Investment in Incomplete Electricity Markets

A Stochastic Discount Rate Equilibrium

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The views expressed in this presentation are those of the authors and not necessarily of ENGIE.

Outline

Introduction

Investment/mothballing in power generation

- Problems of inadequate investment in the European power generation system. Massive uncertainty is suggested as one of the causes.
- Risk intervenes through many facets in investment problems.
  - Plants are facing their own risk
  - Plant portfolio changes forthcoming risk.
  - Instruments to incentivise investment here formalized as contracts and capacity markets.
- These instruments interact in an incomplete market (there is a residual risk/ missing markets).
- We try to model these interactions through a endogeneous stochastic equilibrium model focusing on the stochastic discount factors.

General characteristics of the models (1)

- Standard representation of the technologies (cf. old capacity expansion models).
- Two stages: one invests in physical and financial assets in stage 0. One collects revenue in stage 1.
- Standard scenario tree representing uncertainty. States of the world in the second stage are noted ω occuring with $prob(\omega)$.
- Two types of agents: *Producers* and *a consumer* trading physical quantities (electricity) on the spot market and financial contracts in forward market.
- The consumer has a price inelastic demand.
  - Methodology transposable to price elastic demand
  - which does not mean that usual results (in particular on uniqueness) would obtain.

General characteristics of the model (2)

- Risk averse agents modeled by risk functions in the sense of Artzner et al. [2]: terminology for investment is *risk-adjusted-value*.
- Sub-gradient of risk functions have an interpretation of pricing kernel or stochastic discount factors.
- Sub-gradient of risk functions with all positive components have an interpretation of "equivalent risk measure" in finance.
- With the result that stochastic discount factors are endogenously determined in the model depending on the physical asset, global or agents' portfolios.
  - Theoretical results: existence and uniqueness.
  - Illustration on a toy problem, interpretation in terms of implied risk premium.

Results subject to restrictions

- Some analytic simplifications (hopefully relaxed in future work):
  - Continuous differentiability imposed in several parts of the theory.
  - Extension to multistage should be done at some stage.
- An alternative version, more economically oriented (in terms of long-run and short-run marginal cost) exists subject to additional differentiability properties.

Reminder on risk functions

Reminder on risk functions (1)

The notation and the theory comes from Shapiro et al. [12].

An agent i optimizes its portfolio of assets by choosing a strategy that maximizes the risk-adjusted value ρ_i of its random profit Z_i.

Definition

A coherent risk-adjusted value is a function $\rho : \mathcal{Z} \to \mathbb{R}$ satisfying the following axioms.

- *Monotonicity:* $\forall Z_1, Z_2 \in \mathcal{Z}$: if $Z_1 \preceq Z_2$, then $\rho(Z_1) \leq \rho(Z_2)$.
- *Cash invariance:* $\forall Z \in \mathcal{Z}$: if $a \in \mathbb{R}$ then $\rho(Z + a) = \rho(Z) + a$.
- *Concavity:* $\forall Z_1, Z_2 \in \mathcal{Z}, \forall t \in [0, 1] : \rho(tZ_1 + (1 - t)Z_2) \geq t\rho(Z_1) + (1 - t)\rho(Z_2)$.
- *Positive Homogeneity* $\forall Z \in \mathcal{Z}, \forall \lambda \in \mathbb{R}^+ : \rho(\lambda Z) = \lambda\rho(Z)$.

Reminder on risk functions (2)

Theorem (Artzner et al. [1])

Any coherent risk-adjusted value ρ has a dual representation:

$$\rho(Z) = \min_{Q \in \mathcal{M}} \mathbb{E}_Q[Z(\omega)] ,$$

where $\mathcal{M} \subseteq \mathcal{P}$ is a closed and convex set of probability measures.

The sub-gradient of such coherent risk-adjusted value is given by

- $\partial\rho(Z) = \arg\min_{Q \in \mathcal{M}} \mathbb{E}_Q[Z]$

Reminder on risk functions (3)

Theorem

For agent i:

- $Z_i(\omega) = F_i(x_i, \omega)$: *random pay-off of an agent i resulting from portfolio decision x_i.*
- $\rho_i(\cdot)$: *coherent risk-adjusted value*

Then $0 \in \partial\left(\rho_i \circ F_i\right)(x_i)$ is written as :

$$0 = \mathbb{E}_{\bar{Q}_i(Z_i)}\left[\nabla_{x_i} F(x_i, \omega)\right] \ , \tag{1}$$

where $\bar{Q}_i(Z_i) = \{Q_i \in \partial\rho_i(Z_i)\}$ and equals to the singleton $\nabla\rho_i(Z_i)$ when the risk measure is differentiable.

Reminder on risk functions (4)

We make the assumption that risk-adjusted values $\rho_i()$ are sufficiently continuously differentiable.

Definition

The good-deal risk-adjusted value (from Cochrane [3]) is defined as

$$\rho^{\mathrm{GD}}(Z) = \min_{Q \in \mathcal{Q}_{\mathrm{GD}}} \mathbb{E}_Q[Z(\omega)] \tag{2}$$

where $\mathcal{Q}_{\mathrm{GD}}$ is the following convex and compact set:

$$\mathcal{Q}_{\mathrm{GD}} = \left\{ Q \in \mathcal{P} \;\middle|\; \begin{array}{ll} q(\omega) \geq 0 & \forall \omega \in \Omega \\ \mathbb{E}_Q[p_c^s(\omega)] = p_c^f & \forall c = 1, ..., C \\ \mathbb{E}_P\left[\left(\frac{q_(\omega)}{prob_\omega}\right)^2\right] \leq H^2 & \end{array} \right\}, \tag{3}$$

the scalar H^2 is equal to $(1 + h^2)$, where h is the maximal admissible Sharpe ratio (cf. Hansen-Jagannathan bound [8]).
The problem has two kind of inequality constraints and we limit our self to value of H, such that the volatility constraint is binding and the nonnegativity constraints for $q(\omega)$ are slack.

The Agents

The Consumer

Definition

Given the electricity price $\mathbf{p_{el}} := (p_{el}(\omega))$ the price taking consumer d solves the following problem $\mathcal{C}_d$

$$\mathcal{C}_d \equiv \underset{\mathbf{s}:s(\omega)\geq 0}{\text{Max}} \ \rho_d \left((PC - p_{\text{el}}(\omega)) \cdot (\text{LOAD}(\omega) - s(\omega)) + \pi_d^s(\omega) \right)$$

- Consumer faces a load $LOAD(\omega)$ in state of the world ω.
- He values its load at a cap PC (ideally VOLL).
- He curtails it by $s(\omega)$ when the electricity price $p_{el}(\omega)$ is too high
- He can receive an external pay-off $\pi_d^s(\omega)$ *(for theoretical reason - latter)*

- This problem does not involve any first-stage decision; the KKT conditions are

$$0 \leq PC - p_{el}(\omega) \ \perp \ s(\omega) \geq 0 \qquad \forall \omega \in \Omega$$

The producers (1)

Definition

An electricity company ν ($\nu = 1, \ldots, N$) invests $u_{\nu,k}$ in time $t = 0$ in $k = 1, \ldots, K$ new capacities (differing by technologies, e.g. nuclear, coal, gas,...) that are available in the next time period $t = 1$.

- $I = (I_1, \ldots, I_K)$: annual values of the investment costs
- $C = (C_1(\omega), \ldots, C_K(\omega))$: operating costs.
- The company operates its plant k at level $y_{\nu,k}$ subject to $0 \leq y_{\nu,k} \leq u_{\nu,k}$.
- The company can receive an external pay-off $\pi_\nu^s(\omega)$ *(for theoretical reason)*

Given the electricity price $\mathbf{p_{el}}$, the price-taking company solves

$$\mathcal{G}_\nu = \underset{\mathbf{u}_\nu : u_{\nu,k} \geq 0}{\text{Max}} - \sum_k I_k u_{\nu,k} +$$

$$+ \rho_\nu \left(\max_{\substack{\mathbf{y}_\nu : y_{\nu,k}(\omega) \geq 0 \\ y_{\nu,k}(\omega) \leq u_{\nu,k}}} \left\{ \sum_k \Big(p_{el}(\omega) - C_{\nu,k}(\omega) \Big) \cdot y_{\nu,k}(\omega) \right\} + \pi_\nu^s(\omega) \right)$$

The producers (2)

Let $\mu_{\nu,k}(\omega)$ be the dual variables associated with $y_{\nu,k}(\omega) \leq u_{\nu,k}$.

- At the optimum of the second stage ("tolling agreement")

$$\sum_k \Big(p_{el}(\omega) - C_{\nu,k}(\omega)\Big) \cdot y_{\nu,k}(\omega) = \sum_k u_{\nu,k} \cdot \mu_{\nu,k}(\omega) \tag{4}$$

- and the producer's problem can be reformulated as

$$\underset{\mathbf{u}_\nu : u_{\nu,k} \geq 0}{\text{Max}} \left\{ \min_{Q_\nu \in \mathcal{M}_\nu} \mathbb{E}_{Q_\nu} \left[\sum_k u_{\nu,k} \cdot \mu_{\nu,k}(\omega) + \pi_\nu^s(\omega) \right] \right\} - \sum_k I_k \cdot u_{\nu,k} \ .$$

- from which one infers the investor's behavior

$$0 \leq I_k - \mathbb{E}_{\bar{Q}_\nu(Z_\nu)} \left[\mu_{\nu,k}(\omega)\right] \perp u_{\nu,k} \geq 0 \qquad \forall k = 1, \ldots, K \tag{5}$$

- where $\bar{Q}_\nu(Z_\nu) \in \partial \rho^\nu(Z_\nu)$ is evaluated at

$$Z_\nu(\omega) = \sum_k u_{\nu,k} \cdot \mu_{\nu,k}(\omega) + \pi_\nu^s(\omega)$$

The producers (3)

The second stage optimization problem of the producer is

$$\begin{aligned} \underset{\mathbf{y}_\nu}{\text{Max}} \quad & \sum_k \Big(p_{el}(\omega) - C_{\nu,k}(\omega)\Big) \cdot y_{\nu,k}(\omega) & \\ \text{s.t.} \quad & y_{\nu,k}(\omega) \leq u_{\nu,k} & (\mu_{\nu,k}(\omega)) \\ & y_{\nu,k} \geq 0 & \end{aligned}$$

giving the conditions

$$\begin{aligned} 0 &\leq u_{\nu,k} - y_{\nu,k}(\omega) & \perp \mu_{\nu,k}(\omega) \geq 0 \\ 0 &\leq C_{\nu,k}(\omega) + \mu_{\nu,k}(\omega) - p_{el}(\omega) & \perp y_{\nu,k}(\omega) \geq 0 \end{aligned}$$

that needs to be added to the investment condition.

An alternative producer model: "Project Finance"

Definition

Each investor ν invests in some new capacity on the basis of the sole merits of this capacity and independently of any portfolio effect.
Given the electricity price $\mathbf{p_{el}}$, the price-taking company solves

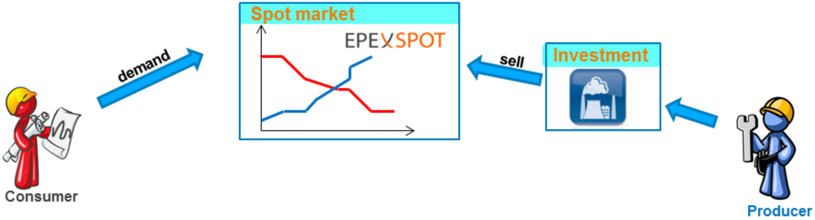
$$\mathcal{G}_\nu^2 \equiv \underset{\mathbf{u}_\nu : u_{\nu,k} \geq 0}{\text{Max}} \sum_k \rho_\nu \left(\underset{\substack{y_{\nu,k} \in \mathbb{R}^{|\Omega|} : y_{\nu,k}(\omega) \geq 0 \\ y_{\nu,k}(\omega) \leq u_{\nu,k}}}{\max} \left\{ \Big(p_{el}(\omega) - C_{\nu,k}(\omega) \Big) \cdot y_{\nu,k}(\omega) \right\} - I_k u_{\nu,k} \right)$$

By homogeneity of the risk adjusted value, the risk adjusted value of the gross margin becomes

$$\mathbb{E}_{Q_\nu(u_{\nu,k}\mu_{\nu,k})}[\mu_{\nu,k}(\omega)] = \mathbb{E}_{Q_\nu(\mu_{\nu,k})}[\mu_{\nu,k}(\omega)] = \rho_\nu \Big(\mu_{\nu,k}(\omega) \Big)$$

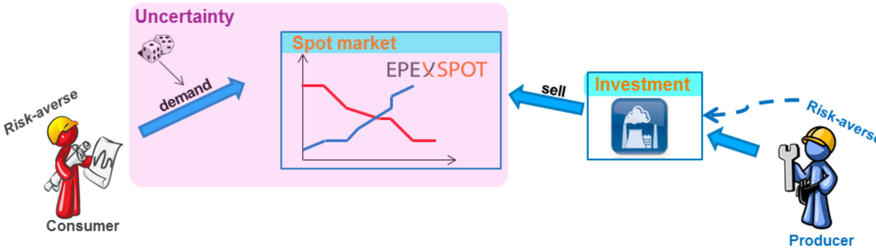
Markets without financial instruments

– **Two market players**, a producer and a consumer.



– **Two period model**: The producer invests before knowing the realisation of the demand.

– **Two market players**, a producer and a consumer.



– **Two period model**: The producer invests before knowing the realisation of the demand.

A "Project Finance" equilibrium model

Principle: Each plant is valued separately with no concern for the portfolio diversification effect of owning a mix of generation assets.

Definition

A solution of the "Project finance" equilibrium problem P consist of a tuple $(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu})$ satisfying

- The KKT conditions of the producers

$$0 \leq I_k - \rho_\nu \Big(\mu_{\nu,k}(\omega) \Big) \perp u_{\nu,k} \geq 0$$

$$0 \leq u_{\nu,k} - y_{\nu,k}(\omega) \perp \mu_{\nu,k}(\omega) \geq 0$$

$$0 \leq C_{\nu,k}(\omega) + \mu_{\nu,k}(\omega) - p_{el}(\omega) \perp y_{\nu,k}(\omega) \geq 0$$

- The KKT conditions of the consumer

$$0 \leq PC - p_{el}(\omega) \perp s(\omega) \geq 0$$

- A market clearing condition for the electricity spot market

$$0 \leq -LOAD(\omega) + \sum_{\nu,k} y_{\nu,k}(\omega) + s(\omega) \perp p_{el}(\omega) \geq 0$$

Agents hedging with financial contracts

Structure of the financial market

- Agents can hedge their profit/surplus by taking positions in $c = 1, \dots, C$ financial contracts.
  - p_c^f: the endogenous price of a contract c in $t = 0$
  - $p_c^s(\omega)$: the stochastic payoff in $t = 1$.
  - $x_{i,c}$: the agent's position in contract c.
- The financial instrument can be of two types :
  - Nominal asset: payoffs are determined by the occurence of the scenario (eg. Arrow-Debreu)
  - Real asset: payoffs depend on the outcome of the spot market. The contract prices and payoffs are endogenous to the equilibrium problem (e.g. contracts are written on electricity prices).

$$p_c^s(\omega) = h_{c,\omega}(p_{el}(\omega))$$

- Agents are also price taker in the financial market.

The Consumer with pure financial contracts (1)

Definition

Given the electricity price $\mathbf{p_{el}}$ and the contract prices (p_c^f, p_c^s), the price taking consumer d solves the following problem $\mathcal{C}_d^f$

$$\mathcal{C}_d^f \equiv \underset{\substack{\mathbf{s}:s(\omega)\geq 0 \\ \mathbf{x_d}}}{\text{Max}} \ \rho^d \Bigg((PC - p_{\mathrm{el}}(\omega)) \cdot (\mathrm{LOAD}(\omega) - s(\omega)) - \sum_c x_{d,c} \cdot (p_c^f - p_c^s(\omega)) + \pi_d^s(\omega) \Bigg)$$

- The financial contracts do not change the KKT conditions for cutailment

$$0 \leq PC - p_{el}(\omega) \perp s(\omega) \geq 0 \qquad \forall \omega \in \Omega \tag{6}$$

The consumer with pure financial contracts (2)

The consumer trading a risk contract values this contract at market price p_c^f (via his subjective risk-neutral probability measure, that is usually different from the real probability measure). :

$$p_c^f = \mathbb{E}_{\bar{Q}_d(Z_d^f)}\left[p_c^s(\omega)\right] - p_c^f$$

This is done by the change of probability embedded in the subgradient of the coherent risk function $\bar{Q}_d(Z_d^f) \in \partial\rho_d(Z_d^f)$ evaluated at the payoff Z_d^f.

$$Z_d^f(\omega) = (PC - p_{\mathrm{el}}(\omega)) \cdot (\mathrm{LOAD}(\omega) - s(\omega)) + \sum_c x_{d,c} \cdot p_c^s(\omega) + \pi_d^s(\omega)$$

The producer with pure financial contracts (1)

Definition

Given the electricity price $\mathbf{p_{el}}$ and the contract prices (p_c^f, p_c^s), a price taking company ν investing in a portfolio of contracts solves

$$\mathcal{G}_\nu^f = \underset{\substack{\mathbf{u}_\nu : u_{\nu,k} \geq 0 \\ \mathbf{x}_\nu}}{\text{Max}} \quad - \sum_k I_k \cdot u_{\nu,k} - \sum_c p_c^f \cdot x_{\nu,c}$$
$$+ \rho_\nu \left(\max_{\substack{\mathbf{y}_\nu : y_{\nu,k}(\omega) \geq 0 \\ y_{\nu,k}(\omega) \leq u_{\nu,k}}} \left\{ \sum_k \Big(p_{el}(\omega) - C_{\nu,k}(\omega) \Big) \cdot y_{\nu,k} \right\} + \right.$$
$$\left. + \pi_s^\nu(\omega) + \sum_c x_{\nu,c} \cdot p_c^s(\omega) \right)$$

The producer with pure financial contracts (2)

- The following conditions are ALMOST identical to those obtained without contracts

$$0 \leq I_k - \mathbb{E}_{\bar{Q}_\nu(Z^f_\nu)}\left[\mu_{\nu,k}(\omega)\right] \;\perp\; u_{\nu,k} \geq 0$$

$$0 \leq u_{\nu,k} - y_{\nu,k}(\omega) \;\perp\; \mu_{\nu,k}(\omega) \geq 0$$

$$0 \leq C_{\nu,k}(\omega) + \mu_{\nu,k}(\omega) - p_{el}(\omega) \;\perp\; y_{\nu,k}(\omega) \geq 0$$

- Differences come from the positions where the risk functions are evaluated

$$Z^f_\nu = \sum_k u_{\nu,k} \cdot \mu_{\nu,k}(\omega) + \pi^\nu_s + \sum_c x_{\nu,c} \cdot p^s_c(\omega)\ .$$

- and from the addition of the pricing conditions of the financial markets:

$$p^f_c = \mathbb{E}_{\bar{Q}_\nu(Z^f_\nu)}\ \left[p^s_c(\omega)\right]$$

Remark: financial contracts linked to physical decisions

To incentivize investment, some contracts $c' \in \mathrm{PHY}$ need to be "supported" by a physical delivery.

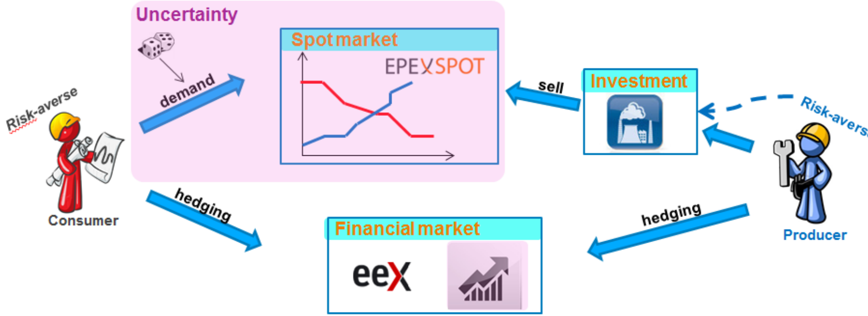
- It has been argued that "reliability options" (see later) should be linked to the available capacity (Oren [9], Vazquez et al. [13]).
  This means that producers cannot sell more contract than their capacity, should be able to produce the amount contracted or otherwise have to pay a penalty $\mathtt{plt}_\ell(\omega)$.

The problem becomes:

$$\mathcal{G}_\nu^f = \underset{\substack{\mathbf{u}_\nu : u_{\nu,k} \geq 0 \\ \mathbf{x}_\nu : x_{\nu,c'} \leq \sum_k u_{\nu,k}}}{\mathrm{Max}} \quad - \sum_k I_k \cdot u_{\nu,k} - \sum_c p_c^f \cdot x_{\nu,c}$$
$$+ \rho_\nu \Bigg(\max_{\substack{\mathbf{y}_\nu : y_{\nu,k}(\omega) \geq 0 \\ y_{\nu,k}(\omega) \leq u_{\nu,k}}} \left\{ \sum_k \Big(p_{el}(\omega) - C_{\nu,k}(\omega) \Big) \cdot y_{\nu,k} \right\}$$
$$+ \pi_s^\nu(\omega) + \sum_c x_{\nu,c} \cdot p_c^s(\omega) - \sum_{c \in C'} \sum_{\ell=1}^{L} \tau_\ell m_{\nu,c',\ell}(\omega) \mathtt{plt}_\ell(\omega) \Bigg)$$

Market equilibrium with financial instruments

– **Two market players**, a producer and a consumer.



– **Two period model**: The producer invests before knowing the realisation of the demand.
The producer/the consumer take financial positions to hedge the spot market outcomes
The payoff of a financial contract is also uncertain (based on the spot market)

The spot-financial equilibrium problems (P^f) (1)

Models of different types can be constructed to reflect particular situations.
We here consider a set of companies each investing in portfolios

- Consumers consume fixed quantities but they hedge their risk
- Producers invest in portfolios, well aware of their diversification needs. They also try to hedge the risk that they could not reduce by diversification by concluding contracts

The mechanisms underpinning the different models is to clearly distinguish between the risk position of the agent and the risk of the asset that it wants to evaluate.
An agent assesses the risk of a plant based on the risk position implied by that plant (same as in "project finance") but it assesses it on its global risk position implied by the portfolio.

A market of portfolios of hedged plants (P^f) (2)

Definition

A solution of the spot-financial equilibrium problem P^f consists of a tuple $(\mathbf{u}, \mathbf{p_{el}}, \mathbf{s}, \mathbf{y}, \boldsymbol{\mu}, \mathbf{x}, \mathbf{p^f}, \mathbf{p}^s)$ satisfying the following complementary conditions

$$
\begin{aligned}
&0 = \mathbb{E}_{\bar{Q}_d(Z_d^f)}[p_c^s(\omega)] - p_c^f \\
&0 = \mathbb{E}_{\bar{Q}_\nu(Z_\nu^f)}[p_c^s(\omega)] - p_c^f \\
&0 = \sum_\nu x_{\nu,c} + x_{d,c} \\
&0 \leq I_k - \mathbb{E}_{\bar{Q}_\nu(Z_\nu^f)}[\mu_{\nu,k}(\omega)] \perp u_{\nu,k} \geq 0 \\
&0 \leq u_{\nu,k} - y_{\nu,k}(\omega) \perp \mu_{\nu,k}(\omega) \geq 0 \\
&0 \leq C_{\nu,k}(\omega) + \mu_{\nu,k}(\omega) - p_{el}(\omega) \perp y_{\nu,k}(\omega) \geq 0 \\
&0 \leq PC - p_{el}(\omega) \perp s(\omega) \geq 0 \\
&0 \leq -LOAD(\omega) + \sum_{\nu,k} y_{\nu,k}(\omega) + s(\omega) \perp p_{el}(\omega) \geq 0 \\
&0 = p_c^s(\omega) - h_{c,\omega}(p_{el}(\omega))
\end{aligned}
$$

Project finance and portfolio of plants(1)

Risk functions and their derivatives are evaluated for different positions

- The risk exposure only involves a single plant (a vector of dual variables) in project finance. It thus only depends on the investment in the plant that is assessed
- The risk exposure involves all plants and financial positions (primal and dual variables) of the agents in "portfolio of plants". It thus depends on the decisions made in all the assets of the portfolios

This difference has technical consequences as seen when discussing uniqueness.

Some theory

Natural questions about an equilibrium model

- The questions
  - Do the models have a solution?
  - If so are they isolated? (GNE are also used to represent some incomplete markets and have non isolated equilibria.)
  - If isolated do they come in odd number?
  - If they come in odd number are they unique?
- And the answers (except if errors)
  - The two models have equilibria under some reasonable assumptions.
  - The proof of existence should be easily applicable to other models.
  - We do not know where the model with portfolio has a unique solution (not fully surprising as the overall exposure is a quadratic function of primal and dual variables.)
  - But uniqueness of solution for the project finance problem seems to derive from considerations that can be generalized.

The methodology: reminder to degree theory (1)

- We resort to topoligical degree theory in order to prove the existence (and structure) of a solution, using the presentation of Facchinei and Pang [7].

Theorem

- *Let Ξ be a nonempty, bounded open subset of R^n*
- *Let $\Phi = cl\Xi \rightarrow R^n$ be a continuous function.*
- *Assume that $p \notin \Phi(bd\Xi)$.*

If $deg(\Phi, \Omega, p) \neq 0$, then there exists an $\bar{x} \in \Xi$ such that $\Phi(\bar{x}) = p$.
Conversely if $p \notin \Phi(\mathrm{cl}\Xi)$, then $deg(\Phi, \Omega, p) = 0$.

- It is well-known that $0 \leq x \perp f(x) \geq 0$ can be rewritten as an element-wise minimization $\min(x, f(x)) = 0$.
- These are continuous functions if the risk functions are continuously differentiable

The methodology: reminder of degree theory (2)

We use in this paper the standard idea (based on homotopy invariance) presented in Facchinei and Pang [7] to prove existence of a solution of $\Phi(x) = 0$.

1. Construct a "homotopy function": $H : cl\Xi \times [0,1] \to \mathbb{R}^n$ such that
   - $H(x,0) = \Phi(x)$ for all $x \in cl\Xi$ and the degree of $H(x,1)$ is known and nonzero.
2. If 0 does not belong to $H(bd\Xi, t)$ for all $t \in [0,1]$, then by the homotopy invariance property, the degree of the original triple $(\Phi, \Xi, 0)$ is equal to the degree of the auxiliary triple $(H(\cdot,1), \Xi, 0)$; hence there exist a solution to $\Phi(x) = 0$.

The auxilary "Project Finance" problem AP

We construct the auxiliary problem AP, by modifying problem P:

- All agents have the same continuously differentiable coherent risk function $\bar{\rho}$ and receive an exogeneous payoff such that they value risk at the same global payoff

$$Z_{\text{tot}}^{AP}(\omega) = \sum_{\nu} Z_{\nu}(\omega) + Z_d(\omega)$$

Problem AP describes a purely fictive economy but has an economic interpretation.

- Transfers are such that the agents value their power plants as they would do after risk trading in a complete market [11]

Proposition

AP is equivalent to the following risk-averse optimization problem:

$$\begin{aligned}\underset{\mathbf{u}:u_{\nu,k}\geq 0}{\text{Max}} \sum_{\nu,k} -I_k \cdot u_{\nu,k} + \overline{\rho}\Bigg(\max_{\mathbf{y},\mathbf{s}} \quad & PC \cdot (LOAD(\omega) - s(\omega)) - \sum_{k} C_{\nu,k}(\omega) \cdot y_{\nu,k}(\omega)\Bigg) \\ s.t. \quad & 0 \leq y_{\nu,k}(\omega) \leq u_{\nu,k} \\ & \sum_{\nu,k} y_{\nu,k} + s(\omega) = LOAD(\omega)\end{aligned}$$

Barring degeneracy this problem has a unique primal-dual solution.

The "Project Finance" homotopy $H^P(\lambda)$

We construct the homotopy function $H^P(\lambda)$ obtained from P and AP by replacing the KKT on investment in P by

$$0 \leq I_k - \lambda \mathbb{E}_{\bar{Q}(Z^{AP}_{\text{tot}})}\left[\mu_{\nu,k}(\omega)\right] - (1-\lambda)\rho_\nu(\mu_{\nu,k}(\omega)) \ \perp \ u_{\nu,k} \geq 0$$

We can find an bounded open set that does not contain, for every $\lambda \in [0,1]$, a solution of $H^P(\lambda) = 0$ on it closure. The scalar Δ is a positive number.

$$\Xi^P := \left\{ \begin{array}{l} u_{\nu,k} \in \ ]-\Delta, \max_{\omega' \in \Omega} \ LOAD(\omega') + \Delta[\\ p_{el}(\omega) \in \ ]-\Delta, PC + \Delta[\\ s(\omega) \in \ ]-\Delta, \max_{\omega' \in \Omega} \ LOAD(\omega') + \Delta[\\ y_{\nu,k}(\omega) \in \ ]-\Delta, \max_{\omega' \in \Omega} \ LOAD(\omega') + \Delta[\\ \mu_{\nu,k}(\omega) \in \ ]-\Delta, PC + \Delta[\end{array} \right\}$$

Following Facchinei and Pang [7] and related assumptions (isolated solutions), we have that the number of solution of the project finance problem is odd.

$$\deg(\Phi, \Xi, p) = \sum_{\bar{x} \in \Phi^{-1}(p)} \operatorname{sgn} \det J\Phi(\bar{x}) \tag{7}$$

The auxilary spot-financial problem AP^f

We construct the auxiliary problem AP^f, by modifying the spot-financial problem P^f:

- All agents have the same continuously differentiable risk function $\bar{\rho}$, that also ensure uniqueness of the position x_i in the financial contract.
- The agents receive an exogeneous payoff such that they value their risk exposure at the same global payoff

$$Z_{\texttt{tot}}^{AP^f}(\omega) = \sum_{\nu} Z_{\nu}^f(\omega) + Z_d^f(\omega)$$

Problem AP^f describes a purely fictive economy but has an economic interpretation.

- Transfers are such that the agents value their power plants as they would do after risk trading in a complete market Ralph and Smeers [11]
- AP^f has a solution iff $x_{\nu,c} = x_{d,c} = 0$.
- Under usual technical assumptions, this problem has a unique solution (cf. optimization problem).

The spot-financial homotopy $H^{P^f}(\lambda)$

We construct the homotopy function $H^{P^f}(\lambda)$ obtained from P^f and AP^f by replacing the KKT on investment decision in P^f by

$$0 \leq I_k - \lambda \cdot \mathbb{E}_{\bar{Q}(Z_{\text{tot}}^{AP^f})}\left[\mu_{\nu,k}(\omega)\right] - (1-\lambda)\mathbb{E}_{\bar{Q}_\nu(Z_\nu^f)}\left[\mu_{\nu,k}(\omega)\right] \quad \perp \quad u_{\nu,k} \geq 0$$

One similarly modifies the conditions relative to financial decisions by

$$0 = \lambda \cdot \mathbb{E}_{\bar{Q}(Z_{\text{tot}}^{AP^f})}\left[p_c^s(\omega)\right] + (1-\lambda)\mathbb{E}_{\bar{Q}_d(Z_d^f)}\left[p_c^s(\omega)\right] - p_c^f$$

We can bound the position in the financial contract by using the same argument than de Maere d'Aertrycke and Smeers [4]:

- The agents' risk measures are *sufficiently similar*:

$$\textbf{int} \ \ \overline{\mathcal{M}} \neq \emptyset$$

- In that case, we cannot have unbounded optimal solution

We have again that the number of solution of the spot-financial problem is odd.

Illustration

A toy problem

A two-stage model: invest in stage 0; operate in stage 1

- Uncertainty: demand only; mainly in peak; 15 scenarios.

Risk behaviour

- Consumers are exposed to spikes in price peaks; these create consumer's surplus volatility.
- Producers are exposed to idle capacity risk and may not recover their fixed costs.
- Modelled by $E - CVaR$ for both.

Two plants

- CAPEX: BASE 110 $euro/MW$; PEAK: 60 $euro/MW$.
- OPEX: BASE 30 $euro/MWh$; PEAK: 60 $euro/MWh$.

Cases

Two references

- Full risk trading (complete markets, cf. Ralph and Smeers [11], Philpott et al. [10]) .
- "Fully" incomplete market: no risk trading; each agent adapts its pricing kernel (Ehrenmann and Smeers [5]).

Three cases of risk trading (a smooth move from energy only to capacity markets): agents adapt their pricing kernel subject to the constraint that they price the financial instruments

- Yearly futures
- Reliability options (proposed in the literature as a substitute to capacity markets).
- Forward capacity market.

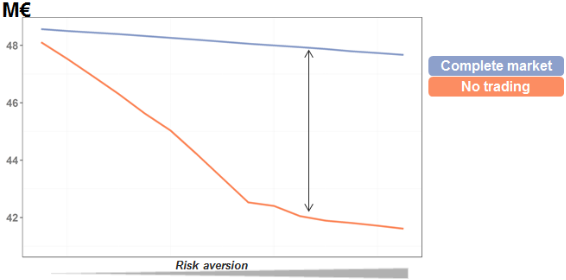
Reference cases: welfare

Two "extreme" cases:

1. One can trade every risk in the market: Complete market.
2. Risk trading is totally impossible: No trading

None of these situations is realistic (benchmark) but they "bound our ignorance".

- Welfare in the complete market is the highest possible.
- Producer and consumer cannot optimally share the risk. The welfare is significantly destroyed as they become risk averse.



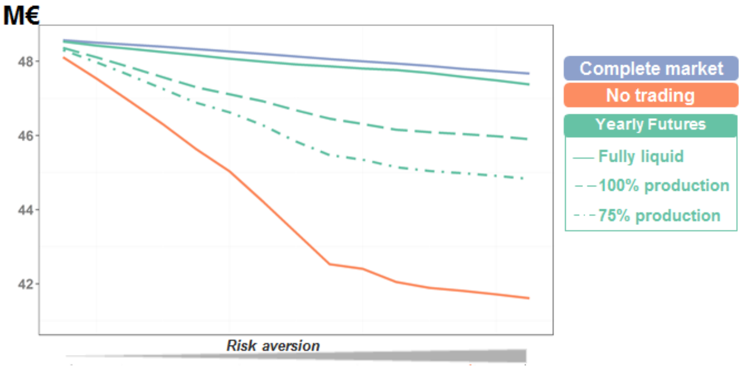
Reference cases: investment

In both cases, investment decreases with risk aversion

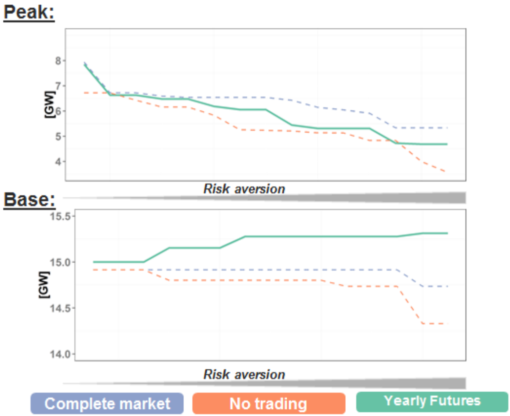
- **Complete market**: A risk-averse system tends to avoid overcapacity for low-demand scenario.
- **No-market**: Underinvestment is exacerbated by producer's risk aversion. Peak units are particularly at risk.

Yearly futures: welfare

- Yearly futures are contracts to hedge against the yearly average of electricity prices (calendar product- CfD).
- Popular contract, liquid up to 3 or 4 years (useless for investment but possibly useful for mothballing).
- Sensitivity on liquidity (through Bid-ask spread)



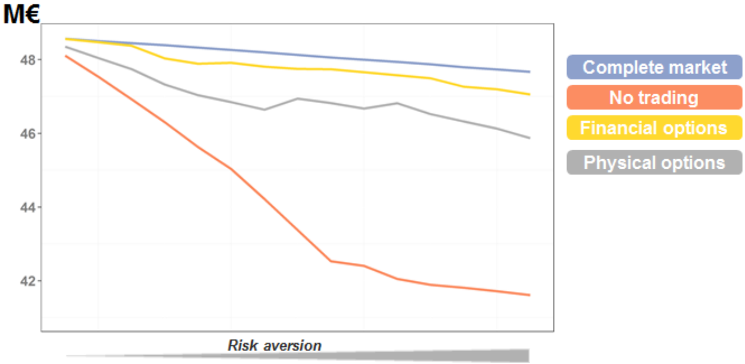
Yearly futures: investment



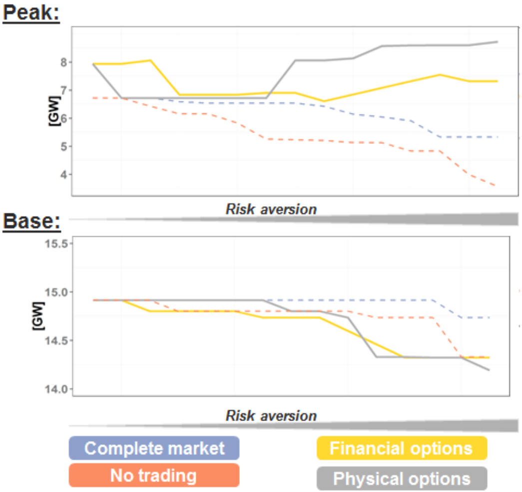
Yearly futures incentivize investment but lead to the "wrong" technology mix by promoting the base technology.

Reliability options: welfare

- Financial reliability options are classical European options with a rather high strike price;
- Physical option: quantity limit. Oren [9] argues that such product would incentivize investment.



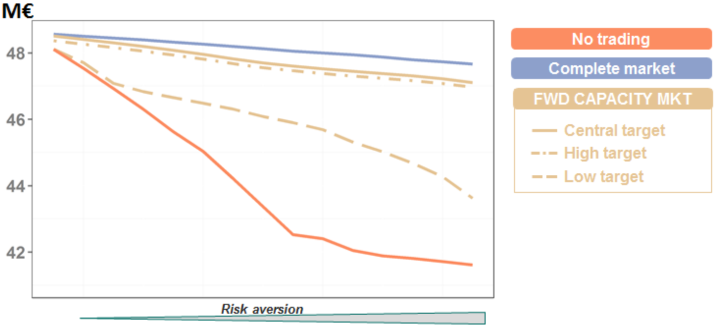
Reliability options: investment



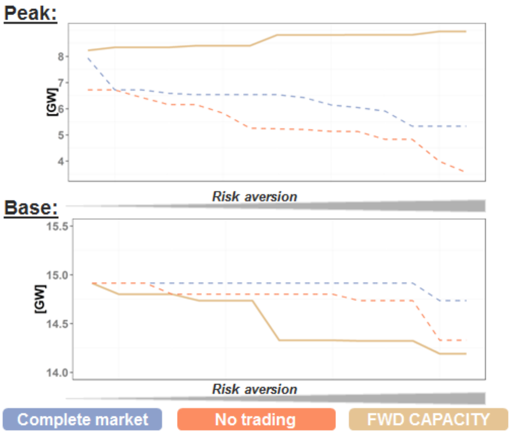
Reliability options penalize base and incentivize too much peak units.

Forward capacity market: welfare

- The SO offers a demand for capacity certificates (exogeneous demand)
- Producers sell capacity certificates.
- The capacity price is then charged to the consumers.
- Sensitivity on the SO demand



Forward capacity market: investment



The forward capacity market promotes investment in peak.
Choosing the socially optimal capacity target requires to know the consumer utility (in the case of an inelastic demand: assymetric risk).

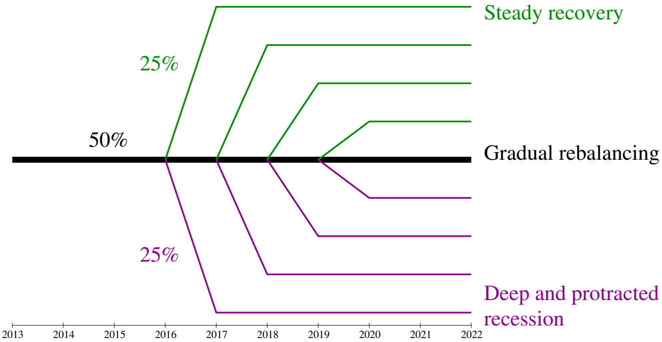
Implicit hurdle rates

	A- Investment cost *[k€]*	**B** – Expected Gross profit from energy trading *[k€]*	**C**- Expected profit from financial trading *[k€]*	**D**- Risk premium *(B + C -A) / (A)*	**E**- Total Welfare *[k€]*
Complete market	2491	3245	-717	1.5%	48359
Yearly futures	2515	2941	-348	3.1%	48243
Illiquid Yearly futures	2460	3753	-964	13.4%	47870
Reliability options	2491	3066	-212	14.6%	47898
Physical options	2499	3061	245	32.3%	47324
Fwd capacity market - central	2591	1146	1549	4.0%	48192
No trading	2445	3838	0	57.0%	46301

Can this be of some interest in practice?

The following tree is easily interpretable today

A common situation with common words: three scenarios



The model is constructed for a large system, focusing on decommissioning. The producer should take its decision a year in advance.

The following introduces new items of discussion.

Handling only the complete and incomplete market cases.

- Do results differ with completeness?
  - Risk averse producers will tend to mothball more in an incomplete market.
  - The average price of electricity increases in the incomplete market
  - And the risk of the producers also increases.

Results also depend on market design and overall market attitude towards risk

- Market design: Price cap
- Market mood: risk aversion characterized by Sharpe ratio (also a familiar notion)

Impact of market incompleteness

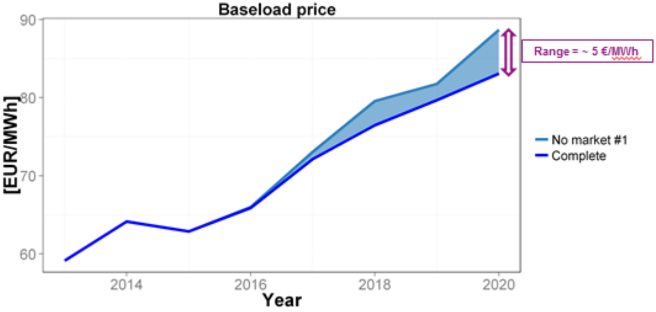
Calculation for the example of a European country

We model the entire production park.
Differences in prices are explained by asset management decisions.

Runs settings:

Good-deal measure calibrated with
– Sharpe ratio = 0.52

Recourse option
– demand curtailment = 3000 [€/MWh]



A numerical note

How to solve

The toy problems and the contracts

- Solved with PATH
- using a good starting point

The "corporate model" in the two extreme cases.

- Iteration between a LP version of the model (with fixed price kernel) both for the complete and incomplete markets.
- and update of the new pricing kernel by a non linear optimizer code (the good deal is a conic measure)
- Converges in a few iterations.

Conclusion

Did we learn something? (1)

In terms of investment: Yes but not very positive!

- Advocated remedies to deterministic market incompleteness may fail in an uncertain market
  - this can already be seen on toy problems
  - but this does not tell how to remedy the remedies

Market incompleteness seems to also be important on large scale models

- In terms of risk compared to the complete market
- Standard stochastic programs with risk functions will only tells us the residual risk assuming completeness:
- and a higher risk compared to the complete market, as observed in the model, is definitely not what we need today.

Did we learn something? (2)

In terms of methods: Yes possibly more positive!

- One can construct models that contain important economic notions of investment theory.
  - representing portfolios by agent
  - and discount factors that depend on portfolios
  - not discussed here: the discount factors can be made compatible with the CAPM for the systematic risk
  - and do not need to set to idiosyncratic risk to zero

Market incompleteness can be tackled by degree theory

- both in terms of existence and uniqueness of solution
- not discussed here (multiplicity of solutions appear in other (short term) power problems)

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