A new stochastic program to facilitate intermittent renewable generation

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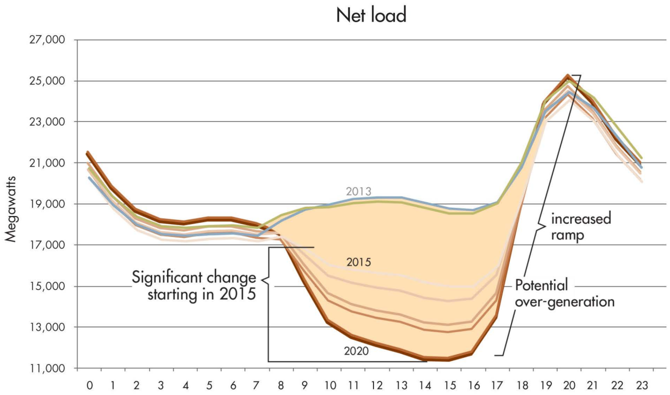
Premise for our model

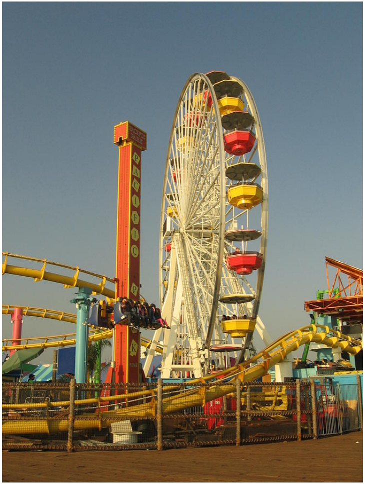
- Growing need for flexibility (renewable penetration)
- Short notice deviations from "pre-dispatch" have costs.

Growing need for flexibility starting 2015



California ISO
Shaping a Renewed Future

Local attraction: solar powered ferris wheel



Don't interrupt the power supply!!

Renewable generation is intermittent

- With high levels of uncertainty in the forecast the pre-dispatch of generation is likely to be inefficient.
- Expensive real time adjustments must be made to meet variations to the forecast.
- So if we look at this from a stochastic programmer's perspective we see a problem of first stage planning followed by recourse.

PZSP: Tying two markets

- A basic electricity market trades in energy.
- New market in “regulation” or “flexibility”.
- Similar markets in ancillary services exist but our approach is to tie these markets together.
- If we have a distribution of future outcomes (e.g. wind generation or demand realization) then we can adapt the pre-dispatch plan to suit this future distribution.
- In 2010 we proposed tying the energy and regulation markets together through a stochastic program.

Recall the conventional OPF

$$
\begin{array}{lll}
\text{[DCOPF]: } \min & c^T X & \\
\text{s.t.} & MX + AF = d & [\pi] \\
 & LF = 0 & [\lambda] \\
 & -F \geq -K & [\eta_U] \\
 & F \geq -K & [\eta_V] \\
 & -X \geq -G & [\mu] \\
 & X \geq 0 & [\psi]
\end{array}
$$

Note that this is deterministic. We have ignored losses and reserves in the above model.

DCOPF notation

- M denotes the matrix that maps tranches to the nodes of the electricity network. In particular $M_{nt} = 1$ if tranche t is offered at none n and $M_{nt} = 0$ otherwise.
- A is the network node-arc incidence matrix for our electricity grid.
- L denotes the matrix for the loop flow constraints that capture Kirchhoff's laws.
- X is the vector of dispatches.
- F is the vector of flows.
- Vectors G and K represent the upper limits for the tranches and the line thermal limits respectively.

PZSP: make DCOPF stochastic

- Allow for variations in demands and offers and capture this as a two stage SP.
- The first stage represents an initial dispatch computed in advance, with only probabilistic estimates of some quantities available.
- This could be thought of as a "day-ahead" dispatch, although the same ideas may apply to shorter time scales.
- The first stage is a bit like a contract.
- The second stage represents "real time", i.e. the actual dispatches over a short period.
- This period is meant to coincide with the market trading period. During this period some quantities will take on realized values unknown at the first stage.
- Adapting to these changes will require re-dispatch.
- We use the term "regulation" for differences between first-stage and second-stage dispatches.

How would the offers work?

Offer t has an associated ask or bid price c_t, which applies to power dispatched at the first stage. In addition, the participant making the offer also offers to, in real time,

- sell (back) additional power to the system at an asking price $p_t^+ = c_t + r_t$, or
- buy back power from the system at a bid price $c_t^- = p_t - r_t$,

wherever this is permitted or required by the capacity constraints. The margin $r_t \geq 0$ is an additional offer parameter to be chosen by the participant.

The SO's stochastic OPF problem can then be stated as:

$$
\begin{array}{llll}
\min & \sum_{t\in T}\left(c_t x_t + E[p_t^+(X_t - x_t)_+ - p_t^-(X_t - x_t)_-]\right) & & \\
\text{s.t.} & \tau_n(f) + \sum_{t\in T(n)} x_t = 0 & \forall n & [\pi_n^1] \\
 & \tau_n(F) - \tau_n(f) + \sum_{t\in T(n)}(X_t - x_t) = 0 & \forall n\ \forall \omega \in \Omega & [\theta^\omega \pi_n^2(\omega)] \\
 & (x_t, X_t \in C_t & \forall t\ \forall \omega \in \Omega & \\
 & f \in U & & \\
 & F \in U & \forall \omega \in \Omega. &
\end{array}
$$

Nodal pricing with regulation

- The first-stage nodal price π_n^1 can be interpreted as the marginal cost of serving a deterministic additional load at node n, present in the first stage and in every second-stage scenario.
- It is therefore an appropriate price at which to trade non-random (i.e. notified in advance) quantities of electricity at node n.
- The second-stage nodal price $\pi_n^2(\omega)$ can be interpreted as the marginal cost of serving an additional load at node n which is present at the second stage in scenario ω only.
- This price is itself a random variable.
- It is an appropriate price at which to trade random (i.e. not foreseen in advance) quantities of electricity in real time at node n.

What the participants pay/get paid?

Based on the above, in PZSP we combine the effects of first- and second-stage trading, and proposed to pay to the market participant responsible for offer i

$$x_t^* \pi_{\nu(t)} + (X_t^* - x_t^*)\pi^2_{\nu(t)}. \tag{1}$$

This can also be written

$$x_t^*(\pi_{\nu(t)} - \pi^2_{\nu(t)}) + X_i^* \pi^2_{\nu(t)},$$

which leaves the first stage to be viewed as a market for hedges (contracts for differences); the second stage as a spot market in which all power is ultimately traded.

Question regarding $\pi_n^2(\omega)$

When computing $\pi_n^2(\omega)$, we allow the perturbation in scenario ω to be met by perturbing the first-stage variables x, f as well as $X(\omega)$, $F(\omega)$. Is this the right price or should we instead consider the real-time problem for the particular ω:

$$\begin{array}{llll} \min & \sum_{t\in T}\left(p_t^+(y_t - x_t^*)_+ - p_i^-(y_t - x_t^*)_-\right) & & \\ \text{s.t.} & \tau_n(g) + \sum_{t\in T(n)} y_t = 0 & \forall n & [\pi_n^R(\omega)] \\ & (x_t^*, y_t) \in C_t(\omega) & \forall t & \\ & g \in U, & & \end{array}$$

where x^* is the already-determined optimal solution of SPOPF.
Turns out that $(\pi_n^2(\omega))_{n\in N}$ are also dual-optimal for RT, so it is valid to use them as second-stage prices under this interpretation also.

Revenue adequacy of PZSP?

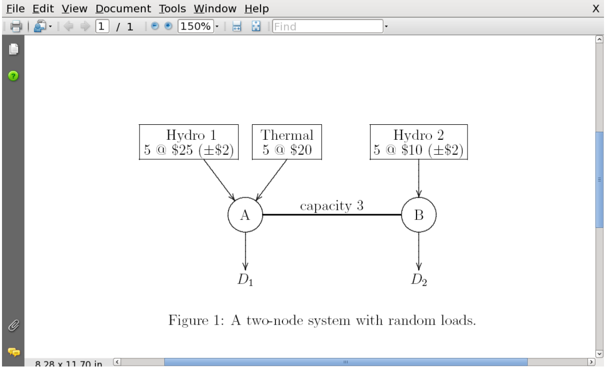
- PZSP is provably revenue adequate in expectation.
- Mathematically this means

$$E\left[\sum_t \left(x_t^* \pi_{\nu(t)} + (X_t^* - x_t^*)\pi_{\nu(t)}^2\right)\right] \leq 0. \qquad (2)$$

(recall payout is positive and collection is negative).

- Intuitively this means that, if this type of market is used repeatedly over many trading periods, the SO will not run a financial deficit over time.
- There may be a deficit in an individual trading period.

A 2 node example



Scenario	probability	D_1	D_2
ω_1	0.6	2	6
ω_2	0.4	7	1

2 node discussion PZSP

- For our example the optimal (first stage) decision is to adapt to scenario 1, that is: $x^* = (0, 3, 5, -2, -6)$ and $f^* = 1$.
- However if the second scenario eventuates then lack of transmission capacity will force a re-dispatch.
- $X^*(\omega_2) = (1, 3, 4, -7, -1)$ and $F^*(\omega_2) = -3$.
- Here $\pi_A = 20$, $\pi_B = 12.4$ even though, in the first-stage primal solution, the transmission line between them is not constrained.
- The price difference reflects a contingent transmission constraint that may come into play upon re-dispatch.

2 node discussion

- If scenario 1 occurs then SO pays out $3 \times 20 + 5 \times 12.4 = 122$, but collects only $2 \times 20 + 6 \times 12.4 = 114.4$ leaving a deficit of 7.6.
- If scenario 2 occurs however there will be additional payments from the re-dispatch.
- We can go through the calculation and demonstrate a surplus of $76 in this scenario.

Our new SP

- In PZSP we require the first stage decision to meet physical conditions such as network flow conditions on dispatch. This is neither necessary nor intuitive.
- We can instead think about a news-vendor type of problem.

New SPOPF

$$
\begin{array}{llll}
\text{[NewSP]:} \min & E_\omega \left[c^T X^\omega + r_U u^\omega + r_V v^\omega\right] & & \\
\text{s.t.} & MX^\omega + AF^\omega = d^\omega & \forall\omega & [\theta^\omega \pi^\omega] \\
& LF^\omega = 0 & \forall\omega & [\theta^\omega \lambda^\omega] \\
& -F^\omega \geq -K & \forall\omega & [-\theta^\omega \eta_U^\omega] \\
& F^\omega \geq -K & \forall\omega & [-\theta^\omega \eta_V^\omega] \\
& -X^\omega \geq -G & \forall\omega & [-\theta^\omega \mu^\omega] \\
& X^\omega \geq 0 & \forall\omega & [\theta^\omega \psi^\omega] \\
& u^\omega \geq 0 & \forall\omega & [\theta^\omega \xi_U^\omega] \\
& v^\omega \geq 0 & \forall\omega & [\theta^\omega \xi_V^\omega] \\
& x - X^\omega + u^\omega \geq 0 & \forall\omega & [\theta^\omega \rho_U^\omega] \\
& -x + X^\omega + v^\omega \geq 0 & \forall\omega & [\theta^\omega \rho_V^\omega] \\
& -x \geq -G & & [-\zeta] \\
& x \geq 0 & &
\end{array}
$$

What the participants pay/get paid?

The new payment mechanism is very simple. Each tranche gets paid

$$(X_t^*)\pi_{\nu(t)}^{\omega}. \tag{3}$$

For any flexible demand, the above is what is charged. The inflexible demands pay

$$d_{\nu(t)}^{\omega}\pi_{\nu(t)}^{\omega}.$$

Revenue adequacy of the new SP

- NewSP is provably revenue adequate in every scenario.
- Mathematically this means for any scenario ω, at optimality

$$\sum_t \left((X_t^*)\pi^{\omega}_{\nu(t)} - d^{\omega}_{\nu(t)}\pi^{\omega}_{\nu(t)} \right) \leq 0. \tag{4}$$

(recall payout is positive and collection is negative).

- This is natural and the argument is very similar to revenue adequacy of the DCOPF.

Previous 2 node example revisited

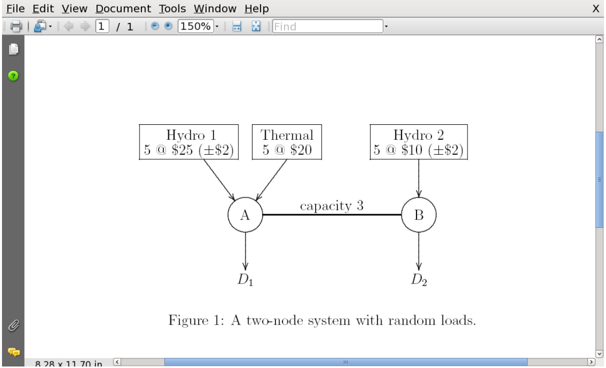


Figure 1: A two-node system with random loads.

Scenario	probability	D_1	D_2
ω_1	0.6	2	6
ω_2	0.4	7	1

2 node discussion new SP

- For our example the optimal (first stage) decision is similar to PZSP, that is: $x^* = (0, 3, 5)$, note that we no longer have first stage flows.
- The real time dispatches are $X^*(\omega_1) = (0, 3, 5)$ (with no unders and overs), and $X^*(\omega_2) = (1, 3, 4)$ implying some adjustments.
- Here $\pi_A^{\omega_1} = 15.33333$, $\pi_A^{\omega_2} = 27$, $\pi_B^{\omega_1} = 15.33333$ and $\pi_B^{\omega_2} = 8$.
- Total payments under ω_1 amount to \$122.667, which is exactly the system revenue.
- Total payments under ω_2 is \$140. Here the collected revenue includes congestion rents and is \$197.

Cost recovery: An intuitive note

For a marginal tranche j, if this tranche has been adjusted upward then

$$\pi_{n(j)} = c_j + (\rho_U)_j.$$

This is exactly what we expect. The cost of producing one more unit in the short run plus marginal cost of deviation. It is intuitive that in this case the generators will recover their cost.
On the other hand if our marginal tranche has been adjusted downward then

$$\pi_{n(j)} = c_j - (\rho_V)_j.$$

This says if one more unit is *consumed*, then we'd save the short run cost of generation, but we will save the downward deviation cost. In this case the generator may not recover cost (by being paid the real time price).

Cost recovery in expectation

- NewSP is provably recovers cost for each tranche, including the deviation penalties, in expectation.
- Mathematically this means at optimality

$$E_\omega\left[\pi_{n(t)}^{\hat{\omega}}\hat{X}_t^\omega - c_t\hat{X}_t^\omega - r_{U\,t}\hat{u}_t^\omega - r_{V\,t}\hat{v}_t^\omega\right] \geq 0. \qquad (5)$$

- This result follows from SP duality and the technical lemma discussed earlier regarding validity of duals in real time.

Another example

$$[\text{Ex1nd}] \ \min \sum_{\omega\in\{1,2\}} \rho(\omega)(10X_1^\omega + 20X_2^\omega + 0.01X_3^\omega$$
$$+1(X_1^\omega - x_1)_+ + 1(X_1^\omega - x_1)_-$$
$$+5(X_2^\omega - x_2)_+ + 0.001(X_2^\omega - x_2)_-$$
$$+1(X_3^\omega - x_3)_+ + 0.001(X_3^{\omega 1} - x_3)_-)$$
$$\text{s/t } x_1 + x_2 + x_3 = 100$$
$$x_i \geq 0 \qquad i \in \{1,2,3\}$$
$$X_1^\omega + X_2^\omega + X_3^\omega = 100 \qquad \omega \in \{1,2\}$$
$$0 \leq X_1^\omega \leq 50 \qquad \omega \in \{1,2\}$$
$$0 \leq X_2^\omega \leq 80 \qquad \omega \in \{1,2\}$$
$$0 \leq X_3^1 \leq 20$$
$$0 \leq X_3^1 \leq 50$$

Welfare enhancing

The optimal cost with all constraints included (PZSP1) is 815.37. If the market clearing equation for the first stage is removed (NewSP2), the cost decreases to 800.38 (resulting in a better social welfare).

Optimal "pre-dispatch" property

Suppose that $\overline{X}^\omega$ are the optimal solution to our new SP. Then the optimal pre-dispatch quantity x is found by

$$\begin{array}{ll} x \in \text{argmin} & E_\omega[c^T X^\omega + r_U^T u^\omega + r_V^T v^\omega] \\ \text{s.t.} & X^\omega = x + u^\omega - v^\omega \\ & \text{non-negativity} \end{array}$$

It is simple to see that the solution is given by x such that

$$P(\overline{X} \leq x) = \frac{r_u}{r_u + r_v}.$$

So x is componentwise a quantile of the distribution of $\overline{X}^\omega$.
So in particular box constraints need not be imposed on x.

Competitive equilibrium is equivalent to system optimization

$$
\begin{array}{llll}
\text{[SNNSP]: } \min & E_\omega\left[c^T X^\omega + r_U u^\omega + r_V v^\omega\right] & & \\
\text{s.t.} & \sum_{t\in\mathcal{T}} X_j^\omega \geq d^\omega & \forall\omega & [\theta^\omega \pi^\omega] \\
& -X^\omega \geq -G & \forall\omega & [-\theta^\omega \mu^\omega] \\
& X^\omega \geq 0 & \forall\omega & [\theta^\omega \psi^\omega] \\
& u^\omega \geq 0 & \forall\omega & [\theta^\omega \xi_U^\omega] \\
& v^\omega \geq 0 & \forall\omega & [\theta^\omega \xi_V^\omega] \\
& x - X^\omega + u^\omega \geq 0 & \forall\omega & [\theta^\omega \rho_U^\omega] \\
& -x + X^\omega + v^\omega \geq 0 & \forall\omega & [\theta^\omega \rho_V^\omega] \\
& -x \geq -G & & [-\zeta] \\
& x \geq 0 & &
\end{array}
$$

Write the Lagrangian ...

$$
\begin{array}{llll}
\text{[SNNSP]: } \min & E_\omega \left[c^T X^\omega + r_U u^\omega + r_V v^\omega + \pi^\omega (d^\omega - \sum_{t\in\mathcal{T}} X_j^\omega)\right] & & \\
\text{s.t.} & -X^\omega \geq -G & \forall\omega & [-\theta^\omega \mu^\omega] \\
& X^\omega \geq 0 & \forall\omega & [\theta^\omega \psi^\omega] \\
& u^\omega \geq 0 & \forall\omega & [\theta^\omega \xi_U^\omega] \\
& v^\omega \geq 0 & \forall\omega & [\theta^\omega \xi_V^\omega] \\
& x - X^\omega + u^\omega \geq 0 & \forall\omega & [\theta^\omega \rho_U^\omega] \\
& -x + X^\omega + v^\omega \geq 0 & \forall\omega & [\theta^\omega \rho_V^\omega] \\
& -x \geq -G & & [-\zeta] \\
& x \geq 0 & &
\end{array}
$$

where

$$0 \leq \sum_{t\in\mathcal{T}} X_j^\omega - d^\omega \perp \pi^\omega \geq 0.$$

The problem decouples into agent (expected) optimization problems.

And now, the MOPEC!

For each agent (in fact tranche):

$$
\begin{array}{llll}
\text{[AgentOpt]:} & \min & E_\omega \left[c^T X^\omega + r_U u^\omega + r_V v^\omega - \pi^\omega X_j^\omega\right] & \\
\text{s.t.} & & -X^\omega \geq -G & \forall\omega \quad [-\theta^\omega \mu^\omega] \\
& & X^\omega \geq 0 & \forall\omega \quad [\theta^\omega \psi^\omega] \\
& & u^\omega \geq 0 & \forall\omega \quad [\theta^\omega \xi_U^\omega] \\
& & v^\omega \geq 0 & \forall\omega \quad [\theta^\omega \xi_V^\omega] \\
& & x - X^\omega + u^\omega \geq 0 & \forall\omega \quad [\theta^\omega \rho_U^\omega] \\
& & -x + X^\omega + v^\omega \geq 0 & \forall\omega \quad [\theta^\omega \rho_V^\omega] \\
& & -x \geq -G & [-\zeta] \\
& & x \geq 0 &
\end{array}
$$

Together with Walrasian prices, we recover the new SP.

What if agents are not risk neutral

- If the market is complete, the problem converts to a system optimization in expectation, but with risk adjusted probabilities (Ralph and Smeers).
- In this case everything we have laid out persists.
- We are interested in studying what practical instruments would complete the market in this context.

Expected revenue adequacy with sampled distribution

Let

$$F(x, \mu) = E[\pi^T X - c^T X - r_u^T U - r_v^T V]$$

Let $x^*(\nu_n)$ be the optimal first stage decision with respect to distribution ν_n. Then if $\nu_n \to \mu$ (in distribution), we need $F(x^*(\nu_n), \nu_n) \to F(x^*(\mu), \mu)$.

So in particular if we use sample average approximation this will work.

Real world estimates

- We want to have a measure of implementability of this mechanism.
- I would like to solve the 2 stage SP using a large sample of wind scenarios and on the NZEM full system. Then simulate and measure cost recovery.
- We can use Benders decomposition.
- Our SP has a special structure so I'd like to use cleverer methods.
- Please stay tuned!

Questions and comments most welcomed

My question for Roger

Could the following algorithm work in finding the optimal solution to my new SP assuming complete recourse?

- Start with some x^0.
- Pass x^0 to all scenarios in stage 2 and solve to optimality.
- Set $x^{k+1} =$ appropriate quantiles of the real time Xs and repeat step 2 till convergence.