Mechanism design and allocation algorithms for energy-network markets with piece-wise linear costs and quadratic externalities

Alejandro Jofré

Center for Mathematical Modeling & DIM
Universidad de Chile

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1 In collaboration with N. Figueroa and B. Heymann
Outline

- Introduction and motivation
- Modeling market and Equilibrium. Discontinuous Games
- Nash and beyond
- Intrinsic market Power
- Efficient regulations and Extended Mechanism Design
- Conclusions
Introduction and motivation

Modeling Market
- Equilibrium: Nash

Intrinsic Market Power

Efficient regulations and mechanism design
- The benchmark game
- Comparing Benchmark with Optimal Mechanism
Motivations

- Most of ISOs have few generation companies: oligopoly
- Transmission networks highly congested in some areas
- Intrinsic market power produced by externalities and information asymmetries
Transmission Europe

European high voltage transmission grid
Transmission US

United States transmission grid
Source: FEMA
Transmission Chile
Introduction and motivation

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A generation short term market: day-ahead mandatory pool

- Today: generators taking into account an estimation of the demand bid *increasing piece-wise linear cost functions or equivalently piece-wise constant "price"*. Even general convex cost functions.
- Tomorrow: the (ISO) using this information and knowing a realization of the demand, minimizes the sum of the costs to satisfy demands at each node considering all the transmission constraints: "dispatch problem".
- Tomorrow: the (ISO) sends back to generators the optimal quantities and "prices" (multipliers associated to supply = demand balance equation at each node)
The (ISO) knows a realization of the demand $d \in \mathbb{R}^V$, receives the costs functions bid $(c_i)_{i \in G}$ and compute: $(q_i)_{i \in G}, (\lambda_i)_{i \in G}$

$$\min_{(h, q)} \sum_{i \in G} c_i(q_i). \quad (1)$$

$$\sum_{e \in K_i} \frac{r_e}{2} h_e^2 + d_i \leq q_i + \sum_{e \in K_i} h_e \text{sgn}(e, i), \quad i \in G \quad (2)$$

$$q_i \in [0, \bar{q}_i], \quad i \in G, \quad (3)$$

$$0 \leq h_e \leq \bar{h}_e \quad (4)$$
We denote $Q(c, d) \subset \mathbb{IR}^G$ the generation component of the optimal solution set associated to each cost vector submitted $c = (c_i)$ and demand $d$. We denote $\Lambda(c, d) \subset \mathbb{IR}^G$ the set of multipliers associated to the supply=demand in the ISO problem.
Modeling Generators

1. At each node $i \in G$ we have a generator with payoff

$$u_i(\lambda, q) = \lambda q - \bar{c}_i(q)$$

$\bar{c}_i$ is the real cost.

2. The strategic set for each player $i$ denoted $S_i$:

$$\{c_i : \mathbb{R} \to \mathbb{R}_+ | \text{convex, nondecreasing, bounded subgradients or } \partial c_i \subset [0, p^*], p^* \text{ is a price cap.} \}$$
Equilibrium

An equilibrium is \((q, \lambda, m)\) such that \(q\) is a selection of \(Q(\cdot, \cdot)\) and \(\lambda\) is a selection of \(\Lambda(\cdot, \cdot)\) and \(m = (m_i)_{i \in G}\) is a mixed-strategy equilibrium of the generator game in which each generator submits costs \(c_i \in S_i\) with a payoff

\[
\mathbb{E}u_i(\lambda_i(c, \cdot), q_i(c, \cdot)) = \int_D \left[ \lambda_i(c, d)q_i(c, d) - \bar{c}_i(q_i(c, d)) \right] d\mathbb{P}(d),
\]

Neutral or risk averse
In some cases, for example, using a supply function equilibria approach there are previous works by Anderson, Philpott, or using variational inequality approach by Pang, Ralph, Ferris or also using game theory by Hogan, Smeers, Wilson, Joskow, Tirole, Hobbs, Oren, Borestein, Wolak...
Nash equilibrium

Consider a game \( G = (X_i, u_i)^N \) that consists of \( N \) players where each player \( i = 1, \ldots, N \) has a strategy set \( X_i \) and a payoff function \( u_i : X \to \mathbb{R} \), where \( X = \Pi_{i \in N} X_i \).

Nash equilibrium \( (x^*_i) \)

\[
x^*_i \in \argmax \{ u_i(x_i, x^*_{-i}) | x_i \in X_i \}
\]
Nash equilibrium

For the sake of simplicity, we assume that each $X_i$ is contained in a metric vectorial space:

- If for all $i$ the strategy set $X_i$ is a compact set, and $u_i$ is a bounded function, we say that $G$ is a *compact game*.

- If for all $i$ the set $X_i$ is convex and for each $x_{-i} \in X_{-i}$, $u_i(\cdot, x_i)$ is a (concave) quasiconcave function, then we say that $G$ is a *convex game* (*quasiconvex game*)
Nash equilibrium existence

A convex compact game \( G = (X_i, u_i)^N \) satisfying:
- \( u_i(\cdot, \cdot) \) is upper semicontinuous
- \( u_i(x_i, \cdot) \) is lower semicontinuous for all \( x_i \)

has a Nash equilibrium point.

Extensions: generalized games, convergence-stabilty lopsided convergence
Discontinuous games: tie-breaking rules

Consider the following two-player game: Let the payoff for the \( i \) player be given by

\[
    u_i(x_i, x_{-i}) = \begin{cases} 
    l_i(x_i) & \text{if } x_i < x_{-i}, \\
    \varphi(x_i) & \text{if } x_i = x_{-i}, \\
    m_i(x_{-i}) & \text{if } x_i > x_{-i}, 
    \end{cases}
\]

where \( x_i \in [0, 1] \). Assume that for all \( i \) and \( x \in [0, 1] \) (a) \( l_i \) and \( m_i \) are continuous functions, \( l_i \) is nondecreasing \( \varphi(x) \) is a convex combination of \( l_i(x) \) and \( m_i(x) \); \n
\[
    \text{sign } [l_i(x) - \varphi(x)] = \text{sign } [\varphi_{-i}(x) - m_{-i}(x)].
\]
Existence discontinuous games

Reny (1999) Econometrica

**Theorem**

A compact quasiconcave game possesses a Nash equilibrium if it is also a better reply secure game.

Bagh and Jofre (2006) Econometrica

**Theorem**

If \((X_i, u_i)^N\) is weakly reciprocally upper semicontinuous and payoff secure, then it is better reply secure.
Assumptions

S1. For all $d \in D$, there exists $\delta_d > 0$ such that

$$\Omega(d) \neq \emptyset, \quad \|\hat{d} - d\| \leq \delta_d.$$ 

S2. $D$ is compact  
S3. (1) Either $P$ is non atomic; or (2) given two convex sets $M, N \subset \mathbb{R}^G$, $u(M \times N)$ is convex.  
S4 $u_i : \mathbb{R}^2 \to \mathbb{R}$ is continuous.
Equilibrium existence

**Theorem**

*If each $S_v$ is a nonempty closed set for the point-wise convergence, then there exists an equilibrium $(q, \lambda, m)$ for the bid-based generator pool game.*

*Example: In real system... increasing piece-wise constant cost functions*
1. Introduction and motivation

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3. Intrinsic Market Power

4. Efficient regulations and mechanism design
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Two nodes case

Symmetric Nash equilibrium

Profit = multiplier × quantity – cost × quantity

Nash = \( \frac{c}{1-2rd} \)

d < \frac{1}{2r}
the ISO Problem: two-node case

Given that each generator reveals a cost \( c_i \), the (ISO) solves:

\[
\min_{q,h} \sum_{i=1}^{2} c_i q_i \\
\text{s.t.} \quad q_i - h_i + h_{-i} \geq \frac{r}{2} [h_1^2 + h_2^2] + d \quad \text{for} \quad i = 1, 2 \\
q_i, h_i \geq 0 \quad \text{for} \quad i = 1, 2
\]
Escobar and J. (ET (2010)) equilibrium exists but producers charge a price above marginal cost:

\[ Nash = \bar{c} / (1 - 2rd) \]
Sensitivity formula

**Proposition**

Let $c \in \prod_{i \in G} S_i$ and $c_i - \hat{c}_i$ a Lipschitz function with constant $\kappa$. Then,

$$|Q_i(c, d) - Q_i(\hat{c}_i, c_{-i}, d)| \leq \kappa \eta,$$

where $\eta = 2 \frac{(1 + r_i h_i)^2}{\min_{i \in G} r_i c_i^+(0)} \in ]0, +\infty]$ and

$$c_i^+(0) = \lim_{y \to 0^+} \frac{c_i(y) - c_i(0)}{y}.$$

**Why?** losses $\Rightarrow$ the second-order growth
Market Power formula

**Proposition**

The equilibrium prices $p_i$ satisfy

\[
\mathbb{E}|p_i - \gamma| \geq \frac{\mathbb{E}[Q_i(p_i, p_{-i}, d)]}{\bar{\eta}_i}
\]

where $\bar{\eta}_i = 2|K_i|^2 \left(1 + \max\{r_e \bar{h}_e : e \in K_i\}\right)^2$

$\gamma(p_{-i}, d)$ is a measurable selection of $\partial \bar{c}_i(Q_i(p_i, p_{-i}, d))$. 

\[
\gamma(p_{-i}, d)
\]
Market Power formula

**Proposition**

*Linear case:* $\bar{c}_i(q) = \bar{c}_i q$, then

$$p_i - \bar{c}_i \geq \mathbb{E}[Q_i(p_i, p_{-i}, d)] \frac{\bar{\eta}}{\bar{\eta}}.$$
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The Questions

In an electric network with transmission costs and private information:

- Does the usual (price equal Lagrange multiplier) regulation mechanism minimize costs for the society?
- If not, what is the mechanism that achieves this objective?
- How does the performance of both systems compare?

Methodology:

- Bayesian Game Theory
- Mechanism Design
Framework

- A network with demand $d$ at each node.
- One producer at each node, with piece-wise linear cost of production $c_i \sim F_i[c_i, \bar{c}_i]$. Common knowledge!
- Transmission costs $rh^2$, with $h$ the amount sent from one node to another.
ISO for piece-wise linear cost functions

Problem

minimize \( (q,h) \)
\[
\sum_{i=1}^{n} \sum_{j=1}^{N} q_{i,j} c_{i,j} \\
\sum_{j=1}^{N} q_{i,j} + \sum_{i' \in V(i)} h_{i',i} - h_{i,i'} - \frac{h_{i,i'}^2 + h_{i',i}^2}{2} r_{i,i'} \geq d_i \quad (\lambda_i)
\]
\[
\forall (i, i') \in E : h_{i,i'} \geq 0 \quad (\gamma_{i,i'}) \\
\forall i \in I, j \in J : q_{i,j} \geq 0 \quad (\mu_{i,j}) \\
\forall i \in I, j \in J : q_{i,j} \leq \bar{q} \quad (\nu_{i,j}).
\]

(6)
100 nodes network
The ISO Problem: two-node case

Given that each generator reveals a cost $c_i$, the ISO solves:

$$\min_{q,h} \sum_{i=1}^{2} c_i q_i$$

subject to:

$$q_i - h_i + h_{-i} \geq \frac{r}{2}[h_1^2 + h_2^2] + d \quad \text{for} \quad i = 1, 2$$

$$q_i, h_i \geq 0 \quad \text{for} \quad i = 1, 2$$
The Solution for ISO problem

If we define

\[ H(x, y) = d + \frac{1}{2r} \left( \frac{x - y}{x + y} \right)^2 - \frac{1}{r} \left( \frac{x - y}{x + y} \right) \]

and

\[ \bar{q} = 2 \left[ 1 - \sqrt{1 - 2dr} \right] \frac{r}{r} \]

then the solution to this problem can be written as

\[
q_i(c_i, c_{-i}) = \begin{cases} 
H(c_i, c_{-i}) & \text{if } H(c_i, c_{-i}) \geq 0 \text{ and } H(c_{-i}, c_i) \geq 0 \\
\bar{q} & \text{if } H(c_{-i}, c_i) < 0 \\
0 & \text{if } H(c_i, c_{-i}) < 0 
\end{cases}
\]

\[
\lambda_i(c_i, c_{-i}) \equiv p_i(c_i, c_{-i}) = c_i \text{ if } H(c_i, c_{-i}) \geq 0
\]
The Bayesian Game: benchmark

The game:

- 2 players. Strategies $c_i \in C_i = [\underline{c}_i, \overline{c}_i]$, i=1,2.
- Payoff $u_i(c_i, c_{-i}) = (\lambda_i(c_i, c_{-i}) - c_i) q_i(c_i, c_{-i})$, where $c_i$ is the real cost. The Equilibrium:
  - A strategy $b_i : [\underline{c}_i, \overline{c}_i] \rightarrow \mathbb{R}^+$ (convex at equilibrium!)
  - In a Nash equilibrium

$$
\bar{b}(c) \in \arg \max \int_{C_{-i}} \left[ \lambda_i(x, \bar{b}(c_{-i})) - c \right] q_i(x, \bar{b}(c_{-i})) f_{-i}(c_{-i}) dc_{-i}
$$

(7)
Numerical Approximation

- For simplicity, $C_i = [1, 2]$.
- Let $k \in \{0, ..., n - 1\}$, and $b(c) = b_k$ for $c \in \left[\frac{k}{n}, \frac{k+1}{n}\right]$.
- The weight of each interval is given by $w_k = F\left(\frac{k+1}{n}\right) - F\left(\frac{k}{n}\right)$.
- The approximate equilibrium is characterized by:

$$b_k \in \arg \max_x \sum_{l=0}^{n-1} [\lambda_i(x, b_l) - r_k] q_i(x, b_l) w_l \quad \text{for all} \quad k \in \{0, ..., n-1\}$$

(8)
The benchmark game

**Optimal Mechanism. Principal Agent Model (Myerson)**

- A *direct revelation mechanism* \( M = (q, h, x) \) consists of an *assignment rule* \( (q_1, q_2, h_1, h_2) : C \rightarrow R^4 \) and a *payment rule* \( x : C \rightarrow R^2 \).

- The ex-ante expected profit of a generator of type \( c_i \) when participates and declares \( c'_i \) is

\[
U_i(c_i, c'_i; (q, h, x)) = E_{c-i}[x_i(c'_i, c_{-i}) - c_i q_i(c'_i, c_{-i})]
\]

- A mechanism \( (q, h, x) \) is feasible iff:

\[
\begin{align*}
U_i(c_i, c_i; (q, h, x)) & \geq U_i(c_i, c'_i; (q, h, x)) \quad \text{for all } c_i, c'_i \in C_i \\
U_i(c_i, c_i; (q, h, x)) & \geq 0 \quad \text{for all } c_i \in C_i \\
q_i(c) - h_i(c) + h_{-i}(c) & \geq \frac{r}{2} [h_1^2(c) + h_2^2(c)] + d \quad \text{for all } c \in C \\
q_i(c), h_i(c) & \geq 0 \quad \text{for all } c \in C
\end{align*}
\]
The Regulator’s Problem

Using the revelation principle, the regulator’s problem can be written as:

\[
\min \int \sum_{i=1}^{2} x_i(c) f(c) dc \\
\text{subject to } (q, h, x) \text{ being “feasible”}
\]

Existence: Knuster-Tarski fixed point theorem (monotone relations)
The Regulator’s Problem (II)

It can be rewritten as

\[
\begin{align*}
\min & \quad \int \sum_{i=1}^{2} q_i(c)[c_i + \frac{F_i(c_i)}{f_i(c_i)}]f(c)dc \\
\text{s.t} & \quad \int_{C_{-i}} q_i(c_i, c_{-i})f_{-i}(c_{-i})dc_{-i} \quad \text{is non-increasing in} \quad c_i \\
& \quad q_i(c) - h_i(c) + h_{-i}(c) \geq \frac{r}{2}[h_1^2(c) + h_2^2(c)] + d \quad \text{for all} \quad c \in C \\
& \quad q_i(c), h_i(c) \geq 0 \quad \text{for all} \quad c \in C
\end{align*}
\]

We denote by \( J_i(c_i) = c_i + \frac{F_i(c_i)}{f_i(c_i)} \) the virtual cost of agent \( i \). We assume it is increasing (Monotone likelihood ratio property: true for any log concave distribution)
Solution

An optimal mechanism is given by

$$\hat{q}_i(c_i, c_{-i}) = \begin{cases} 
H(J_i(c_i), J_{-i}(c_{-i})) & \text{if } H(J_i(c_i), J_{-i}(c_{-i})) \geq 0 \\
\bar{q} & \text{if } H(J_{-i}(c_{-i}), J_i(c_i)) < 0 \\
0 & \text{if } H(J_i(c_i), J_{-i}(c_{-i})) < 0
\end{cases}$$

$$\hat{x}_i(c_i, c_{-i}) = c_i \hat{q}_i(c_i, c_{-i}) + \int_{c_i}^{\bar{c}_i} \hat{q}_i(s, c_{-i}) ds$$

Such mechanism is dominant strategy incentive compatible.
We consider the family of distributions with densities

\[ f_a(x) = \begin{cases} 
  a(x - 1) + (1 - \frac{a}{4}) & \text{if } x \leq 1.5 \\
  -a(x - 1) + (1 + \frac{3a}{4}) & \text{if } x \geq 1.5 
\end{cases} \]
Comparing Benchmark with Optimal Mechanism

Asymmetric information

```
Marginal Cost
Density Functions ga
Marginal Cost
Cumulatives Functions Ga
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- $a = 0$
- $a = 1$
- $a = 2$
- $a = 3$
Comparing Benchmark with Optimal Mechanism

Social costs for different mechanisms
Robustness and Practical Implementation

- The optimal mechanism is detail free. If the designer is wrong about common beliefs, then the mechanism is still not bad:

\[ |X_f - X_{\tilde{f}}| \leq |x|_1 |f - \tilde{f}|_\infty \leq c\bar{q}|f - \tilde{f}|_\infty \]

- The assignment rule is computationally simple to implement. It requires solving once the dispatcher problem, with modified costs.

- However, the payments are computationally difficult

\[ c_i\hat{q}_i(c_i, c_{-i}) + \int_{c_i}^{c_i} \hat{q}_i(s, c_{-i}) ds \]
Comparing Benchmark with Optimal Mechanism


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