Hierarchical Optimization and Equilibrium Problems: Applications in Liberalized Electricity Markets

IPAM Workshop

Optimization and Equilibrium in Energy Economics

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About me

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Research interests:

Operations research, energy storage, bilevel programming, investment under uncertainty.

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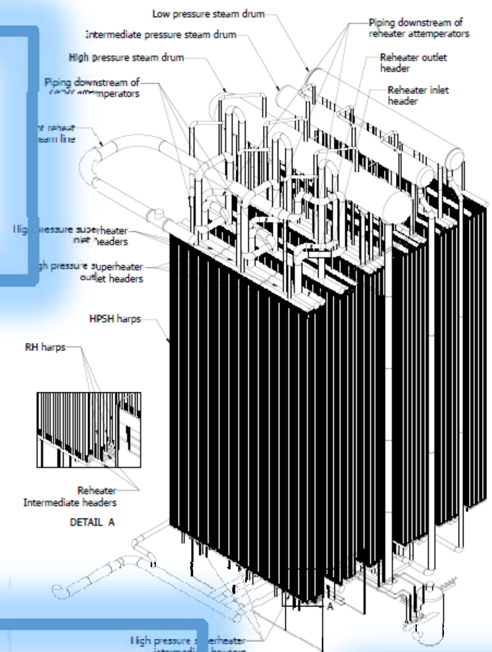
Areas of Research

UC Models with Fatigue Impact



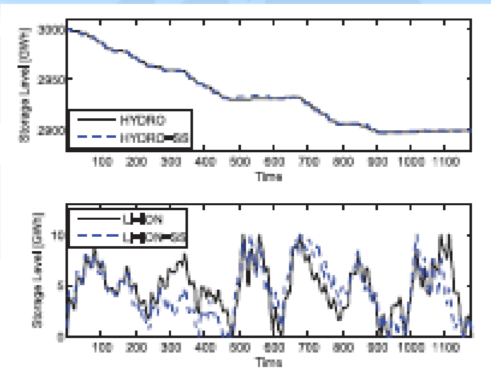
Bilevel Programming, MPECs and EPECs

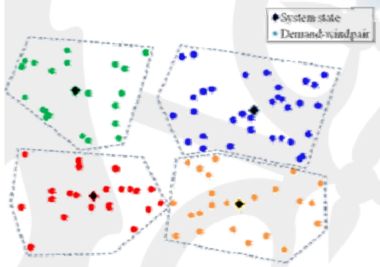
$$\forall i \begin{cases} \max_{x_i, q_i} & t(p(q_i, q_{-i}) - \delta) q_i - \beta x_i \\ s.t. & \text{Second stage} \end{cases}$$

$$\forall i \begin{cases} \max_{x_i, q_i} & t(p(q_i, q_{-i}) - \delta) q_i \\ s.t. & 0 \le q_i \le x_i \end{cases}$$

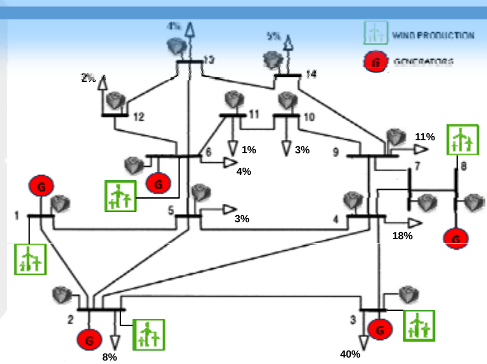
$$d = q_i + q_{-i}, \; d = D_0 - \alpha p$$

Improving LDC Models with high Renewable Penetration





Storage Allocation and Investment



Outline

- Introduction & Motivation
- Generation Expansion Planning (GEP)
- Approximating Bilevel GEP Equilibria
- Ramp Bidding in Electricity Markets
- Conclusions and Work in Progress

Introduction & Motivation

Generation Expansion Planning (GEP)

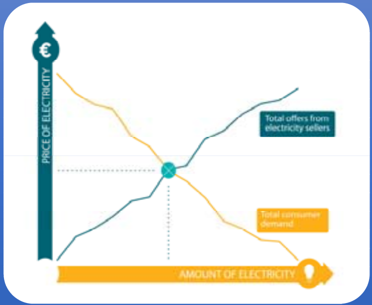
Approximating Bilevel GEP Equilibria

Ramp Bidding in Electricity Markets

Conclusions and Work in Progress

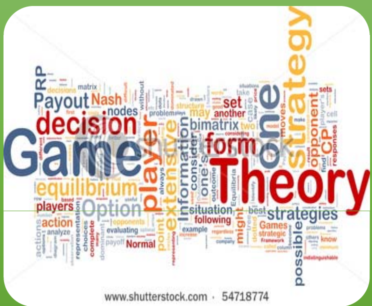
Introduction and Motivation

Motivation



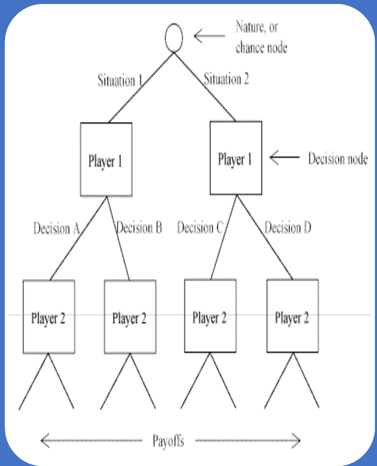
The liberalization of the electricity sector and the introduction of electricity markets have greatly complicated the organization of the electricity sector, especially for generation companies.

Under a centralized framework a central planner took decisions maximizing social welfare, whereas in electricity markets the responsibility of taking many decisions, such as generation expansion for example, lies with the generation companies.

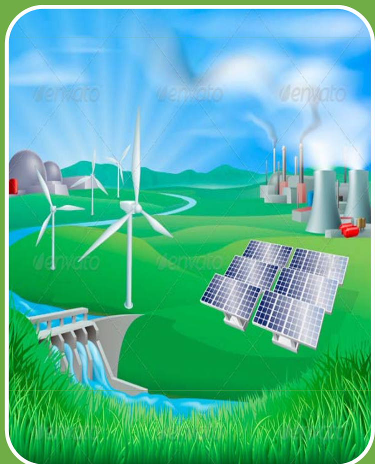


From a game-theoretic point of view many decision-making problems in a liberalized power sector can be regarded and analyzed as a game among strategic competitors in search of equilibrium solutions.

Motivation



The sequence in which decisions are taken, can convert simple equilibrium games into complicated hierarchical equilibrium problems whose outcomes can diverge significantly depending on the type of game.



This talk discusses two applications of such hierarchical games in electricity markets: generation expansion planning; and, generation flexibility in ramp rates. The results indicate that the market structure, i.e., the set-up of the game, can drastically influence outcomes.

Introduction and Motivation

Basic Concepts

Bilevel Problem

- A bilevel programming problem is a hierarchical optimization problem which is constrained by another optimization problem.

MPEC

- Mathematical Program with Equilibrium Constraints – this is a bilevel optimization problem

EPEC

- Equilibrium Problem with Equilibrium Constraints – this is a bilevel equilibrium problem

Conjectured price response θ

- A type of conjectural variation which allows to express GENCO i's belief concerning its influence on price as a result of a change in its output. This allows us to model different strategic behavior in the market.

Conjectured Price Response

The conjectured price response parameter θ_i is defined as company i 's belief concerning its influence on price *p* as a result of a change in its output q_i :

$$\theta_i := -\frac{dp(q_i, q_{-i})}{dq_i} \geq 0$$

- θ_i allows us to represent various ways of strategic behavior.
- It can easily be translated into conjectural variations, and vice versa.
- It can represent more complex dynamic games, for example the (Allaz and Vila, 1993) game.

- Introduction & Motivation
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- Approximating Bilevel GEP Equilibria
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Generation Expansion Planning

Problem Definition

Long-term Electricity Generation Expansion Planning:

- Managing available generation assets and deciding upon the construction of new capacity (time horizon up to 40 years).

Liberalized Systems - Approaches:

- *Uncertainty emphasis*: decision theory, risk management, scenario analysis, real options theory.
- *Market emphasis*: system dynamics, multi-agent based simulation**, game theory**.

Generation Expansion Planning

Game Theoretic Approach

Open Loop Models

Investment and production decisions taken at the same time.

Simplification; easier to formulate and solve; less realistic.

Examples: (Ventosa et al., 2003), (Murphy and Smeers, 2005).

Closed Loop Models

First investment decisions are taken; then energy production in the spot market is determined.

More difficult to formulate and solve; more realistic.

MPECs and EPECs: (Murphy and Smeers, 2005), (Hobbs et al., 2000), (Wogrin et al., 2011).

When does the additional modeling effort make sense?

Generation Expansion Planning

Problem Statement

We have two identical firms with perfectly substitutable products, facing either a one-stage or a two-stage competitive situation.

One-stage situation (open loop model)	Two-stage situation (closed loop model)	
Investment and operation decisions are made simultaneously.	First, firms choose capacities that maximize their profit anticipating the second stage, where…	…quantities and prices are determined by a conjectured price response market equilibrium.

Generation Expansion Planning

Price and Demand

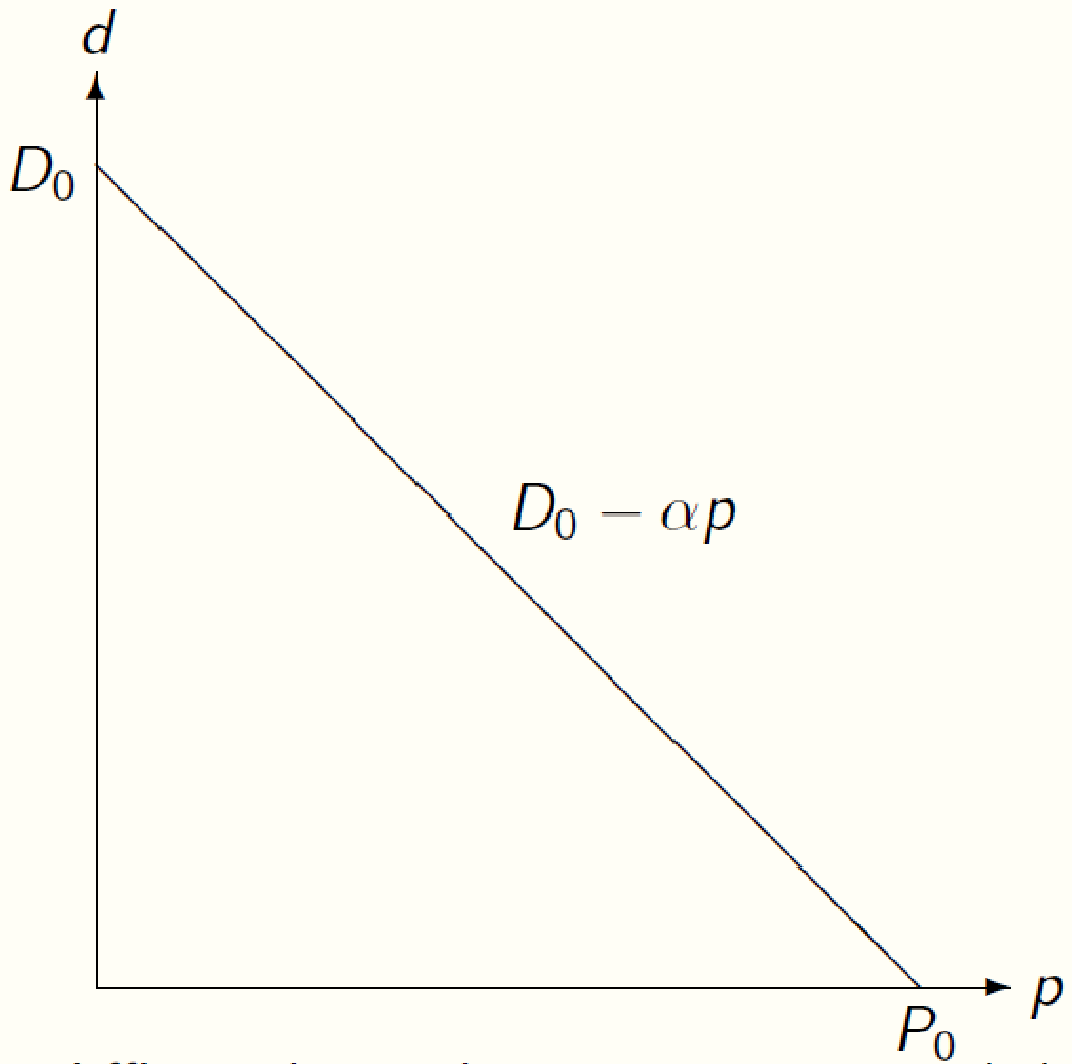
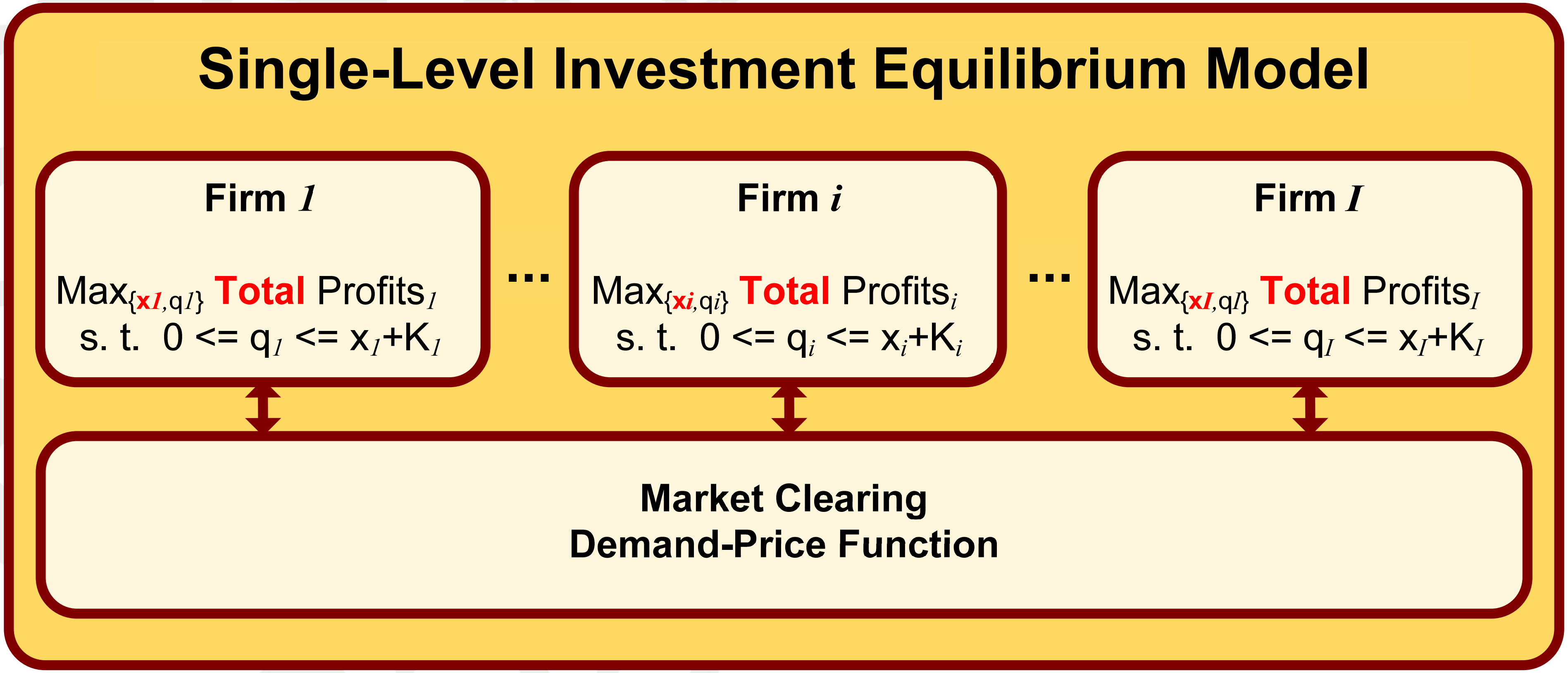


Figure: Affine relation between price p and demand d.

Generation Expansion Planning
Single-Level (SL) Investment Equilibrium

- All GENCOs simultaneously maximize their total profits (market revenues minus investment costs minus production costs) subject to lower and upper bounds on production and a demand balance.



Generation Expansion Planning
SL Investment Formulation

Concept:

$$\forall i \begin{cases} \max_{x_i, q_i} & t(p(q_i, q_{\text{-}i}) - \delta) q_i - \beta x_i \\ s.t. & q_i \leq x_i \end{cases}$$

$$d = q_i + q_{\text{-}i}, \quad d = D_0 - \alpha p(q_i, q_{\text{-}i})$$

KKT-conditions:

$$\forall i \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_i} = tp(q_i, q_{\text{-}i}) - t\theta q_i - t\delta - \lambda_i = 0 \\ \frac{\partial \mathcal{L}_i}{\partial x_i} = \beta - \lambda_i = 0 \\ q_i \leq x_i \\ \lambda_i \geq 0 \\ \lambda_i (x_i - q_i) = 0 \end{cases}$$

$$d = q_i + q_{\text{-}i}, \quad d = D_0 - \alpha p(q_i, q_{\text{-}i})$$

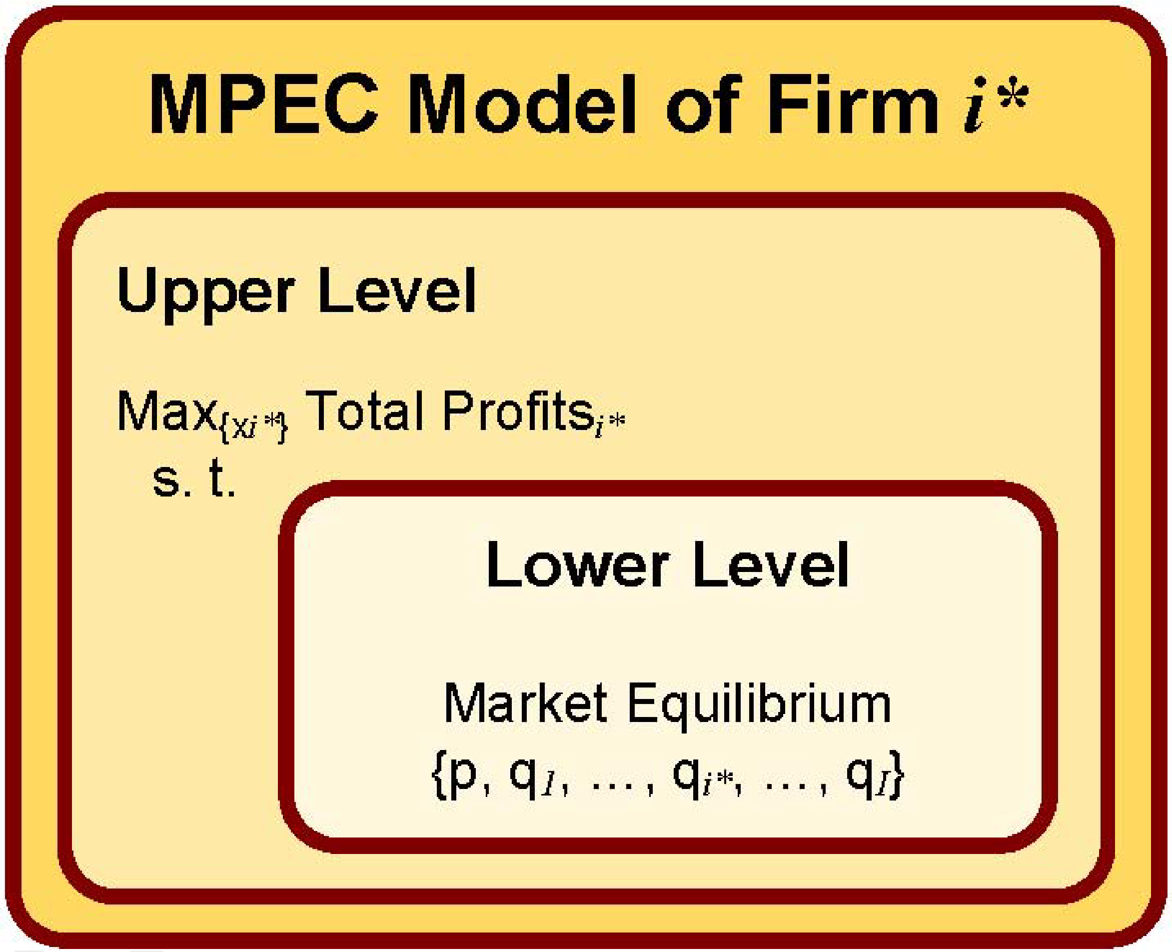
Generation Expansion Planning
Bilevel Investment Optimization

- This model **assists one GENCO** in taking capacity decisions while considering the competitors' investments as fixed.

This model is an **MPEC**.

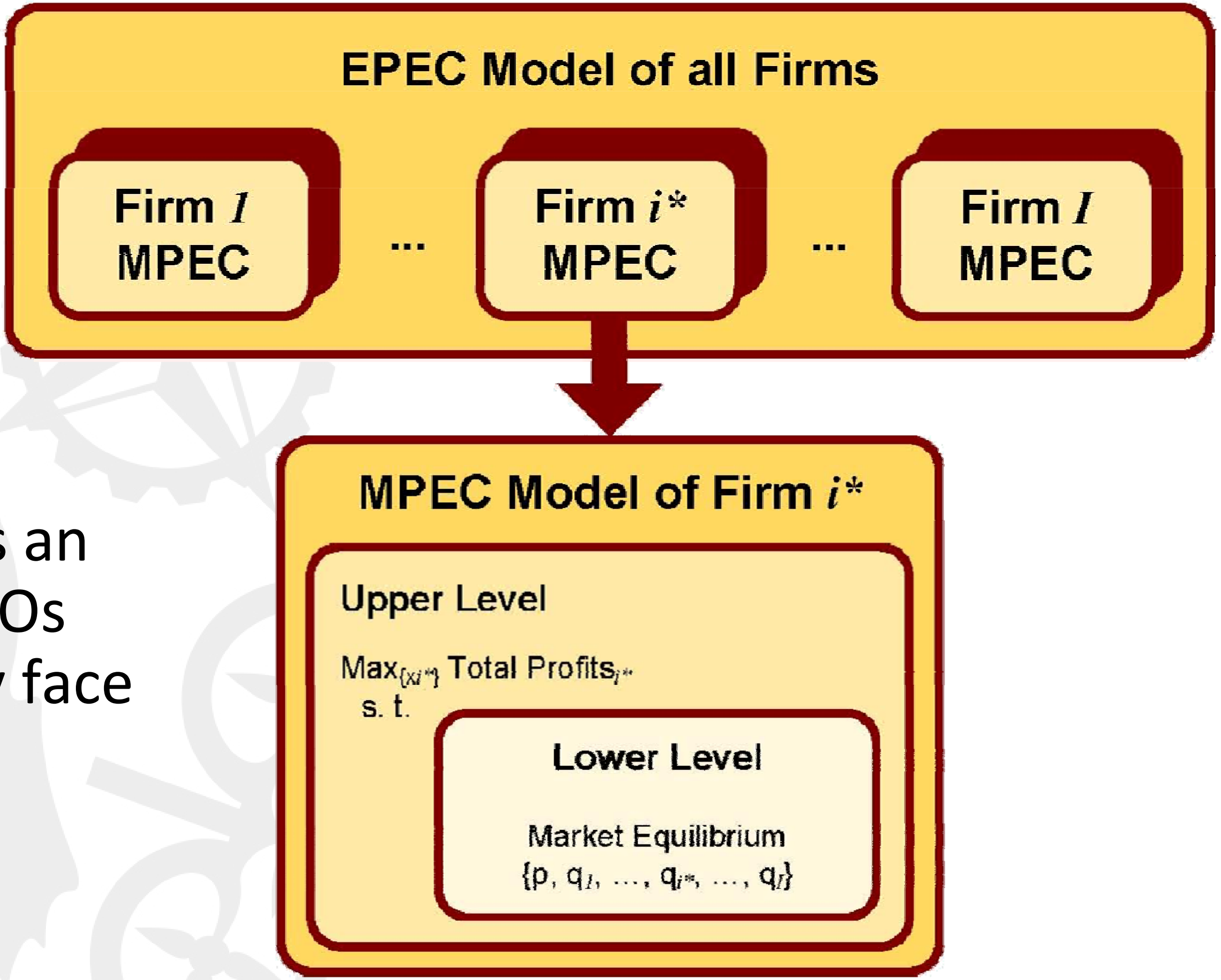
In the **upper level** investment decisions of firm i* are taken.

The **lower level** corresponds to the previously defined **market equilibrium**.



Generation Expansion Planning
Bilevel Investment Equilibrium

- This model **assists ALL GENCOs** in taking capacity decisions.



This problem is an **EPEC**: all GENCOs simultaneously face an **MPEC**.

Generation Expansion Planning
Bilevel Investment Formulation

First Stage (Investment):

$$\forall i \begin{cases} \max_{x_i} & t(p(q_i, q_{\text{-}i}) - \delta)q_i - \beta x_i \\ s.t. & \text{Second Stage} \end{cases}$$

Second Stage (Production):

$$\forall i \begin{cases} \max_{q_i} & t(p(q_i, q_{\text{-}i}) - \delta)q_i \\ s.t. & q_i \leq x_i \end{cases}$$

$$d = q_i + q_{\text{-}i}, \quad d = D_0 - \alpha p(q_i, q_{\text{-}i}),$$

Generation Expansion Planning

Comparison Single- and Bilevel Equilibria

- We compare (Wogrin et al, 2013) two generation expansion models:
  - A single-level model where investment and production decisions are considered to be taken simultaneously.
  - A bilevel model where first investment decisions are taken and then sequentially production decisions are decided in the market.
- The intensity of competition among producers in the energy market is represented using conjectural variations.
- For simplicity, in each of the models we consider two identical generation companies, a one-year time horizon and investment in one technology.

Comparison Single- and Bilevel Equilibria

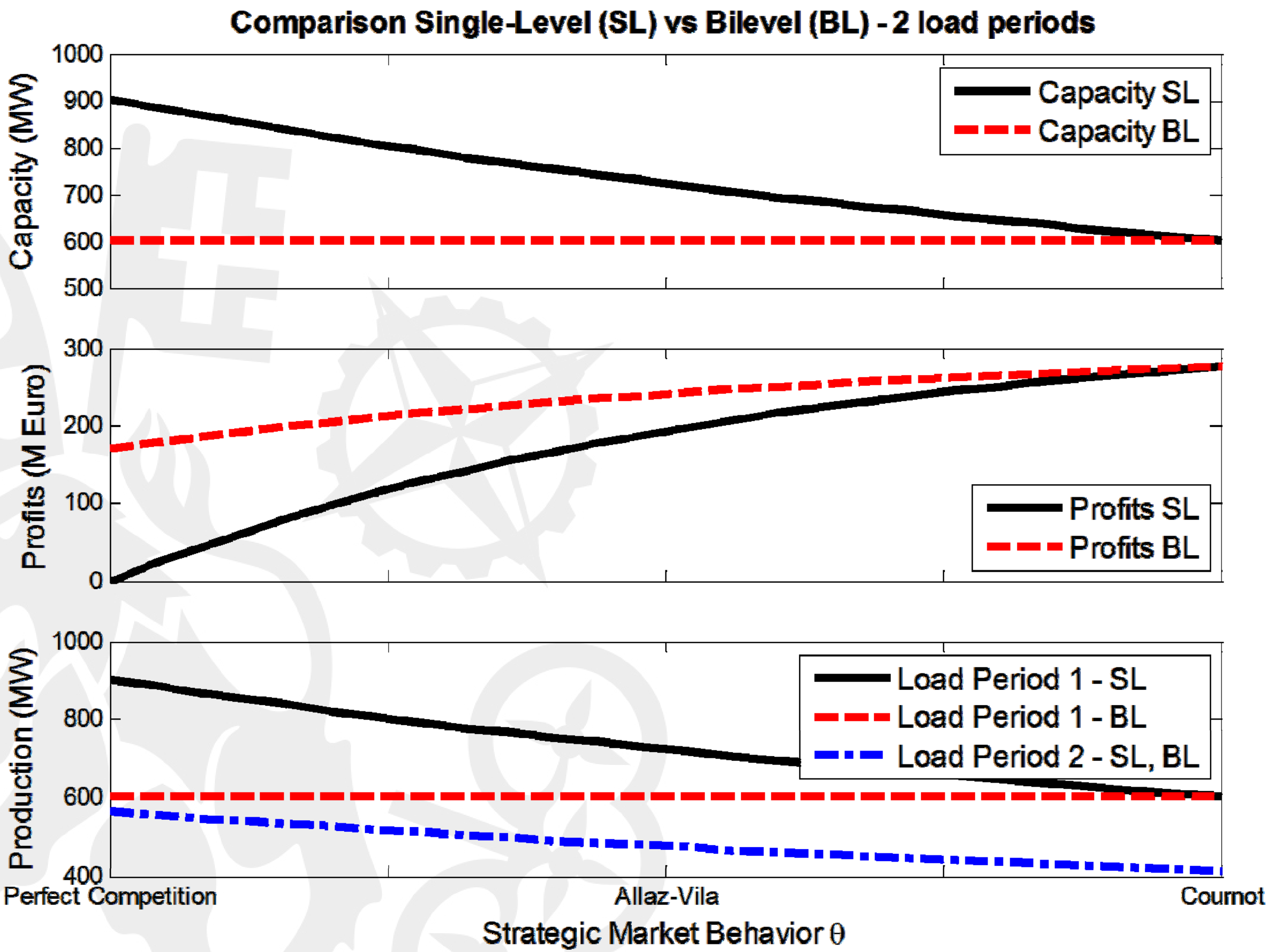
Theorem

Let there be two identical firms with perfectly substitutable products and one load period and let the affine price $p(d)$ and the parameters be as previously defined. When comparing the open and closed loop competitive equilibria for two firms, we find the following:
The open loop Cournot solution, is a solution to the closed loop conjectured price response equilibrium for any choice of the conjectured price response parameter θ from perfect competition to Cournot competition.

- The result extends to multiple load periods and – under certain circumstances – to asymmetric firms.
- This is an extension of (Kreps and Scheinkman, 1983).

Comparison Single- and Bilevel Equilibria

Comparison example 2 periods



Comparison Single- and Bilevel Equilibria
Comparison example 2 periods

These results are related to the findings of (Kreps and Scheinkman, 1983).

The closed loop model is capable of depicting a feature that the open loop model fails to capture, which is that generation companies would not voluntarily build all the capacity that might be determined by the spot market equilibrium if that meant less profits for themselves.

Thus the closed loop model could be useful to evaluate the effect of alternative market designs for mitigating market power in spot markets and incenting capacity investments in the long run, e.g., capacity mechanisms.

Comparison Single- and Bilevel Equilibria
Counter-intuitive Results

- It is possible for the BL model that assumes perfectly competitive behavior in the market to actually result in lower market efficiency, lower consumer surplus, and higher average prices than under Cournot.
- For a 20 load period example, we obtain the following results for the bilevel model:

[Billion Euro]	Perfect competition	Intermediate competition	Cournot
Market Efficiency	1.24	**1.30**	1.28
Consumer Surplus	0.62	0.72	0.64
Total Profits	0.62	0.58	0.64

Comparison Single- and Bilevel Equilibria
Summary of Results

1. The **bilevel** model always **yields Cournot capacities** independent of strategic spot market behavior.

2. This makes them more realistic than single-level models whose capacity decisions vary with market behavior.

3. Therefore bilevel models are very useful to study realistic generation expansion decisions and to evaluate the effect of alternative market designs for mitigating market power.

4. Under certain circumstances (Cournot market behavior) both **single-level and bilevel results can coincide**.

5. In bilevel models, **more competition can lead to less consumer surplus and less overall market efficiency**, depending on the model parameters.

- Introduction & Motivation
- Generation Expansion Planning (GEP)
- **Approximating Bilevel GEP Equilibria**
- Ramp Bidding in Electricity Markets
- Conclusions and Work in Progress

Approximating Bilevel Equilibria
Overview of BL Equilibrium Models

This type of model is usually formulated as an Equilibrium Problem with Equilibrium Constraints (**EPEC**).

From numerical examples it becomes apparent that:

- EPECs are **very hard to solve**.
- Even for small examples (2 GENCOs, 2 years, 2 load periods, 2 technologies) there can be **multiple equilibria**.
- MIP approaches to EPECs allow to choose equilibria but only allow to solve small case studies and they take a **long time to solve**.
- NLP approaches to EPECs allow to solve larges case studies but only yield an arbitrary local solution.

Approximating Bilevel Equilibria Using SL Equilibria

- We propose (Wogrin et al., 2013b) an **approximation scheme** of bilevel equilibria (EPECs) using only single-level equilibria (alternative version as a QCP).

- This approximation allows us to **reduce computational time** by two orders of magnitude.

- Approximation is based on theoretical results of (Wogrin et al., 2013).

Approximating Bilevel Equilibria
Approximation Scheme

- In order to approximate a bilevel equilibrium assuming market behavior θ we carry out the following:

1. Solve the single-level equilibrium model, assuming Cournot behavior in the market. This yields capacity decisions x.
2. Fix the capacity decisions x to values of the previous step.
3. Solve the single-level equilibrium model again but this time with strategic spot market behavior θ which yields market prices p, demand d and production decisions q.

Small Example
Application of Approximation Scheme

2 GENCOs, 1 technology, 1 year, 6 load periods, θ=0.3

Actual solution of bilevel problem:

Load period	1	2	3	4	5	6
Production [MW]	13.67	13.67	11.66	13.67	13.24	9.19
Prices [Euro/MWh]	291.82	117.81	54.21	87.96	56.27	50.99

Approximation after first step:

Load period	1	2	3	4	5	6
Production [MW]	13.74	13.74	8.94	12.87	10.15	7.05
Prices [Euro/MWh]	291.19	117.18	77.88	94.94	83.14	69.64

Approximation after final step:

Load period	1	2	3	4	5	6
Production [MW]	13.74	13.74	11.66	13.74	13.24	9.19
Prices [Euro/MWh]	291.19	117.18	54.21	87.32	56.27	50.99

Small Example
Results of Approximation Scheme

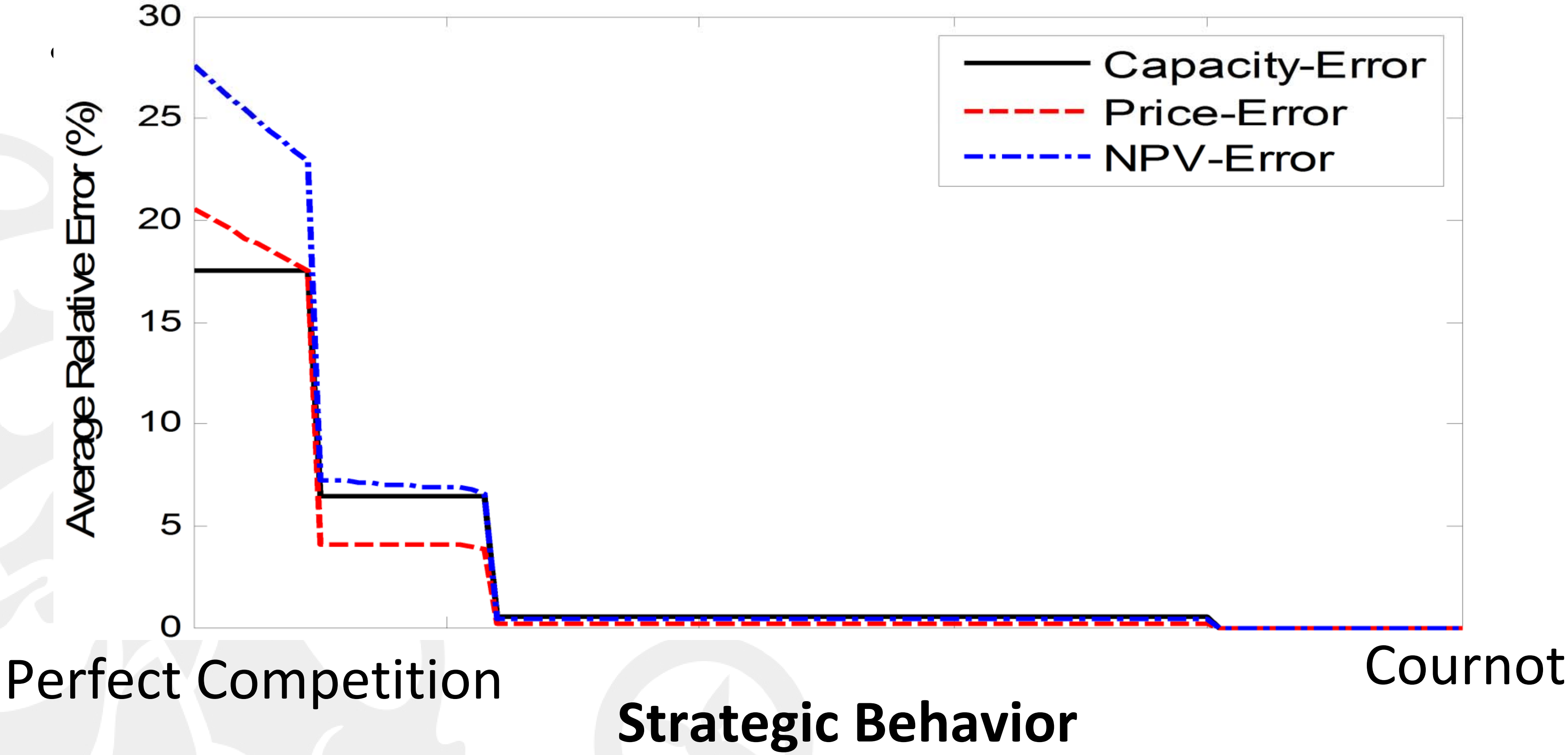
Approximation works **very well** for the small case study

- Relative error in capacities 0.5%;
- Maximum relative error in prices 0.7%;
- Relative error in production decisions is 0% in non-binding load periods and 0.5% in binding load periods.

Computational time **two orders of magnitude faster** than standard EPEC method (diagonalization).

- Approximation takes 0.5 seconds.
- Computational time of diagonalization depends on the initial point (ranges from 6.5 seconds to 144 seconds).

Small Example
Sensitivity Analysis



The closer strategic behavior is to Cournot, the better the approximation.

Large-Scale Example of Approximation Scheme

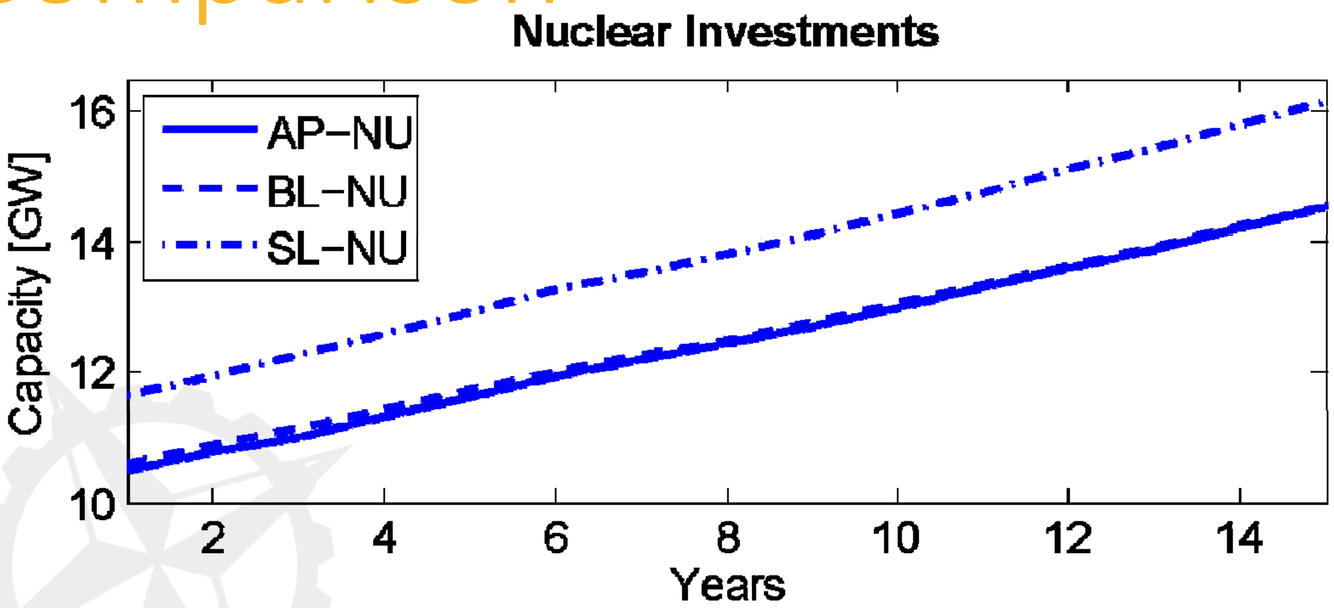
- 2 GENCOs, 4 technology (NU, CO, CC, GT), 15 years, 6 load periods, θ=0.7

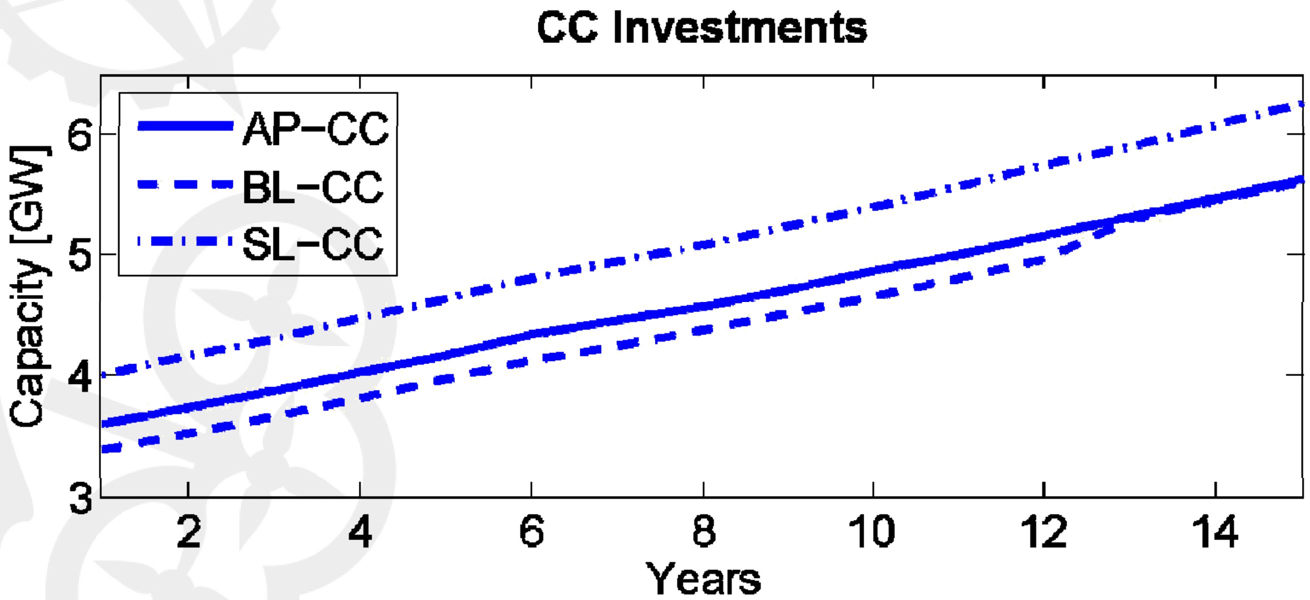
- We want to approximate the bilevel equilibrium (BL) and compare the approximation scheme (AP) to the naïve single-level approach (SL).

- Approximation scheme, as well as the presented case study have been published in (Wogrin et al., 2013b) in IEEE Transactions on power systems.

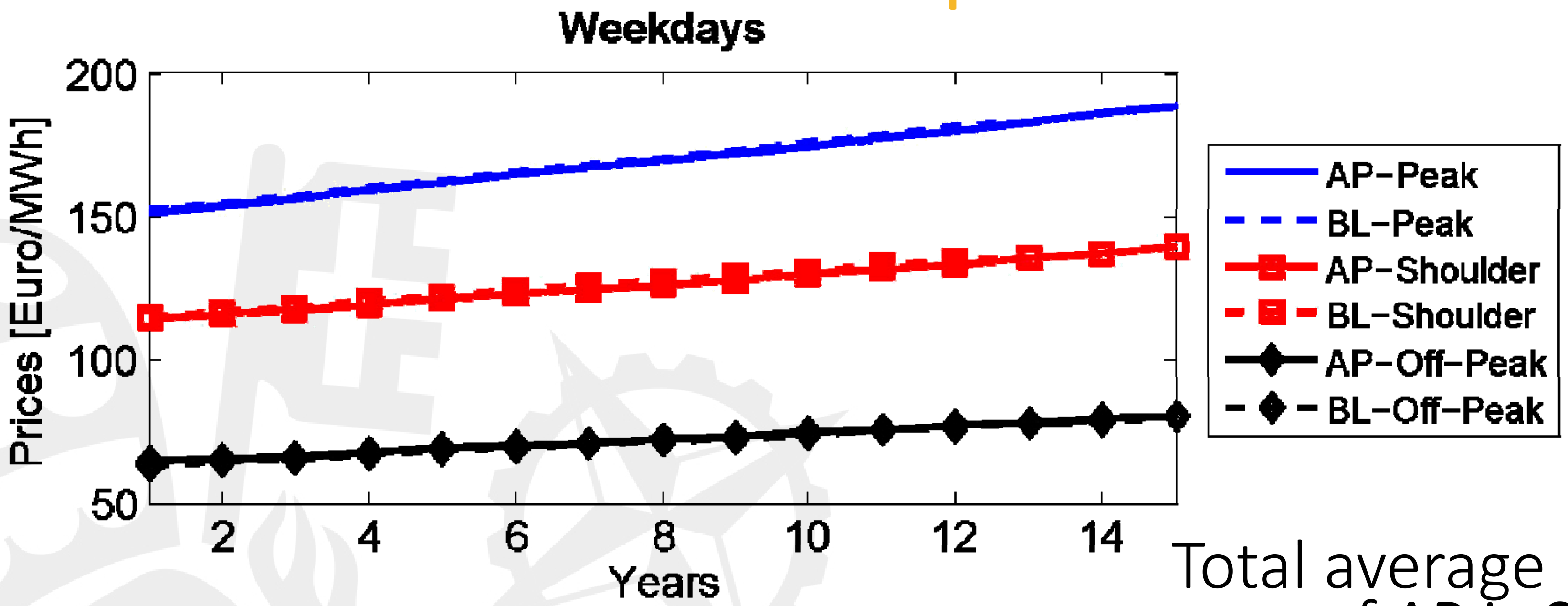
Large-Scale Example
Capacity Comparison

- ★Total average relative error of **AP** is **2.26%**.
- ★Total average relative error of **SL** is **11.54%**.
- ★AP is five times better than the naïve SL approach.

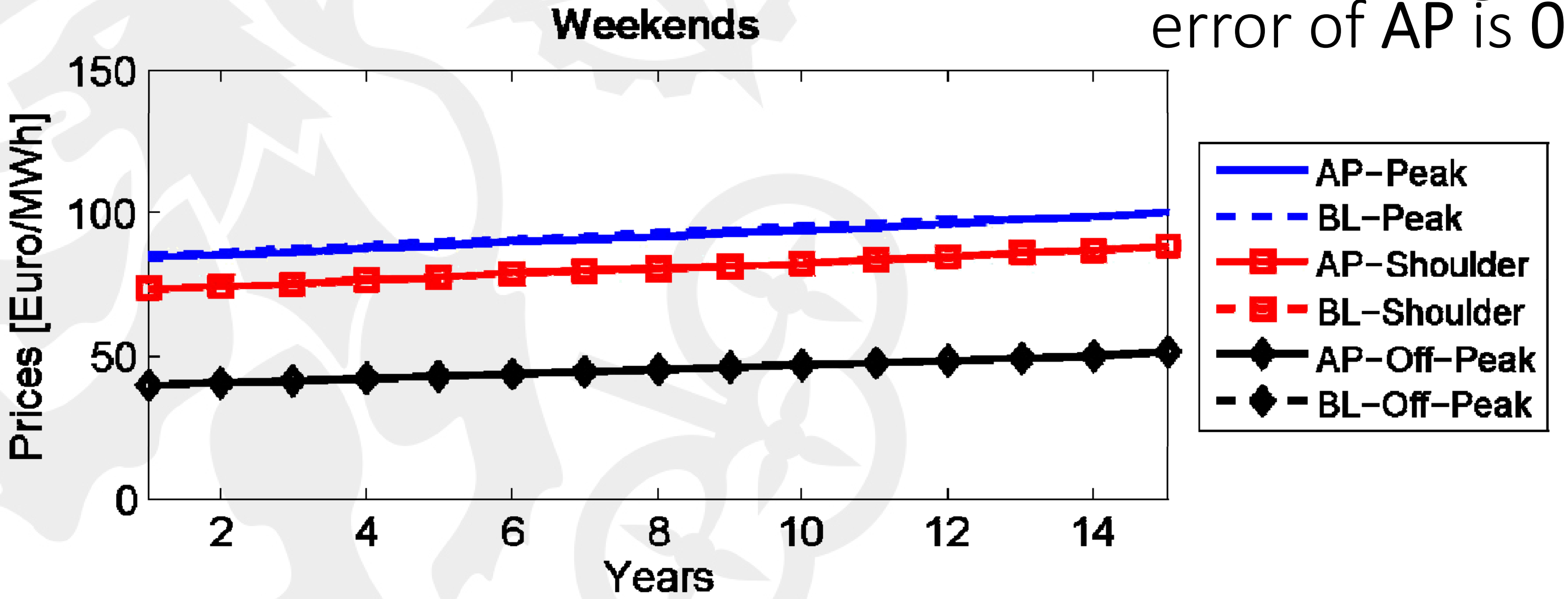




Large-Scale Example
Market Prices Comparison

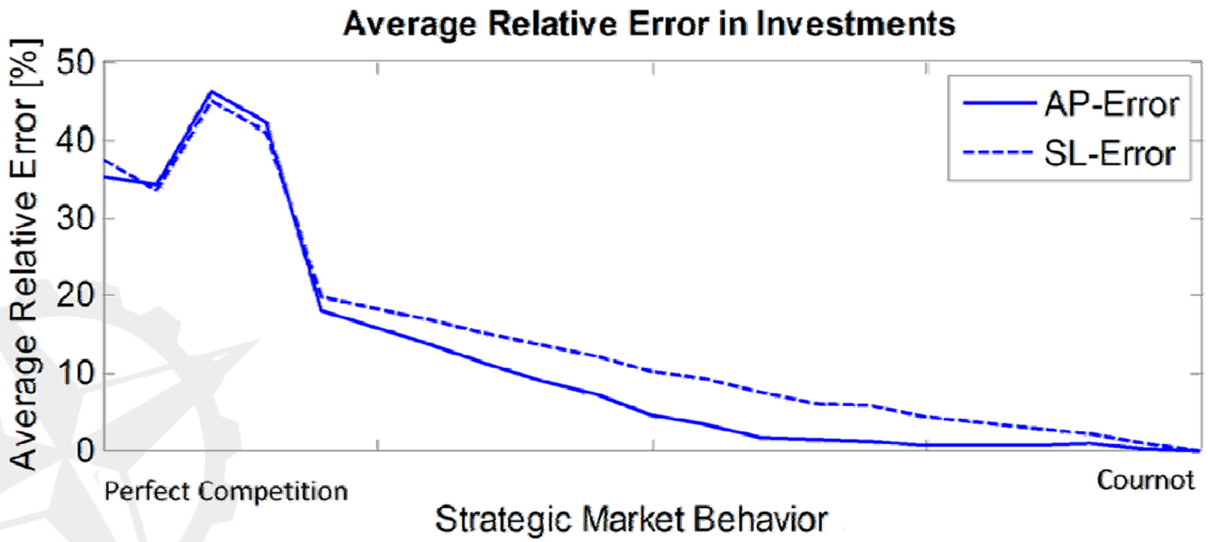


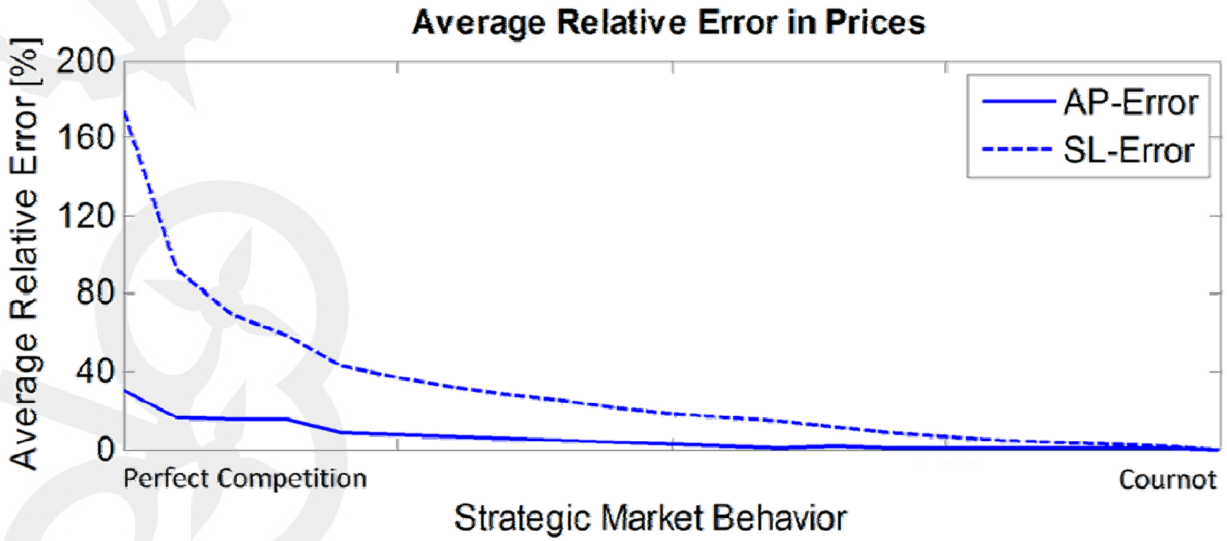
Total average relative error of AP is 0.54%.



Large-Scale Example
Sensitivity Analysis

- ★ These results are confirmed for large-scale examples:
- ★ The approximation scheme (AP) works very well when close to Cournot.
- ★ The AP still is two orders of magnitude faster than diagonalization.





Introduction & Motivation

Generation Expansion Planning (GEP)

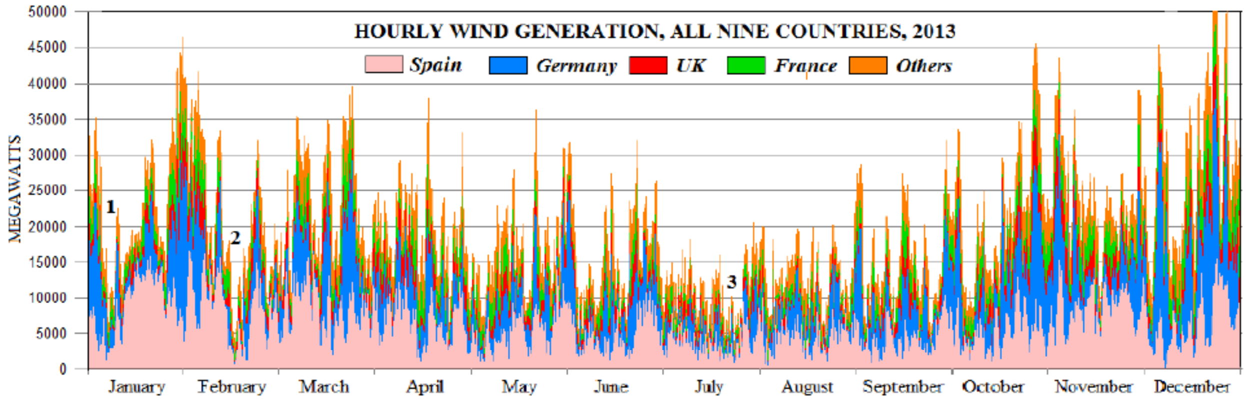
Approximating Bilevel GEP Equilibria

Ramp Bidding in Electricity Markets (work with E. Moiseeva)

Conclusions and Work in Progress

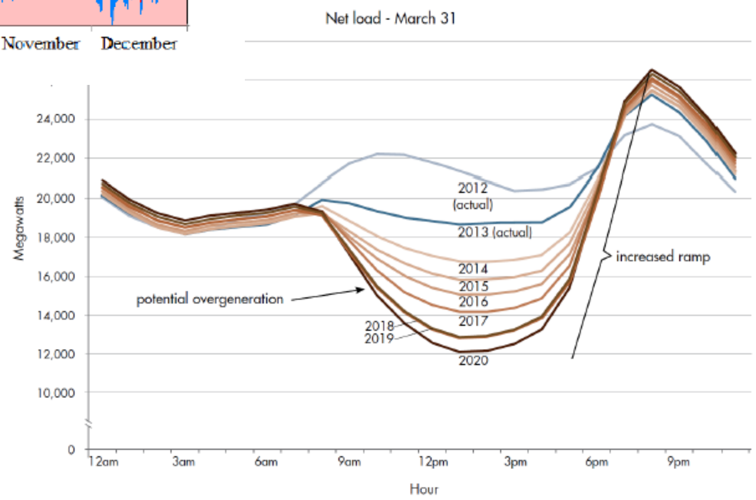
Ramp Bidding in Electricity Markets
Motivation

- Recent years have shown a rapid increase in renewable production:
  - Wind power - hard to predict and volatile



(Energy Matters)

  - Solar – increasingly low net demand during day.



(CAISO)

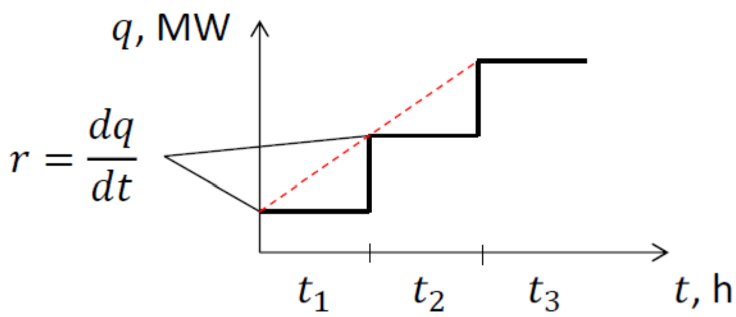
Ramp Bidding in Electricity Markets
Need for Flexibility

Flexibility expresses the capability of power system to maintain continuous service, even exposed to rapid and large swings in supply or demand

Flexibility can be offered on different levels:

- Flexibility of transmission and distribution
- Demand side flexibility
- Flexibility of generation resources

Ramp rate is how fast an electricity generator can reach a required production level, flexibility of the generator:



Ramp Bidding in Electricity Markets
Strategic Ramp Rate Bidding

We investigate (Moiseeva et al., 2015) how market design affects the strategic behavior of generators by creating two models:

- **One-stage** model - where generators choose the level of their production and ramp bids simultaneously,
- **Two-stage** model - where generators choose their ramp levels first, and compete in quantities in the second stage.

We use an illustrative example to show our findings, and analyze the strategic behavior if the market model becomes more complex.

Ramp Bidding in Electricity Markets

Comparison of SL and Bilevel

1-stage model

Choosing ramp and production levels **simultaneously**

Formulated as a **Mixed-Complementarity Problem (MCP)**

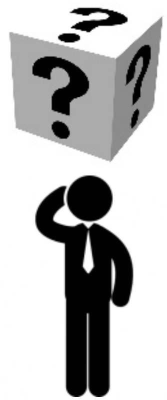
vs

2-stage model

Choosing ramp and production levels **sequentially**

Formulated as an **Equilibrium Problem with Equilibrium Constraints (EPEC)**

- In which set-up do generators exercise more market power?
- How does level of competition affect bidding strategies?
- Which set-up is better for the social welfare?



Ramp Bidding in Electricity Markets
Single-Level (SL) Ramp Rate Model

Company 1

$maximize_{q_i, r_i}$ profit

s.t. capacity limits

ramp constraints

…

Company *i*

$maximize_{q_i, r_i}$ profit

s.t. capacity limits

ramp constraints

Market equilibrium: system balance

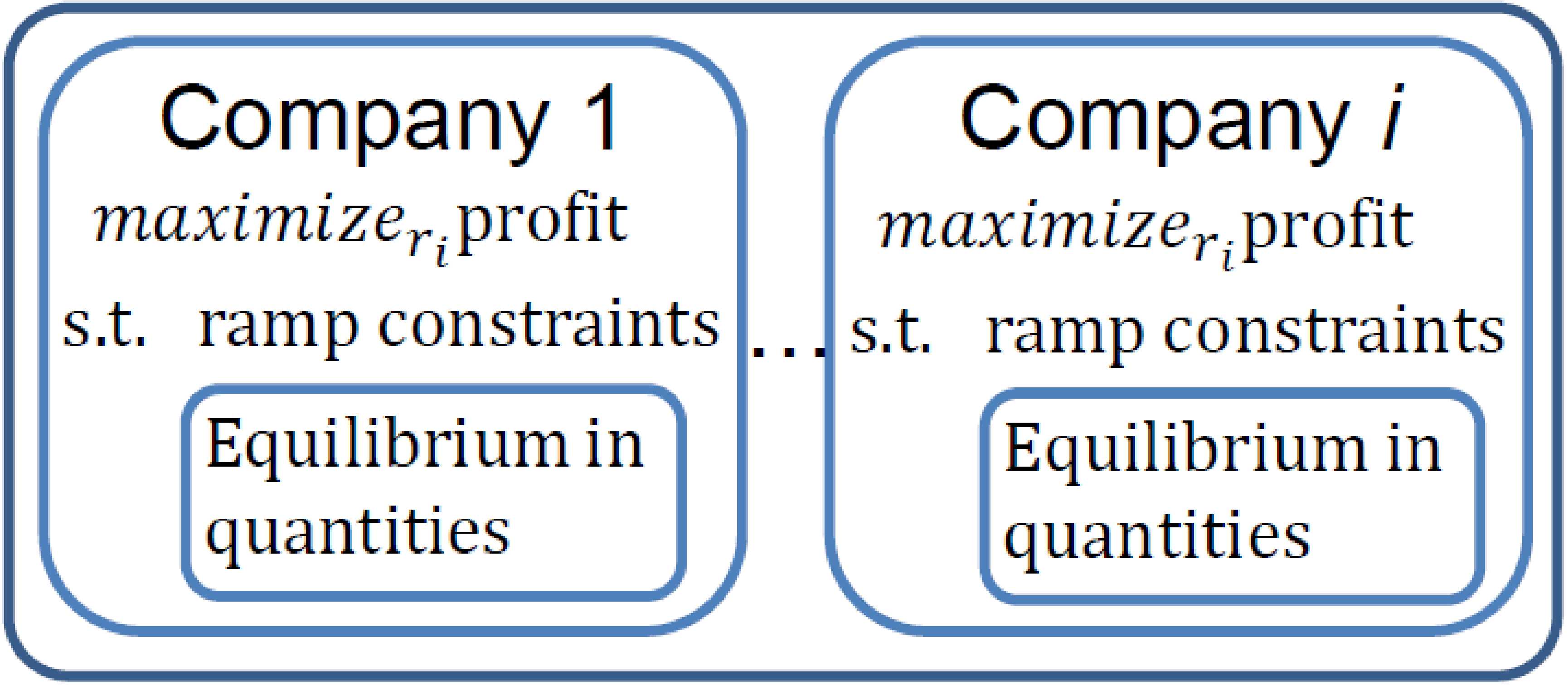
Ramp Bidding in Electricity Markets
SL Ramp Rate Formulation

$$\forall i: \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_i} = p_{t1}(q_i, q_{-i}) + p_{t2}(q_i, q_{-i}) - 2\theta q_i - 2c + \\ \qquad \mu_i^{A1} - \mu_i^{A2} = 0 \\ \frac{\partial \mathcal{L}_i}{\partial r_i} = p_{t2}(q_i, q_{-i}) - c + \mu_i^{D3} - \mu_i^{D4} = 0 \\ 0 \leq \mu_i^{A2}, \mu_i^{A1} \perp \hat{Q} \geq q_i \geq 0 \\ 0 \leq \mu_i^{D4}, \mu_i^{D3} \perp \hat{R} \geq r_i \geq 0 \end{cases}$$

$$d_{t1} = \sum_i q_i, \quad d_{t2} = \sum_i (q_i + r_i)$$

$$d_t = D_t^0 - \alpha p_t(q_i, q_{-i}) \quad \forall t.$$

Ramp Bidding in Electricity Markets
Bilevel Ramp Rate Equilibrium



Ramp Bidding in Electricity Markets
Bilevel Ramp Rate Formulation

$$\forall i : \begin{cases} \underset{r_i}{\text{maximize}} & (p_{t1}(q_i, q_{-i}) - c)q_i + (p_{t2}(q_i, q_{-i}) - c)(q_i + r_i) \\ \text{subject to:} & \hat{R} \geq r_i \geq 0 \\ & q_i \in \Omega^{LL}. \end{cases}$$

Quantity q_i is an outcome of the lower-level Ω^{LL} market equilibrium:

$$\forall i : \begin{cases} \underset{q_i}{\text{maximize}} & (p_{t1}(q_i, q_{-i}) - c)q_i + (p_{t2}(q_i, q_{-i}) - c)(q_i + r_i) \\ \text{subject to:} & \hat{Q} \geq q_i \geq 0. \end{cases}$$

The market equilibrium conditions link together the optimization problems of the generators:

$$\text{ME} : \quad d_{t1} = \sum_i q_i, \quad d_{t2} = \sum_i (q_i + r_i), \quad d_t = D_t^0 - \alpha p_t(q_i, q_{-i}) \quad \forall t.$$

Ramp Bidding in Electricity Markets
Theoretical Results

Proposition 1

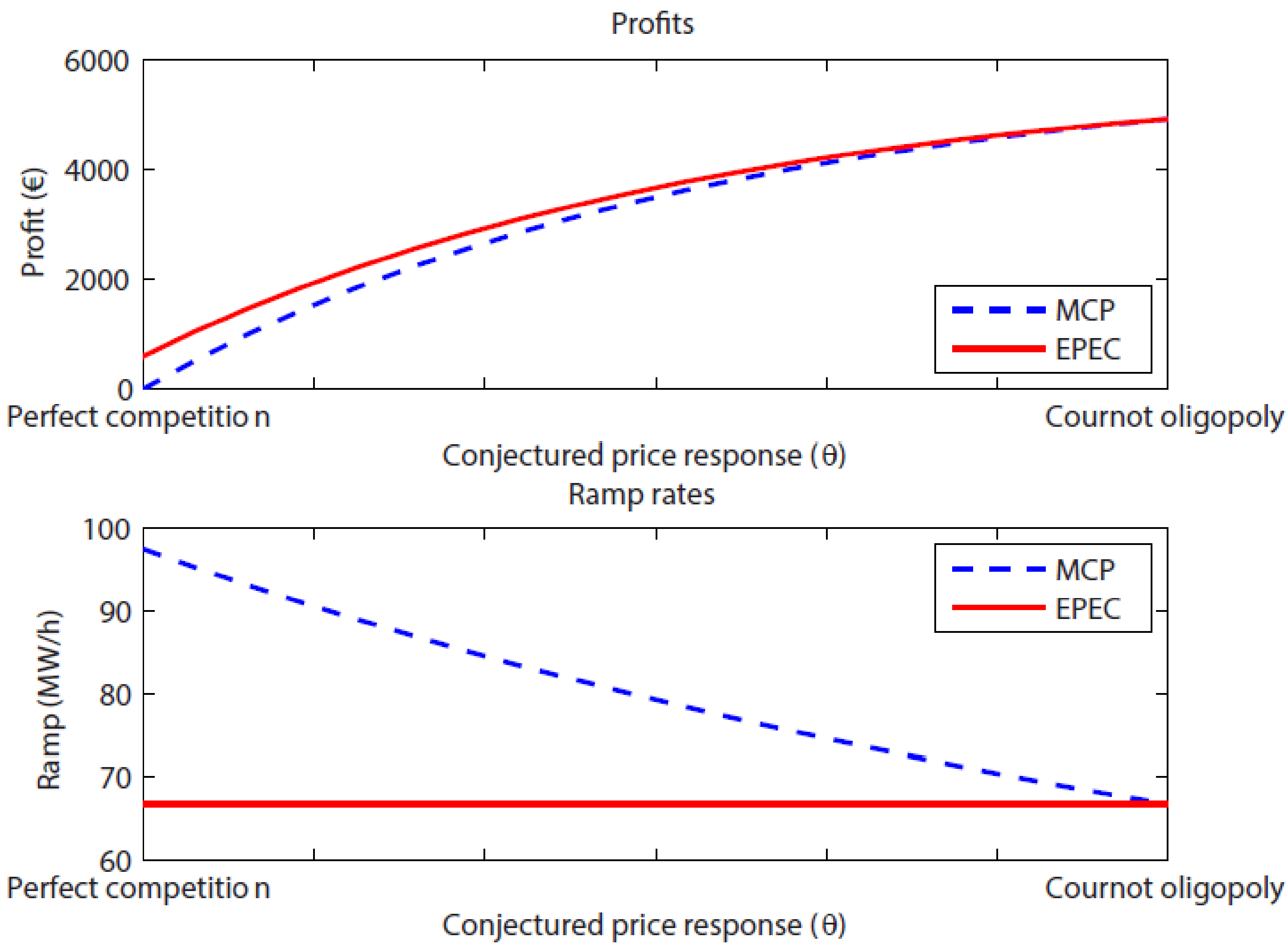
For two symmetric agents with affine cost functions and perfectly substitutable products we find that the **optimal level of flexibility** for the two-stage model is **independent from the conjectured price response parameter**, representing any market structure from the perfect competition to the Cournot oligopoly.

Proposition 2

In the one-stage model the level of flexibility offered to the market varies with the level of competition, represented by the conjectured price response:

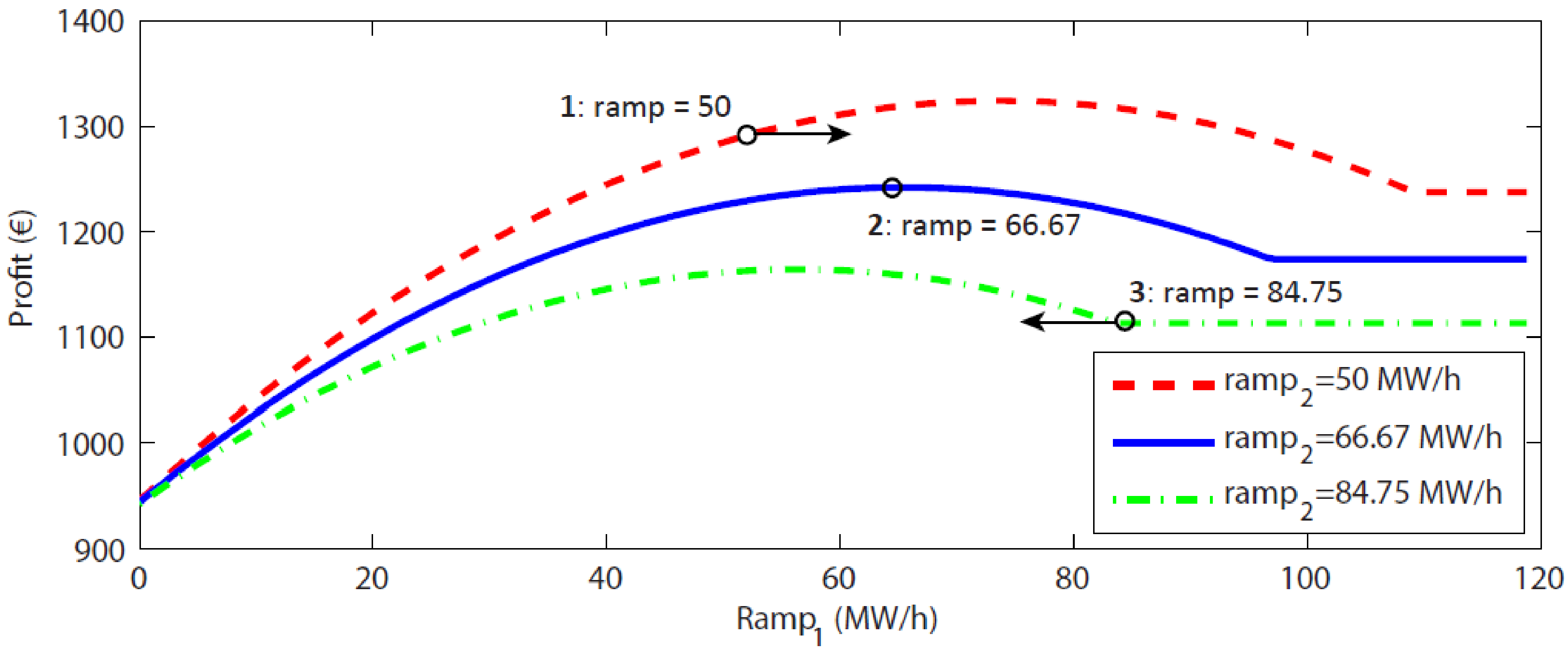
- The **levels of flexibility of single- and two-stage set-ups coincide** when the market structure approaches **Cournot**
- In the case of perfect competition the **flexibility level in a single-stage model is higher** than in a two-stage model.

Ramp Bidding in Electricity Markets
Duopoly Results (I)



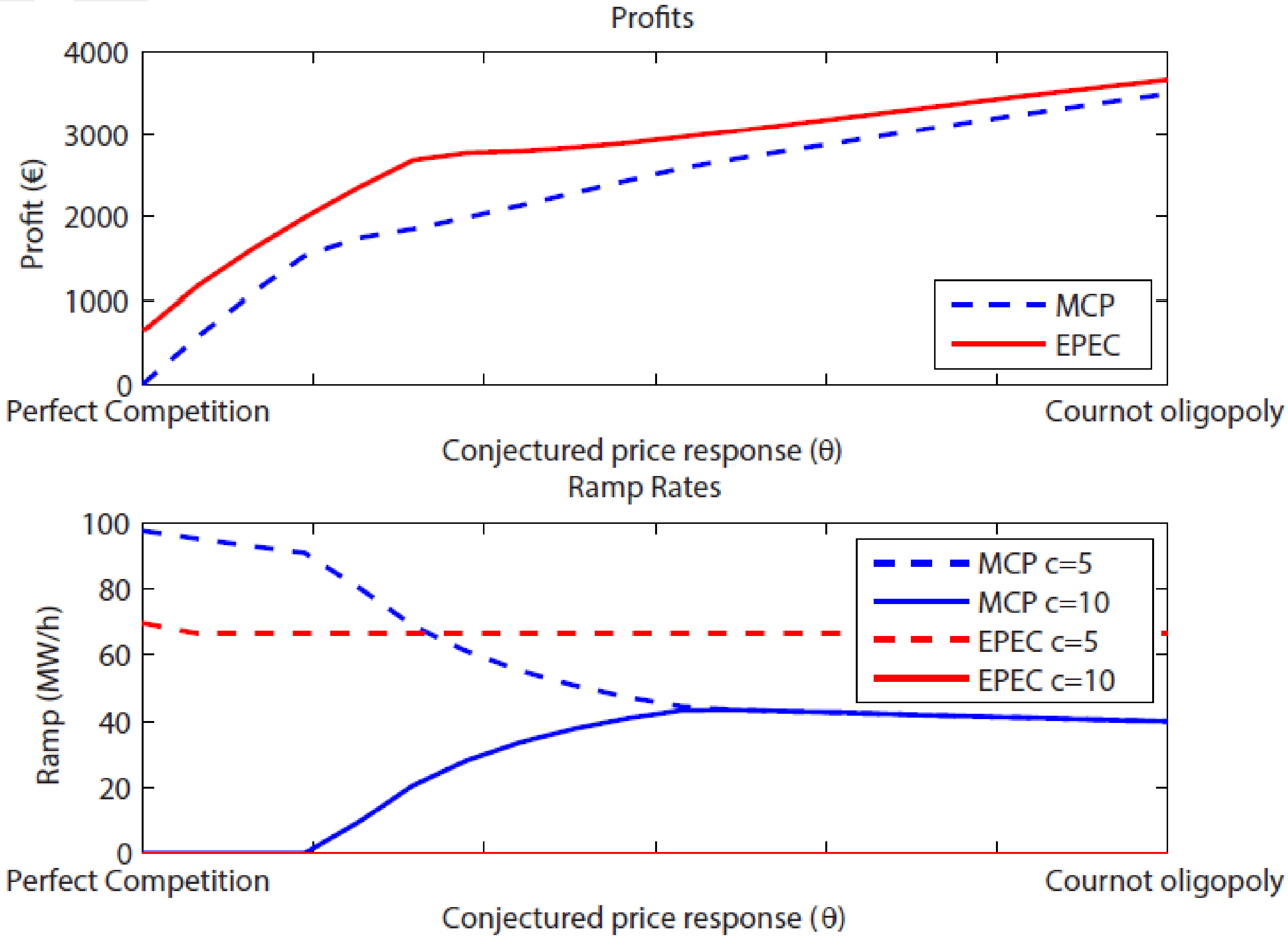
Ramp Bidding in Electricity Markets
Duopoly Results (II)

- Profit planes generated for different combinations of generators' ramp levels (1: collaborative equilibrium, 2: 2-stage model equilibrium, 3: 1-stage model equilibrium)



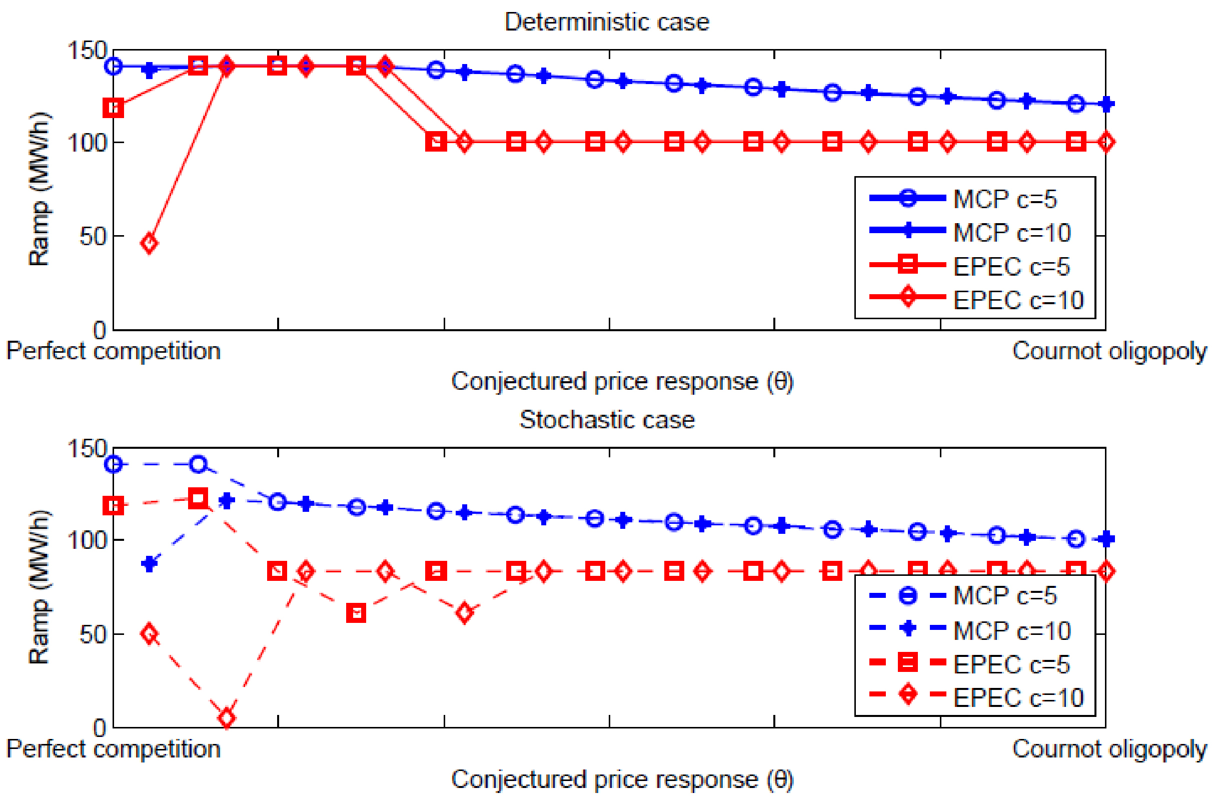
Ramp Bidding in Electricity Markets
Extensions – Generating Portfolios

- Similar trends: solution of the BL model is constant, the level of ramp in a SL model steadily decreases



Ramp Bidding in Electricity Markets
Extensions – Uncertainty

- Trends of optimal ramp levels are similar
- In certain cases we observe that generators withhold more ramp rate, when facing uncertainty!



Ramp Bidding in Electricity Markets
Summary

1. Contrary to regulatory intuition, for the markets that are more competitive than Cournot oligopolies separating the flexibility from production decisions leads to a higher level of withholding

2. In case of duopoly, when the conjectured price response corresponds to Cournot both models give the same results

3. We can observe similar trends for the extended models: portfolio bidding, bidding under uncertainty

4. New methods are needed to prevent flexibility withholding.

Introduction & Motivation

Generation Expansion Planning (GEP)

Approximating Bilevel GEP Equilibria

Ramp Bidding in Electricity Markets

Conclusions and Work in Progress

5 – Conclusions and Work in Progress

Hierarchical equilibrium models are important when analyzing liberalized electricity markets.

They provide dynamic insight that single-level models cannot capture.

Applications in generation expansion planning and ramp rate bidding have been presented.

<u>Work in progress</u>: explore games with integer variables in electricity markets (Nogales et al., 2015).

Questions…

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