Stochastic Equilibrium Problems arising in the energy industry

Claudia Sagastizábal

(visiting researcher IMPA)

mailto:sagastiz@impa.br http://www.impa.br/~sagastiz

ENEC workshop, IPAM, Los Angeles, January 2016

joint work with J.P. Luna (UFRJ) and M. Solodov (IMPA)

What this talk is about?

For equilibrium models of energy markets (including stochastic versions with risk aversion);

- Modelling issues

- Existence and other theoretical issues

- Solution techniques

Energy markets can be large

Strategic sectors:

- subject to regulations in quality, price and entry
- couple several regions and markets

Electric Power:

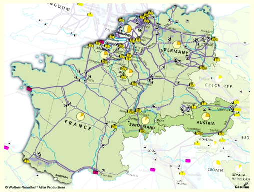
(source EPEX)

Energy markets can be large

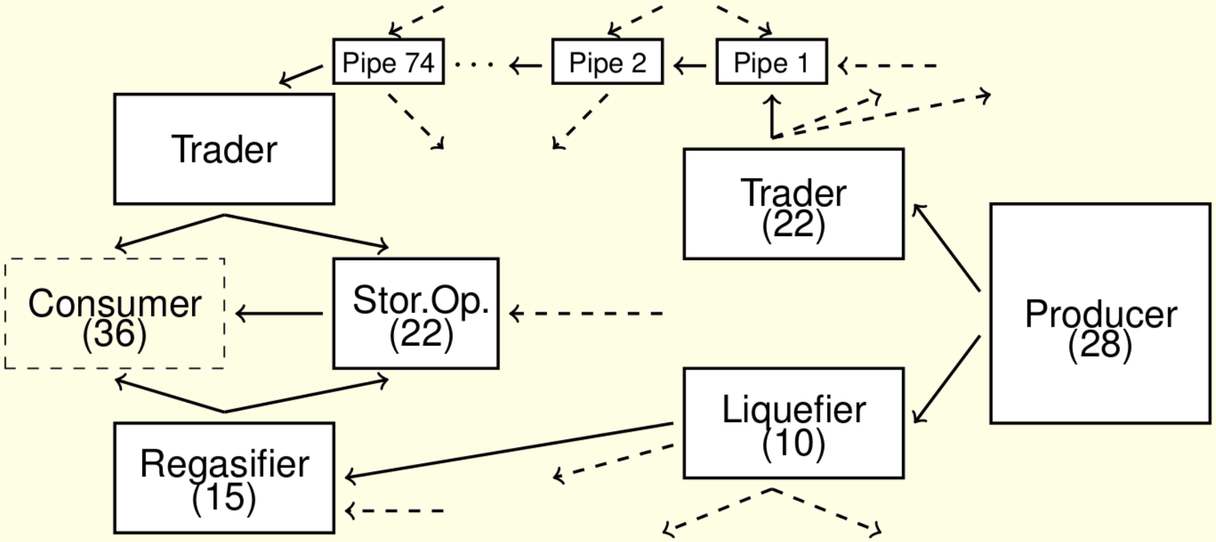
Strategic sectors:

- subject to regulations in quality, price and entry
- couple several regions and markets

Natural Gas: Energy Policy, 36:2385–2414, 2008. Egging, Gabriel, Holtz, Zhuang,



A complementarity model for the European natural gas market



Market: Premises

- Agents (producers, traders, logistics)
  - -take unilateral decisions
  - -behave competitively
- A representative of the consumers (the ISO)
  - -focuses on the benefits of consumption
  - -seeking a price that matches supply and demand
  - -while keeping prices “low”
- Agents’ actions coupled by some relations, clearing the market.

Market: Premises

- Agents (producers, traders, logistics)
  - -take unilateral decisions
  - -behave competitively
- A representative of the consumers (the ISO)
  - -focuses on the benefits of consumption
  - -seeking a price that **matches supply and demand**
  - -while keeping prices "low"
- Agents' actions coupled by some relations, clearing the market, **(MC)**

Market: Premises

- Agents (producers, traders, logistics)
  - -take unilateral decisions
  - -behave competitively
- A representative of the consumers (the ISO)
  - -focuses on the benefits of consumption
  - -seeking a price that **matches supply and demand**
  - -while keeping prices “low”
- Agents’ actions coupled by some relations, clearing the market, (MC)

Today, models from game theory or complementarity leading to Variational Inequalities (VIs) (i.e., sufficiently "convex")

Different models

– **Mixed Complementarity formulations**

– **Models from game theory**

Different models

- **Mixed Complementarity formulations**

  Agents maximize profit independently

  Supply$\geq$Demand: Market Clearing constraint (MC)

  multiplier$\equiv$equilibrium price

- **Models from game theory**

Different models

- **Mixed Complementarity formulations**

  Agents maximize profit independently

  Supply $\geq$ Demand: Market Clearing constraint (MC)

  multiplier $\equiv$ equilibrium price

- **Models from game theory**

  Agents minimize cost s.t. MC

  MC multiplier $\equiv$ (variational) equilibrium price

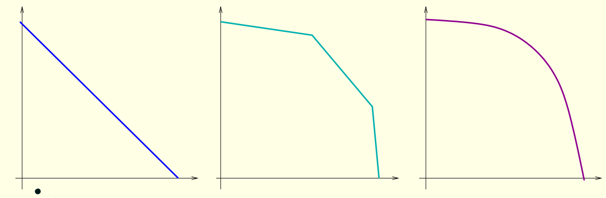
Different models

– **Mixed Complementarity formulations**

Agents maximize profit independently

Supply≥Demand: Market Clearing constraint (MC)

multiplier≡equilibrium price



Price is an exogenous concave

function of the total offer: $\pi = \pi(\sum_i q^i)$

– **Models from game theory**

Agents minimize cost s.t. MC

MC multiplier≡(variational) equilibrium price

Different models

How different are these models?

– **Mixed Complementarity formulations**

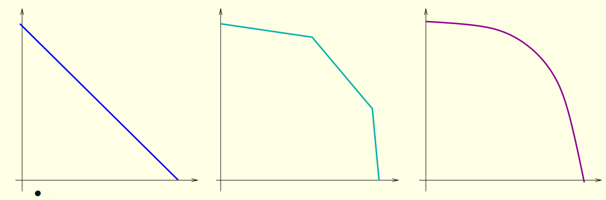
Agents maximize profit independently

Supply≥Demand: Market Clearing constraint (MC)

multiplier≡equilibrium price

Price is an exogenous concave

function of the total offer: $\pi = \pi(\sum_i q^i)$



– **Models from game theory**

Agents minimize cost s.t. MC

MC multiplier≡(variational) equilibrium price

Consumers indirectly represented

Notation: $q = (q^i, q^{-i})$, in particular $\pi = \pi(q^i, q^{-i})$

Market: Example as a Mixed Complementarity Problem

- Agents (producers, traders, logistics)

ith producer problem $\begin{cases} \max & r^i(q^i) \\ \text{s.t.} & q^i \in Q^i \end{cases}$

- Revenue $r^i(q^i) = \pi^\top q^i - c^i(q^i)$

Market: Example as a Mixed Complementarity Problem

- Agents (producers, traders, logistics)

ith producer problem $\left\{\begin{array}{ll} \max & r^i(q^i, \mathbf{q^{-i}}) \equiv \min \mathbf{c^i}(\mathbf{q^i}, \mathbf{q^i}) \; (c^i = -r^i) \\ \text{s.t.} & q^i \in Q^i \end{array}\right.$

- Revenue $r^i(q^i) = \pi^\top q^i - c^i(q^i) = r^i(q^i, \mathbf{q^{-i}})$

(price is an exogenous function $\pi(q)$ of **all** the offer)

Market: Example **as a Mixed Complementarity Problem**

- Agents (producers, traders, logistics)

ith producer problem $\begin{cases} \max & r^i(q^i, \mathbf{q^{-i}}) \equiv \min \mathbf{c^i}(\mathbf{q^i}, \mathbf{q^i}) \; (c^i = -r^i) \\ \text{s.t.} & q^i \in Q^i \end{cases}$

- Revenue $r^i(q^i) = \pi^\top q^i - c^i(q^i) = r^i(q^i, \mathbf{q^{-i}})$

(price is an exogenous function $\pi(q)$ of **all** the offer)

- Agents' actions coupled by a market clearing constraint $\mathrm{MC}(q^i, q^{-i}) = 0$ (mult. π)

Market: Example **as a Mixed Complementarity Problem**

- Agents (producers, traders, logistics)

  ith producer problem $\begin{cases} \max & r^i(q^i, \mathbf{q^{-i}}) \equiv \min \mathbf{c^i}(\mathbf{q^i}, \mathbf{q^i}) \; (c^i = -r^i) \\ \text{s.t.} & q^i \in Q^i \end{cases}$

- Revenue $r^i(q^i) = \pi^\top q^i - c^i(q^i) = r^i(q^i, \mathbf{q^{-i}})$

  (price is an exogenous function $\pi(q)$ of **all** the offer)

- Agents' actions coupled by a market clearing constraint $\text{MC}(q^i, q^{-i}) = 0$ (mult. π)

- Equilibrium price $\bar{\pi}$ coincides with the exogenous $\pi(\bar{q})$

Market: Equilibrium price: $\bar{\pi}$

Mixed Complementarity Model

Agents problems
$$\begin{cases} \min & c^i(q^i, q^{-i}) \\ \text{s.t.} & q^i \in Q^i \end{cases}$$

and, at equilibrium, $\text{MC}(q^i, q^{-i}) = 0 \quad (\bar{\pi} = \pi(\bar{\mathbf{q}}))$

Market: Equilibrium price: $\bar{\pi}$

Mixed Complementarity Model

Agents problems
$$\begin{cases} \min & c^i(q^i, q^{-i}) \\ \text{s.t.} & q^i \in Q^i \end{cases}$$

and, at equilibrium, $\mathrm{MC}(q^i, q^{-i}) = 0 \quad (\bar{\pi} = \pi(\bar{q}))$

Generalized Nash Game

Agents problems
$$\begin{cases} \min & c^i(q^i) \\ \text{s.t.} & q^i \in Q^i \\ & \mathrm{MC}(q^i, \tilde{q}^{-i}) = 0 \qquad \bar{\pi}^i \end{cases}$$

Market: Equilibrium price: $\bar{\pi}$

Mixed Complementarity Model

Agents problems $\left\{ \begin{array}{ll} \min & c^i(q^i, q^{-i}) \\ \text{s.t.} & q^i \in Q^i \end{array} \right.$

and, at equilibrium, $\quad \text{MC}(q^i, q^{-i}) = 0 \quad (\bar{\pi} = \pi(\bar{q}))$

Generalized Nash Game

Agents problems $\left\{ \begin{array}{ll} \min & c^i(q^i) \\ \text{s.t.} & q^i \in Q^i \\ & \text{MC}(q^i, \tilde{q}^{-i}) = 0 \quad \text{(same } \bar{\pi} \text{ for all } i) \end{array} \right.$

A Variational Equilibrium of the game is a Generalized Nash Equilibrium satisfying $\bar{\pi}^i = \bar{\pi}$

Both models give same equilibrium

Mixed Complementarity Model

Agents problems
$$\begin{cases} \min & c^i(q^i, q^{-i}) \\ \text{s.t.} & q^i \in Q^i \end{cases}$$

and, at equilibrium, $\mathrm{MC}(q^i, q^{-i}) = 0 \quad (\bar{\pi} = \pi(\bar{q}))$

Generalized Nash Game

Agents problems
$$\begin{cases} \min & c^i(q^i) \\ \text{s.t.} & q^i \in Q^i \\ & \mathrm{MC}(q^i, \tilde{q}^{-i}) = 0 \quad (\text{same } \bar{\pi} \text{ for all } i) \end{cases}$$

J.P. Luna, C. Sagastizábal, M. Solodov. Complementarity and game-theoretical models for equilibria in energy markets: deterministic and risk-averse formulations. Ch. 10 in Risk Management in Energy Production and Trading, (R. Kovacevic, G. Pflüg and M. T. Vespucci), "Int. Series in Op. Research and Manag. Sci.", Springer, 2013.

Both models give same equilibrium

Both models yield equivalent VIs

Mixed Complementarity Model

Agents problems $\begin{cases} \min & c^i(q^i, q^{-i}) \\ \text{s.t.} & q^i \in Q^i \end{cases}$

and, at equilibrium, $\qquad \text{MC}(q^i, q^{-i}) = 0 \quad (\bar{\pi} = \pi(\bar{q}))$

Generalized Nash Game

Agents problems $\begin{cases} \min & c^i(q^i) \\ \text{s.t.} & q^i \in Q^i \\ & \text{MC}(q^i, \tilde{q}^{-i}) = 0 \quad (\text{same } \bar{\pi} \text{ for all } i) \end{cases}$

J.P. Luna, C. Sagastizábal, M. Solodov. Complementarity and game-theoretical models for equilibria in energy markets: deterministic and risk-averse formulations. Ch. 10 in Risk Management in Energy Production and Trading, (R. Kovacevic, G. Pflüg and M. T. Vespucci), "Int. Series in Op. Research and Manag. Sci.", Springer, 2013.

Market: VI reformulation

$$
i\text{th problem}\begin{cases}\min & c^i(q^i)\\ \text{s.t.} & q^i \in Q^i\\ & \text{MC}(q^i, \tilde{q}^{-i}) = 0\end{cases}
$$

Variational Inequality follows from optimality conditions

Market: VI reformulation

ith problem
$$\begin{cases} \min & c^i(q^i) \\ \text{s.t.} & q^i \in Q^i \\ & \text{MC}(q^i, \tilde{q}^{-i}) = 0 \end{cases}$$

1st order OC

(primal form)

$$\left\langle \nabla_{q^i} c^i(\bar{q}^i), q^i - \bar{q}^i \right\rangle \geq 0$$

$$\forall q^i \in Q^i \cap \text{MC}$$

Variational Inequality follows from optimality conditions

Market: VI reformulation

ith problem $\begin{cases} \min & c^i(q^i) \\ \text{s.t.} & q^i \in Q^i \\ & \text{MC}(q^i, \tilde{q}^{-i}) = 0 \end{cases}$

1st order OC
(primal form)
$\left\langle \nabla_{q^i} c^i(\bar{q}^i), q^i - \bar{q}^i \right\rangle \geq 0$
$\forall q^i \in Q^i \cap \text{MC}$

Variational Inequality follows from optimality conditions

In VI$(F, C) : \langle F(\bar{q}), q - \bar{q} \rangle \geq 0 \; \forall$ feasible q

- the VI operator $F(q) = \prod_{i=1}^{N} F^i(q)$ for $F^i(q) = \nabla_{q^i} c^i(q^i)$
- the VI feasible set $C = \prod_{i=1}^{N} Q^i \bigcap \{q : \text{MC}(q) = 0\}$

Market: VI reformulation

$$i\text{th problem} \begin{cases} \min & c^i(q^i) \\ \text{s.t.} & q^i \in Q^i \\ & \text{MC}(q^i, \tilde{q}^{-i}) = 0 \end{cases}$$

1st order OC

(primal form)

$$\left\langle \nabla_{q^i} c^i(\bar{q}^i), q^i - \bar{q}^i \right\rangle \geq 0$$

$$\forall q^i \in Q^i \cap \text{MC}$$

Variational Inequality follows from optimality conditions

In $\text{VI}(F, C) : \langle F(\bar{q}), q - \bar{q} \rangle \geq 0 \; \forall$ feasible q

- the VI operator $F(q) = \prod_{i=1}^{N} F^i(q)$ for $F^i(q) = \nabla_{q^i} c^i(q^i)$

decomposability

- the VI feasible set $C = \prod_{i=1}^{N} Q^i \bigcap \{q : \text{MC}(q) = 0\}$

Market: VI reformulation

ith problem $\begin{cases} \min & c^i(q^i) \\ \text{s.t.} & q^i \in Q^i \\ & \mathrm{MC}(q^i, \tilde{q}^{-i}) = 0 \end{cases}$

1st order OC
(primal form)
$\left\langle \nabla_{q^i} c^i(\bar{q}^i), q^i - \bar{q}^i \right\rangle \geq 0$
$\forall q^i \in Q^i \cap \mathrm{MC}$

Variational Inequality follows from optimality conditions

In $\mathrm{VI}(F, C) : \langle F(\bar{q}), q - \bar{q} \rangle \geq 0 \ \forall$ feasible q

- the VI operator $F(q) = \prod_{i=1}^{N} F^i(q)$ for $F^i(q) = \nabla_{q^i} c^i(q^i)$

decomposability

- the VI feasible set $C = \prod_{i=1}^{N} Q^i \bigcap \{q : \mathrm{MC}(q) = 0\}$

NOTE: MC does not depend on i: constraint is **shared**

Incorporating a Capacity Market

Suppose producers pay $I^i(z^i)$

to invest in an increase z^i in production capacity

Production bounds go	from	$0 \le q^i \le q^i_{max}$	$(\equiv q^i \in Q^i)$
	to	$0 \le q^i \le q^i_{max} + \mathbf{z^i}$	$(z^i, q^i) \in X^i$

Incorporating a Capacity Market

Suppose producers pay $I^i(z^i)$

to invest in an increase z^i in production capacity

Production bounds go	from	$0 \leq q^i \leq q^i_{max}$	$(\equiv q^i \in Q^i)$
	to	$0 \leq q^i \leq q^i_{max} + z^i$	$(\mathbf{z^i}, \mathbf{q^i}) \in \mathbf{X^i}$

Incorporating a Capacity Market

Suppose producers pay $I^i(z^i)$

to invest in an increase z^i in production capacity

Production bounds go from $0 \leq q^i \leq q^i_{max}$ $\quad (\equiv q^i \in Q^i)$

to $0 \leq q^i \leq q^i_{max} + z^i$ $\quad (\mathbf{z^i}, \mathbf{q^i}) \in \mathbf{X^i}$

ith problem
$$\begin{cases} \min & I^i(z^i) + c^i(q^i) \\ \text{s.t.} & (z^i, q^i) \in X^i \\ & \text{MC}(q^i, q^{-i}) = 0 \end{cases}$$

$$\equiv \quad \min I^i(z^i) + \mathcal{V}^i(z^i)$$

$$\mathcal{V}^i(z^i) = \begin{cases} \min c^i(q^i) \\ (z^i, q^i) \in X^i \\ \text{MC} \end{cases}$$

Incorporating a Capacity Market

Suppose producers pay $I^i(z^i)$

to invest in an increase z^i in production capacity

Production bounds go from $0 \leq q^i \leq q^i_{max}$ $(\equiv q^i \in Q^i)$

to $0 \leq q^i \leq q^i_{max} + z^i$ $\quad (\mathbf{z^i}, \mathbf{q^i}) \in \mathbf{X^i}$

ith problem
$$\left\{ \begin{array}{ll} \min & I^i(z^i) + c^i(q^i) \\ \text{s.t.} & (z^i, q^i) \in X^i \\ & MC(q^i, q^{-i}) = 0 \end{array} \right. \equiv \begin{array}{l} \min I^i(z^i) + \mathcal{V}^i(z^i) \\ \mathcal{V}^i(z^i) = \left\{ \begin{array}{l} \min c^i(q^i) \\ (z^i, q^i) \in X^i \\ MC(q^i, q^{-i}) = 0 \end{array} \right. \end{array}$$

Incorporating a Capacity Market

Suppose producers pay $I^i(z^i)$

to invest in an increase z^i in production capacity

Production bounds go from $0 \le q^i \le q^i_{max}$ $(\equiv q^i \in Q^i)$

to $0 \le q^i \le q^i_{max} + z^i$ $(\mathbf{z^i}, \mathbf{q^i}) \in \mathbf{X^i}$

ith problem $\left\{ \begin{array}{ll} \min & I^i(z^i) + c^i(q^i) \\ \text{s.t.} & (z^i, q^i) \in X^i \\ & MC(q^i, q^{-i}) = 0 \end{array} \right.$ $\equiv$ $\min I^i(z^i) + \mathcal{V}^i(z^i)$, $\mathcal{V}^i(z^i) = \left\{ \begin{array}{l} \min c^i(q^i) \\ (z^i, q^i) \in X^i \\ MC(q^i, q^{-i}) = 0 \end{array} \right.$

can this problem be rewritten as a 2-level problem?

Incorporating a Capacity Market

When trying to rewrite $\min \quad I^i(z^i) + \mathcal{V}^i(z^i)$ using

$$\mathcal{V}^i(z^i) \qquad = \begin{cases} \min c^i(q^i) \\ (z^i, q^i) \in X^i \\ \textsc{mc}(q^i, q^{-i}) = 0 \end{cases}$$

a difficulty arises.

Incorporating a Capacity Market

When trying to rewrite $\min \quad I^i(z^i) + \mathcal{V}^i(z^i)$ using

$$\mathcal{V}^i(z^i, q^{-i}) = \begin{cases} \min c^i(q^i) \\ (z^i, q^i) \in X^i \\ \text{MC}(q^i, q^{-i}) = 0 \end{cases}$$

a difficulty arises.

The function $\mathcal{V}^i$ depends on (z^i, q^{-i}), the second-level problem is a Generalized Nash Game (hard!)

Incorporating a Capacity Market

When trying to rewrite $\min \quad I^i(z^i) + \mathcal{V}^i(z^i)$ using

$$\mathcal{V}^i(z^i, q^{-i}) = \begin{cases} \min c^i(q^i) \\ (z^i, q^i) \in X^i \\ \text{MC}(q^i, q^{-i}) = 0 \end{cases}$$

a difficulty arises.

The function $\mathcal{V}^i$ depends on (z^i, q^{-i}), the second-level problem is a Generalized Nash Game (hard!)

Consistent with reality: Agents will keep competing after capacity expansion

Incorporating a Capacity Market

When trying to rewrite $\min \quad I^i(z^i) + \mathcal{V}^i(z^i)$ using

$$\mathcal{V}^i(z^i, q^{-i}) = \begin{cases} \min c^i(q^i) \\ (z^i, q^i) \in X^i \\ \text{MC}(q^i, q^{-i}) = 0 \end{cases}$$

a difficulty arises.

The function $\mathcal{V}^i$ depends on (z^i, q^{-i}), the second-level problem is a Generalized Nash Game (hard!)

Consistent with reality: Agents will keep competing after capacity expansion. **Similarly for Mixed Complementarity model**

Incorporating a Capacity Market

When trying to rewrite $\min \quad I^i(z^i) + \mathcal{V}^i(z^i)$ using

$$\mathcal{V}^i(z^i, q^{-i}) = \begin{cases} \min c^i(q^i) \\ (z^i, q^i) \in X^i \\ \text{MC}(q^i, q^{-i}) = 0 \end{cases}$$

a difficulty arises.

The function $\mathcal{V}^i$ depends on (z^i, q^{-i}), the second-level problem is a Generalized Nash Game (hard!)

Consistent with reality: Agents will keep competing after capacity expansion. **Similarly for Mixed Complementarity model and 2 stage with recourse, even without expansion**

What about uncertainty?

Given $k = 1, \ldots, K$ uncertain scenarios (demand, costs, etc)

Investment variables are (naturally) the same for all realizations: z^i

Production variables are (naturally) different for each realization: q^i_k

ith problem for scenario k

$$\begin{cases} \min & I^i(z^i) + c^i_k(q_k) \\ \text{s.t.} & (z^i, q^i_k) \in X^i_k \\ & \text{MC}_k(q^i_k, q^{-i}_k) = 0 \end{cases}$$

Two-stage formulation with recourse not possible

What about uncertainty?

Given $k = 1, \dots, K$ uncertain scenarios (demand, costs, etc)

Investment variables are (naturally) the same for all realizations: z^i

Production variables are (naturally) different for each realization: q^i_k

ith problem for scenario k
$$\begin{cases} \min & I^i(z^i) + c^i_k(q_k) \\ \text{s.t.} & (z^i, q^i_k) \in X^i_k \\ & \text{MC}_k(q^i_k, q^{-i}_k) = 0 \end{cases}$$

Two-stage formulation with recourse not possible

Single-stage formulation instead: find a capacity expansion compatible with K scenarios of competition.

What about uncertainty?

Given $k = 1, \ldots, K$ uncertain scenarios (demand, costs, etc)

Investment variables are (naturally) the same for all realizations: z^i

Production variables are (naturally) different for each realization: q^i_k

ith problem for scenario k

$$\begin{cases} \min & I^i(z^i) + c^i_k(q_k) \\ \text{s.t.} & (z^i, q^i_k) \in X^i_k \\ & \text{MC}_k(q^i_k, q^{-i}_k) = 0 \end{cases}$$

Two-stage formulation with recourse not possible

Single-stage formulation instead: find a capacity expansion compatible with K scenarios of competition (likewise for generation-only market)

Which Stochastic VI?

Risk-neutral agents

Derive VI from

ith problem

using expected value

$$\left\{\begin{array}{ll} \min & I^i(z^i) + \mathbb{E}[c_k^i(q_k^i)] \\ \text{s.t.} & (z^i, q_k^i) \in X_k^i \text{ for } k = 1:K \\ & \text{MC}_k(q_k^i, q_k^{-i}) = 0 \text{ for } k = 1:K \end{array}\right.$$

Which Stochastic VI?

Risk-neutral agents

Derive VI from

ith problem

using expected value

$$\left\{ \begin{array}{ll} \min & I^i(z^i) + \mathbb{E}[c^i_k(q^i_k)] \\ \text{s.t.} & (z^i, q^i_k) \in X^i_k \text{ for } k = 1:K \\ & \text{MC}_k(q^i_k, q^{-i}_k) = 0 \text{ for } k = 1:K \end{array} \right.$$

- a VI operator F involving $\nabla I^i(z^i) \times \nabla_{q^i_{1:K}} \mathbb{E}\left[c^i_{1:K}(q)\right]$
- a VI feasible set $C = \prod_{k=1}^{K} \prod_{i=1}^{N} X^i_k \bigcap \{q_k : \text{MC}_k(q_k) = 0\}$

Which Stochastic VI?

Risk-neutral agents

Derive VI from

ith problem

using expected value

$$\begin{cases} \min & I^i(z^i) + \mathbb{E}[c^i_k(q^i_k)] \\ \text{s.t.} & (z^i, q^i_k) \in X^i_k \text{ for } k = 1:K \\ & \text{MC}_k(q^i_k, q^{-i}_k) = 0 \text{ for } k = 1:K \end{cases}$$

- a VI operator F involving $\nabla I^i(z^i) \times \nabla_{q^i_{1:K}} \mathbb{E}\left[c^i_{1:K}(q)\right]$
- a VI feasible set $C = \prod_{k=1}^{K} \prod_{i=1}^{N} X^i_k \bigcap \{q_k : \text{MC}_k(q_k) = 0\}$

decomposability

Which Stochastic VI?

Risk-neutral agents

Derive VI from

ith problem

using expected value

$$\begin{cases} \min & I^i(z^i) + \mathbb{E}[c_k^i(q_k^i)] \\ \text{s.t.} & (z^i, q_k^i) \in X_k^i \text{ for } k = 1:K \\ & \mathrm{MC}_k(q_k^i, q_k^{-i}) = 0 \text{ for } k = 1:K \end{cases}$$

- a VI operator F involving $\nabla I^i(z^i) \times \nabla_{q^i_{1:K}} \mathbb{E}\left[c^i_{1:K}(q)\right]$
- a VI feasible set $C = \prod_{k=1}^{K} \prod_{i=1}^{N} X_k^i \bigcap \{q_k : \mathrm{MC}_k(q_k) = 0\}$

decomposability

there is no coupling between scenarios ($\mathbb{E}$ is linear)

Which Stochastic VI?

Risk-averse agents, risk measure ρ

Derive VI from

ith problem

using risk measure

$$\begin{cases} \min & I^i(z^i) + \rho[c_k^i(q_k^i)] \\ \text{s.t.} & (z^i, q_k^i) \in X_k^i \text{ for } k = 1:K \\ & \text{MC}_k(q_k^i, q_k^{-i}) = 0 \text{ for } k = 1:K \end{cases}$$

Which Stochastic VI?

Risk-averse agents, risk measure ρ

Derive VI from

ith problem

using risk measure

$$\left\{\begin{array}{ll} \min & I^i(z^i) + \rho[c^i_k(q^i_k)] \\ \text{s.t.} & (z^i, q^i_k) \in X^i_k \text{ for } k = 1:K \\ & \mathrm{MC}_k(q^i_k, q^{-i}_k) = 0 \text{ for } k = 1:K \end{array}\right.$$

Difficulties arise: The risk measure is in general nonsmooth $\rho(\mathcal{Z}) := AV@R_\varepsilon(\mathcal{Z}) = \min_u \left\{u + \frac{1}{1-\varepsilon}\mathbb{E}\left([\mathcal{Z}_k - u]^+\right)\right\}$: it is a value-function and $[\cdot]^+$ is nonsmooth

Which Stochastic VI?

Risk-averse agents, risk measure ρ

Derive VI from

ith problem using risk measure
$$\begin{cases} \min & I^i(z^i) + \rho[c^i_k(q^i_k)] \\ \text{s.t.} & (z^i, q^i_k) \in X^i_k \text{ for } k = 1:K \\ & \mathrm{MC}_k(q^i_k, q^{-i}_k) = 0 \text{ for } k = 1:K \end{cases}$$

Difficulties arise: The risk measure is in general nonsmooth $\rho(\mathcal{Z}) := AV@R_\varepsilon(\mathcal{Z}) = \min_u \left\{u + \frac{1}{1-\varepsilon}\mathbb{E}\Big([\mathcal{Z}_k - u]^+\Big)\right\}$: it is a value-function and $[\cdot]^+$ is nonsmooth

- the VI operator F involves $\nabla I^i(z^i) \times \partial_{\mathbf{q}^i_{1:K}} \rho\left[c^i_{1:K}(q)\right]$, **multivalued**

Two ways of handling multivalued VI operator

Reformulation:

Introduce $AV@R$ directly into the agent problem, by rewriting $[\,]^+$ in

$$\rho(\mathcal{Z}) := \min_u \left\{ u + \frac{1}{1-\varepsilon}\mathbb{E}\left([\mathcal{Z}_k - u]^+\right)\right\}$$

by means of new variables and constraints

Two ways of handling multivalued VI operator

Reformulation:

Introduce $AV@R$ directly into the agent problem, by rewriting $[]^+$ in

$$\rho(\mathcal{Z}) := \min_u \left\{ u + \frac{1}{1-\varepsilon} \mathbb{E}\left([\mathcal{Z}_k - u]^+ \right) \right\}$$

by means of new variables and constraints

Smoothing:

Smooth the $[\cdot]^+$-function and solve the smoothed VI

$$\rho^\ell(\mathcal{Z}) := \min_u \left\{ u + \frac{1}{1-\varepsilon} \mathbb{E}\left(\sigma_\ell (\mathcal{Z}_k - u) \right) \right\},$$

for smoothing $\sigma_\ell \to [\cdot]^+$ uniformly as $\ell \to \infty$

Reformulation

$$\rho(\mathcal{Z}) = \min_u \left\{ u + \frac{1}{1-\varepsilon}\mathbb{E}\left([\mathcal{Z}_k - u]^+\right)\right\}$$

FROM

$$\begin{cases} \min & I^i(z^i) + \rho[c_k^i(q_k^i)] \\ \text{s.t.} & (z^i, q_k^i) \in X_k^i \text{ for } k = 1:K \\ & \mathrm{MC}_k(q_k^i, q_k^{-i}) = 0 \text{ for } k = 1:K \end{cases}$$

TO:

$$\begin{cases} \min & I^i(z^i) + \mathbf{u^i} + \frac{1}{1-\varepsilon}\mathbb{E}\left(\mathbf{T_k^i}\right) \\ \text{s.t.} & (z^i, q_k^i) \in X_k^i \text{ for } k = 1:K \\ & \mathrm{MC}_k(q_k^i, q_k^{-i}) = 0 \text{ for } k = 1:K \\ & T_k^i \geq c_k^i(q_k^i) - u^i, T_k^i \geq 0 \text{ for } k = 1:K, \mathbf{u} \in \mathbb{R} \end{cases}$$

Reformulation

$$\rho(\mathcal{Z}) = \min_u \left\{ u + \frac{1}{1-\varepsilon}\mathbb{E}\left([\mathcal{Z}_k - u]^+\right)\right\}$$

FROM

$$\begin{cases} \min & I^i(z^i) + \rho[c^i_k(q^i_k)] \\ \text{s.t.} & (z^i, q^i_k) \in X^i_k \text{ for } k = 1:K \\ & \text{MC}_k(q^i_k, q^{-i}_k) = 0 \text{ for } k = 1:K \end{cases}$$

TO:

$$\begin{cases} \min & I^i(z^i) + \mathbf{u^i} + \frac{1}{1-\varepsilon}\mathbb{E}\left(\mathbf{T^i_k}\right) \\ \text{s.t.} & (z^i, q^i_k) \in X^i_k \text{ for } k = 1:K \\ & \text{MC}_k(q^i_k, q^{-i}_k) = 0 \text{ for } k = 1:K \\ & T^i_k \geq c^i_k(q^i_k) - u^i, T^i_k \geq 0 \text{ for } k = 1:K, \mathbf{u} \in \mathbb{R} \end{cases}$$

NOTE: new constraint is **NOT shared**: no longer a generalized Nash game, but a bilinear CP (how to show $\exists$?).

Assessing both options

PATH can be used for the two variants.

- Reformulation
  - eliminates nonsmoothness
  - Non-separable feasible set

- Smoothing
  - To drive smoothing parameter to 0: repeated VI solves
  - Keeps feasible set separable by scenarios: easier VI

Assessing both options

PATH can be used for the two variants.

- Reformulation

  eliminates nonsmoothness

  Non-separable feasible set

- Smoothing

  To drive smoothing parameter to 0: repeated VI solves

  Keeps feasible set separable by scenarios: easier VI

  Provides existence result!

Smoothing

We use smooth approximations ρ^ℓ

$$\rho^\ell(\mathcal{Z}) := \min_u \left\{ u + \frac{1}{1-\varepsilon} \mathbb{E}\left[\sigma_\ell(\mathcal{Z}_k - u) \right] \right\},$$

for smoothing $\sigma_\ell \to [\cdot]^+$ uniformly as $\ell \to \infty$. For instance,

$$\sigma_\ell(t) = (t + \sqrt{t^2 + 4\tau_\ell^2})/2$$

for $\tau_\ell \to 0$.

Since ρ^ℓ is smooth, $\mathbf{VI}(F^\ell, C)$ has a a single-valued VI operator involving $\nabla_{q^i} \rho^\ell \left[(c_k^i(q_k))_{k=1}^K \right]$

Theorems

- like $\mathrm{AV@R}$, ρ^ℓ is a risk-measure
  - convex, monotone, and translation equi-variant,
  - but not positively homogeneous (only coherent in the limit).
- ρ^ℓ is C^2 for strictly convex smoothings such as $\sigma^\ell(t) = (t + \sqrt{t^2 + 4\tau_\ell^2})/2$
- Any accumulation point of the smoothed problems solves the original risk-averse (non-smooth) problem as $\ell \to \infty$.

Theorems

- like AV@R, ρ^ℓ is a risk-measure
  - convex, monotone, and translation equi-variant,
  - but not positively homogeneous (only coherent in the limit).
- ρ^ℓ is C^2 for strictly convex smoothings such as $\sigma^\ell(t) = (t + \sqrt{t^2 + 4\tau_\ell^2})/2$
- Any accumulation point of the smoothed problems solves the original risk-averse (non-smooth) problem as $\ell \to \infty$.

  existence result!

Reference: An approximation scheme for a class of risk-averse stochastic equilibrium problems. Luna, Sagastizábal, Solodov

Numerical performance of smoothing

$$\tau_\ell \Rightarrow \ \mathrm{VI}^\ell \Rightarrow \tau_{\ell+1} \Rightarrow \ \mathrm{VI}^{\ell+1} \ldots \text{until stabilization}$$

$$\text{for } x = (z^{1:N}, q_{1:K}^{1:N}) \text{ stop if } \frac{\left|\bar{x}_{j+1} - \bar{x}_j\right|}{\max\left(1, \left|\bar{x}_{j+1}\right|\right)} \leq 0.01$$

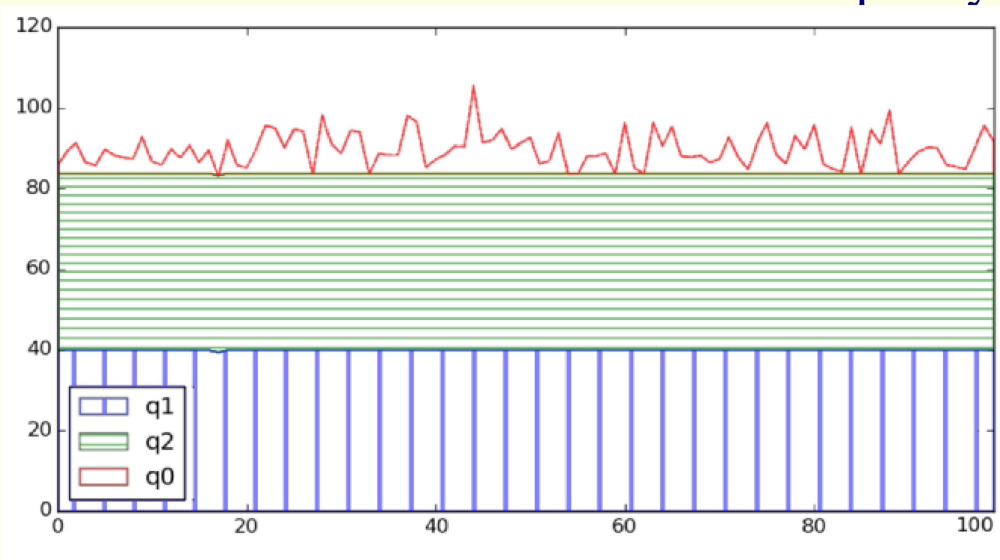
2 players and a consumer representative, player 0. Player 2 has higher generation costs. Less than 5 solves in average, each solve takes 45 seconds. Excellent solution quality

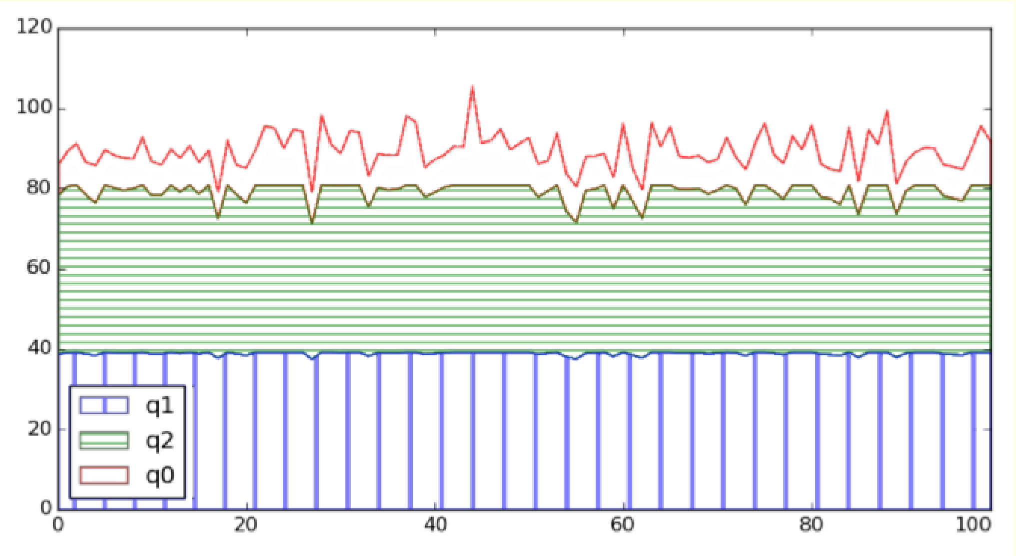
Numerical performance of smoothing

$$\tau_\ell \Rightarrow \mathrm{VI}^\ell \Rightarrow \tau_{\ell+1} \Rightarrow \mathrm{VI}^{\ell+1} \ldots \text{until stabilization}$$

$$\text{for } x = (z^{1:N}, q_{1:K}^{1:N}) \text{ stop if } \frac{|\bar{x}_{j+1} - \bar{x}_j|}{\max\left(1, |\bar{x}_{j+1}|\right)} \leq 0.01$$

2 players and a consumer representative, player 0. Player 2 has higher generation costs. Less than 5 solves in average, each solve takes 45 seconds. Excellent solution quality



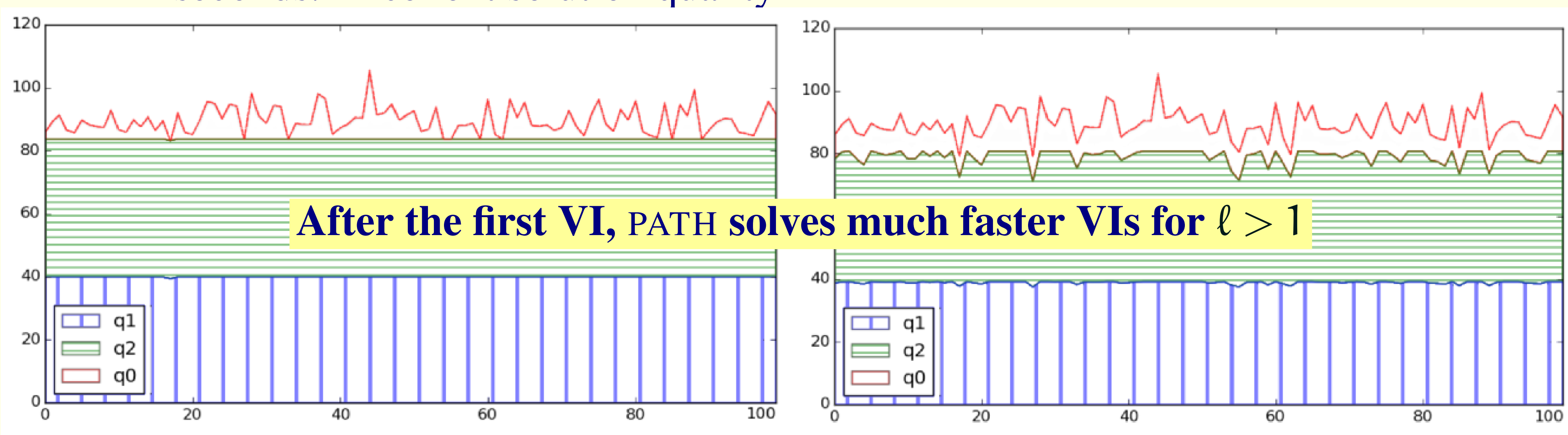


Numerical performance of smoothing

$$\tau_\ell \Rightarrow \text{VI}^\ell \Rightarrow \tau_{\ell+1} \Rightarrow \text{VI}^{\ell+1} \ldots \text{until stabilization}$$

$$\text{for } x = (z^{1:N}, q_{1:K}^{1:N}) \text{ stop if } \frac{|\bar{x}_{j+1} - \bar{x}_j|}{\max\left(1, |\bar{x}_{j+1}|\right)} \leq 0.01$$

2 players and a consumer representative, player 0. Player 2 has higher generation costs. Less than 5 solves in average, each solve takes 45 seconds. Excellent solution quality



After the first VI, PATH **solves much faster VIs for $\ell > 1$**

Numerical performance of smoothing

For nonconvex generation costs, reformulation becomes slower with nonconvex generation costs.

Smoothing needs less than 6 solves in average. Once again, after the first VI solve, PATH much faster for consecutive smoothed VIs:

$$\text{time of PATH}^{\text{smoothing}} \leq 2 \times \text{time of PATH}^{\text{reformulation}}$$

but: Total time of reformulation increases a lot, it scales less well

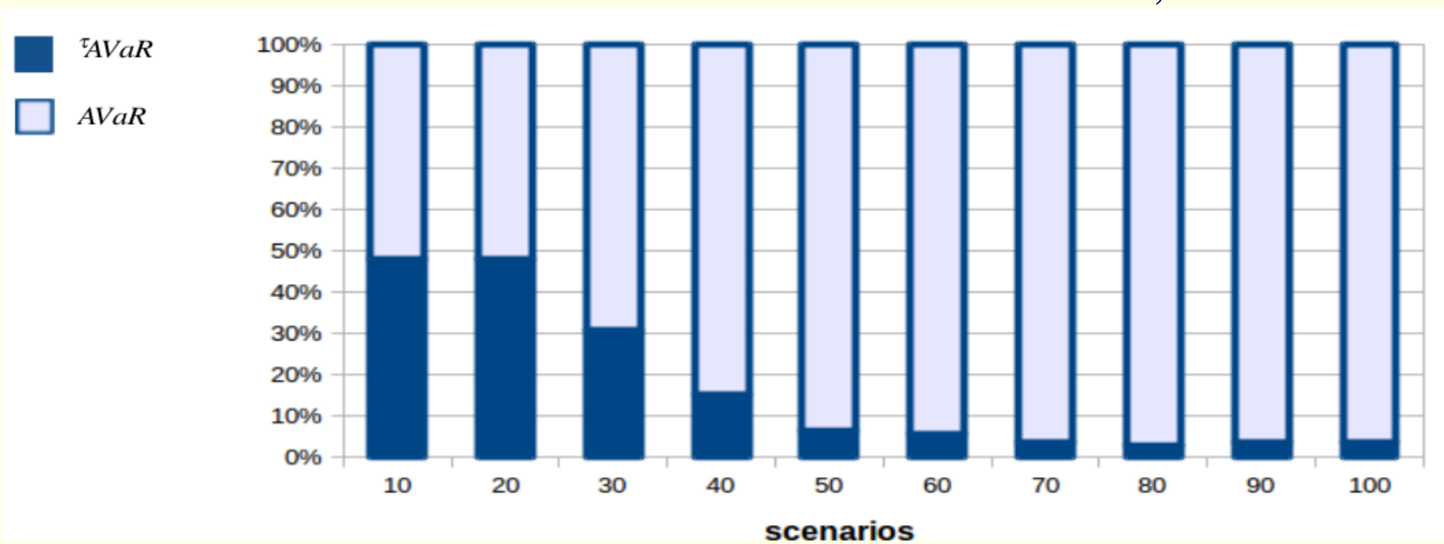
Numerical performance of smoothing

For nonconvex generation costs, reformulation becomes slower with nonconvex generation costs.

Smoothing needs less than 6 solves in average. Once again, after the first VI solve, PATH much faster for consecutive smoothed VIs:

$$\text{time of PATH}^{\text{smoothing}} \leq 2 \times \text{time of PATH}^{\text{reformulation}}$$

but: Total time of reformulation increases a lot, it scales less well



Final Comments

- When in the agents' problems the objective or some constraint depends on actions of other agents, writing down the stochastic game/VI can be tricky (which selection mechanism in a 2-stage setting?)

- Handling nonsmoothness via reformulation seems inadequate for large instances

- Smoothing solves satisfactorily the original risk-averse nonsmooth problem for moderate τ (no need to make $\tau \to 0$)

- Smoothing preserves separability; it is possible to combine
  - Benders' techniques (along scenarios) with
  - Dantzig-Wolfe decomposition (along agents)

- Decomposition matters: for European Natural Gas network
  - Solving VI directly with PATH solver S. Dirkse, M. C. Ferris, and T. Munson
  - Using DW-decomposition **saves 2/3 of solution time**

SAVE THE DATES! June 25th-July 1st, 2016