Modeling and Decomposing Multi-Stage and Multi-Scale Stochastic Optimization Problems

Ramteen Sioshansi

Integrated Systems Engineering
The Ohio State University

Institute for Pure and Applied Mathematics
University of California, Los Angeles
12 January, 2016

Work supported by the National Science Foundation under Grant No. CBET-1029337

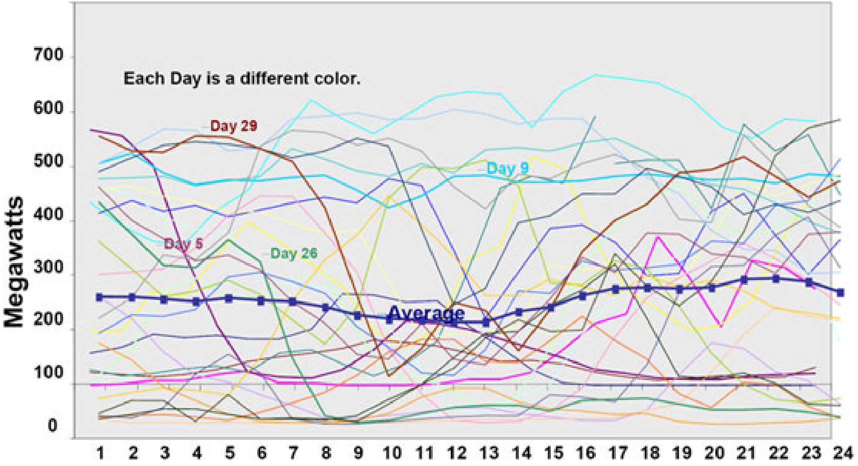
Overview

Background

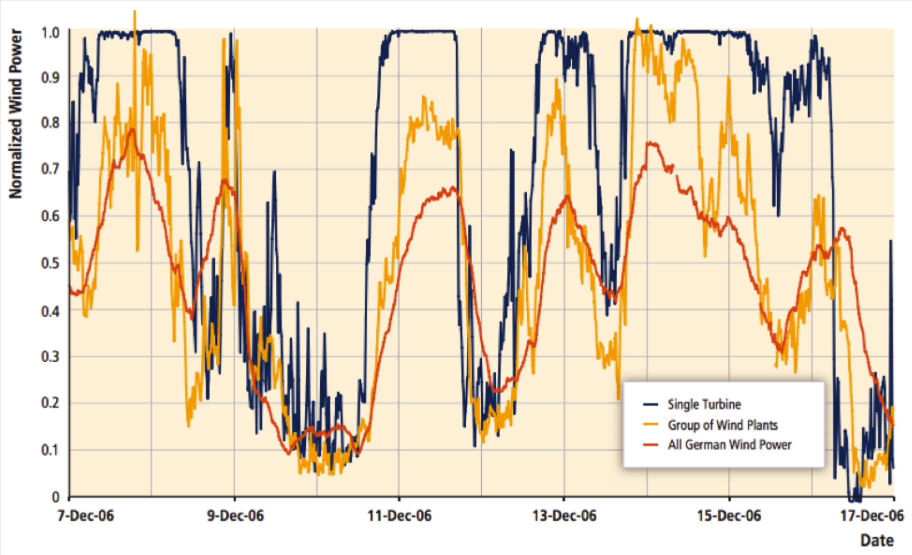
How to satisfy electricity demands with minimum costs?

- Scope: consider long investment periods, multiple electricity-generating technologies, and uncertainties
- Policy: renewable portfolio standards, carbon limits, *etc.* may necessitate the use of variable renewable technologies
- Perspective: a centralized model where generation and transmission investments are planned together

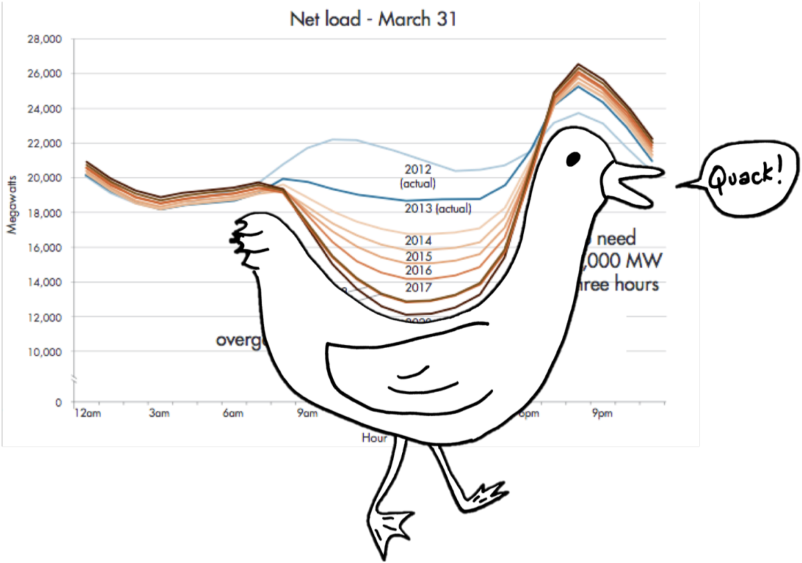
Supply Uncertainty



Supply Variability



Operational Flexibility

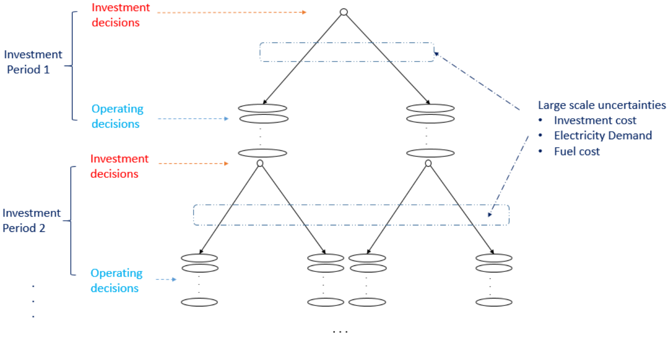


Challenges

- Multistage forward-looking investments with recourse
- Multiscale short- and long-run uncertainties
  - Long-run: investment costs, technology development, policy changes, fuel prices, demand growth
  - Short-run: demand pattern, generation availability, wind speeds, solar insolation
- Problem size explodes if we model multiple stages and all of the uncertainties explicitly

Model Structure

Stochastic, multistage, multiscale model



- investment-stage decisions: coarse timescale
- operating-stage decisions: fine timescale

Outline

Objective Function

$$\min \sum_{\omega} \overbrace{\alpha_{\omega}}^{\texttt{probability}} \Bigg(\sum_{\tau} \sum_{t} \sum_{n} c^{\tau}_{t,n,\omega} X^{\tau}_{t,n,\omega} \quad \texttt{// gen. invest. cost}$$

$$+ \sum_{t} \sum_{l} c^{L}_{l,t,\omega} Y_{l,t,\omega} \quad \texttt{// trans. invest. cost}$$

$$+ \sum_{\tau} \sum_{t} \sum_{n} \sum_{r} N_r \cdot \sum_{d} \bar{c}^{\tau}_{t,n,\omega} P^{\tau}_{t,n,r,d} \quad \texttt{// operating cost}$$

$$\Bigg)$$

Investment-Stage Constraints

$$0 \leq X^{\tau}_{t,n,\omega} \leq X^{\tau,max}_{n}, \forall \omega, \tau, t, n$$ // investment

$$0 \leq Y_{t,l,\omega} \leq Y^{max}_{l}, \forall \omega, t, n$$ limit

$$\sum_{\tau} \sum_{n} c^{\tau}_{t,n,\omega} X^{\tau}_{t,n,\omega} + \sum_{l} c^{L}_{l,t,\omega} Y_{t,l,\omega} \leq c^{max}_{t}, \forall \omega, t$$ // investment budget

$$X^{\tau}_{t,n,\omega_k} = X^{\tau}_{t,n,\omega_{\bar{k}}} : \Omega_m(\omega_k) = \Omega_m(\omega_{\bar{k}}), \forall m < t, \tau, \omega, t, n$$ // investment

$$Y_{t,l,\omega_k} = Y_{t,l,\omega_{\bar{k}}} : \Omega_m(\omega_k) = \Omega_m(\omega_{\bar{k}}), \forall m < t, \omega, t, n$$ nonanticipativity

Operating-Stage Constraints

System Constraints

$$\sum_{\tau} P^{\tau}_{t,n,r,d,\omega} + P^{STD}_{t,n,r,d,\omega} - P^{STC}_{t,n,r,d,\omega} + UD_{t,n,r,d,\omega}$$ // load

$$- \sum_{l|O(l)=n} f_{t,l,r,d,\omega} + \sum_{l|D(l)=n} f_{t,l,r,d,\omega} = D_{t,n,r,d,\omega}, \forall \omega, t, n, r, d$$ balance

$$f_{t,l,r,d,\omega} = B_l \cdot (\theta_{t,O(l),r,d,\omega} - \theta_{t,D(l),r,d,\omega}), \forall \omega, t, l, r, d$$ // flow def.

$$- f^{max}_{l,ES} - \sum_{m=0}^{t} Y_{m,l,\omega} \leq f_{t,l,r,d,\omega} \leq f^{max}_{l,ES}$$ // flow

$$+ \sum_{m=0}^{t} Y_{m,l,\omega}, \forall \omega, t, l, r, d$$ limit

$$- \pi \leq \theta_{t,n,r,d,\omega} \leq \pi, \forall \omega, t, n, r, d$$ // phase angle

Operating-Stage Constraints

Generator Constraints

$$0 \leq P^{\tau}_{t,n,r,d,\omega} \leq F^{\tau}_{t,n,r,d} \cdot \left(X^{\tau}_{ES,n} + \sum_{m=0}^{t} X^{\tau}_{m,n,\omega} \right), \forall \omega, \tau, t, n, r, d \qquad \texttt{// gen. limit}$$

$$-R_{\tau} \cdot \left(X^{\tau}_{ES} + \sum_{m=0}^{t} X^{\tau}_{m,n,\omega} \right) \leq P^{\tau}_{t,n,r,d,\omega} - P^{\tau}_{t,n,r,d-1,\omega} \qquad \texttt{// ramp}$$

$$\leq R_{\tau} \cdot \left(X^{\tau}_{ES} + \sum_{m=0}^{t} X^{\tau}_{m,n,\omega} \right), \forall \omega, \tau, t, n, r, d \qquad \texttt{limit}$$

Operating-Stage Constraints

Storage Constraints

$$P^{ST}_{t,n,r,d,\omega} = P^{ST}_{t,n,r,d-1,\omega} - P^{STD}_{t,n,r,d,\omega} + \eta P^{STC}_{t,n,r,d,\omega}, \forall \omega, t, n, r, d$$

`// SoC balance`

$$0 \leq P^{ST}_{t,n,r,d,\omega} \leq h \cdot \left(X^{ST}_{ES,n} + \sum_{m=0}^{t} X^{ST}_{m,n,\omega} \right), \forall \omega, t, n, r, d$$

`// SoC limit`

$$0 \leq P^{STC}_{t,n,r,d,\omega} \leq X^{ST}_{ES,n} + \sum_{m=0}^{t} X^{ST}_{m,n,\omega}, \forall \omega, t, n, r, d$$

`// charge limit`

$$0 \leq P^{STD}_{t,n,r,d,\omega} \leq X^{ST}_{ES,n} + \sum_{m=0}^{t} X^{ST}_{m,n,\omega}, \forall \omega, t, n, r, d$$

`// discharge limit`

Two Challenges

1. Many operating stages to capture fine-scale uncertainties
2. Many investment-stage scenarios to capture coarse uncertainties

Outline

Operating-Stage Periods

- Problem: Model is intractable if solving dispatch decisions for every hour in the operating stage
- Standard Solution: Use representative hours, based on LDC, to represent operating stage
  - ➥ Loses correlations between load, wind, and solar
  - ➥ Cannot model intertemporal constraints (*e.g.*, storage, ramping)
- Our Solution: Use representative days with intact correlation structures and intertemporal constraints in operating stage

Representative Days

- Each representative day contains one day's hourly load, solar, and wind data in each region
- Cluster to generate representative days that respects the correlation among variables, locations, and time

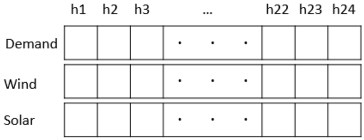


Figure: There are 72 data points in a representative day for each region

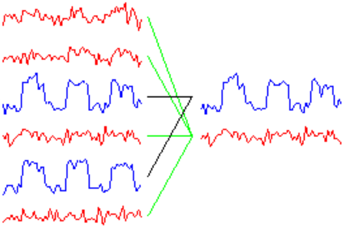
Clustering Methods

Method 1

- Hierarchical Clustering using Dynamic Time Warping
- Dynamic Time Warping: measures similarity between two time series, which may vary in time

Method 2

- Step 1: Use *k*-means clustering, with Euclidean distance as a metric, to find a starting set of clusters
- Step 2: Apply Method 1 within each cluster to find representative days



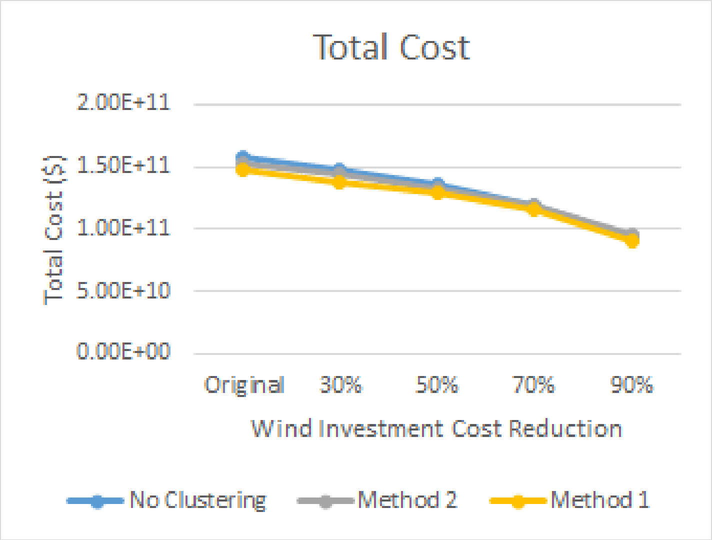
Clustering Test

- Dataset: One year's hourly wind, solar, and load data for three cities in Texas
- Model: An investment model with one investment stage and 20 years' operations
- Method:
  - ➥ Run the model with original dataset and with representative days from Methods 1 and 2
  - ➥ Compare investment decisions and total cost
  - ➥ Investigate how decisions change as a function of model inputs

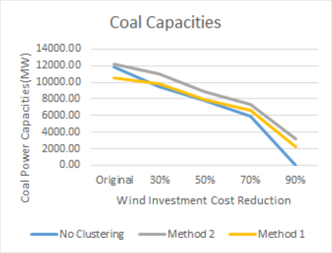
k-Means Clustering

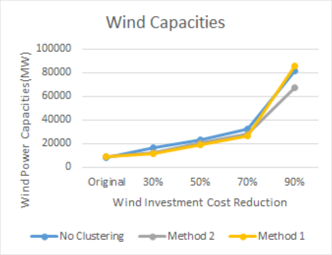
- *k*-means clustering is fast, but provides no representative days for the clusters
- Using cluster centroids gives poor results—variable renewables are overbuilt because they are modeled as having 'average' performance
- Percentiles within clusters could overcome this
- Hierarchical clustering within each *k*-means cluster provides an actual day from the underlying data

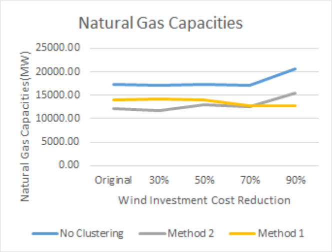
Investment Cost

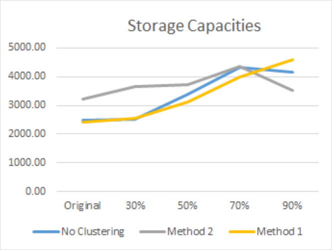


Investment Capacities









Clustering Results

- The two clustering methods perform similarly well overall
- Method 2 takes less time to implement: Method 1 takes about 15 minutes in R studio as opposed to 2 minutes for Method 2
- 30 clusters (representative days) gives a good approximation of original dataset

Outline

Investment-Stage Scenarios

- Problem: Model may need many investment-stage scenarios to capture coarse-grain uncertainties
- Some Solutions:
  1. Lagrangian relaxation: multiplier updates highly sensitive to problem data
  2. Progressive hedging algorithm[1]
  3. Linear decision rules (Kuhn): Fallback option—model decisions as being a linear function of problem data:

$$x = A\xi$$

     need to find coefficients, A

[1] Rockafellar and Wets (1991): “Scenario and policy aggregation in optimization under uncertainty”.

Progressive Hedging

- Suppose we have the following stochastic problem:

$$\min_{x} \sum_{s \in S} p_s f_s(x_s) \tag{1}$$

$$\text{s.t. } x_s \in C_s \qquad \texttt{// } x_s \texttt{ is admissible}$$

$$x_s \text{ is implementable}$$

- Relax nonanticipativity to get scenario-s problem:

$$\min_{x_s} f_s(x_s) \tag{2}$$

$$\text{s.t. } x_s \in C_s$$

Progressive Hedging

- Add penalty for nonanticipativity violations:

$$\begin{aligned} \min_{x_s}\ & f_s(x_s) + \left[W^\top x_s + \frac{\rho}{2} ||x_s - \hat{x}||^2 \right] \\ \text{s.t.}\ & x_s \in C_s \end{aligned} \tag{3}$$

- W: Lagrange multiplier vector
- ρ: positive penalty parameter, introduced to attain convergence stability in an algorithmic sense
- $\hat{x} = \sum_{s \in S} p_s x_s$: average of x_s's

Algorithm

for $s \in S$ **do**
    Solve Problem (2) for scenario s
end for
$\hat{x} \leftarrow \sum_{s \in S} p_s x_s$
$W_t \leftarrow \rho \cdot (x_t - \hat{x}_t)$
while $|x - \hat{x}| > \epsilon$ **do**
    for $s \in S$ **do**
        Solve Problem (3) for scenario s
    end for
    $\hat{x} \leftarrow \sum_{s \in S} p_s x_s$
    $W_t \leftarrow \rho \cdot (x_t - \hat{x}_t)$
end while

Lower Bound

- A feasible solution gives an upper bound
- The dual of the non-anticipativity constraints in two-stage stochastic MIPs define implicit lower bounds[2]

$$\sum_s p_s[\min f_s(x_s) + W^\top x_s]$$

- We show a similar bound for multi-stage stochastic problems
- Allows us to assess the quality of a progressive hedging solution
- LB obtained with the same effort as one PH iteration

[2] Gade et al. (2014): "Obtaining Lower Bounds from the Progressive Hedging Algorithm for Stochastic Mixed-Integer Programs".

Performance of PHA

Table: Performance of PHA with Different Number of Representative Days in Operating Stage

			Full Problem		**PHA**		
Days	**Variables**	**Constraints**	CPU Time [s]	Objective [$ billion]	CPU Time [s]	Upper Bound [$ billion]	Lower Bound [$ billion]
3	1337856	2248967	2968	95.096	1161	95.122	95.095
9	3992064	6727943	16535	100.988	5898	101.011	100.988

- Tested on a three-region system
- Four investment periods, each operating stage lasts 10 years
- 128 scenarios in total

Investment Decisions

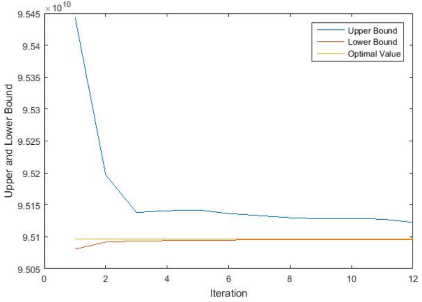
- The investment decisions from the original model and the decomposed model are similar
- Maximum and average absolute differences $< 2\%$ and 0.3%

Table: Investment Capacities [MW]

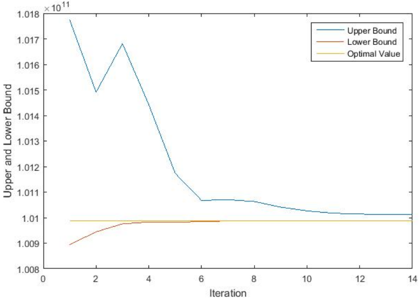
Investment Period	**No Decomposition** Bus 1	Bus 2	Bus 3	**Decomposed Model** Bus 1	Bus 2	Bus 3
1	37717	3495	7781	37672	3502	7791
2	1985	2427	3634	1982	2423	3632
3	4509	2621	6728	4528	2621	6731
4	8402	949	2082	8407	951	2042

Upper Bounds and Lower Bounds

3 Representative Days



9 Representative Days



Outline

Conclusions

- Decisions and uncertainties occur at different stages and scales
- Long-term uncertainties are explicitly modeled in the scenario tree
- Short-term uncertainty are implicitly modeled through different operating-stage problems
- The resulting multistage multiscale stochastic model can be effectively solved using PHA
- Representative days allows intertemporal constraints to be captured in long-term investment decisions

Future Work

- Comprehensive numerical case study
- Study effects of variable renewable energy sources on price signals and investment cost remuneration

Questions?