Accurate global machine learning force fields for molecules with hundreds of atoms

IPAM Workshop: EMS2023

Stefan Chmiela









Force field reconstruction with ML A unique set of challenges



Classical force fields







- Learning on sets (of indistinguishable atoms)
- Invariant / equivariant outputs
- Stringent speed / accuracy requirements



Ab initio approximations to Schrödinger's equation

Atomistic modeling with ML Historical perspective



Behler 2011, Bartók et al. 2013, Hirn et al. 2013, Cohen-Welling 2016, Gilmer et al. 2017, Duvenaud et al. 2015, Schütt et al. 2017, Thomas-Smidt et al. 2018, Weiler et al. 2018, Anderson et al. 2019, Unke et al. 2021, Frank et al. 2021





Numerical analysis

- exploit model structure in training (e.g. correlations, PDE constraints)
- remove uncontrolled approximations

2018

Architecture engineering

- sophisticated filter parameterizations (e.g. YLM)
- "geometric deep learning"





Atomistic modeling with ML Efficient modeling of global force fields

All atoms interact in quantum many-body systems.





strong



Global atomic interactions

An emerging field without universal solution

TensorMol Hybrid nearsighted NN potential with screened long-range electrostatic and van der Waals physics Yao, K., Herr, J. E., Toth, D. W., Mckintyre, R. & Parkhill, J. The TensorMol-0.1 model chemistry: a neural network augmented with long-range physics. Chem. Sci. 9, 2261 (2018).

IPML

Physics-based mechanistic descriptions combined with environment-dependent ML corrections Bereau, T., DiStasio Jr, R. A., Tkatchenko, A. & Von Lilienfeld, O. A. Non-covalent interactions across organic and biological subsets of chemical space: physics-based potentials parametrized from machine learning. J. Chem. Phys. 148, 241706 (2018).

Long-distance equivariant (LODE) representation + SOAP Atom-density potential folds global structural and compositional information information into a local representation Grisafi, A. & Ceriotti, M. Incorporating long-range physics in atomic-scale machine learning. J. Chem. Phys. 151, 204105 (2019).

Fourth-generation Behler-Parinello neural network (4G-BPNN) ML-FF with non-local charge transfer correction using independent ML model Ko, T. W., Finkler, J. A., Goedecker, S., & Behler, J. (2021). A fourth-generation high-dimensional neural network potential with accurate electrostatics including non-local charge transfer. Nat. Commun., 12(1), 1-11.

SpookyNet

ML force field augmented with physically motivated corrections for long-ranged electrostatic and dispersion interactions Unke, O. T., Chmiela, S., Gastegger, M., Schütt, K. T., Sauceda, H. E., & Müller, K. R. (2021). SpookyNet: Learning force fields with electronic degrees of freedom and nonlocal effects. Nat. Commun., 12(1), 1-14.





Symmetries / conservation laws

Physical

Frank et al. 2021, Schmitz et al. 2022, Chmiela et al. 2023, Blücher et al. 2023

Physically feasible subspace is much smaller.



e.g. Noether's theorem, geometric priors etc.

Physical

Frank et al. 2021, Schmitz et al. 2022, Chmiela et al. 2023, Blücher et al. 2023





Symmetries / conservation laws

Correlation patterns in the reference data

Physical

Statistical (ML) • • • • •

Frank et al. 2021, Schmitz et al. 2022, Chmiela et al. 2023, Blücher et al. 2023





Data driven interactions

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Efficient training of constrained models

Architectural (ML)







Symmetries / conservation laws

Correlation patterns in the reference data

Physical

Statistical (ML) • • • • •

Frank et al. 2021, Schmitz et al. 2022, Chmiela et al. 2023, Blücher et al. 2023





Data driven interactions

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Efficient training of constrained models

Architectural (ML)





Correlation patterns in the reference data



Force field reconstruction with ML Representing global interactions

All atoms interact:

quadratic complexity (minimum)





Force field reconstruction with ML Representing global interactions

All atoms interact:

quadratic complexity (minimum)











"Donor-Bridge-Acceptor"

(E)-N,N-dimethyl-4-(4-nitrostyryl)aniline

All atoms need to interact to make accurate predictions.

Chmiela et al. 2023







Limit interaction length

"Donor-Bridge-Acceptor"

(E)-N,N-dimethyl-4-(4-nitrostyryl)aniline

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Chmiela et al. 2023







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"Donor-Bridge-Acceptor"

(E)-N,N-dimethyl-4-(4-nitrostyryl)aniline

All atoms need to interact to make accurate predictions.

Chmiela et al. 2023



Localization can remove important interaction-scales!





Force field reconstruction with ML The most immediate application of ML in QM

All atoms interact:

quadratic memory complexity (minimum)









Correlation matrices

Numerical challenges in training

Involves matrix diagonalization: $\alpha = (\mathbf{X}\mathbf{X}^{\mathsf{T}} + \lambda \mathbb{I})^{\mathsf{T}}$

Alternative approach: iterative optimization using n



 Krylov subspace solver (conjugate gr Dynamic learning rate: $\gamma_t = \frac{\mathbf{p}_t^{\mathsf{T}} \mathbf{y}}{\mathbf{p}_t^{\mathsf{T}} \mathbf{K}_{\lambda} \mathbf{j}}$

Conjugate optimization steps: $\mathbf{p}_{t} =$

Rank-1 decomposition of $\mathbf{K} = \sum_{i=1}^{N}$





$$Complexity:$$

$$Memory: \mathcal{O}(N^{2})$$

$$Time: \mathcal{O}(N^{3})$$

$$T^{1}\mathbf{y} = (\mathbf{L}\mathbf{L}^{\top})^{-1}\mathbf{y}$$

$$\mathbf{X}: \text{ data points}$$

$$\mathbf{K} = \mathbf{X}\mathbf{X}^{\top}: \text{ symmetric, F}$$

$$\mathbf{Memory: } \mathcal{O}(N^{3})$$

$$\mathbf{X}: \text{ data points}$$

$$\mathbf{K} = \mathbf{X}\mathbf{X}^{\top}: \text{ symmetric, F}$$

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$$\mathbf{Memory: } \mathcal{O}(N^{3})$$

$$\mathbf{X}: \mathbf{Memory: } \mathcal{O}(N^{3})$$

$$\mathbf{Memory: } \mathcal{O}(N^{3}$$







Correlation matrices

Strongly correlated reference geometries

Spectrum



d: atoms per molecule



Dominant eigenvectors





Correlation matrices

Strongly correlated reference geometries

Spectrum



d: atoms per molecule



Dominant eigenvectors



Effective preconditioners for correlation matrices Strongly correlated reference geometries

General idea:
$$\mathbf{K} = \sum_{t=1}^{k} \mathbf{u}_t \mathbf{u}_t^{\mathsf{T}} + \sum_{t=k+1}^{n} \mathbf{p}_t \mathbf{p}_t^{\mathsf{T}}$$

 $\overrightarrow{\mathsf{Precond.}}$
 $\overrightarrow{\mathsf{CG}}$ decomp.
 $\overline{\mathbf{K}} = \mathbf{P}^{-1}\mathbf{K}$ of $\overline{\mathbf{K}}$









Effective preconditioners for correlation matrices Preconditioning methods (MD17 Aspirin)

Preconditioner dim.: k = 50



Chmiela et al. 2023, Blücher et al. 2023

Incomplete Cholesky

Leverage score sampling

Effective preconditioners for correlation matrices Strongly correlated reference geometries

General idea:
$$\mathbf{K} = \sum_{t=1}^{k} \mathbf{u}_t \mathbf{u}_t^{\top} + \sum_{t=k+1}^{n} \mathbf{p}_t \mathbf{p}_t^{\top}$$

Precond.
 $\mathbf{\bar{K}} = \mathbf{P}^{-1}\mathbf{K}$ of $\mathbf{\bar{K}}$



Chmiela et al. 2023, Blücher et al. 2023





Woodbury identity: $\mathbf{P}^{-1} = \lambda^{-1} \left[\mathbf{I} - \mathbf{K}_{mk} \left(\lambda \mathbf{K}_{kk} + \mathbf{K}_{mk}^{\mathsf{T}} \mathbf{K}_{mk} \right)^{-1} \mathbf{K}_{mk}^{\mathsf{T}} \right]$

> Stable decomposition into Cholesky factors, indirectly using $\mathbf{K}_{mk}^{\mathsf{T}}\mathbf{K}_{mk} = \mathbf{R}^{\mathsf{T}}\mathbf{Q}^{\mathsf{T}}\mathbf{Q}\mathbf{R} = \mathbf{Q}\mathbf{Q}^{\mathsf{T}}$



- 0



Global force fields with hundreds of atoms A new frontier for machine learning



Conjugated system with Examples: delocalized electrons.

> Chmiela, S., Vassilev-Galindo, V., Unke, O. T., Kabylda, A., Sauceda, H. E., Tkatchenko, A., & Müller, K. R. (2023). Accurate global machine learning force fields for molecules with hundreds of atoms. Science Advances, 9(2), eadf0873.





Long timescale (PI)MD of supramolecular complexes







Efficient contraction of constrained models



Differential constraints in ML models with AD Stronger inductive biases

Differential equations: $\mathcal{L}u(\mathbf{x}) = \mathbf{f}(\mathbf{x})$, where $\mathscr{L} = \sum a_i \mathscr{D}^j$ is a finite linear combination of differential operators \mathscr{D}^j or order *n*. $|j| \leq n$

- PDE's describe causal systems with interactions
- Learn from few examples and generalize

Challenges:

- Tedious manual implementation: "one new constraint per paper"
- Constraints often increase model complexity: e.g. $\mathscr{L} = \nabla_{\mathbf{x}}$

Algorithmic differentiation: Express operators in terms of JVPs $\mathbf{J}_{\mu}(\mathbf{x})\mathbf{v}$ and VJPs $\mathbf{J}_{\mu}^{\top}(\mathbf{x})\mathbf{w}$ at $\mathcal{O}(C_{\mu})$!

Example: Hessian-vector products in only $\mathcal{O}(C_{\mu})!$

Schmitz, N. F., Müller, K. R., & Chmiela, S. (2022). Algorithmic Differentiation for Automated Modeling of Machine Learned Force Fields. The Journal of Physical Chemistry Letters, 13(43), 10183-10189.

 C_{u} : time complexity of function *u*



Differential constraints in ML models with AD $\mathbf{f}(\mathbf{x}) \sim \mathscr{GP}\left(\mu(\mathbf{x}), k(\mathbf{x}, \mathbf{x}')\right)$ Gaussian process example

Constrained GP: $\mathbf{f}(\mathbf{x}) \sim \mathscr{GP}(\mathscr{L}\mu(\mathbf{x}), \mathscr{L}_{\mathbf{x}} \otimes \mathscr{L}_{\mathbf{x}'}^{\mathsf{T}}k(\mathbf{x}, \mathbf{x}'))$ mean prior 🗕

Example: $\mathscr{L} = (1, \nabla, \nabla^2)$

Constraints: function values, gradients & Hessians

Schmitz et al. 2022

covariance prior



Full instantiation of kernel can be avoided with AD: $\mathbf{f}(\mathbf{x}) = \sum \mathscr{L}_{\mathbf{x}} \left[\mathscr{L}_{\mathbf{x}_{i}}^{\mathsf{T}} \alpha_{i} k\left(\mathbf{x}, \mathbf{x}_{i}\right) \right]$



Differential constraints in ML models with AD Gaussian process example

Constrained GP: $\mathbf{f}(\mathbf{x}) \sim \mathscr{GP}(\mathscr{L}\mu(\mathbf{x}), \mathscr{L}_{\mathbf{x}} \otimes \mathscr{L}_{\mathbf{x}'}^{\mathsf{T}}k(\mathbf{x}, \mathbf{x}'))$

Factor N reducing in complexity after contraction! \neg

 $\mathcal{L}_{\mathbf{x}} \quad \mathcal{L}_{\mathbf{x}'} \mid \left[\mathcal{L}_{\mathbf{x}} \otimes \mathcal{L}_{\mathbf{x}'}^{\top} k(\mathbf{x}, \mathbf{x}') \right]$ $| \operatorname{Jac}_{\mathbf{x}}(\operatorname{Grad}_{\mathbf{x}'}(k))^{\top}$ ∇ ∇ $|\operatorname{Jac}_{\mathbf{x}}(\operatorname{Hess}_{\mathbf{x}'}(k))^{\top}$ ∇ $abla^2$ $\operatorname{Hess}_{\mathbf{x}}(\operatorname{Hess}_{\mathbf{x}'}(k))^{\top}$ ∇^2 $abla^2$

Schmitz, N. F., Müller, K. R., & Chmiela, S. (2022). Algorithmic Differentiation for Automated Modeling of Machine Learned Force Fields. The Journal of Physical Chemistry Letters, 13(43), 10183-10189.

$\cos t$	$\mathcal{L}_{\mathbf{x}}\left[\mathcal{L}_{\mathbf{x}'}^{ op} oldsymbol{lpha}\left[k(\mathbf{x},\mathbf{x}') ight] ight]$	cost
NC_k	$\operatorname{Grad}_{\mathbf{x}'}(\operatorname{Grad}_{\mathbf{x}'}(k)\boldsymbol{\alpha})$	C_k
N^2C_k	$\operatorname{Grad}_{\mathbf{x}}(\operatorname{Hess}_{\mathbf{x}'}(k)\boldsymbol{lpha})$	NC_k
N^3C_k	$\operatorname{Hess}_{\mathbf{x}'}(\operatorname{Hess}_{\mathbf{x}'}(k)\boldsymbol{\alpha})$	N^2C_k

Cost of evaluating the (scalar) base kernel.

Differential constraints in ML models with AD Gaussian process example

Constrained GP: $\mathbf{f}(\mathbf{x}) \sim \mathscr{GP}(\mathscr{L}\mu(\mathbf{x}), \mathscr{L}_{\mathbf{x}} \otimes \mathscr{L}_{\mathbf{x}'}^{\mathsf{T}}k(\mathbf{x}, \mathbf{x}'))$

$\mathcal{L}_{\mathbf{x}}$	$\mathcal{L}_{\mathbf{x}'}$	$\left \left[\mathcal{L}_{\mathbf{x}} \otimes \mathcal{L}_{\mathbf{x}'}^{ op} k(\mathbf{x},\mathbf{x}') ight] ight.$	$\cos t$	$\int \mathcal{L}_{\mathbf{x}} \left[\mathcal{L}_{\mathbf{x}'}^{\top} \boldsymbol{\alpha} \left[k(\mathbf{x}, \mathbf{x}') \right] \right]$
$egin{array}{c} abla \ abl$	$egin{array}{c} abla^2 \ abla^2 \end{array} \ abla^2 \end{array}$	$\begin{vmatrix} \operatorname{Jac}_{\mathbf{x}}(\operatorname{Grad}_{\mathbf{x}'}(k))^{\top} \\ \operatorname{Jac}_{\mathbf{x}}(\operatorname{Hess}_{\mathbf{x}'}(k))^{\top} \\ \operatorname{Hess}_{\mathbf{x}}(\operatorname{Hess}_{\mathbf{x}'}(k))^{\top} \end{vmatrix}$	NC_k N^2C_k N^3C_k	$\begin{bmatrix} \operatorname{Grad}_{\mathbf{x}}(\operatorname{Grad}_{\mathbf{x}'}(k)\mathbf{a})\\ \operatorname{Grad}_{\mathbf{x}}(\operatorname{Hess}_{\mathbf{x}'}(k)\mathbf{a})\\ \operatorname{Hess}_{\mathbf{x}}(\operatorname{Hess}_{\mathbf{x}'}(k)\mathbf{a}) \end{bmatrix}$
				((/

Gradient constraints in GPs:

Same complexity class as the unconstrained model!

Schmitz, N. F., Müller, K. R., & Chmiela, S. (2022). Algorithmic Differentiation for Automated Modeling of Machine Learned Force Fields. The Journal of Physical Chemistry Letters, 13(43), 10183-10189.



Differential constraints in ML models with AD $\mathbf{f}(\mathbf{x}) \sim \mathscr{GP}\left(\mathscr{L}\mu(\mathbf{x}), \mathscr{L}_{\mathbf{x}} \otimes \mathscr{L}_{\mathbf{x}'}^{\mathsf{T}} k\left(\mathbf{x}, \mathbf{x}'\right)\right)$ Gaussian process example



2D Rosenbrock function: u(x)

Schmitz, N. F., Müller, K. R., & Chmiela, S. (2022). Algorithmic Differentiation for Automated Modeling of Machine Learned Force Fields. The Journal of Physical Chemistry Letters, 13(43), 10183-10189.

$$(x_1, x_2) = (1 - x_1)^2 + 100(x_2 - x_1^2)^2$$



Differential constraints in ML models with AD Combining differential equation constraints in GP's



Schmitz, N. F., Müller, K. R., & Chmiela, S. (2022). Algorithmic Differentiation for Automated Modeling of Machine Learned Force Fields. The Journal of Physical Chemistry Letters, 13(43), 10183-10189.

Solving Laplace's equation $\Delta u(x) = 0$ on the unit disk

Neumann boundary condition ∇u(x) ⋅ n(x) = cos(5φ)
φ: radial angle of x
n(x): boundary normal vector

Differential constraints in ML models with AD Combining differential equation constraints in GP's



Schmitz, N. F., Müller, K. R., & Chmiela, S. (2022). Algorithmic Differentiation for Automated Modeling of Machine Learned Force Fields. The Journal of Physical Chemistry Letters, 13(43), 10183-10189.

vave equation
$$\Box u(x,t) = 0$$
, $\Box = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}$

- Dirichlet boundary conditions: u(0,t) = u(1,t) = 0, u(x,0) = x(1 - x)
- Initial Neumann boundary condition: $\partial_t u(x,0) = 0$



Interactions from self-attention



-
$$\alpha_{ij} = \langle \boldsymbol{v}_i, \boldsymbol{k}_j \rangle$$

Self-attention mechanism Basic definition

 $\mathbf{q}_i = \mathbf{W}_q \mathbf{x}_i$ $\mathbf{k}_i = \mathbf{W}_k \mathbf{x}_i$ $\mathbf{v}_i = \mathbf{W}_v \mathbf{x}_i$ "queries" "keys" "values"

x: atom embeddings W: learnable weights

 $\mathbf{A} = \text{Attention}(\mathbf{Q}, \mathbf{K}, \mathbf{V}) = \text{softmax}\left(\frac{\mathbf{Q}\mathbf{K}^T}{\sqrt{d}}\right)\mathbf{V}$

Q, **K**, **V**: query, key and value embeddings for each atom *d*: embedding dimension

Frank, Thorben et al. "GeoPaTra: An Equivariant Transformer for Atomic Interactions on Arbitrary Length-Scales", 2021



Interactions from self-attention Neural network architecture



Frank, Thorben et al. "GeoPaTra: An Equivariant Transformer for Atomic Interactions on Arbitrary Length-Scales", 2021

Emerging non-linear couplings between atoms Attention coefficients vs. bond distances



Frank, Thorben et al. "GeoPaTra: An Equivariant Transformer for Atomic Interactions on Arbitrary Length-Scales", 2021

Want to know more? Articles, datasets & code

sGDML Symmetric Gra	adient Domain Machine Learning	<> Code	↓, Datasets
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■ Articles	Method Software Analysis Machine Learning of Accurate E Force Fields Chmiela, S., Tkatchenko, A., Sauceda, I	Applications Energy-Conservin	Reviews g Molecular hütt, K. T.,



$\longleftarrow MD22 \, benchmark \, dataset$

- four major classes of biomolecules and supramolecules
- up to 370 atoms



Want to know more? Articles, datasets & code



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209	22,3632	6.3800	Model Training Lap

Upload your dataset and let us do the training.



Parametrize your own datasets and use them as a force field.

OR



