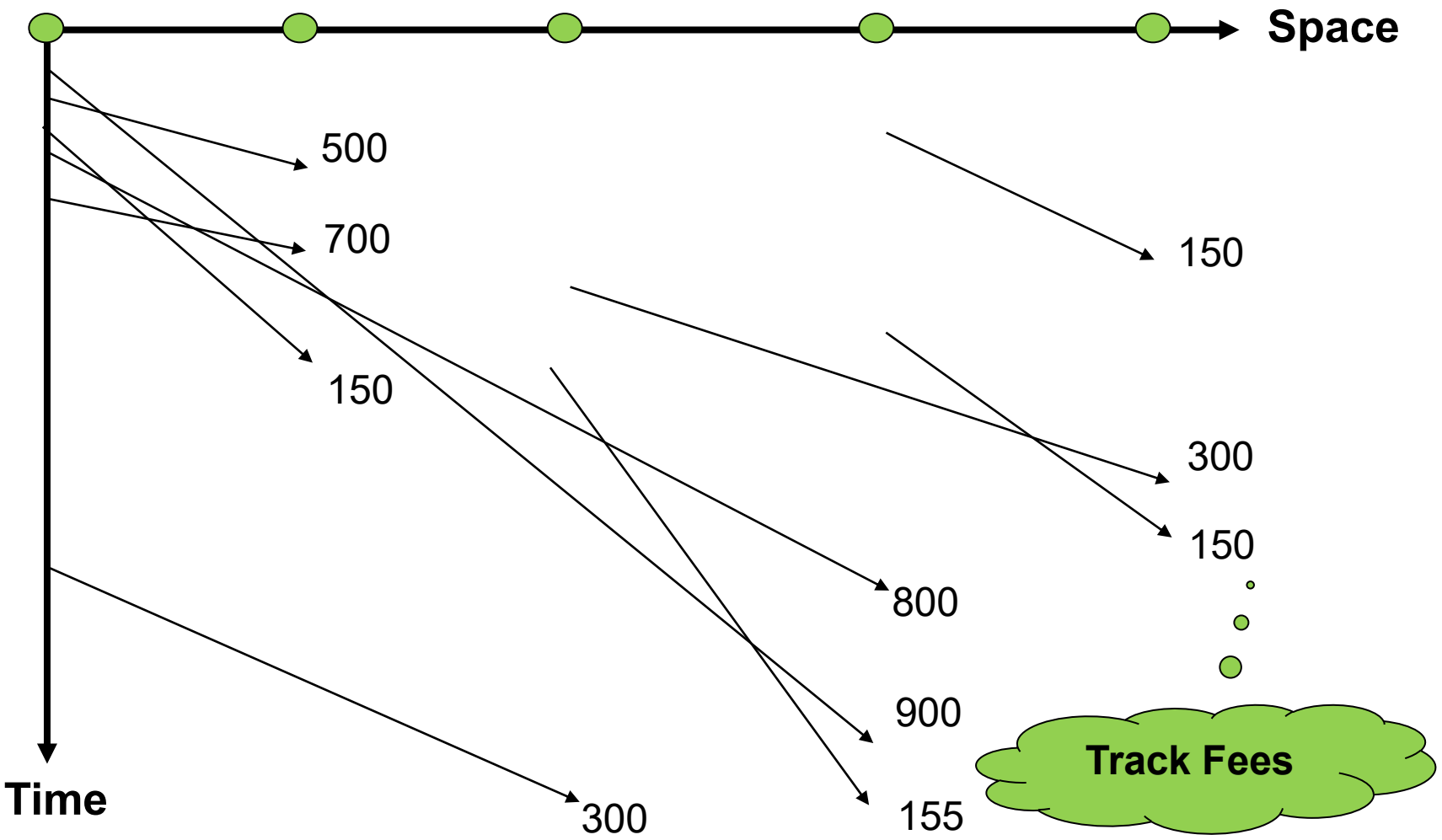


Resolving Conflicts by Negotiation or Auctioning

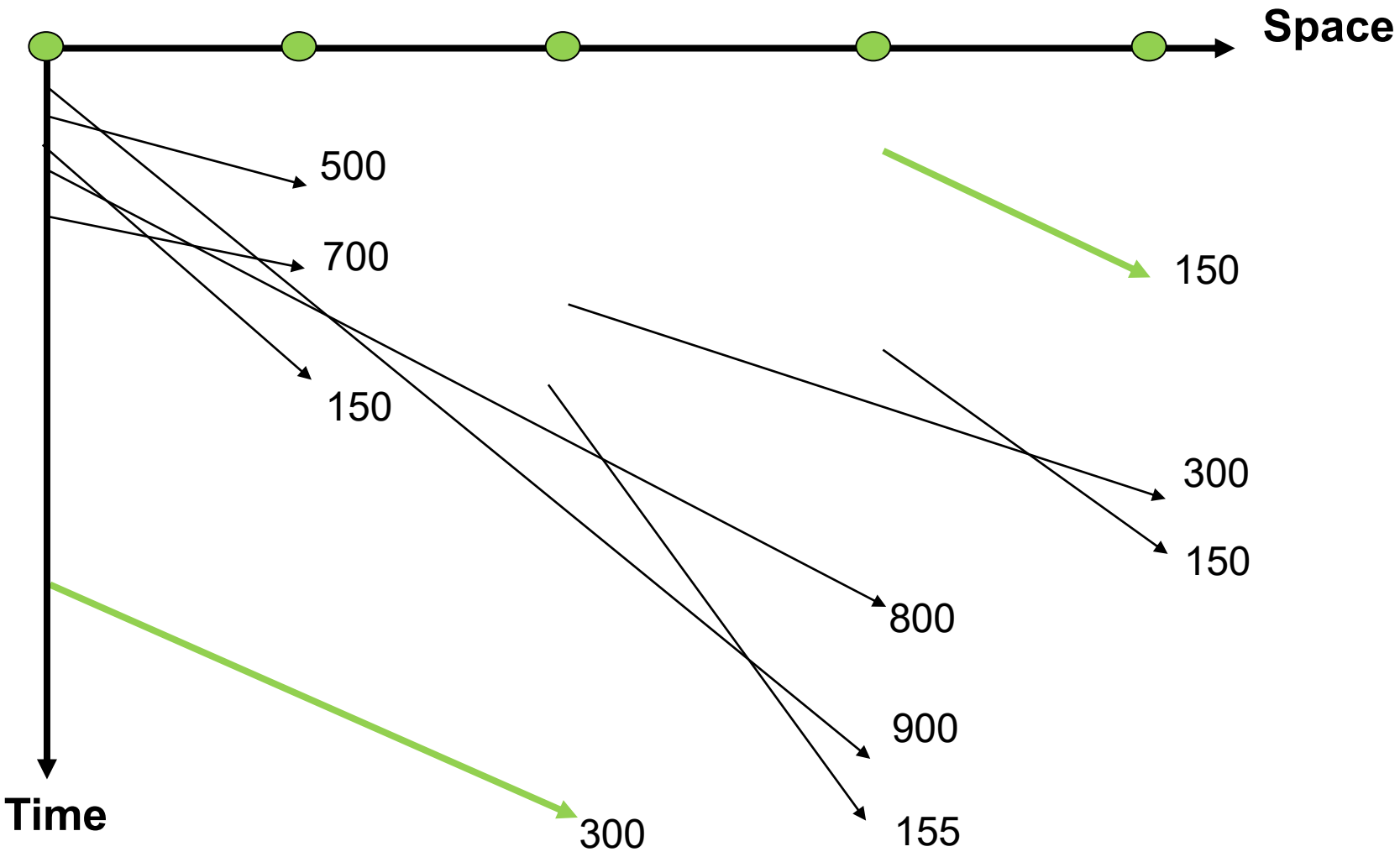


*Besondere Daten, z. B. fahrdynamische Daten von Triebfahrzeugen, müssen 14 Tage vor der Trassenanmeldung abgegeben werden.

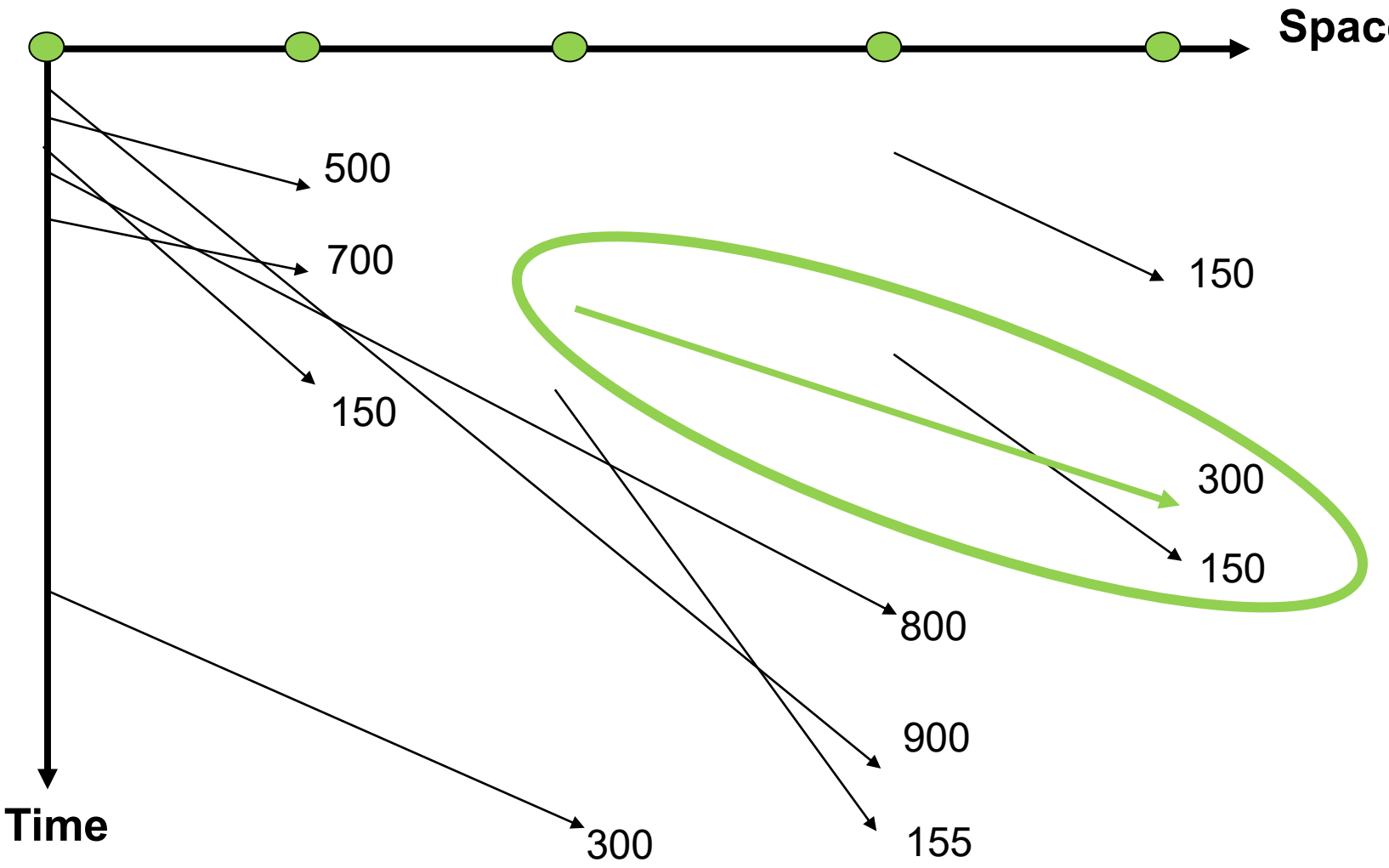
Route Requests



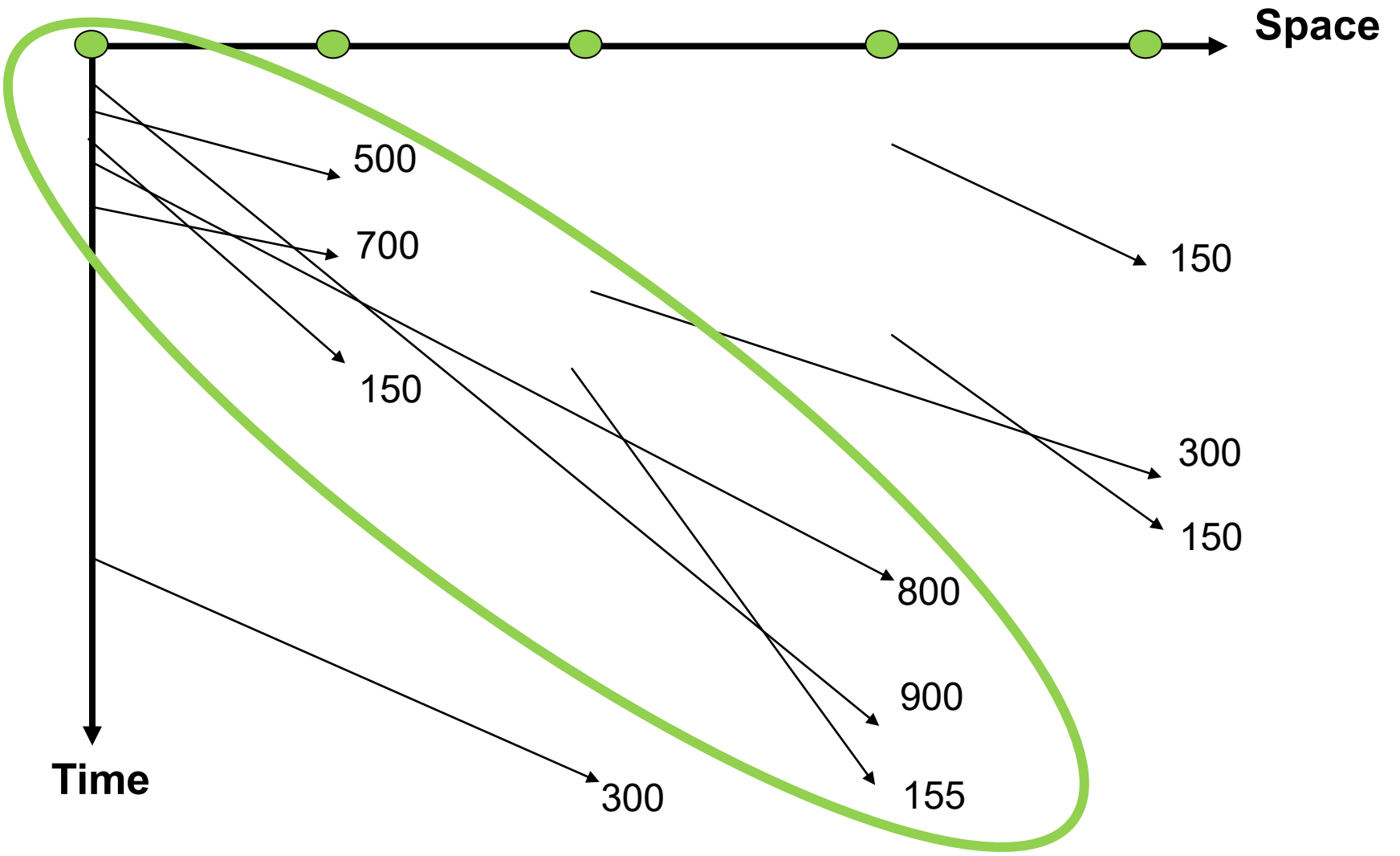
Routes without Conflicts



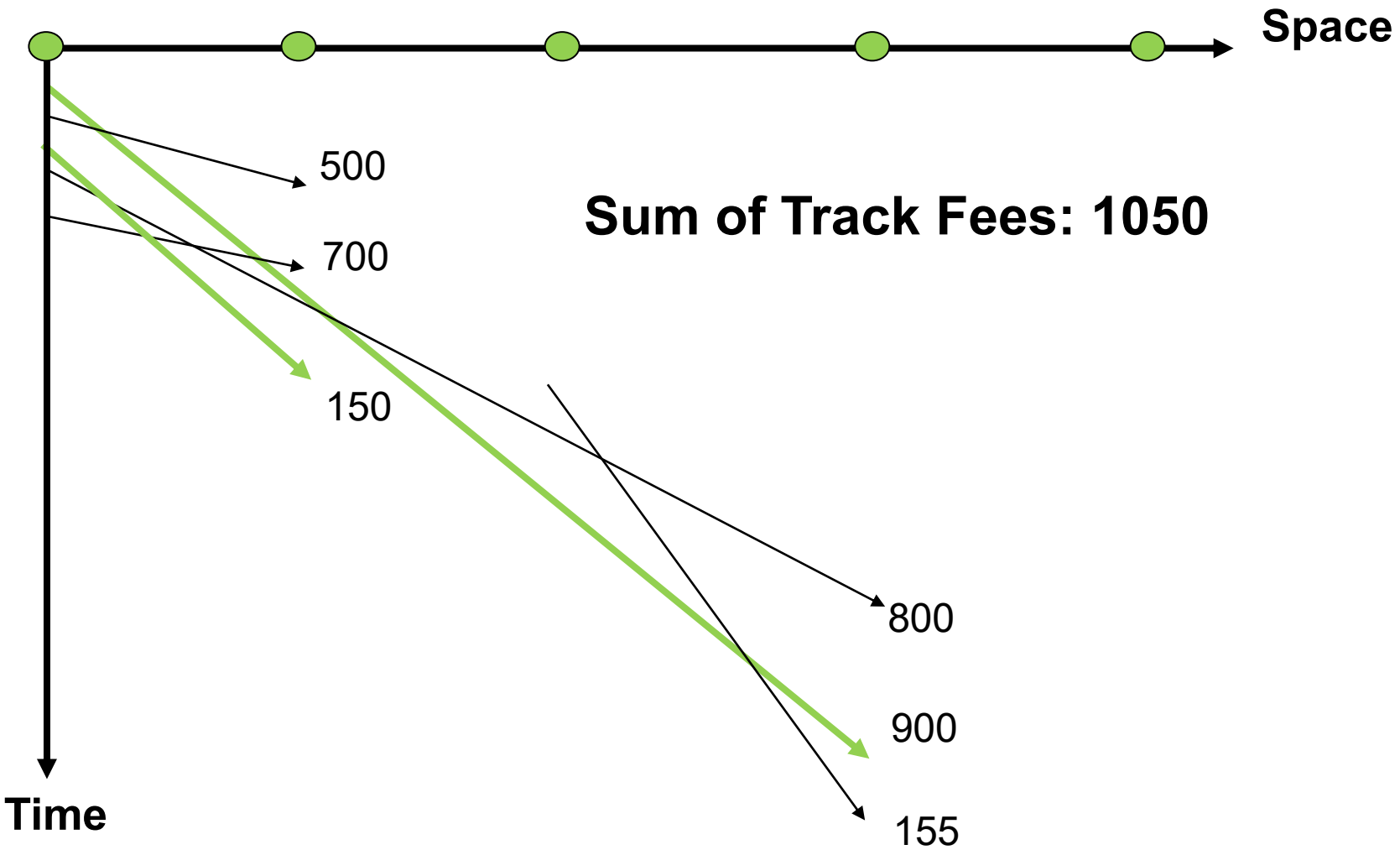
Two Routes in Conflict



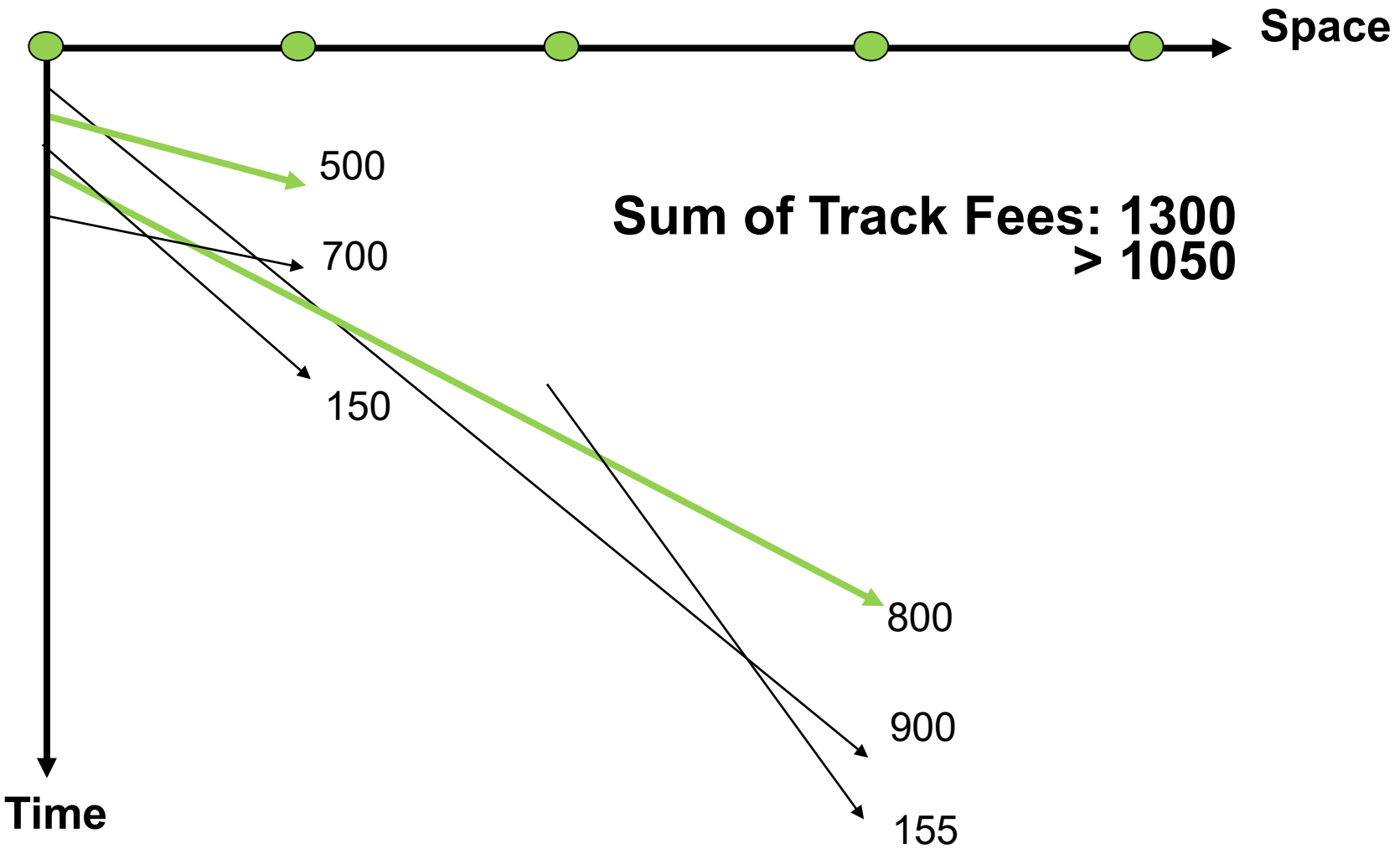
A Complex Conflict: Where is it?



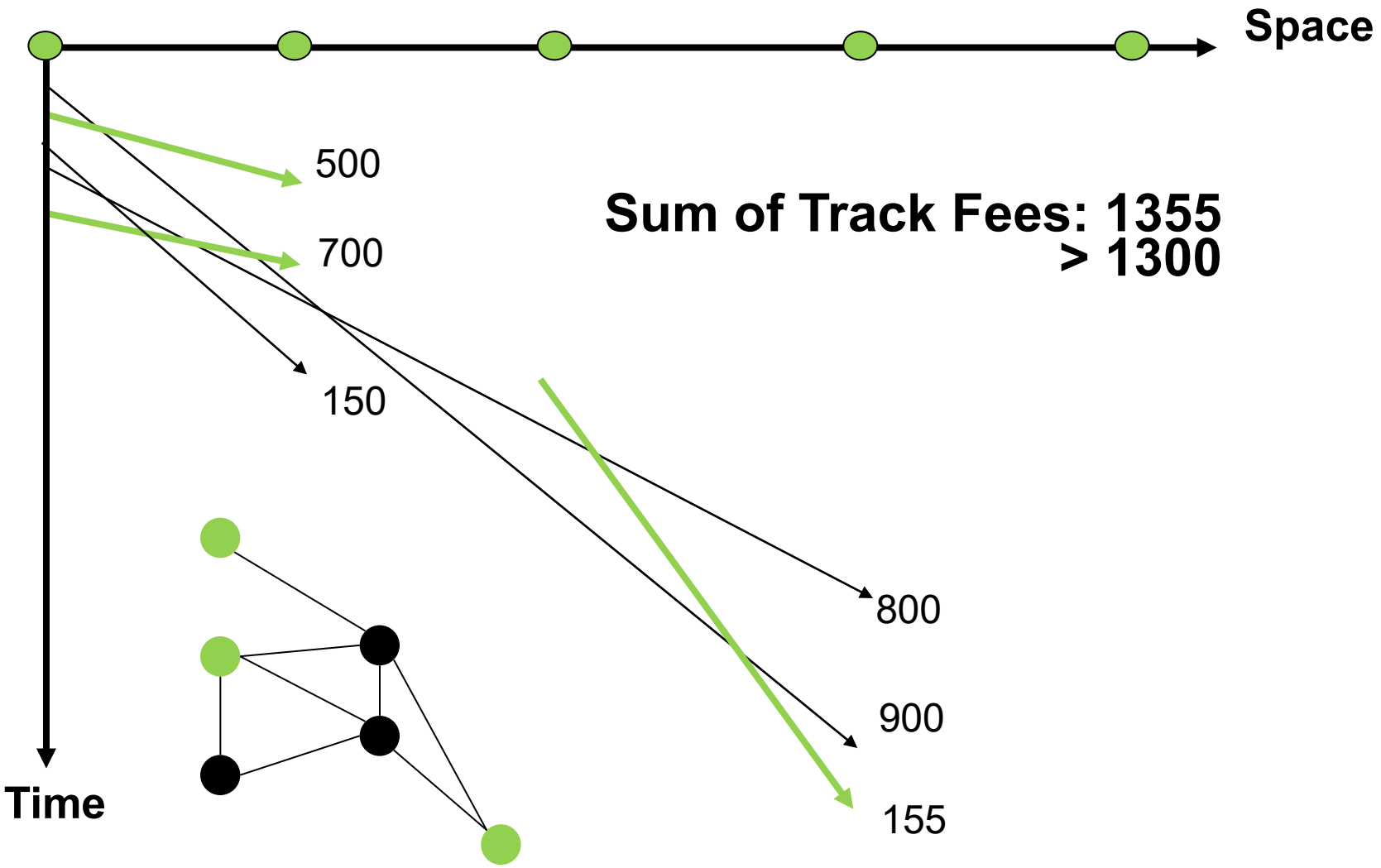
Greedy Solution



Another Solution

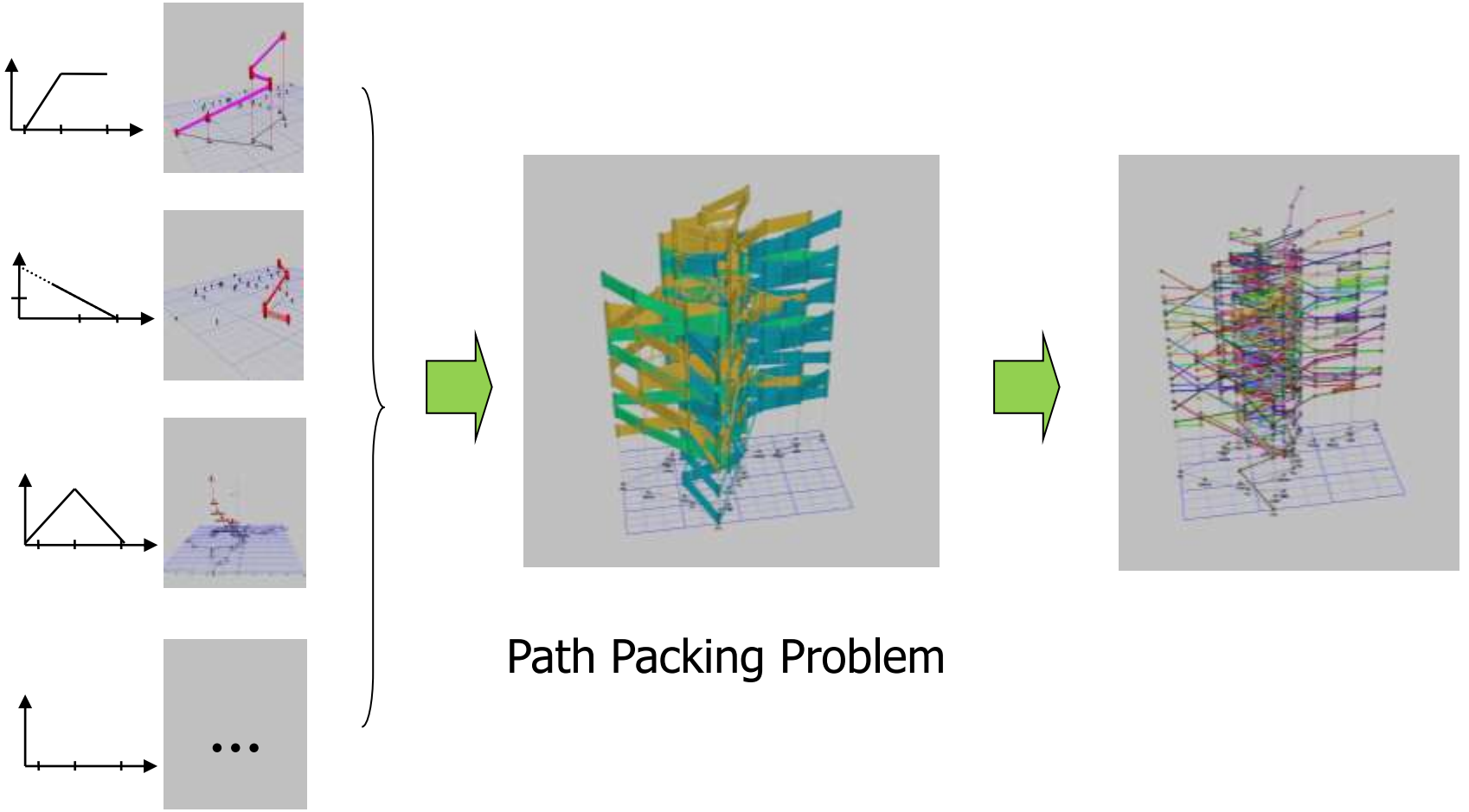


Optimal Solution

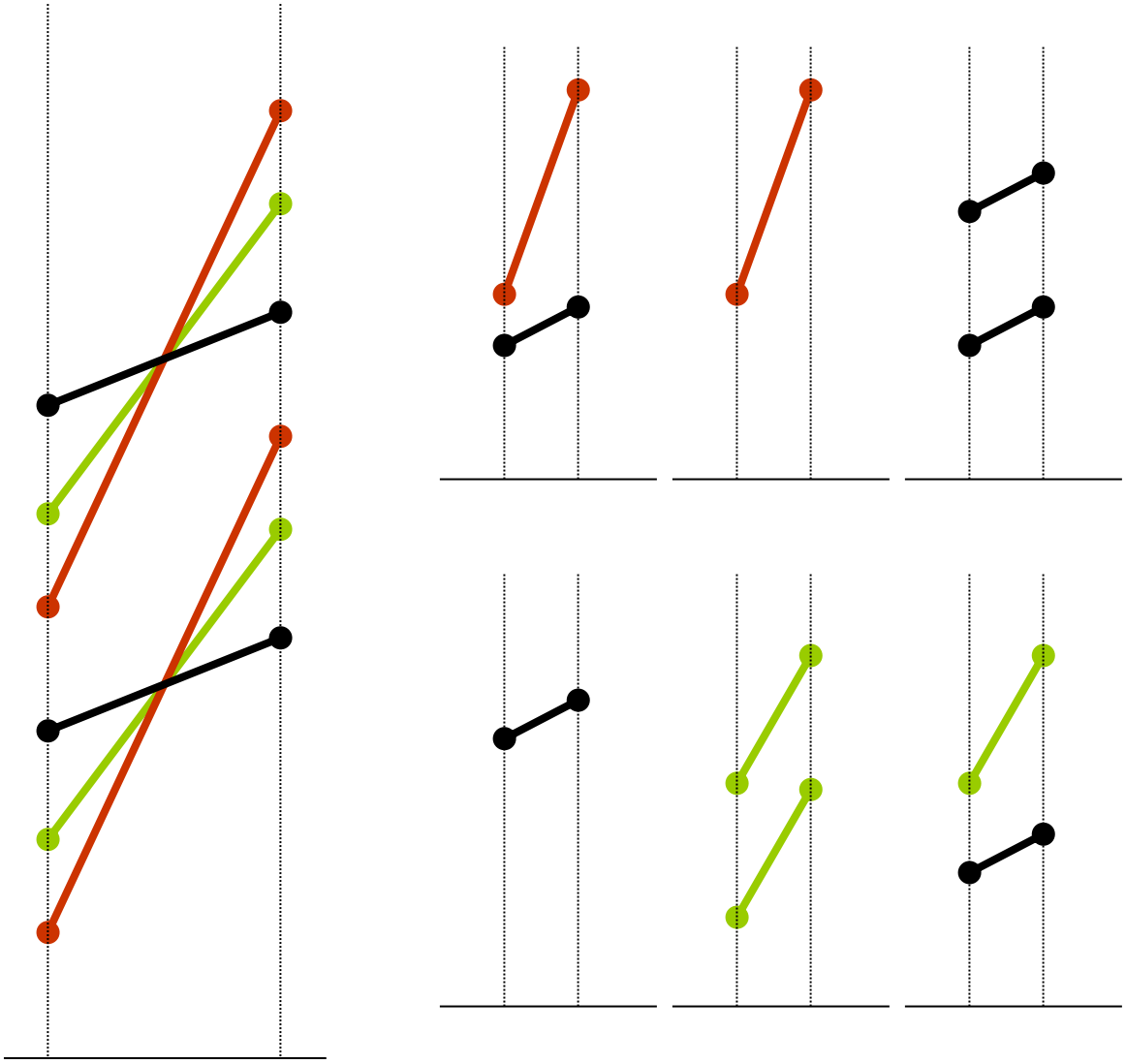


Adding Flexibility in Time (and Space)

Track Allocation Problem



Track Occupation Configuration Model



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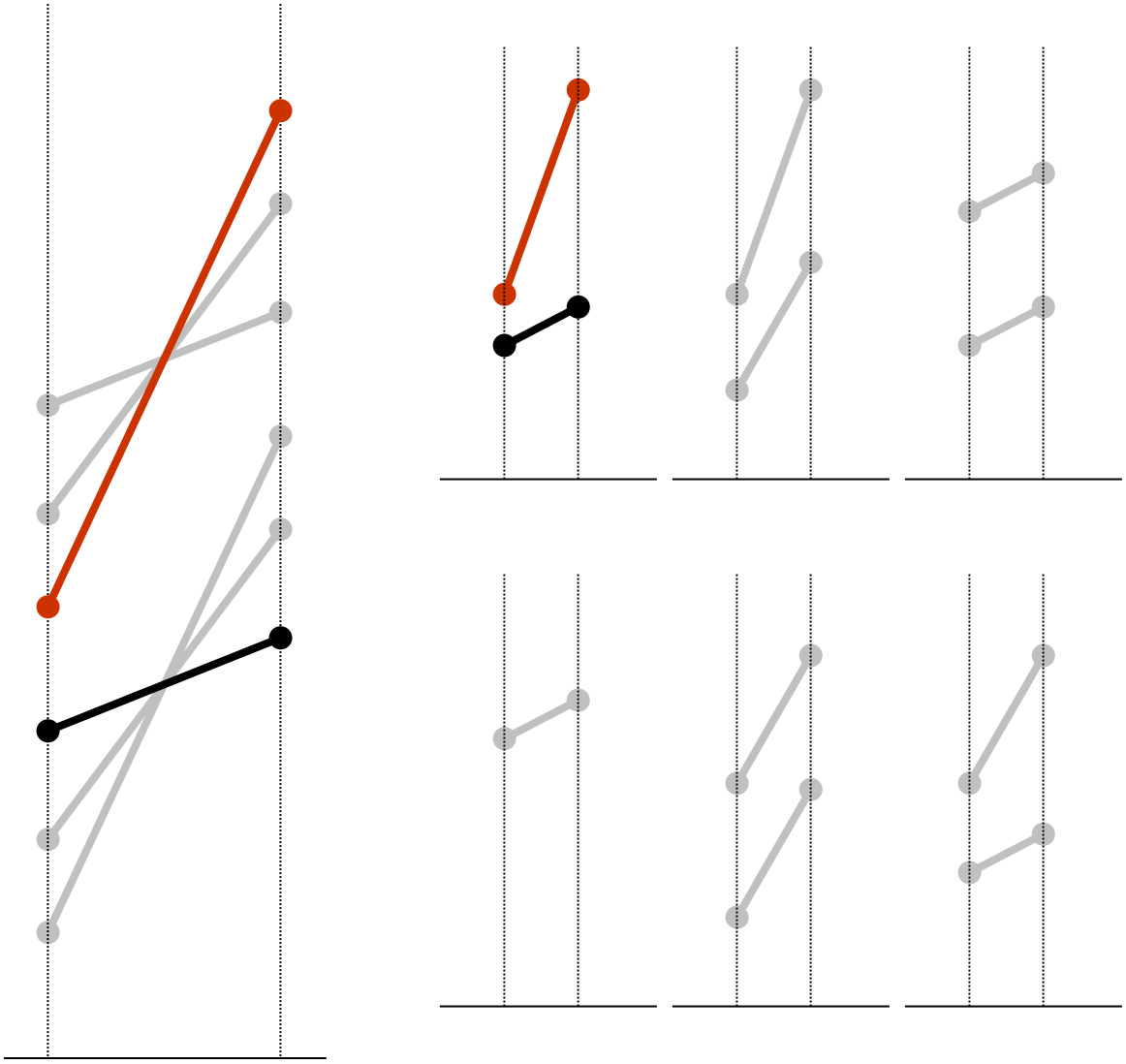
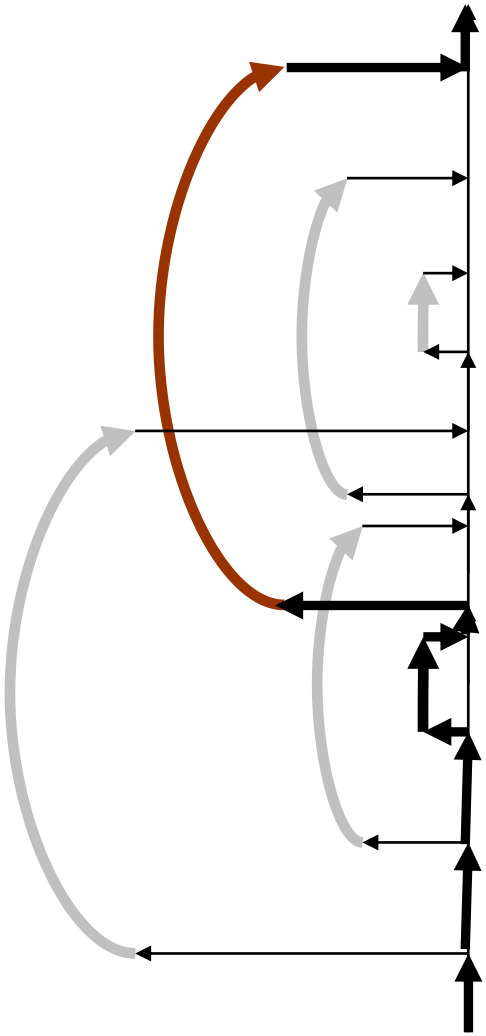




Path and Configuration Packing Model

$$\text{(APP)} \quad \max \sum_{i \in I} \sum_{a \in A} c_a^i x_a^i$$

$$\text{(i)} \quad \sum_{a \in \delta_i^+(v)} x_a^i - \sum_{a \in \delta_i^-(v)} x_a^i = \beta_i(v) \quad \forall v \in V, i \in I \quad \text{Flow}$$

$$\text{(ii)} \quad \sum_{(a,i) \in k} x_a^i \leq 1 \quad \forall k \in K \quad \text{Conflicts}$$

$$\text{(iii)} \quad x_a^i \in \{0,1\} \quad \forall a \in A, i \in I \quad \text{Integ.}$$

$$\text{(PCP)} \quad \max \sum_{i \in I} \sum_{p \in P_i} \sum_{a \in p} c_a^i x_p$$

$$\text{(i)} \quad \sum_{p \in P_i} x_p \leq 1 \quad \forall i \in I \quad \text{Trains}$$

$$\text{(ii)} \quad \sum_{q \in Q_j} y_q \leq 1 \quad \forall j \in J \quad \text{Configs}$$

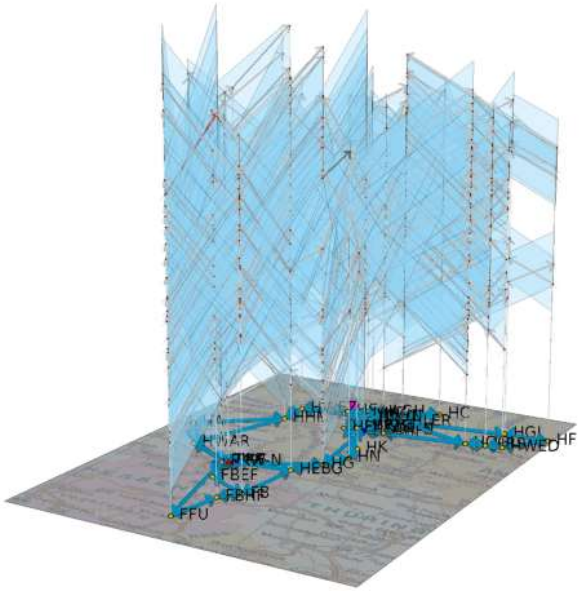
$$\text{(iii)} \quad \sum_{a \in p \in P} x_p - \sum_{a \in q \in Q} y_q \leq 0 \quad \forall a \in A \quad \text{Coupling}$$

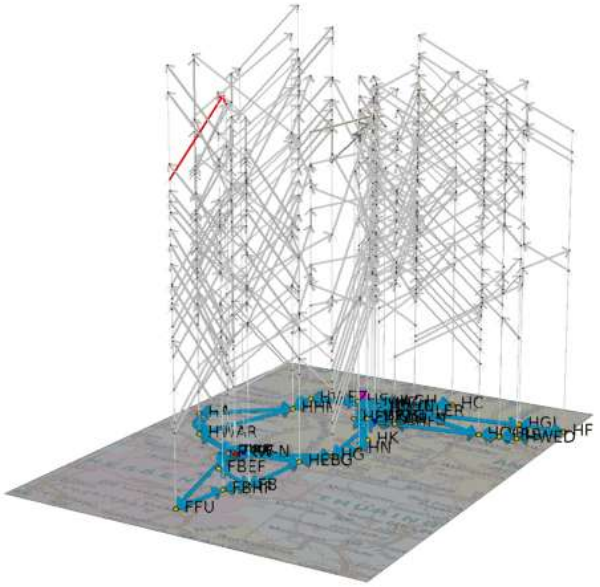
$$\text{(iv)} \quad x_p \in \{0,1\} \quad \forall p \in P \quad \text{Integ.}$$

$$\text{(v)} \quad y_q \in \{0,1\} \quad \forall q \in Q \quad \text{Integ.}$$

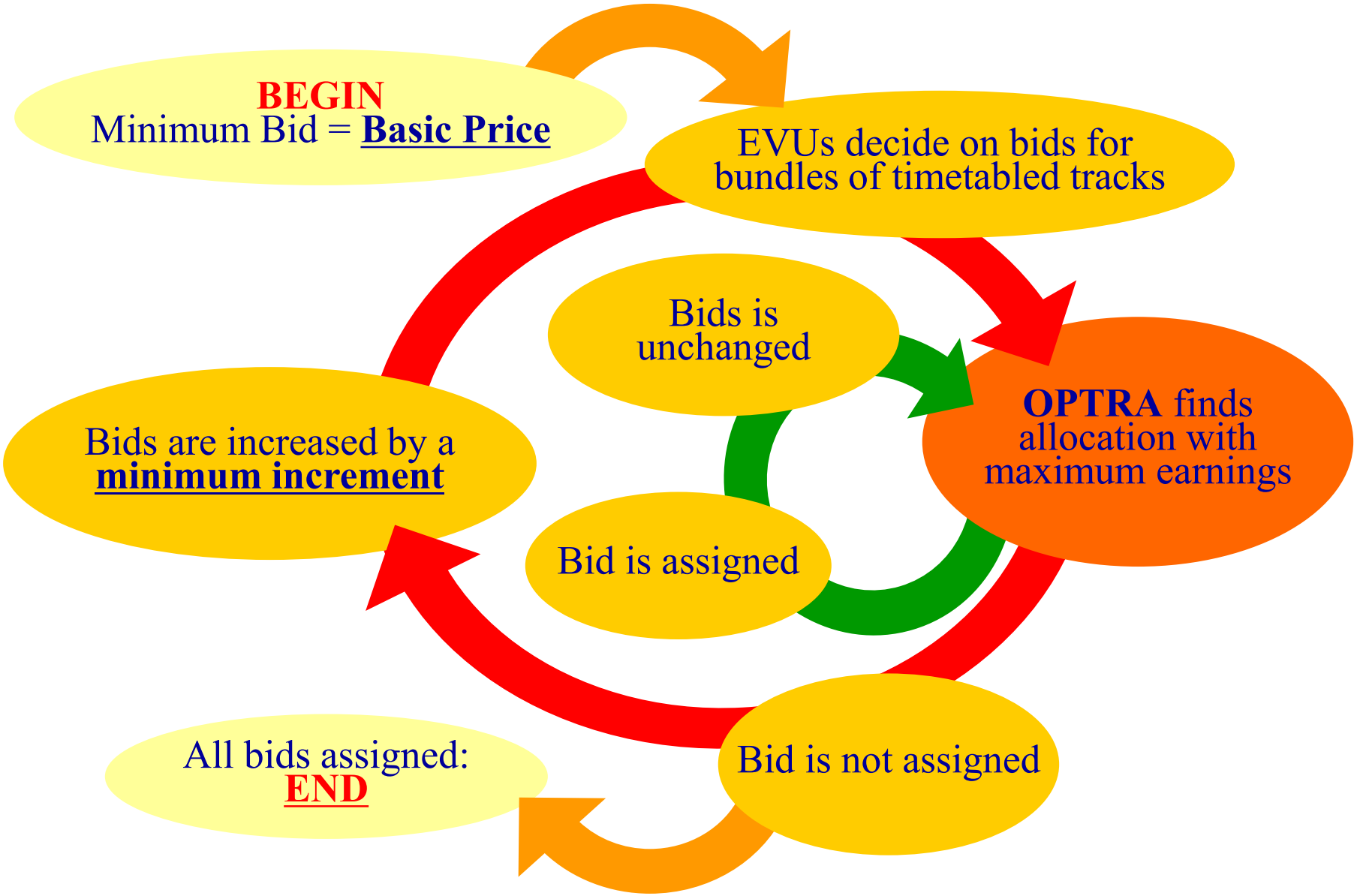
Track Allocation Takes Time

instance	#trains	#reqs	#rows	#cols	v(LP)	v^*	gap in %	t_Σ in s	#bbn
REQ_506	218	506	30213	282463	274.55	266.79	2.91	70186.90	2188
REQ_567	247	567	30595	259003	369.47	360.58	2.46	63573.24	1875
REQ_813	215	813	32287	225482	441.45	418.58	5.46	37627.05	157
REQ_875	239	875	36206	248922	395.10	368.22	7.30	46128.19	228
REQ_906	235	906	35155	265837	441.16	409.06	7.85	51234.58	471





Rail Track Auction



Why Auctions?

Auctions can ...

- resolve user conflicts in such a way that the bidder with the highest willingness to pay receives the commodity (efficient allocation, welfare maximization)
- maximize the auctioneer's earnings
- reveal the bidders' willingness to pay
- reveal bottlenecks and the added value if they are removed.

Economists argue ...

- that a "working auctioning system" is usually superior to alternative methods such as bargaining, fixed prices, etc.

Successful (?) auctions ...

- eBay
- Sears, Roebuck & Co.
- Frequency auctions in mobile telecommunications
- Regional monopolies (franchising) at British Rail

Hannover-Kassel-Fulda (HaKaFu) Network

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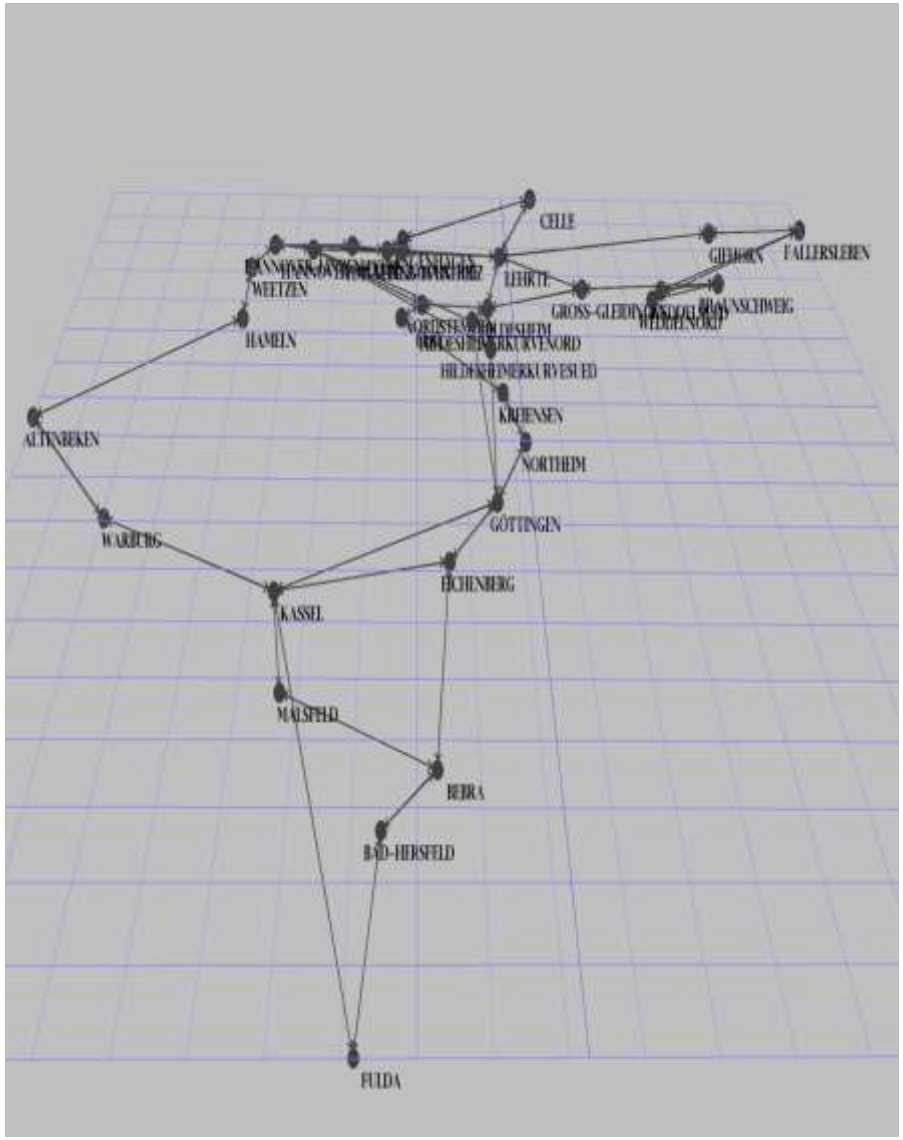
- 45 tracks
- 32 stations
- 6 train types
- 10 train sets
- 122 nodes
- 659 arcs
- 3-12 hours
- 96 station capacities
- 612 headway times
- 320 trains in reference timetable



Auctioning Experiments

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# Trains (Rev.)	ICE		IC		RE		RB		S		ICG	#
Type	**ind**	**sync**	**ind**	**sync**	**ind**	**sync**	**ind**	**sync**	**ind**	**sync**	**ind**	
Ref. (302)	**27**	**0**	**27**	**0**	**34**	**19**	**83**	**23**	**0**	**61**	**26**	**—**
+30 IC/ICE ind	33	0	30	0	32	19	77	23	0	57	24	41
+45 IC/ICE sync	24	9	27	9	32	19	79	19	0	58	26	43
+45 R*/S ind	27	0	26	0	43	19	85	23	8	46	25	29
+45 R*/S sync	27	0	26	0	34	22	81	29	0	61	26	26
+15 ICG	27	0	27	0	34	19	83	23	0	61	42	9
+33	28	0	26	0	37	22	82	23	1	52	30	29

# Trains (TPS)	ICE		IC		RE		RB		S		ICG	#
Type	**ind**	**sync**	**ind**	**sync**	**ind**	**sync**	**ind**	**sync**	**ind**	**sync**	**ind**	
Ref. (310)	**27**	**0**	**27**	**0**	**38**	**19**	**87**	**23**	**0**	**61**	**28**	**—**
+30 IC/ICE ind	30	0	31	0	38	19	85	23	0	61	25	20
+45 IC/ICE sync	24	12	27	10	36	19	83	19	0	58	27	22
+45 R*/S ind	27	0	25	0	44	19	93	23	8	50	27	23
+45 R*/S sync	27	0	26	0	34	22	85	35	0	65	25	26
+15 ICG	27	0	27	0	38	19	87	23	0	61	42	19
+33	27	0	26	0	37	22	86	26	0	60	30	29
+66	28	0	25	3	38	25	85	29	2	55	31	29

Auctioning Experiments

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€ (Rev.)	ICE		IC		RE		RB		S		ICG
Type	**ind**	**sync**	**ind**	**sync**	**ind**	**sync**	**ind**	**sync**	**ind**	**sync**	**ind**
+30 IC/ICE ind	24995	—	6784	—	4263	1703	3405	1214	—	2123	3911
+45 IC/ICE sync	12695	3577	3645	6513	3125	1405	3900	872	—	1772	4074
+45 R*/S ind	10070	—	3681	—	6419	1419	4457	1214	779	1817	3857
+45 R*/S sync	10300	—	3593	—	4211	1644	3847	1598	—	3057	4165
+15 ICG	9831	—	3710	—	3036	1421	3613	1214	—	1805	8499
+33	17256	—	3834	—	4914	2497	4022	1214	290	2321	5172

€ (TPS)	ICE		IC		RE		RB		S		ICG
Type	**ind**	**sync**	**ind**	**sync**	**ind**	**sync**	**ind**	**sync**	**ind**	**sync**	**ind**
+30 IC/ICE ind	13436	—	6361	—	3809	1404	4341	1216	—	1962	3937
+24 IC/ICE sync	11045	3258	3611	5043	3163	1440	3594	872	—	1646	4123
+45 R*/S ind	9415	—	3355	—	4887	1426	4938	1216	195	2148	4488
+45 R*/S sync	11605	—	3500	—	2628	1704	4023	3102	—	3678	4027
+15 ICG	9137	—	3515	—	3298	1408	3852	1216	—	1803	8765
+33	11590	—	3554	—	3319	2157	3959	1831	—	2728	5460
+66	13140	—	4128	1680	3485	2376	4080	2086	196	3109	7082

Auctioning Experiments

Freie Universität Berlin ZIB

€/km (Rev.)	ICE		IC		RE		RB		S		ICG
Type	**ind**	**sync**	**ind**	**sync**	**ind**	**sync**	**ind**	**sync**	**ind**	**sync**	**ind**
+30 IC/ICE ind	3,83	—	2,01	—	1,55	1,30	0,93	0,85	—	1,10	1,20
+45 IC/ICE sync	2,52	2,11	1,47	4,23	1,22	1,07	1,02	0,82	—	0,96	1,14
+45 R*/S ind	1,83	—	1,53	—	1,53	1,09	0,96	0,86	1,26	1,04	1,11
+45 R*/S sync	1,89	—	1,53	—	1,47	1,10	0,90	0,93	—	1,08	1,14
+15 ICG	1,82	—	1,52	—	1,06	1,09	0,86	0,90	—	0,88	1,54
+33	2,98	—	1,60	—	1,51	1,41	0,95	0,88	3,02	1,21	1,23

€/km (TPS)	ICE		IC		RE		RB		S		ICG
Type	**ind**	**sync**	**ind**	**sync**	**ind**	**sync**	**ind**	**sync**	**ind**	**sync**	**ind**
+30 IC/ICE ind	2,21	—	1,83	—	1,25	1,07	1,00	0,85	—	0,96	1,14
+45 IC/ICE sync	2,19	1,94	1,47	3,27	1,15	1,10	0,88	0,82	—	0,89	1,15
+45 R*/S ind	1,70	—	1,27	—	1,15	0,98	0,96	0,87	1,58	1,13	0,86
+45 R*/S sync	2,12	—	1,49	—	1,04	1,14	0,90	1,38	—	1,25	1,14
+15 ICG	1,45	—	1,44	—	1,08	1,08	0,87	0,90	—	0,88	1,03
+33	2,04	—	1,49	—	1,09	1,22	0,87	1,08	—	1,32	1,33
+66	2,21	—	1,88	2,87	1,03	1,10	0,89	1,11	1,53	1,47	1,60

Three approaches to …

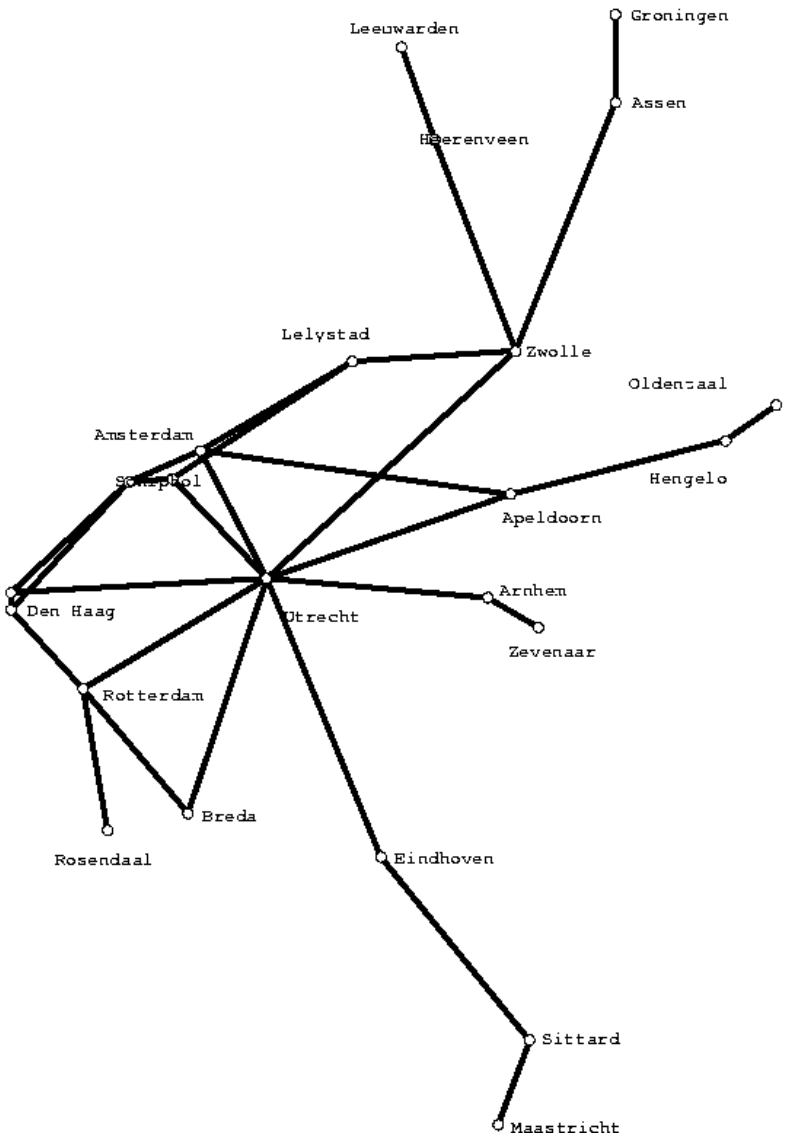
- a fair **operation** of transportation systems:
  employee preferences (equitable rosters)
  with Thomas Breugem, Christof Schulz, Thomas Schlechte

- a fair **access** to transportation systems:
  market access for **operators** (track auctions)
  with Thomas Schlechte and Anke Reuter

- a fair **design** of transportation systems:
  cost sharing for **users** (ticket prices)
  with Martin Grötschel and Nam Dũng Hoàng

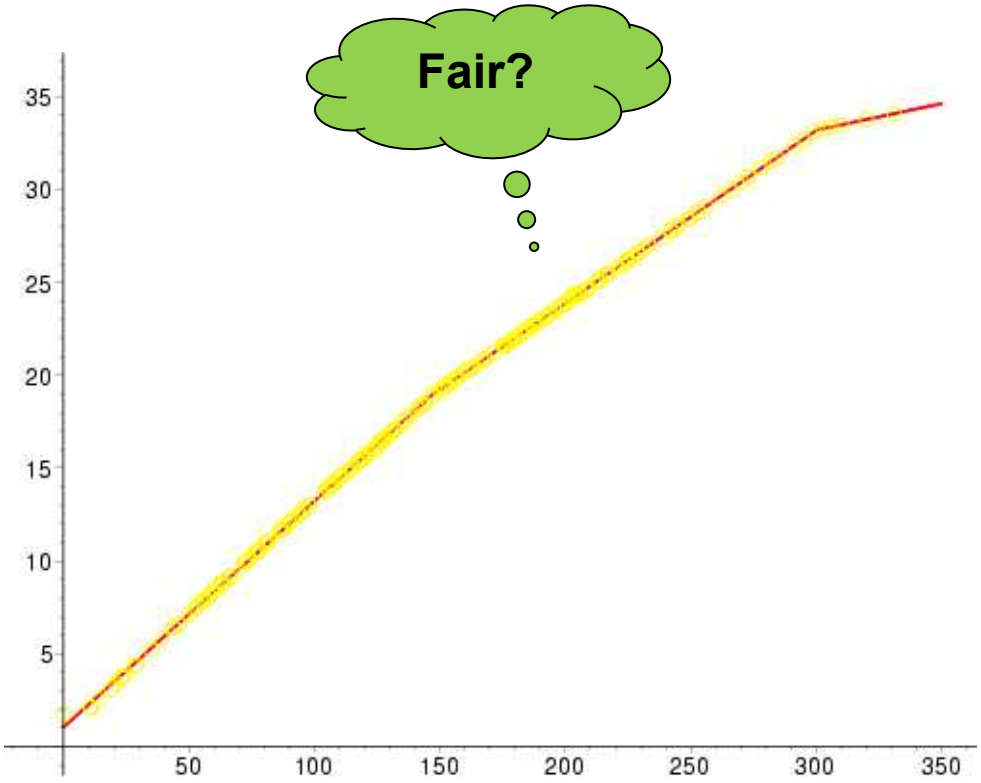
Dutch Intercity Network

(Bussieck [1998], Bussieck, Kreuzer, Zimmermann [1996], Claessens, van Dijk, Zwaneveld [1998])



	Hr	Asn	Zl	Hgl	Ah	Ut	Shl	Asdz	Asd	Gv	Gvc	Rtd	Bd	Ehv	Std	Mt	Lls	Rsdg	Zvg	Odzg
Lw	478		380		13	145	20	21	90	6	26	36	14	9	9	4	77	7	14	
Gn		1720	720			331	48	88	205	12	73	75	34	28	29	13	200	33	14	
Hr			511		11	209	20	16	115	10	48	58	16	11	8	4	77	10	19	
Asn			854		16	502	32	58	235	13	117	125	42	33	28	14	152	48	19	
Zl					56	1112	64	171	400	33	163	182	79	47	46	21	390	100	32	
Apd				468		1160	32	76	917	21	202	143	57	62	10	5		47	83	71
Hgl						422	11	24	287	20	81	52	39	28	20	12		24		75
Ah						4244	60	721	726	109	741	180	136	101			8	320	602	
Ut							278	5826	4919	225	3138	2260	1165	3109	720	359	89	325	996	21
Shl								1456	6469	1339	1503	509	7	99	44	29	103	164		
Asdz										461	207	369	138	542	203	149	819	6	155	
Asd										730	2540	1756	154	437	155	37	2783	2258	489	22
Gv											785	4586	531	35	22	8	29	890		
Gvc												2829	228	335	104	41	31	3	229	7
Rtd													1829	569	179	73	46	1077	157	11
Bd														950	157	79	6	329	14	5
Ehv															936	404	8	75	11	3
Std																863	2	19		
Mt																	1	22		
Lls																		15		

	Hr	Asn	Zl	Hgl	Ut	Shl	Asdz	Asd	Gv	Gvc	Rtd	Bd	Ehv	Std	Mt	Lls	Rsdg	Zvg	Odzg
Lw	29																		
Gn		28																	
Hr			66																
Asn			78																
Zl				85													50		
Apd				69	64			89											
Hgl																			18
Ah					58													19	
Ut						34	39		61	57	92	81							
Shl						9	19	43	43										
Asdz				9													56		
Asd																	54		
Gv									1	23									
Rtd										49								18	
Ehv															78				
Std																21			



Cost Allocation Games

- $N = [n]$ — players
- N — grand coalition
- $\Sigma \subseteq 2^N, \Sigma^+ = \Sigma \setminus \emptyset$ — coalitions
- $c: \Sigma^+ \to \mathbb{R}_{\geq 0}$ — cost function
- $P \subseteq \mathbb{R}^N_{\geq 0}$ — feasible prices (polyhedron)
- $\Gamma = (N, c, P, \Sigma)$ — cost allocation game
- $f: \Sigma^+ \to \mathbb{R}_{>0}$ — weight function $(1, |\cdot|, c)$
- $e_f(S, x) = \frac{c(S) - x(S)}{f(S)}, S \in \Sigma^+, x \in P$ — f-excess of S at price x
- $\mathcal{X}(\Gamma) = \{x \in P: x(N) = c(N)\}$ — imputation set
- $\mathcal{C} := \{x \in \mathcal{X}: e_f(\cdot, x) \geq 0\}$ — core
- $\mathcal{C}_{\epsilon,f} := \{x \in \mathcal{X}: e_f(\cdot, x) \geq \epsilon\}$ — (ϵ, f)-core
- $\epsilon_f := \max\ \epsilon: \mathcal{C}_{\epsilon,f} \neq \emptyset$ — f-least core radius
- $\mathcal{LC}_f := \mathcal{C}_{\epsilon_f,f}$ — f-least core
- $\mathcal{N}_f := \operatorname{lexmax} \mathcal{LC}_f$ — f-nucleolus
- $\phi: \Gamma \to P$ — cost allocation method

Desirable Properties

1. $\phi(\Gamma)(N) = c(N)$ — efficiency
2. $\phi(\Gamma)(S) \leq \phi(\Gamma_S)(S) \ \forall S \in \Sigma^+$ — coalitional stability
3. $\phi(\Gamma) \in \mathcal{C}$ — core price
4. $\exists K, \alpha > 0 : |\tilde{c}(\cdot) - c(\cdot)| \leq \alpha c(\cdot)$
   $\Longrightarrow |\phi(\tilde{c})_i - \phi(c)_i| \leq K\alpha\phi(c)_i \forall i$ — bounded variation

Cost allocation methods

- $\phi(\Gamma)_i = c(i) \ \forall i \in N$ — fixed-price ($\neg 1, \neg 3$)
- $\phi(\Gamma)_i = c(i) \cdot \frac{c(N)}{\sum_{j \in N} c(j)} \ \forall i \in N$ — proportional ($\neg 2, \neg 3$)
- $\phi(\Gamma) = \mathcal{LC}_f$ — f-least core ($\neg 2, \neg 4$)

$$= \operatorname{argmax}_x \epsilon$$
$$x(S) + \epsilon f(S) \leq c(S) \ \forall S \in \Sigma^+ \setminus N$$
$$x(N) = c(N)$$
$$x \in P$$

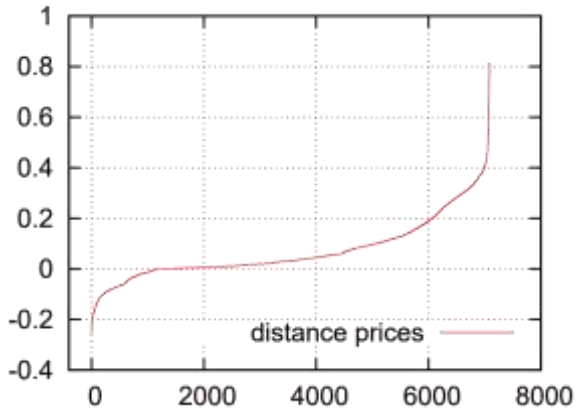
Desirable Properties

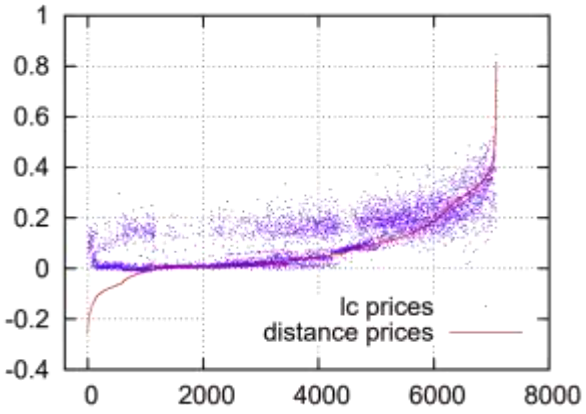
1. $\phi(\Gamma)(N) = c(N)$ — efficiency
2. $\phi(\Gamma)(S) \leq \phi(\Gamma_S)(S) \ \forall S \in \Sigma^+$ — coalitional stability
3. $\phi(\Gamma) \in \mathcal{C}$ — core price
4. $\exists K, \alpha > 0 : |\tilde{c}(\cdot) - c(\cdot)| \leq \alpha c(\cdot)$
   $\Longrightarrow |\phi(\tilde{c})_i - \phi(c)_i| \leq K\alpha\phi(c)_i \forall i$ — bounded variation

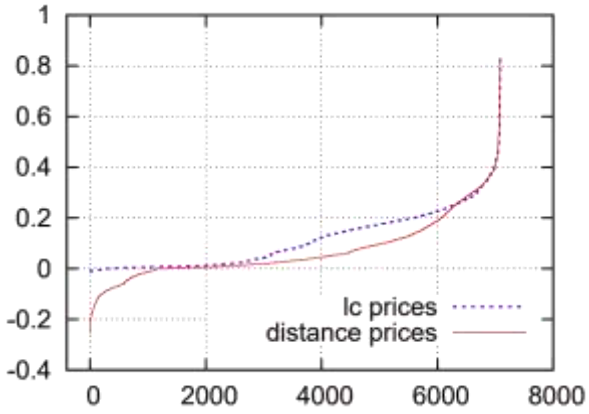
Proposition (Hoang [2010]): There is no (general) cost allocation method that can guarantee more than 2 out of the above 4 properties (for all games), even for cost allocation games with monotone, subadditive cost functions.

Theorem (Hoang [2010]): For weight functions $f = \alpha g + \beta c$, where $\alpha, \beta \in \mathbb{Q}, \alpha + \beta > 0, g: \Sigma \to \mathbb{Q}$ modular and positive on Σ^+, the f-nucleolus of a "strongly bounded" cost allocation game can be computed in time that is polynomial in oracles for computing $c(S)$, $g(S)$, membership for Σ, and separation for P.

Fairness Diagram







- $\Lambda = \{\{i\}: i \in N\} \cup N$
- $\mathrm{R}: \Lambda \to \mathbb{R}_{>0}, \sum_i R(\{i\}) = c(N)$
- $r_i = R(\{i\}), i \in N$
- $\mathcal{LC}_{f,r} := \mathcal{N}_R(N, R, \mathcal{LC}_f(\Gamma), \Lambda)$

$$= \mathrm{arglexmin}_{x \in \mathcal{LC}_f} \left(\frac{x_i}{r_i}\right)_{i \in N}$$

reference price function

reference price for player i

(f, r)-least core

as $e_R(\{i\}, x) = \frac{r_i - x_i}{r_i} = 1 - \frac{x_i}{r_i}$

Cost Function (Line Planning Problem)

$$c(S) = \min \sum_{(r,f)\in\mathcal{R}\times\mathcal{F}} \left(c^1_{r,f}\xi_{r,f} + c^2_{r,f}\rho_{r,f}\right)$$

arc capacities for shortest path routing

$$\text{s.t.} \sum_{e\in r\in\mathcal{R}} \sum_{f\in\mathcal{F}} c_{cap} f\left(m\xi_{r,f} + \rho_{r,f}\right) \geq \sum_{i\in S} P_e^i \qquad \forall e \in E$$

min frequencies for shortest path routing

$$\sum_{e\in r\in\mathcal{R}} \sum_{f\in\mathcal{F}} f\xi_{r,f} \geq F_e^i \qquad \forall (e,i) \in S \times E$$

$$\rho_{r,f} - (M-m)\xi_{r,f} \leq 0 \qquad \forall (r,r) \in \mathcal{R}\times\mathcal{F}$$

≤ 1 frequencies per route

$$\sum_{f\in\mathcal{F}} \xi_{r,f} \leq 1 \qquad \forall r \in \mathcal{R}$$

$$\xi \in \{0,1\}^{\mathcal{R}\times\mathcal{F}}, \rho \in \mathbb{Z}_{\geq 0}^{\mathcal{R}\times\mathcal{F}}$$

route r has frequency f

route r with frequency f has $m + \rho_{r,f}$ coaches

Separation problem extends to OD-pair choice plus objective change to excess.

Setup

- Players

  $n = 81$ OD-pairs of highest demand, $2^{81} - 1$ coalitions

- Reference price

$$P_{st} = \frac{c(N)}{\sum_{st \in N} d_{st}} \cdot d_{st}$$

- Monotonicity

$$0 \leq \frac{x_{uv}}{P_{uv}} \leq \frac{x_{st}}{P_{st}} \leq \sum_{uv \in \mathcal{P}_{st}} \frac{x_{uv}}{P_{uv}} \qquad \forall st \in S, uv \in \mathcal{P}_{st}$$

- Distance-likeness

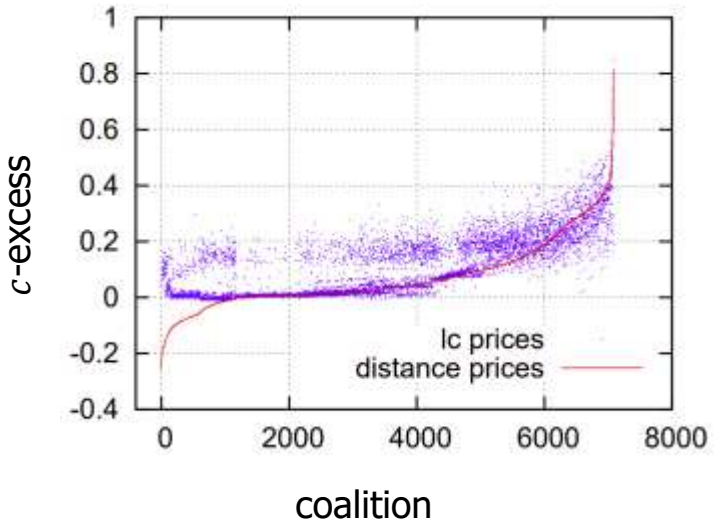
$$\max_{st} \frac{x_{st}}{d_{st} P_{st}} \leq K \min_{st} \frac{x_{st}}{d_{st} P_{st}} \qquad \forall st \in S$$

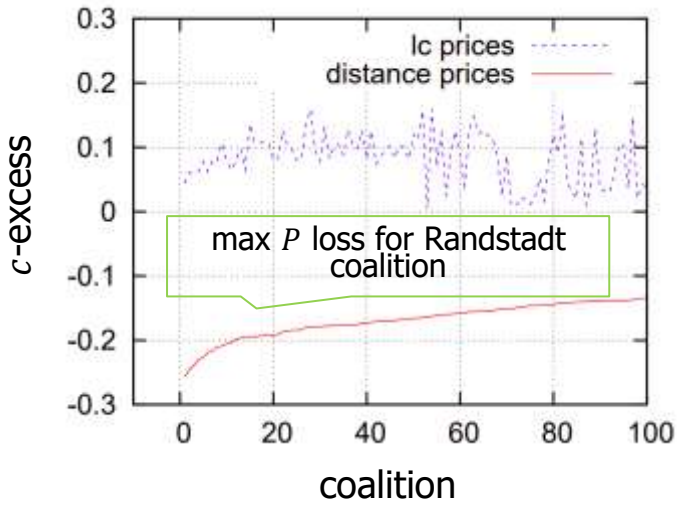
- Weight function

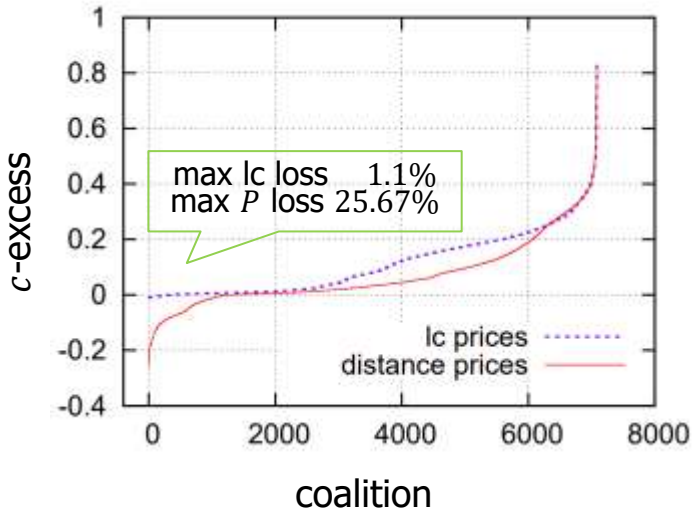
$$f = c,$$

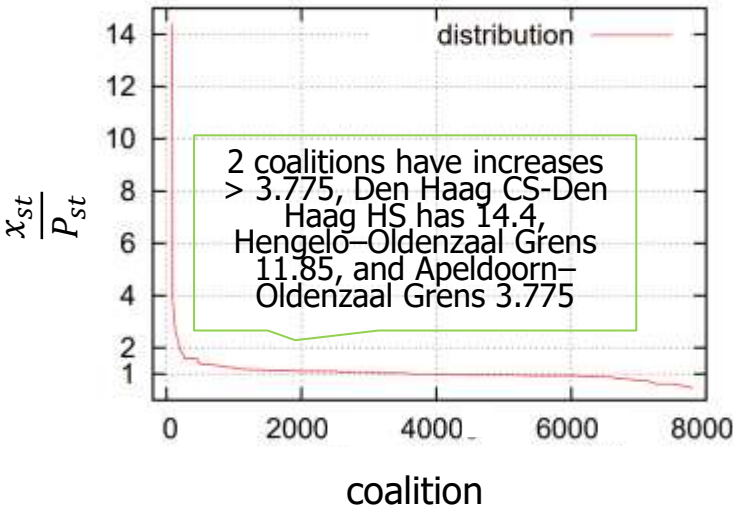
  i.e, $e_f(S, x) = e_c(S, x) = \frac{c(S) - x(S)}{c(S)}$ = relative savings

Least-core Prices ($K = +\infty$)

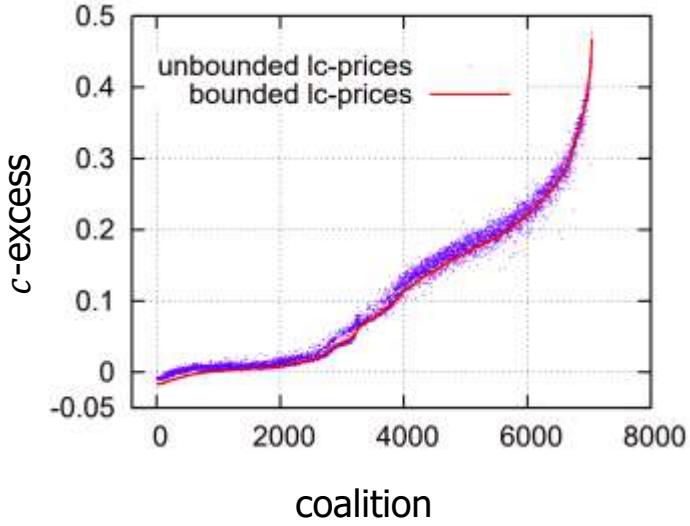


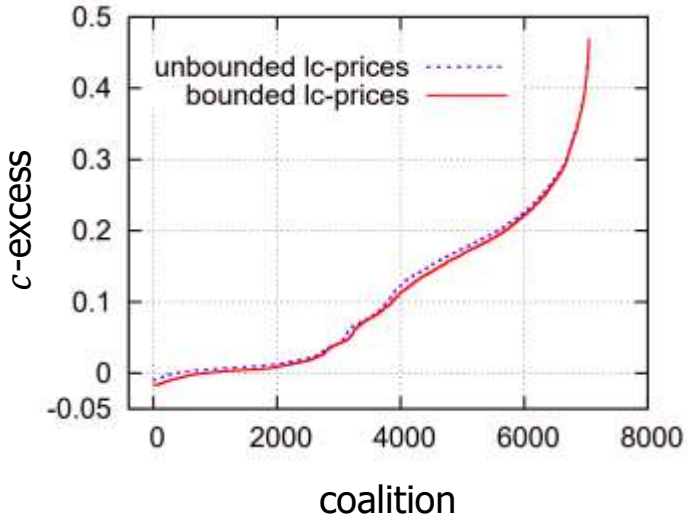




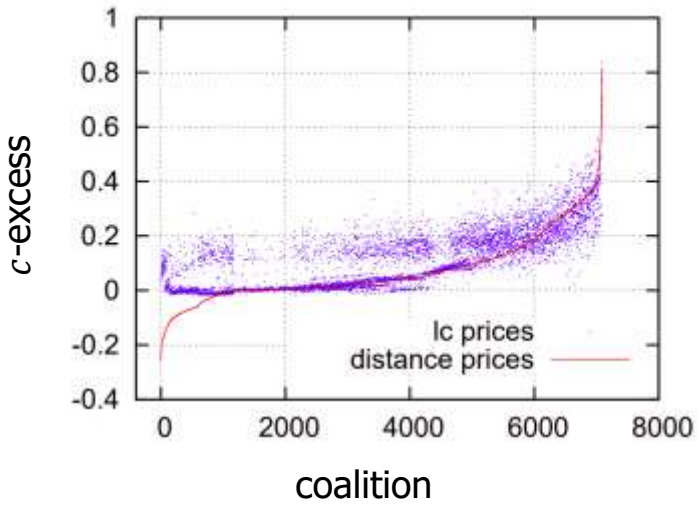


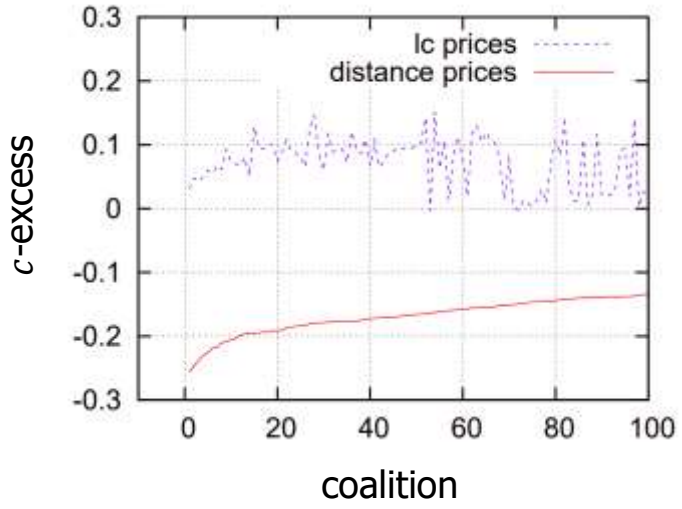
Least-core Prices ($K = +\infty$ vs. $K = 3$)

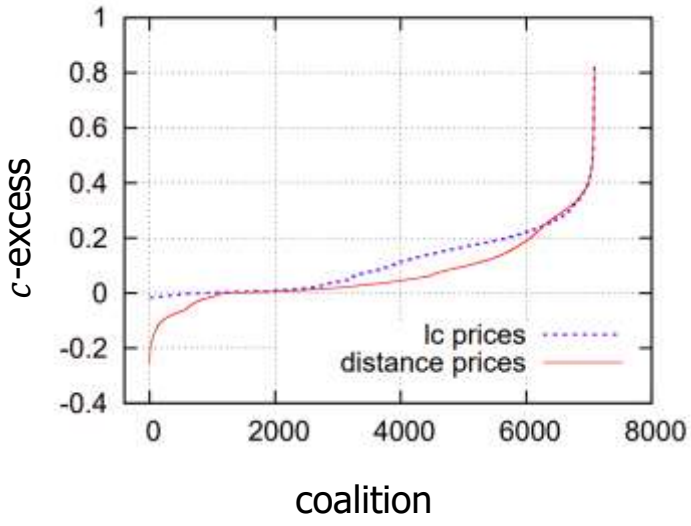


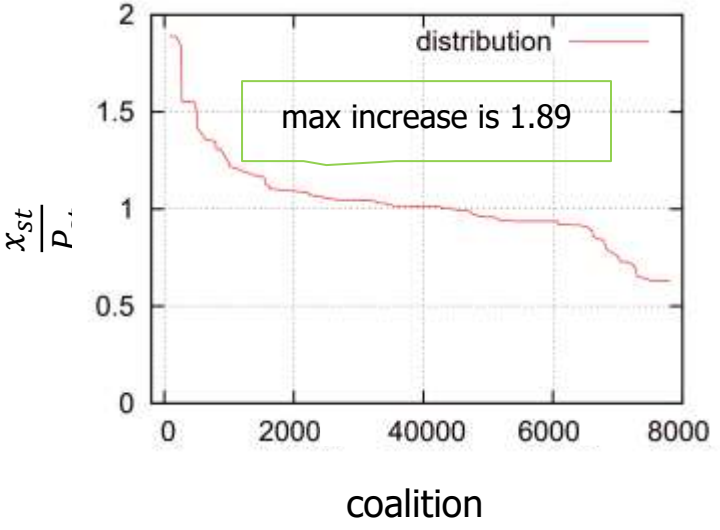


Least-core Prices ($K = 3$)



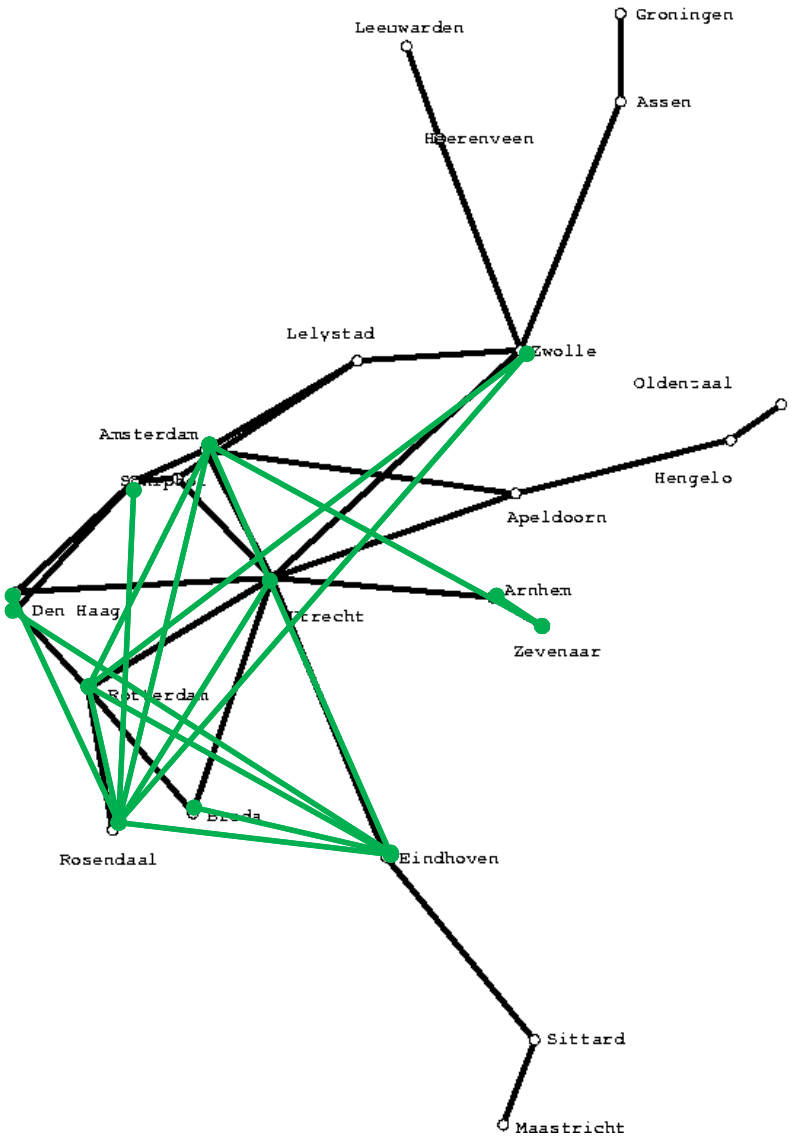






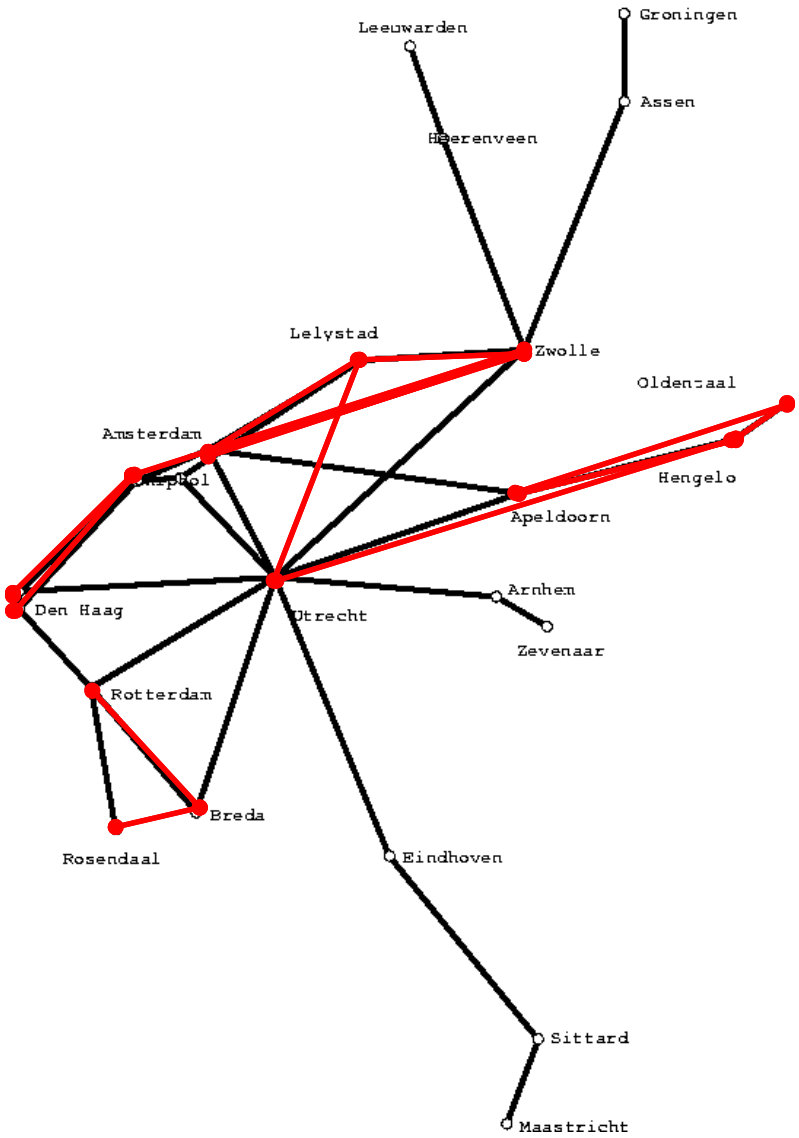
Best 15 (x_{st}/P_{st} for $K = 3$ and $K = +\infty$)

Arnhem-Zevenaar Grens	0.63
Amsterdam CS-Roosendaal	0.63
Breda-Eindhoven	0.63
Roosendaal-Rotterdam CS	0.63
Roosendaal-Zwolle	0.65
Eindhoven-Den Haag CS	0.65
Roosendaal-Schiphol	0.69
Eindhoven-Roosendaal	0.70
Eindhoven-Rotterdam CS	0.71
Amsterdam CS-Eindhoven	0.72
Den Haag HS-Roosendaal	0.73
Roosendaal-Utrecht CS	0.73
Rotterdam CS-Zwolle	0.74
Amsterdam CS-Rotterdam CS	0.76
Amsterdam CS-Zevenaar	0.79

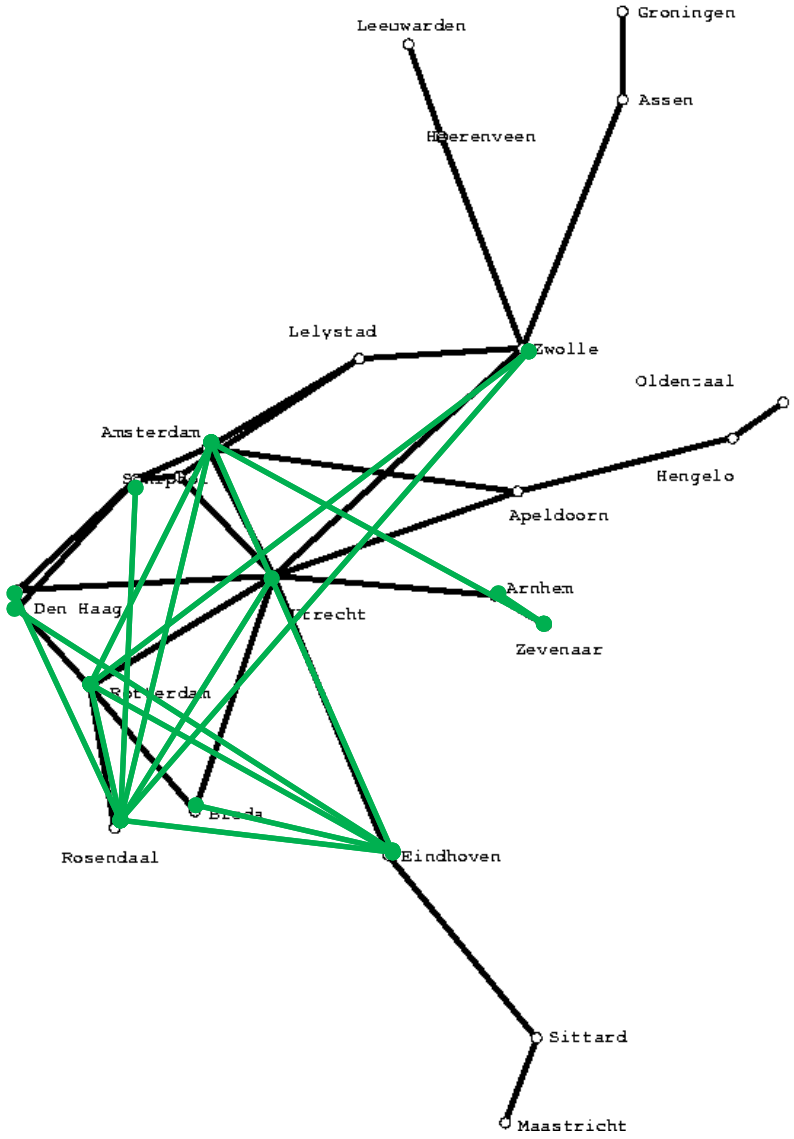


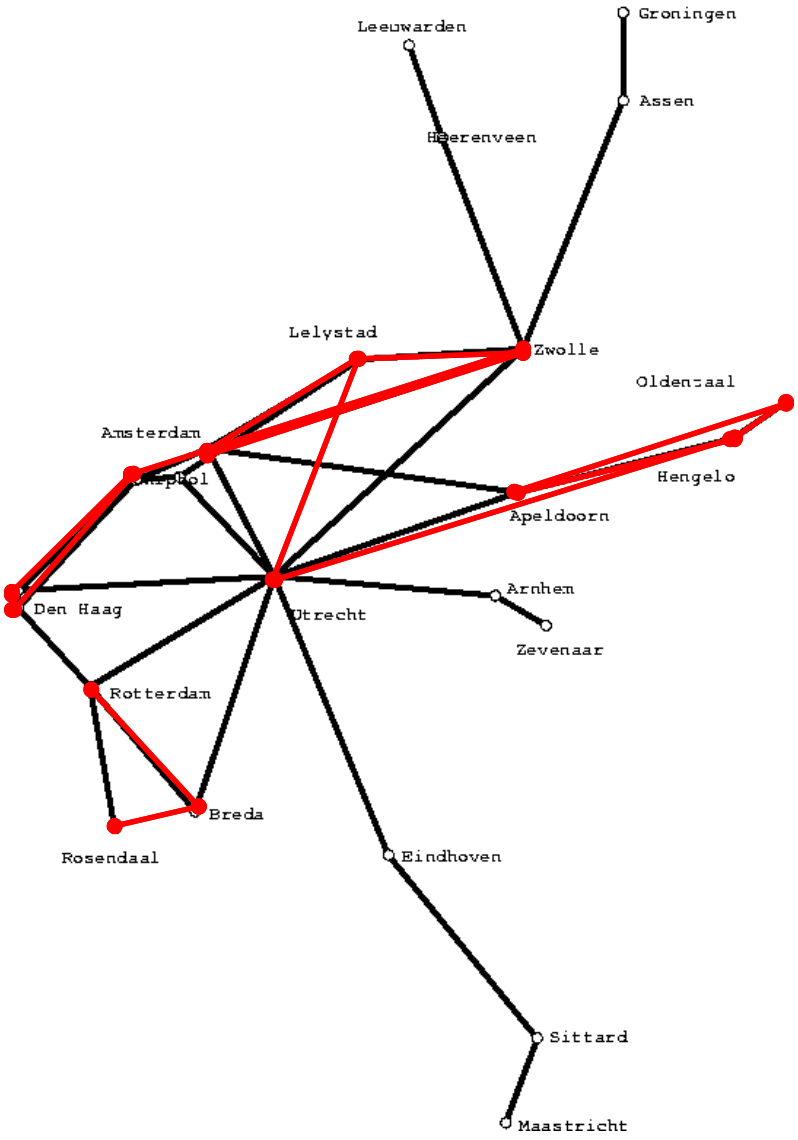
Worst 15 (x_{st}/P_{st} for $K = 3$ and $K = +\infty$)

Breda-Roosendaal	1.30
Hengelo-Utrecht CS	1.30
Schiphol-Zwolle	1.30
Den Haag CS-Schiphol	1.35
Den Haag HS-Schiphol	1.36
Amsterdam Zuid-Zwolle	1.42
Amsterdam CS-Zwolle	1.52
Breda-Rotterdam CS	1.55
Leylstad-Utrecht CS	1.55
Amsterdam Zuid-Leylstadt	1.84
Apeldoorn-Hengelo	1.89
Apeldoorn-Oldenzaal	1.89
Den Haag HS-Den Haag CS	1.89
Hengelo-Oldenzaal	1.89
Leylstad-Zwolle	1.89



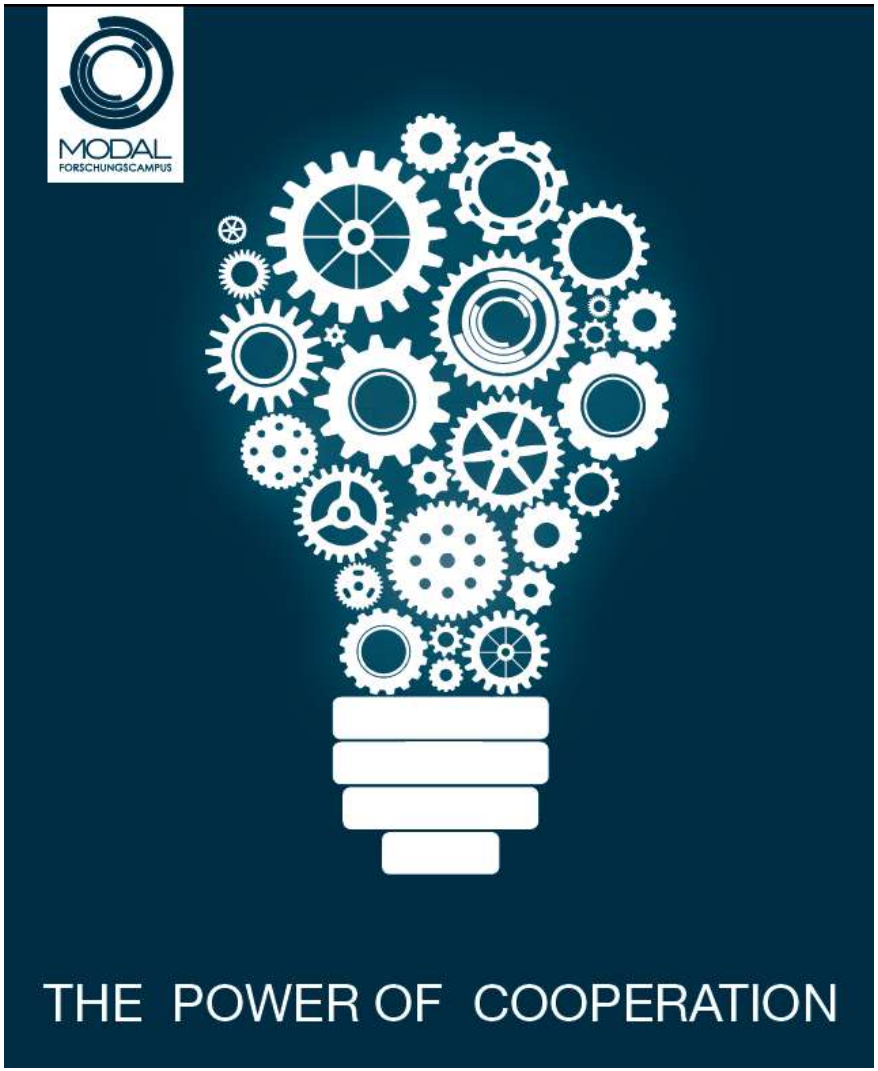
Worst 15 (x_{st}/P_{st} for $K = 3$ and $K = +\infty$)





G-RIPS

Thank you very much for your attention!



https://www.zib.de/research/efficient-infrastructure-and-logistics

http://www.zib.de/optimization/mobilitylab

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