Emission Markets III. Pricing Options on CO_2

René Carmona

ORFE, Bendheim Center for Finance
Princeton University

IPAM

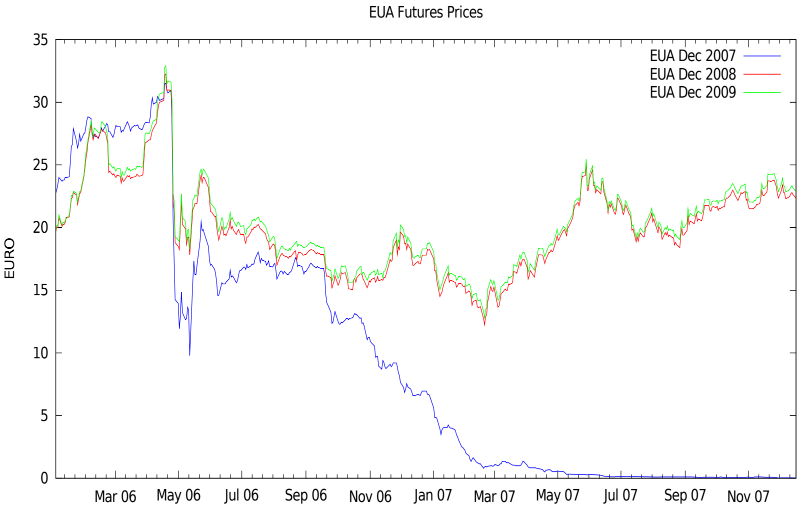
Cap-and-Trade Schemes for Emission Control

- **Cap & Trade Schemes for CO_2 Emissions**
  - Kyoto Protocol
  - Mandatory Carbon Markets **(EU ETS, RGGI since 01/01/09)**
  - Lessons learned from the EU Experience
- **Mathematical (Equilibrium) Models**
  - Price Formation for Goods and Emission Allowances
  - New Designs and Alternative Schemes
  - Calibration & Option Pricing
- **Computer Implementations**
  - Several case studies (Texas, Japan)
  - Practical Tools for Regulators and Policy Makers

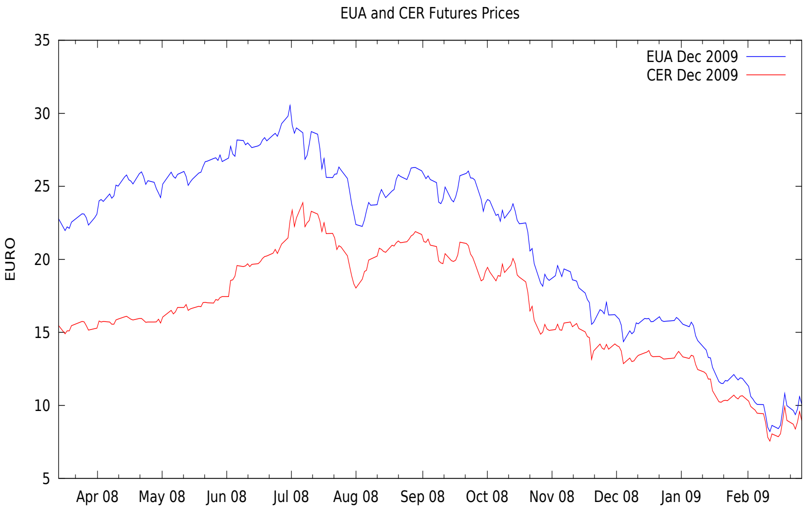
EU ETS First Phase: Main Criticism

- **No (Significant) Emissions Reduction**
  - DID Emissions go down?
  - Yes, but as part of an existing trend
- **Significant Increase in Prices**
  - Cost of Pollution passed along to the ”end-consumer”
  - Small proportion (40%) of polluters involved in EU ETS
- **Windfall Profits**
  - Cannot be avoided
  - Proposed Remedies
    - Stop Giving Allowance Certificates Away for Free !
    - Auctioning

Falling Carbon Prices: What Happened?



CDM: Can we Explain CER Prices?



Description of the Economy

- **Finite set** $\mathcal{I}$ of **risk neutral firms**
- **Producing a finite set** $\mathcal{K}$ of **goods**
- Firm $i \in \mathcal{I}$ can use **technology** $j \in \mathcal{J}^{i,k}$ to produce good $k \in \mathcal{K}$
- **Discrete time** $\{0, 1, \cdots, T\}$
- **No Discounting** Work with T-Forward Prices
- **Inelastic Demand**

$$\{D^k(t);\ t = 0, 1, \cdots, T-1,\ k \in \mathcal{K}\}.$$

- $\cdots\cdots\cdots\cdots$

Regulator Input (EU ETS)

At inception of program (i.e. time $t = 0$)

- **INITIAL DISTRIBUTION** of **allowance certificates**

$$\theta_0^i \qquad \text{to firm } i \in \mathcal{I}$$

- Set **PENALTY** π for emission unit **NOT** offset by allowance certificate at end of **compliance period**

Extensions (not discussed in this talk)

- **Risk aversion** and agent preferences (existence theory easy)
- **Elastic** demand (e.g. smart meters for electricity)
- **Multi-period models** with lending, borrowing and withdrawal (more realistic)
- . . . .

Goal of Equilibrium Analysis

Find **two stochastic processes**

- **Price of one allowance**

$$A = \{A_t\}_{t \geq 0}$$

- **Prices of goods**

$$S = \{S_t^k\}_{k \in K,\, t \geq 0}$$

satisfying the usual conditions for the existence of a

competitive equilibrium

(to be spelled out below).

Individual Firm Problem

During each time period $[t, t+1)$

- Firm $i \in \mathcal{I}$ **produces** $\xi_t^{i,j,k}$ of good $k \in \mathcal{K}$ with technology $j \in \mathcal{J}^{i,k}$
- Firm $i \in \mathcal{I}$ **holds** a position θ_t^i in emission credits

$$L^{A,S,i}(\theta^i, \xi^i) := \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{J}^{i,k}} \sum_{t=0}^{T-1} (S_t^k - C_t^{i,j,k}) \xi_t^{i,j,k}$$
$$+ \theta_0^i A_0 + \sum_{t=0}^{T-1} \theta_{t+1}^i (A_{t+1} - A_t) - \theta_{T+1}^i A_T$$
$$- \pi(\Gamma^i + \Pi^i(\xi^i) - \theta_{T+1}^i)^+$$

where

$$\Gamma^i \text{ random}, \qquad \Pi^i(\xi^i) := \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{J}^{i,k}} \sum_{t=0}^{T-1} e^{i,j,k} \xi_t^{i,j,k}$$

Problem for (risk neutral) firm $i \in I$

$$\max_{(\theta^i, \xi^i)} \mathbb{E}\{L^{A,S,i}(\theta^i, \xi^i)\}$$

Business As Usual (i.e. $\pi = 0$)

The corresponding prices of the goods are

$$S_t^{*k} = \max_{i \in \mathcal{I},\, j \in \mathcal{J}^{i,k}} C_t^{i,j,k} \mathbf{1}_{\{\xi_t^{*i,j,k} > 0\}},$$

Classical **MERIT ORDER**

- At each time t and for each good k
- Production technologies ranked by increasing production costs $C_t^{i,j,k}$
- Demand D_t^k met by producing from the cheapest technology first
- Equilibrium spot price is the marginal cost of production of the most expansive production technoligy used to meet demand

Business As Usual

(typical scenario in Deregulated **electricity markets**)

Equilibrium Definition for Emissions Market

The processes $A^* = \{A_t^*\}_{t=0,1,\cdots,T}$ and $S^* = \{S_t^*\}_{t=0,1,\cdots,T}$ form an equilibrium if for each agent $i \in \mathcal{I}$ there exist strategies $\theta^{*i} = \{\theta_t^{*i}\}_{t=0,1,\cdots,T}$ (**trading**) and $\xi^{*i} = \{\xi_t^{*i}\}_{t=0,1,\cdots,T}$ (**production**)

- **(i) All financial positions are in constant net supply**

$$\sum_{i \in I} \theta_t^{*i} = \sum_{i \in I} \theta_0^i, \qquad \forall\, t = 0, \dots, T+1$$

- **(ii) Supply meets Demand**

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}^{i,k}} \xi_t^{*i,j,k} = D_t^k, \qquad \forall\, k \in \mathcal{K}, \quad t = 0, \dots, T-1$$

- **(iii) Each agent $i \in I$ is satisfied by its own strategy**

$$\mathbb{E}[L^{A^*,S^*,i}(\theta^{*i}, \xi^{*i})] \geq \mathbb{E}[L^{A^*,S^*,i}(\theta^i, \xi^i)] \qquad \text{for all } (\theta^i, \xi^i)$$

Necessary Conditions

Assume

- (A^*, S^*) is an equilibrium
- (θ^{*i}, ξ^{*i}) optimal strategy of agent $i \in I$

then

- The allowance price A^* is a **bounded martingale** in $[0, \pi]$
- Its terminal value is given by

$$A^*_T \;=\; \pi \mathbf{1}_{\{\Gamma^i + \Pi(\xi^{*i}) - \theta^{*i}_{T+1} \geq 0\}} \;=\; \pi \mathbf{1}_{\{\sum_{i \in \mathcal{I}} (\Gamma^i + \Pi(\xi^{*i}) - \theta^{*i}_0) \geq 0\}}$$

- The **spot prices** S^{*k} of the goods and the **optimal production strategies** ξ^{*i} are given by the **merit order** for the equilibrium with **adjusted costs**

$$\tilde{C}^{i,j,k}_t = C^{i,j,k}_t + e^{i,j,k} A^*_t$$

Existence by Social Cost Minimization

- **Overall production costs**

$$C(\xi) := \sum_{t=0}^{T-1} \sum_{(i,j,k)} \xi_t^{i,j,k} C_t^{i,j,k}.$$

- **Overall cumulative emissions**

$$\Gamma := \sum_{i \in I} \Gamma^i \qquad \Pi(\xi) := \sum_{t=0}^{T-1} \sum_{(i,j,k)} e^{i,j,k} \xi_t^{i,j,k},$$

- **Total allowances**

$$\theta_0 := \sum_{i \in I} \theta_0^i$$

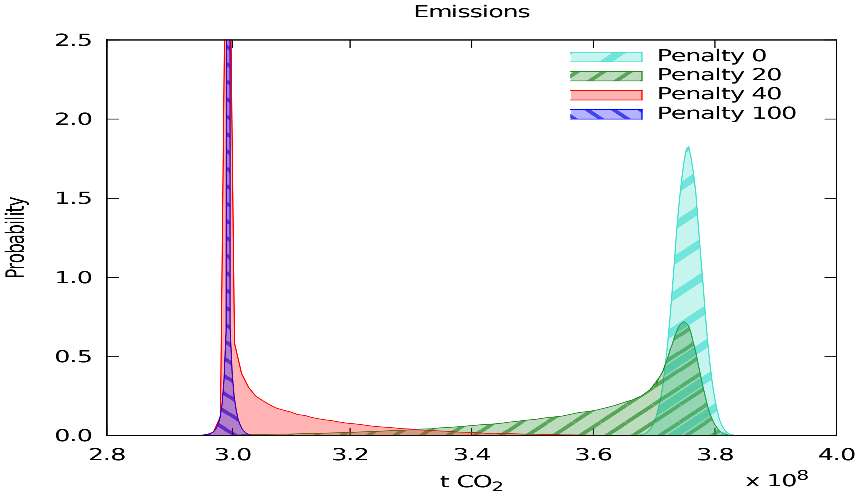
The **total social costs from production and penalty payments**

$$G(\xi) := C(\xi) + \pi(\Gamma + \Pi(\xi) - \theta_0)^+$$

We introduce the global optimization problem

$$\xi^* = \arg \inf_{\xi \text{meets demands}} \mathbb{E}[G(\xi)],$$

Effect of the Penalty on Emissions



Costs in a Cap-and-Trade

- **Consumer Burden**

$$SC = \sum_t \sum_k (S_t^{k,*} - S_t^{k,BAU*}) D_t^k.$$

- **Reduction Costs** (producers' burden)

$$\sum_t \sum_{i,j,k} (\xi_t^{i,j,k*} - \xi_t^{BAU,i,j,k*}) C_t^{i,j,k}$$

- **Excess Profit**

$$\sum_t \sum_k (S_t^{k,*} - S_t^{k,BAU*}) D_t^k - \sum_t \sum_{i,j,k} (\xi_t^{i,j,k*} - \xi_t^{BAU,i,j,k*}) C_t^{i,j,k} - \pi (\sum_t \sum_{ijk} \xi_t^{ijk} e_t^{ijk} - \theta_0)^+$$

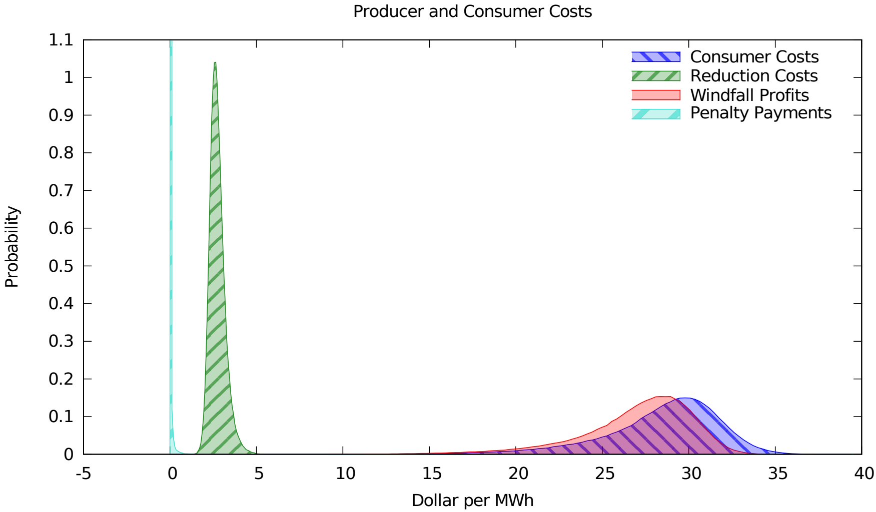
- **Windfall Profits**

$$WP = \sum_{t=0}^{T-1} \sum_{k \in K} (S_t^{*k} - \hat{S}_t^k) D_t^k$$

where

$$\hat{S}_t^k := \max_{i \in I, j \in J^{i,k}} C_t^{i,j,k} \mathbf{1}_{\{\xi_t^{*i,j,k} > 0\}}.$$

Costs in a Cap-and-Trade Scheme



Histograms of consumer costs, social costs, windfall profits and penalty payments of a standard cap-and-trade scheme calibrated to reach the emissions target with 95% probability and BAU.

One of many Possible Generalizations

Introduction of **Taxes** / **Subsidies**

$$\ddot{L}^{A,S,i}(\theta^i,\xi^i) = -\sum_{t=0}^{T-1} G_t^i + \sum_{k\in K}\sum_{j\in J^{i,k}}\sum_{t=0}^{T-1}(S_t^k - C_t^{i,j,k} - H_t^k)\xi_t^{i,j,k}$$
$$+\sum_{t=0}^{T-1}\theta_t^i(A_{t+1}-A_t) - \theta_T^i A_T$$
$$-\pi(\Gamma^i + \Pi^i(\xi^i) - \theta_T^i)^+.$$

In this case

- In equilibrium, **production** and **trading** strategies remain the same $(\theta^\dagger,\xi^\dagger)=(\theta^*,\xi^*)$
- **Abatement costs** and **Emissions reductions** are also the same
- New equilibrium prices $(A^\dagger, S^\dagger)$ given by

$$A_t^\dagger = A_t^* \quad \text{for all } t = 0,\dots,T \tag{1}$$
$$S_t^{\dagger k} = S_t^{*k} + H_t^k \quad \text{for all } k\in K, t=0,\dots,T-1 \tag{2}$$

- Cost of the tax passed along to the end consumer

Alternative Market Design

- **Currently Regulator Specifies**
  - Penalty π
  - Overall Certificate Allocation θ_0 $(= \sum_{i\in I} \theta_0^i)$
- **Alternative Scheme (Still) Controlled by Regulator**
  (i) **Sets penalty level** π
  (ii) **Allocates allowances**
    - θ_0' at inception of program $t = 0$
    - then **proportionally to production**

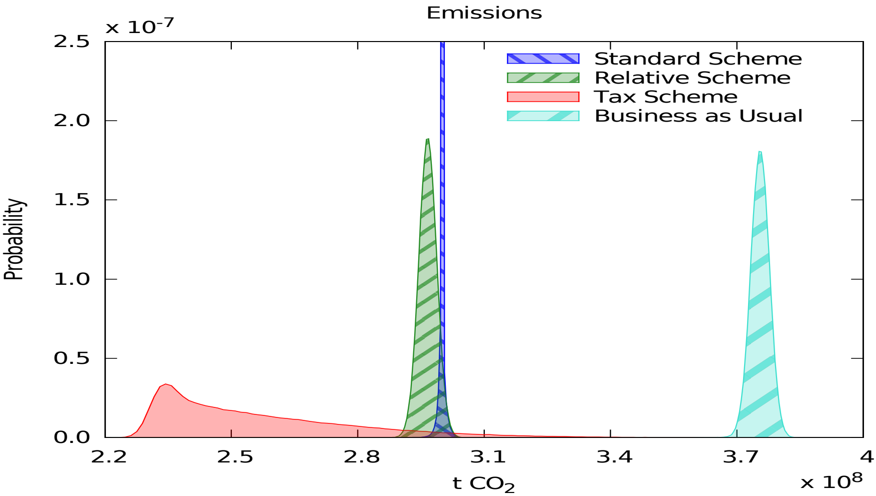
  $y\xi_t^{i,j,k}$ to agent i for producing $\xi_t^{i,j,k}$ of good k with technology j

  (iii) **Calibrates** y, e.g. in **expectation**.

$$y = \frac{\theta_0 - \theta_0'}{\sum_{t=0}^{T-1} \sum_{k\in K} \mathbb{E}\{D_t^k\}}$$

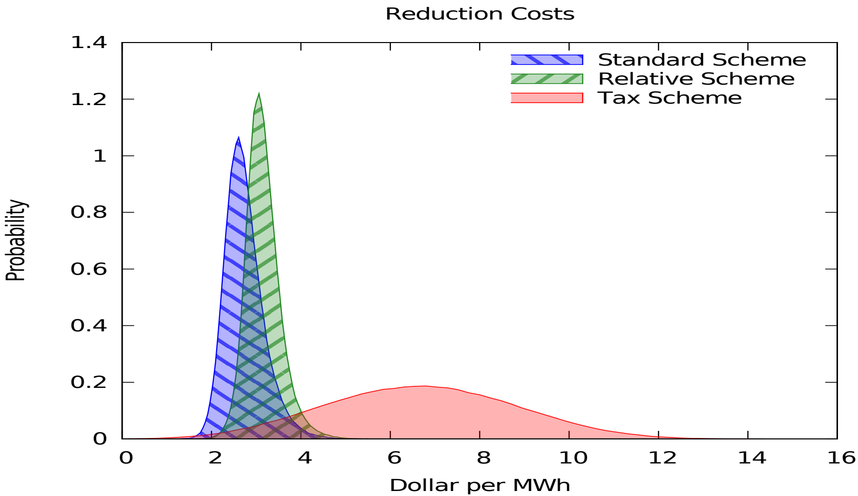
So total number of credit allowance is the same in expectation, i.e. $\theta_0 = \mathbb{E}\{\theta_0' + y \sum_{t=0}^{T-1} \sum_{k\in K} D_t^k\}$

Yearly Emissions Equilibrium Distributions



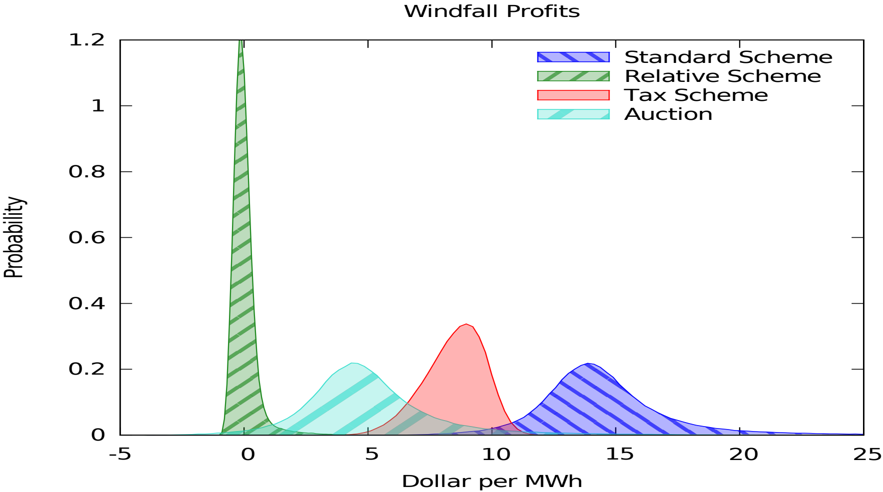
Yearly emissions from electricity production for the Standard Scheme, the Relative Scheme, a Tax Scheme and BAU.

Abatement Costs



Yearly abatement costs for the Standard Scheme, the Relative Scheme and a Tax Scheme.

Windfall Profits



Histograms of the yearly distribution of windfall profits for the Standard Scheme, a Relative Scheme, a Standard Scheme with 100% Auction and a Tax Scheme

Reduced Form Models & Option Pricing

(**Uhrig-Homburg-Wagner, R.C - Hinz**)

- Emissions Cap-and-Trade Markets **SOON** to exist in the US (and Canada, Australia, Japan, ...)
- Liquid **Option** Market **ALREADY** exists in Europe
  - Underlying $\{A_t\}_t$ non-negative martingale with **binary terminal value**
  - Think of A_t as of a binary option
  - Underlying of binary option should be *Emissions*
- Need for **Formulae** (closed or computable)
  - Prices and Hedges difficult to compute (only numerically)
  - to study effect of announcements (**Cetin** et al.)
- **Reduced Form Models**

Option quotes on Jan. 3, 2008

Option Maturity	Option Type	Volume	Strike	Allowance Price	Implied Vol	Settlement Price
Dec-08	Call	150,000	24.00	23.54	50.50%	4.19
Dec-08	Call	500,000	26.00	23.54	50.50%	3.50
Dec-08	Call	25,000	27.00	23.54	50.50%	3.20
Dec-08	Call	300,000	35.00	23.54	50.50%	1.56
Dec-08	Call	1,000,000	40.00	23.54	50.50%	1.00
Dec-08	Put	200,000	15.00	23.54	50.50%	0.83

Option quotes on Jan. 4, 2008

Option Maturity	Option Type	Volume	Strike	Allowance Price	Implied Vol	Settlement Price
Dec-08	Cal	200,000	22.00	23.55	51.25%	5.06
Dec-08	Call	150,000	26.00	23.55	51.25%	3.57
Dec-08	Call	450,000	27.00	23.55	51.25%	3.27
Dec-08	Call	100,000	28.00	23.55	51.25%	2.99
Dec-08	Call	125,000	29.00	23.55	51.25%	2.74
Dec-08	Call	525,000	30.00	23.55	51.25%	2.51
Dec-08	Call	250,000	40.00	23.55	51.25%	1.04
Dec-08	Call	700,000	50.00	23.55	51.25%	0.45
Dec-08	Put	1,000,000	14.00	23.55	51.25%	0.64
Dec-08	Put	200,000	15.00	23.55	51.25%	0.86
Dec-08	Put	200,000	15.00	23.55	51.25%	0.86
Dec-08	Put	400,000	16.00	23.55	51.25%	1.13
Dec-08	Put	100,000	17.00	23.55	51.25%	1.43
Dec-08	Put	1,000,000	18.00	23.55	51.25%	1.78
Dec-08	Put	500,000	20.00	23.55	51.25%	2.60
Dec-08	Put	200,000	21.00	23.55	51.25%	3.07
Dec-08	Put	200,000	22.00	23.55	51.25%	3.57

Reduced Form Models and Calibration

Allowance price should be of the form

$$A_t = \pi \mathbb{E}\{\mathbf{1}_N \,|\mathcal{F}_t\}$$

for a non-compliance set $N \in \mathcal{F}_t$. Choose

$$N = \{\Gamma_T \geq 1\}$$

for a random variable Γ_T representing the normalized emissions at compliance time. So

$$A_t = \pi \mathbb{E}\{\mathbf{1}_{\{\Gamma_T \geq 1\}} \,|\mathcal{F}_t\}, \qquad t \in [0, T]$$

We choose Γ_T in a parametric family

$$\Gamma_T = \Gamma_0 \exp\Big[\int_0^T \sigma_s dW_s - \frac{1}{2}\int_0^T \sigma_s^2\, ds\Big]$$

for some square integrable deterministic function

$$(0, T) \ni t \hookrightarrow \sigma_t$$

Dynamic Price Model for $a_t = \frac{1}{\pi} A_t$

- a_t is given by

$$a_t = \Phi\left(\frac{\Phi^{-1}(a_0)\sqrt{\int_0^T \sigma_s^2 ds} + \int_0^t \sigma_s dW_s}{\sqrt{\int_t^T \sigma_s^2 ds}}\right) \qquad t \in [0, T)$$

  where Φ is standard normal c.d.f.
- a_t solves the SDE

$$da_t = \Phi'(\Phi^{-1}(a_t))\sqrt{z_t}dW_t$$

  where the positive-valued function $(0, T) \ni t \hookrightarrow z_t$ is given by

$$z_t = \frac{\sigma_t^2}{\int_t^T \sigma_u^2 du}, \qquad t \in (0, T)$$

Risk Neutral Densities

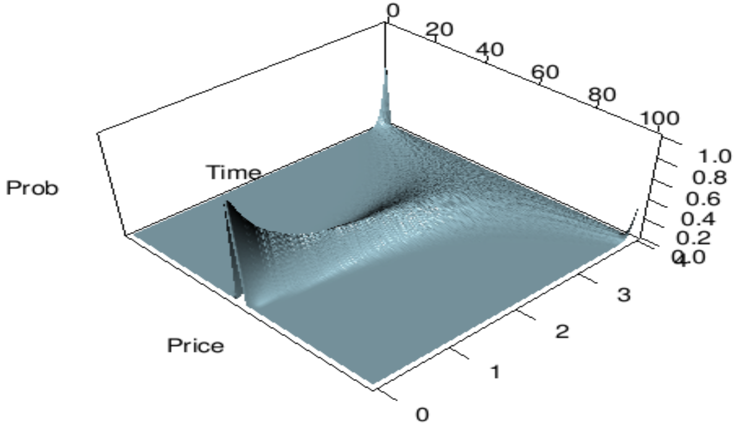


Figure: Histograms for each day of a 4 yr compliance period of 10^5 simulated risk neutral allowance price paths.

Aside: Binary Martingales as Underliers

Allowance prices are given by $A_t = \pi a_t$ where $\{a_t\}_{0 \le t \le T}$ satisfies

- $\{a_t\}_t$ is a martingale
- $0 \le a_t \le 1$
- $\mathbb{P}\{\lim_{t \to T} a_t = 1\} = 1 - \mathbb{P}\{\lim_{t \to T} a_t = 0\} = p$ for some $p \in (0, 1)$

The model

$$da_t = \Phi'(\Phi^{-1}(a_t))\sqrt{z_t} dW_t$$

suggests looking for martingales $\{Y_t\}_{0 \le t < \infty}$ satisfying

- $0 \le Y_t \le 1$
- $\mathbb{P}\{\lim_{t \to \infty} Y_t = 1\} = 1 - \mathbb{P}\{\lim_{t \to \infty} T_t = 0\} = p$ for some $p \in (0, 1)$

and do a **time change** to get back to the (compliance) interval $[0, T)$

Gives conditions for the SDE

$$da_t = \Theta(a_t)dW_t$$

for $x \hookrightarrow \Theta(x)$ satisfying

- $\Theta(x) > 0$ for $0 < x < 1$
- $\Theta(0) = \Theta(1) = 0$

to

- Converge to the boundaries 0 and 1
- NOT explode (i.e. NOT reach the boundaries in finite time)

Interestingly enough the solution of

$$dY_t = \Phi'(\Phi^{-1}(Y_t))dW_t$$

IS ONE OF THEM !

Explicit Examples

The SDE

$$dX_t = \sqrt{2}dW_t + X_t dt$$

has the solution

$$X_t = e^t \left(x_0 + \int_0^t e^{-s} dW_s\right)$$

and

$$\lim_{t\to\infty} X_t = +\infty \qquad \text{on the set } \left\{\int_0^\infty e^{-s} dW_s > -x_0\right\}$$

$$\lim_{t\to\infty} X_t = -\infty \qquad \text{on the set } \left\{\int_0^\infty e^{-s} dW_s < -x_0\right\}$$

Moreover Φ is **harmonic** so if we choose

$$Y_t = \Phi(X_t)$$

we have a martingale with the desired properties.

Another (explicit) example can be constructed from Ph. Carmona, Petit and Yor on Dufresne formula.

Calibration

Has to Be Historical !!!!

- Choose **Constant** Market Price of Risk
- **Two-parameter** Family for Time-change

$$\{z_t(\alpha,\beta) = \beta(T-t)^{-\alpha}\}_{t\in[0,T]}, \qquad \beta > 0, \alpha \geq 1.$$

Volatility function $\{\sigma_t(\alpha,\beta)\}_{t\in(0,T)}$ given by

$$\begin{aligned} \sigma_t(\alpha,\beta)^2 &= z_t(\alpha,\beta)e^{-\int_0^t z_u(\alpha,\beta)du} \\ &= \begin{cases} \beta(T-t)^{-\alpha}e^{\beta\frac{T^{-\alpha+1}-(T-t)^{-\alpha+1}}{-\alpha+1}} & \text{for } \beta > 0, \alpha > 1 \\ \beta(T-t)^{\beta-1}T^{-\beta} & \text{for } \beta > 0, \alpha = 1 \end{cases} \end{aligned}$$

Maximum Likelihood

Sample Data

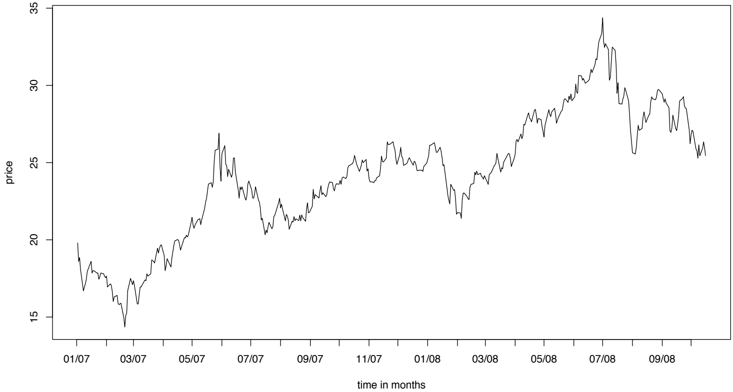


Figure: Future prices on EUA with maturity Dec. 2012

Call Option Price in One Period Model

for $\alpha = 1$, $\beta > 0$, the price of an European call with strike price $K \geq 0$ written on a one-period allowance futures price at time $\tau \in [0, T]$ is given at time $t \in [0, \tau]$ by

$$
\begin{aligned}
C_t &= e^{-\int_t^\tau r_s ds} \mathbb{E}\{(A_\tau - K)^+ \mid \mathcal{F}_t\} \\
&= \int (\pi \Phi(x) - K)^+ N(\mu_{t,\tau}, \nu_{t,\tau})(dx)
\end{aligned}
$$

where

$$
\begin{aligned}
\mu_{t,\tau} &= \Phi^{-1}(A_t/\pi) \sqrt{\left(\frac{T-t}{T-\tau}\right)^\beta} \\
\nu_{t,\tau} &= \left(\frac{T-t}{T-\tau}\right)^\beta - 1.
\end{aligned}
$$

Price Dependence on T and Sensitivity to β

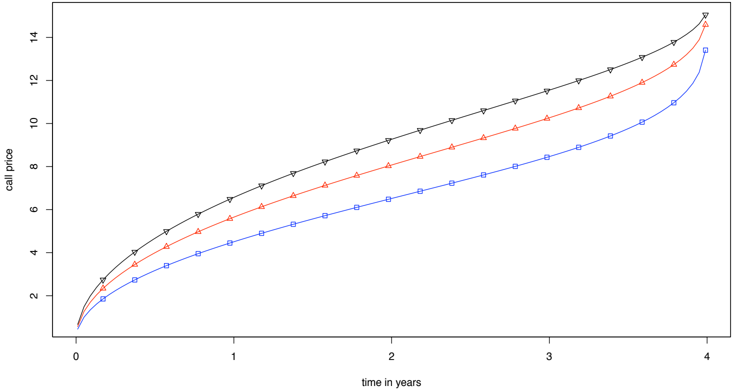
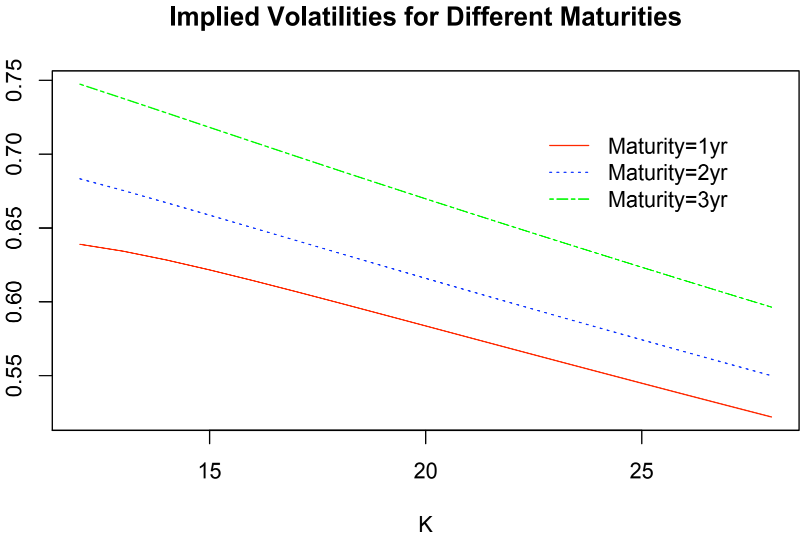
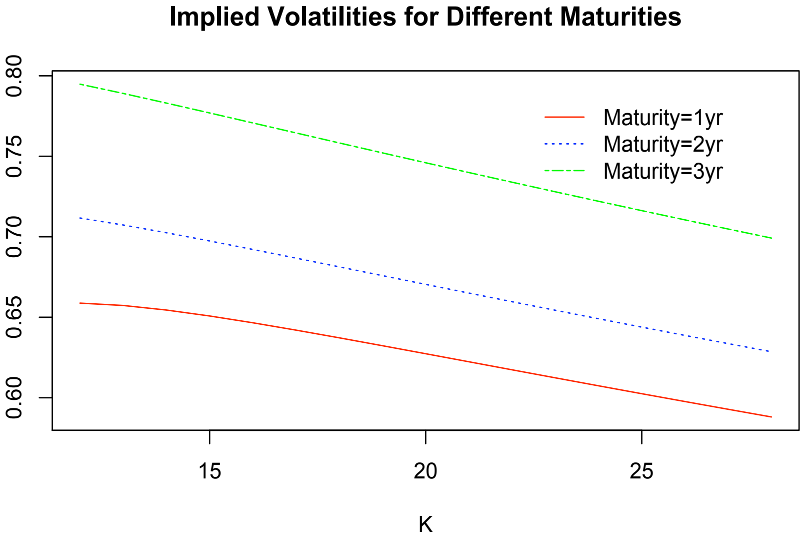


Figure: Dependence $\tau \mapsto C_0(\tau)$ of Call prices on maturity τ. Graphs $\square$, $\triangle$, and $\triangledown$ correspond to $\beta = 0.5$, $\beta = 0.8$, $\beta = 1.1$.

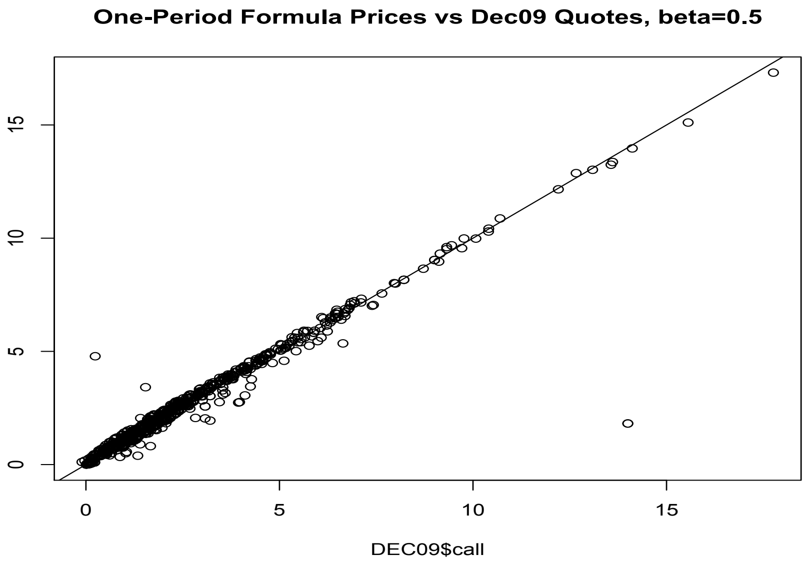
Implied Volatilities $\beta = 1.2$



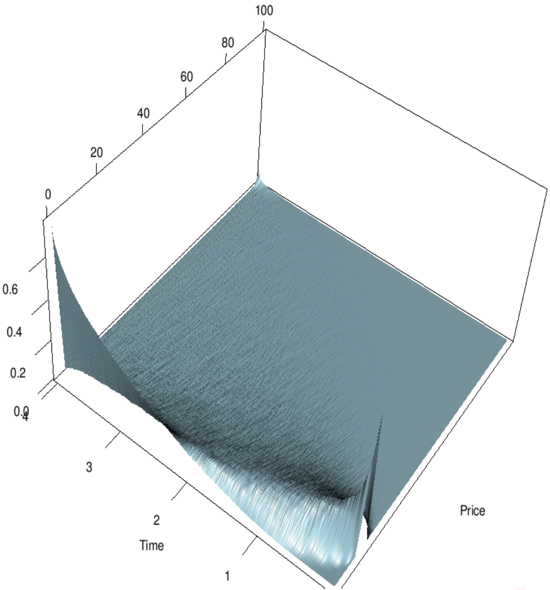
Implied Volatilities $\beta = 0.6$, $\pi = 100$



More Calibration away from End of Period



Risk Neutral Densities



Lectures based on)

1. R.C., M. Fehr and J. Hinz: Mathematical Equilibrium and Market Design for Emissions Markets Trading Schemes. *SIAM J. Control and Optimization* (2009)
2. R.C., M. Fehr, J. Hinz and A. Porchet: Mathematical Equilibrium and Market Design for Emissions Markets Trading Schemes. *SIAM Review* (2010)
3. R.C., M. Fehr and J. Hinz: Properly Designed Emissions Trading Schemes do Work! (working paper)
4. R.C., and J. Hinz: Risk-Neutral Modeling of Emission Allowance Prices and Option Valuation (working paper)
5. R.C. & M. Fehr: Auctions and Relative Allocation Mechanisms for Cap-and-Trade Schemes (working paper)
6. R.C. & M. Fehr: The Clean Development Mechanism: a Mathematical Model. (submitted *Proc. 2008 Ascona Conf.*)