Mathematical models in public transportation

. . . and their extension to equity objectives

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Content of the talk

1. Stages in Public Transport Planning
   - Stop location
   - Line Planning
   - Timetabling
   - Vehicle scheduling
   - Delay management
   - Tariff Planning

2. Some more remarks on equity

Optimize public transport

General goal:

Under a restricted budget make public transport as **good** as possible.

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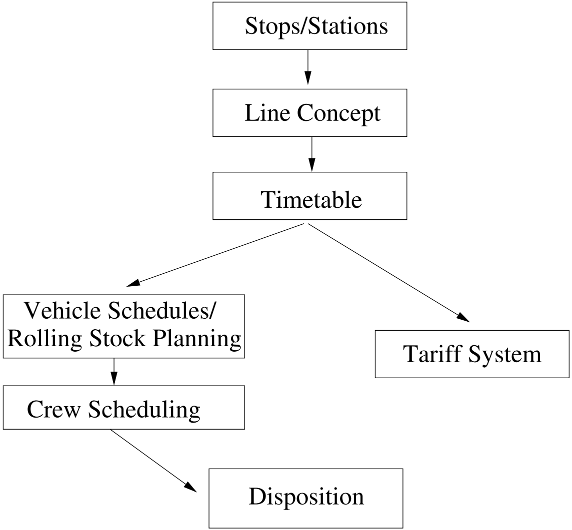
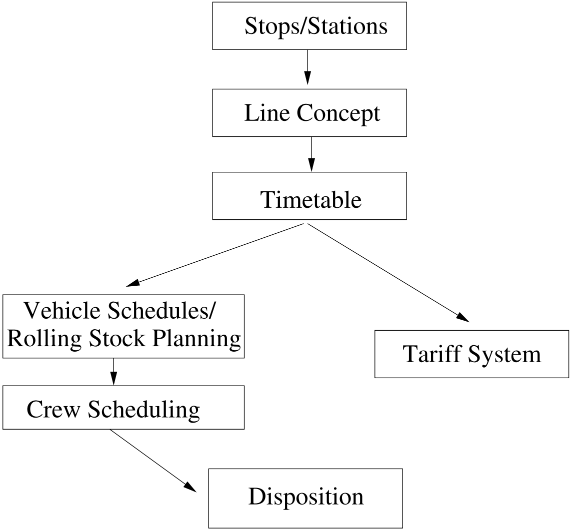
Why?

- Provide sufficient accessibility to everybody (school kids, people without car, ...)
- Environmental goals (pollution)

The planning process in public transportation



A note on integration



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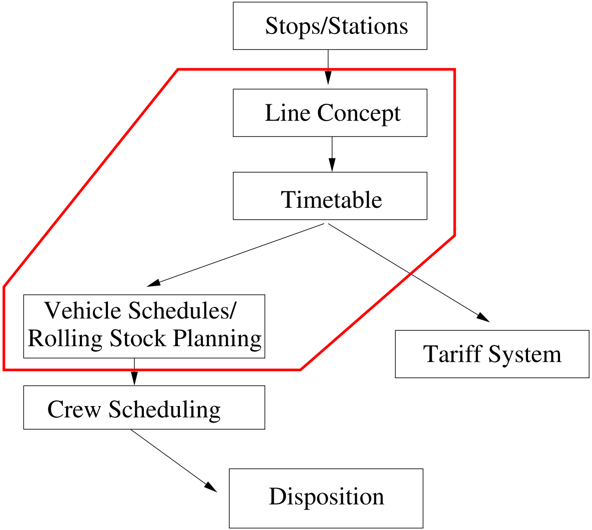

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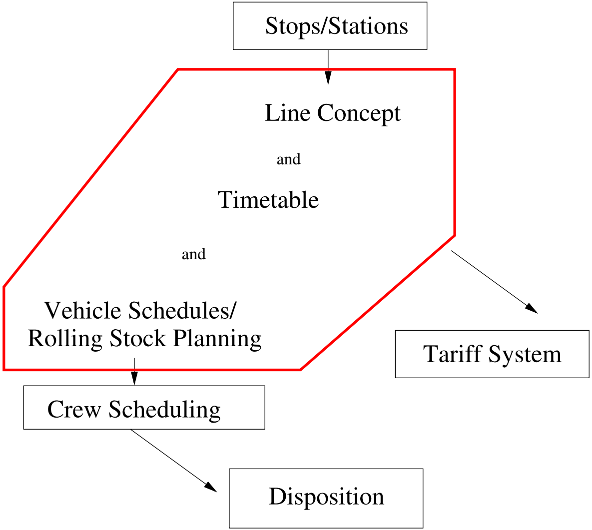

Literature

. . . is numerous and still increasing

. . . see many books and many more research papers!

A try of a short overview

- will be given in this talk
- see also: Hickman and Schöbel: 50 years of Operations Research in Public Transport, 2025

Commercial Break

Commercial Break

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Commercial Break

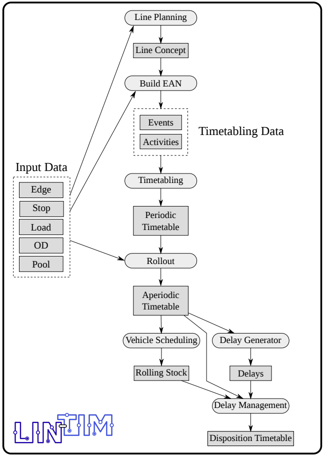
Academic **Open Source Library** for optimization in public transportation

for planning stops, lines, timetables, vehicle schedules, tariffs and for delay management.

LinTim

`https://www.lintim.net/`

LinTim: Open Source Library for public transport



Data sets:

- Toy example
- Lower Saxony (railway)
- Grid Network
- Metro in Athens
- Long-distance trains of Germany
- Göttingen Bus System
- more to come!

LinTim: Open Source Library for public transport

Team: **Anita Schöbel**, **Philine Schiewe**, **Sven Jäger**, Alexander Schiewe, Vera Grafe, Reena Urban, Sebastian Albert, Felix Spühler, Moritz Stinzendörfer, Julius Pätzold, Christopher Scholl, and many former (PhD)students of Kaiserslautern and Göttingen

LinTim: Open Source Library for public transport

Don't forget:

LinTim

https://www.lintim.net/

End of Commercial Break

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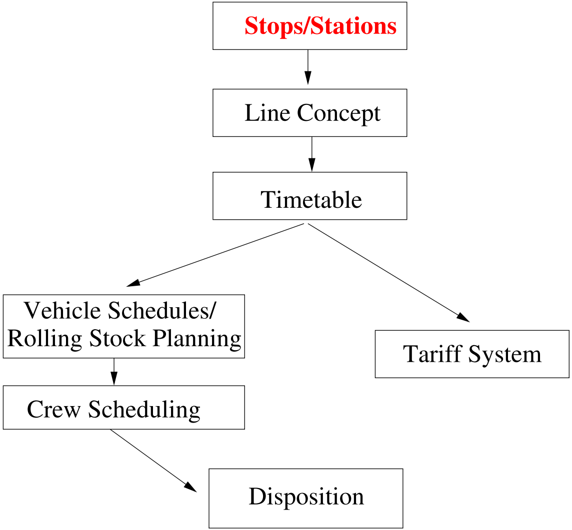
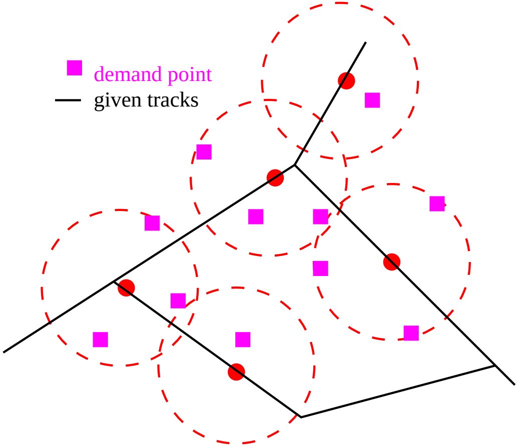
Stop Location



Stop location

Problem: Where should stops be located such that every (potential) passenger lives in a 400-m-radius (2 km radius) and the number of stops is minimal?



Stop location

Problem: Where should stops be located such that every (potential) passenger lives in a 400-m-radius (2 km radius) and the number of stops is minimal?

Variables?

- discrete version: A finite number of potential locations $\{l_1, \ldots, l_L\}$

$$x_i = \begin{cases} 1 & \text{if location } i \text{ is chosen} \\ 0 & \text{otherwise} \end{cases}$$

- continuous version: everywhere along the tracks
  $\rightarrow$ find a finite candidate set and use discrete version

Objective function: minimize number (costs) of the stops min $\sum_{i=1}^{L} \mathrm{cost}_i x_i$

Constraints: Cover all passengers
$\sum_{i=1}^{L} a_{pi} x_i \geq 1$ for all passengers p
where $a_{pi} = 1$ if and only if station l_i is close enough at passengers p.

Stop location problem as integer program

Problem (SL): Where should stops be located such that every (potential) passenger lives in a 400-m-radius (2 km radius) and the number of stops is minimal?

$$\min \sum_{i=1}^{L} \mathrm{cost}_i x_i$$

$$\text{s.t.} \quad \sum_{i=1}^{L} a_{pi} x_i \geq 1 \text{ for all passengers } p$$

$$x_i \in \{0, 1\} \text{ for all } i = 1, \ldots, L$$

where $a_{pi} = 1$ if and only if station l_i is close enough at passengers p.

Analysis

Theorem

(SL) is NP-hard in general, but polynomially solvable if all stations have to be located on a straight line.

Observation: (SL) is a *covering problem* and can also be treated as *location problem*.

Other objectives functions

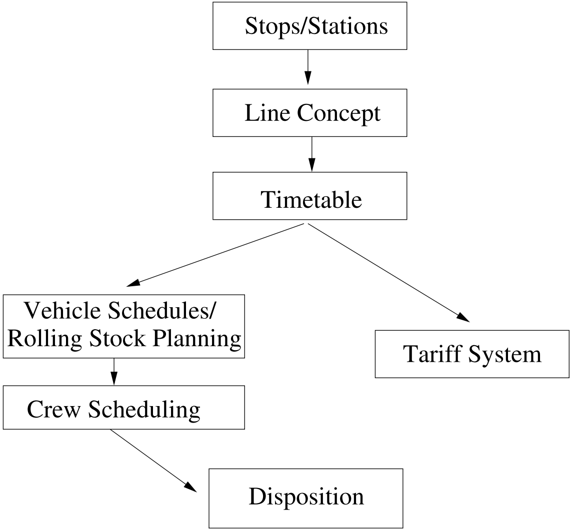
- Given a budget, cover as many potential passengers (in a 400-m-radius, 2 km radius) as possible. many papers, Murray (2001)
- Given a budget, minimize the average access time Poetranto et al (2009)
- Minimize average the door-to-door traveling time including
  - the access time
  - the time passengers in the trains lose by stopping

Schöbel, Hamacher, Liebers and Wagner (2009) within a project with German railways

Equity?

- Cover all potential passengers $\rightarrow$ justice in accessibility
- One could focus on people that have to rely on public transport (no car, no driving license) $\rightarrow$ even more "equity"?
- Considering the door-to-door traveling time is another option.

How to plan the lines?



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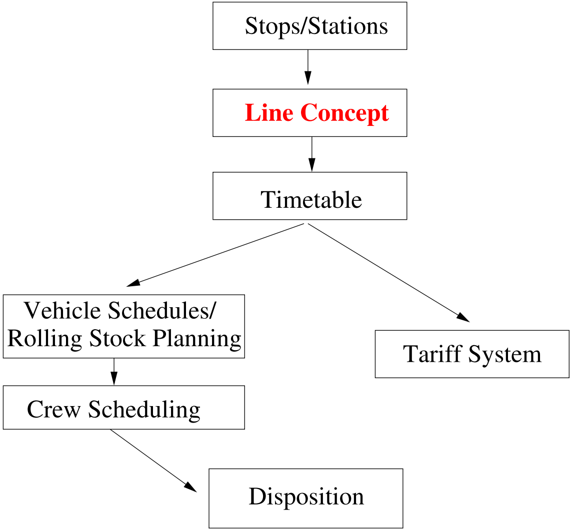
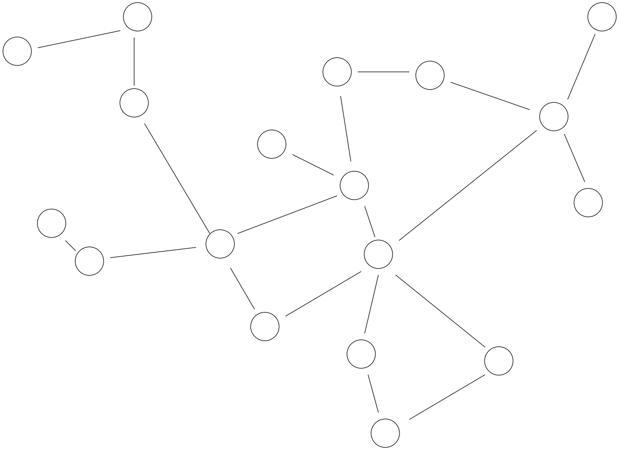


Example



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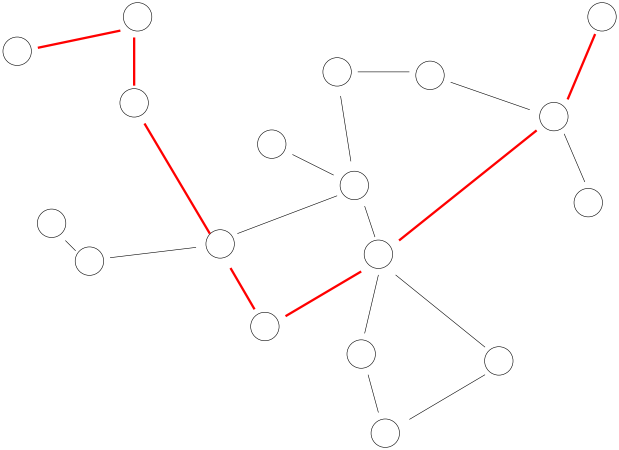

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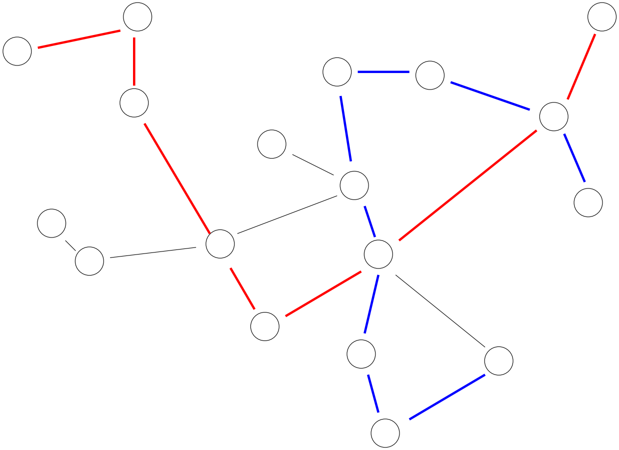

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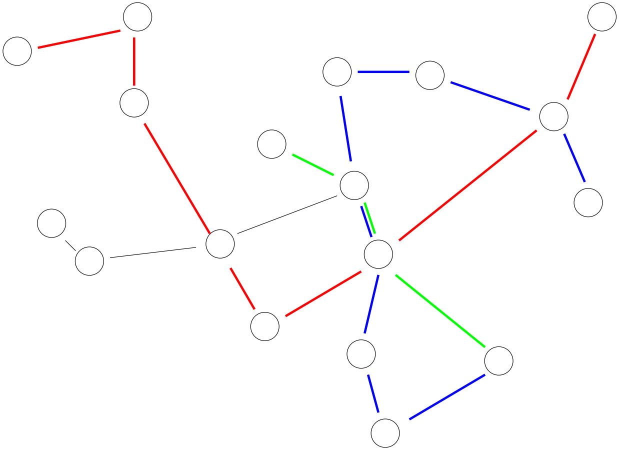

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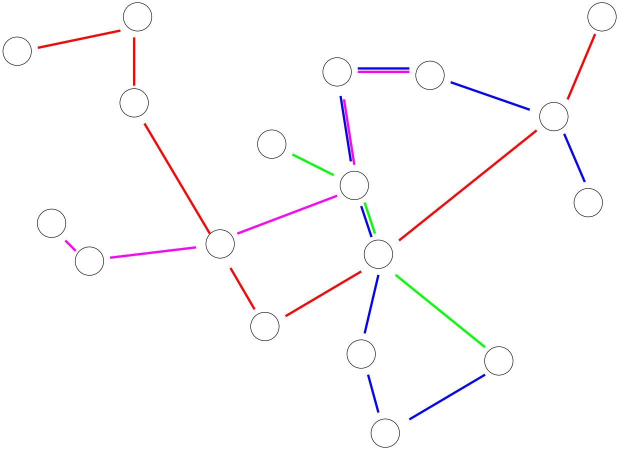

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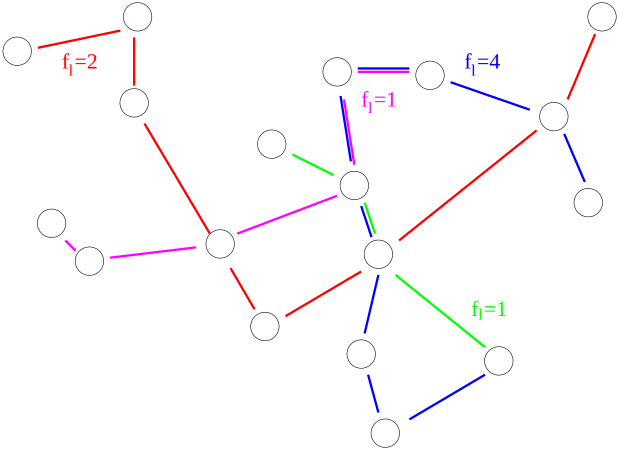

Line planning in the literature

- many different models:
  - how to measure costs
  - how to measure traveling time
  - and which behaviour of passengers is used for the latter
- even different names: line planning, transit network design, with/without frequency setting
- many different algorithms (depending on the community)

first paper Patz (1925)

many papers by Ralf Borndörfer and co-authors

surveys: Guihare and Hao (2008), Kepaptsoglou and Karlaftis (2009), Schöbel (2012), Farahani et al (2013), Schöbel and Schmidt (2025)

Variables

Notation

The **Public Transportation Network PTN** is a graph PTN$=(V, E)$ with stops/stations V and direct connections between them as edges E.

Let a **planning period T** be given.

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The **Public Transportation Network PTN** is a graph PTN$=(V,E)$ with stops/stations V and direct connections between them as edges E.

Let a **planning period T** be given.

A **line** P is a path in the public transportation network.

The **frequency** f_l of a line l says how often service is offered along line l within a (given) time period T.

A **line concept** $(\mathcal{L}, f)$ is a set of lines $\mathcal{L}$ together with their frequencies f_l for all $l \in \mathcal{L}$.

Variables

Find a set of lines, i.e.,

- determine the number of lines,
- the routes of the lines,
- and their frequencies

Variables

We distinguish two cases:

Line Pool We assume a predefined set of potential lines. $\mathcal{L}^0 = \{l_1, \ldots, l_N\}$
Then we define:

$$x_l = \begin{cases} 1 & \text{if } l \text{ is chosen} \\ 0 & \text{otherwise} \end{cases}$$

or even better:

$$f_l = \text{frequency of line } l$$

Lines from Scratch

- Can be seen as "*full*" line pool and modeled as above
- lines constructed in the IP by column generation Borndörfer et al (2007)

From now on: Let $\mathcal{L}^0$ be a given line pool.

Constraints

Notation

Let $f_e^{\min}, f_e^{\max}$ denote the minimal and maximal allowed frequency on edge $e \in E$, i.e., the minimal and maximal number of vehicles which should pass edge e within our time period T.

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Example:

- $f_e^{\min}$ is due to the minimal number of vehicles needed to transport all passengers.
- $f_e^{\max}$ often due to security headways (minimal distances) or noise avoidance

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We require :

$$\text{For all} e \in E: \quad f_e^{\min} \leq \sum_{l: e \in l} f_l \leq f_e^{\max}$$

Constraints

Many other constraints are possible:

- Not more than K lines: $\sum_{l \in \mathcal{L}^0} x_l \leq K$
- Not more than K_v arrivals at station v: $\sum_{l : v \in l} f_l \leq K_v$
- Kilometers driven in the line concept smaller than B: $\sum_{l \in \mathcal{L}^0} f_l \mathrm{length}_l \leq B$
- ...

Some models for line planning

(LP1) Cost model

(LP2) Extended cost model Claessens, van Dijk, Zwaneveld (1998),

(LP3) Direct travelers model Patz (1925), Bussieck et al (1996)

(LP4) Travel time model Schöbel and Scholl (2006)

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Cost-oriented model (LP1)

(LP1) (Cost-oriented line concept)

Given a PTN, a set $\mathcal{L}^0$ of potential lines, lower and upper frequencies $f_e^{\min} \leq f_e^{\max}$ for all $e \in E$, and parameters $cost_l$ for all $l \in \mathcal{L}^0$, find a feasible line concept $(\mathcal{L}, f)$ with minimal overall costs

$$cost(\mathcal{L}, f) = \sum_{l \in \mathcal{L}} f_l \; cost_l.$$

where

$$cost_l = Time_l \; cost_{time} + length_l \; cost_{km}$$

Integer programming formulation for (LP1)

$$
\begin{aligned}
\min \sum_{l \in \mathcal{L}^0} f_l \quad & cost_l \\
\text{s.t.} \sum_{l \in \mathcal{L}^0, e \in l} f_l \quad & \geq \quad f_e^{\min} \ \forall\ e \in E \\
\sum_{l \in \mathcal{L}^0, e \in l} f_l \quad & \leq \quad f_e^{\max} \ \forall\ e \in E \\
f_l \quad & \in \quad \mathbb{N}.
\end{aligned}
$$

Note: solution f_l for all $l \in \mathcal{L}^0$, then line concept $(\mathcal{L}, f)$ is given as $\mathcal{L} = \{l \in \mathcal{L}^0 : f_l > 0\}$.

Analysis of (LP1)

Theorem

(LP1) is NP-hard, even without considering upper frequencies and with $cost_l = 1$ for all $l \in \mathcal{L}^0$ and $f_e^{\min} = 1$ for all $e \in E$.

Observations: (LP1) without upper frequencies is a *multi covering problem*.

In our case:

$$a_{el} = \begin{cases} 1 & \text{if } e \in l \\ 0 & \text{else} \end{cases}$$

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Travel time model (LP4)

Most models use fixed passengers' weights on the edges!

But: Passengers choose their paths dependent on the line concept, we want to compute!

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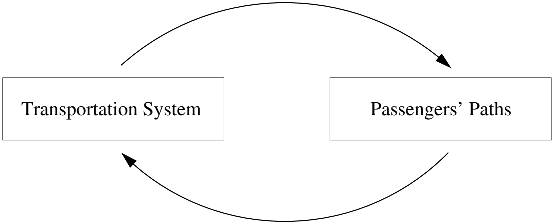
But: Passengers choose their paths dependent on the line concept, we want to compute!

More precisely: Passengers choose a specific path, if

- it is short,
- it has few transfers,
- it has a high frequency.

but these properties depend on the line concept to be determined !

Integrating the Passengers' Routes



Question: With which should we start?

Integrating the Passengers' Routes

Transportation System	AND	Passengers' Paths

Question: With which should we start?

Do both steps simultaneously!

Objective function

Passengers like

- short riding time
- few transfers

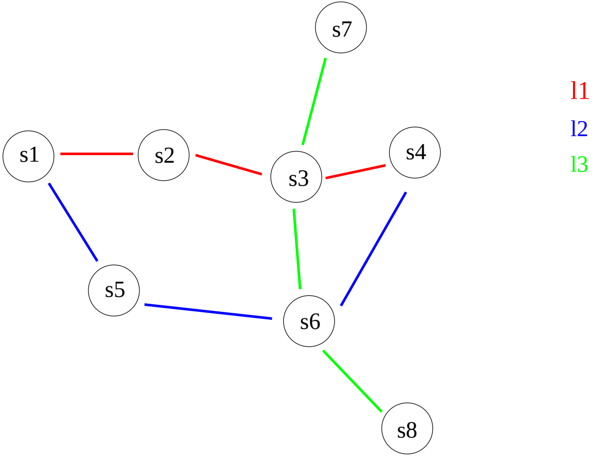
Take both effects into account! $\rightarrow$ “perceived traveling times”

$$\textbf{PerTravelTime} = k_1 \cdot \textbf{ Riding Time} + k_2 \cdot \textbf{ number of transfers}$$

k_2 is an estimate for the transfer time (which is not known exactly without a timetable).

Can we deteremine the lines and the passenger routes simultaneously?

The change & go graph Schöbel and Scholl (2006)



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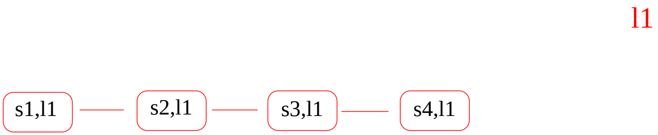

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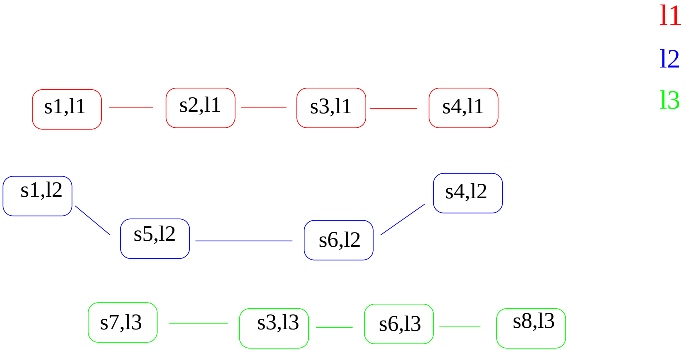

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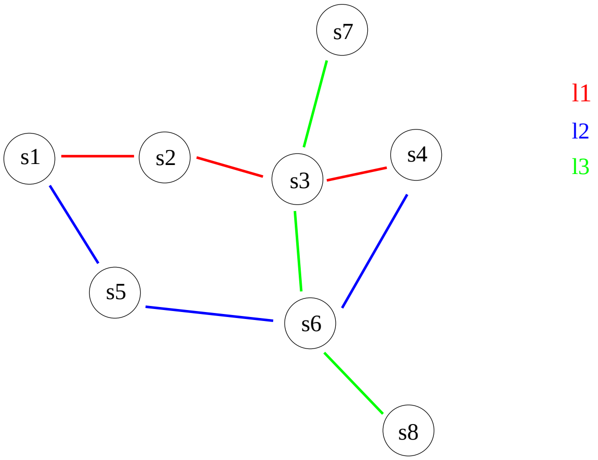

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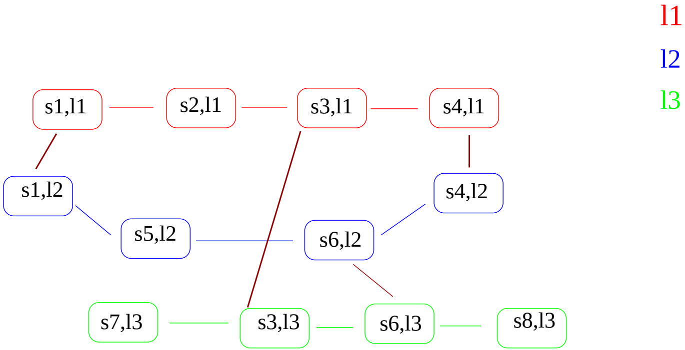

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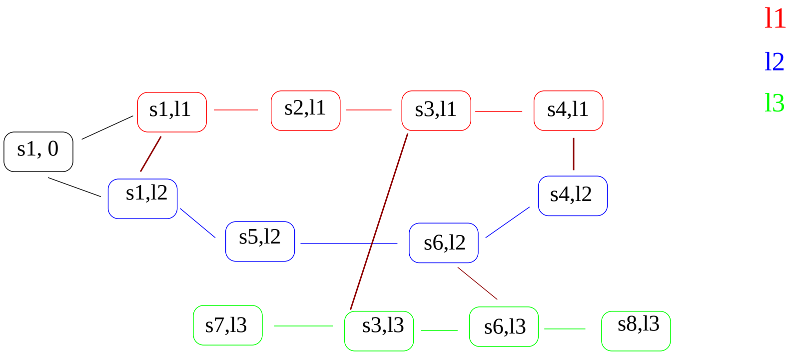

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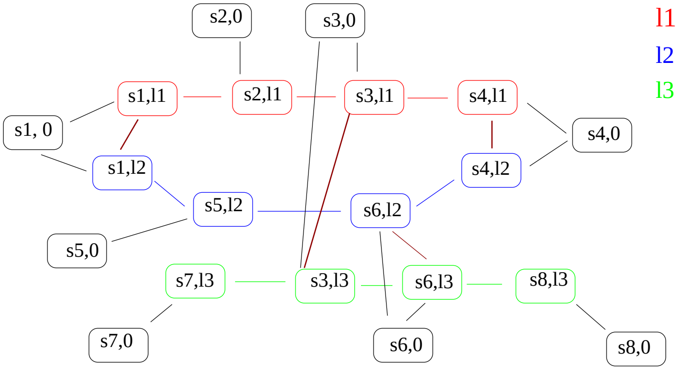

Result: Change & Go Graph $\mathcal{N} = (\mathcal{E}, \mathcal{A})$

$\rightarrow$ paths with minimal perceived traveling time can be computed as shortest paths!

Integer programming model for (LP4)

Variables:

$$x^a_{st} = \begin{cases} 1 & \text{if activity } a \in \mathcal{A} \text{ is used on a shortest path from } s \text{ to } t \text{ in } \mathcal{N} \\ 0 & \text{otherwise} \end{cases}$$

$$y_l = \begin{cases} 1 & \text{if line } l \text{ is established} \\ 0 & \text{otherwise} \end{cases}$$

Parameters: Θ as node-arc-incidence matrix of $\mathcal{N}$,

$$b^i_{st} = \begin{cases} 1 & \text{if } i = (s, 0) \\ -1 & \text{if } i = (t, 0) \\ 0 & \text{otherwise} \end{cases}$$

Integer programming model for (LP4)

$$
\begin{array}{lrcll}
\min & \sum_{s,t \in V} \sum_{a \in \mathcal{A}} C_{st} w_a x_{st}^a & & & \\
\text{s.t.} & x_{st}^a & \leq & y_l & \text{for all } s,t \in V, l \in \mathcal{L}, a \in l \\
 & \Theta x_{st} & = & b_{st} & \text{for all } s,t \in V \text{ with } C_{st} > 0 \\
 & \sum_{l \in \mathcal{L}} y_l cost_l & \leq & B & \\
 & x_{st}^a, y_l \in \{0,1\} & & &
\end{array}
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- this model assumes unlimited capacity of the vehicles
- with limited capacity A of the trains:

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$$

- this model assumes unlimited capacity of the vehicles
- with limited capacity A of the trains:

relax x_{st}^a and $f_l = y_l$ to integers and replace

$$x_{st}^a \leq y_l \text{ by } \sum_{s,t \in V} x_{st}^a \leq f_l A \text{ for all } l \in \mathcal{L}, a \in l$$

and $\Theta x_{st} = b_{st}$ is a network flow problem.

Analysis of (LP4)

Theorem

(LP4) is NP-hard, even if

- only the number of transfers is counted in the objective
- the network is a linear graph
- and all costs are equal to one, see Schöbel and Scholl (2006)

or if

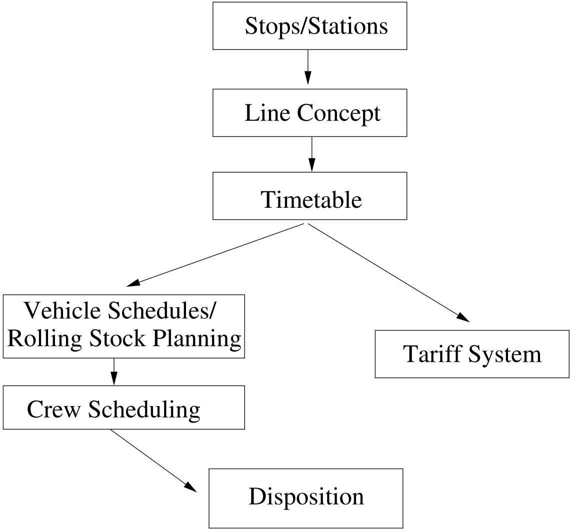
- all passengers depart from the same origin, see Schmidt and Schöbel (2014)

Problem has one block for each OD-pair s, t and a y-variable block
$\rightarrow$ Dantzig-Wolfe decomposition as solution technique

Equity?

- Restrict the maximum number of transfers?
- The absolute perceived traveling time is longer for long relations. But we could minimize the ratio between individual car and public transport over all passengers.
- Improve the line concept in particular for OD-pairs with low income.

How to plan the timetable?



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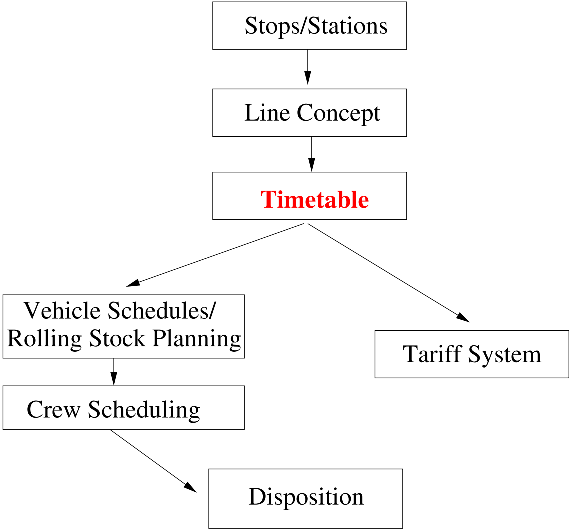


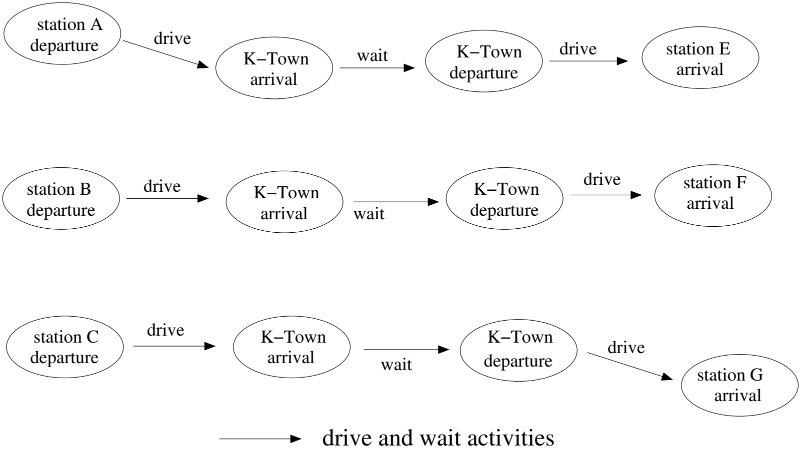
Timetabling

Assign a time to every arrival and departure of every bus/train.

- An event is the arrival or the departure of a bus/train at a station
- The timetable may be periodic or aperiodic.
- Constraints: Driving times and dwell times have to be respected.
  Also headway times in rail-traffic.
- Goal: minimize the riding and transfer times of the passengers.
  Maybe also: plan for robustness

Modeling timetables

Use event-activity network $(\mathcal{E}, \mathcal{A})$:



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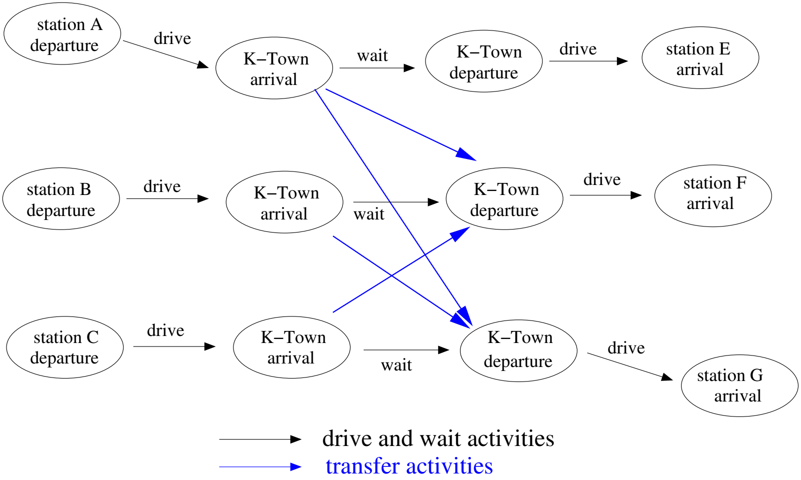

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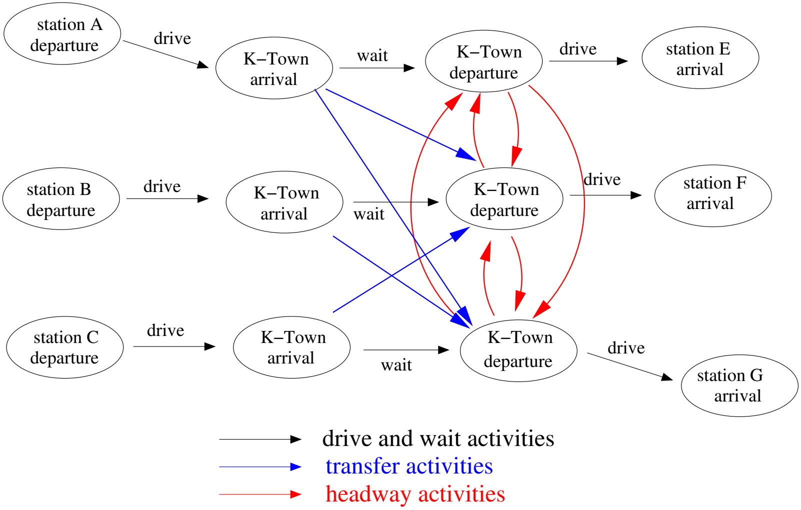

Modeling timetables

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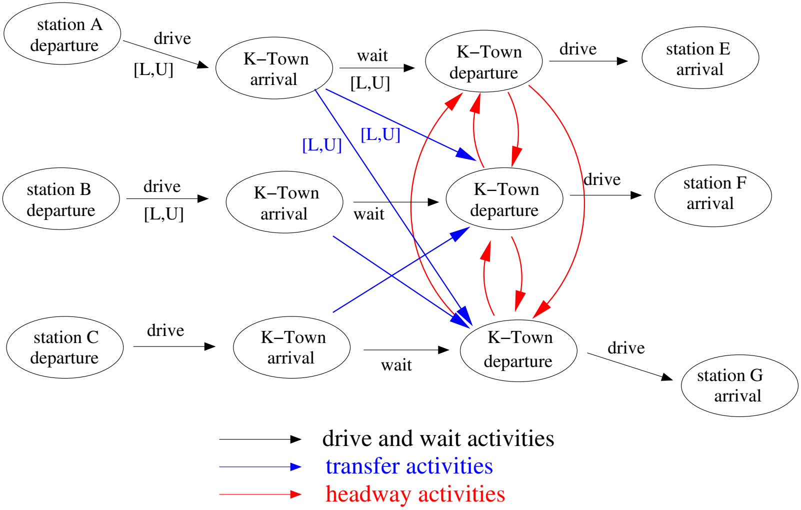

Aperiodic timetables

Given event-activity network $\mathcal{N} = (\mathcal{E}, \mathcal{A})$

- with lower and upper bounds $[L_a, U_a]$ for every activity $a \in \mathcal{A}$

A timetable $\pi_i, i \in \mathcal{E}$ assigns a time to every event $i \in \mathcal{E}$.

Aperiodic timetables

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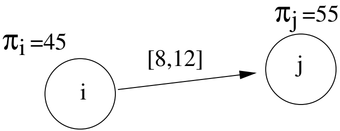
Variables: π_i=time for event i for all $i \in \mathcal{E}$

Aperiodic timetables

When is a timetable feasible?



$$\pi_j - \pi_i \in [L_a, U_a]$$

Note: Instances with circles may be infeasible!

Aperiodic timetabling

Minimize the traveling time of the passengers.

$$\min \sum_{a=(i,j)\in\mathcal{A}} c_a(\pi_j - \pi_i)$$

Note: Passengers' weights c_a are fixed beforehand.

Model (ATT)

$$\begin{array}{lrcll} \min & \sum_{a=(i,j)\in\mathcal{A}} c_a(\pi_j - \pi_i) & & & \\ \text{s.t.} & L_a \leq \pi_j - \pi_i & \leq & U_a & \text{for all } a=(i,j)\in\mathcal{A} \\ & \pi_i & \in & \mathbb{Z} & \text{for all } i \in \mathcal{E}. \end{array}$$

Periodic timetables

Given event-activity network $\mathcal{N} = (\mathcal{E}, \mathcal{A})$

- with lower and upper bounds $[L_a, U_a]$ for every activity $a \in \mathcal{A}$
- and a period T

A (periodic) timetable $\pi_i \in \{0, 1, \ldots, T-1\}, i \in \mathcal{E}$ assigns a time to every event $i \in \mathcal{E}$.

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Periodic timetables

When is a aperiodic timetable feasible?

Periodic timetables

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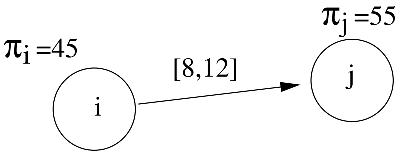

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Periodic timetables

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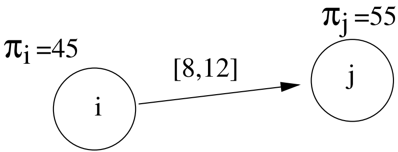

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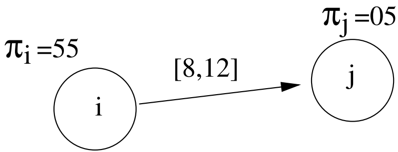
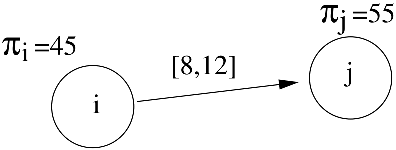


$\pi_j - \pi_i + zT \in [L_a, U_a]$ for some $z \in \mathbb{Z}$

Periodic timetables

When is a aperiodic timetable feasible?



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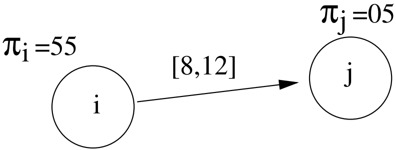

$\pi_j - \pi_i + zT \in [L_a, U_a]$ for some $z \in \mathbb{Z}$

$$\pi_j - \pi_i + zT \in [L_a, U_a] \text{ for some } z \in \mathbb{Z} \iff (\pi_j - \pi_i - L_a) \mod T \in [0, U_a - L_a]$$

Note: More instances are feasible as in the aperiodic case, but infeasible instances still exist.

Periodic Event Scheduling Problem PESP

Minimize the traveling time of the passengers.

$$\min \sum_{a=(i,j)\in\mathcal{A}} c_a(\pi_j - \pi_i - L_a) \mod T$$

Note: Passengers' weights c_a are fixed beforehand.

PESP (Serafini & Ukovich, 1989)

$$\min \sum_{a=(i,j)\in\mathcal{A}} c_a[\pi_j - \pi_i - L_a]_T$$

$$\begin{aligned} \text{s.t. } [\pi_j - \pi_i - L_a]_T &\in [0, U_a - L_a] \text{ for all } a=(i,j)\in\mathcal{A} \\ \pi_i &\in \{0,1,\ldots,T-1\} \text{ for all } i \in \mathcal{E}. \end{aligned}$$

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PESP (Serafini & Ukovich, 1989)

$$\begin{array}{llcl}
\min & \displaystyle\sum_{a=(i,j)\in\mathcal{A}} c_a(\pi_j - \pi_i + z_a T) & & \\
\text{s.t.} & L_a \leq \pi_j - \pi_i + z_a T & \leq & U_a \text{ for all } a=(i,j)\in\mathcal{A} \\
 & \pi_i & \in & \{0,1,\ldots,T-1\} \text{ for all } i \in \mathcal{E}. \\
 & z_a & \in & \mathbb{Z} \text{ for all } a \in \mathcal{A}.
\end{array}$$

z_a are called *modulo parameters*

Properties

Theorem Serafini and Ukovich (1989)

Aperiodic timetabling (ATT) is polynomially solvable.
The problem of deciding if a feasible solution for PESP exists is NP-complete.

Theorem

- The tension along all cycles in a feasible aperiodic timetable is zero $\rightarrow$ existence of an aperiodic timetable can be checked in a modified network by shorts path techniques.
- The tension along all cycles modulo T is zero in every feasible periodic timetable.

Basic algorithm for PESP

1. For all feasible modulo-parameter solve the aperiodic problem.
2. Choose the best solution.

Basic algorithm for PESP

1. For all feasible modulo-parameter solve the aperiodic problem.
2. Choose the best solution.

We need not consider all possible modulo-parameter:

Theorem of Odijk 1996

PESP is feasible if and only if there exists $z \in \mathbb{Z}^{|A|}$ such that for every cycle C in $\mathcal{N}$:

$$\lceil a_C \rceil \leq \sum_{a \in C^+} z_a - \sum_{a \in C^-} z_a \leq \lfloor b_C \rfloor$$

$$\text{with } a_C = \frac{1}{T}\left(\sum_{a \in C^+} L_a - \sum_{a \in C^-} U_a\right) \text{ and } b_C = \frac{1}{T}\left(\sum_{a \in C^+} U_a - \sum_{a \in C^-} L_a\right)$$

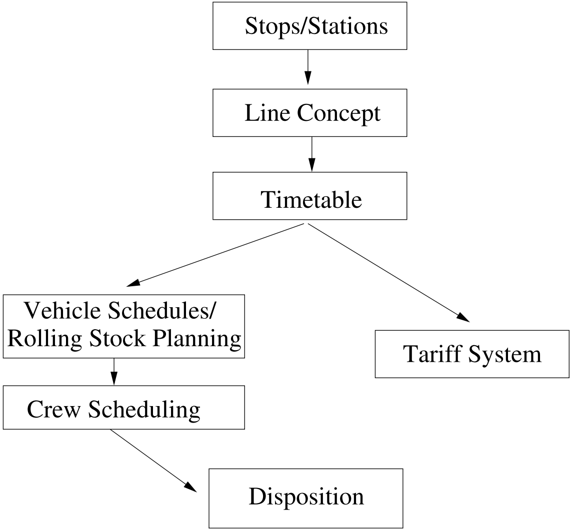
Other algorithms for PESP

- Integer programming, in particular cycle-based formulation e.g. Borndörfer et al (2016)
- SAT solving Großmann et al. (2012), Gattermann et al (2016)
- Modulo-Simplex Nachtigall and Opitz (2008), Goerigk and Schöbel (2013)
- Matching-approach Pätzold and Schöbel (2016)

Equity?

- Having good waiting times at all transfers?
- Minimizing the largest transfer time?
- Minimizing the maximum difference between the real traveling time and the lower bound traveling time over all journeys?

How to plan the vehicle schedule?



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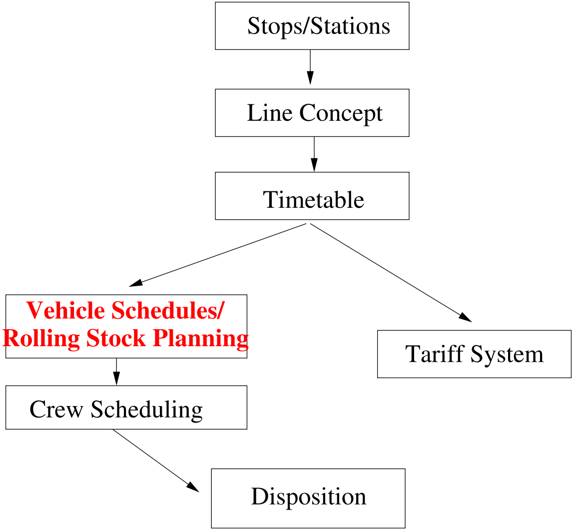

Vehicle scheduling

We now have given:

- A line plan,
- a timetable.

Question: How many vehicles are needed to operate this line plan with its corresponding timetable? How many kilometers need to be driven by these vehicles?
$\Longrightarrow$ Goal is to minimize the costs.

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Note: An easy solution is a line-based vehicle schedule. Every vehicle stays on the same line the whole day.

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Clear: A line from its start station to its end station has to be operated by the same vehicle.

Note: An easy solution is a line-based vehicle schedule. Every vehicle stays on the same line the whole day. But this need not be the best solution!

Trips

Notation

A **trip**

$$t = (l_t, v_t^{start}, v_t^{end}, \tilde{\pi}_t^{start}, \tilde{\pi}_t^{end})$$

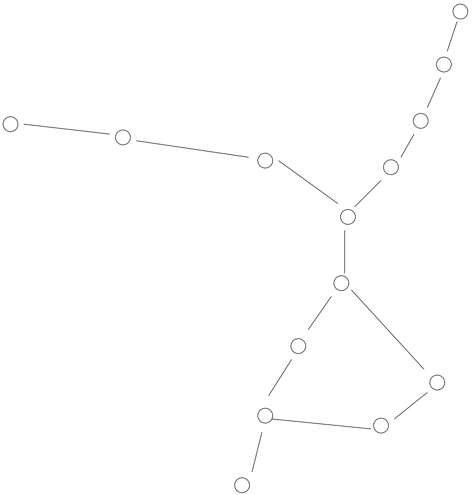
is specified by

- a line l_t,
- the first and last station v_t^{start} and v_t^{end} of line l_t,
- the corresponding start time $\tilde{\pi}_t^{start}$ and end time $\tilde{\pi}_t^{end}$ of the trip.

The set of all trips for a line plan/timetable is denoted as $\mathcal{T}$.

Remark: A line with frequency f_l leads to $2f_l$ trips, namely f_l forward and f_l backward trips per period.

Trips: Example



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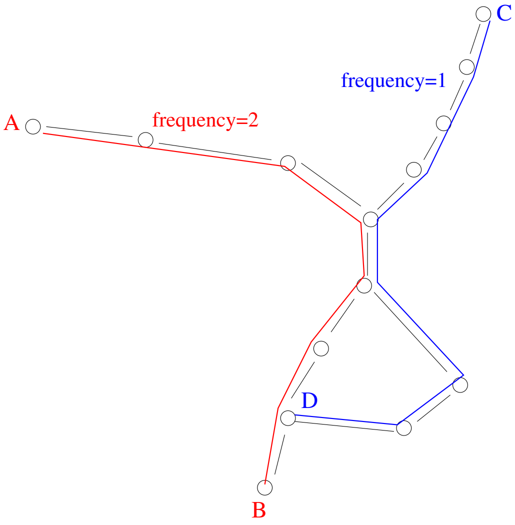

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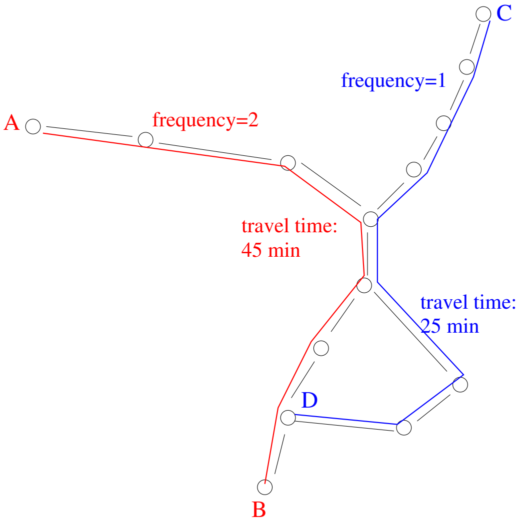

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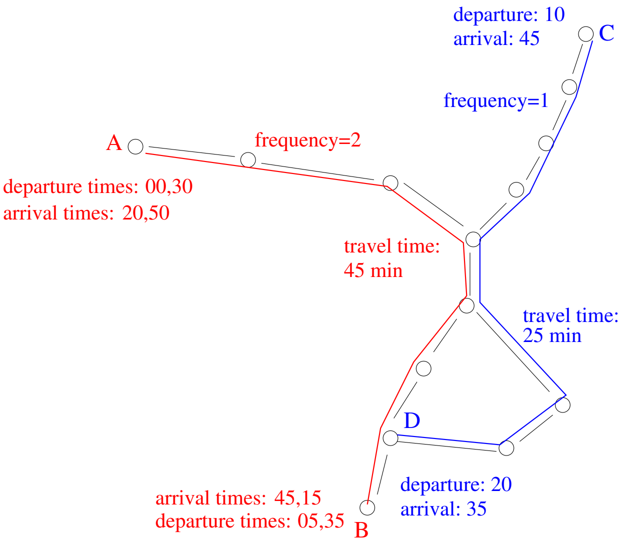
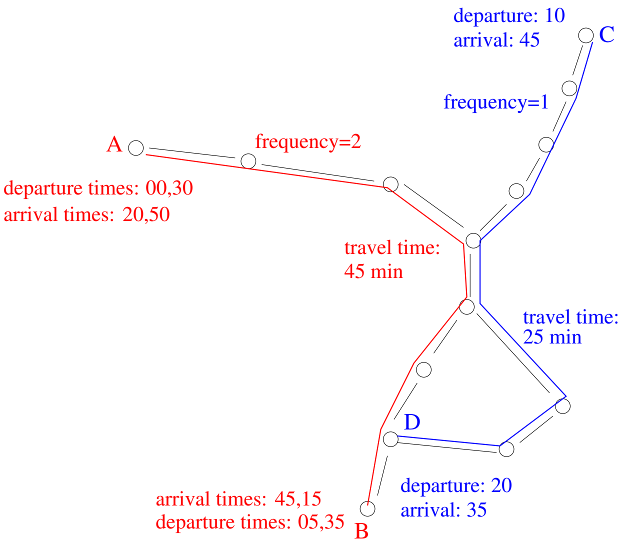


Trips: Example



Trips starting between 8:00 and 9:00

$(l^1, A, B, 8{:}00, 8{:}45)$

$(l^1, A, B, 8{:}30, 9{:}15)$

$(l^2, B, A, 8{:}05, 8{:}50)$

$(l^2, B, A, 8{:}35, 9{:}20)$

$(l^1, C, D, 8{:}10, 8{:}35)$

$(l^2, D, C, 8{:}20, 8{:}45)$

Compatibility of trips

Question: When can a vehicle operate two trips consecutively?

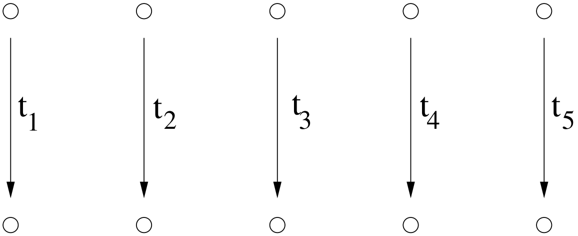
For $u, v \in V$ let $time(u, v)$ denote the time if a vehicle drives directly between u and v.

Notation

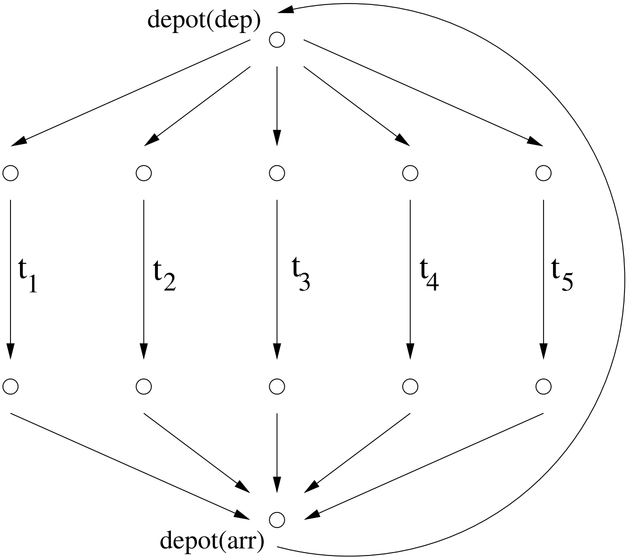
Two trips t_1 and t_2 are called **compatible** if

$$\tilde{\pi}_{t_2}^{start} - \tilde{\pi}_{t_1}^{end} \geq \text{time}(v_{t_1}^{end}, v_{t_2}^{start})$$

The vehicle scheduling problem as circulation problem



The vehicle scheduling problem as circulation problem



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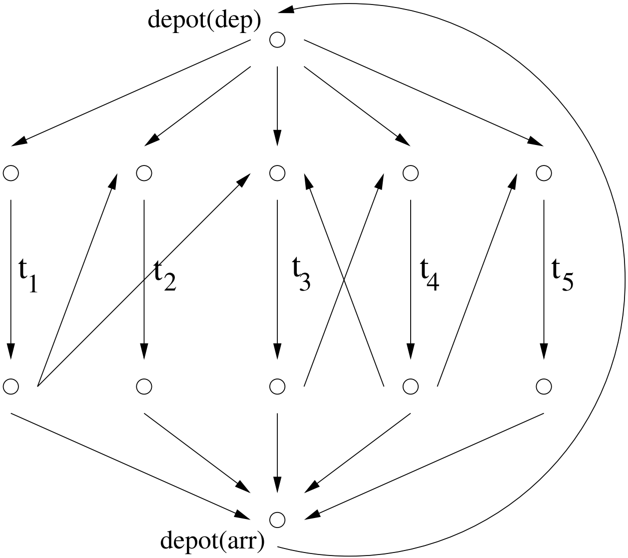

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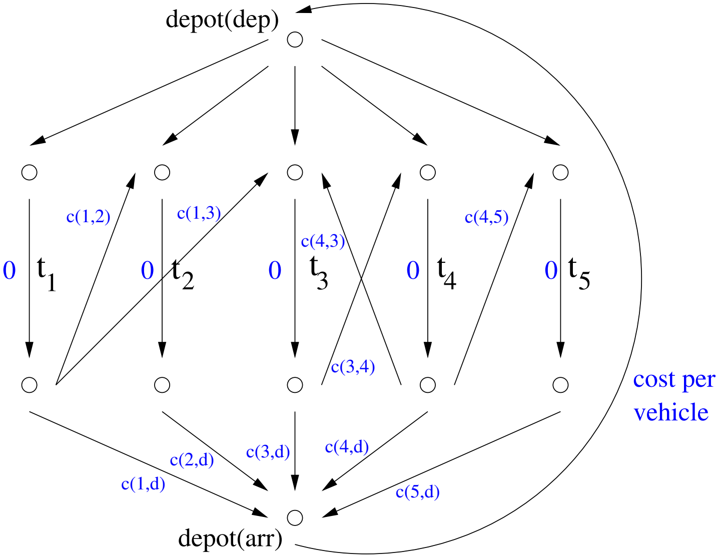
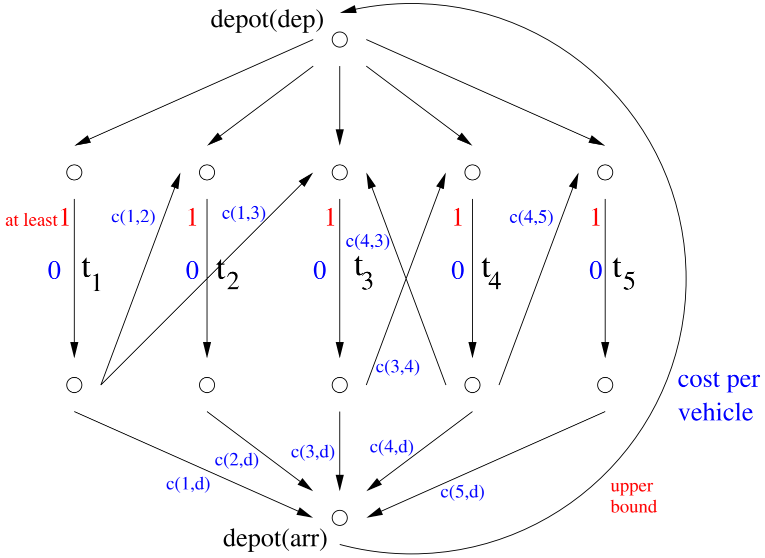


The vehicle scheduling problem as circulation problem



The vehicle scheduling problem as circulation problem

Theorem

A flow in the network represents a feasible vehicle schedule with the correct costs.

Vehicle scheduling problem as integer program

$$\min \sum_{e \in E} cost_e x_e$$

such that

$$Ax = b$$

$$l_e \leq x_e \leq u_e \text{ for all } e \in E$$

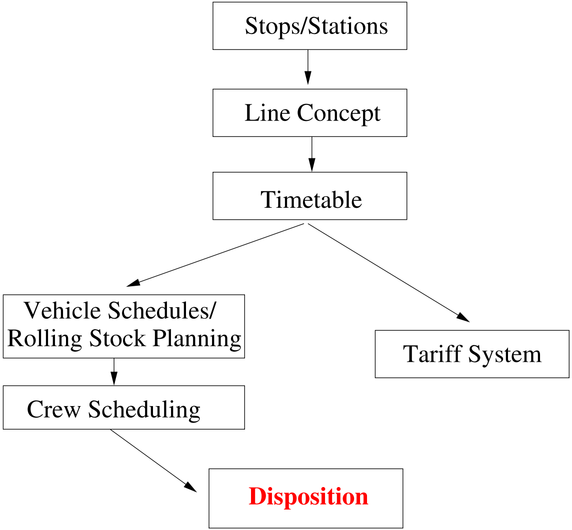
Theorem

Vehicle scheduling with one depot is solvable in polynomial time.
Vehicle scheduling with several depots is NP-hard.

Equity?

- Probably not in vehicle scheduling, since passengers do not even see the vehicle schedules.
- Crew scheduling is important for drivers $\rightarrow$ tackled in rostering problems.

Delay management



Delay management

Problem: In case of delayed trains (or buses), at least two decisions have to be made:

- wait for transferring passengers or depart on time?
- which train can go first if two trains would like to use the same track?

Delay management

Variables?

$$x_i = \text{ disposition timetable for event } i$$

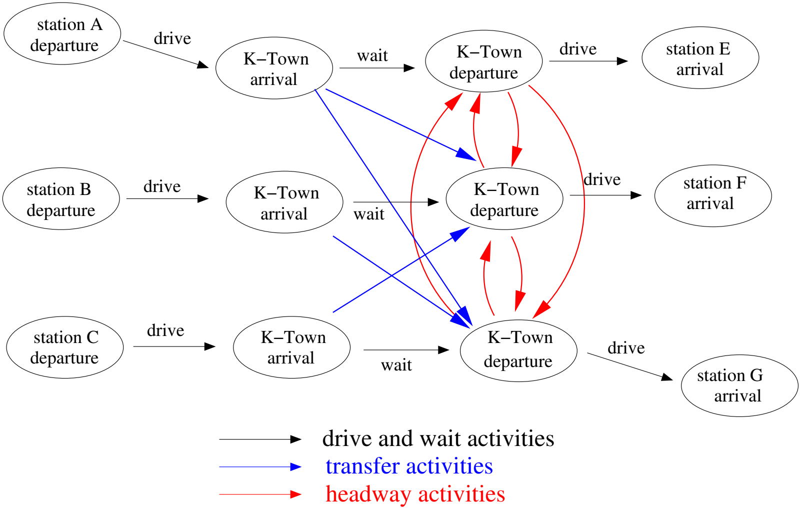
$$z_a = \begin{cases} 1 & \text{if transfer activity } a \text{ is maintained} \\ 0 & \text{otherwise} \end{cases} \qquad \eta_{ij} = \begin{cases} 1 & \text{if train } i \text{ goes before train } j \\ 0 & \text{otherwise} \end{cases}$$

Objective function: minimize sum of delays over all passengers

Constraints: Delays need to be propagated correctly:
For all $a = (i, j)$: If i is delayed, then also j should is delayed (if buffer time is not sufficient).

Modeling delays

Use event-activity network $(\mathcal{E}, \mathcal{A})$:



Delay management as integer program

$$\min \sum_{i \in \mathcal{E}} w_i x_i \quad + \sum_{a=(i,j) \in \mathcal{A}_{transfer}} w_a z_a T$$

$$\begin{array}{lrcll}
\text{s.t.} & x_i & \geq & \pi_i + d_i & \text{for all } i \\
 & x_j - x_i & \geq & L_a & \text{for all } a = (i,j) \in \mathcal{A}_{wait} \cup \mathcal{A}_{drive} \\
 & Mz_a + x_j - x_i & \geq & L_a & \text{for all } a = (i,j) \in \mathcal{A}_{transfer} \\
 & x_i \in \mathbb{N}^0 & , & z_a \in \{0,1\} &
\end{array}$$

Literature: Schöbel (2006), Rückert et al (2016), Dollevoet et al (2018), Grafe (2020)

Analysis

Theorem

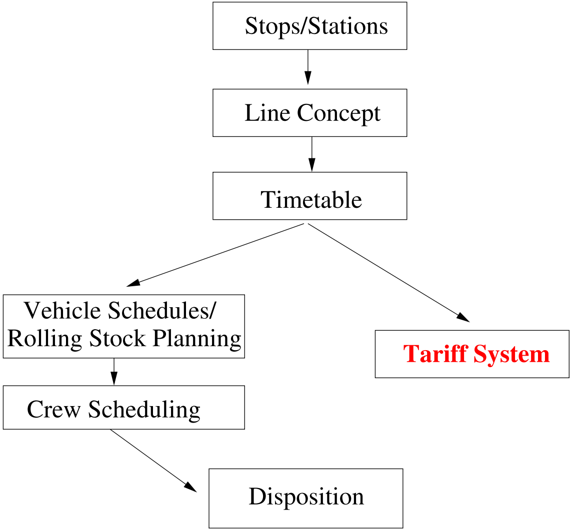
(DM) is NP-hard, even in special cases and even the approximation given here.

Can the spreading of delays be avoided by robust timetables? $\rightarrow$ ongoing topic of research!

Equity?

- I would not advise to minimize the maximum delay because this lets other passengers suffer.
- Delays are a kind of "emergency" situation. Respecting each single passenger might bring the whole system out of control.
- Instead: Passengers can claim money back or get overnight stays or taxis paid.
- Maybe try to avoid the spreading of delays in the same way on any line/region.

Tariff planning



Tariff planning

Problem: Find the prices for the passengers!

Which tariff structure?

- A tariff structure is a flat tariff if all journeys cost the same.
- A tariff system is a distance tariff if the price for journey J is determined by its length. Often distance tariffs are affine linear: $\text{Price}(J) = \alpha\text{Length}(J) + \beta$.
- A tariff system is a zone tariff, if the price for a journey is determined by the number of zones it passes. Often the price for k is defined by a list.

We would expect a tariff structure to satisfy

- the no-stopover property
- the no-elongation property

Tariff structure design problem

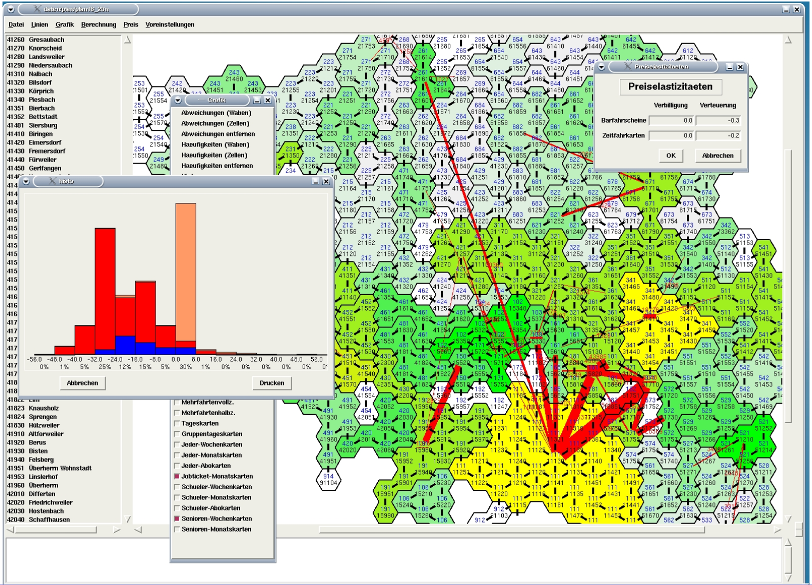
Given a tariff strategy (e.g., zone tariff, distance tariff, flat tariff) and reference prices for every journey, find a tariff structure which is as close as possible to the reference prices.

Applications:

- The reference prices are fair and should be realized with a tariff structure.
- Transition of tariff systems (The old prices are then the reference prices.)

Modeling the tariff structure design problem: Hamacher and Schöbel (2004), Urban and Schöbel (2021)

Application



Analysis

Given reference prices, find a new tariff structure which minimizes the maximum absolute deviation or the average absolute deviation to the reference prices.

Theorem

- Designing a flat tariff or an affine linear distance tariff is polynomially solvable.
- Finding zone prices when the zones are given is polynomially solvable.
- The zone tariff design problem is NP-hard.

Equity?

Tariffs are considered as <u>the</u> application for equity.

Directly in the tariff structure design problem:

- Reference prices should be fair!
- When finding a tariff structure, it should minimize the deviations to the reference prices.

Thoughts outside of these problems:

- Choosing the tariff structure is important: In a flat tariff passengers who have short journeys pay for the ones with long journeys. One has to observe which type of trips passengers with low income usually take.
- The tariff system should depend on the passengers (which is usually the case: school kids have other tariffs when adults, there exist social passes, passes for refugees, ...)
- New development in Germany: a flat tariff for <u>all</u> buses and regional trains all over Germany now available.

Content of the talk

1. Stages in Public Transport Planning
   - Stop location
   - Line Planning
   - Timetabling
   - Vehicle scheduling
   - Delay management
   - Tariff Planning
2. Some more remarks on equity

Equity

We already have seen some ideas on equity for every planning stage.

- Many equity objectives may be included as objective function or as constraints in the presented models.
- Easy thing, no new modeling necessary: Concentrate on the part of demand that cannot use cars.
- Other aspects are (from a mathematical part) more challenging and can raise new research topics.

But is equity on a local level what we want?

A global perspective

I am currently leading the following project about urban mobility.

Given is a city and a demand matrix: from where to where wish people to travel?

Task: Determine the best transport modes to be installed in the city.

where we consider:

- traveling times for the passengers
- the budget
- and CO_2 emissions and energy

A global perspective

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Given is a city and a demand matrix: from where to where wish people to travel?

Task: Determine the best transport modes to be installed in the city.

where we consider:

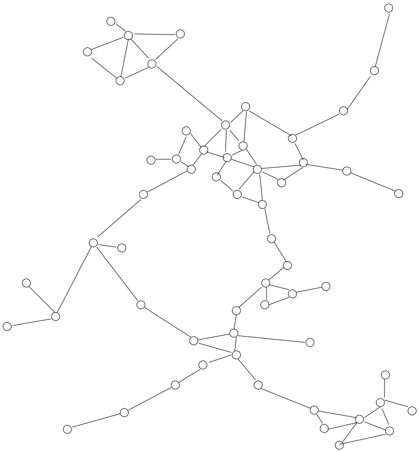
- traveling times for the passengers
- the budget
- and CO_2 emissions and energy

Questions:

- Do we need a metro?
- Where in the city should we offer regular bus transportation? With which lines?
- Where is shared mobility (ride pooling) more appropriate? with how many vehicles?
- To whom do we recommend to use the private car?

Illustration

Given: City und demand. Goal: transport modes



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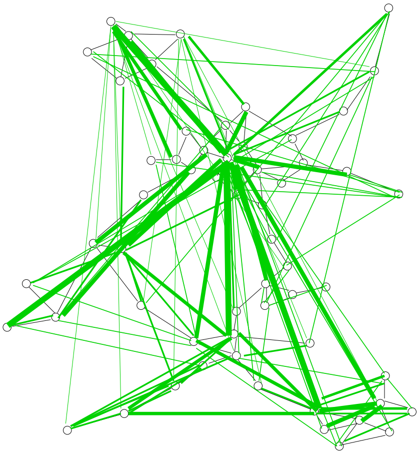

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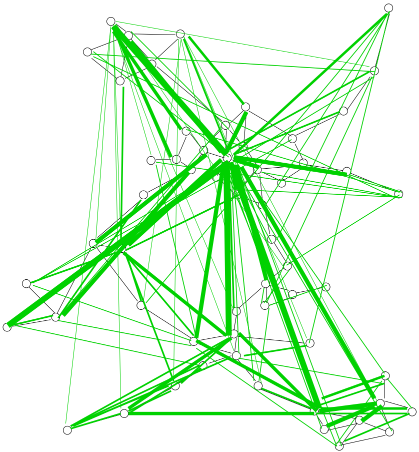
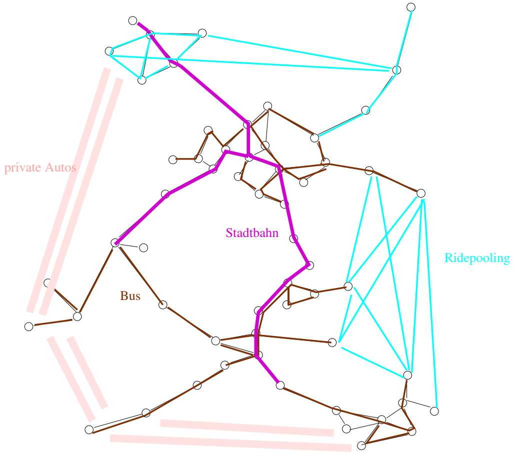




Particularities

Multimodal planning

- Metro
- Bus
- Shared mobility (ride pooling)
- Car

Multicriteria evaluation

- Travel times
- budget needed
- CO_2 emissions, energy

Equilibrium-constraints

- to model the behavior of the passengers
- compute their traveling times including traffic jams

Not easy :-(

We use a decomposition approach into a surrogate model and detailed models.

for planning a metro

for planning buses

for planning shared mobility

Our first results

The surrogate model does a routing and recommends modes of transport due to the number of people who want to use a specific edge.
The detailed models plan accordingly.

Then we compute the equilibrium. Result:

Our first results

The surrogate model does a routing and recommends modes of transport due to the number of people who want to use a specific edge.
The detailed models plan accordingly.

Then we compute the equilibrium. Result: **Everybody (who can) takes the car!**

Why? Because the budget is not enough to be competitive.

Question: Wouldn't it be better for all of us to have some really good lines that people love to take such that they leave space on the streets instead of treating all links equally (bad)?

But is this equity?

Optimize public transport

Maybe we need a shift of goals:

So far:

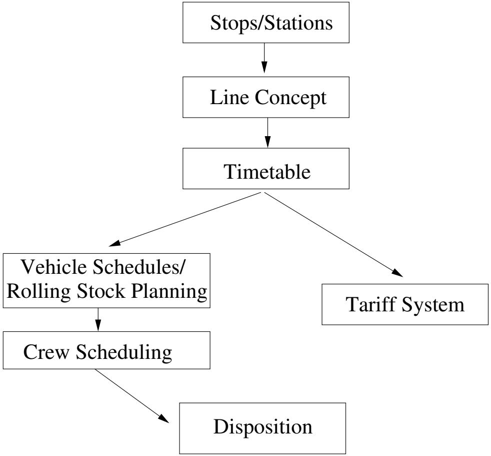
Under a restricted budget make public transport as **good** as possible for the given demand.

maybe better?

Under a restricted budget ensure some sufficient public transport for everybody but make only a small part as **good** as possible.

and keep equity goals in both approaches in mind!

Thank you!


Stops/Stations
Line Concept
Timetable
Vehicle Schedules/
Rolling Stock Planning
Tariff System
Crew Scheduling
Disposition