Emission Markets I. Cap-and-Trade Schemes: First Equilibrium Models

René Carmona

ORFE, Bendheim Center for Finance
Princeton University

IPAM January 5–8, 2010

Schematic of the Course

- Descriptive Introduction
  - Zoology of the Carbon Markets: **EU ETS**, and those soon to exist in the **US**
  - **Lessons** learned from the EU Experience
- First **Mathematical (Equilibrium) Models**
  - Joint Price Formation for Electricity and Emission Allowances
  - **Computer Implementations**
  - Case studies
- **Allocation Mechanisms**
  - **Costs** associated to a Cap-and-Trade scheme
  - **Comparisons** of grandfathering, auctions, output based allocations
- **Information Structure** and **Partial Equilibrium Models**
- **EUA Option Markets**
  - Are traders **really** using Black-Scholes formula?
  - Alternative models and pricing formulas

First Emission Trading Market

- Established in the United States **Clean Air Act of 1990**
- **Acid Rain Program**
- **Program to reduce the primary causes of acid rain**
  - sulfur dioxide (SO_2)
  - nitrogen oxides (NO_x)
- **Program based on BOTH**
  - **regulatory approach**
  - **market mechanisms**
- To achieve this goal **at the lowest cost to society**
- **SOx and NOx Trading**: Great learning experience!
  - Liquidity and Price Collapse Issues
  - Do they create Pollution Hot Spots?
- **TOO SMALL** a scale (Montgomery *flip-flop*)

Kyoto Protocol

- Kyoto Conference **1997**
- Assign **MANDATORY** Green House Gas (GHG) emission limits to signatory nations
  - Reduce emissions of CO_2 and 5 other gases in 2008 - 2012
  - Target level: 95% of 1990 levels
- Set up **Cap & Trade for Green House Gases**
- Clean Development Mechanism (CDM) and Joint Initiative (JI)
- **ENFORCEMENT?** (theory of self-enforced treaties)

Flexible Mechanisms of Kyoto Protocol

- Stimulate sustainable development and emission reductions, when and where it is cheapest to do
- Projects must qualify through a rigorous and public registration and issuance process
  - Ensure **real**, **measurable** and **verifiable** emission reductions
  - **Additional** to what would have occurred without the projects
- **Clean Development Mechanism** (CDM)
  - Projects located in **developing countries**
- **Joint Initiative** (JI)
  - Projects located in economies **in transition**
- We'll use **same mathematical models!**
- Approved projects earn **Certified Emission Reduction** (CER)

CERs

- Using CERs to meet emission reduction targets
  - 1 CER = 1 ton of CO_2 equivalent to meet emission reduction
  - traded and sold on **ANY** market, **NO** date limitation
  - discount due to moral hazard, political, project completion, . . . **RISK**
- **Trading**
  - Program started in 2006
  - More than 1,000 projects already registered
  - Anticipated to produce CERs amounting to more than 2.7 billion tons of CO_2 equivalent for 2008 – 2012
- **Speculative Trading of Spread** between EUAs and CERs
- Role of **CERs** in EUA option prices still a **mystery**

EUAs vs CERs

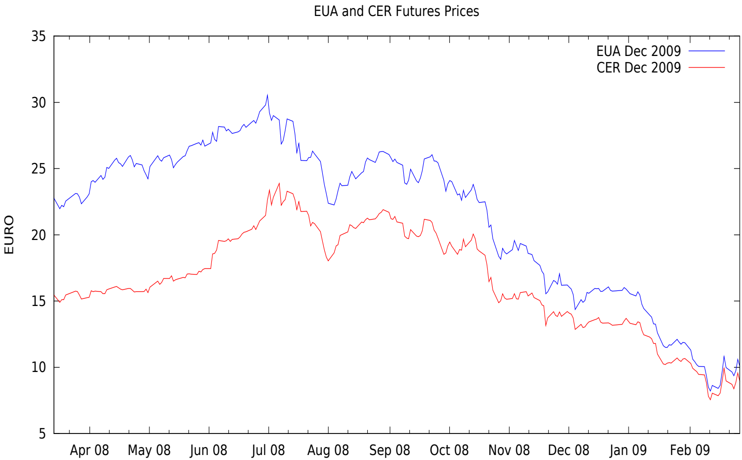


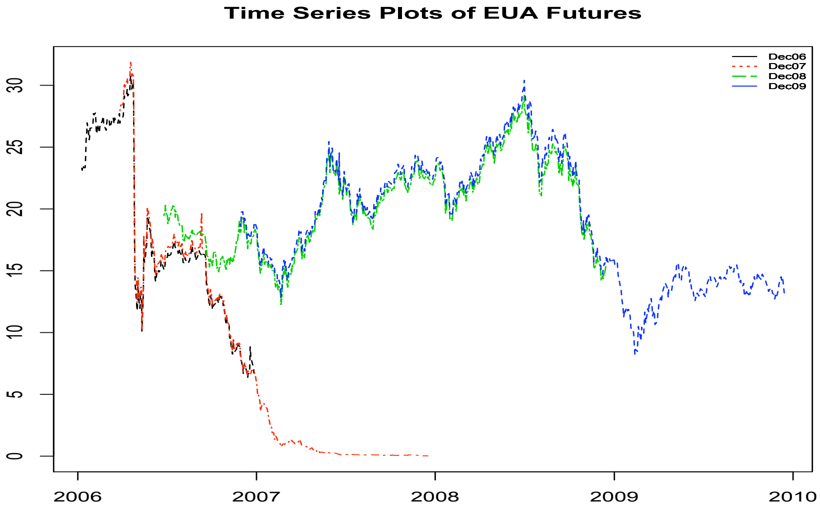
Figure: Prices of the December 2012 EUA futures contract (EU-ETS second phase), together with the price of the corresponding CER futures contract.

EU and the Kyoto Protocol

- **European Climate Change Programme (ECCP)** June 2000
- **All** 25 EU countries ratify Kyoto Protocol on 31 May 2002
- **Directive 2003/87/ec** of the European Parliament of October 13, 2003: *establishment of a scheme for greenhouse gas emission allowance trading.*
- **Each** EU member state proposes a **National Allocation Plan** (NAP) with a **cap**
- **Permit Allocation:**
  Installations covered by ETS are given **allowances for FREE**
  - power plants (capacity $>$ 20MW)
  - steel manufacturers
  - cement factories
  - . . . . . . (1200 installations in EU during first phase)

EU Emission Trading Scheme (ETS)

- Actual trading in EU ETS started **January 2005**
- **400 million** tons of CO_2 equivalent traded the first year
- EU ETS structured in **Three Phases**
  - **Phase I**: January 2005 - December 2007 (trial)
  - **Phase II**: January 2008 - December 2012 (current)
  - **Phase III**: January 2013 - December 2018 (unclear – Copenhagen)
- Exchange traded (standardized & cleared) futures contracts (Dec-05, Dec-06, Dec-07, Dec-08, . . ., Dec-12)
- 1 **contract** = 1 **lot** = 1000 EUAs of 1 ton CO_2 equivalent each
- **Liquid** Front End contract
- Vibrant **option** market on these futures contracts



Prices of the EUA futures contracts.

How Do Things Work?

- Each year, installation receive allowances according to NAP
- Each year, cumulative emissions are tallied up to Dec. 31
- Each installation has up to Apr. 30 to cover its emissions
  - by **surrendering** allowances
  - paying a **penalty** of π euros per ton not covered by an allowance
  - $\pi = 40$ euros in **Phase I**; $\pi = 100$ euros in **Phase II**
  - Paying the penalty is not enough: the corresponding amount of allowances is **withdrawn** from the next allocation
- Phase I was a *trial balloon*
- Phase I allowances **COULD NOT BE USED** beyond their maturities
- Phase II allowances **CAN BE BANKED** for later use

(Simplified) Cap-and-Trade Scheme: Data

- Regulator Input at inception of program (i.e. time $t = 0$)
  - **INITIAL ALLOCATION** of θ_0 **allowances**
  - Set **PENALTY** π per ton of CO_2 equivalent emitted and **NOT** offset by allowance certificate at **time of compliance**
- Given **exogenously**
  - $\{D_t\}_{t=0,1,\cdot,T}$ daily **demand** for electricity
    - $\{C_t^n\}_{t=0,1,\cdot,T}$ **production cost** for 1MWh of electricity from **nuclear** plant
    - $\{C_t^g\}_{t=0,1,\cdot,T}$ **production cost** for 1MWh of electricity from **gas** plant
    - $\{C_t^c\}_{t=0,1,\cdot,T}$ **production cost** for 1MWh of electricity from **coal** plant
- **Known** physical characteristics
  - e^n **emission** (in CO_2 ton-equivalent) for 1MWh from **nuclear** plant
  - e^g **emission** (in CO_2 ton-equivalent) for 1MWh from **gas** plant
  - e^c **emission** (in CO_2 ton-equivalent) for 1MWh from **coal** plant

(Simplified) Cap-and-Trade Scheme: Data

- Regulator Input at inception of program (i.e. time $t = 0$)
  - **INITIAL ALLOCATION** of θ_0 **allowances**
  - Set **PENALTY** π per ton of CO_2 equivalent emitted and **NOT** offset by allowance certificate at **time of compliance**
- Given **exogenously**
  - $\{D_t\}_{t=0,1,\cdot,T}$ daily **demand** for electricity
    - $\{C^n_t\}_{t=0,1,\cdot,T}$ **production cost** for 1MWh of electricity from **nuclear** plant
    - $\{C^g_t\}_{t=0,1,\cdot,T}$ **production cost** for 1MWh of electricity from **gas** plant
    - $\{C^c_t\}_{t=0,1,\cdot,T}$ **production cost** for 1MWh of electricity from **coal** plant
- **Known** physical characteristics
  - e^n **emission** (in CO_2 ton-equivalent) for 1MWh from **nuclear** plant
  - e^g **emission** (in CO_2 ton-equivalent) for 1MWh from **gas** plant
  - e^c **emission** (in CO_2 ton-equivalent) for 1MWh from **coal** plant

(Simplified) Cap-and-Trade Scheme: Data

- Regulator Input at inception of program (i.e. time $t = 0$)
  - **INITIAL ALLOCATION** of θ_0 **allowances**
  - Set **PENALTY** π per ton of CO_2 equivalent emitted and **NOT** offset by allowance certificate at **time of compliance**
- Given **exogenously**
  - $\{D_t\}_{t=0,1,\cdot,T}$ daily **demand** for electricity
    - $\{C_t^n\}_{t=0,1,\cdot,T}$ **production cost** for 1MWh of electricity from **nuclear** plant
    - $\{C_t^g\}_{t=0,1,\cdot,T}$ **production cost** for 1MWh of electricity from **gas** plant
    - $\{C_t^c\}_{t=0,1,\cdot,T}$ **production cost** for 1MWh of electricity from **coal** plant
- **Known** physical characteristics
  - e^n **emission** (in CO_2 ton-equivalent) for 1MWh from **nuclear** plant
  - e^g **emission** (in CO_2 ton-equivalent) for 1MWh from **gas** plant
  - e^c **emission** (in CO_2 ton-equivalent) for 1MWh from **coal** plant

(Simplified) Cap-and-Trade Scheme: Data

- Regulator Input at inception of program (i.e. time $t = 0$)
  - **INITIAL ALLOCATION** of θ_0 **allowances**
  - Set **PENALTY** π per ton of CO_2 equivalent emitted and **NOT** offset by allowance certificate at **time of compliance**
- Given **exogenously**
  - $\{D_t\}_{t=0,1,\cdot,T}$ daily **demand** for electricity
    - $\{C_t^n\}_{t=0,1,\cdot,T}$ **production cost** for 1MWh of electricity from **nuclear** plant
    - $\{C_t^g\}_{t=0,1,\cdot,T}$ **production cost** for 1MWh of electricity from **gas** plant
    - $\{C_t^c\}_{t=0,1,\cdot,T}$ **production cost** for 1MWh of electricity from **coal** plant
- **Known** physical characteristics
  - e^n **emission** (in CO_2 ton-equivalent) for 1MWh from **nuclear** plant
  - e^g **emission** (in CO_2 ton-equivalent) for 1MWh from **gas** plant
  - e^c **emission** (in CO_2 ton-equivalent) for 1MWh from **coal** plant

(Simplified) Cap-and-Trade Scheme: Outcome

- $\{S_t\}_{t=0,1,\cdot,T}$ daily **price** of electricity
- $\{A_t\}_{t=0,1,\cdot,T}$ daily **price** of a credit allowance
- **Production schedules**
  - $\{\xi_t^n\}_{t=0,1,\cdot,T}$ daily **production** of electricity from **nuclear** plant
  - $\{\xi_t^g\}_{t=0,1,\cdot,T}$ **production** of electricity from **gas** plant
  - $\{\xi_t^c\}_{t=0,1,\cdot,T}$ **production** of electricity from **coal** plant
- Inelasticity constraint

$$\xi_t^n + \xi_t^g + \xi_t^c = D_t \qquad t = 0, 1, \cdots, T$$

- **Daily Production Profits & Losses**

$$\xi_t^n(S_t - c_t^n) + \xi_t^g(S_t - c_t^g) + \xi_t^c(S_t - c_t^c) = \left(D_t S_t - (\xi_t^n c_t^n + \xi_t^g c_t^g + \xi_t^c c_t^c)\right)$$

- (possible) **Pollution Penalty**

$$\pi \left(\sum_{t=0}^{T} \xi_t^n e^n + \xi_t^g e^g + \xi_t^c e^g - \theta_0 \right)^+$$

(Simplified) Cap-and-Trade Scheme: Outcome

- $\{S_t\}_{t=0,1,\cdot,T}$ daily **price** of electricity
- $\{A_t\}_{t=0,1,\cdot,T}$ daily **price** of a credit allowance
- **Production schedules**
  - $\{\xi_t^n\}_{t=0,1,\cdot,T}$ daily **production** of electricity from **nuclear** plant
  - $\{\xi_t^g\}_{t=0,1,\cdot,T}$ **production** of electricity from **gas** plant
  - $\{\xi_t^c\}_{t=0,1,\cdot,T}$ **production** of electricity from **coal** plant
- Inelasticity constraint

$$\xi_t^n + \xi_t^g + \xi_t^c = D_t \qquad t = 0, 1, \cdots, T$$

- **Daily Production Profits & Losses**

$$\xi_t^n(S_t - c_t^n) + \xi_t^g(S_t - c_t^g) + \xi_t^c(S_t - c_t^c) = \left(D_t S_t - (\xi_t^n c_t^n + \xi_t^g c_t^g + \xi_t^c c_t^c)\right)$$

- (possible) **Pollution Penalty**

$$\pi\left(\sum_{t=0}^{T} \xi_t^n e^n + \xi_t^g e^g + \xi_t^c e^g - \theta_0\right)^+$$

(Simplified) Cap-and-Trade Scheme: Outcome

- $\{S_t\}_{t=0,1,\cdot,T}$ daily **price** of electricity
- $\{A_t\}_{t=0,1,\cdot,T}$ daily **price** of a credit allowance
- **Production schedules**
  - $\{\xi_t^n\}_{t=0,1,\cdot,T}$ daily **production** of electricity from **nuclear** plant
  - $\{\xi_t^g\}_{t=0,1,\cdot,T}$ **production** of electricity from **gas** plant
  - $\{\xi_t^c\}_{t=0,1,\cdot,T}$ **production** of electricity from **coal** plant
- Inelasticity constraint

$$\xi_t^n + \xi_t^g + \xi_t^c = D_t \qquad t = 0, 1, \cdots, T$$

- **Daily Production Profits & Losses**

$$\xi_t^n(S_t - c_t^n) + \xi_t^g(S_t - c_t^g) + \xi_t^c(S_t - c_t^c) = \left(D_t S_t - (\xi_t^n c_t^n + \xi_t^g c_t^g + \xi_t^c c_t^c)\right)$$

- (possible) **Pollution Penalty**

$$\pi \left(\sum_{t=0}^{T} \xi_t^n e^n + \xi_t^g e^g + \xi_t^c e^g - \theta_0 \right)^+$$

EU ETS First Phase: Main Criticism

- **No (Significant) Emissions Reduction**
  - DID Emissions go down?
  - Yes, but as part of an existing trend
- **Significant Increase in Prices**
  - Cost of Pollution passed along to the "end-consumer"
  - Small proportion (40%) of polluters involved in EU ETS
- **Obscene Windfall Profits**
  - Remedy: Stop Giving Allowance Certificates Away for Free !
  - Auctioning
- **Market Malfunctions**
  - Prices (Dec-06 & Dec-07) **collapsed** (April 2006 from 30 to 10 !)
  - Prices converged to 0 at end of Phase I

EU ETS First Phase: Main Criticism

- **No (Significant) Emissions Reduction**
  - DID Emissions go down?
  - Yes, but as part of an existing trend
- **Significant Increase in Prices**
  - Cost of Pollution passed along to the "end-consumer"
  - Small proportion (40%) of polluters involved in EU ETS
- **Obscene Windfall Profits**
  - Remedy: Stop Giving Allowance Certificates Away for Free !
  - Auctioning
- **Market Malfunctions**
  - Prices (Dec-06 & Dec-07) **collapsed** (April 2006 from 30 to 10 !)
  - Prices converged to 0 at end of Phase I

EU ETS First Phase: Main Criticism

- **No (Significant) Emissions Reduction**
  - DID Emissions go down?
  - Yes, but as part of an existing trend
- **Significant Increase in Prices**
  - Cost of Pollution passed along to the ”end-consumer”
  - Small proportion (40%) of polluters involved in EU ETS
- **Obscene Windfall Profits**
  - Remedy: Stop Giving Allowance Certificates Away for Free !
  - Auctioning
- **Market Malfunctions**
  - Prices (Dec-06 & Dec-07) **collapsed** (April 2006 from 30 to 10 !)
  - Prices converged to 0 at end of Phase I

A First Mathematical Model

Description of the Economy

- **Finite set** $\mathcal{I}$ of **risk neutral agents/firms**
- **Producing a finite set** $\mathcal{K}$ of **goods**
- Firm $i \in \mathcal{I}$ can use **technology** $j \in \mathcal{J}^{i,k}$ to produce good $k \in \mathcal{K}$
- **Discrete time** $\{0, 1, \cdots, T\}$
- **Inelastic Demand**

$$\{D^k(t);\ t = 0, 1, \cdots, T-1,\ k \in \mathcal{K}\}.$$

- . . . .

Regulator Input (EU ETS)

At inception of program (i.e. time $t = 0$)

- **INITIAL ALLOCATION** of **allowances**

$$\theta_0^i \quad \text{to firm } i \in \mathcal{I} \quad \text{according to NAP}$$

  What will really matter to us is $\theta_0 = \sum_{i \in \mathcal{I}} \theta_0^i$

- Set **PENALTY** π for emission unit **NOT** offset by allowance certificate at end of **compliance period**

Variations (not addressed yet)

- **Risk aversion** and agent preferences
- **Auctioning** of allowances
- Distribution**over time** of allowances (game theory)
- **Elastic** demand (e.g. smart meters)
- **Multi-period period** lending and borrowing
- . . . .

Goal of Equilibrium Analysis

Find **two stochastic processes**

- **Price of one allowance**

$$A = \{A_t\}_{t \geq 0}$$

- **Prices of goods**

$$S = \{S_t^k\}_{k \in K,\, t \geq 0}$$

satisfying the usual equilibrium conditions to be spelled out later.

Individual Firm Problem

During each time period $[t, t+1)$

- Firm $i \in \mathcal{I}$ **produces** $\xi_t^{i,j,k}$ of good $k \in \mathcal{K}$ with technology $j \in \mathcal{J}^{i,k}$
- Firm $i \in \mathcal{I}$ **holds** a position θ_t^i in emission credits

$$
\begin{aligned}
L^{A,S,i}(\theta^i, \xi^i) := \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{J}^{i,k}} \sum_{t=0}^{T-1} (S_t^k - C_t^{i,j,k}) \xi_t^{i,j,k} \\
+ \theta_0^i A_0 + \sum_{t=0}^{T-1} \theta_{t+1}^i (A_{t+1} - A_t) - \theta_{T+1}^i A_T \\
- \pi(\Gamma^i + \Pi^i(\xi^i) - \theta_{T+1}^i)^+
\end{aligned}
$$

where

$$
\Gamma^i \text{ random}, \qquad \Pi^i(\xi^i) := \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{J}^{i,k}} \sum_{t=0}^{T-1} e^{i,j,k} \xi_t^{i,j,k}
$$

Problem for agent $i \in I$

$$
\max_{(\theta^i, \xi^i)} \mathbb{E}\{L^{A,S,i}(\theta^i, \xi^i)\}
$$

Business As Usual (BAU)

If $\pi = 0$, **Business As Usual** & Classical **merit order**.

- At each time t and for each good k
- Production technologies ranked by increasing production costs $C_t^{i,j,k}$
- Demand D_t^k met by producing from the cheapest technology first
- Equilibrium spot price is the marginal cost of production of the most expansive production technoligy used to meet demand

Deregulated **electricity markets**

If (A^*, S^*) is an equilibrium, the optimization problem of firm i is

$$\sup_{(\theta^i,\xi^i)} \mathbb{E}\left[\sum_{k\in\mathcal{K}}\sum_{j\in\mathcal{J}^{i,k}}\sum_{t=0}^{T-1}(S_t^k - C_t^{i,j,k})\xi_t^{i,j,k} + \theta_0^i A_0 + \sum_{t=0}^{T-1}\theta_{t+1}^i(A_{t+1} - A_t) - \theta_{T+1}^i A_T\right]$$

Business As Usual (cont.)

We have $A_t^* = \mathbb{E}_t[A_{t+1}^*]$ for all t and $A_T^* = 0$ (hence $A_t^* \equiv 0$!)

Classical competitive equilibrium problem where each agent maximizes

$$\sup_{\xi^i \in \mathcal{U}^i} \mathbb{E}\left[\sum_{k\in\mathcal{K}}\sum_{j\in\mathcal{J}^{i,k}}\sum_{t=0}^{T-1}(S_t^k - C_t^{i,j,k})\xi_t^{i,j,k}\right], \qquad (1)$$

and the equilibrium prices S^* are set so that supply meets demand.

Production in **MERIT ORDER**

$$((\xi_t^{*i,j,k})_{j,k})_i = \arg\max_{((\xi_t^{i,j,k})_{\mathcal{J}^{i,k}})_{i\in\mathcal{I}}} \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}^{i,k}} -C_t^{i,j,k}\xi_t^{i,j,k}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}^{i,k}} \xi_t^{i,j,k} = D_t^k$$

$$\xi_t^{i,j,k} \le \kappa^{i,j,k} \quad \text{for } i\in\mathcal{I}, j\in\mathcal{J}^{i,k}$$

$$\xi_t^{i,j,k} \ge 0 \quad \text{for } i\in\mathcal{I},\ j\in\mathcal{J}^{i,k}$$

for all times t, and the associated price is

$$S_t^{*k} = \max_{i\in\mathcal{I},\, j\in\mathcal{J}^{i,k}} C_t^{i,j,k}\mathbf{1}_{\{\xi_t^{*i,j,k}>0\}},$$

Equilibrium Definition for Emissions Market

The processes $A^* = \{A^*_t\}_{t=0,1,\cdots,T}$ and $S^* = \{S^*_t\}_{t=0,1,\cdots,T}$ form an equilibrium if for each agent $i \in \mathcal{I}$ there exist strategies $\theta^{*i} = \{\theta^{*i}_t\}_{t=0,1,\cdots,T}$ (**trading**) and $\xi^{*i} = \{\xi^{*i}_t\}_{t=0,1,\cdots,T}$ (**production**)

- **(i) All financial positions are in constant net supply**

$$\sum_{i \in I} \theta^{*i}_t = \sum_{i \in I} \theta^i_0, \qquad \forall\, t = 0, \ldots, T+1$$

- **(ii) Supply of each good meets demand**

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}^{i,k}} \xi^{*i,j,k}_t = D^k_t, \qquad \forall\, k \in \mathcal{K}, \;\; t = 0, \ldots, T-1$$

- **(iii) Each agent $i \in I$ is satisfied by its own strategy**

$$\mathbb{E}[L^{A^*,S^*,i}(\theta^{*i}, \xi^{*i})] \geq \mathbb{E}[L^{A^*,S^*,i}(\theta^i, \xi^i)] \qquad \text{for all } (\theta^i, \xi^i)$$

Necessary Conditions

Assume

- (A^*, S^*) is an equilibrium
- (θ^{*i}, ξ^{*i}) optimal strategy of agent $i \in I$

then

- The allowance price A^* is a **bounded martingale** in $[0, \pi]$
- Its terminal value is given by

$$A^*_T = \pi \mathbf{1}_{\{\Gamma^i + \Pi(\xi^{*i}) - \theta^{*i}_{T+1} \geq 0\}} = \pi \mathbf{1}_{\{\sum_{i \in \mathcal{I}} (\Gamma^i + \Pi(\xi^{*i}) - \theta^{*i}_0) \geq 0\}}$$

- The **spot prices** S^{*k} of the goods and the **optimal production strategies** ξ^{*i} are given by the **merit order** for the equilibrium with **adjusted costs**

$$\tilde{C}^{i,j,k}_t = C^{i,j,k}_t + e^{i,j,k} A^*_t$$

Existence by Social Cost Minimization

- **Overall production costs**

$$C(\xi) := \sum_{t=0}^{T-1} \sum_{(i,j,k)} \xi_t^{i,j,k} C_t^{i,j,k}.$$

- **Overall cumulative emissions**

$$\Gamma := \sum_{i \in I} \Gamma^i \qquad \Pi(\xi) := \sum_{t=0}^{T-1} \sum_{(i,j,k)} e^{i,j,k} \xi_t^{i,j,k},$$

- **Total allowances**

$$\theta_0 := \sum_{i \in I} \theta_0^i$$

The **total social costs from production and penalty payments**

$$G(\xi) := C(\xi) + \pi(\Gamma + \Pi(\xi) - \theta_0)^+$$

We introduce the global optimization problem

$$\xi^* = \arg \inf_{\xi \text{meets demands}} \mathbb{E}[G(\xi)],$$

Social Cost Minimization Problem (cont.)

- **First Theoretical Result**
  - There exists a set $\xi^* = (\xi^{*i})_{i \in I}$ realizing the minimum social cost
- **Second Theoretical Result**

(i) If $\overline{\xi}$ minimizes the social cost, then the processes $(\overline{A}, \overline{S})$ defined by

$$\overline{A}_t = \pi \mathbb{P}_t\{\Gamma + \Pi(\overline{\xi}) - \theta_0 \geq 0\}, \qquad t = 0, \ldots, T$$

and

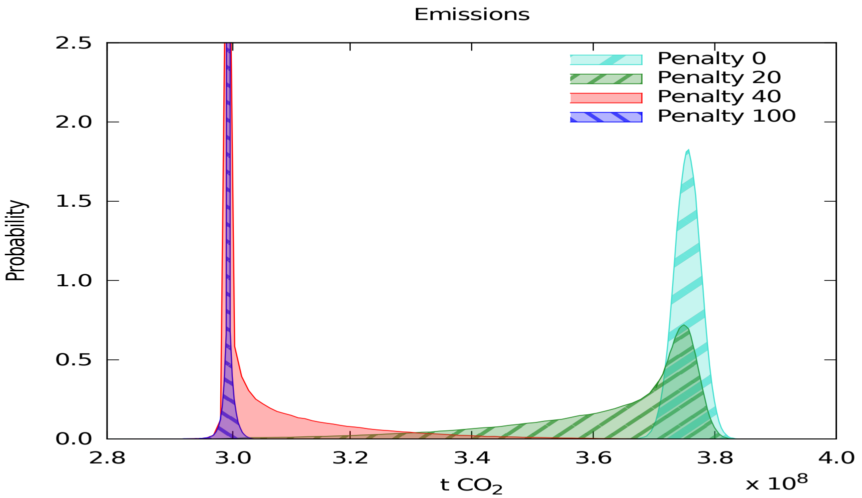
$$\overline{S}_t^k = \max_{i \in I,\, j \in J^{i,k}} (C_t^{i,j,k} + e_t^{i,j,k} \overline{A}_t) 1_{\{\overline{\xi}_t^{i,j,k} > 0\}}, \qquad t = 0, \ldots, T-1 \;\; k \in K,$$

form a **market equilibrium** with associated production strategy $\overline{\xi}$.

(ii) If (A^*, S^*) is an equilibrium with corresponding strategies (θ^*, ξ^*), then ξ^* solves the **social cost minimization problem**

(iii) The equilibrium allowance price is **unique**.

Effect of the Penalty on Emissions



Costs in a Cap-and-Trade

- **Consumer Burden**

$$SC = \sum_t \sum_k (S_t^{k,*} - S_t^{k,BAU*}) D_t^k.$$

- **Reduction Costs** (producers' burden)

$$\sum_t \sum_{i,j,k} (\xi_t^{i,j,k*} - \xi_t^{BAU,i,j,k*}) C_t^{i,j,k}$$

- **Excess Profit**

$$\sum_t \sum_k (S_t^{k,*} - S_t^{k,BAU*}) D_t^k - \sum_t \sum_{i,j,k} (\xi_t^{i,j,k*} - \xi_t^{BAU,i,j,k*}) C_t^{i,j,k} - \pi(\sum_t \sum_{ijk} \xi_t^{ijk} e_t^{ijk} - \theta_0)^+$$

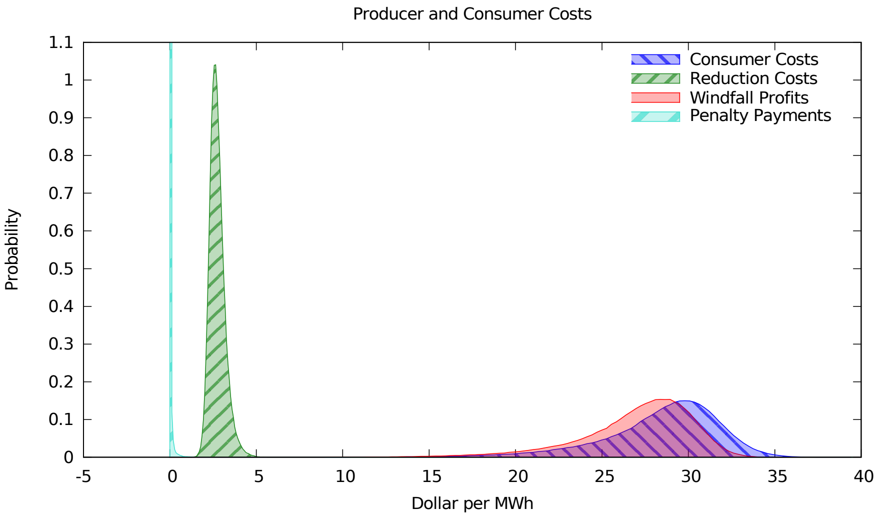
- **Windfall Profits**

$$WP = \sum_{t=0}^{T-1} \sum_{k \in K} (S_t^{*k} - \hat{S}_t^k) D_t^k$$

where

$$\hat{S}_t^k := \max_{i \in I, j \in J^{i,k}} C_t^{i,j,k} \mathbf{1}_{\{\xi_t^{*i,j,k} > 0\}}.$$

Costs in a Cap-and-Trade Scheme



Histograms of consumer costs, social costs, windfall profits and penalty payments of a standard cap-and-trade scheme calibrated to reach the emissions target with 95% probability and BAU.