Optimization and Equilibrium in Energy Economics

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Welcome and thanks

- **What**: Physical system + markets + stochastics + modeling + computation
- **Who**: power system engineers, economists, mathematicians, operations researchers, and computer scientists
- **Why**: create more dialogue between researchers in this area with different expertise
- **How**?
  - IPAM: Christian Ratsch and Roland McFarland
  - Organizing committee: Benjamin Hobbs, Antonio Conejo, Andrew Philpott, Claudia Sagastizabal
  - All of you for attending and preparing presentations
- **Outcomes**: new ideas and models, white paper summary
- **This tutorial aims to provide some context and vocabulary for the meeting**
Determine generators’ output to reliably meet the load

- \( \sum \text{Gen MW} \geq \sum \text{Load MW} \), at all times.
- Power flows cannot exceed lines’ transfer capacity.
Single market, single good: equilibrium

Walras: \[ 0 \leq s(\pi) - d(\pi) \perp \pi \geq 0 \]

Market design and rules to foster competitive behavior/efficiency

- Spatial extension: Locational Marginal Prices (LMP) at nodes (buses) in the network
- Supply arises often from a generator offer curve (lumpy)
- Technologies and physics affect production and distribution
The PIES Model (Hogan) - Optimal Power Flow (OPF)

\[
\begin{align*}
\min_{x} & \quad c(x) \quad \text{cost} \\
\text{s.t.} & \quad Ax \geq q \quad \text{balance} \\
& \quad Bx = b, x \geq 0 \quad \text{technical constr}
\end{align*}
\]
The PIES Model (Hogan) - Optimal Power Flow (OPF)

\[
\begin{align*}
\min_{x} & \quad c(x) \\
\text{s.t.} & \quad Ax \geq d(\pi) \\
& \quad Bx = b, \quad x \geq 0
\end{align*}
\]

- \( q = d(\pi) \): issue is that \( \pi \) is the multiplier on the “balance” constraint
- Such multipliers (LMP’s) are critical to operation of market
- Can solve the problem iteratively or by writing down the KKT conditions of this QP, forming an LCP and exposing \( \pi \) to the model
- Existence, uniqueness, stability from variational analysis
- EMP does this automatically from the annotations
Reformulation details

\[ 0 \leq Ax - d(\pi) \quad \perp \quad \mu \geq 0 \]
\[ 0 = Bx - b \quad \perp \quad \lambda \]
\[ 0 \leq \nabla c(x) - A^T \mu - B^T \lambda \quad \perp \quad x \geq 0 \]

- empinfo: dualvar \( \pi \) balance
- replaces \( \mu \equiv \pi \)
Reformulation details

\[ 0 \leq Ax - d(\pi) \quad \perp \quad \pi \geq 0 \]
\[ 0 = Bx - b \quad \perp \quad \lambda \]
\[ 0 \leq \nabla c(x) - A^T \pi - B^T \lambda \quad \perp \quad x \geq 0 \]

- **empinfo:** dualvar \( \pi \) balance
- replaces \( \mu \equiv \pi \)
- LCP/MCP is then solvable using PATH

\[
z = \begin{bmatrix} \pi \\ \lambda \\ x \end{bmatrix}, \quad F(z) = \begin{bmatrix} A \\ -A^T & -B^T \end{bmatrix} z + \begin{bmatrix} -d(\pi) \\ -b & \nabla c(x) \end{bmatrix}
\]
Reformulation details (VI formulation)

\[ 0 \leq Ax - q \quad \perp \quad \pi \geq 0 \]
\[ 0 = Bx - b \quad \perp \quad \lambda \]
\[ 0 \leq \nabla c(x) - A^T \pi - B^T \lambda \quad \perp \quad x \geq 0 \]
\[ q = d(\pi) \quad 0 = -p(q) + \pi \quad \perp \quad q \]

- Inverse demand \( p(q) \): \( \pi = p(q) \iff q = d(\pi) \)
- \((x, q) \in C, 0 \in \left[ \nabla c(x) \right] + N_c(x, q) \)
- \((x, q)\) solves \( VI(F, C), F(x, q) = (\nabla c(x), -p(q))^T \)
- New solvers for VI: PATHVI, decomposition, distributed solution
- Straightforward to extend to more general production functions and cost functions
Extensions: maximizing profit and multiple agents

\[
\max_{x} \pi^T x - c(x) \quad \text{profit}
\]
\[
s.t. \quad Ax \geq d(\pi) \quad \text{balance}
\]
\[
Bx = b, \quad x \geq 0 \quad \text{technical constr}
\]

- Issue is that there are multiple producers \( i \)
- The price is now determined by total production

\[
\max_{x_i} p\left(\sum_{j} x_j\right)^T x_i - c_i(x_i) \quad \text{profit}
\]
\[
s.t. \quad B_i x_i = b_i, \quad x_i \geq 0 \quad \text{technical constr}
\]
Special case: perfect competition

\[
\max_{x_i} \pi p(\sum_{j} x_j) ^T x_i - c_i(x_i) \quad \text{profit}
\]

s.t. \( B_i x_i = b_i, x_i \geq 0 \quad \text{technical constr} \)

\[
0 \leq \sum_{i} x_i - d(\pi) \perp \pi \geq 0
\]

- When there are many agents, assume none can affect \( \pi \) by themselves
- Each agent is a price taker
- Two agents, \( d(\pi) = \bar{d} - \pi, \bar{d} = 24, c_1 = 3, c_2 = 2 \)
- KKT(1) + KKT(2) + Market Clearing gives Complementarity Problem
- \( x_1 = 0, x_2 = 22, \pi = 2 \)
MOPEC

\[ \min_{x_i} \theta_i(x_i, x_{-i}, p) \text{ s.t. } g_i(x_i, x_{-i}, p) \leq 0, \forall i \]

\[ p \text{ solves } \text{VI}(h(x, \cdot), C) \]

- (Generalized) Nash
- Reformulate optimization problem as first order conditions (complementarity)
- Use nonsmooth Newton methods to solve
- Solve overall problem using “individual optimizations”?

**Trade/Policy Model (MCP)**
- Split model (18,000 vars) via region
- Gauss-Seidel, Jacobi, Asynchronous
- 87 regional subprobs, 592 solves
Let us assume that $1 > 0$ and $p(!) > 0$ for every $!$. This corresponds to a solution of SP meeting the demand constraints exactly, and being able to save money by reducing demand in each time period and in each state of the world. Under this assumption TP($i$) and HP($i$) also have unique solutions. Since they are convex optimization problems their solution will be determined by their Karush-Kuhn-Tucker (KKT) conditions. We define the competitive equilibrium to be a solution to the following variational problem:

$$\text{CE}: \left( u_1(i); u_2(i; !) \right) \in \arg\max_{u_1(i); u_2(i; !)} \text{HP}(i), \quad i \in H \bigg( v_1(j); v_2(j; !) \bigg) \in \arg\max_{v_1(j); v_2(j; !)} \text{TP}(j), \quad j \in T$$

This gives the following result.

**Proposition 2**

Suppose every agent is risk neutral and has knowledge of all deterministic data, as well as sharing the same probability distribution for inflows. Then the solution to SP is the same as the solution to CE.

### 3.1 Example

Throughout this paper we will illustrate the concepts using the hydro-thermal system with one reservoir and one thermal plant, as shown in Figure 1. We let thermal cost be $C(v) = v^2$, and define $U(u) = 1$:

- $V(x) = 30 \cdot 3 \cdot x + 0$:
- $V(x) = 0.15 x^2$:
- $V(x) = 0.025 x^2$

We assume inflow 4 in period 1, and inflows of $1, 2, \ldots, 10$ with equal probability in each scenario in period 2. With an initial storage level of 10 units this gives the competitive equilibrium shown in Table 1. The central plan that maximizes expected welfare (by minimizing expected generation and future cost) is shown in Table 2. One can observe that the two solutions are identical, as predicted by Proposition 2.

- Competing agents (consumers, or generators in energy market)
- Each agent maximizes objective independently (profit)
- Market prices are function of all agents activities
Simple electricity “system optimization” problem

SO: \( \max_{d_k, u_i, v_j, x_i \geq 0} \sum_{k \in K} W_k(d_k) - \sum_{j \in T} C_j(v_j) + \sum_{i \in H} V_i(x_i) \)

s.t. \( \sum_{i \in H} U_i(u_i) + \sum_{j \in T} v_j \geq \sum_{k \in K} d_k \),

\( x_i = x_i^0 - u_i + h_i^1, \quad i \in H \)

- \( u_i \) water release of hydro reservoir \( i \in H \)
- \( v_j \) thermal generation of plant \( j \in T \)
- \( x_i \) water level in reservoir \( i \in H \)
- prod fn \( U_i \) (strictly concave) converts water release to energy
- \( C_j(v_j) \) denote the cost of generation by thermal plant
- \( V_i(x_i) \) future value of terminating with storage \( x \) (assumed separable)
- \( W_k(d_k) \) utility of consumption \( d_k \)
SO equivalent to CE (price takers)

Consumers $k \in K$ solve $CP(k)$: \[
\max_{d_k \geq 0} \quad W_k \left( d_k \right) - \pi^T d_k
\]

Thermal plants $j \in T$ solve $TP(j)$: \[
\max_{v_j \geq 0} \quad \pi^T v_j - C_j \left( v_j \right)
\]

Hydro plants $i \in H$ solve $HP(i)$: \[
\max_{u_i, x_i \geq 0} \quad \pi^T U_i \left( u_i \right) + V_i \left( x_i \right)
\]
\[
\text{s.t.} \quad x_i = x_i^0 - u_i + h_i^1
\]

Perfectly competitive (Walrasian) equilibrium is a MOPEC

CE: \[
d_k \in \arg \max CP(k), \quad k \in K,
\]
\[
v_j \in \arg \max TP(j), \quad j \in T,
\]
\[
u_i, x_i \in \arg \max HP(i), \quad i \in H,
\]
\[
0 \leq \pi \perp \sum_{i \in H} U_i \left( u_i \right) + \sum_{j \in T} v_j \geq \sum_{k \in K} d_k.
\]
General Equilibrium models (static)

\[(C): \max_{x_k \in X_k} U_k(x_k) \text{ s.t. } \pi^T x_k \leq i_k(y, \pi)\]

\[(I): i_k(y, \pi) = \pi^T \omega_k + \sum_j \alpha_{kj} \pi^T g_j(y_j)\]

\[(P): \max_{y_j \in Y_j} \pi^T g_j(y_j)\]

\[(M): \max_{\pi \geq 0} \pi^T \left( \sum_k x_k - \sum_k \omega_k - \sum_j g_j(y_j) \right) \text{ s.t. } \sum_l \pi_l = 1\]

- This is an example of a MOPEC: strategic, top-down, policy analyses
- Can extend these models in several ways: more goods (not just energy), more agents (refineries, farmers), different behavior patterns: who is driving the bus?
Bus or Taxi: two agents (duopoly)

$$\max_{x_i} p\left(\sum_j x_j\right)^T x_i - c_i(x_i)$$  \quad \text{profit}

s.t.  \quad B_i x_i = b_i, x_i \geq 0  \quad \text{technical constr}

- Cournot: assume each can affect $p$ by choice of $x_i$
- Two agents, same data
- KKT(1) + KKT(2) gives Complementarity Problem
- $x_1 = 20/3, x_2 = 23/3, \pi = 29/3$

- Exercise of market power (some price takers, some Cournot)
UBER: Bilevel Program (Stackelberg)

- Assumes one leader firm, the rest follow
- Leader firm optimizes subject to expected follower behavior
- Follower firms act in a competitive Nash manner

Bilevel programs:

\[
\begin{align*}
\min_{x^*, y^*} & \quad f(x^*, y^*) \\
\text{s.t.} & \quad g(x^*, y^*) \leq 0, \\
& \quad y^* \text{ solves } \min_y v(x^*, y) \text{ s.t. } h(x^*, y) \leq 0
\end{align*}
\]

- model bilev /deff,defg,defv,defh/;
  empinfo: bilevel min v y defv defh
- EMP tool automatically creates the MPCC

\[
\begin{align*}
\min_{x^*, y^*, \lambda} & \quad f(x^*, y^*) \\
\text{s.t.} & \quad g(x^*, y^*) \leq 0, \\
& \quad 0 \leq \nabla v(x^*, y^*) + \lambda^T \nabla h(x^*, y^*) \perp y^* \geq 0 \\
& \quad 0 \leq -h(x^*, y^*) \perp \lambda \geq 0
\end{align*}
\]
Representative decision-making timescales in electric power systems

Many interacting levels/hierarchies, with different time scaled decisions at each level - collections of models needed.
Complications and myriad of acronyms

- **Size/integrity:**
  - AC/DC models, reactive power, new devices
  - Day ahead, regulation, FTR’s, co-optimization
  - Semidefinite programming (global), EPEC’s

- **Discrete:**
  - Unit commitment (DAUC, RUC, RT)
  - Topology optimization (e.g. Transmission line switching, siting)
  - Design of flexible computing systems
  - How do we price discrete decisions? (make-good, fix ints, ...)

- **Dynamic:**
  - Design/operation
  - Multi-period, unit commitment, minimum up and down time
  - Demand response, load shedding, demand bidding

- **Stochastic:**
  - Security constraints - failures/reserves (SCED/SCUC)
  - Stochastic demand
  - Renewables/storage/feed-in prices
$N = 3$, $m = 5$. Each player chooses a path from $s$ to $t$. Costs for one, two or three players using arc are given by triplets.

The goal is to find a pure Nash equilibrium, i.e. a state $x = (x^1, \ldots, x^N) \in X$ such that, for each player $i$ and $\bar{x}^i \in X^i$:

$$c^i(x^1, \ldots, x^i, \ldots, x^N) \leq c^i(x^1, \ldots, \bar{x}^i, \ldots, x^N).$$

**Theorem (DelPia/F./Micini)**

*There is a strongly polynomial-time algorithm for finding a pure Nash equilibrium in symmetric TU congestion games.*
Dynamic: PJM buy/sell storage

- Storage transfers energy over time (horizon = \( T \)).
- PJM: given price path \( p_t \), determine charge \( q^+_t \) and discharge \( q^-_t \):

\[
\max_{h_t, q^+_t, q^-_t} \sum_{t=0}^{T} p_t (q^-_t - q^+_t) \\
\text{s.t. } \partial h_t = e q^+_t - q^-_t \\
0 \leq h_t \leq S \\
0 \leq q^+_t \leq Q \\
0 \leq q^-_t \leq Q \\
h_0, h_T \text{ fixed}
\]

- Uses: price shaving, load shifting, transmission line deferral
- What about real-time storage, or different storage technologies?
Stochastic: Agents have recourse?

- Agents face uncertainties in reservoir inflows
- Two stage stochastic programming, $x^1$ is here-and-now decision, recourse decisions $x^2$ depend on realization of a random variable $\rho$
- $\rho$ is a risk measure (e.g. expectation, CVaR)

SP: \[
\begin{align*}
\text{min} \quad & c^T x^1 + \rho[q^T x^2] \\
\text{s.t.} \quad & Ax^1 = b, \quad x^1 \geq 0, \\
& T(\omega)x^1 + W(\omega)x^2(\omega) \geq d(\omega), \\
& x^2(\omega) \geq 0, \forall \omega \in \Omega.
\end{align*}
\]
Risk Measures

- Modern approach to modeling risk aversion uses concept of risk measures
  - \( CVaR_\alpha \): mean of upper tail beyond \( \alpha \)-quantile (e.g. \( \alpha = 0.95 \))

- mean-risk, mean deviations from quantiles, VaR, CVaR
- Much more in mathematical economics and finance literature
- Optimization approaches still valid, different objectives, varying convex/non-convex difficulty
Stochastic price paths (day ahead market)

\[
\min_{x, h, q^+, q^-} c^1(x) + \mathbb{E}_\omega \left[ \sum_{t=0}^T p_{\omega t} (q_{\omega t}^+ - q_{\omega t}^-) + c^2(q_{\omega t}^+ + q_{\omega t}^-) \right]
\]

s.t. \( \partial h_{\omega t} = e q_{\omega t}^+ - q_{\omega t}^- \)

\( 0 \leq h_{\omega t} \leq S x \)

\( 0 \leq q_{\omega t}^+, q_{\omega t}^- \leq Q x \)

\( h_{\omega 0}, h_{\omega T} \) fixed

- First stage decision \( x \): amount of storage to deploy.
- Second stage decision: charging strategy in face of uncertainty
Contingency: a single line failure

A network with $N$ lines can have up to $N$ contingencies

Each contingency case:

- Corresponds to a different network topology
- Requires a different set of equations $g$ and $h$
- E.g., equations $g_k$ and $h_k$ for the $k$-th contingency.
Security-constrained Economic Dispatch

- Base-case network topology $g_0$ and line flow $x_0$.
- If the $k$-th line fails, line flow jumps to $x_k$ in new topology $g_k$.
- Ensure that $x_k$ is within limit, for all $k$.
- SCED model:

$$\begin{align*}
\min \quad & c^T u \\
\text{s.t.} \quad & 0 \leq u \leq \bar{u} \\
& g_0(x_0, u) = 0 \\
& -\bar{x} \leq x_0 \leq \bar{x} \\
& g_k(x_k, u) = 0, \quad k = 1, \ldots, K \\
& -\bar{x} \leq x_k \leq \bar{x}, \quad k = 1, \ldots, K
\end{align*}$$

- ▶ Total cost
- ▶ GEN capacity const.
- ▶ Base-case network eqn.
- ▶ Base-case flow limit
- ▶ Ctgcy network eqn.
- ▶ Ctgcy flow limit
Model structure

Figure: Sparsity structure of the Jacobian matrix of a 6-bus case, considering 3 contingencies and 3 post-contingency checkpoints.

Decomposition approaches allow solution of realistic sized models with many contingencies in minutes.
Contracts in MOPEC (Philpott/F./Wets)

- Can we modify (complete) system to have a social optimum by trading risk?
- How do we design these instruments? How many are needed? What is cost of deficiency?
- Facilitated by allowing contracts bought now, for goods delivered later (e.g. Arrow-Debreu Securities)
- Conceptually allows to transfer goods from one period to another (provides wealth retention or pricing of ancilliary services in energy market)
- Can investigate new instruments to mitigate risk, or move to system optimal solutions from equilibrium (or market) solutions
Example as MOPEC: agents solve a Stochastic Program

Buy $y_i$ contracts in period 1, to deliver $D(\omega)y_i$ in period 2, scenario $\omega$
Each agent $i$:

$$\min \quad C(x^1_i) + \rho_i \left( C(x^2_i(\omega)) \right)$$

s.t. $p^1 x^1_i + vy_i \leq p^1 e^1_i$ (budget time 1)

$$p^2(\omega)x^2_i(\omega) \leq p^2(\omega)(D(\omega)y_i + e^2_i(\omega))$$ (budget time 2)

---

$$0 \leq v \perp - \sum_i y_i \geq 0$$ (contract)

$$0 \leq p^1 \perp \sum_i (e^1_i - x^1_i) \geq 0$$ (walras 1)

$$0 \leq p^2(\omega) \perp \sum_i \left( D(\omega)y_i + e^2_i(\omega) - x^2_i(\omega) \right) \geq 0$$ (walras 2)
Theory and Observations

- Agent problems are multistage stochastic optimization models.
- Perfectly competitive partial equilibrium still corresponds to a social optimum when all agents are risk neutral and share common knowledge of the probability distribution governing future inflows.
- Situation complicated when agents are risk averse.
  - Utilize stochastic process over scenario tree.
  - Under mild conditions a social optimum corresponds to a competitive market equilibrium if agents have time-consistent dynamic coherent risk measures and there are enough traded market instruments (over tree) to hedge inflow uncertainty.
- Otherwise, must solve the stochastic equilibrium problem.
- Research challenge: develop reliable algorithms for large scale decomposition approaches to MOPEC.
Conclusions

- Optimization critical for understanding of power system markets
- Different behaviors are present in practice and modeled here
- Modern optimization within applications requires multiple model formats, computational tools and sophisticated solvers
- Policy implications addressable using MOPEC
- Stochastic MOPEC enables modeling dynamic decision processes under uncertainty
- Modeling, optimization, statistics and computation embedded within the application domain is critical
- Many new settings available for deployment; need for more theoretic and algorithmic enhancements
What is EMP?

Annotates existing equations/variables/models for modeler to provide/define additional structure

- equilibrium
- vi (agents can solve min/max/vi)
- bilevel (reformulate as MPEC, or as SOCP)
- disjunction (or other constraint logic primitives)
- randvar
- dualvar (use multipliers from one agent as variables for another)
- extended nonlinear programs (library of plq functions)

Currently available within GAMS