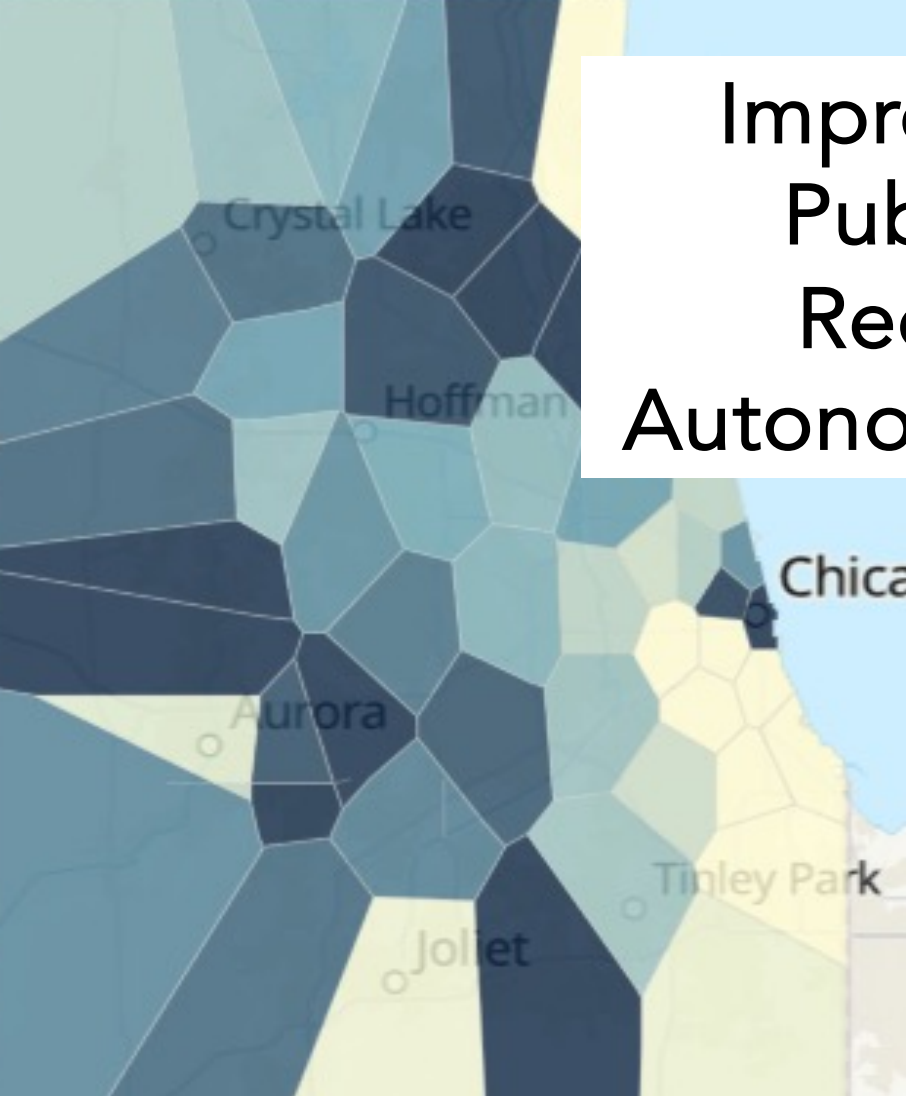
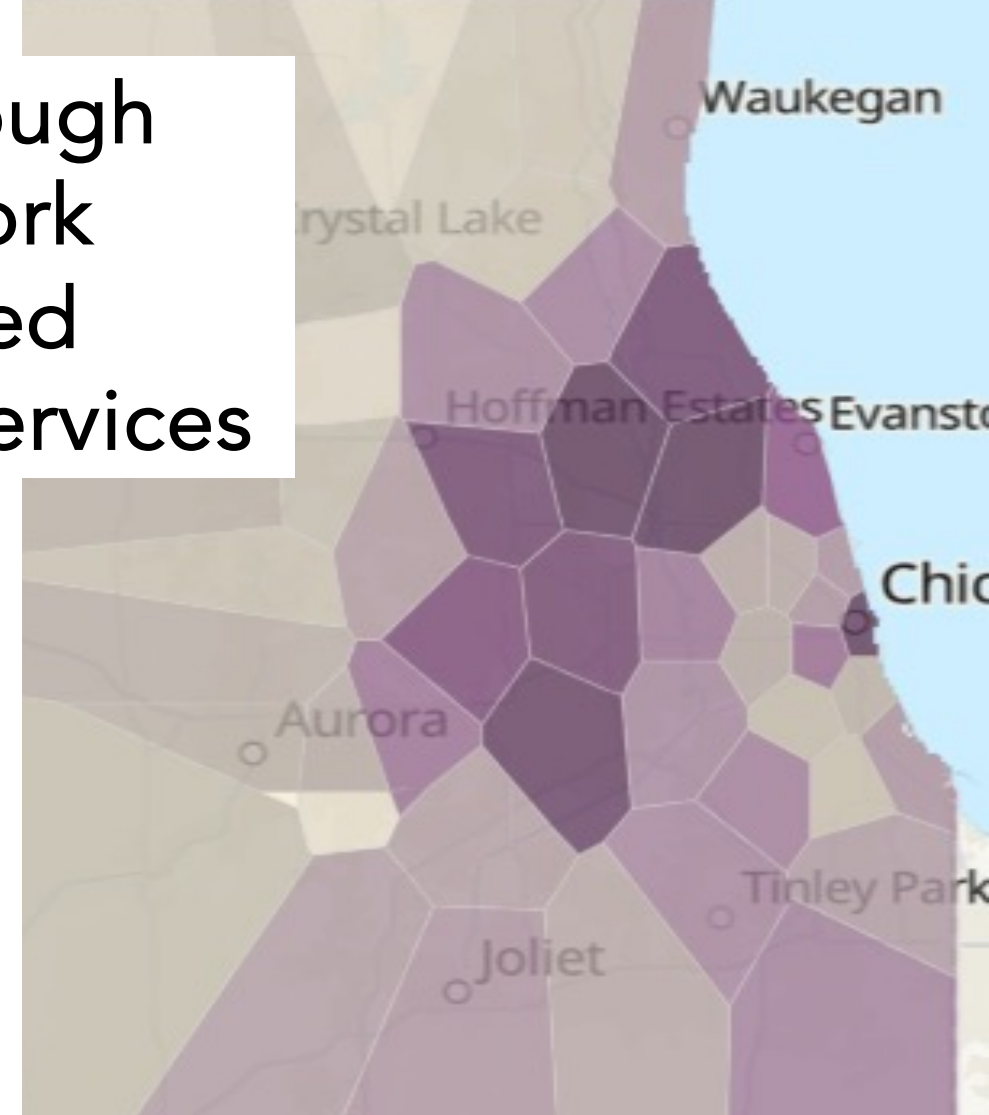


# Improving Equity through Public Transit Network Redesign with Shared Autonomous Mobility Services



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Max Ng

Northwestern University  
Transportation Center



Mathematical Foundations for Equity in Transportation Systems  
IPAM, Los Angeles, CA  
January 22, 2024

# Equity concepts in transportation investment

For given system, we examine level of service offered in terms of several metrics.

Impacts  $C = (C_1, C_2, C_3 \dots)$  distributed across residents—grouped spatially, by income group, ethnicity, etc...

For existing system— we may have given distribution  $F_0(C)$ ; Lorenz curve is a good concept to capture that.

Invest Sum B

After investment, we may examine  $F_1(C)$  – could reflect improved situation, "better" distribution.

# Equity concepts in transportation investment

Does investment improve equity when

1. It brings previously lagging entities to the previous mean (or benchmark) level?
2. The new relative impact (say improvement per \$ invested) is somehow equal?
3. The new distribution is itself more equitable (using similar metric as base case)? – and would that mean that the investment impact is inequitable?

# Transit Equity through Technology and New Mobility Services

Here, we ask:

1. How do we capture and quantify equity in the context of urban transport mobility? What metrics can planners use to characterize existing systems and evaluate proposed design configurations?
2. Can we make transit systems more equitable by integrating new on-demand shared mobility services (with autonomous vehicles) in future redesigned transit systems?

We examine these questions through an application to the Greater Chicago area.

# Transit Equity in Chicago

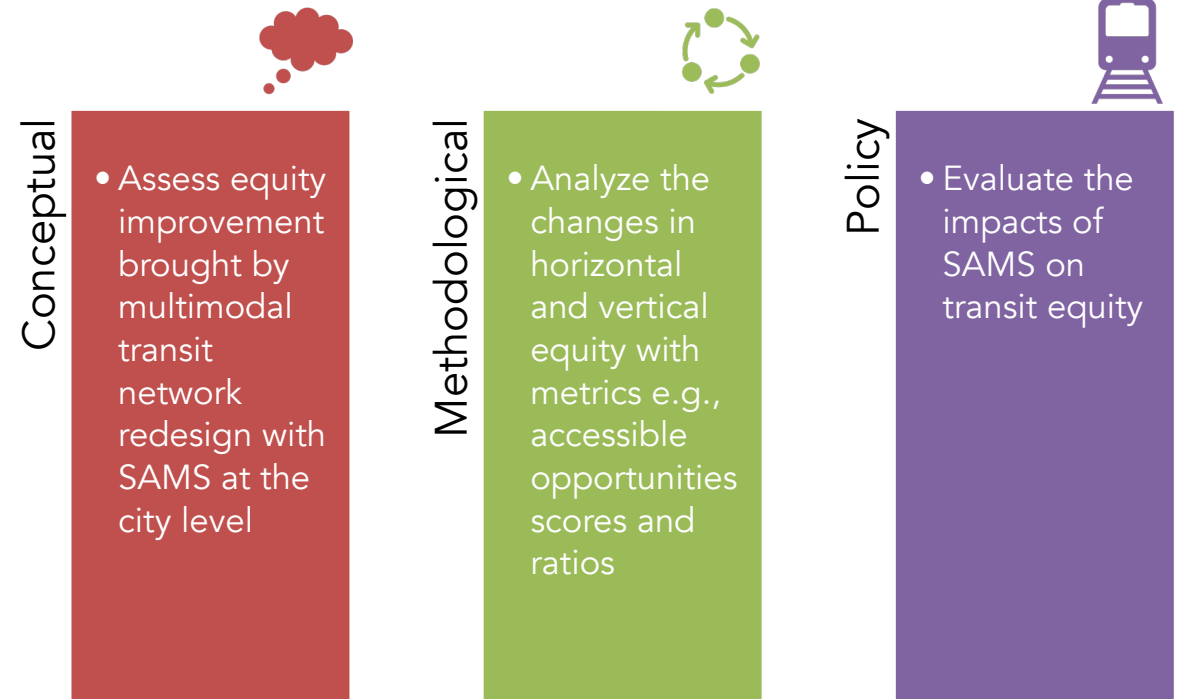
## Research question

Can redesigning transit networks with SAMS, in addition to enhancing overall transit system performance (mobility), improve transit equity?

## Method

Comparing travel times and accessible opportunities of various zones under modeled scenarios.

## Contributions



# Measuring Equity

1. Transit travel time vs. Driving time
  - Weighted by prevailing demand pattern
2. Access to opportunities
  - Within time range (e.g., 45min or 60min)
  - Gravity model (Intervening Opportunities)

## Representation

1. Magnitude difference
2. Transit-to-driving ratio
3. Lorenz curve & Gini's index

*Each measure can be evaluated for both horizontal and vertical equity*

### Spatial equity

*(aka horizontal equity)*

- distribution of impacts between individuals and groups considered equal in ability and need

### Vertical equity

- distribution of impacts between individuals and groups that differ (typically in income)

# Measuring Equity- Travel Time

Travel Time by Zone

$$TT_{s,i} = \frac{\sum_j D_{ij} T_{s,ij}}{\sum_j D_{ij}}$$

Travel Time Ratio (transit to auto)

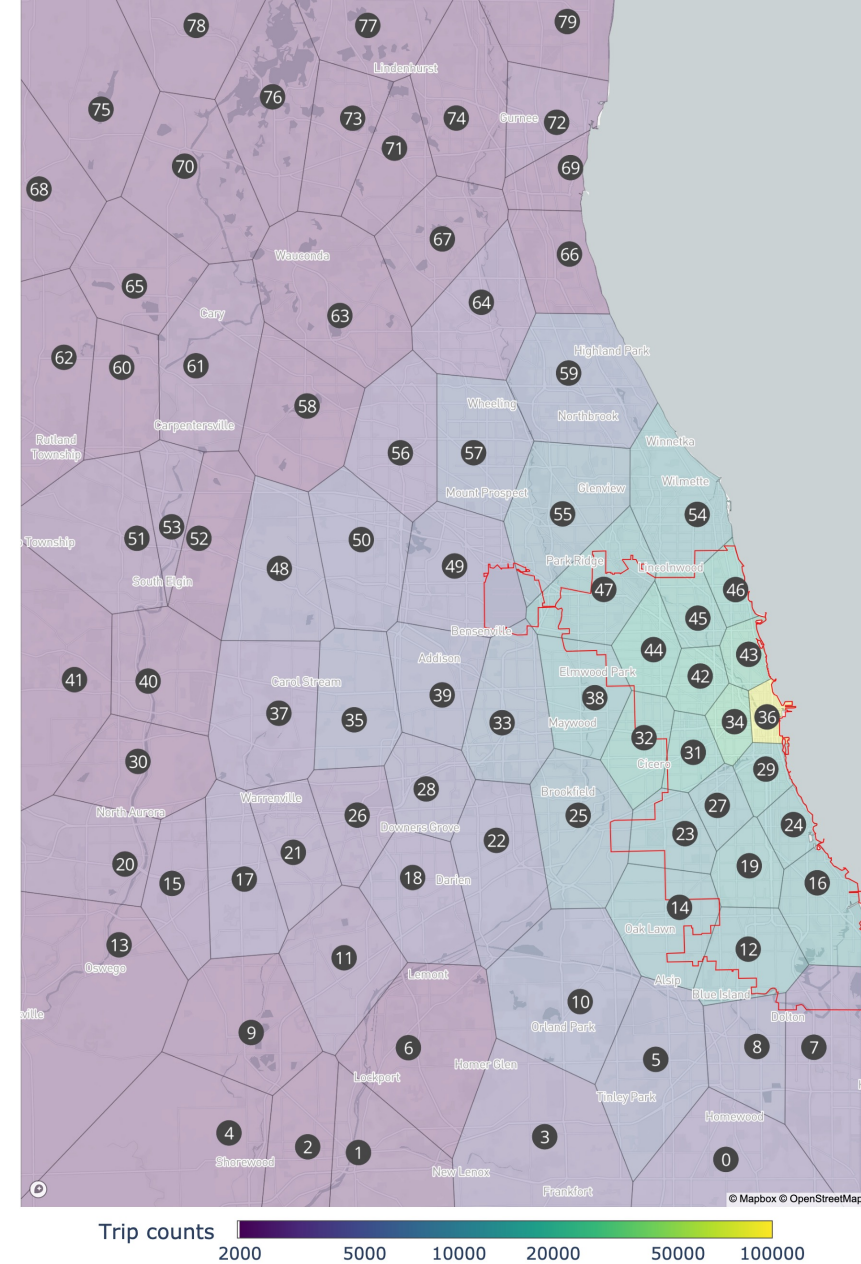
$$TTR_{s,i} = \frac{TT_{s,i}}{TT_{4,i}}$$

where

$T_{s,ij}$  is the travel time from zone  $i$  to destination  $j$  in scenario  $s$

$D_{ij}$  represents the demand from zone  $i$  to  $j$

*Note: Scenario  $s=4$  is auto driving*



# Measuring Equity- Accessible Opportunities

Accessible Opportunities Score

$$AOS_{s,i}^t = \sum_m \frac{\sum_{j|T_{s,ij} \leq t} O_{s,j}^m}{\sum_k O_{s,k}^m}$$

Accessible Opportunities Ratio (relative to auto)

$$AOR_{s,i}^t = \frac{AOS_{s,i}^t}{AOS_{4,i}^t}$$

where

$O_{s,j}^m$  is the opportunities of type  $m$  available at zone  $j$  in scenario  $s$

$t$  denotes the travel time threshold

$\sum_k O_{s,k}^m$  is the opportunities available in the region

Accessible Opportunities Score with Gravity Weighting

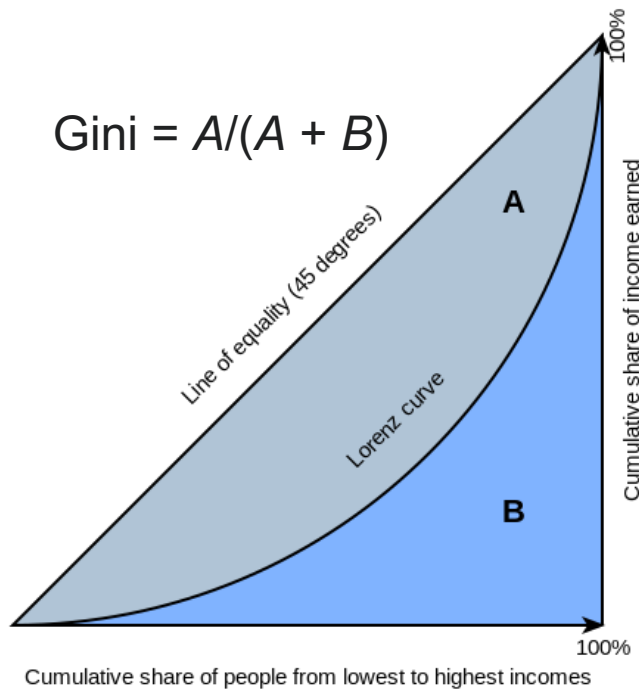
$$AOS_{s,i}^G = \sum_j \sum_m \frac{w_{s,ij} O_{s,j}^m}{\sum_k O_{s,k}^m}$$

where  $w_{s,ij} = \min(\frac{1}{T_{s,ij}^2}, 1)$  is a measure of travel impedance.

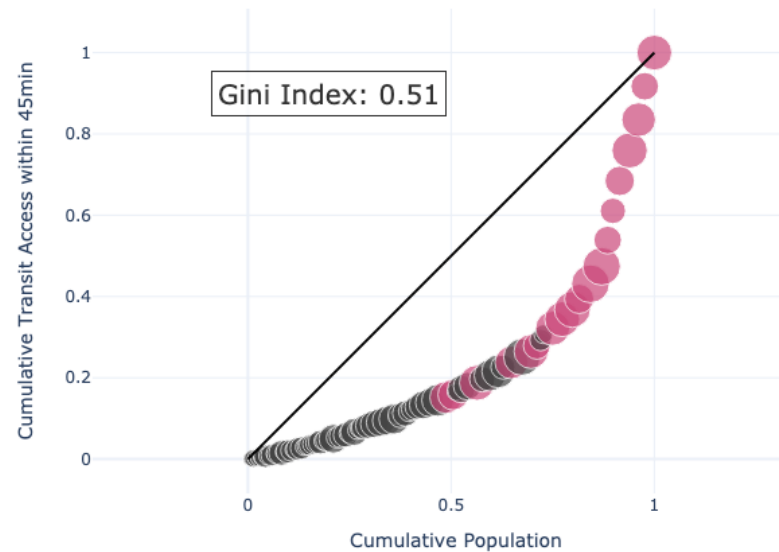


# Measuring Equity in Access to Opportunities

- Lorenz Curve and Gini's Index  
eg: Job access within 45 minutes of transit access

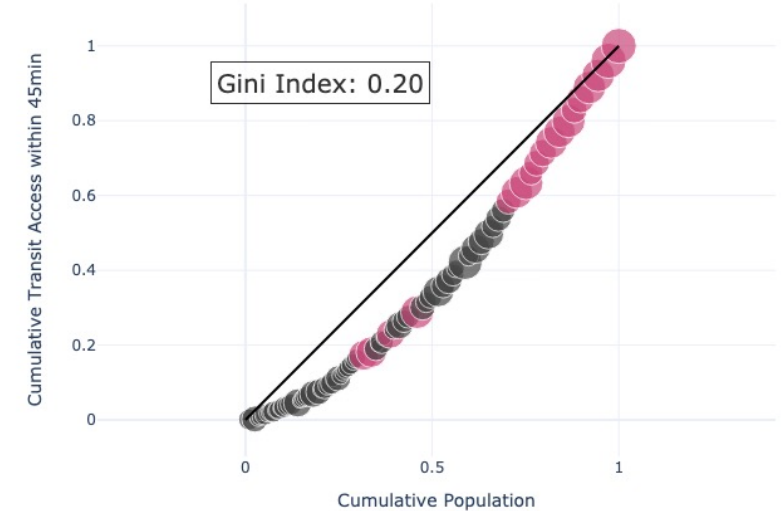


Lorenz Curve for Job Access within 45min of Existing Transit



Scenario 1

Lorenz Curve for Job Access within 45min of Redesigned Transit



Scenario 2

Delbosc, A., Currie, G., 2011. Using Lorenz curves to assess public transport equity. *Journal of Transport Geography*, Special section on Alternative Travel futures 19, 1252–1259. <https://doi.org/10.1016/j.jtrangeo.2011.02.008>

# Measuring Equity- Accessible Opportunities with Gini

Gini Index for Accessible Opportunities Score

$$GI_s^t = \frac{1}{2n^2 AOS_{s,i}^t} \sum_i \sum_j |AOS_{s,i}^t - AOS_{s,j}^t|$$

Gini Index for Accessible Opportunities Ratio

$$GI_s^t = \frac{1}{2n^2 AOR_{s,i}^t} \sum_i \sum_j |AOR_{s,i}^t - AOR_{s,j}^t|$$

# Transit Equity in Chicago

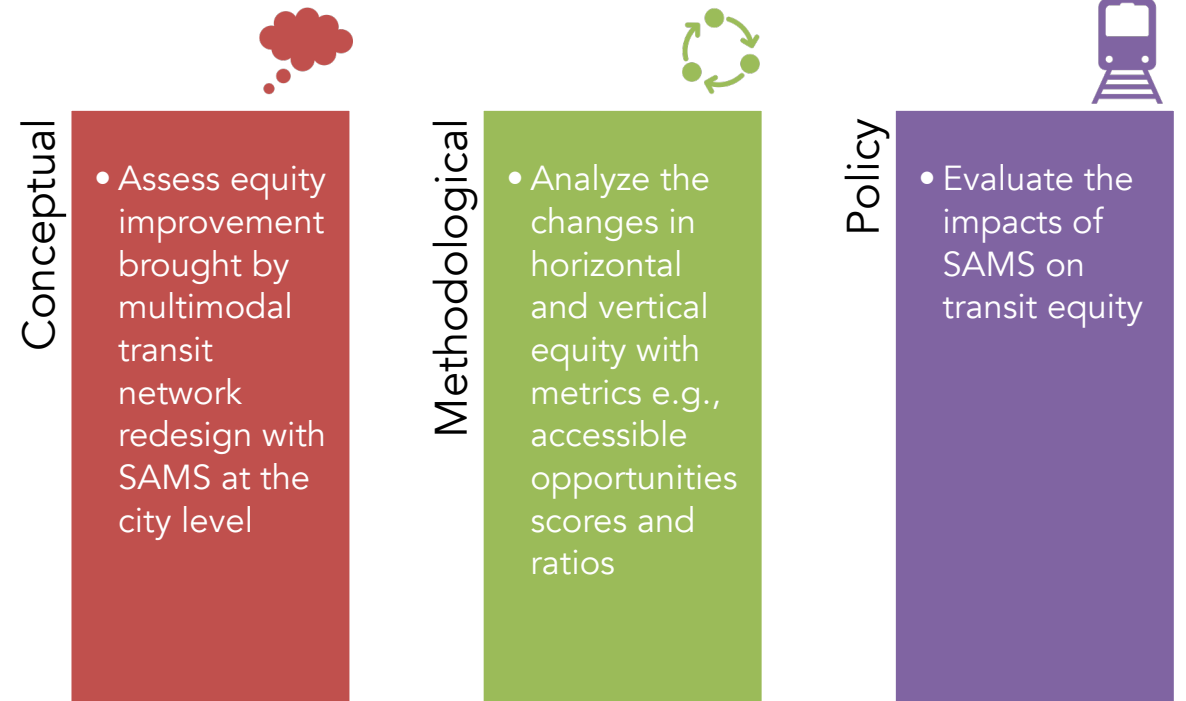
## Research question

Can redesigning transit networks with SAMS, in addition to enhancing overall transit system performance (mobility), improve transit equity?

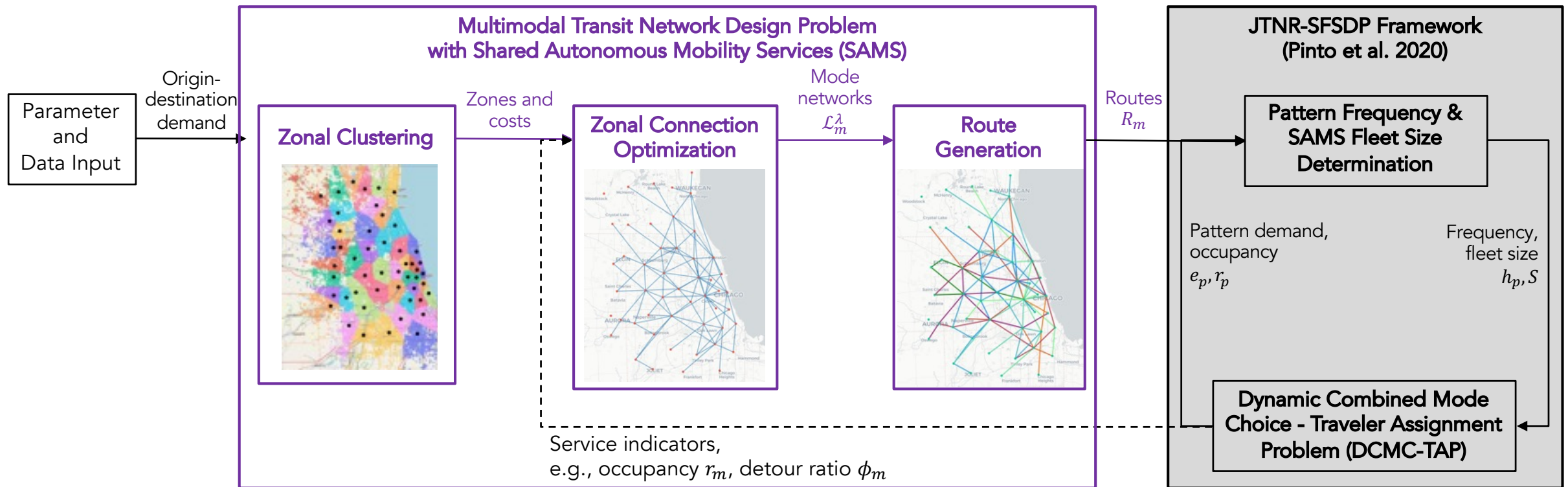
## Method

Comparing travel times and accessible opportunities of various zones under modeled scenarios.

## Contributions



# Solution Approach



# Clustering

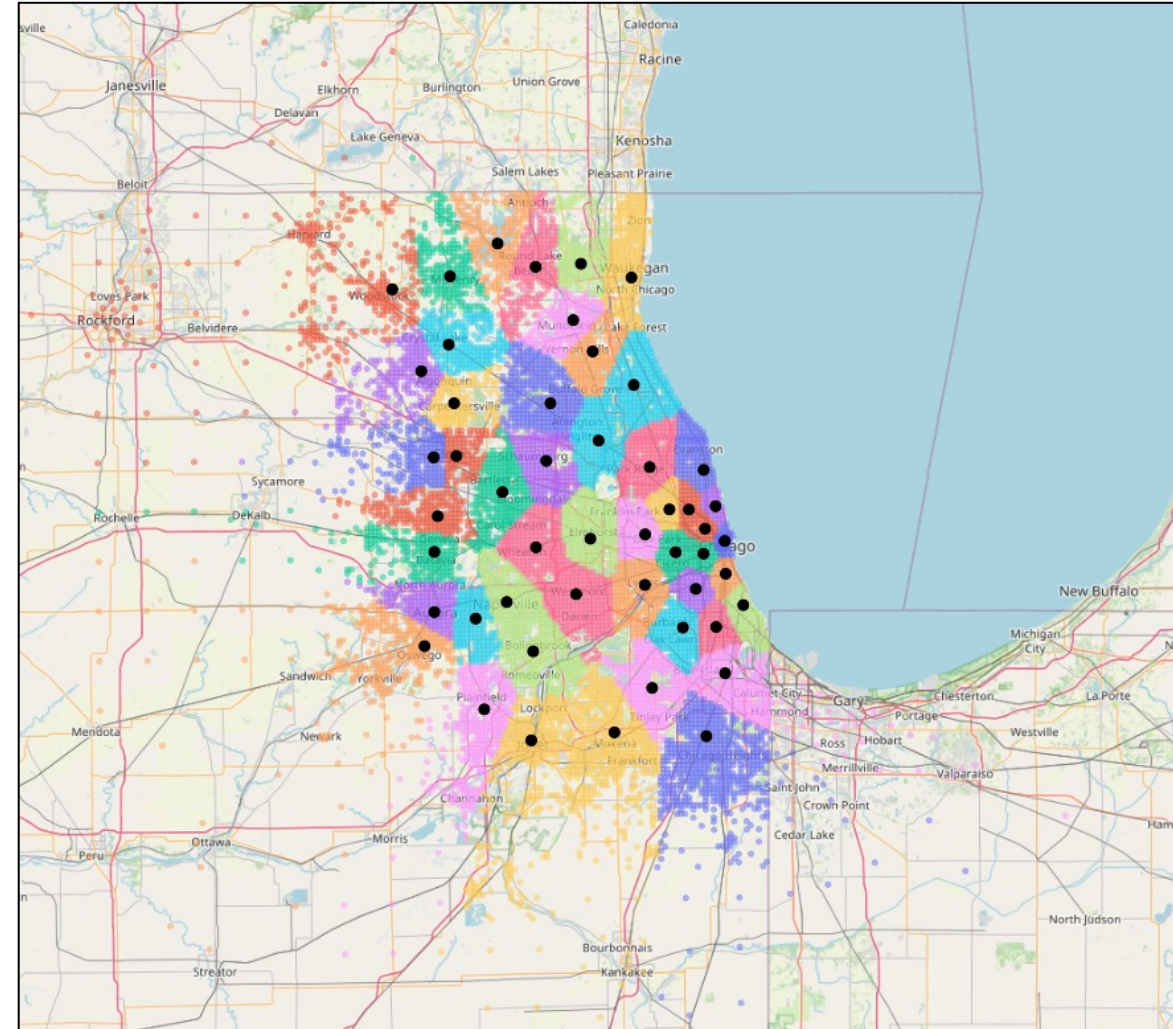
k-means clustering weighted on trips:  
Higher density areas with smaller clusters

## Network Connectivity

Limit possible zone connections based on:

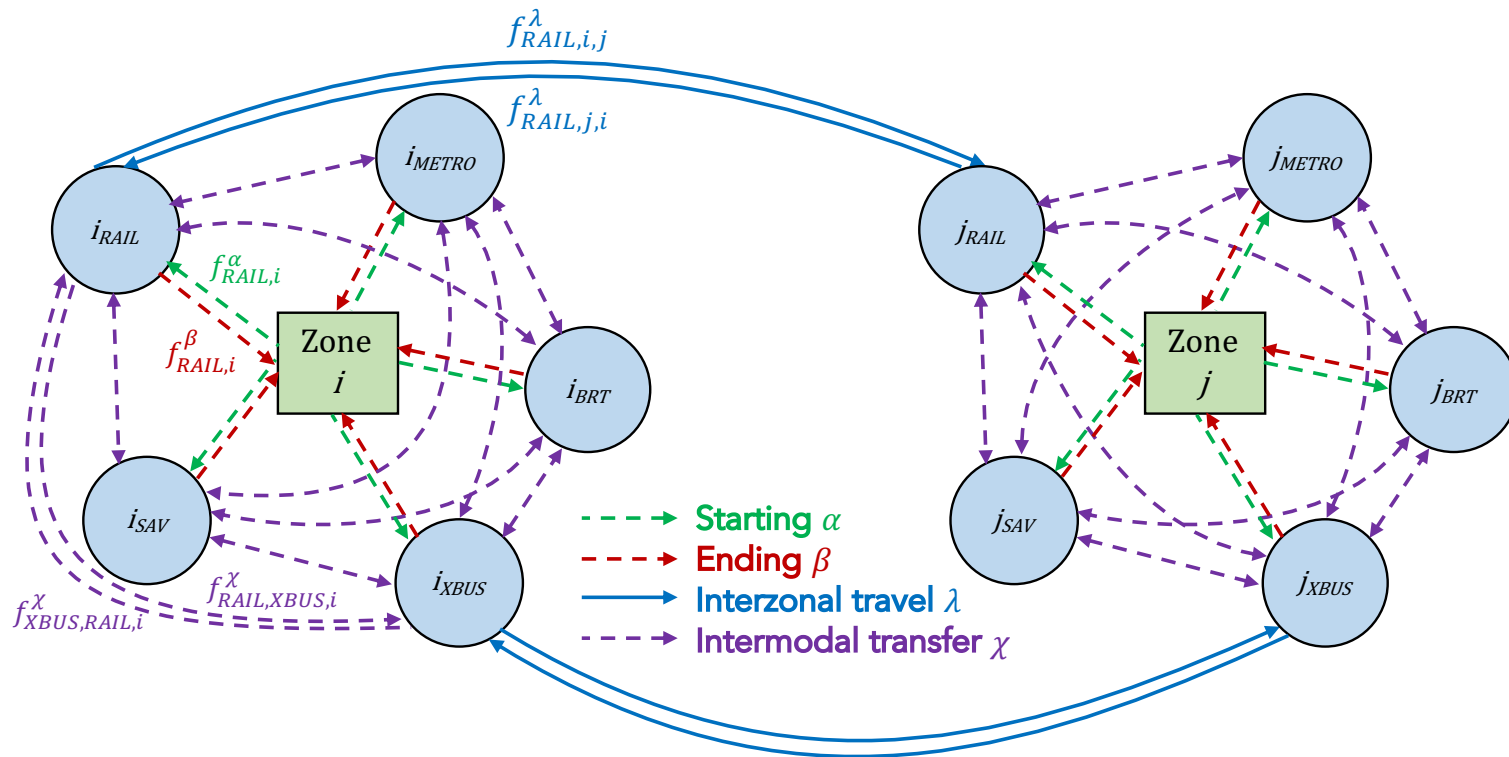
1. Proximity
2. Demand
3. Existing rail connection

Only interzonal trips considered



# Network Representation

## Inter-zonal Di-graph



Examples of two nodes (not all connections shown)

Continuous approximation to compute arc costs from geospatial, demand, and modal characteristics, covering door-to-door costs:

- Access
  - Waiting
  - Transfer
  - Riding
  - Operational
  - Emissions
- } User's (main line & feeders)  
 } Operator's (main line & feeders)

# Zonal Connection Optimization

## Multi-Commodity Network Flow (MCNF) model

- Zonal connectivity and infrastructure decisions determined by integer variables
- System-optimal flows assigned by continuous variables (labeled by origins to limit the number of variables)
- MILP: non-convex problem with linearized constraints
- Number of variables for 50 clusters, 6 modes: approx. 400,000 continuous + 10,000 integer

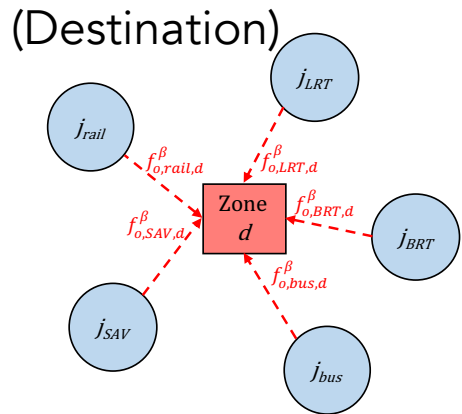
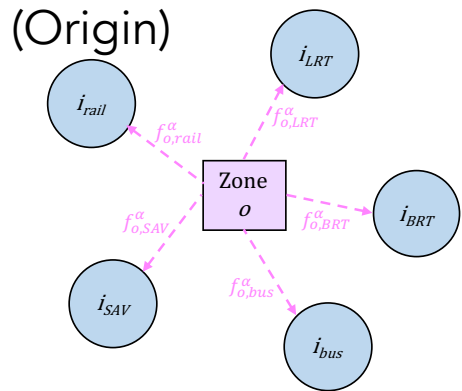
### Objective

$$\min_{\substack{f_{omij}^\lambda, f_{om}^\alpha, f_{omd}^\beta, f_{om_1m_2i}^\chi \\ x_{mij}^\lambda, x_{mij}^\omega}} \sum_{o \in Z} \sum_{m \in M} \left[ \begin{array}{l} \text{Travel costs:} \\ \gamma^{travel} \left( f_{om}^\alpha t_{mo}^\alpha + \sum_{(i,j) \in \mathcal{L}_m} f_{omij}^\lambda t_{mij}^\lambda + \sum_{m_1 \in M} \sum_{i \in Z} f_{omm_1i}^\chi t_{mm_1i}^\chi + \sum_{d \in Z} f_{omd}^\beta t_{md}^\beta \right) \\ + \sum_{(i,j) \in \mathcal{L}_m} \frac{f_{omij}^\lambda}{r_m^{design}} (\gamma_m^{op} t_{mij}^\lambda + \gamma_m^{em} d_{mij}^\lambda) + \left( f_{om}^\alpha + \sum_{i \in Z} f_{imo}^\beta \right) \frac{\eta_{mo}^f}{r_{mf}^{design}} (\gamma_{mf}^{op} t_{mo}^f + \gamma_{mf}^{em} d_{mo}^f) \end{array} \right] \quad (12)$$

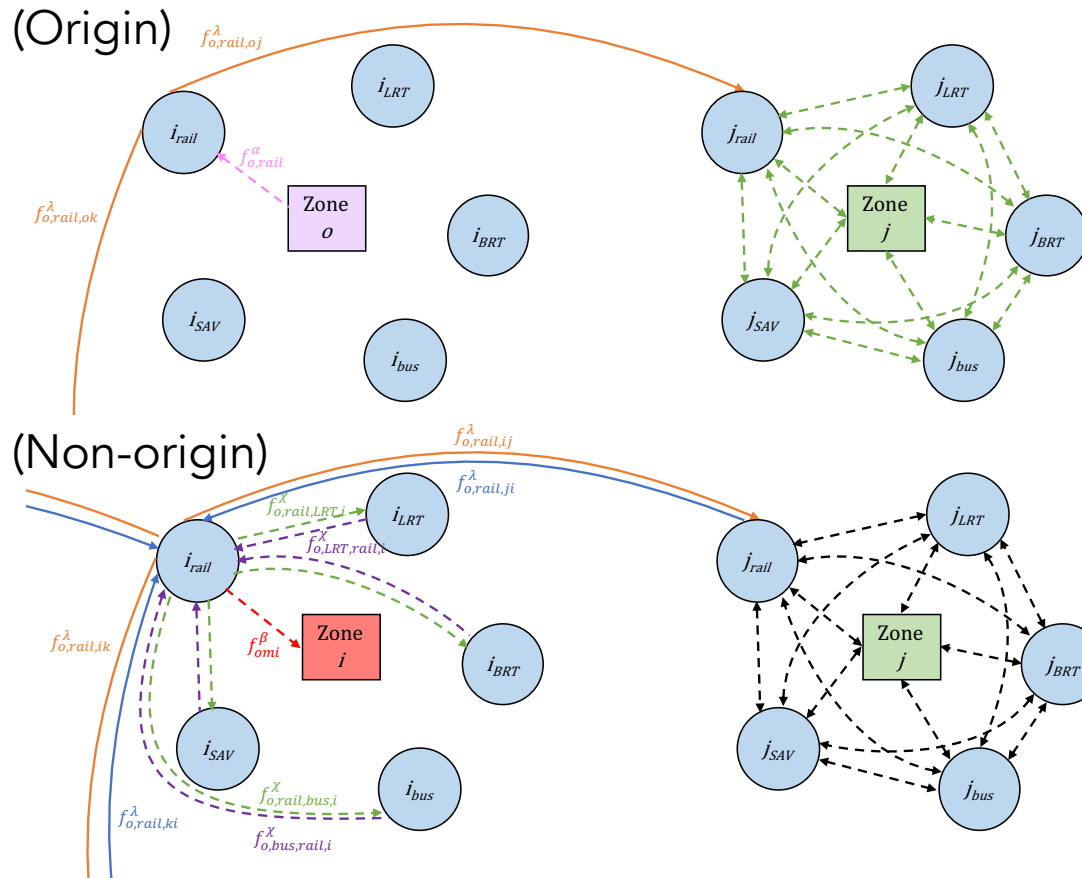
Main lines:
Operating Emissions
Feeders:
Operating Emissions

# Zonal Connection Optimization - Constraints

## Demand-Flow



## Flow Balance



## Connection and Infrastructure

- Infrastructure budget
- Transit capacity
- Infrastructure sufficiency
- Bi-directional link

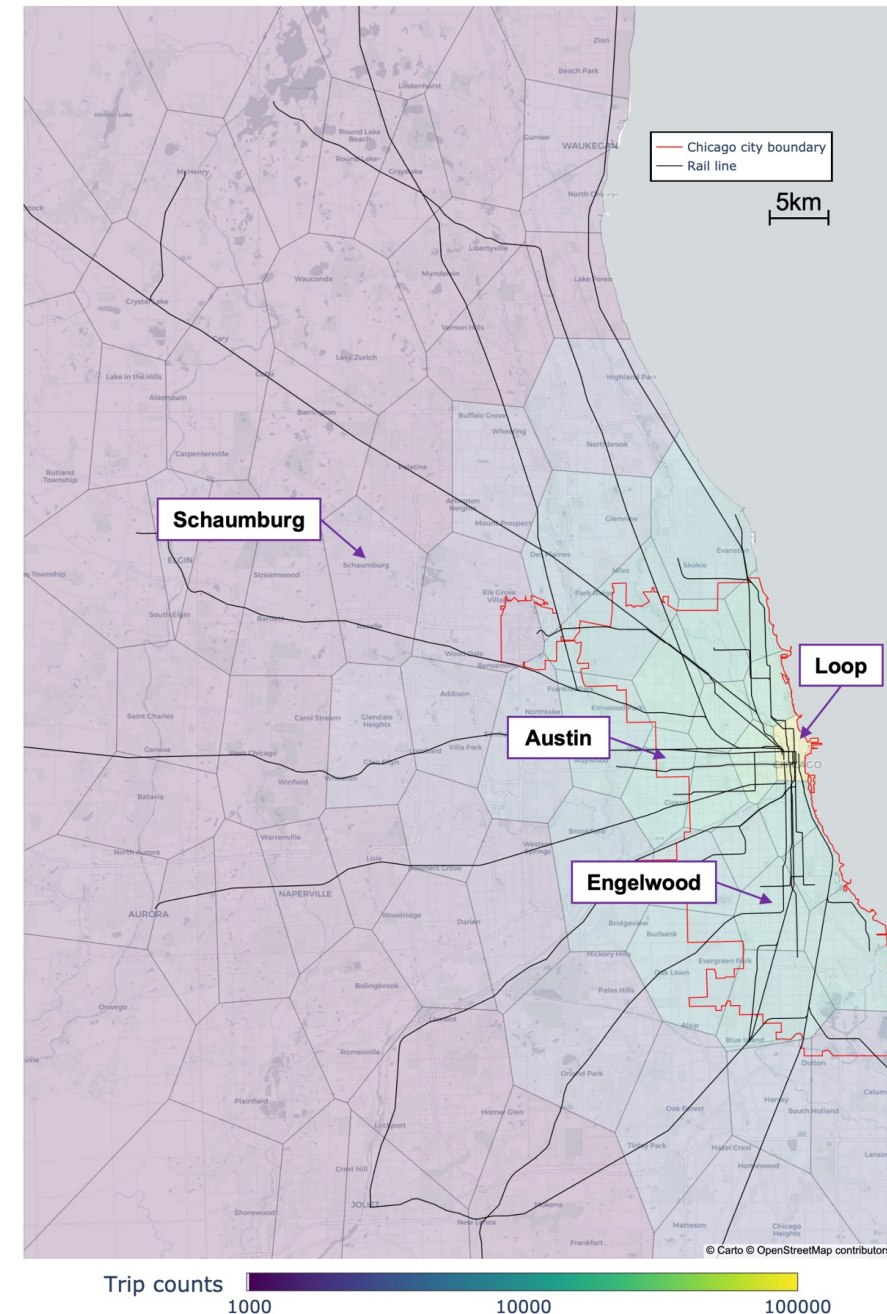
## Interzonal SAVs

- Link capacity
- Fleet size

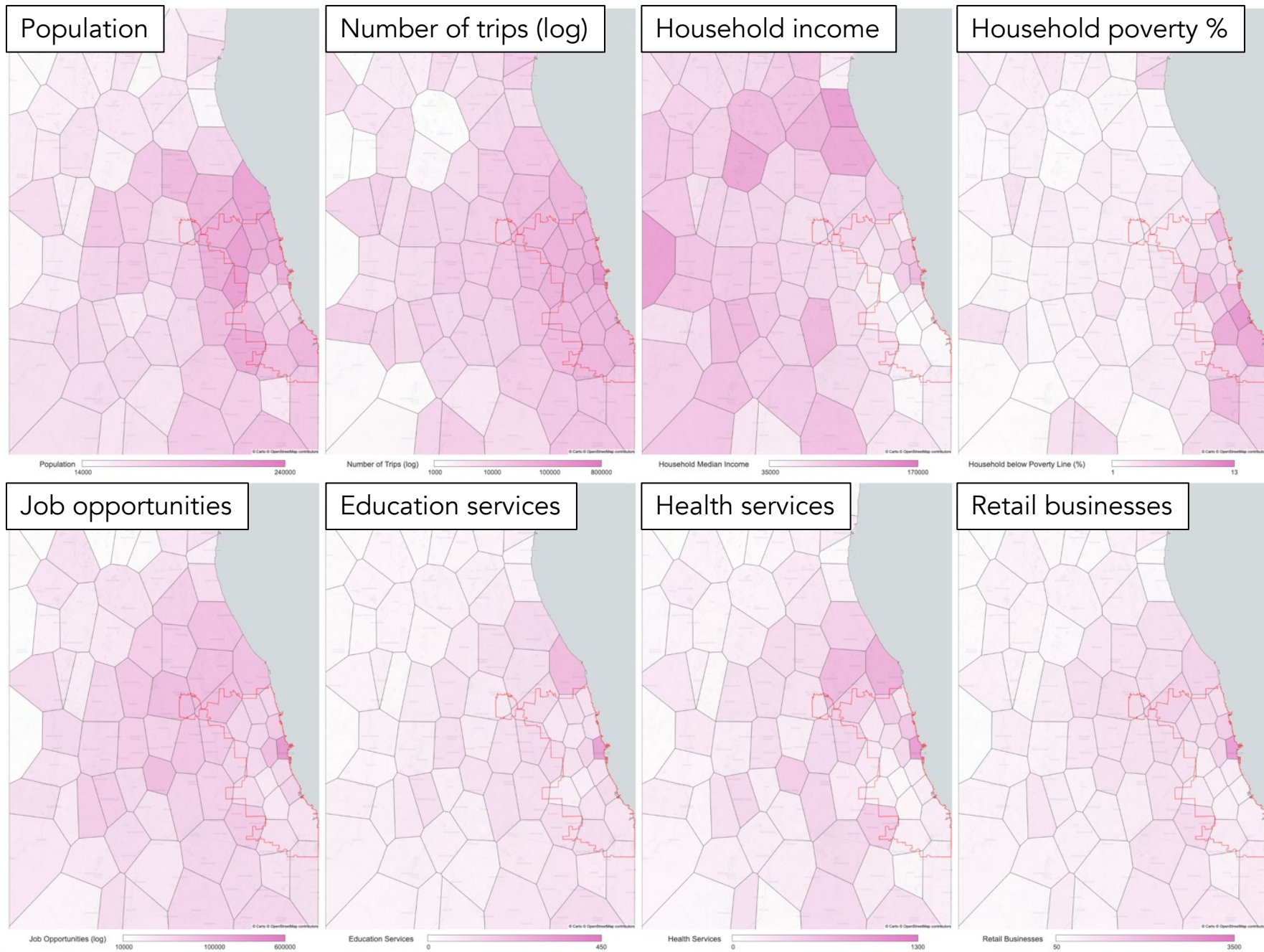


# Case Study

- Chicago Metropolitan Area
  - 80 clusters
  - 8.32M population
- City area:
  - Loop (CBD)
  - Connected with metro rail (CTA “L”)
  - Income disparity: southern & western areas (e.g., Engelwood, Austin) lower median household income
- Suburbs:
  - Mostly connected with commuter rail (Metra)
  - Higher median household income
  - e.g., Schaumburg



# Spatial Distribution of Characteristics



# Redefined Scenarios: OD travel time

Scenario	Scenario 1 - Redesigned Bus Network	Scenario 2 - Redesigned Bus Network and SAVs	Scenario 3 - Redesigned Bus Network and SAVs
Existing infrastructure network	Commuter rail (Metra), metro rail ("L")		
New infrastructure investment	N/A		\$15B on commuter rail (Metra), metro rail ("L"), and bus rapid transit
Other interzonal mode	Express and local buses, Interzonal TNC	Express and local buses, Interzonal SAV	
Feeder within zone	Local buses	SAV	

Scenario 4: Current driving option

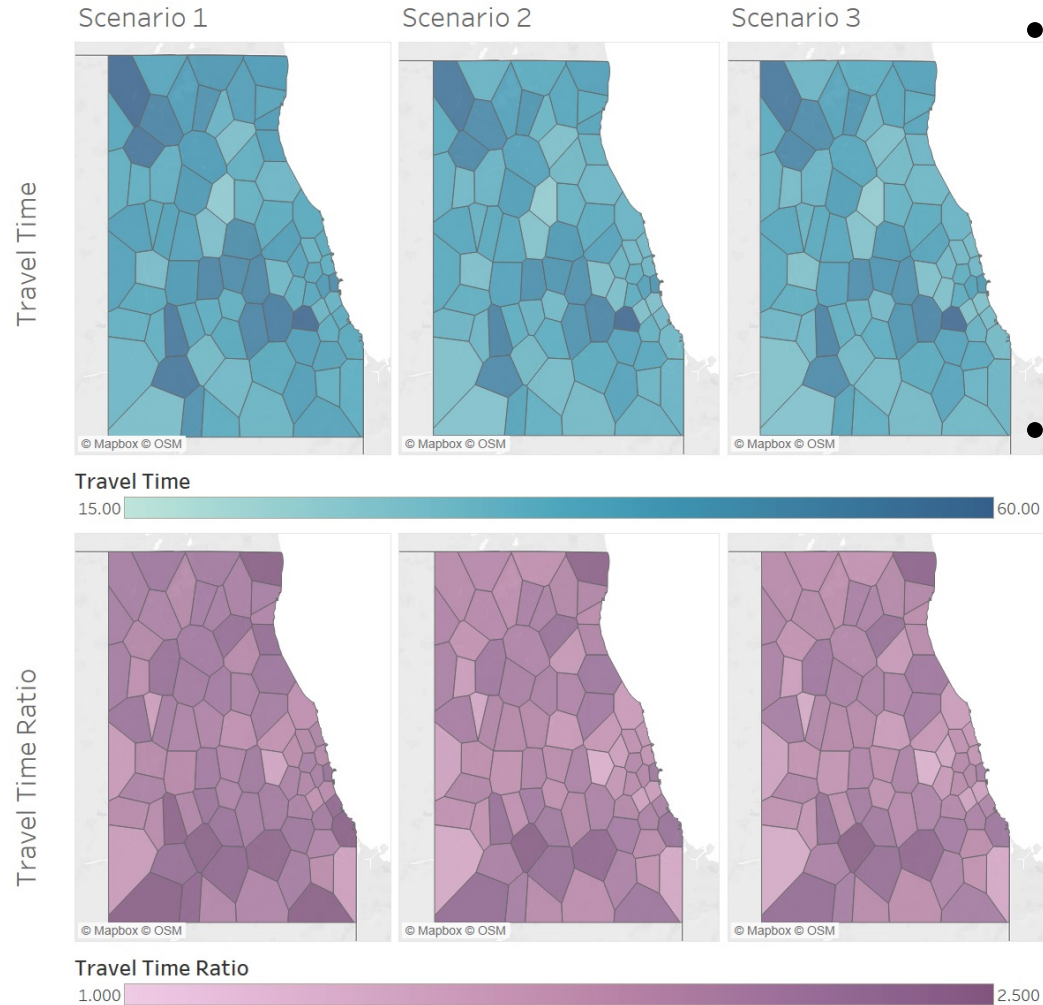
- Travel time at weekday morning peak extracted from Google Maps Distance Matrix API

# Model Demand

## Demand

- CMAP CT-RAMP Activity-Based Model
- Included the following modes:
  - 9 Walk to local transit;
  - 10 Walk to premium transit;
  - 11 Drive to local transit;
  - 12 Drive to premium transit;
- Total weekly demand: 4.16M

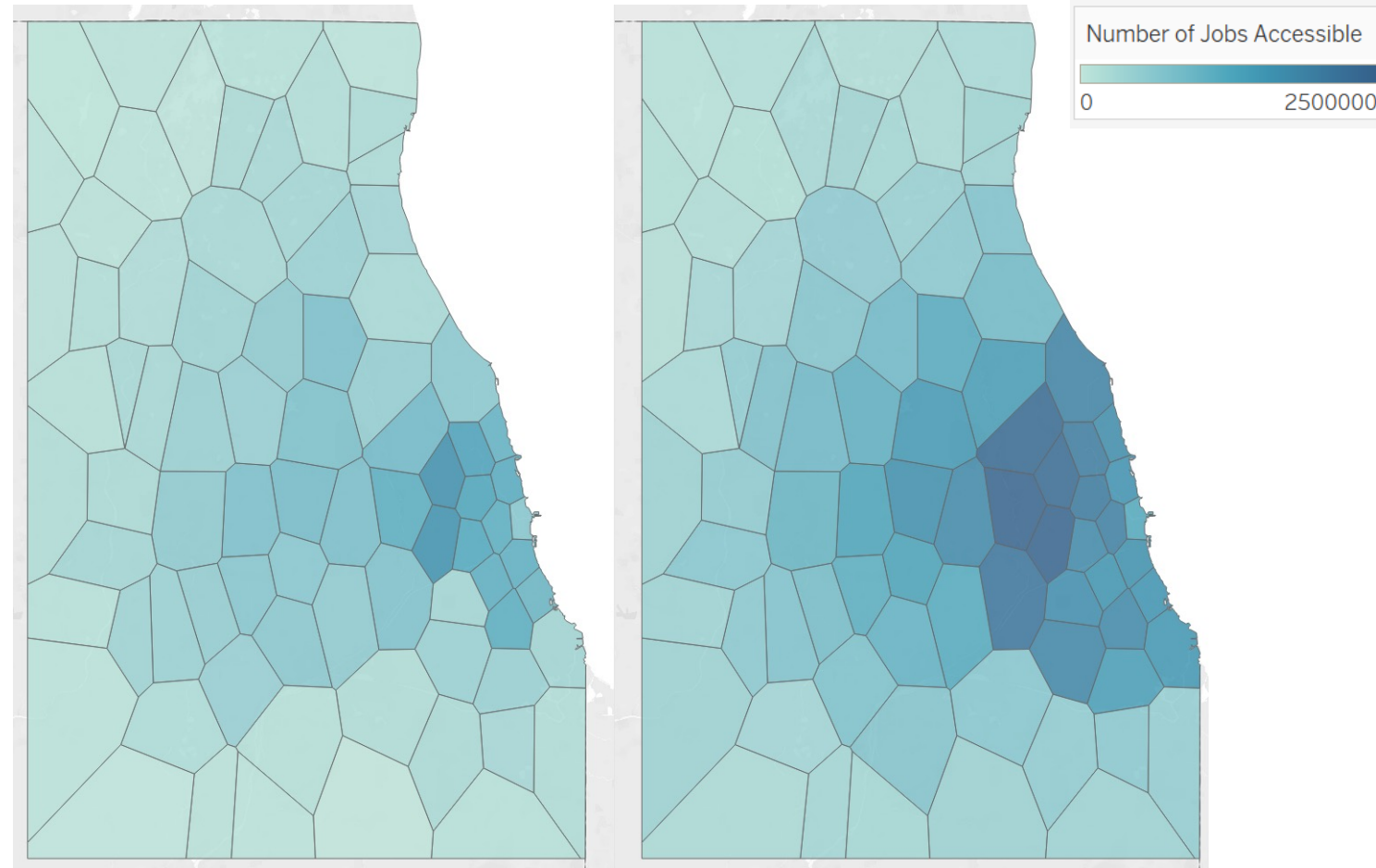
# Spatial Equity-Travel Times



- From the spatial distribution of travel time, there is high variation (inequity) between zones with some outlying suburban zones as well as more central zones having high travel times
- This pattern is exhibited across scenarios and is reflected in the travel time ratio as well

# Measuring Equity in Access to Opportunities

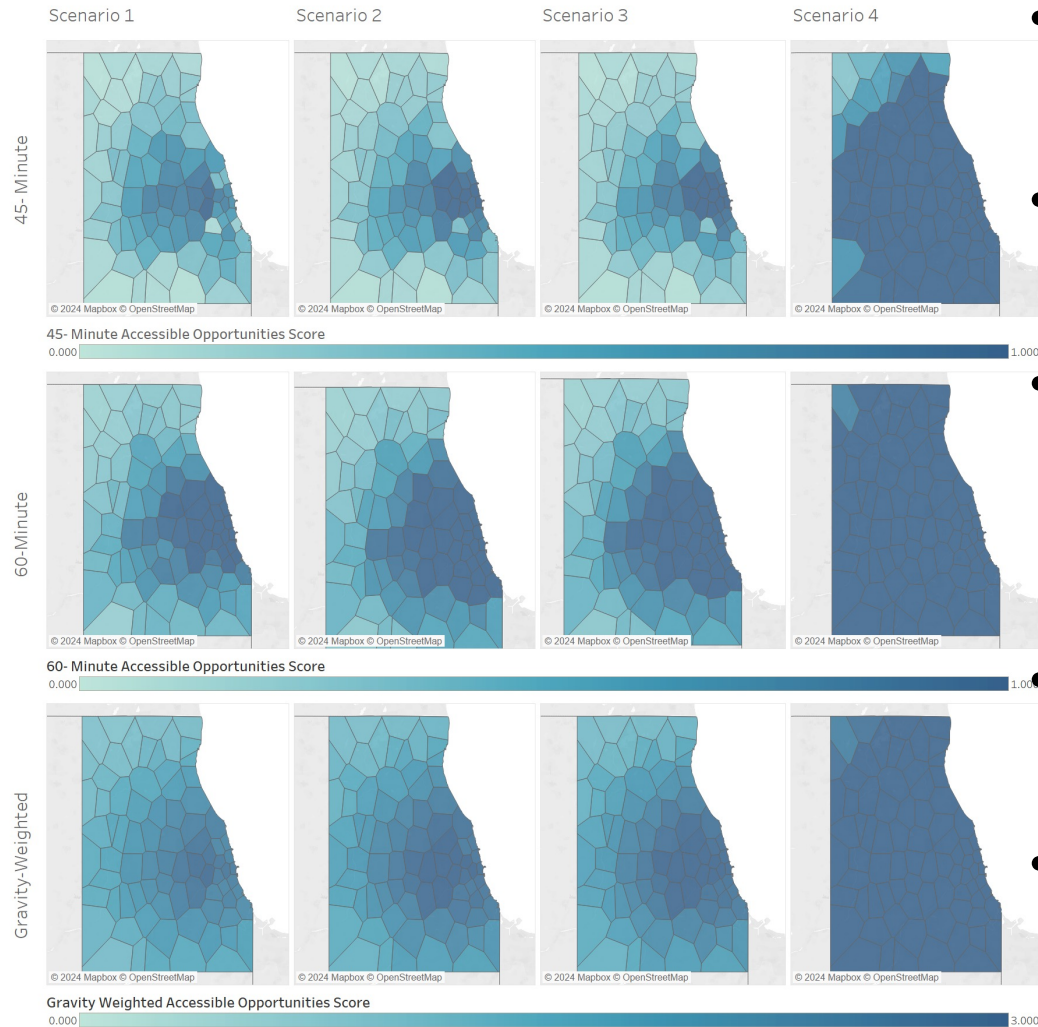
- People travel for a reason- what can reduction in travel time get users to? How are differences in travel time changing the opportunities that users can reach?
- Opportunity Data: Number of Jobs, Health Care Facilities, Retail (and Retail Jobs), and Education Locations
  - Within time bounds of 45 and 60 minute journeys



45 minutes (model - Scenario 2)

60 minutes (model - Scenario 2)

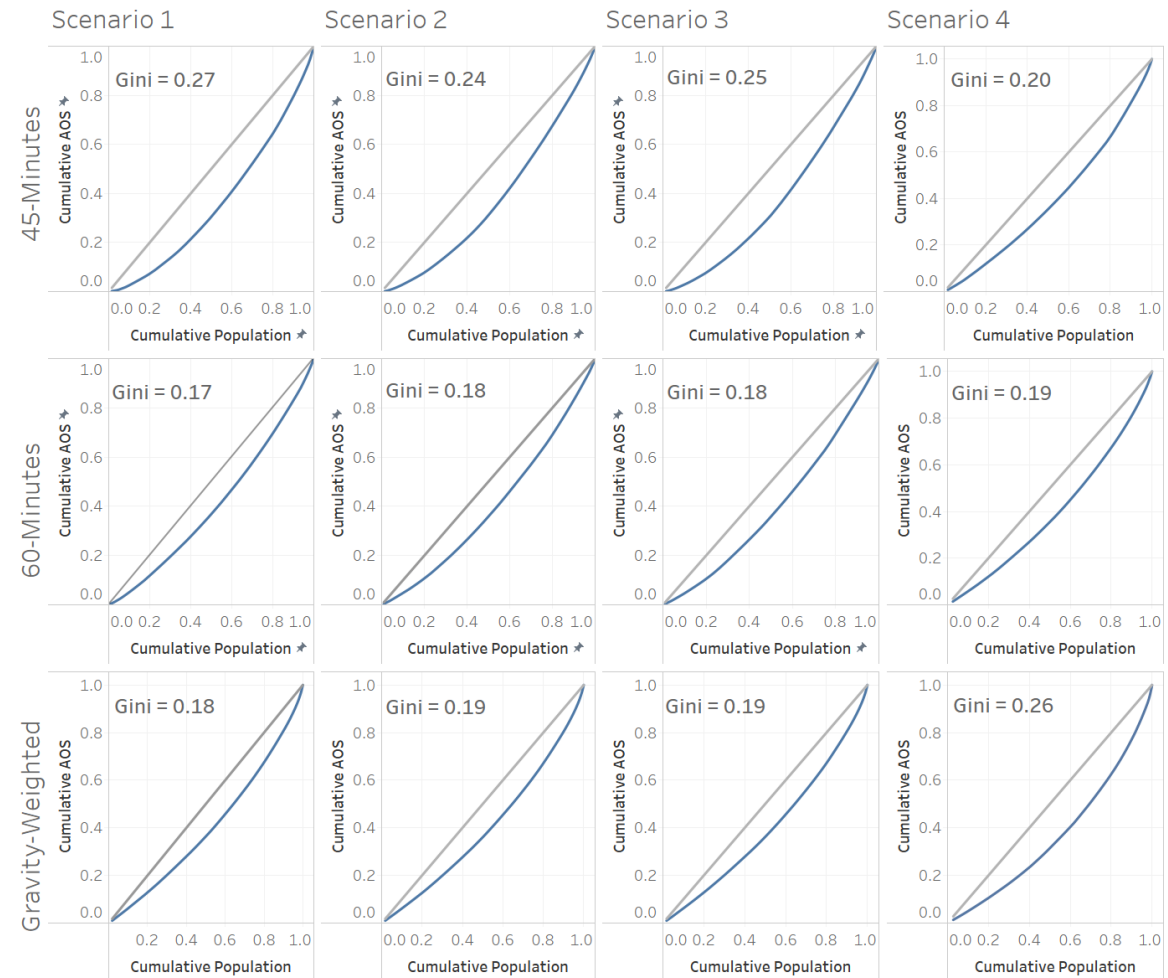
# Spatial Equity-Accessible Opportunities Score



- In the accessible opportunities score, increasing access- shown through increasing spread of the dark blue can be seen across scenarios
- Scenario 4 (driving) displays both the most accessibility as well as the most spatial equity (lowest variation across zones)
- From Scenario 1 to Scenarios 2 and 3, there is increasing access and equity though the core of the city shows the most improvement (there is inequity between core and suburbs but between core zones and between suburban zones there is equity)
- Each metric shows a different variation in equity across scenarios with 45-minute accessibility showing the least accessibility and most spatial inequity
- Across all scenarios, the gravity metric shows the highest equity and accessibility

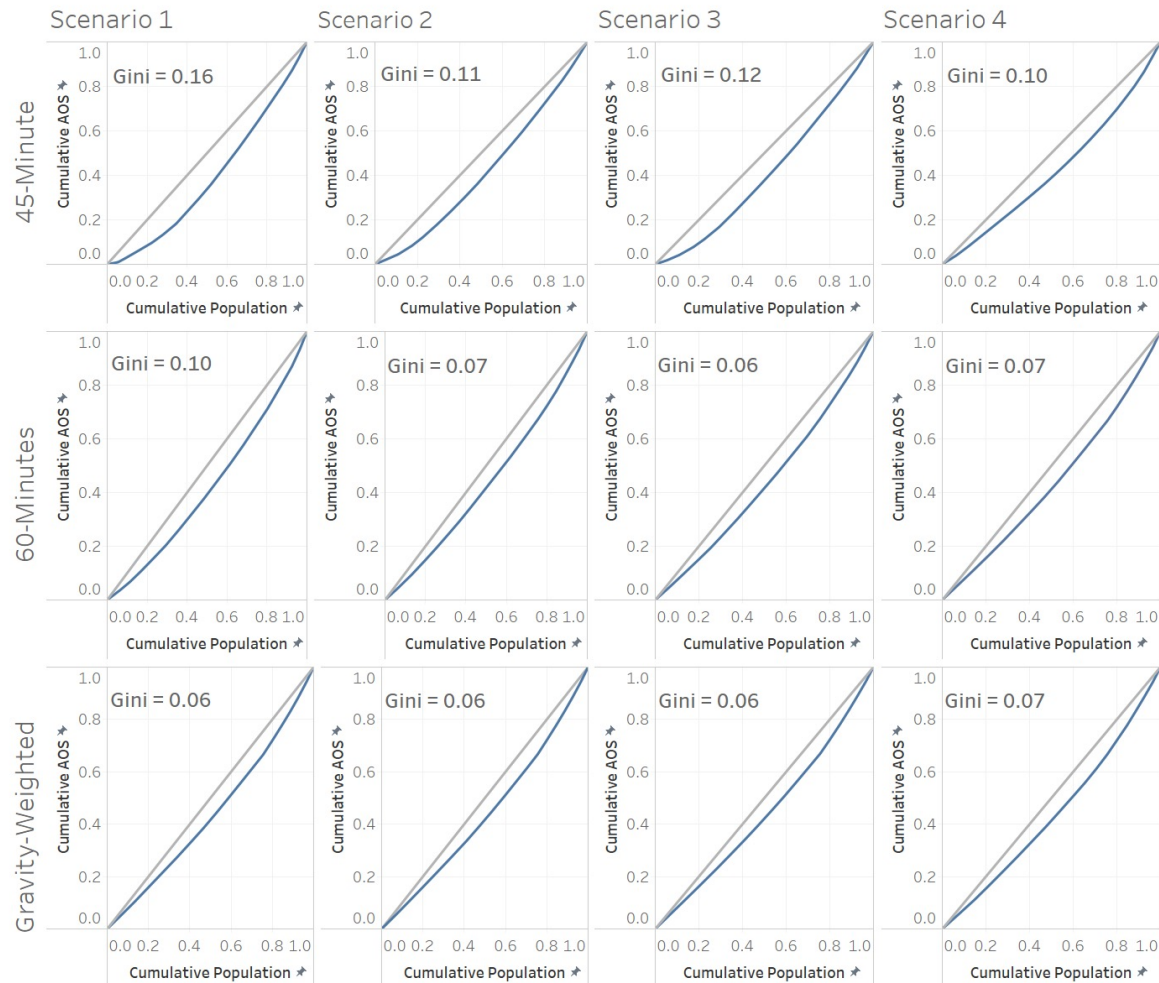
# Spatial Equity-Accessible Opportunities Score with Lorenz Curve

- In the assessment of equity by the Lorenz curve and Gini index, we see a different pattern across scenarios and measures
- In the 45- and 60-minute accessibility score, there is little variation across transit scenarios in the Gini Index
- This is true for the gravity weighting with the exception of Scenario 4, which has a high Gini Index (indicating low equity)
- While visual analysis of the geospatial distribution shows increasing access across scenarios, in the population distribution this conclusion does not hold



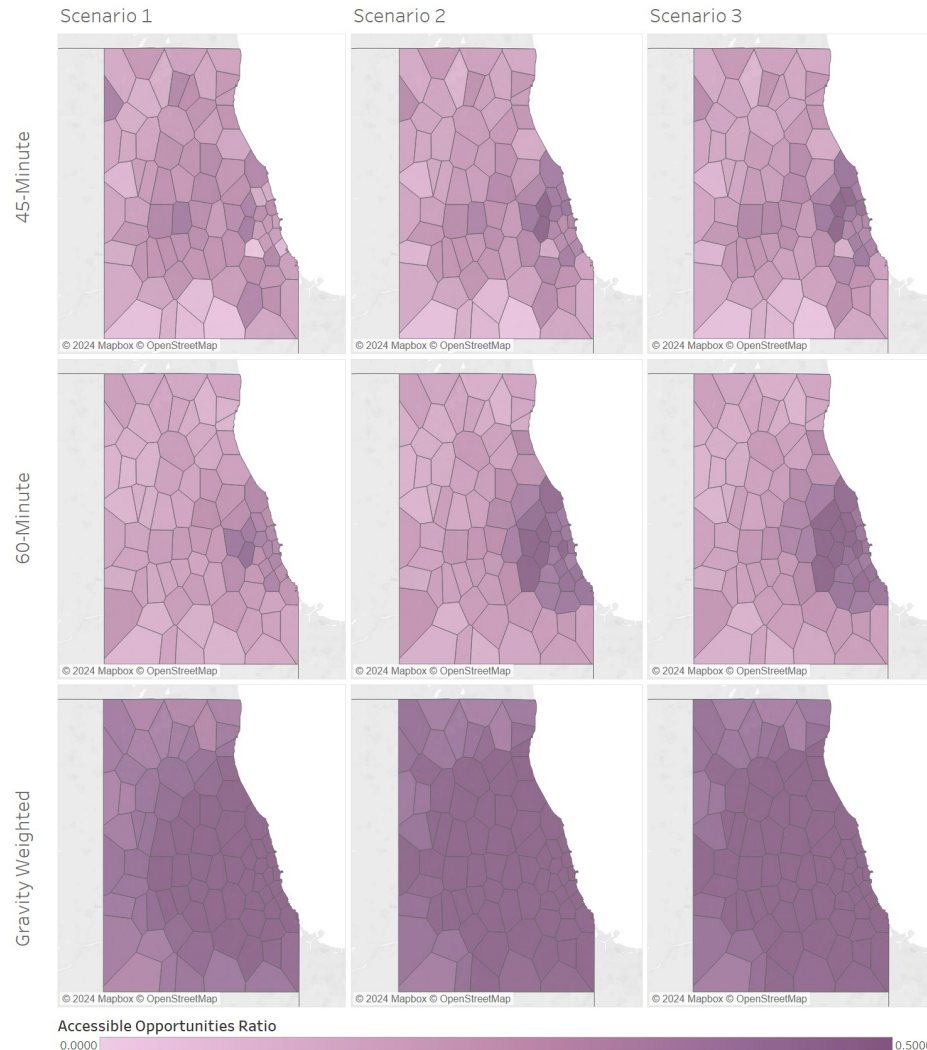


# Spatial Equity-Accessible Opportunities Score with Lorenz Curve (Chicago Only)



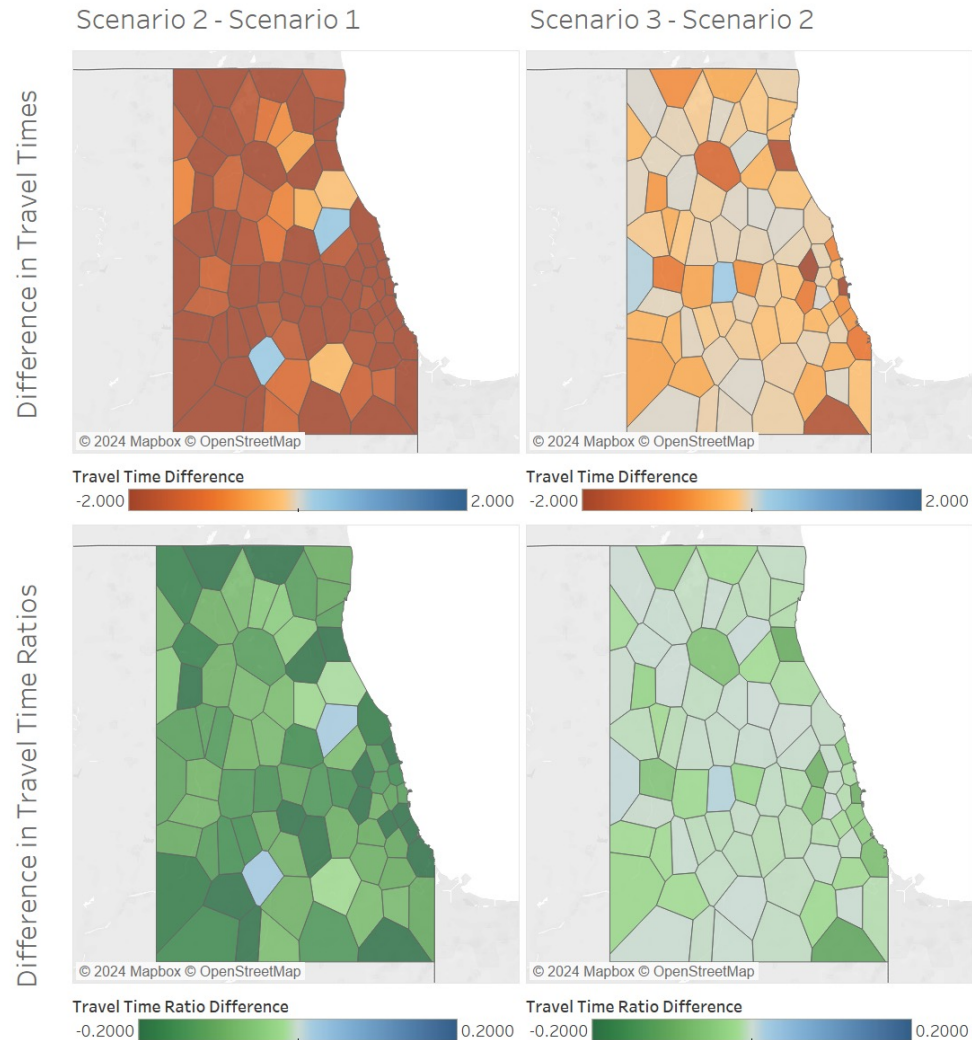
- As shown in the geospatial distributions, accessibility in the core varies much less than in the surrounding suburban zones
- This is reflected in the Lorenz curves and Gini Index (unlike the whole region)

# Spatial Equity-Accessible Opportunities Ratio



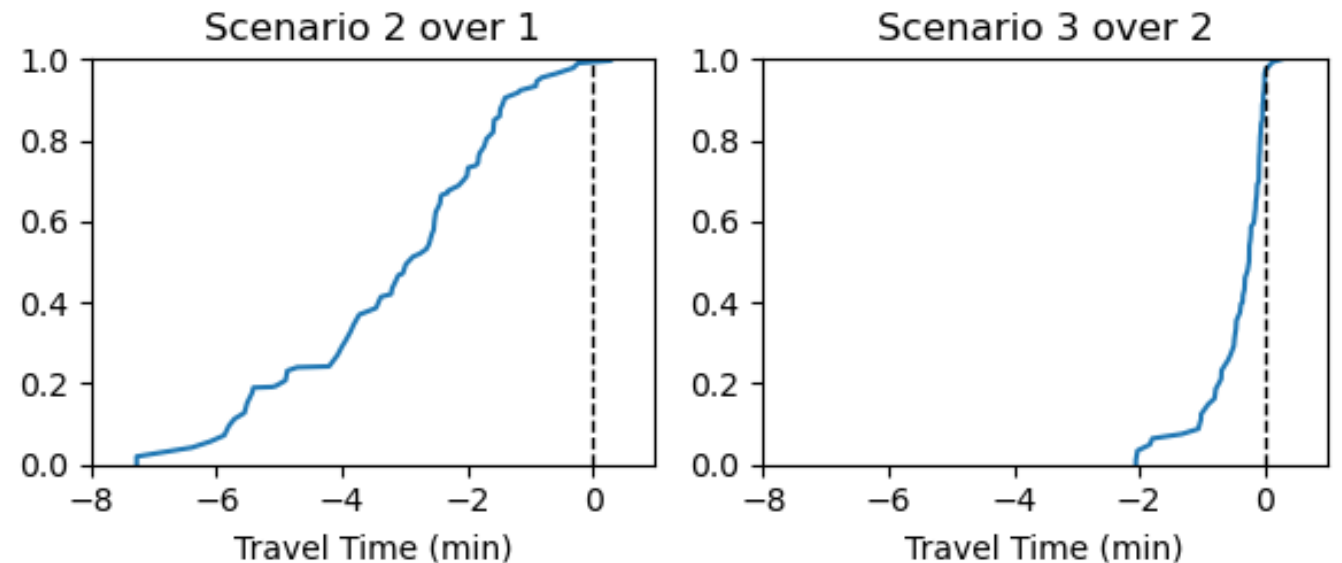
- The geospatial distribution of the Accessible Opportunities Ratio shows some differences compared to the distribution of the AOS
- These differences can be seen in the city where the value of being a central zone, is shown through more access. While the CBD is located to the east of the map, central zones show higher accessibility due to their geography
  - Showing that using the AOR includes the impacts of geography more than AOS

# Changes Spatial Equity-Travel Time



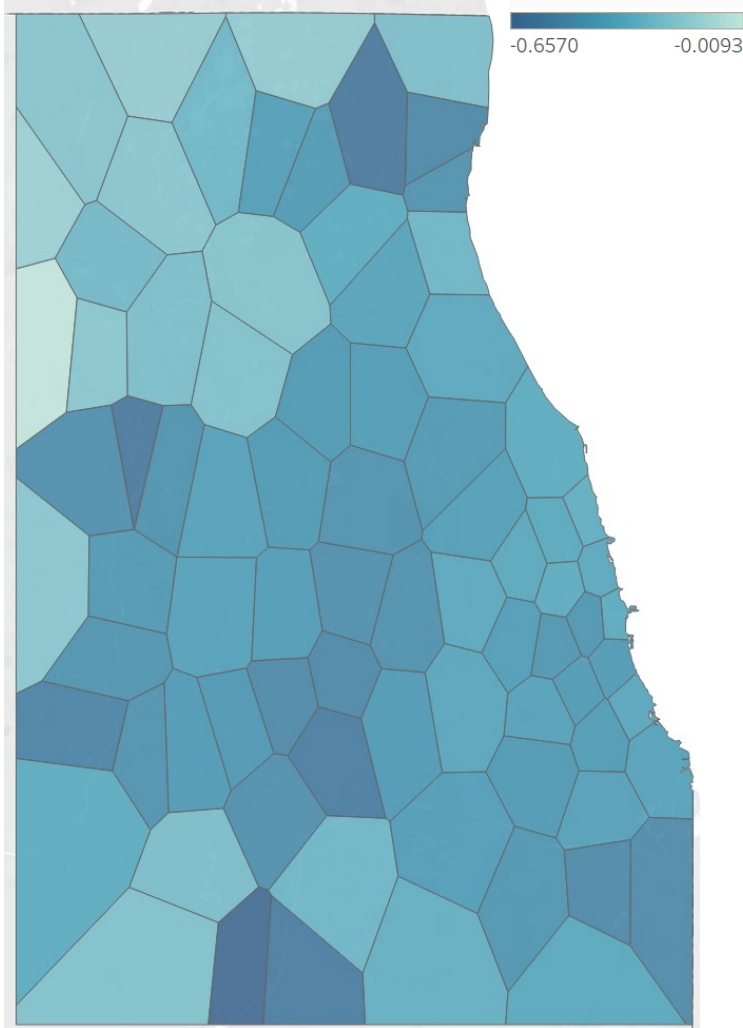
- Across scenarios, most zones saw decreasing travel times however these improvements were not seen equally across the region
- Between Scenario 1 and 2 the distribution of improvement is mostly even with the exception of the outlier zones but between Scenario 2 and 3 there is more variation and less overall improvement

Cumulative Distribution of Weighted Travel Time Change



# Measuring Equity in Travel Times

(Vertical equity)  
Scenario 2 -  
Scenario 1

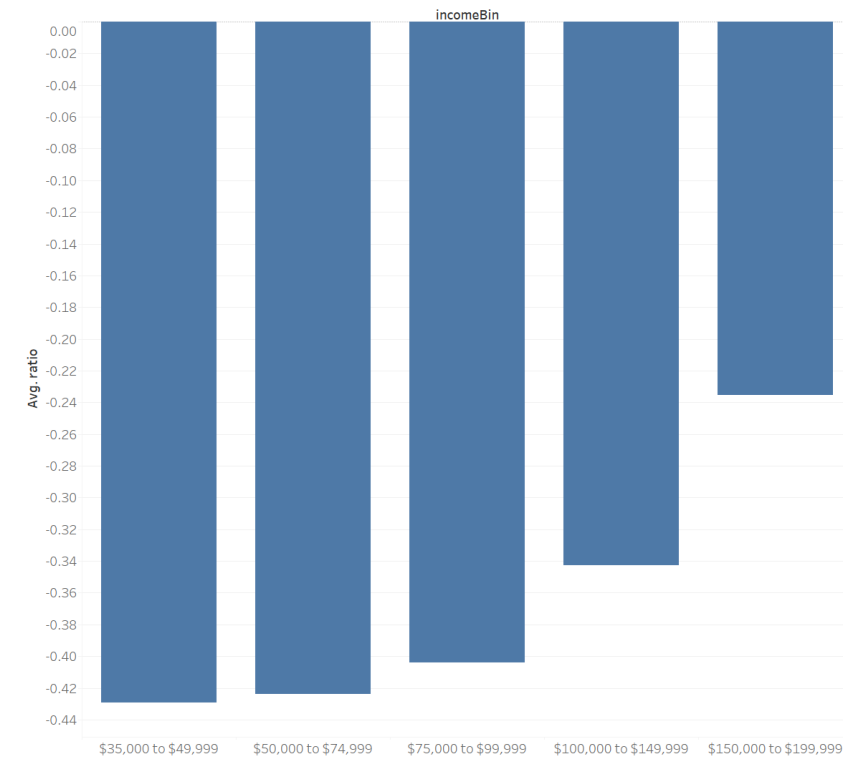


- Equity can be considered not just in the effect of the new system on different income levels but on different modes
- Does the new model reduce the inequity between driving and commute times?

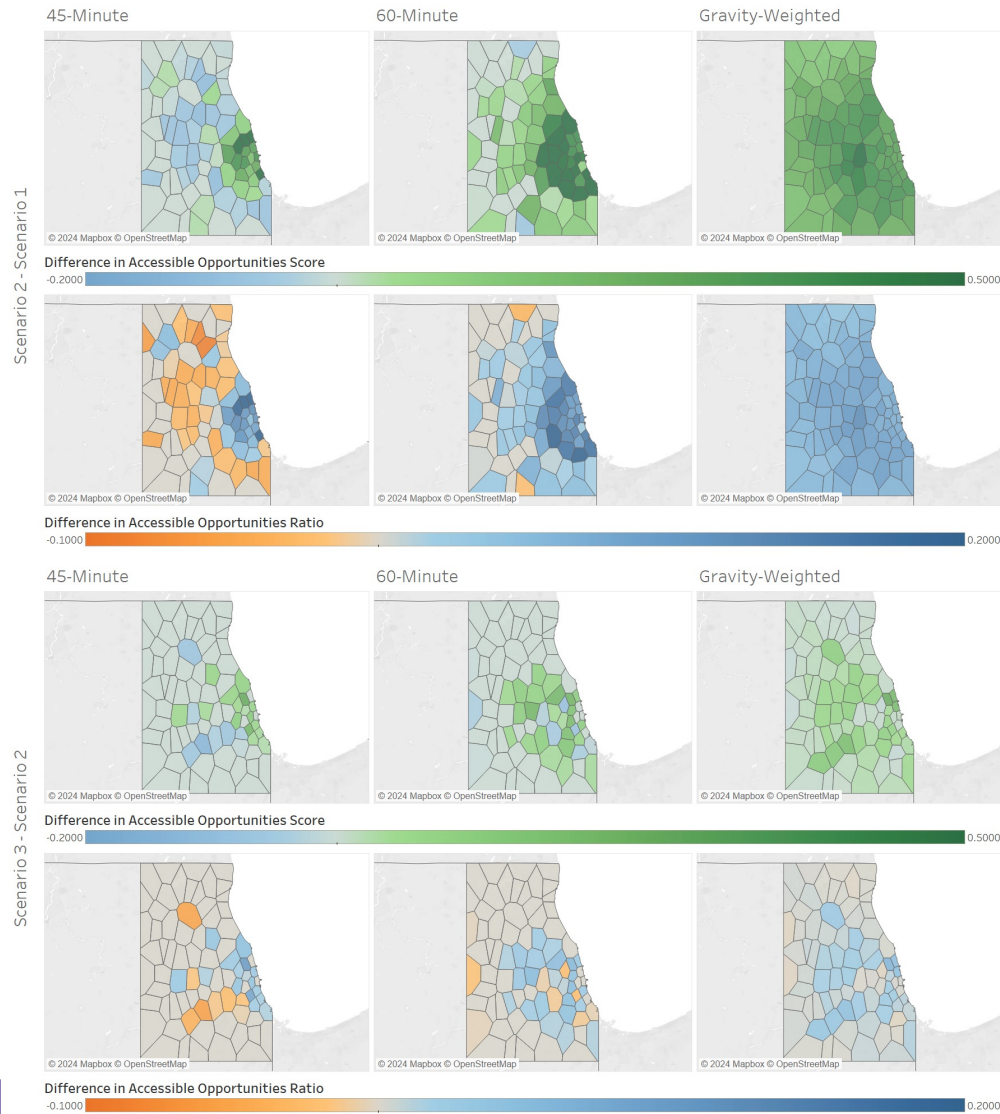
$$\frac{\text{model } TT}{\text{drive } TT} - \frac{\text{transit } TT}{\text{drive } TT}$$

Golub, Aaron, and Karel Martens. "Using principles of justice to assess the modal equity of regional transportation plans." *Journal of Transport Geography* 41 (2014): 10-20.

- How is the reduction distributed across income groups?



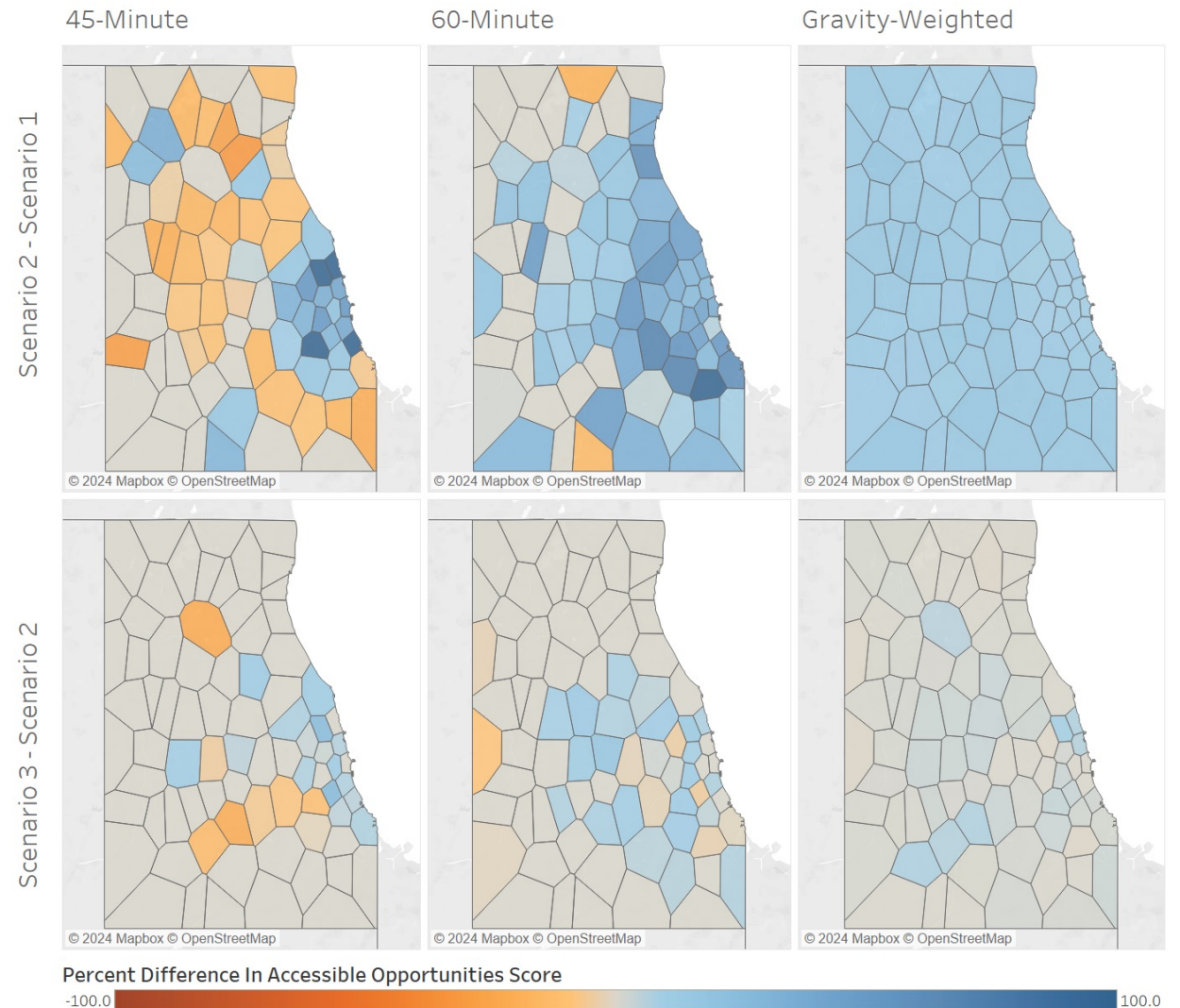
# Changes Spatial Equity-Accessible Opportunities



- The gravity measurement shows increased horizontal equity
- On the other hand, 45-minute and 60-minute accessibility present a muddier pictures with just a 15-minute difference in travel time threshold changing the equity picture completely
- Between Scenario 1 and Scenario 2, there is a significant difference in access and equitable distribution of access- depending on the metric considered
- While there is less improvement in accessibility between Scenarios 2 and 3, the distribution of the improvements is more uniform/spatially equitable

# Changes Spatial Equity-% Change In Accessible Opportunities

- The gravity measurement shows increased horizontal equity and this hold for both the change between Scenario 1 and 2 and the change between Scenario 2 and 3
  - From 1 to 2 there is strong positive improvements
  - From 2 to 3 there are smaller improvements, but slightly positive nonetheless
- In the time thresholds of accessibility, the difference 15-minutes makes in distribution of access is again shown to be significant

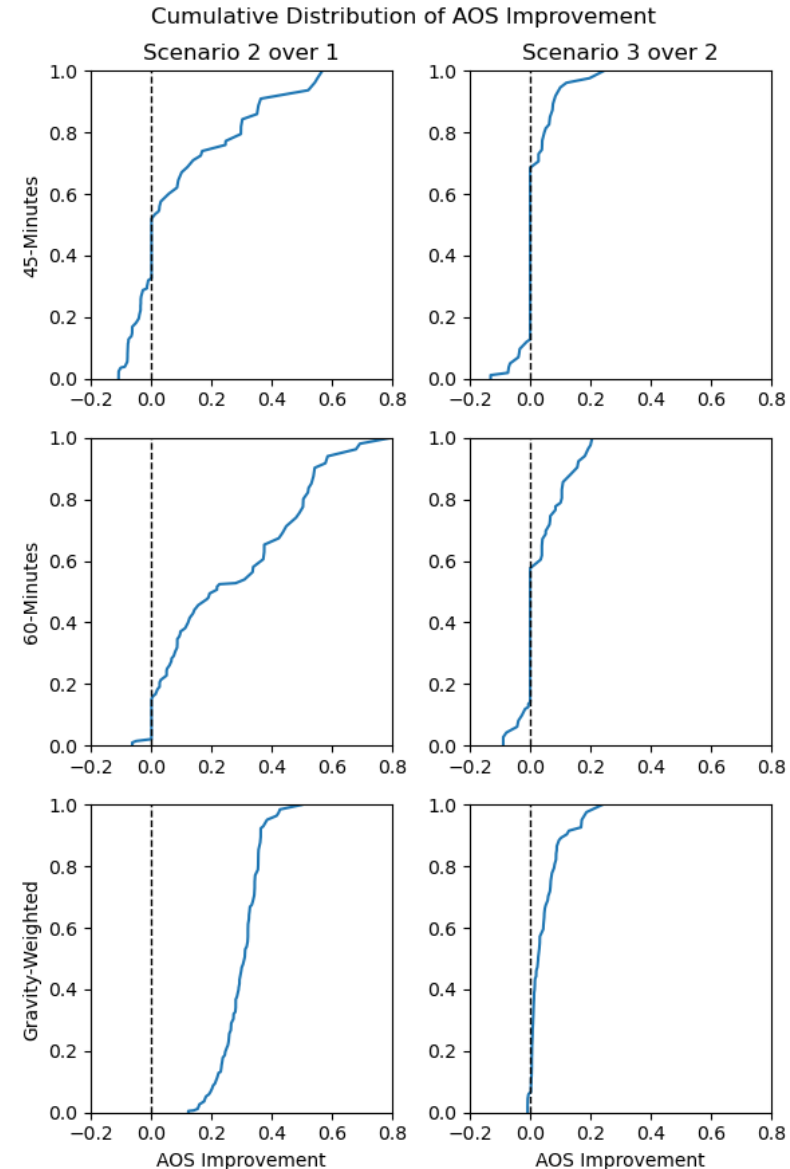


# Changes Spatial Equity

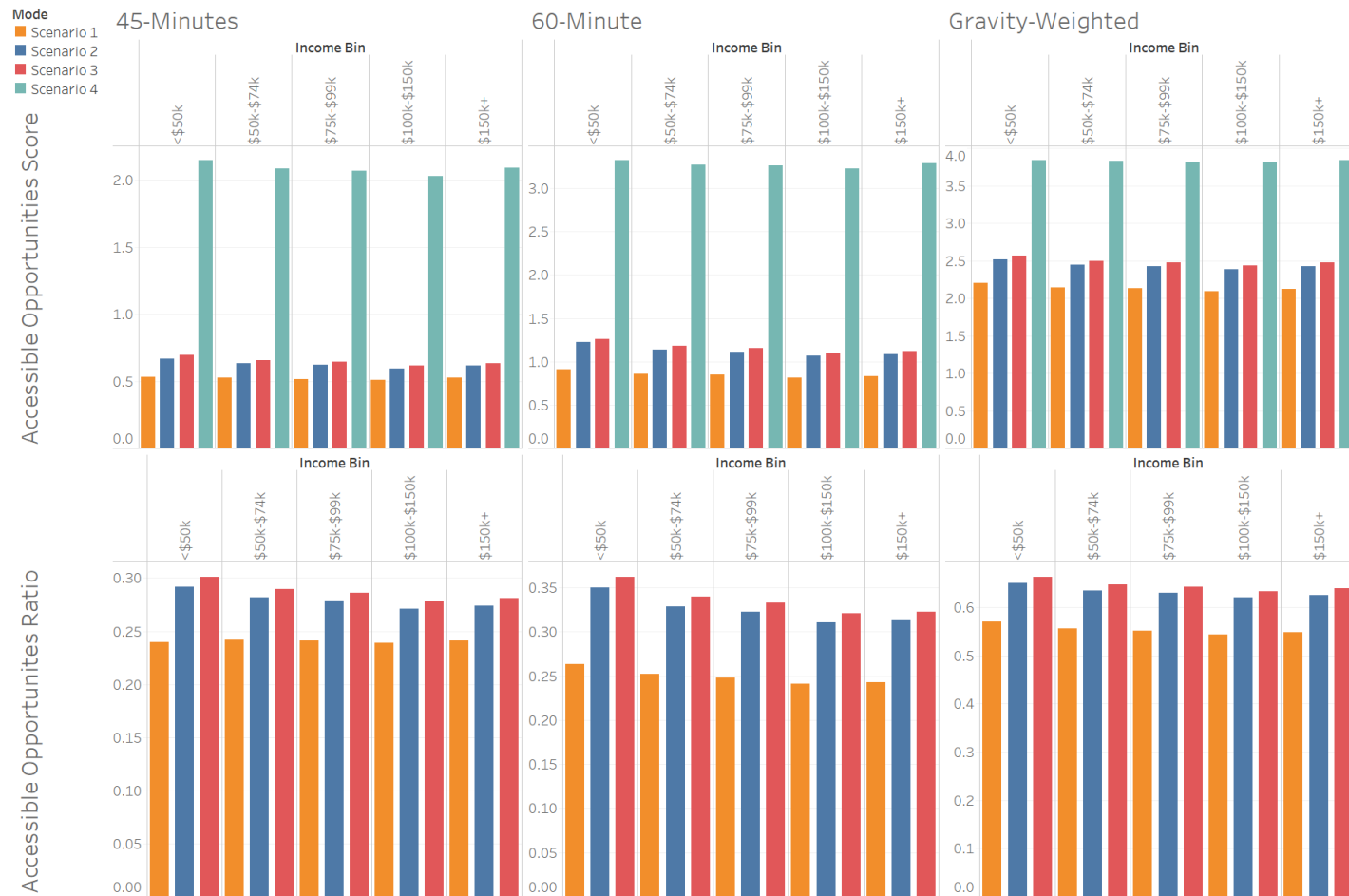
Distribution of gains vs. losses:

CDF of changes in opportunity scores

Overall improvement of 45-minute score  
Is accompanied by small loss for some areas.



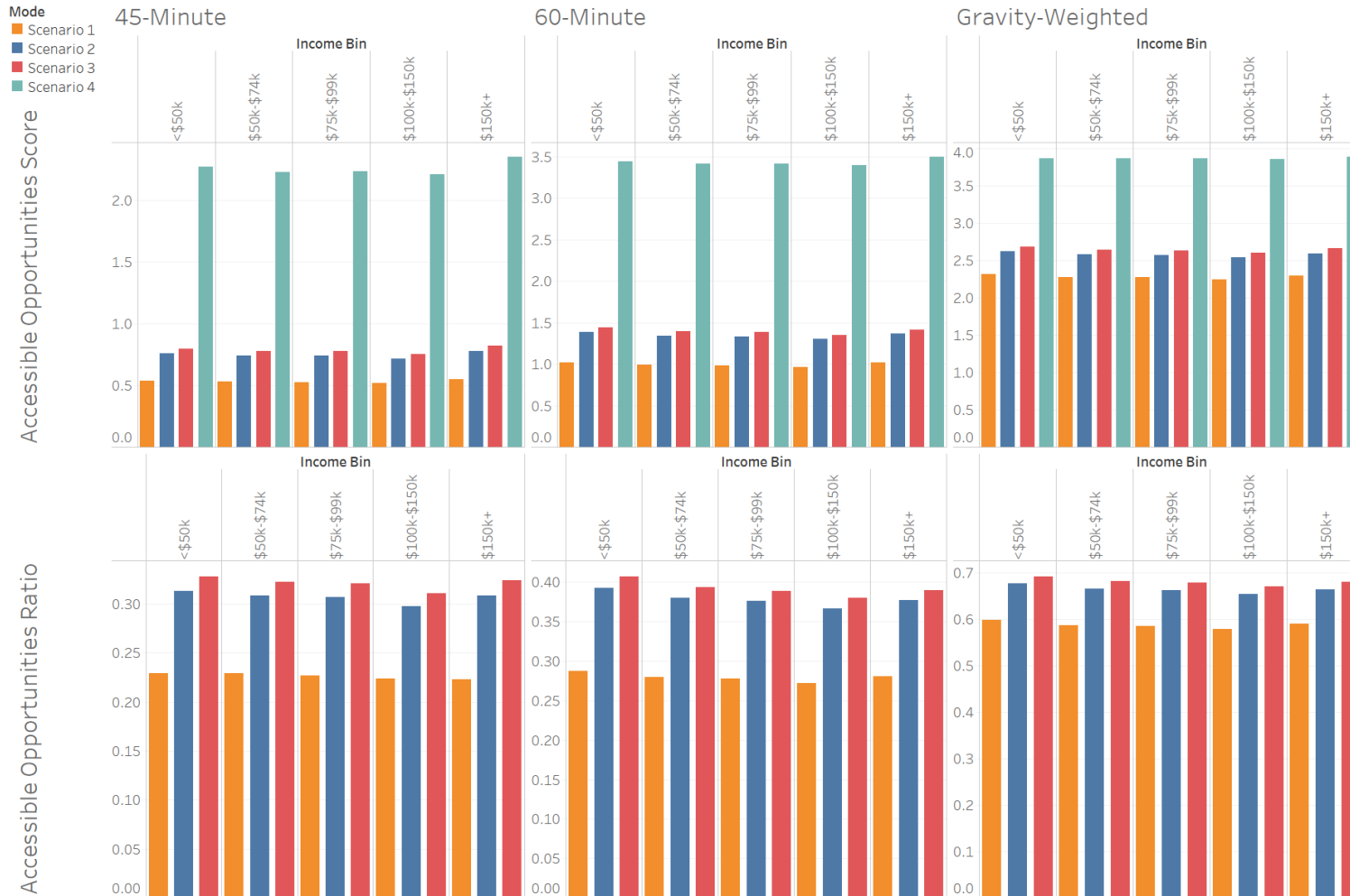
# Vertical Equity- Accessible Opportunities by Income Group



- Not clear improvements in vertical equity in the whole metropolitan area
- Evaluation skewed by wealthy suburbs, which also benefit significantly from redesign (due to limited transit access currently)

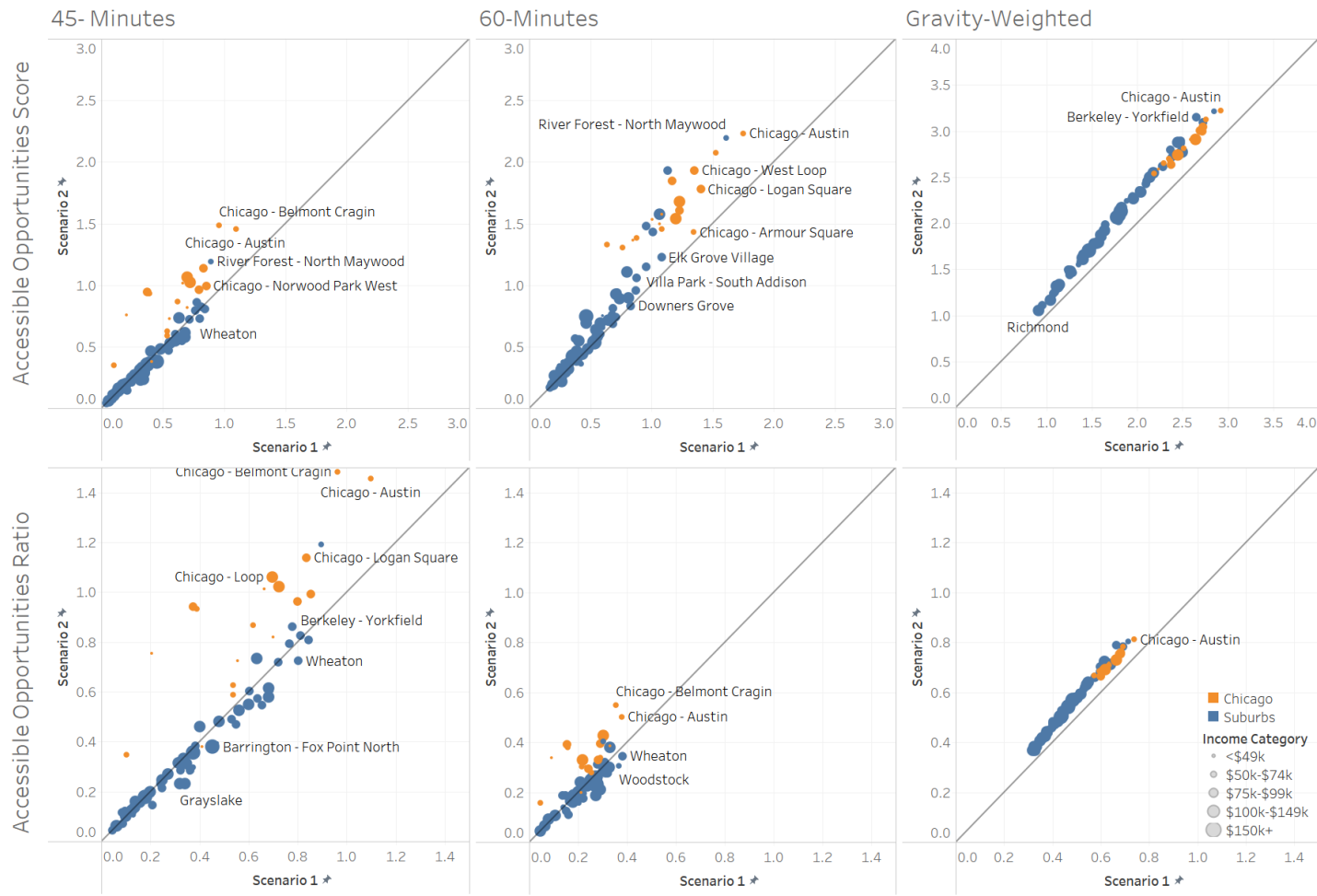


# Vertical Equity- Accessible Opportunities by Income Group (Chicago Only)



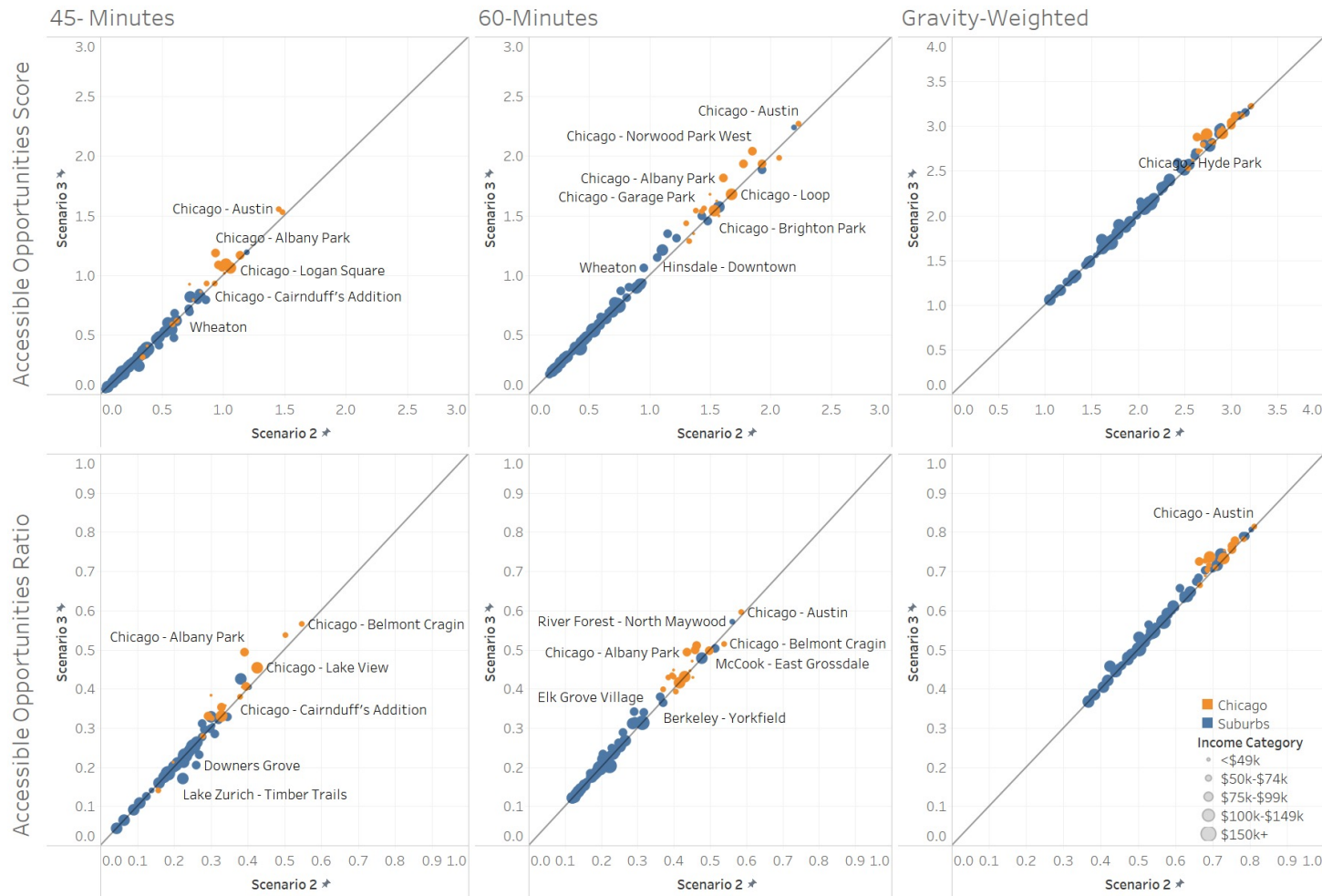
- In the city there are larger gains in vertical equity
- This is mostly seen in the AOR where there is higher accessibility in low-income groups, especially in the 45-minute measure

# Changes in Vertical Equity- Accessible Opportunities Between Scenario 1 and 2



- Each equity metric again shows differences in the changes in accessibility and their distributions
- In all metrics, we see more improvement in the city zones
- Additionally, the areas with the most improvement have lower median incomes indicating an improvement in vertical equity

# Changes in Vertical Equity- Accessible Opportunities Between Scenario 2 and 3



- Between Scenario 2 and 3 there are fewer significant improvements
- The city and especially low-income zones in the city show the most improvements
- This finding is most significant in the time-based accessibility

# Insights

1. Transit redesign with SAMS can improve equity- but how much depends on what type of equity is of interest and the measures used for evaluation
  - It bridges the gap in opportunity access between transit users and drivers (shown in ratios of travel time and opportunity access).
  - It reduces inequality to opportunity access across different zones (shown in Lorenz curve and Gini's index).
2. A discrepancy in equity improvement is shown between suburbs and the city:
  - Suburbs: drastic improvement in opportunity access due to current low levels of coverage, more increase opportunity access for areas closer to the city
  - City: general improvement in opportunity access, areas currently underserved by transit show bigger improvement (southern area)

# Limitations

- Model limitations
  - The model employed focuses primarily on connectivity and has not captured service level improvements e.g., frequency and ease of access which are especially important in assessing transit
  - There is also an inherent gap between modeled scenarios and the real-world conditions.
  - Did not explicitly include equity metrics in objective function of redesign problem.
- Data availability
  - Modal shift from private vehicles to transit usage can be assessed with comprehensive transit demand datasets and considering the impact of reduced transit time on demand patterns
  - Integrating micro-level socio-economic data would facilitate a nuanced understanding of demographics and construction of more meaningful distributional measures.

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# Thank You

## We Love Feedback

### Questions/Comments

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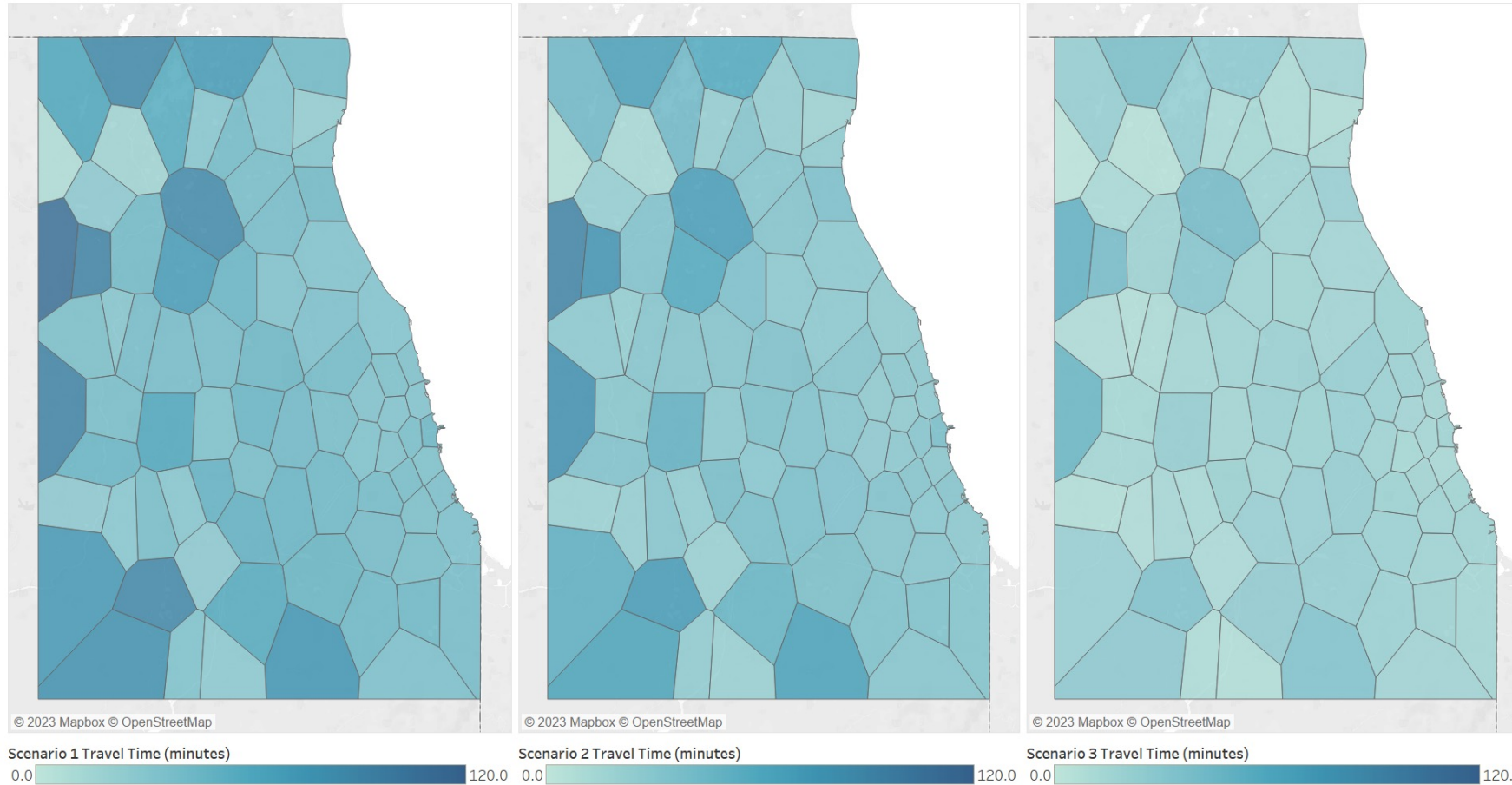
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## Connect with NUTC



# 1. Measuring Equity in Travel Times



Existing Infrastructure with  
Redesigned Bus Routes

Improved Infrastructure with  
Redesigned Bus Routes and SAVs

Current driving option

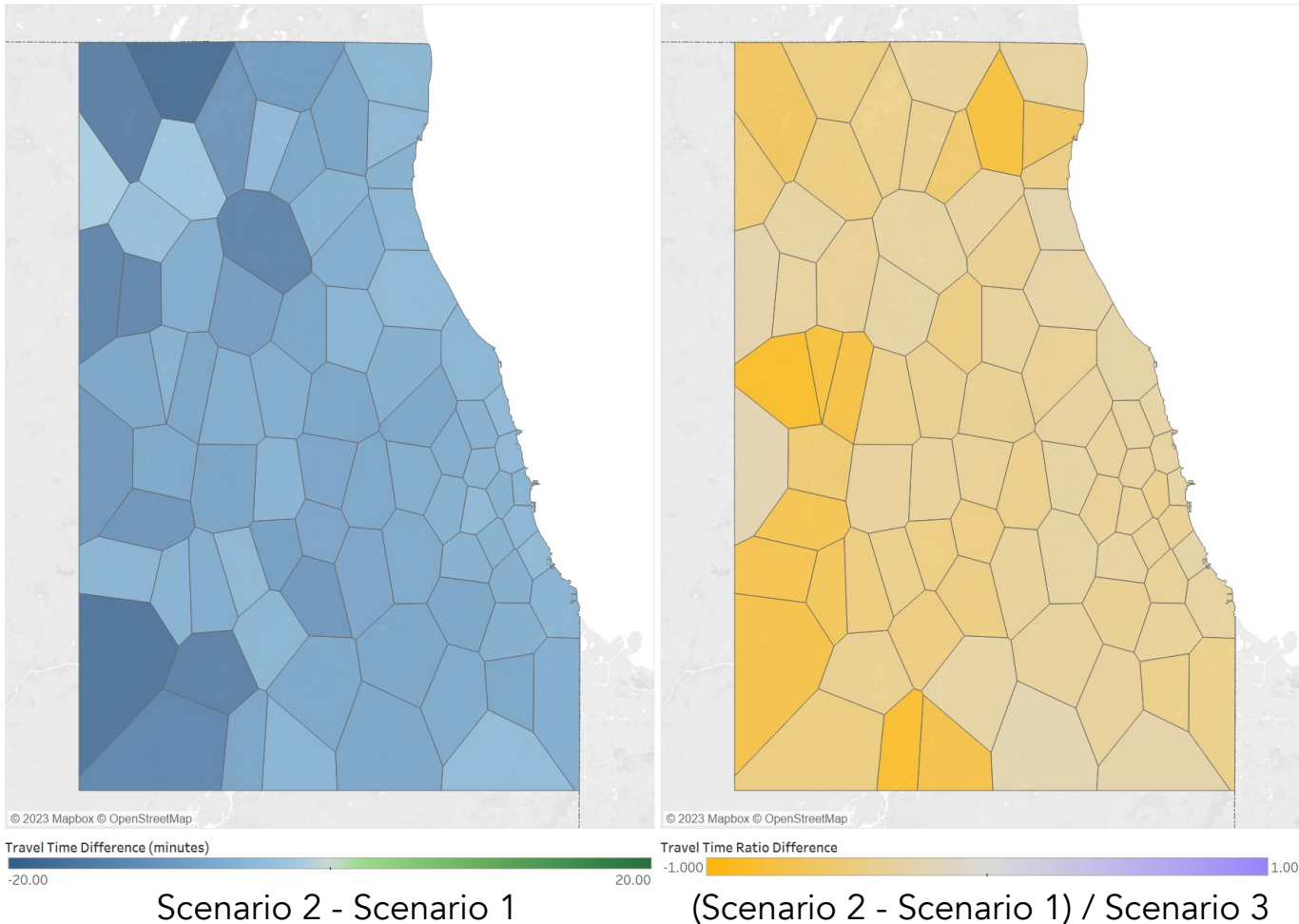
Scenario 1 & 2:

- Average 9-minute reduction in travel time
- Improvement in equity of travel times

Scenario 3:

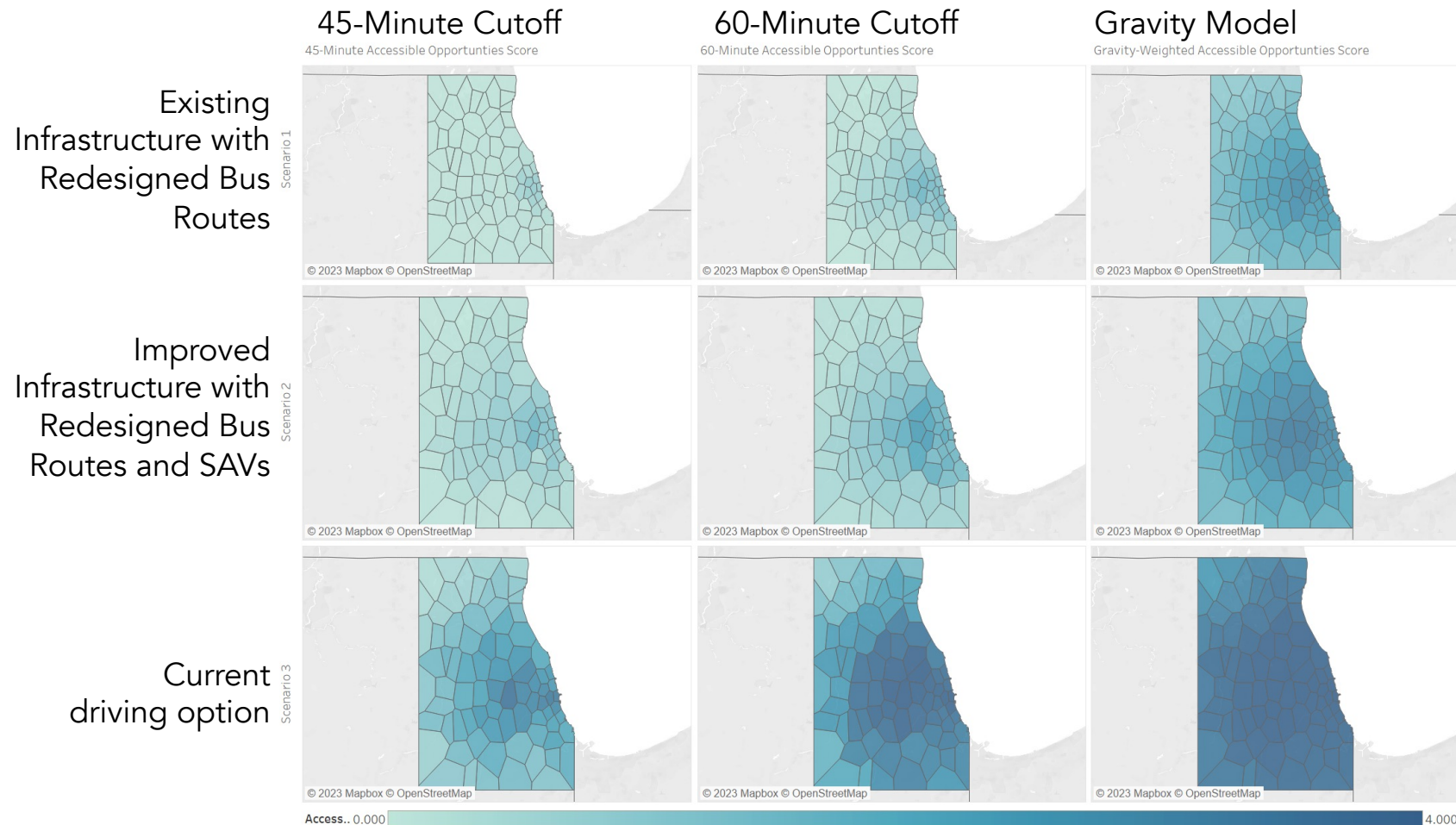
- Greatest geospatial equity for driving (Shen, 1998)

# 1. Measuring Equity in Travel Times & Ratio (Difference)



- Greatest reduction in the furthest suburbs (current transit access is scarce)
- More uniform reduction in the overall city and suburban area

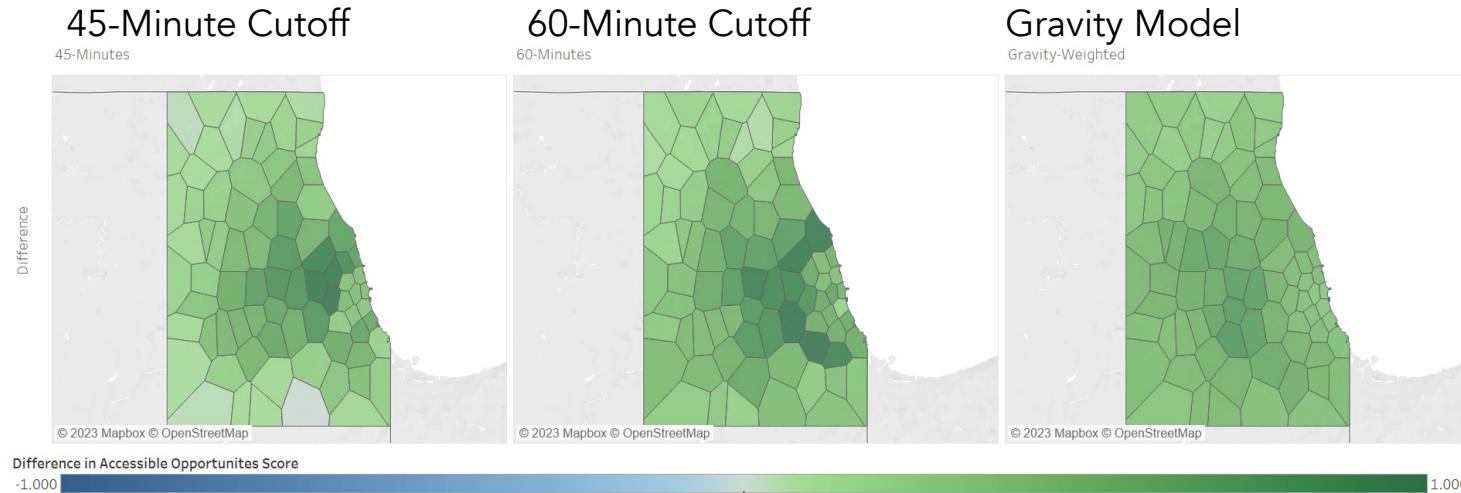
# 2. Measuring Equity with Accessible Opportunities Score



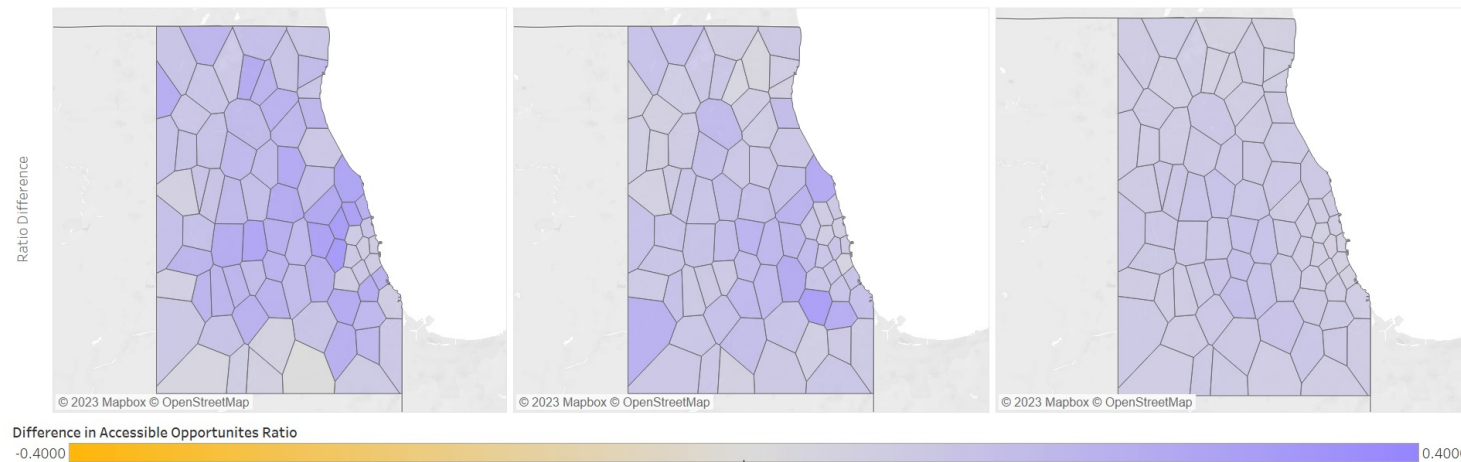
- A clear advantage of driving in being able to reach opportunities
- The most accessible opportunities not in the Loop (due to geography)
- Significant gains by 15 additional minutes of travel between the 45- and 60-minute

# 2. Measuring Equity with Differences in Accessible Opportunities Score and Ratio

Difference in Accessible Opportunities Score (Scenario 2 - Scenario 1)

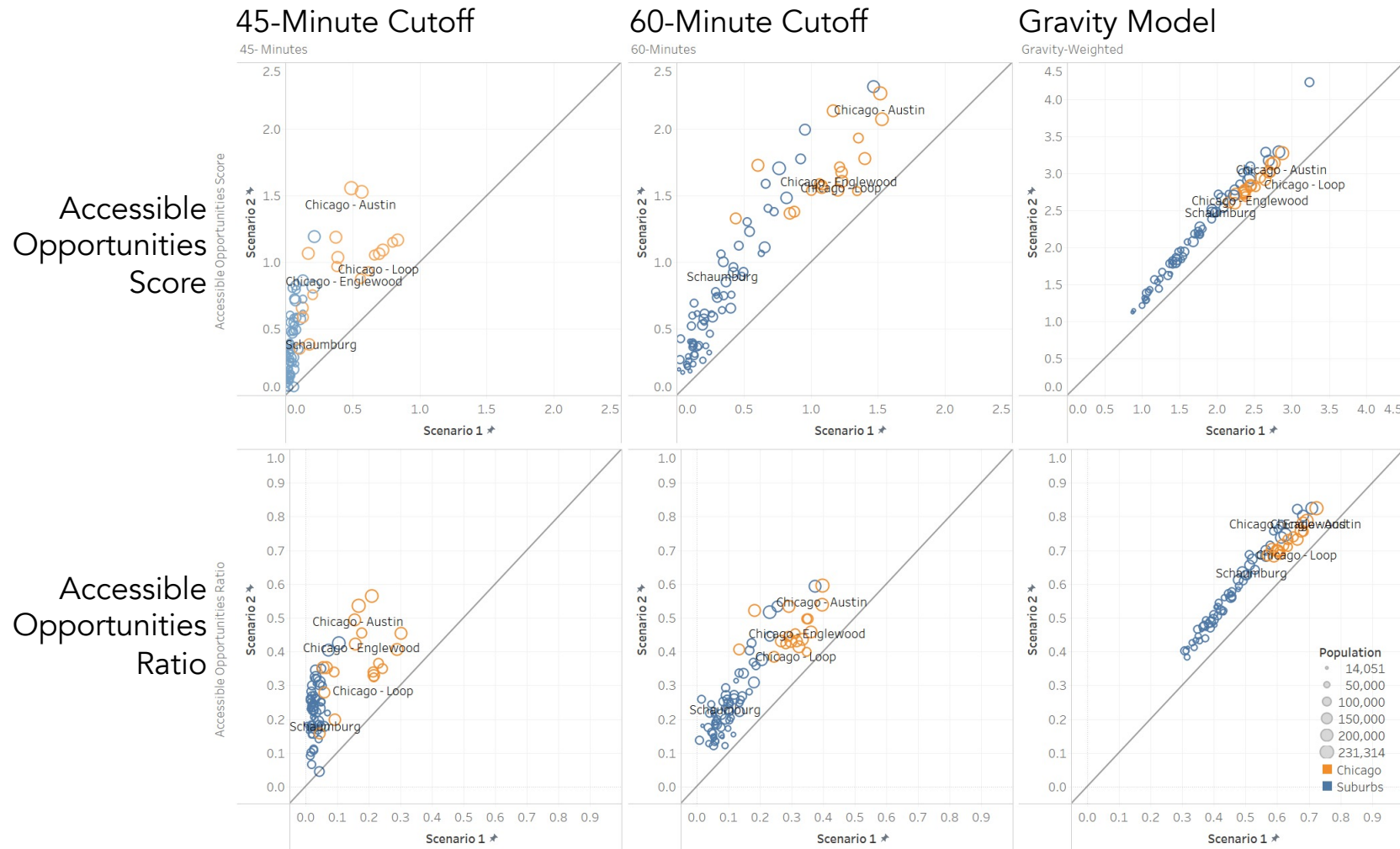


Difference in Accessible Opportunities Ratio (Scenario 2 - Scenario 1) / Scenario 3



- Highest difference in scores for 45- and 60-minute access times for the interior/central zones
- A smoother and more realistic reflection in the gravity weighing metric
- Improvement in horizontal equity in Scenario 2

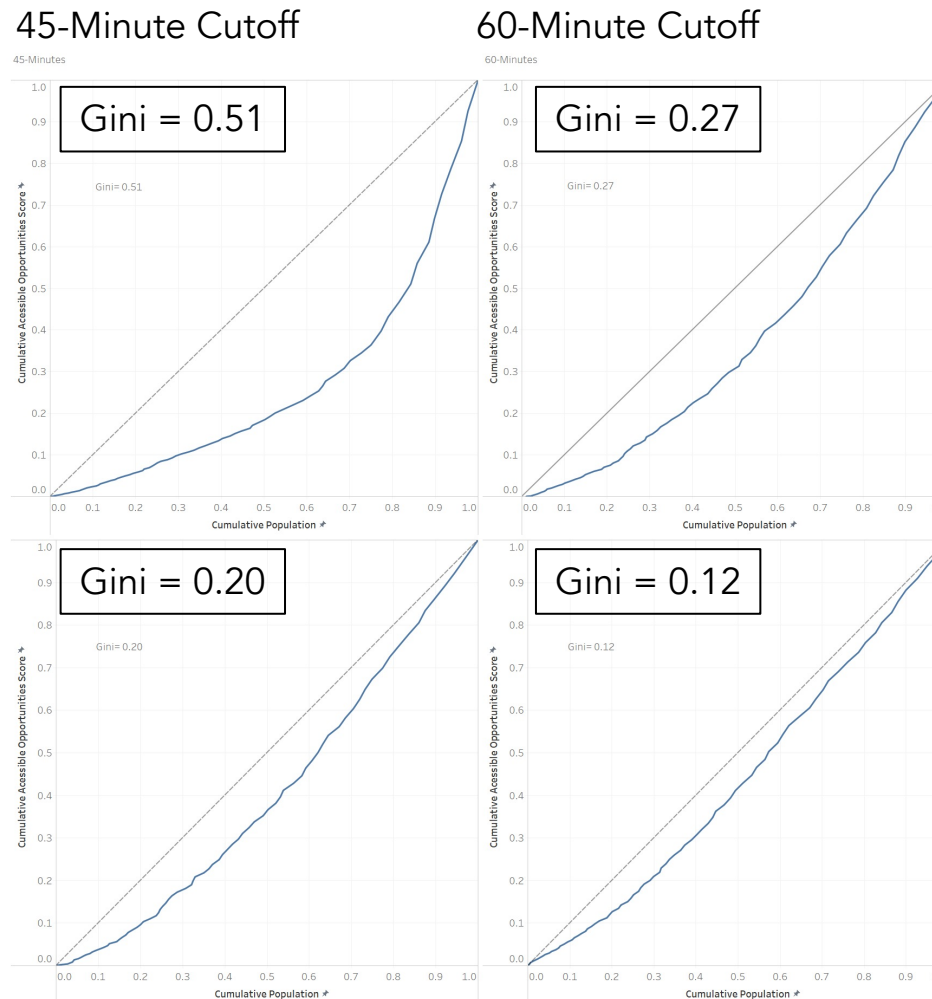
# 2. Scatter Plot of Accessible Opportunities Score and Ratio (Scenario 2 vs Scenario 1)



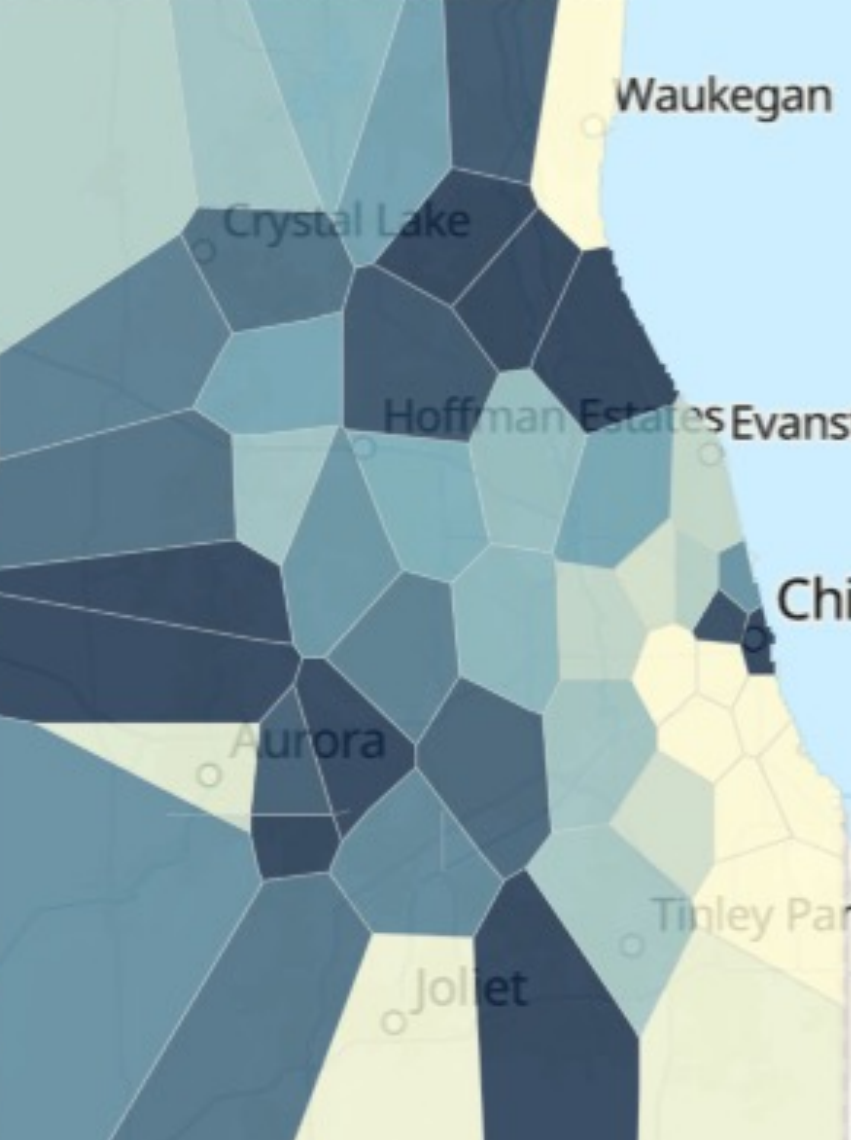
- Improvement in accessible opportunities score (above the 45-deg line)
- 3 groups of points
  1. City zones closest to the 45-deg line (e.g., Loop): affluent, currently well served by transit
  2. Other city zones (e.g., Austin): even greater improvement by redesign
  3. Suburban zones (e.g., Schaumburg): enjoy 2x to 4x more opportunities after redesign

# 2. Measuring Equity with Accessible Opportunities (Lorenz Curves and Gini Index)

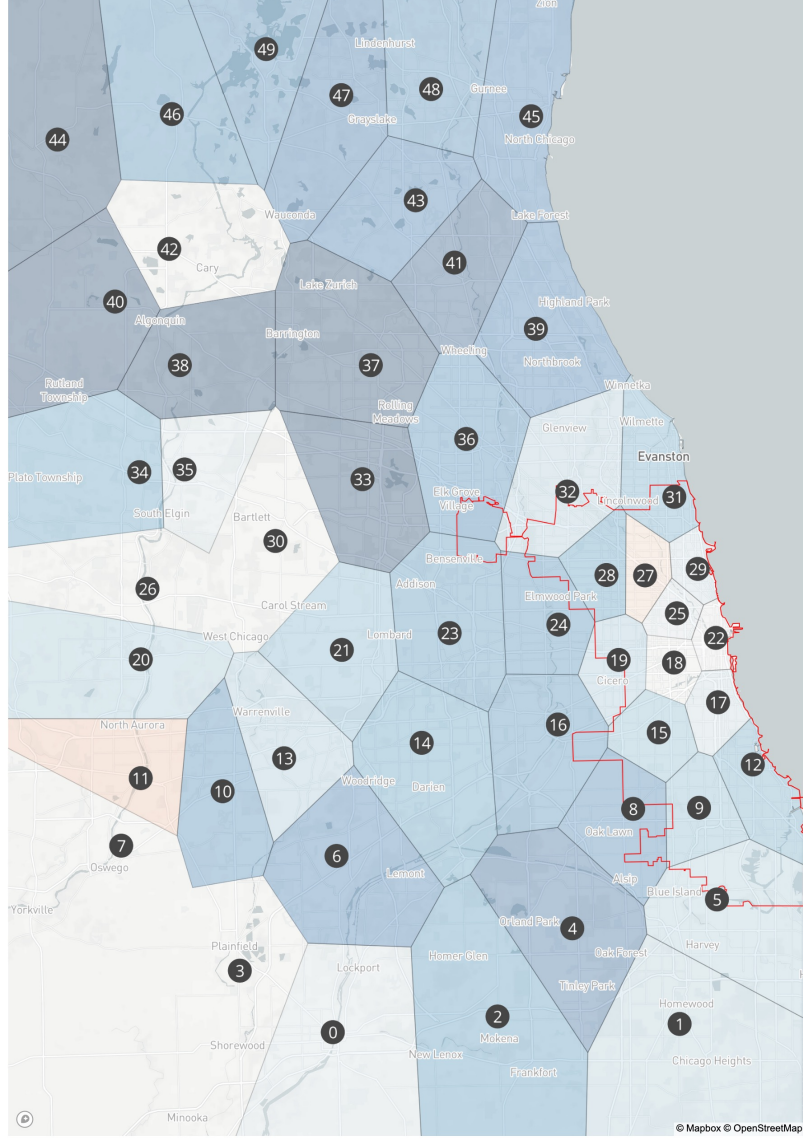
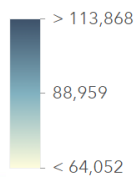
Existing Infrastructure with Redesigned Bus Routes



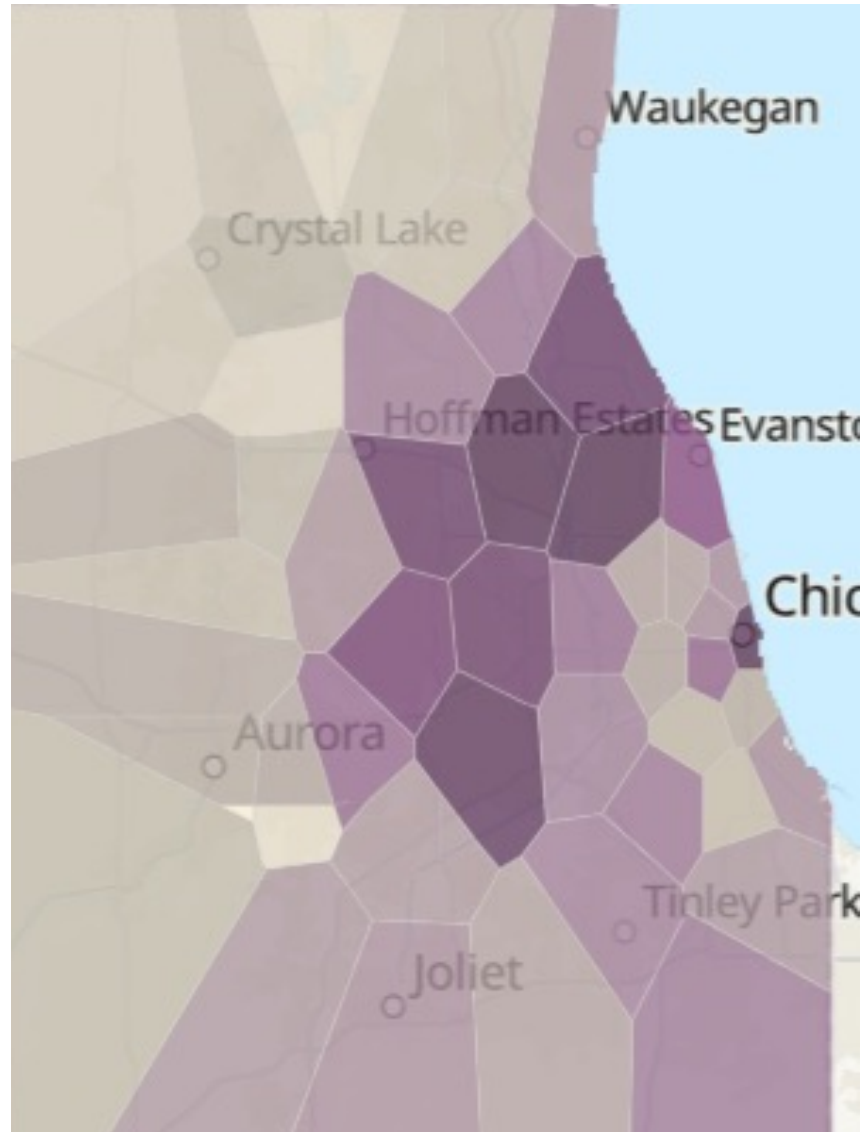
Improved Infrastructure with Redesigned Bus Routes and SAVs



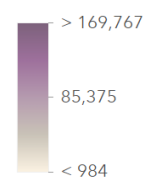
Median Income (2023 Esri)



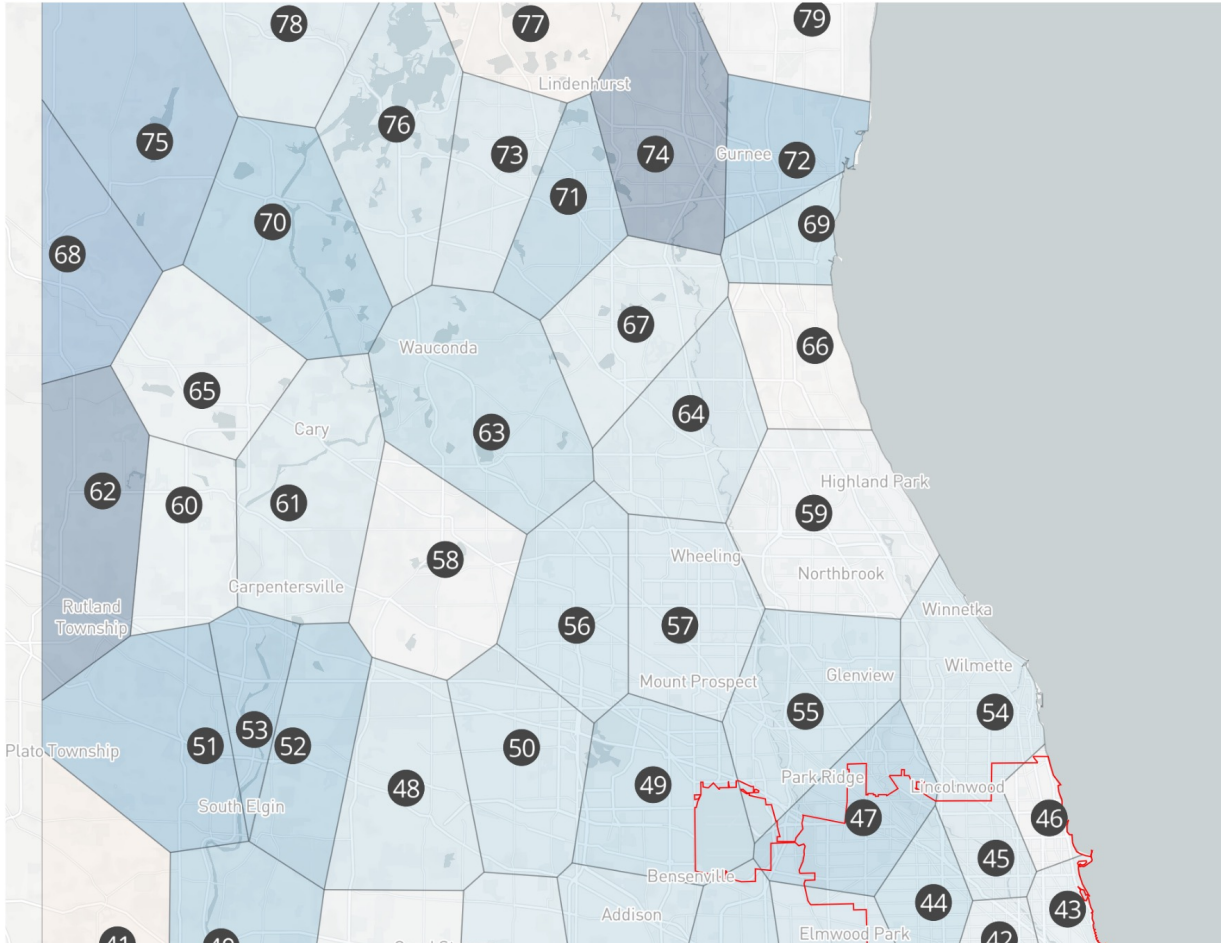
Difference in Travel time (%)



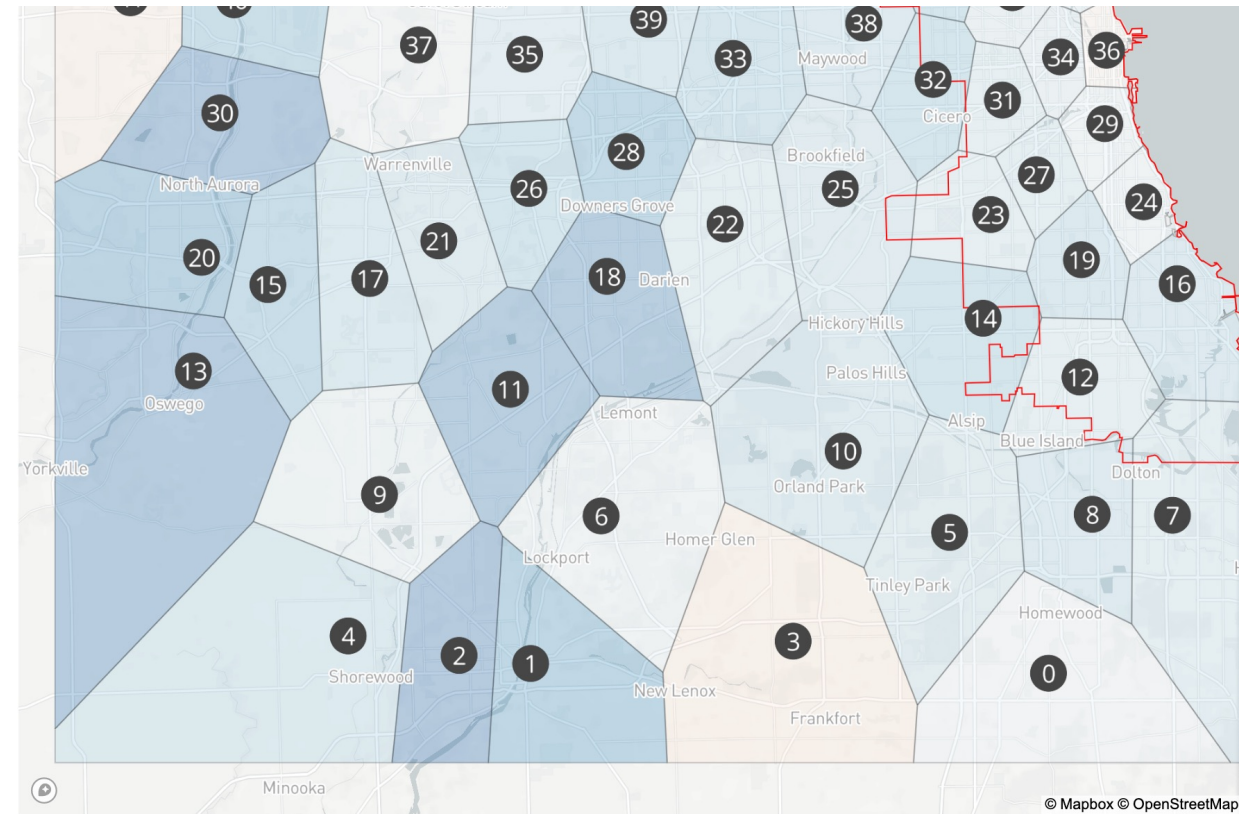
Number of Jobs (2023 SIC)



# 1. Travel Time Difference (Scenario 1 vs 2)



(weighted with existing demand)



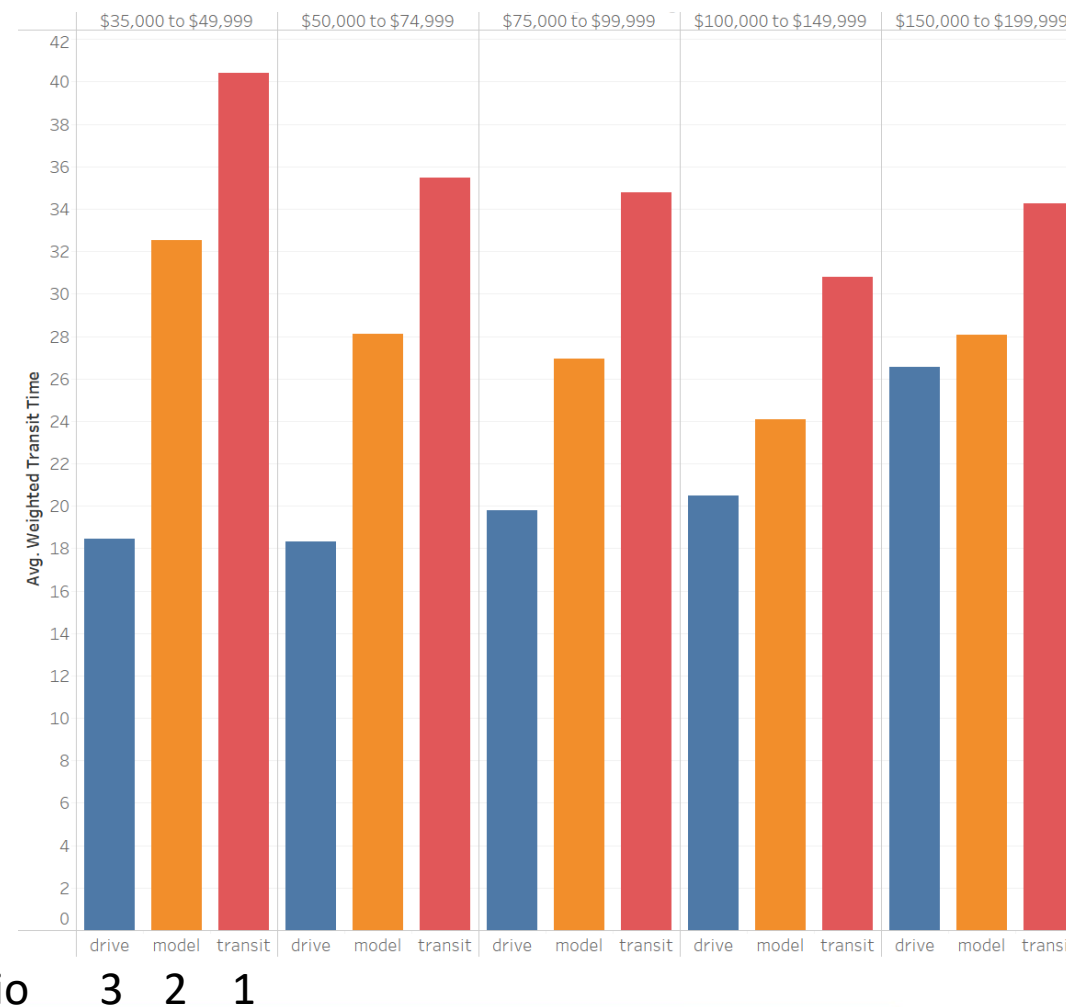
Difference in Travel time (%)



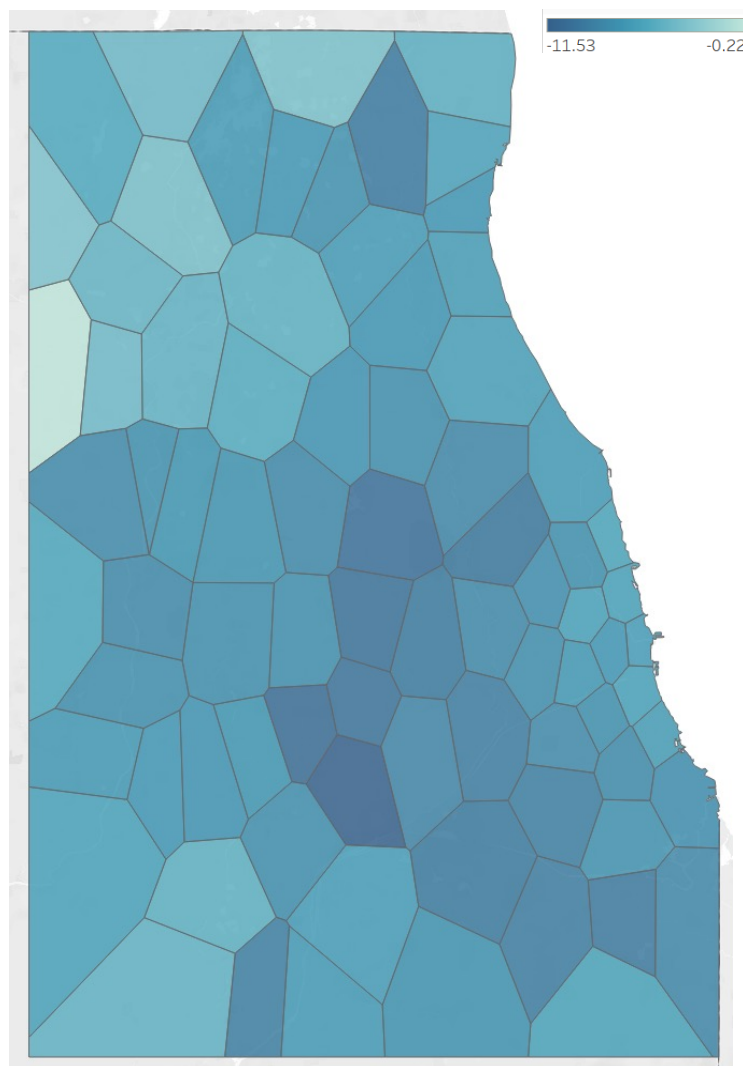


# 1. Measuring Equity in Travel Times

- Are zones with lower median incomes experiencing different travel times than zones with higher median incomes?
- How do those travel time differences vary across modes?
  - Including the new model
- When we introduce the new model, does it have more equal travel times across varying income zones?

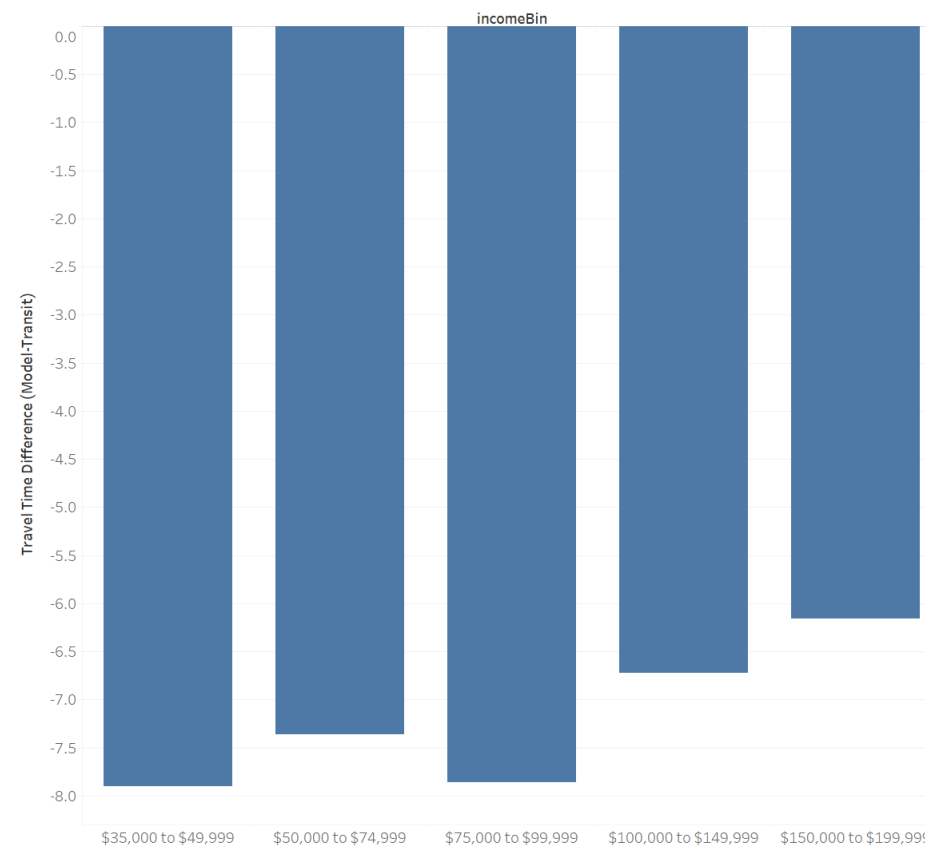


# 1. Measuring Equity in Travel Times



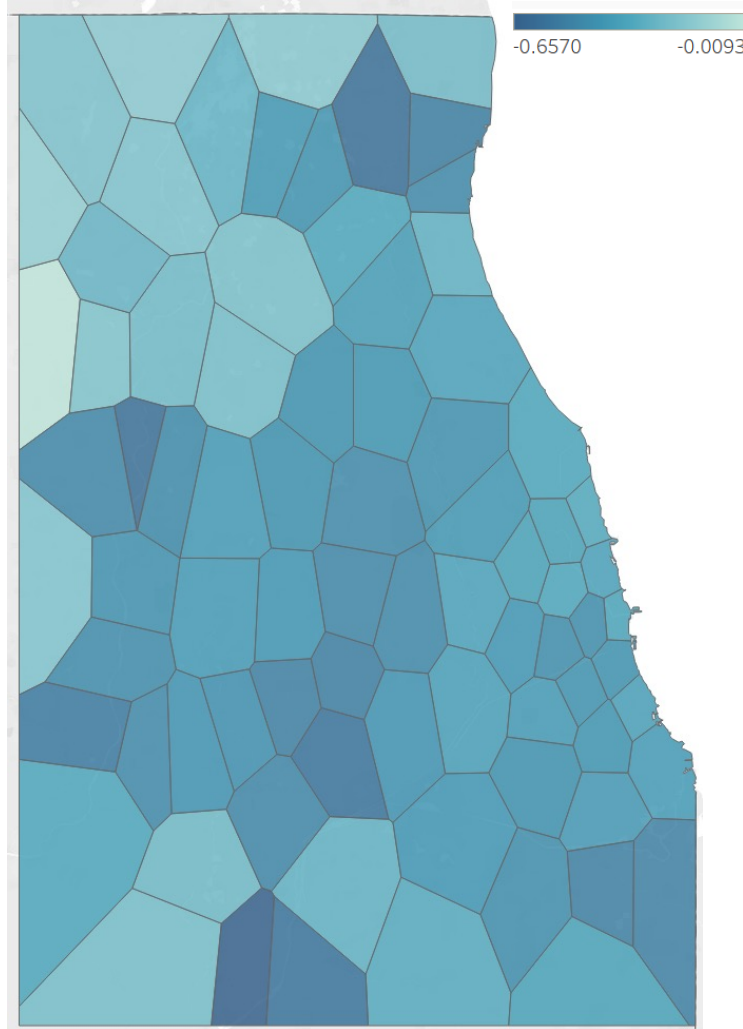
- Does the new model have equal reduction in transit travel time across zones?

Scenario 2 -  
Scenario 1



# 1. Measuring Equity in Travel Times

(Vertical equity)  
Scenario 2 -  
Scenario 1

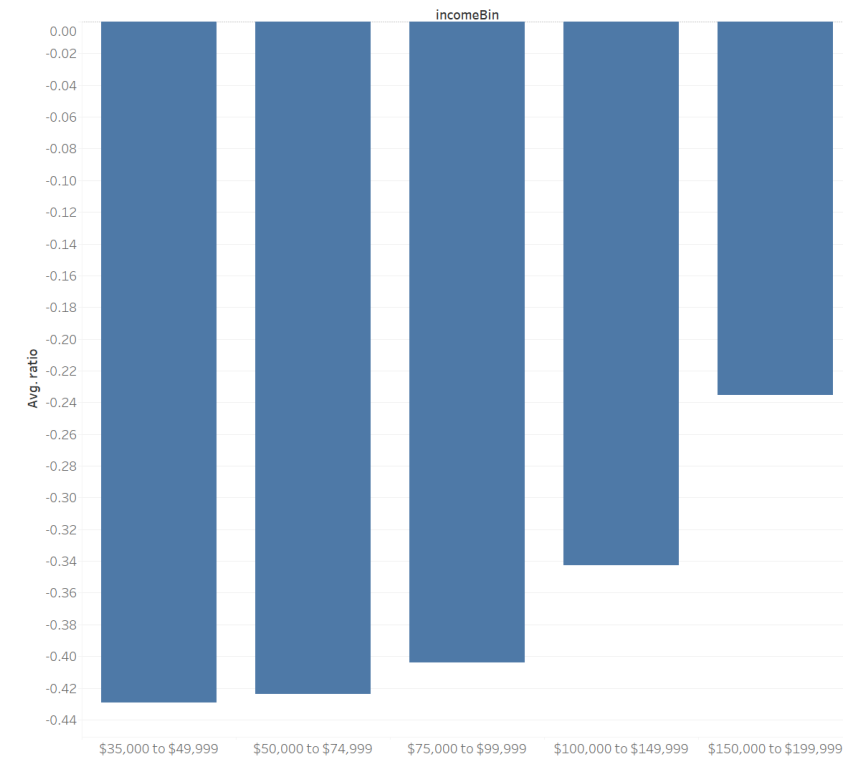


- Equity can be considered not just in the effect of the new system on different income levels but on different modes
- Does the new model reduce the inequity between driving and commute times?

$$\frac{\text{model } TT}{\text{drive } TT} - \frac{\text{transit } TT}{\text{drive } TT}$$

Golub, Aaron, and Karel Martens. "Using principles of justice to assess the modal equity of regional transportation plans." *Journal of Transport Geography* 41 (2014): 10-20.

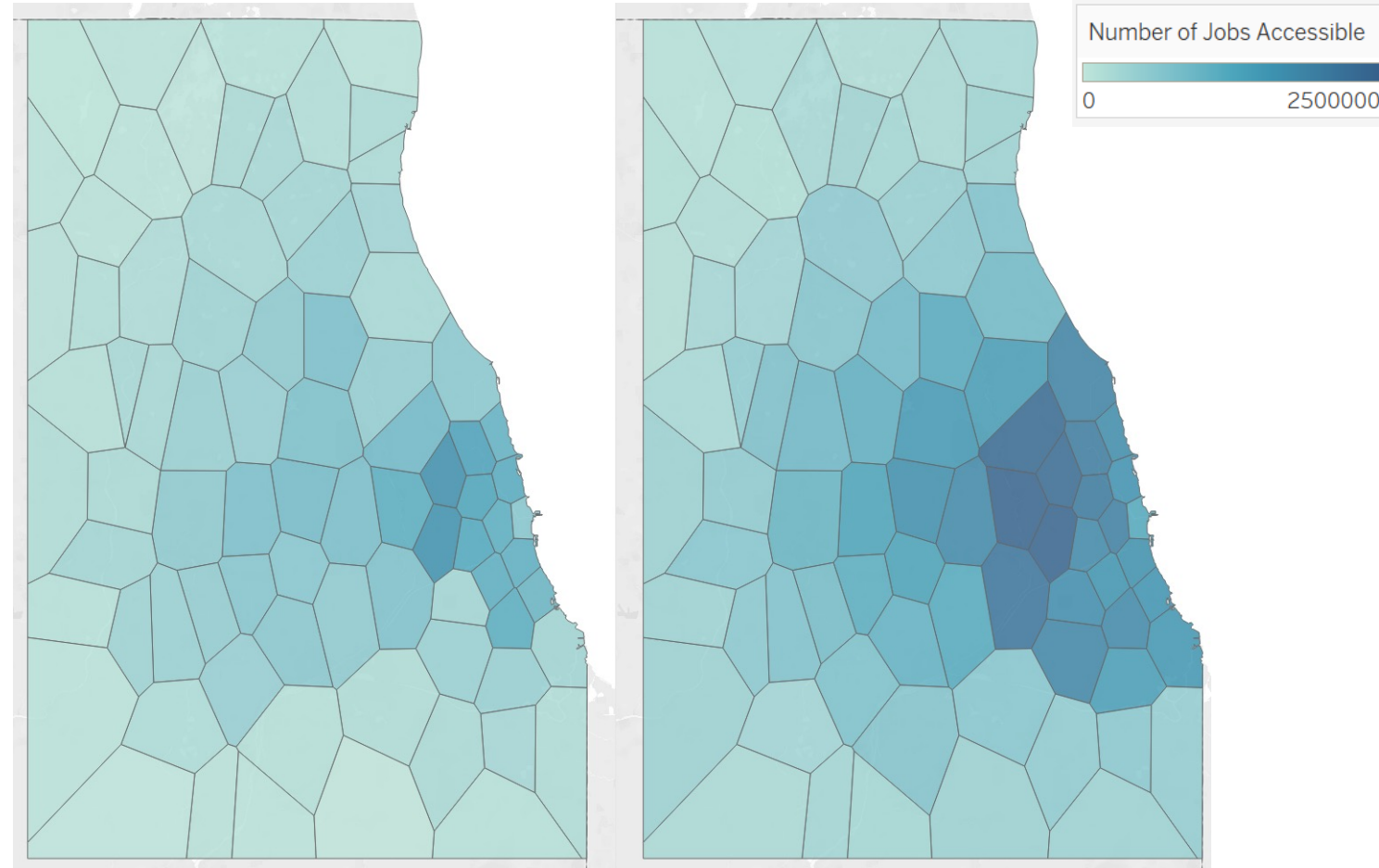
- How is the reduction distributed across income groups?



# 2. Measuring Equity in Access to Opportunities

(Horizontal equity)

- People travel for a reason- what can reduction in travel time get users to? How are differences in travel time changing the opportunities that users can reach?
- Opportunity Data: Number of Jobs, Health Care Facilities, Retail (and Retail Jobs), and Education Locations
  - Within time bounds of 45 and 60 minute journeys



45 minutes (model - Scenario 2)

60 minutes (model - Scenario 2)

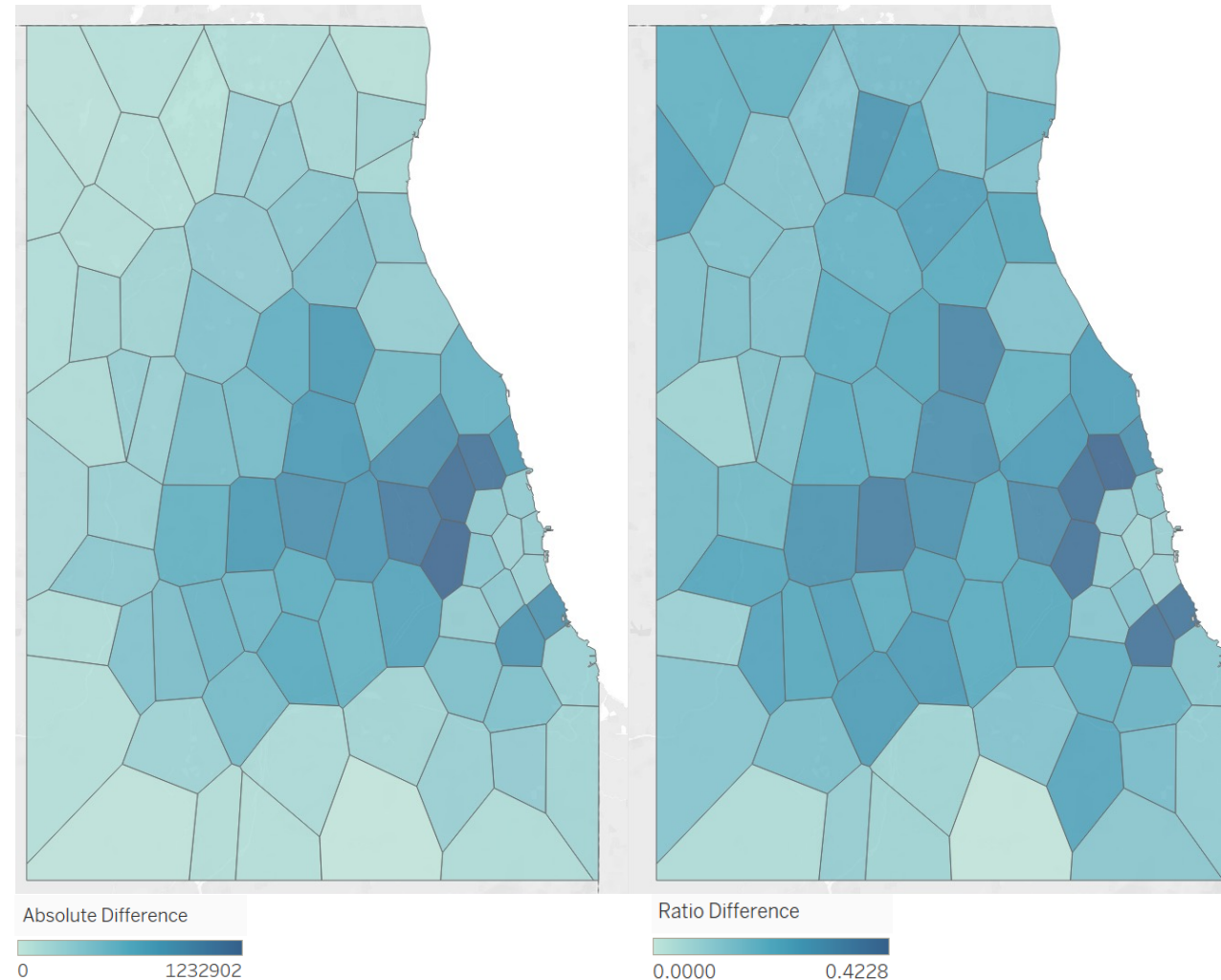
# 2. Measuring Equity in Access to Opportunities

(Horizontal equity)

- Does the model when compared to transit improve access to jobs on an absolute level?
- What about relative to driving access?

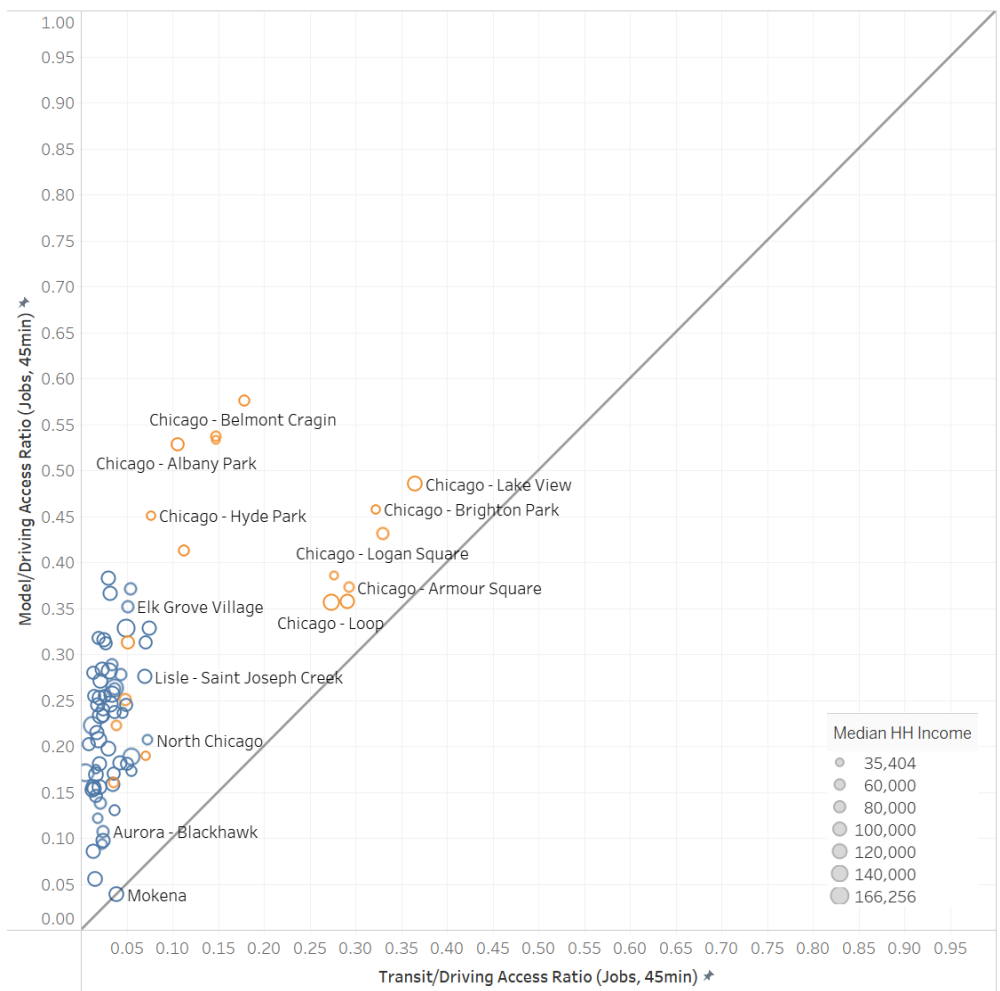
$$\frac{\#jobs\ accessible\ by\ model\ (45min)}{\#jobs\ accessible\ by\ drive\ (45min)} - \frac{\#jobs\ accessible\ by\ transit\ (45min)}{\#jobs\ accessible\ by\ drive\ (45min)}$$

Scenario 2 - Scenario 1



# 2. Measuring Equity in Access to Opportunities

Scenario 2 /  
Scenario 3



Scenario 1 /  
Scenario 3

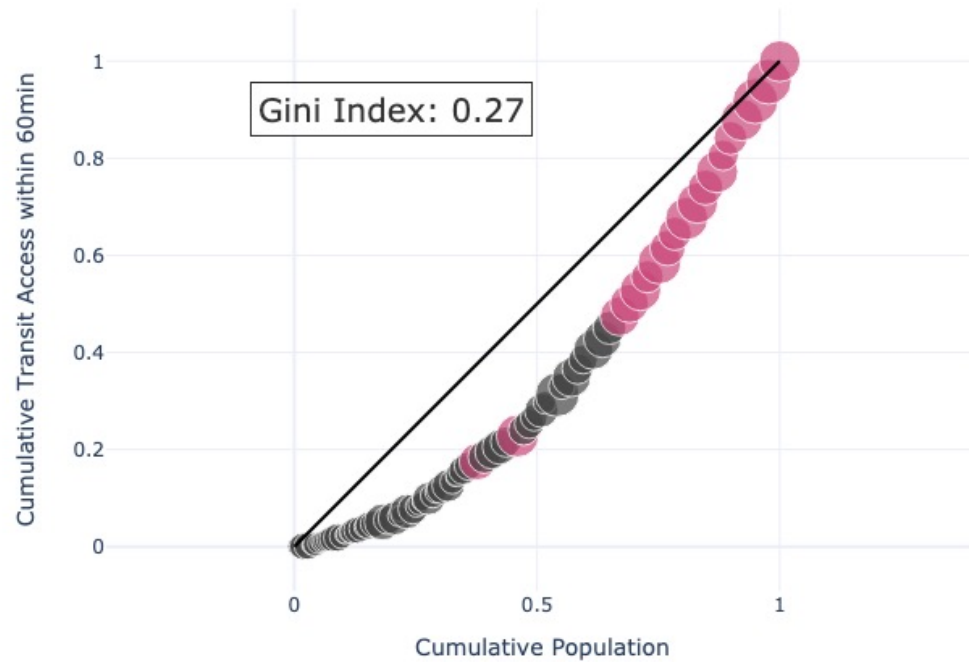


# 2. Measuring Equity in Access to Opportunities

(Horizontal equity)

- Lorenz Curve and Gini's Index  
Job access within 60 minutes of transit access

Lorenz Curve for Job Access within 60min of Existing Transit



Scenario 1

Lorenz Curve for Job Access within 60min of Redesigned Transit



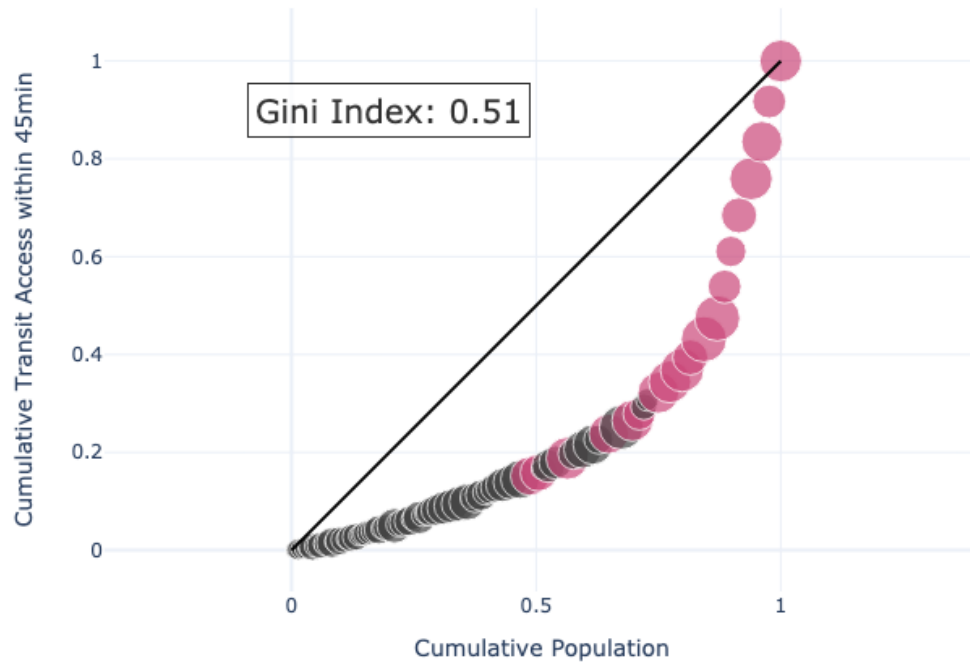
Scenario 2

# 2. Measuring Equity in Access to Opportunities

(Horizontal equity)

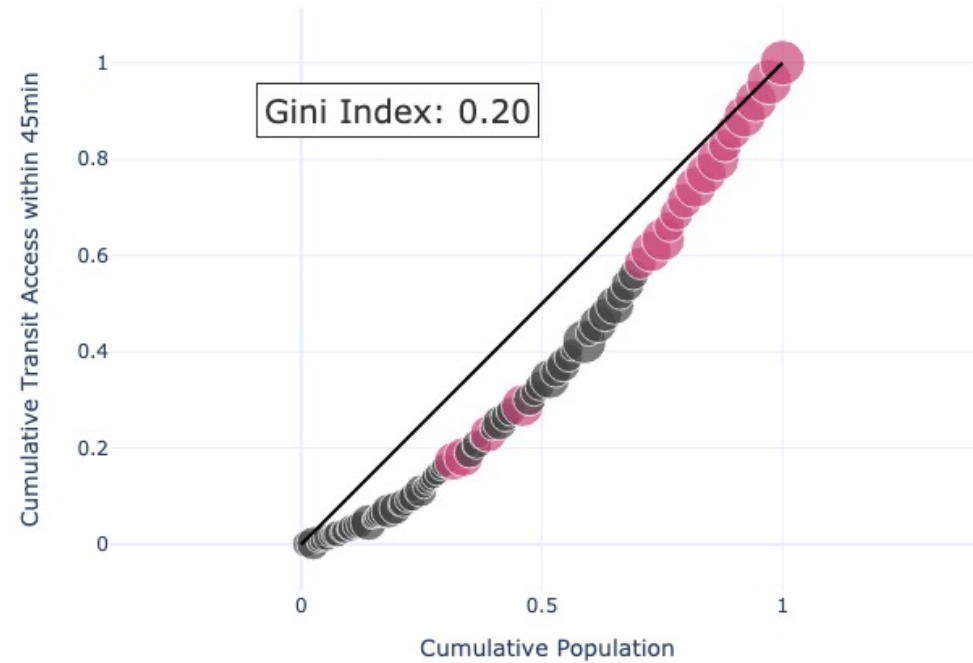
- Lorenz Curve and Gini's Index  
Job access within 45 minutes of transit access

Lorenz Curve for Job Access within 45min of Existing Transit



Scenario 1

Lorenz Curve for Job Access within 45min of Redesigned Transit



Scenario 2



# 2. Measuring Equity in Access to Opportunities

- Lorenz curve and Gini coefficient

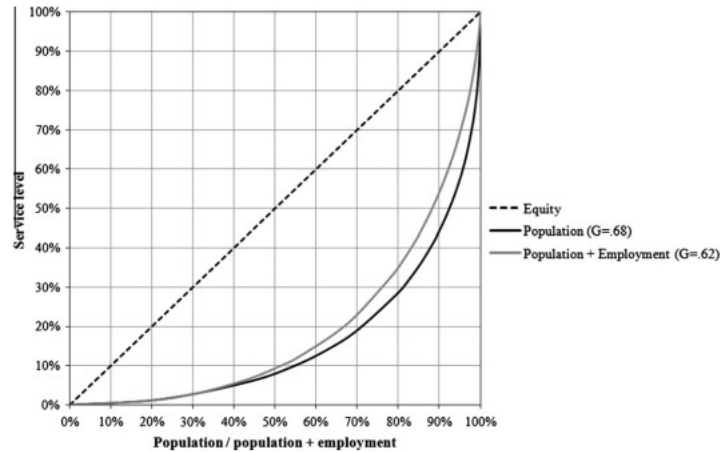
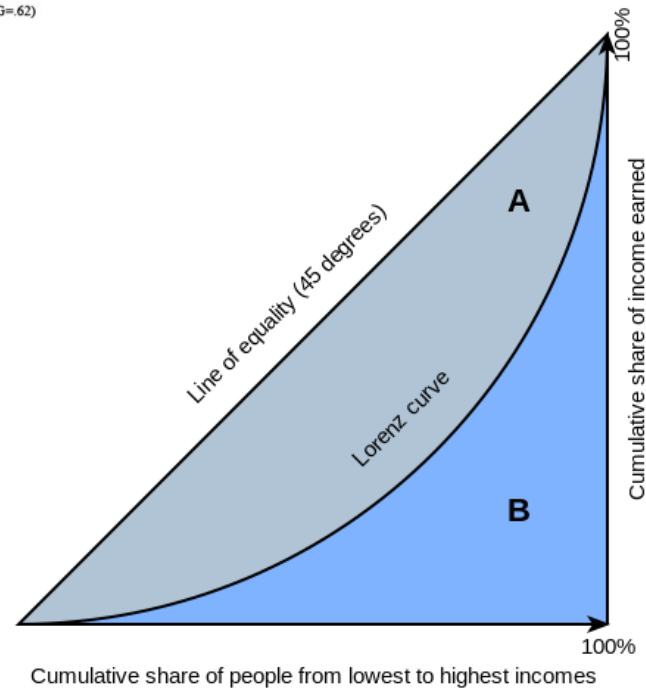


Fig. 4. Lorenz curves of population and employment.

Delbosc, A., Currie, G., 2011. Using Lorenz curves to assess public transport equity. *Journal of Transport Geography*, Special section on Alternative Travel futures 19, 1252–1259. <https://doi.org/10.1016/j.jtrangeo.2011.02.008>



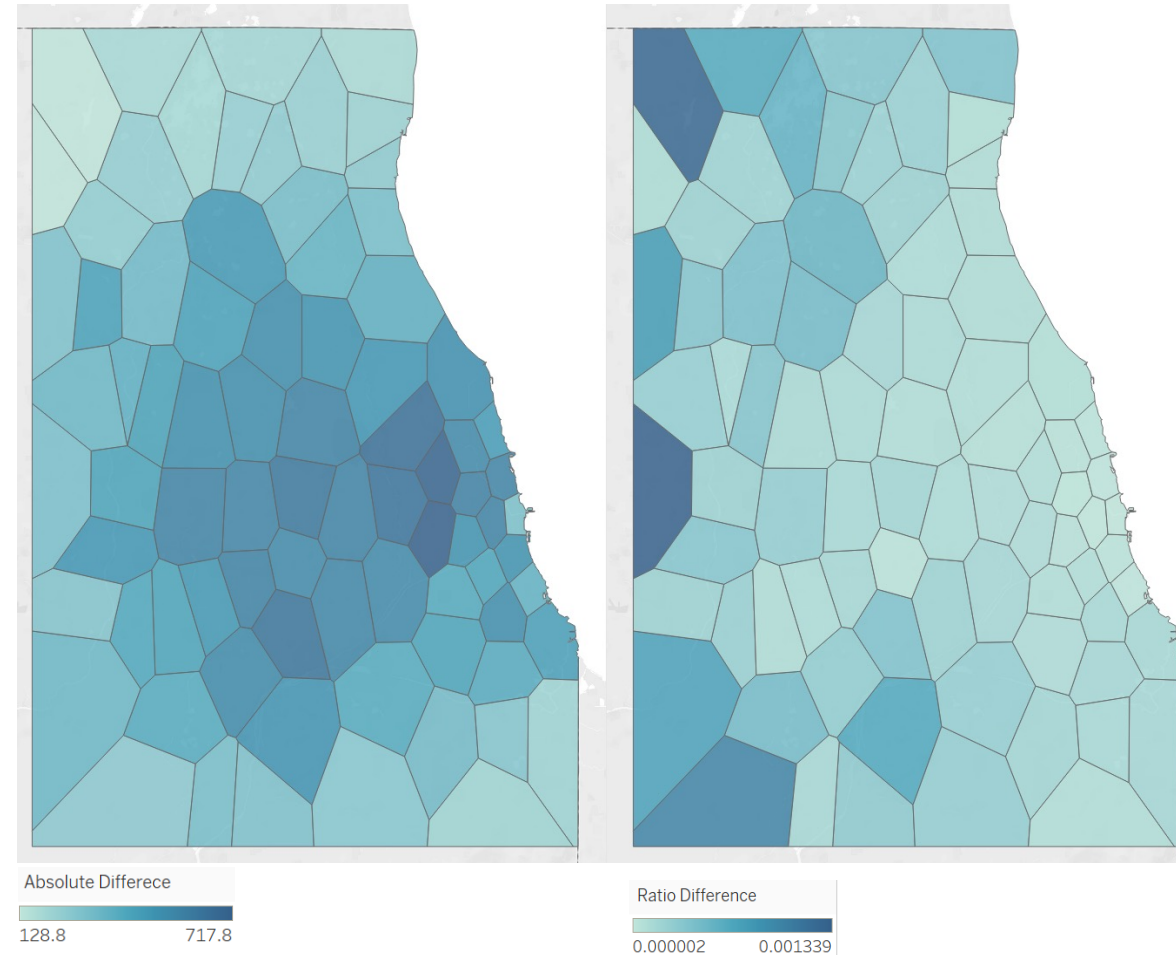
$$\text{Gini} = A/(A + B) = 1 - 2B$$

Wikipedia. 2023. "Gini coefficient." Wikimedia Foundation. Last modified July 13, 2023. [https://en.wikipedia.org/wiki/Gini\\_coefficient](https://en.wikipedia.org/wiki/Gini_coefficient).

## 2. Gravity Model

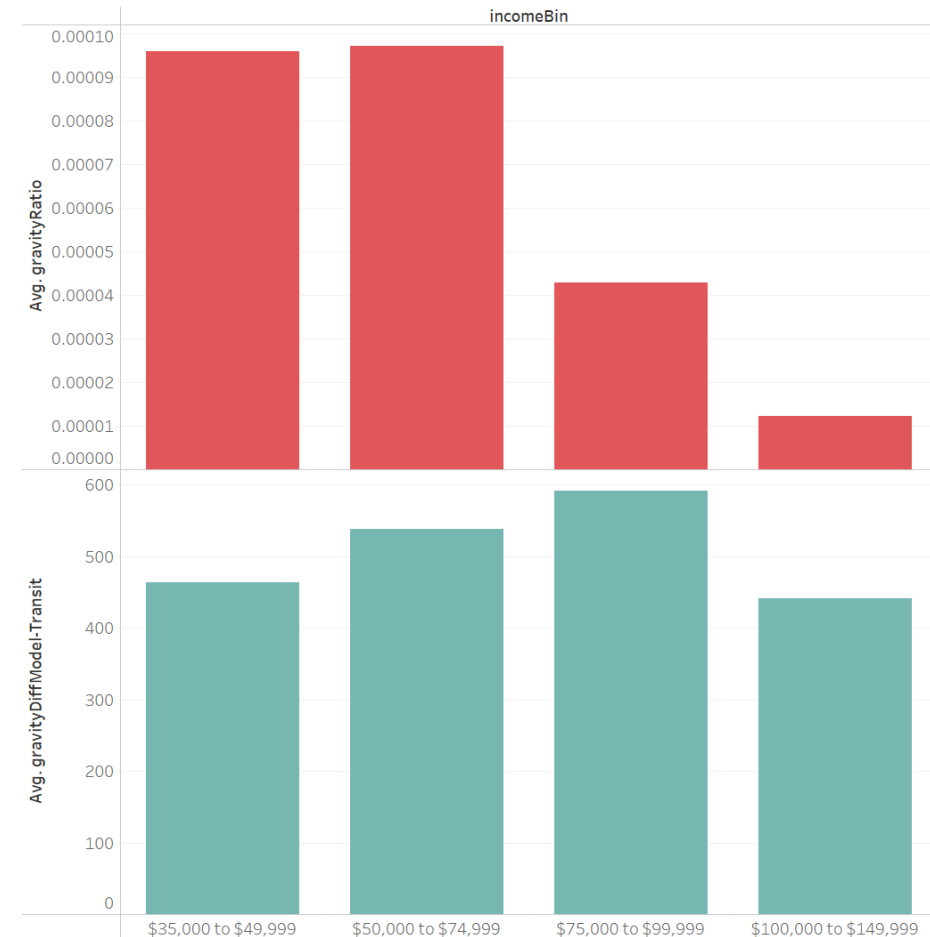
- Accessibility measure inversely proportional to the square of distance
  - $A_i$  - accessibility measure of zone  $i$
  - $O_j$  - opportunity available at zone  $j$
  - $t_{ij}$  - travel time from zone  $i$  to  $j$

$$A_i = \sum_j \frac{O_j}{t_{ij}^2}$$



## 2. Gravity Model

- Wealthy suburbs without transit access going 0 to some access skews the trends of vertical equity, but if we look just in the city, there are large gains for middle and low income zones.



# 2. Gravity Model

- Exponential gravity model

With demographics and travel times calculated, it was possible to develop accessibility measures. We used two versions of the familiar gravity model formulation of accessibility:

$$AT_i^w = \sum_j E_j^w e^{-\beta t_{ij}} \quad (1)$$

$$AW_i^w = W_i^w \frac{AT_i^w - \overline{AT}^w}{\sigma_{AT}^w} \quad (2)$$

where

$AT_i^w$   
= Territorial accessibility at stop  $i$   
for resident workers with wage level  $w$

$AW_i^w$  = Worker  
– weighted accessibility at stop  $i$   
for resident workers with wage level  $w$

$E_j^w$  = Jobs in service area sliver  $j$  with wage level  $w$

$W_i^w$  = Resident workers in service area at stop  $i$  with wage level  $w$

$\overline{AT}^w$  = Mean territorial accessibility for resident workers with wage level  $w$

$\sigma_{AT}^w$  = Standard deviation of territorial accessibility for resident workers with wage level  $w$

$t_{ij}$  = Average peak period travel time (minutes) by transit between stop  $i$  and service area sliver  $j$

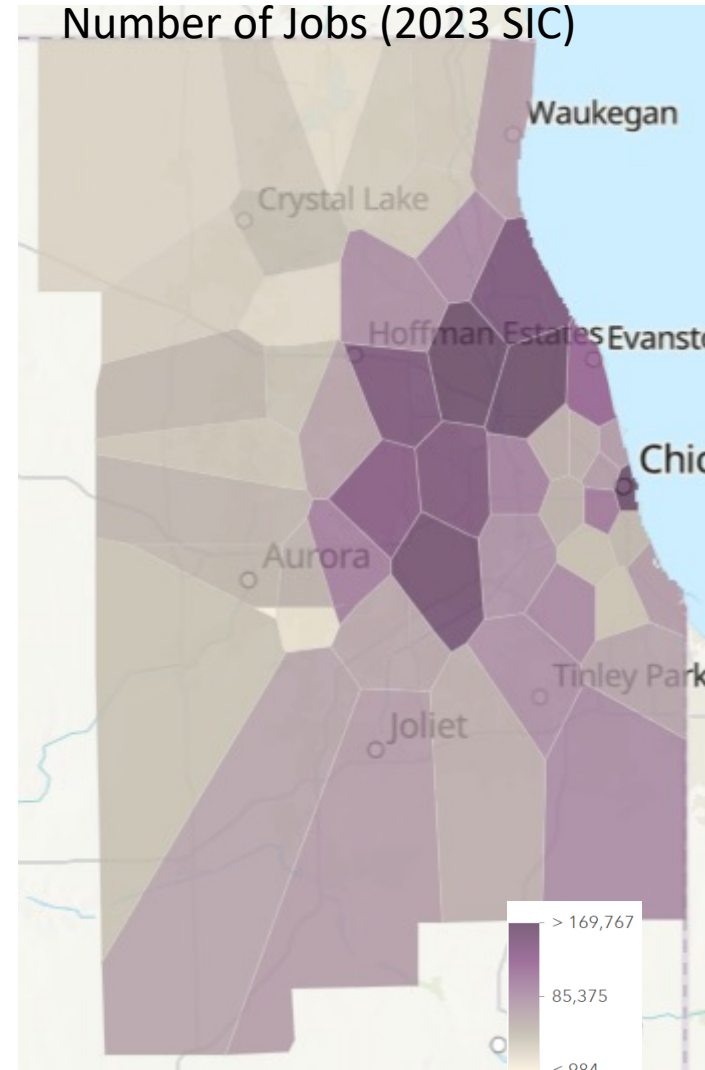
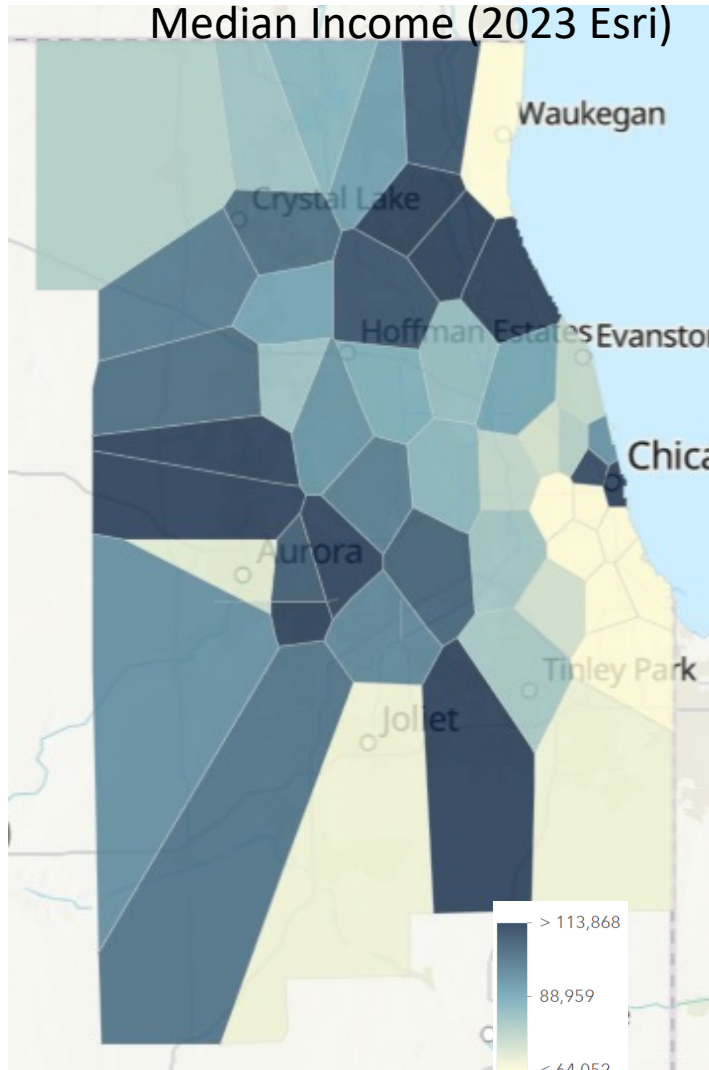
$\beta$  = empirically derived impedance term

Kamer, A., 2018. Assessing public transit service equity using route-level accessibility measures and public data. *Journal of Transport Geography* 67, 24–32. <https://doi.org/10.1016/j.jtrangeo.2018.01.005>

## Gravity model original paper:

Shen, Q., 1998. Location characteristics of inner-city neighborhoods and employment accessibility of low-wage workers. *Environment and planning B: Planning and Design* 25, 345–365.

# Income and Employment Distribution of Zones



# Things to explore

- Method to measure current travel time
  - Multiple points on Google Maps
- Different measures of equity
  - Compatibility with zones

# Other Measures of Equity

- HUD Location Affordability Index's Employment Access Index

*Variable 3: Employment Access Index*

The Employment Access Index is determined using a gravity model which considers both the quantity of and distance to all employment destinations, relative to any given Census tract. Using an inverse-square law, an employment index is calculated by summing the total number of jobs divided by the square of the distance to those jobs. This quantity allows for the examination of both the existence of jobs and the accessibility of these jobs for a given Census tract. Because a gravity model enables consideration of jobs both directly in and adjacent to a given Census tract, the employment access index gives a better measure of job opportunity, and thus a better understanding of job access than a simple employment density measure. This index also serves as a surrogate for access to economic activity.

The Employment Access Index is calculated as:

$$E \equiv \sum_{i=1}^n \frac{p_i}{r_i^2}$$

Where

$E$  = Employment Access for a given Census tract

$n$  = total number of Census tracts

$p_i$  = number of jobs in the  $i^{\text{th}}$  Census tract

$r_i$  = distance (in miles) from the center of the given Census tract to the center of the  $i^{\text{th}}$  Census tract

- Total Cost of Travel Function

Haas, P. M., Gregory L. Newmark, and T. R. Morrison. "Untangling housing cost and transportation interactions: The location affordability index model—Version 2 (LAIM2)." *Housing Policy Debate* 26.4-5 (2016): 568-582.

El-Geneidy, Ahmed, et al. "The cost of equity: Assessing transit accessibility and social disparity using total travel cost." *Transportation Research Part A: Policy and Practice* 91 (2016): 302-316.