Electricity derivative trading: private information and supply functions for contracts

Optimization and Equilibrium in Energy Economics

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13 January 2016

Talk outline

- The context: The role of the forward contract market.
- The model: A simple approach when firms make different forecasts for the spot price.
- Supply functions: What does an equilibrium in supply functions look like?
- Deductions from the other firm's behavior: Will this make things better?
- Conclusions

Risk trading in an electricity market

- Participants in the wholesale electricity market also take part in a derivatives market. In Australia the National Electricity Market operates across the Eastern States and there are 5 minute prices for each State. The most important forward contracts traded are simple contracts for differences on the average price for a whole quarter, whether for peak or base load. The other liquid market is for options ($300 caps) .
- Firms buy and sell contracts with the aim both of maximizing profits and hedging risks. A generator's contract position will often cover the majority of its output.
- Contracts are either "over-the-counter" (OTC) involving a bilateral agreement (possibly arranged through a broker), or they take place in a futures market which provides a transparent price. In Australia the majority of futures trades are block trades negotiated bilaterally before being transacted on the futures exchange.

The hedging context

- Forward contracts are financial instruments with payments depending on the price of electricity. So the "right" price depends on the electricity price time series behavior, which is highly seasonal (daily, weekly and yearly cycles) with a mean reverting behavior and large positive peaks.
- There are many price models suggested:e.g. multi-factor Ornstein-Uhlenbeck model with spikes driven by a Levy-process. (Benth, et al. *Applied Mathematical Finance*, 2007, and Kluppelberg et al. *Quantitative Finance*, 2010.) Or we can use nonlinear functions of load to drive the spike behavior (Berger et al. *Quantitative Finance*, 2004). Alternative models look for an equilibrium price for the forward contracts (Bessembinder and Lemmon, *Journal of Finance*, 2002, Willems and Morbee *Energy Economics* 2010).
- Forward markets are driven by hedging behavior. Retailers sell at a fixed price but buy at the spot price, so forward contracts that fix the price for the contract quantity helps to protect them from price spikes. Generators have an opposite set of incentives.

Some complications

- Generators face uncertainty both in the price that they will be paid and the amount of power they will be dispatched. Retailers are also uncertain of their own demand. So exact hedges are not possible.
- In practice bidding decisions in the physical market will depend on contract positions (with higher contract levels tending to depress prices). But we will concentrate on risk trading and assume that the spot market for electricity is a stochastic process that is not affected by the contracts in place.
- Contracts are signed over time. For a particular quarter some contracts will be fixed more than a year in advance, with more added as time goes by. So there is increasing volume of contract cover as we consider quarters that are closer to the present

The contribution of this paper

- We try to understand how the contract market will operate when individual firms have some private information on future prices. We can think of this as arising from simulation models of the future behavior of the market.
- Thus firms can profit from their private information, at the same time as trading in order to hedge their risks. Sanda, Olsen and Fleten, 'Selective hedging in hydro-based electricity companies', Energy Economics 40 (2013) 326–338, discuss the way that Norwegian Hydro power companies use derivative trading to hedge risk, but at the same time make substantial amounts of money from this activity.
- Since we are looking at bilateral trades it is natural to ask how the trading stance of a firm may indicate its private information. Can one side of the negotiation infer the other's beliefs about future prices and gain from this?

The model

- There are M future scenarios and scenario i results in an electricity price of w_i and a profit from operations of $R_i^{(j)}$ for player j.
- There is a contract (cfd) trading at price f and player j buys an amount $Q^{(j)}$ which may be negative. Then, if scenario i occurs, player j receives a profit from the cfd of $(w_i - f)Q^{(j)}$, in addition to $R_i^{(j)}$.
- We use a utility function U_j to capture the risk aversion for player j. So the expected utility with contract price f and contract quantity Q is given by

$$\Pi_j(f, Q) = \sum_{i=1}^{M} p_i^{(j)} U_j \left(R_i^{(j)} + (w_i - f)Q \right)$$

  where $p_i^{(j)}$ is player j's estimate of the probability of scenario i.
- The contract price f is determined by the market clearing ($\sum Q^{(j)} = 0$).

The simplest interesting case

- Two players (1 = retailer, 2 = generator). Two scenarios with prices w_H and w_L. Write ρ_i for player i's estimate of the probability of outcome H. So

$$\begin{aligned}\Pi_1(f,Q) &= \rho_1 U\left(R_H^{(1)} + (w_H - f)Q\right) + (1-\rho_1) U\left(R_L^{(1)} + (w_L - f)Q\right)\\ \Pi_2(f,Q) &= \rho_1 U\left(R_H^{(2)} + (w_H - f)Q\right) + (1-\rho_1) U\left(R_L^{(2)} + (w_L - f)Q\right).\end{aligned}$$

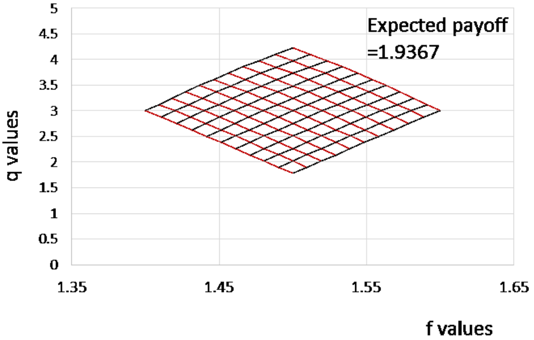
- Each player has the same quadratic utility function $U(x) = x - bx^2/2$.
- Suppose first (**the base model**) that each player maximizes its own expected utility at any given price f. This gives the optimal contract purchase amount for the retailer as a function of price f:

$$Q = \frac{1}{b}\frac{\rho_1(w_H - f)(1 - bR_H^{(1)}) + (1-\rho_1)(w_L - f)(1 - bR_L^{(1)})}{\rho_1(w_H - f)^2 + (1-\rho_1)(w_L - f)^2}.$$

- A similar expression, with ρ_2, gives the optimal contract sell amount for the generator also as a function of f. The market clears where these two supply functions intersect.
- As risk aversion approaches zero ($b \longrightarrow 0$) contract quantities get very large ($Q \longrightarrow \infty$) unless $\rho_1 = \rho_2$.

An example

- Take $w_H = 2$ and $w_L = 1$. At the high price the retailer has net profit in the spot market of $R_H^{(1)} = 1$ and the generator has net profit of $R_H^{(2)} = 4$. At the low price these numbers are reversed. Take $b = 0.18$. The probability estimates ρ_1 and ρ_2 can vary between 0.4 and 0.6. Without contracting the expected payoff (utility) is 1.735. This increases to 1.937 with contracting.

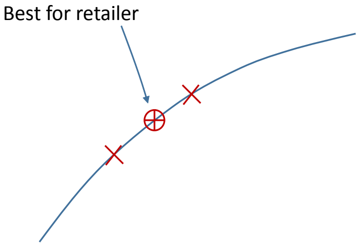


How do we calculate the expected utility?

- Initially both players are aware of Nature's distribution of possible ρ values. This is the prior (We take the prior as uniform on $(0.4, 0.6)$)
- Nature selects the true value of ρ according to the prior.
- Both players take a sample of size N ($N = 5$ here). On the basis of the number of w_H and w_L values in the sample they adjust their prior. The value ρ_i that player i uses in it's contract bidding is the expected value of the posterior distribution of ρ given the sample observed by player i.
- The expected utility is obtained by taking expectations over Nature's choice from the prior. There is correlation between ρ_i and ρ_j (when nature chooses a high ρ both samples are likely to have a large number of w_H values).

Best response to a supply function

- The retailer knows that the generator will sell more at a higher price. This changes his optimal behavior: the retailer wants to find the optimal (f, Q) point on the generator's supply function.
- Suppose the generator has probability estimate ρ_2 and offers a supply function parameterized by t: $(f(t, \rho_2), Q(t, \rho_2))$. The retailer wants to find the point on this curve maximizing its own profit $\Pi_1(f(t, \rho_2),\ Q(t, \rho_2))$



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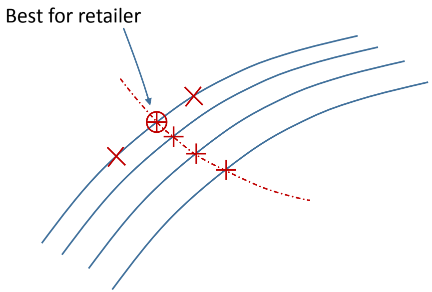


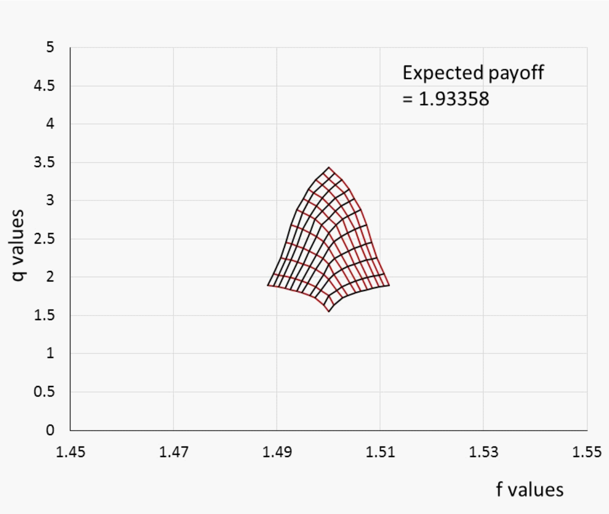
Supply function equilibria

- Look for an equilibria amongst offers. This will be a complete solution $f(\rho_1, \rho_2)$, $Q(\rho_1, \rho_2)$ which describes the joint behavior of retailer and generator.
- Best response means that $\Pi_1(f(t, \rho_2),\ Q(t, \rho_2))$ is maximized at $t = \rho_1$.
- First order conditions imply that at $f(\rho_1, \rho_2)$, $Q(\rho_1, \rho_2)$ we must have:

$$\rho_1 U'\left(R_H^{(1)} + Q(w_H - f)\right)\left(\frac{\partial Q}{\partial \rho_1}(w_H - f) - Q\frac{\partial f}{\partial \rho_1}\right) + (1-\rho_1)U'\left(R_L^{(1)} + Q(w_L - f)\right)\left(\frac{\partial Q}{\partial \rho_1}(w_L - f) - Q\frac{\partial f}{\partial \rho_1}\right) = 0.$$

Back to the example

- There are many supply function equilibria possible. We can search for the one that achieves the highest expected utility. For this example we achieve an expected utility that is close to, but slightly worse than, the base case.



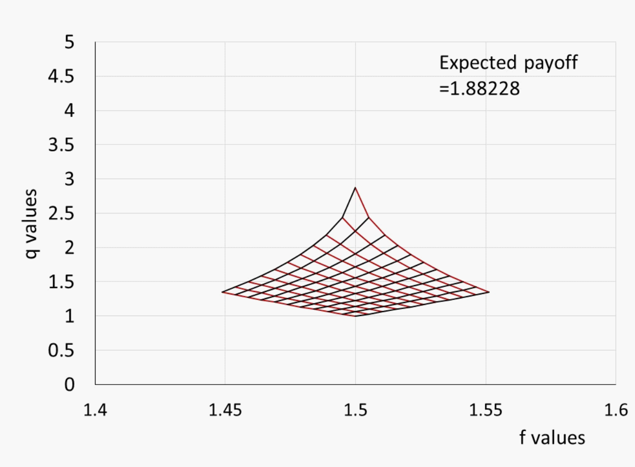
Learning from the other player's information

- If the retailer faces a generator supply function with high contract quantities, then he can deduce that the generator thinks the price will be low. This information can be combined with the retailer's own information.
- We suppose that the two players weight their own information as equivalent to the other player's information. So that given ρ_1 for the retailer and observing a supply function implying ρ_2 for the generator, then the retailer updates his expectation to $(\rho_1+\rho_2)/2$.
- The first order conditions become

$$\frac{\rho_1+\rho_2}{2}U'\left(R_H^{(1)}+Q(w_H-f)\right)\left(\frac{\partial Q}{\partial\rho_1}(w_H-f)-Q\frac{\partial f}{\partial\rho_1}\right)$$
$$+\left(1-\frac{\rho_1+\rho_2}{2}\right)U'\left(R_L^{(1)}+Q(w_L-f)\right)\left(\frac{\partial Q}{\partial\rho_1}(w_L-f)-Q\frac{\partial f}{\partial\rho_1}\right)=0.$$

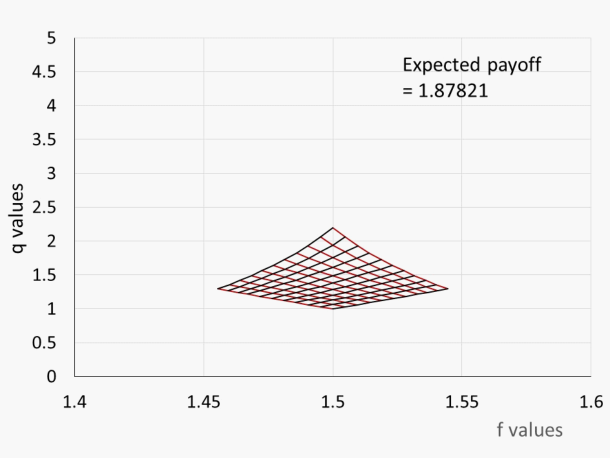
Back to the example again

- Out of the range of possible supply function equilibria, this one achieves a relatively high expected utility.



Other possible equilibrium solutions

- There are two degrees of freedom: we can shift the solutions up and down or expand them in the price range.



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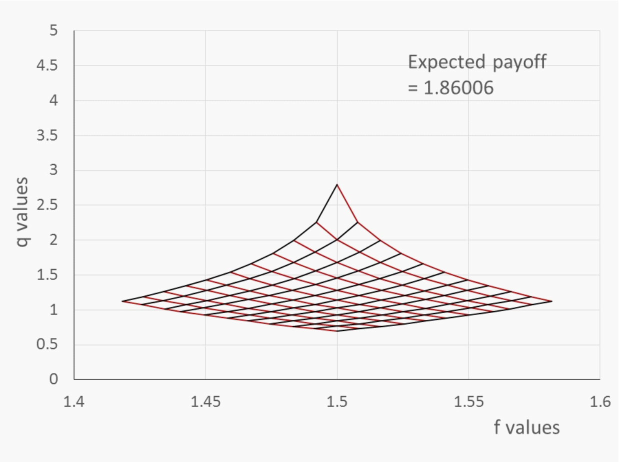


Why are contract amounts less?

- Worse overall performance (Base model = 1.937, SFE model = 1.933, SFE model with learning = 1.882)
- Assume (anti-)symmetric returns $r = R_H^{(2)} = R_L^{(1)}$, $p = R_H^{(1)} = R_L^{(2)}$ with $r > p$.
- When $\rho_1 + \rho_2 = 1$ the first order conditions are (writing $\gamma = (w_H - w_L)/2$)

$$(1 - bp - b\gamma Q)\left(\gamma \frac{\partial Q}{\partial \rho_1} - Q\frac{\partial f}{\partial \rho_1}\right) + (1 - br + b\gamma Q)\left(-\gamma \frac{\partial Q}{\partial \rho_1} - Q\frac{\partial f}{\partial \rho_1}\right) = 0.$$

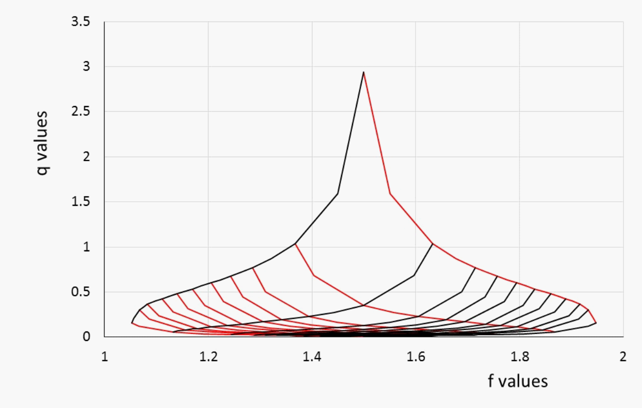
- Thus,

$$\frac{\partial Q}{\partial \rho_1} = \frac{\partial f}{\partial \rho_1} \frac{2Q}{b(w_H - w_L)} \frac{(2 - bp - br)}{(r - p - 2\gamma Q)}.$$

- There is an inflection point (infinite slope) when $Q = (r - p)/(w_H - w_L)$ and this is an upper bound on contract quantities. This leads to lower contract quantities throughout.

Finding a unique SFE

- If we let the range of possible ρ values expand to cover the entire interval $(0, 1)$ then the range of possible equilibria is reduced. But the actual equilibria we find get closer to the case with zero contract volume.



Conclusions

- We believe this is the first model to look at derivative trading in a context where players have different information and trade partly to hedge risks and partly to make profits from their private information.
- There are alternative supply function equilibria possible, so clean results are hard to establish. However we can see there is little overall improvement in outcomes when both players act strategically obtaining a supply function equilibrium.
- When both players combine their private information with deductions made from the other player's offers, there is a reduction in the contract quantities and significantly worse overall results.
- Being clever can leave everyone worse off!.