

# Coarse-graining for classical and quantum systems

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Einstein Stiftung Berlin  
Einstein Foundation Berlin





Wangfei

Tim



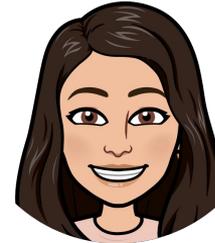
Cecilia



Jacopo



Andrea

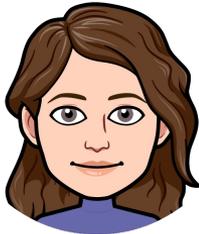


Iryna



Nick

Daria



Klara



Clark



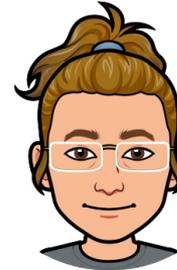
Felix



David



Arvid



Aldo

All models are wrong,  
but some are useful.

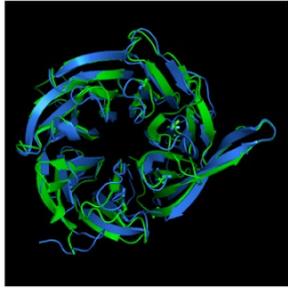
George E. P. Box

quartzfancy

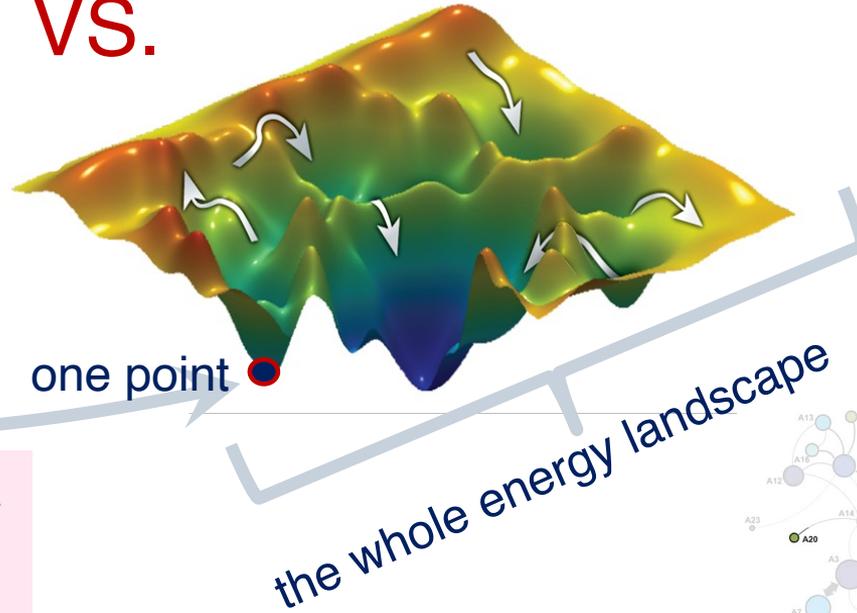
# Ultimate goal

AlphaFold2@DeepMind  
predicts  
static structures  
from sequences

Structures:  
Ground truth (green)  
Predicted (blue)

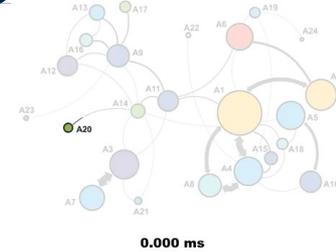


VS.

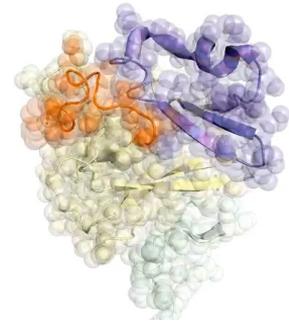


one point

the whole energy landscape



0.000 ms

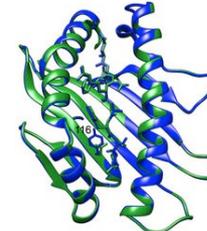
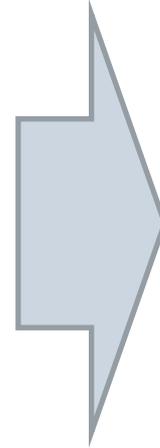
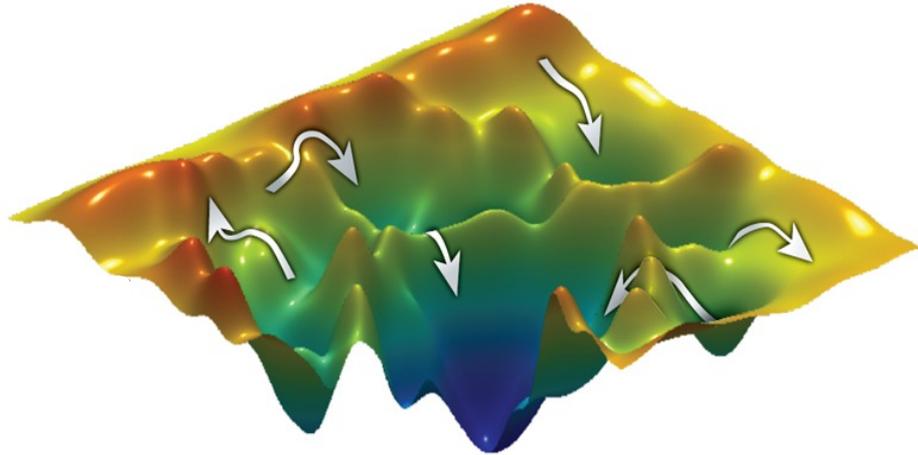


**AlphaFold: a solution to a 50-year-old grand challenge in biology**

Biological function requires dynamics

# Ultimate goal

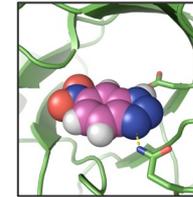
the whole energy landscape



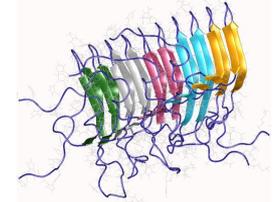
Vaccines



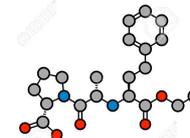
Antibodies



Enzymes

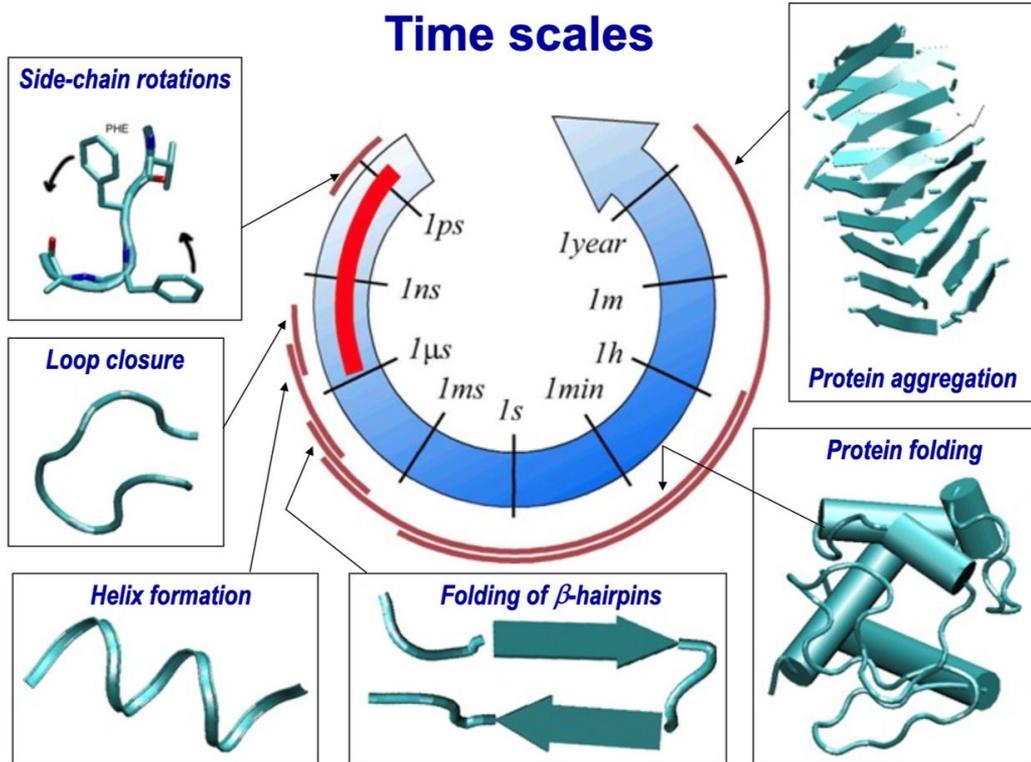


Amyloids



Drugs

# Main challenge: wide range of spatiotemporal scales



## Coarse-graining idea:

Can we design a **minimalist** model speeding up the simulations significantly while preserving the relevant physics?

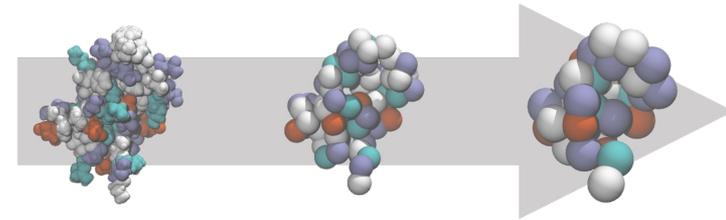
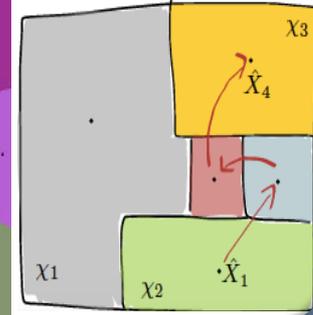
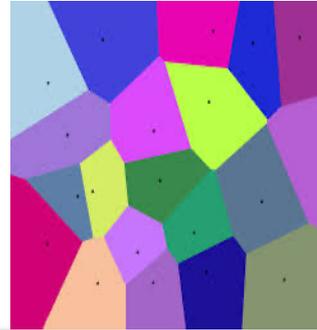


Figure from J.E. Shea

# Coarse-Graining

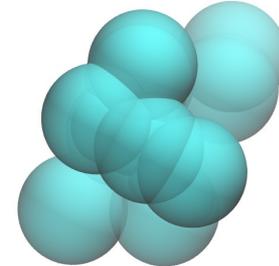
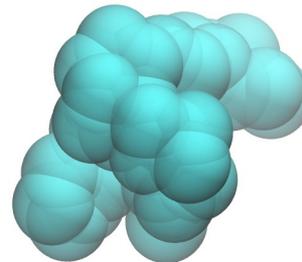
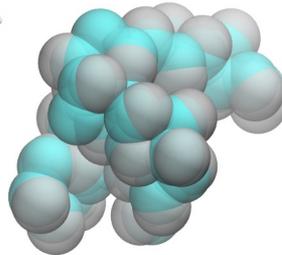
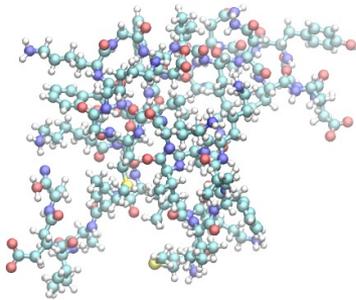
$\mathbb{R}^{3N}$

coarse-graining in conformation space



$\mathbb{R}^3$

coarse-graining in structural space





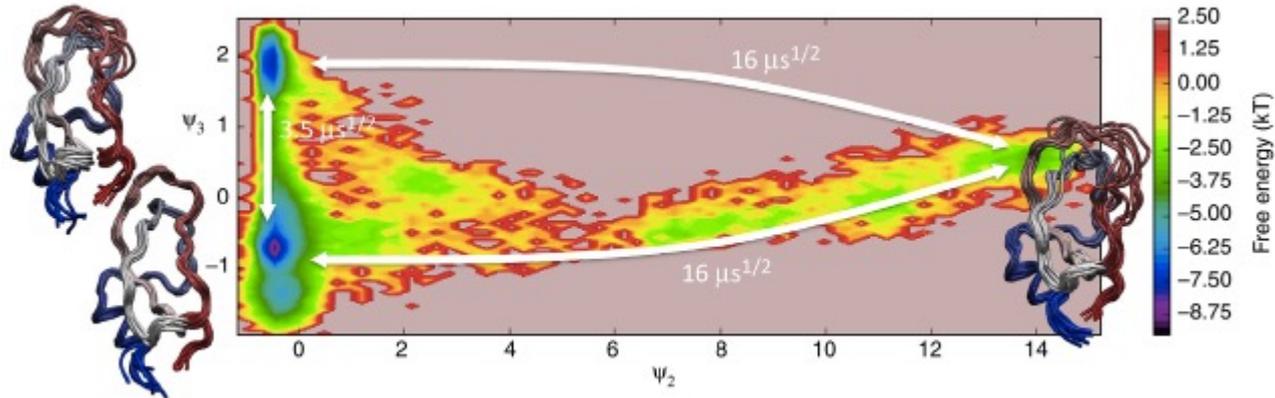
Available online at [www.sciencedirect.com](http://www.sciencedirect.com)

ScienceDirect

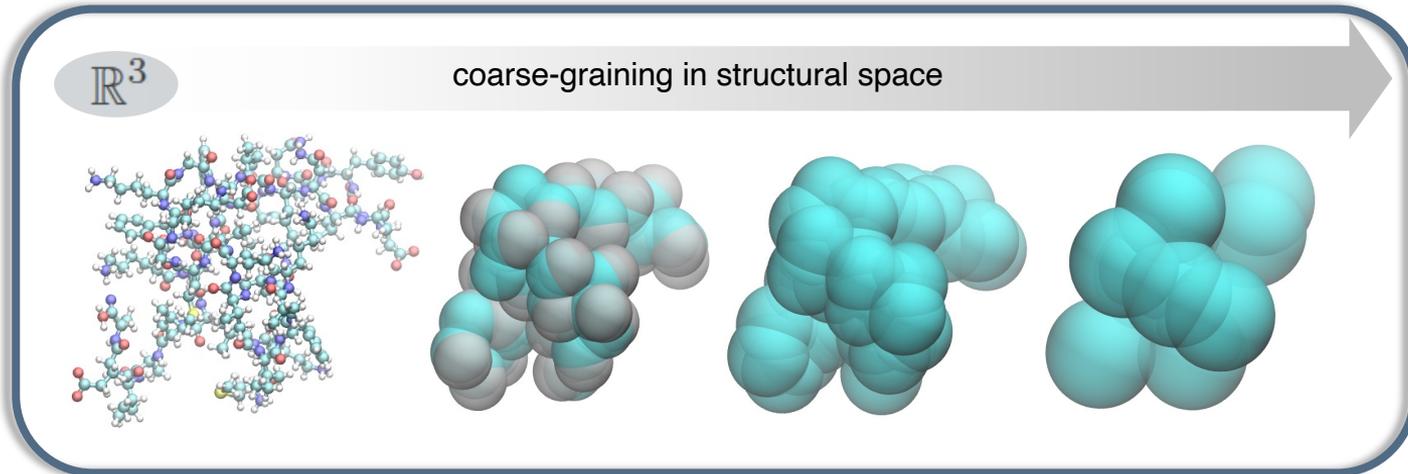
Current Opinion in  
Structural Biology

## Collective variables for the study of long-time kinetics from molecular trajectories: theory and methods

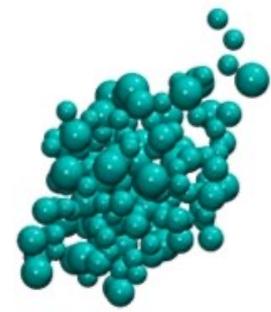
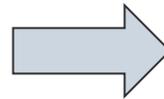
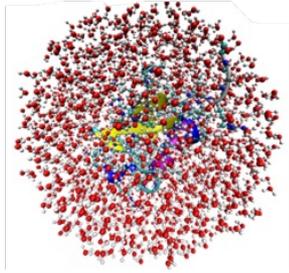
Frank Noé<sup>1</sup> and Cecilia Clementi<sup>2</sup>



# Coarse-Graining

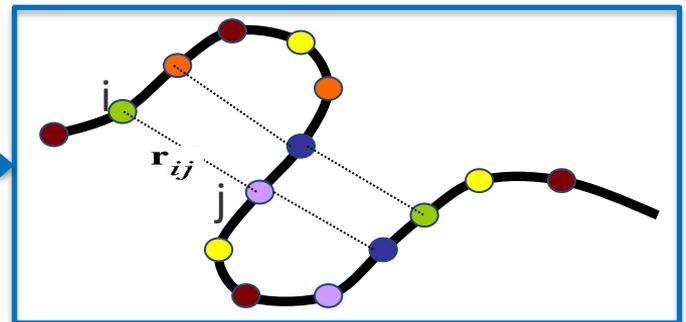
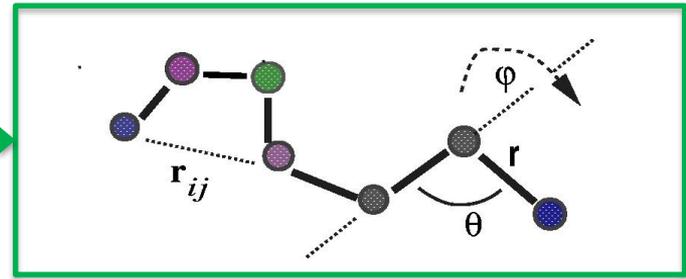


# CG model = mapping + potential

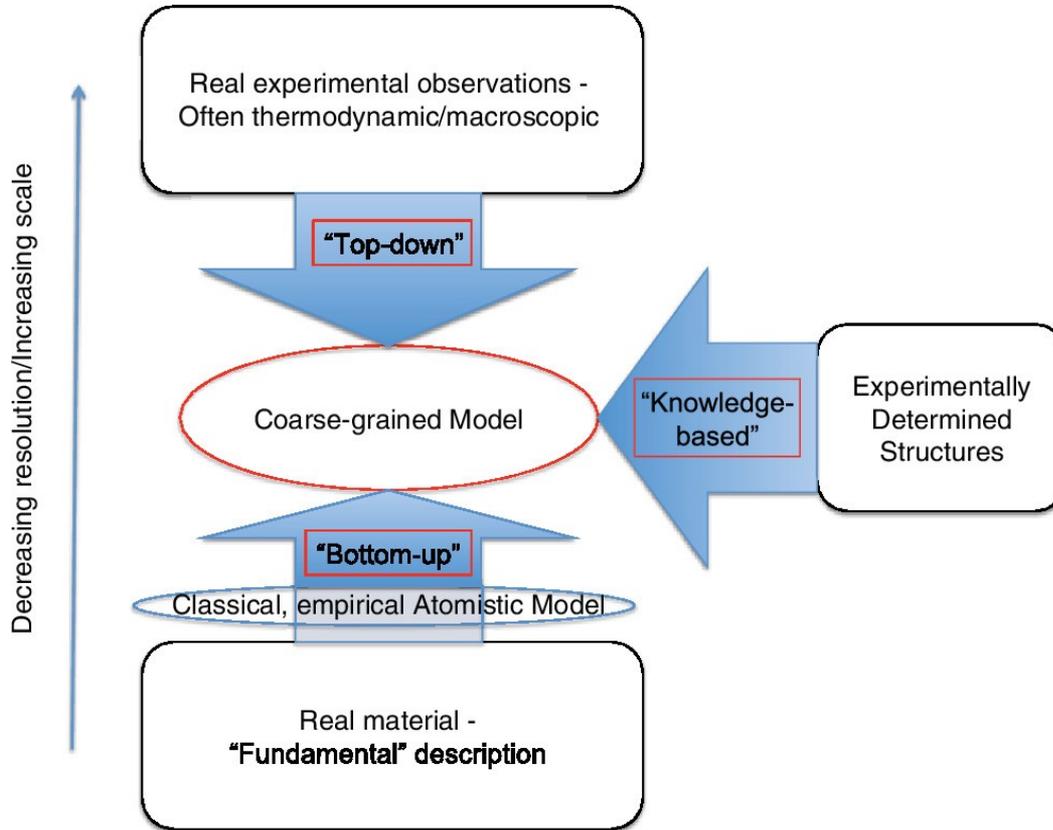


$$\mathbf{H} = \sum_{\text{bonds}} K_r (r - r_0)^2 + \sum_{\text{angles}} K_\theta (\theta - \theta_0)^2 + \sum_{\text{dihedrals}} K_d (1 + \cos(n(\phi - \phi_0))) +$$

$$\sum_{i,j} \epsilon_{ij} \left[ 5 \left( \frac{\sigma_{ij}}{r_{ij}} \right)^{12} - 6 \left( \frac{\sigma_{ij}}{r_{ij}} \right)^{10} \right]$$

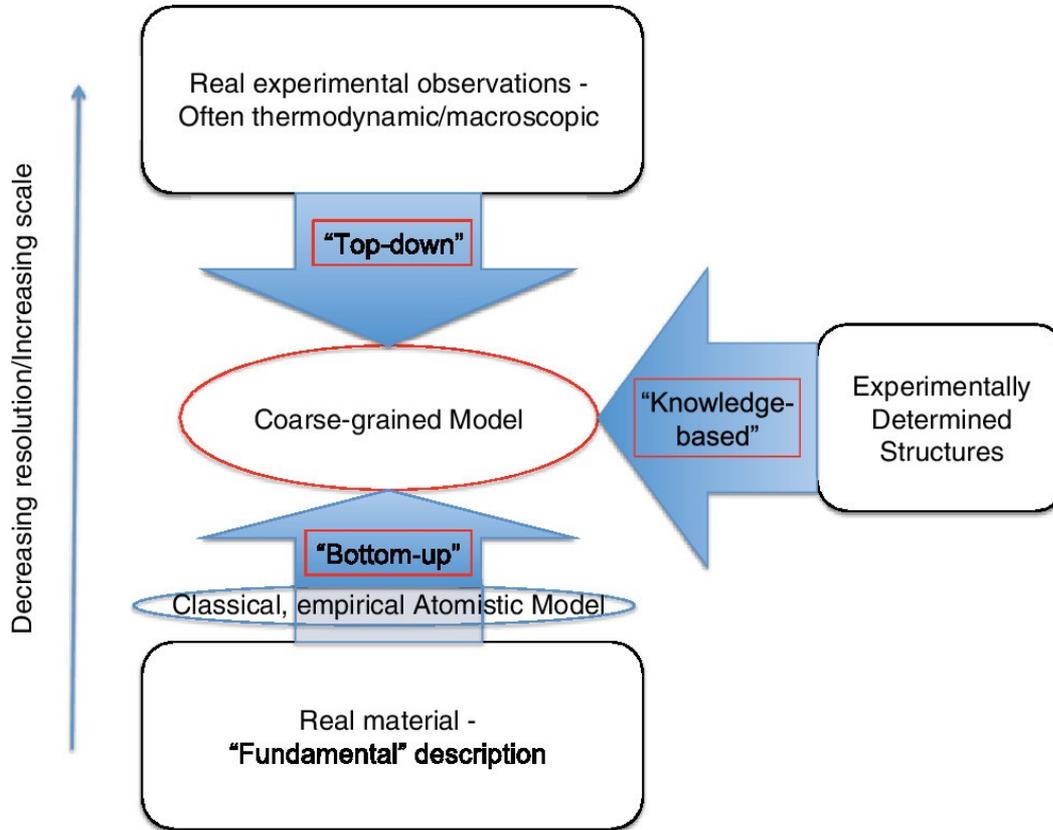


# CG models: which properties to preserve?



1. Learning CG models from all-atom simulations
2. Incorporating nuclear quantum effects into all-atom simulation
3. Learning CG models from experimental data

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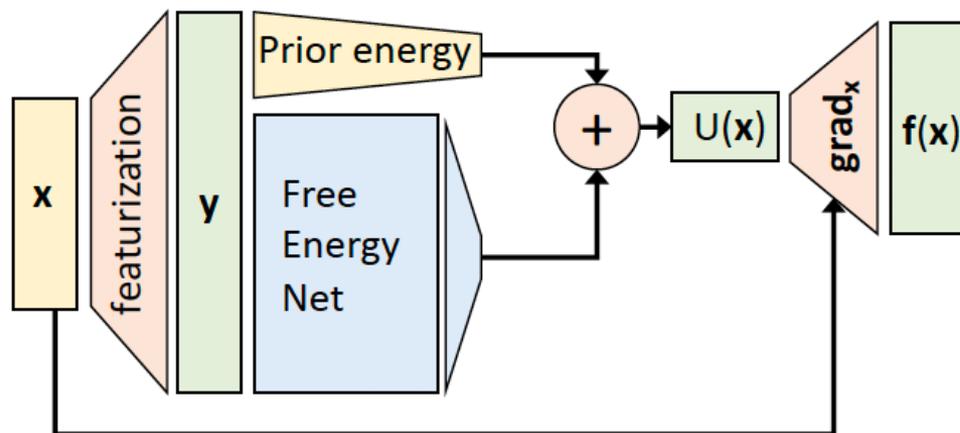


1. Learning CG models from all-atom simulations
2. Incorporating nuclear quantum effects into all-atom simulation
3. Learning CG models from experimental data

# Force-matching

Enforcing the thermodynamic consistency is equivalent to minimize the force matching error:

$$\chi^2(\boldsymbol{\theta}) = \left\langle \|\xi(\mathbf{F}(\mathbf{r})) + \nabla U(\xi(\mathbf{r}); \boldsymbol{\theta})\|^2 \right\rangle_{\mathbf{r}}$$



S. Izvekov and G. A. Voth *J. Phys. Chem. B* 109:2469–2473 (2005)

W. G. Noid, et al, *J. Chem. Phys.* 128, 244114 (2008)

J. Wang, S. Olsson, C. Wehmeyer, A. Perez, N.E. Charron, G. de Fabritiis, F. Noé, C. Clementi, *ACS Cent. Sci.* 5, 755-767 (2019)

# Force-matching

Enforcing the thermodynamic consistency is equivalent to minimizing the **force matching error**:

$$\chi^2(\theta) = \left\langle \left\| \xi_f \mathbf{F}(\mathbf{r}) + \nabla U(\xi_r(\mathbf{r}); \theta) \right\|^2 \right\rangle_{\mathbf{r}}$$

**Instantaneous all-atom force** (points to  $\mathbf{F}(\mathbf{r})$ )

**Model parameters** (points to  $\theta$ )

**CG force mapping** (points to  $\xi_f$ )

**CG configurational mapping** (points to  $\xi_r(\mathbf{r})$ )

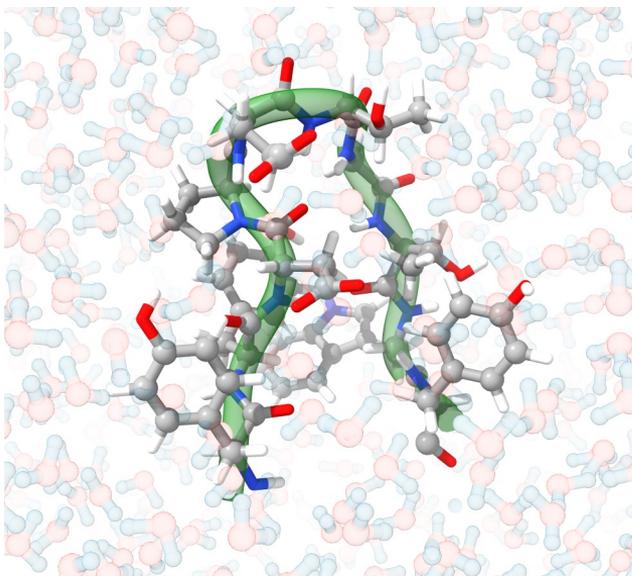
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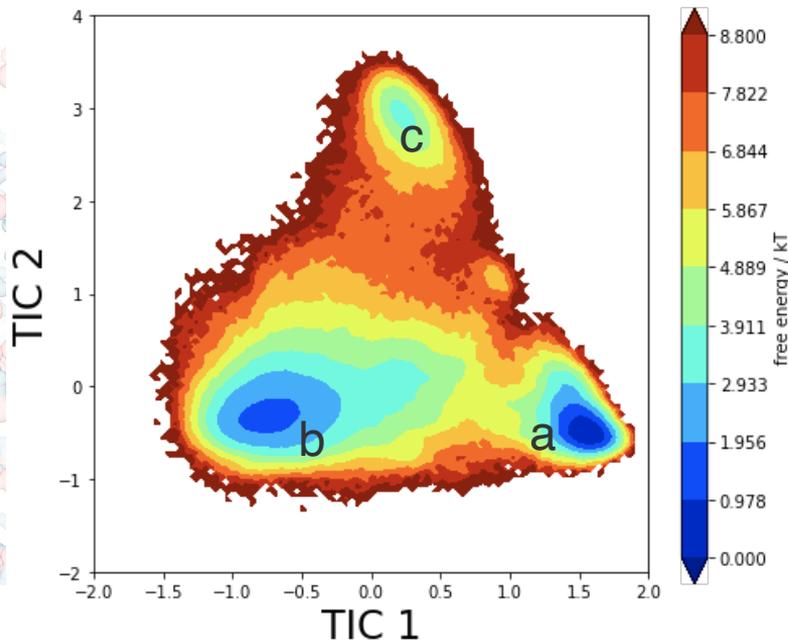
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# Test system: Chignolin folding/unfolding/misfolding

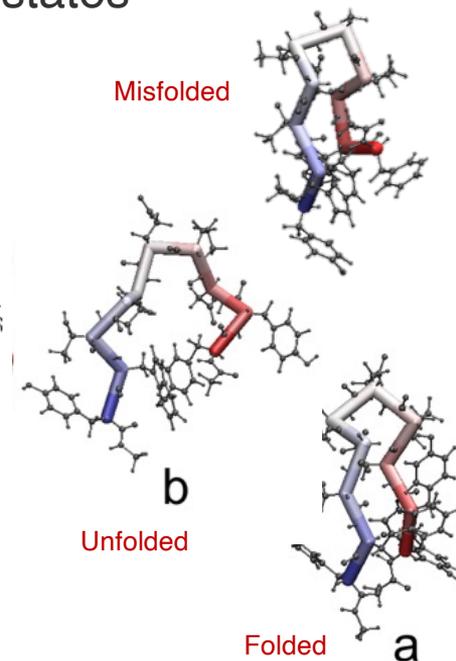
atomistic simulation  
in explicit solvent



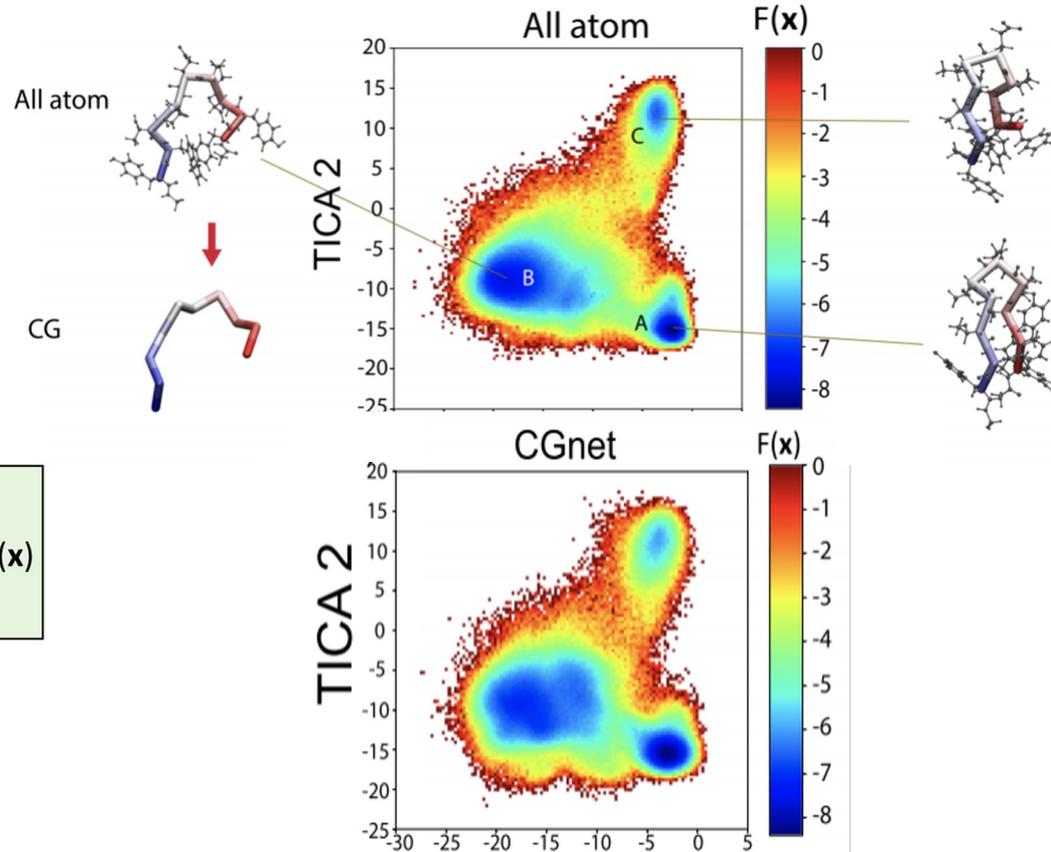
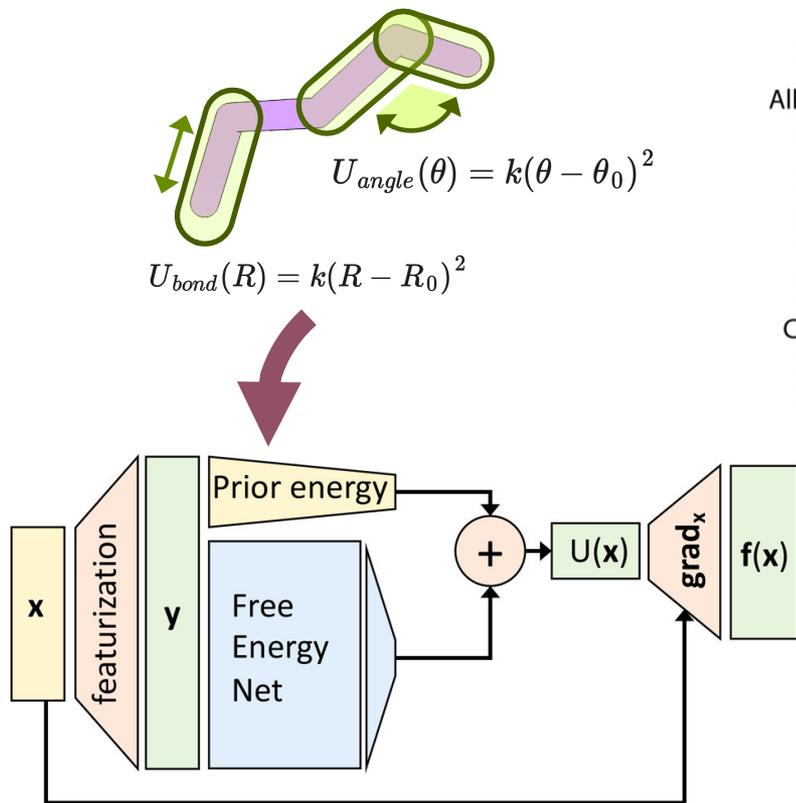
reference free energy



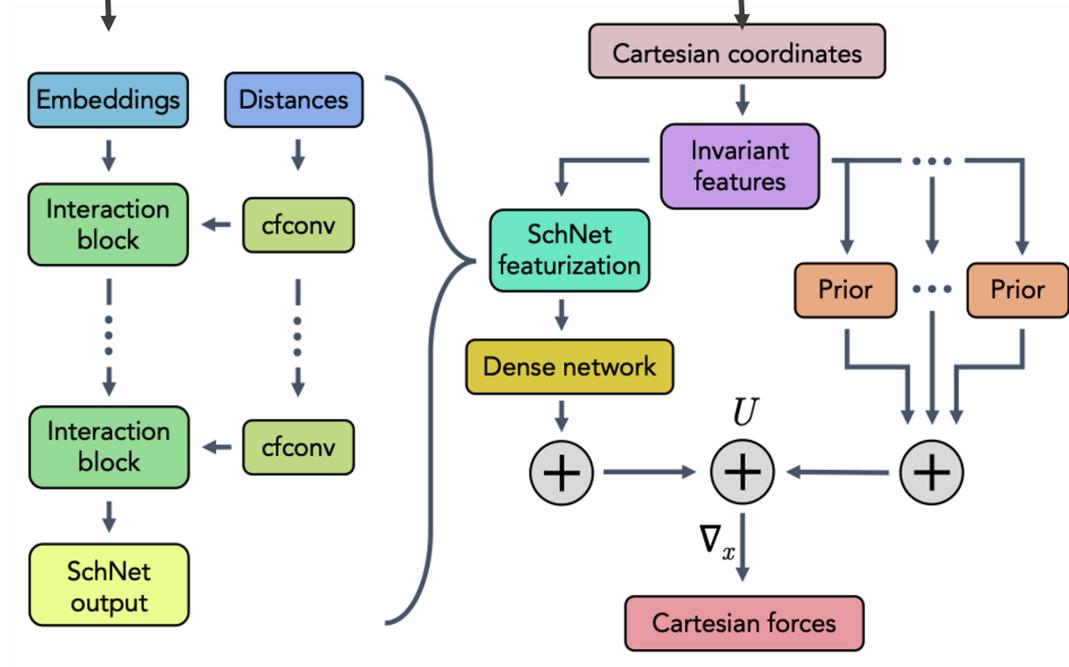
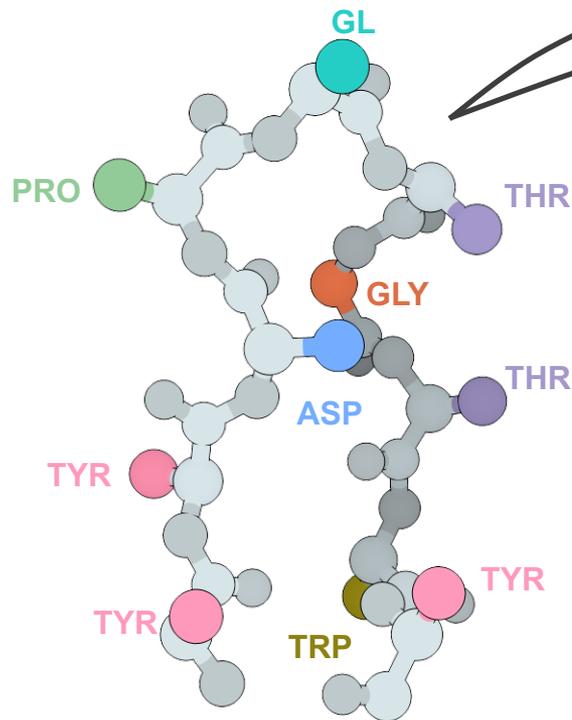
3 metastable  
states



# CGNet: Feedforward, Fully-Connected

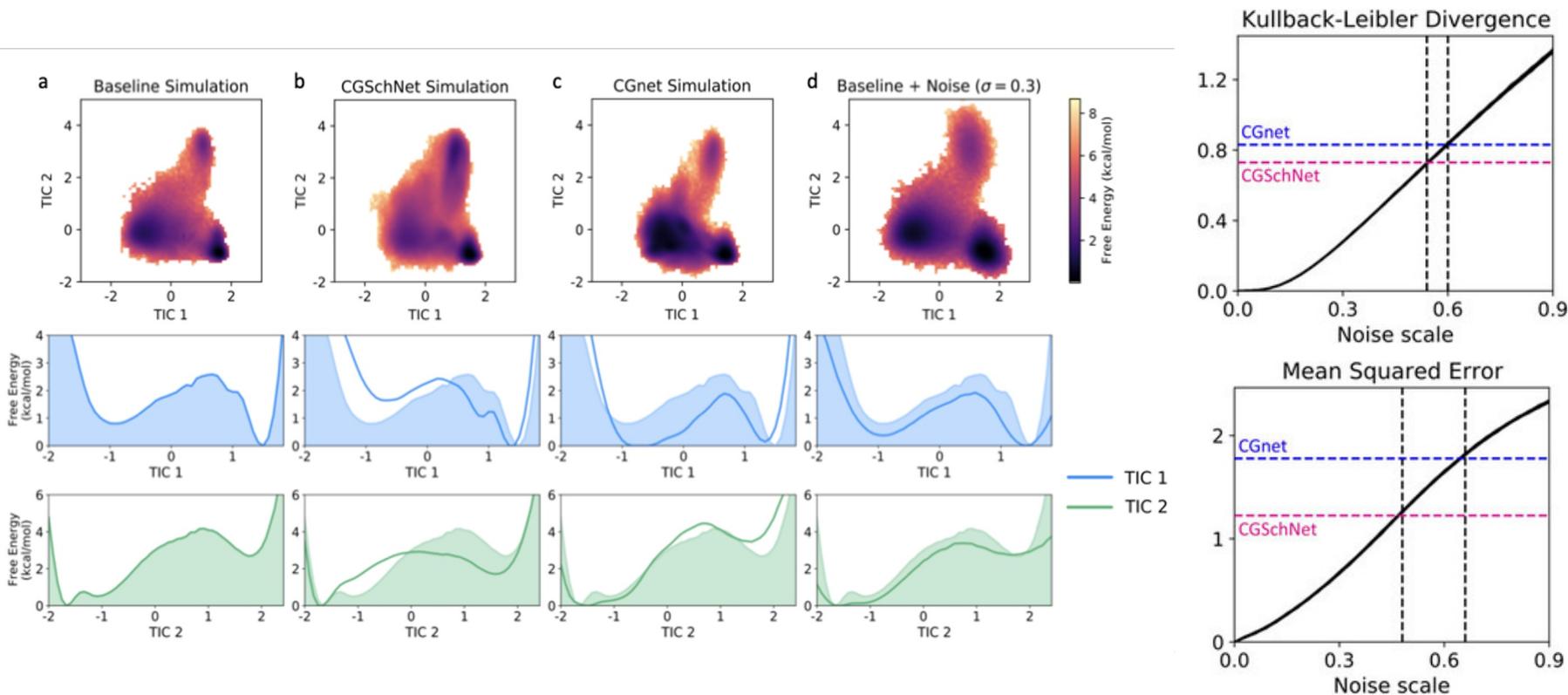


# CGSchNet: A Message-passing, Graph Network

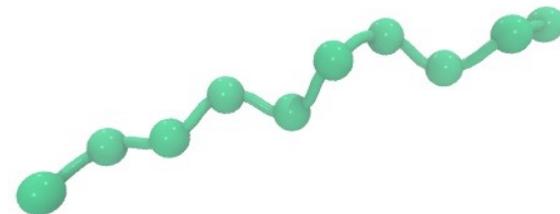
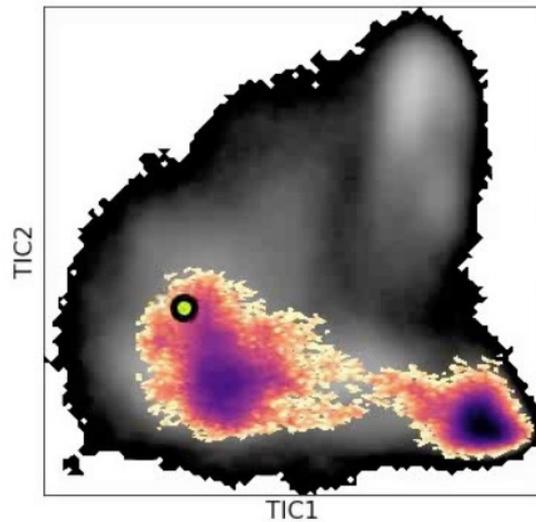
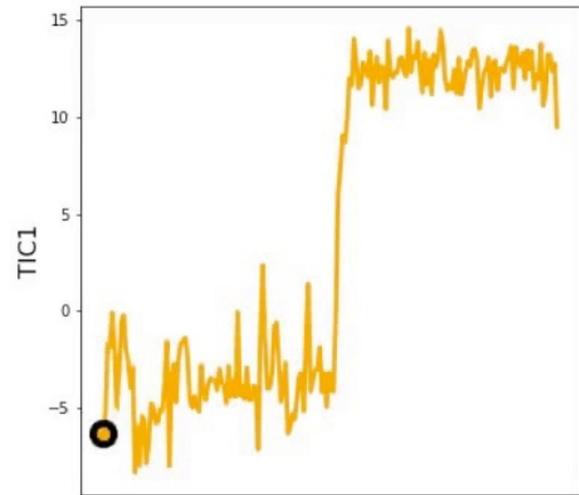


Husic, B. E., Charron, N. E., Lemm, D., Wang, J., Pérez, A., Majewski, M., Krämer, A., Chen, Y., Olsson, S., de Fabritiis, G., Noé, F., & Clementi, C. Coarse graining molecular dynamics with graph neural networks. *J. Chem. Phys.* 153(19), 194101 (2020).  
Schütt, K. T., Sauceda, H. E., Kindermans, P.-J., Tkatchenko, A., & Müller, K.-R. SchNet – A deep learning architecture for molecules and materials. *J. Chem. Phys.* 148(24), 241722 (2018).

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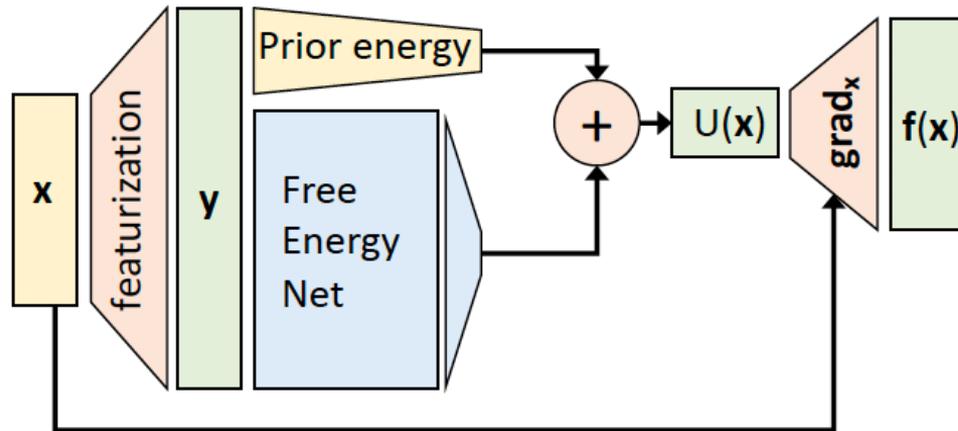
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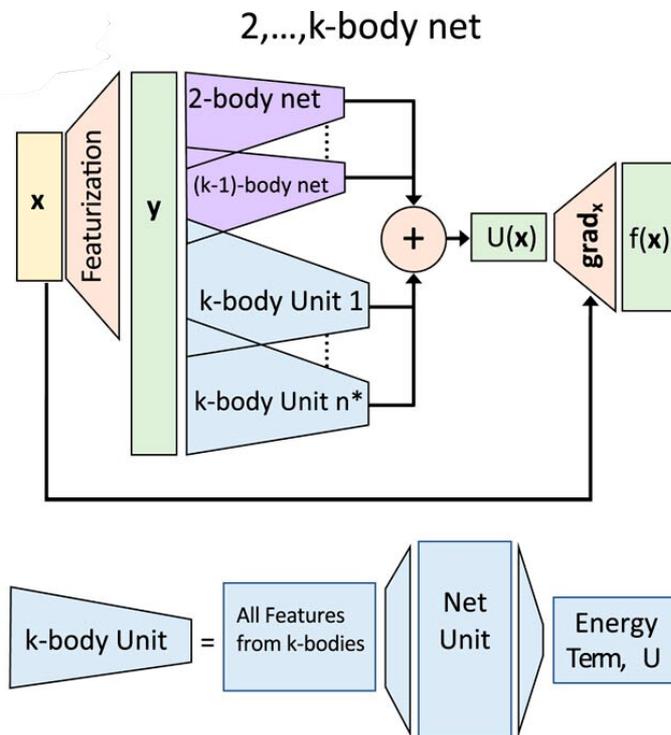
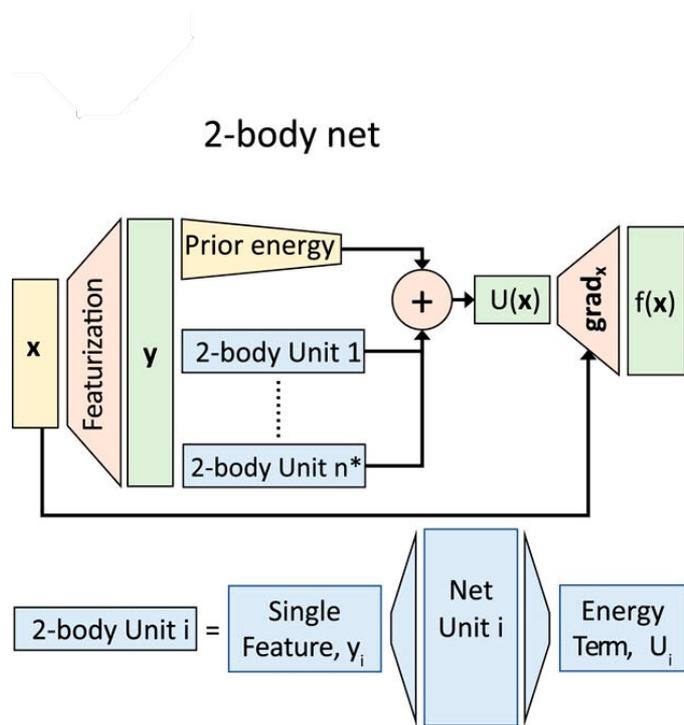


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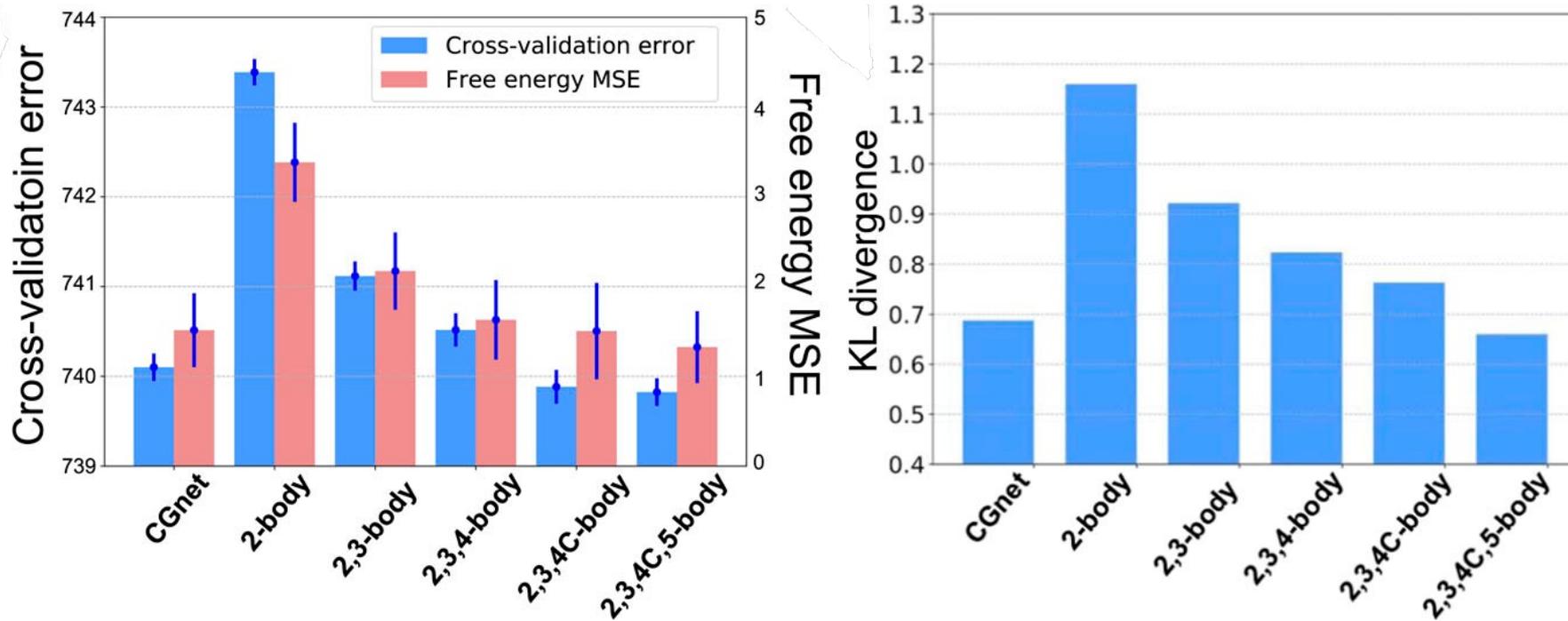
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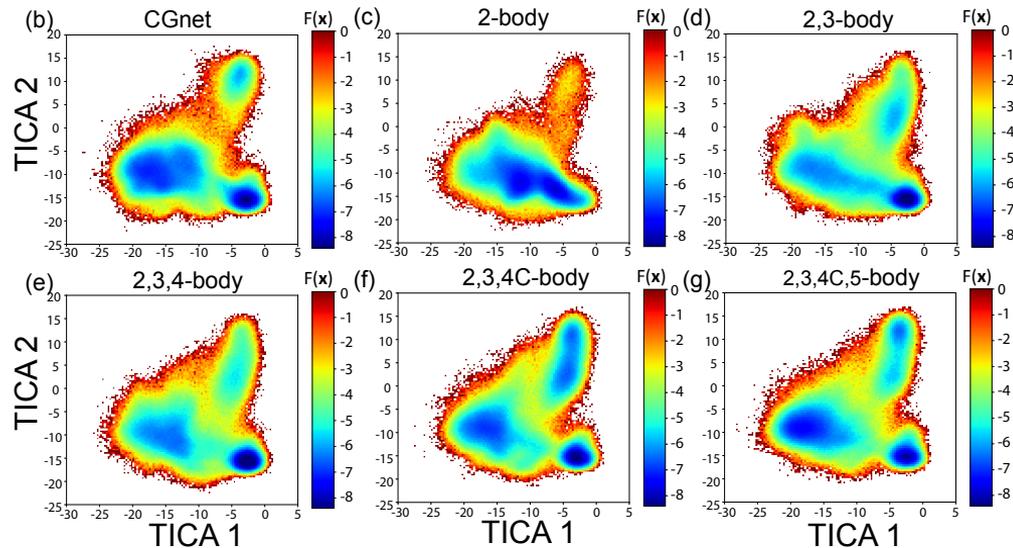
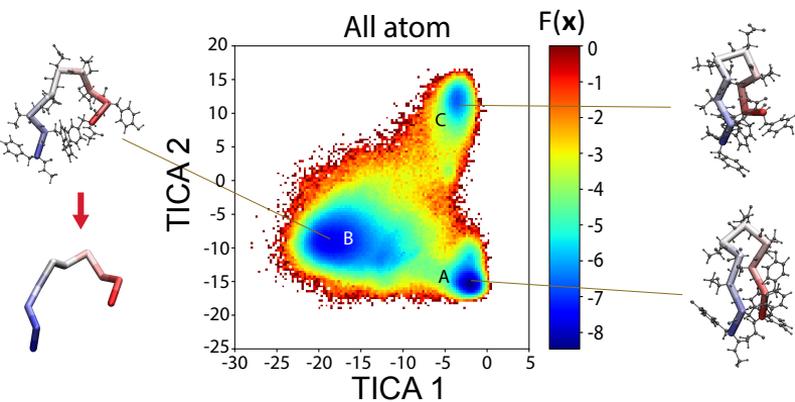
# Multibody potentials: how many bodies?



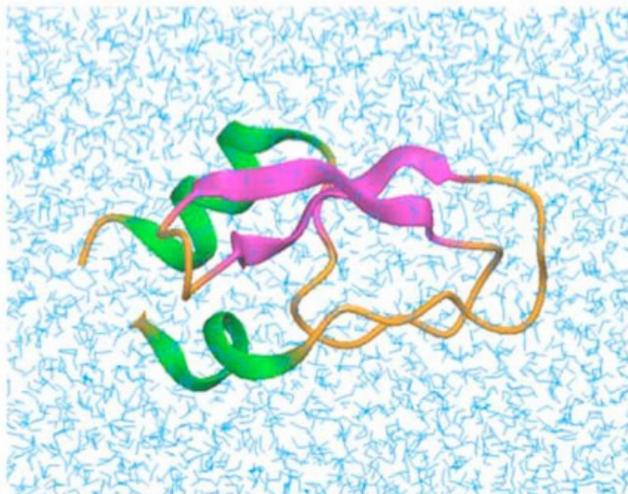
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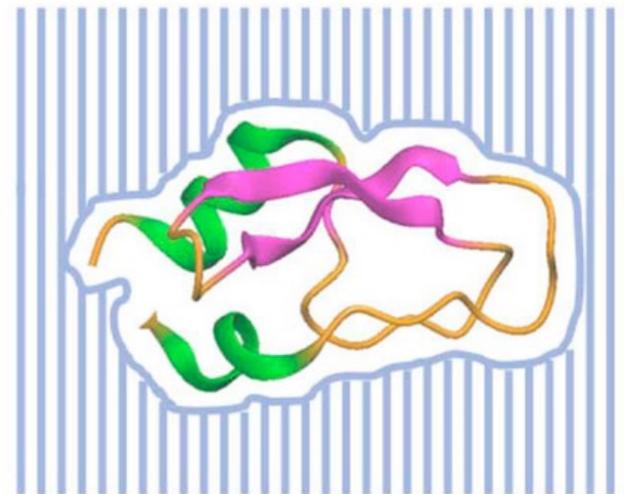


## Explicit Water

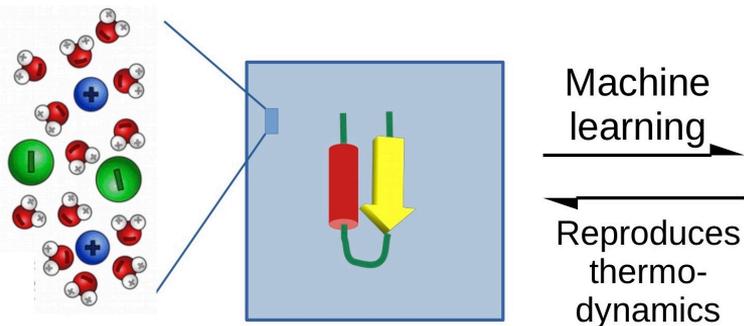


$$H = H_{\text{prot}} + H_{\text{prot-wat}} + H_{\text{wat}}$$

## Implicit Solvent



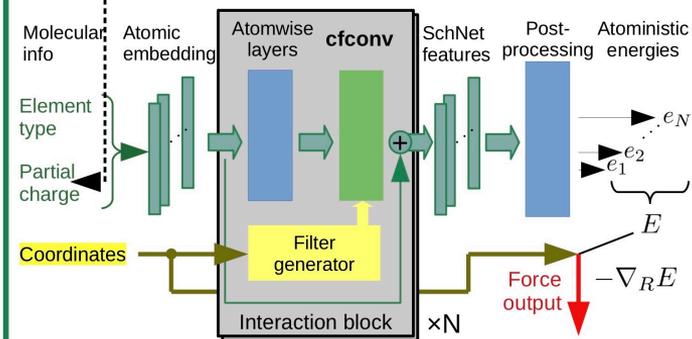
$$H = H_{\text{prot}} + \Delta G_{\text{solv}}$$



## Methods

### Neural network force field

- ISSNet = an extension of SchNet,<sup>[3]</sup> (graph neural network)
- Including electrostatic information by embedding atomic partial charges
- Implicit solvent MD with ISSNet



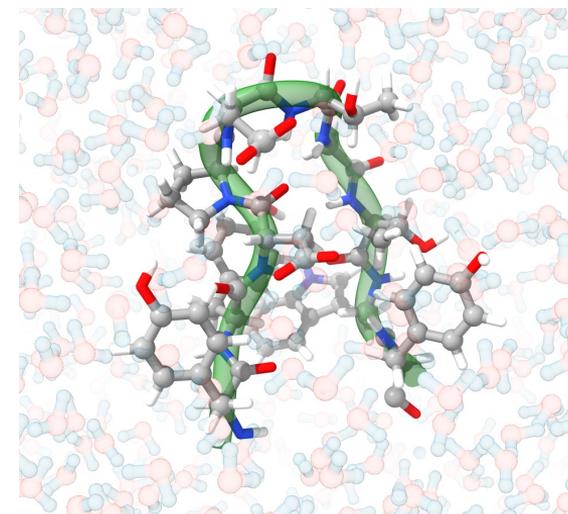
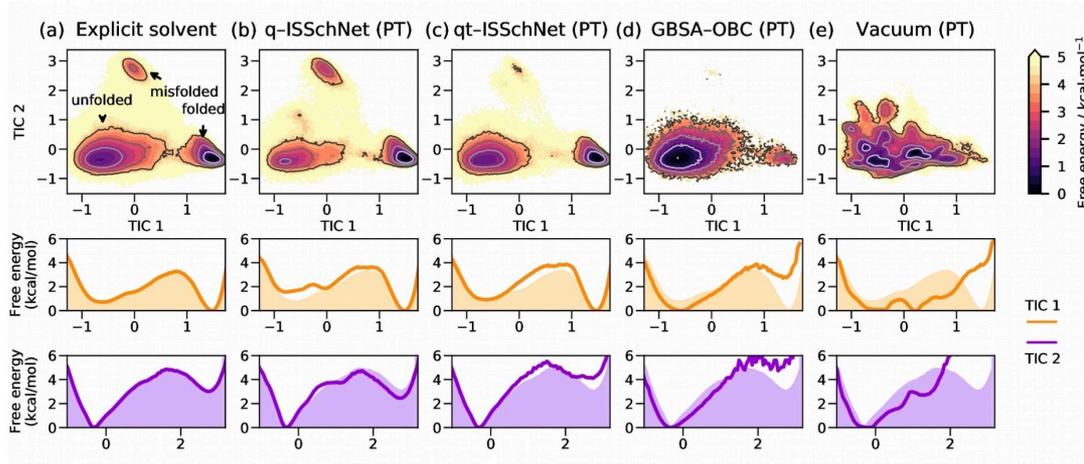
3. Schütt, et al., *J. Chem. Phys.* 148, 241722 (2018).

# ISSchNet: Implicit Solvent CG Models

ISSNet model can correctly reproduce the folding temperature of miniprotein **chignolin**

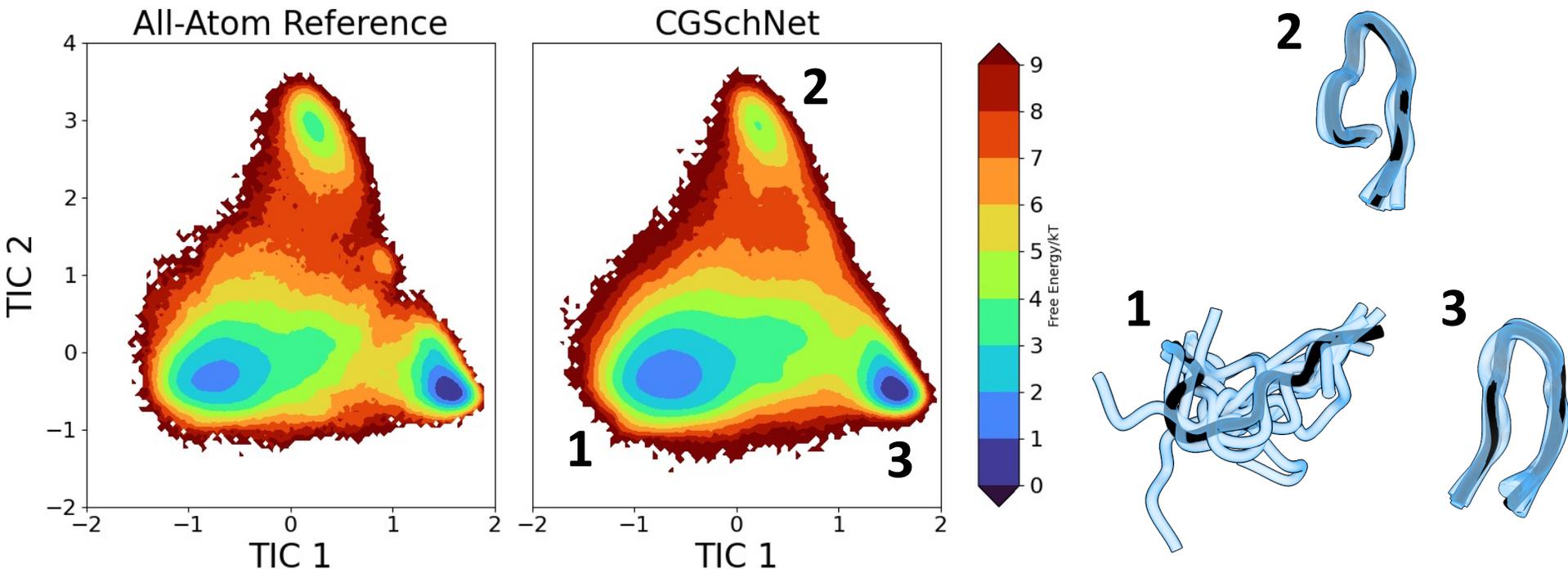
- CLN025 variant: 175 solute atoms
- Comparing to GBSA model, trained ISSNet model
  - more accurately reproduces thermodynamics (shown below), and
  - gives more reasonable folding temperature →

Solvation model	$T_m$ / K
<b>q-ISSNet</b>	<b>~368.3</b>
GBSA-OBC2 (comparable with Anandakrishnan et al., Biophys. J., 2015)	~266.5
(Ref) Explicit solvent (Lindorff-Larsen et al., <i>Science</i> , 2011)	361.0–393.0
(Ref) Experimental (S. Honda et al., <i>J. Am. Chem. Soc.</i> , 2008)	~343



Chen, Y., Krämer, A., Charron, N. E., Husic, B. E., Clementi, C., & Noé, F. Machine learning implicit solvation for molecular dynamics. *J. Chem. Phys.*, 155(8), 084101 (2021).

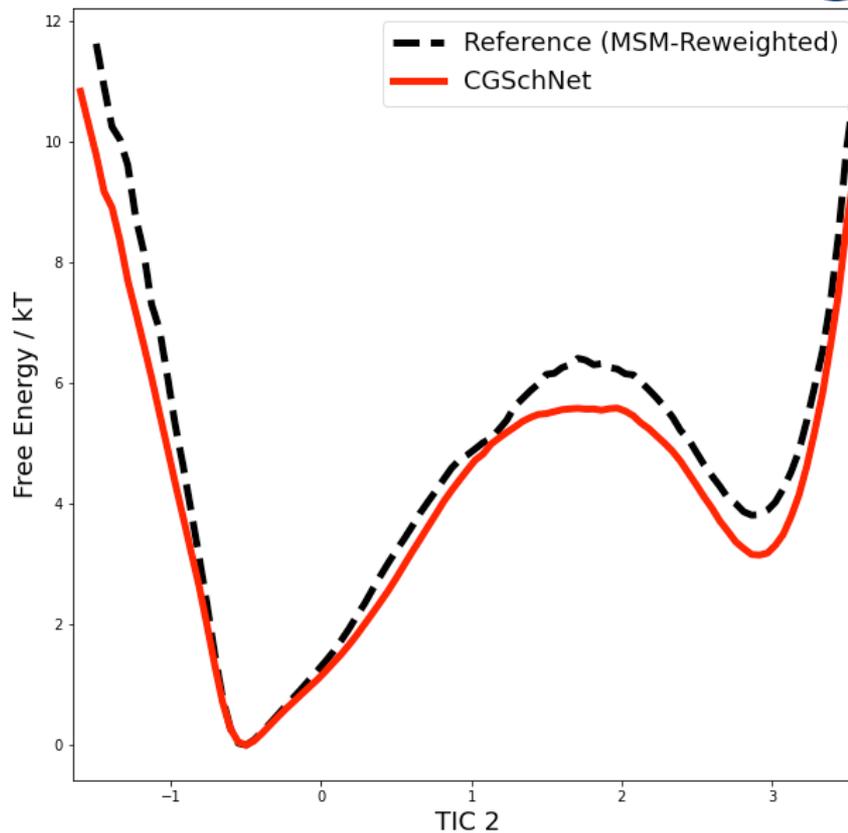
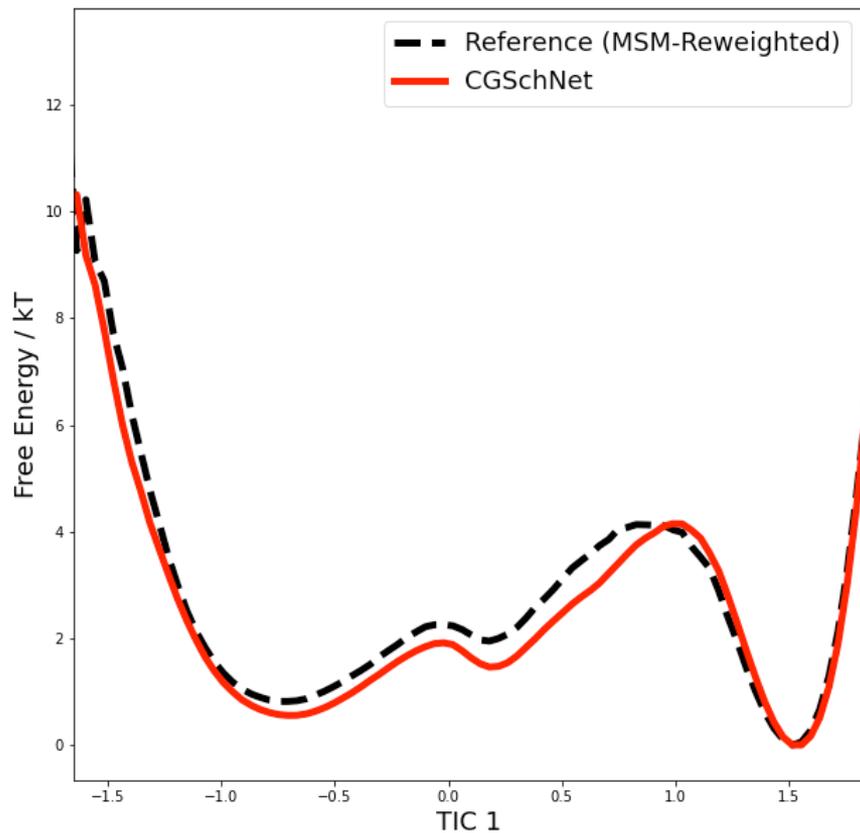
# State of The Art MLCG Model



J. Wang, S. Olsson, C. Wehmeyer, A. Perez, N.E. Charron, G. de Fabritiis, F. Noé, C. Clementi, *ACS Cent. Sci.* 5, 755-767 (2019)

B. E. Husic, N. E. Charron, ..., G. de Fabritiis, F. Noé, C. Clementi *J. Chem. Phys.* 153 (19), 194101 (2020)

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B. E. Husic, N. E. Charron, ..., G. de Fabritiis, F. Noé, C. Clementi *J. Chem. Phys.* 153 (19), 194101 (2020)

# How do we achieve state of the art?

## Dataset Curation

## Model Construction & Training

## Validation & Prediction

Simulation/  
Experiment

CG Mapping

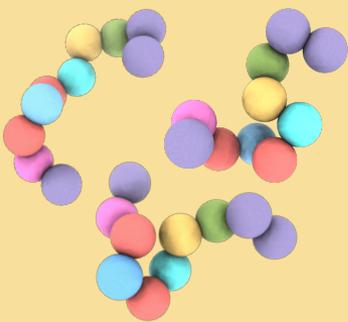
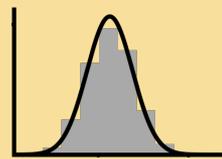
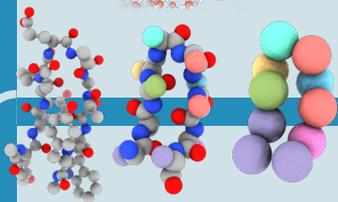
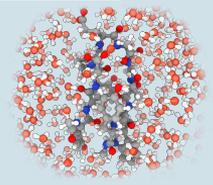
Physical  
Constraints

CG Dataset

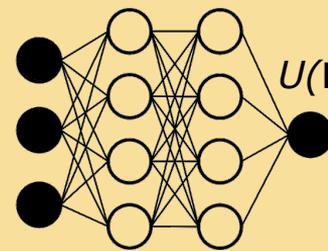
Trained  
CG MLFF

Property  
Prediction

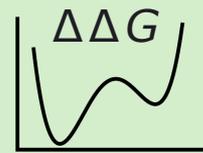
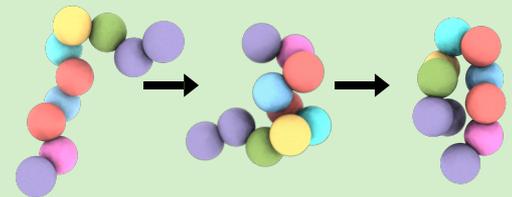
Experimental  
Comparison



$\mathbf{R}(t)$

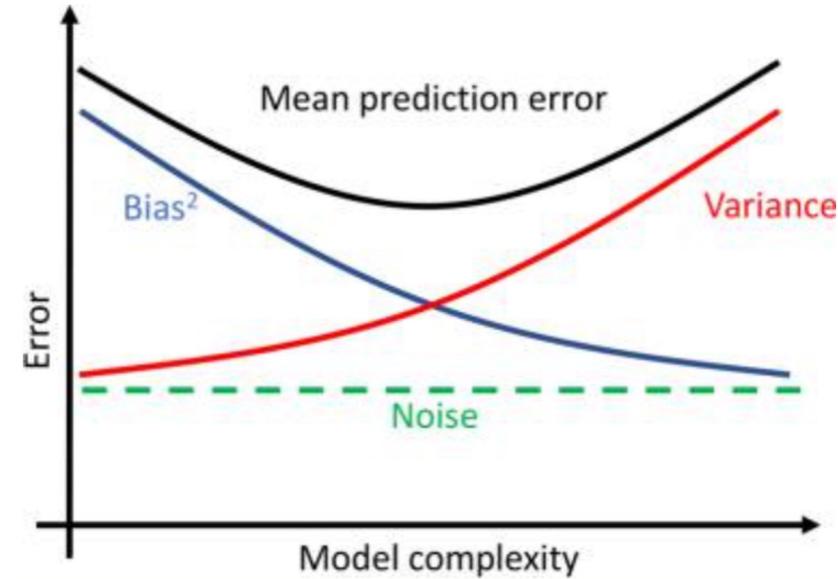


$U(\mathbf{R}(t))$



# Optimization: CG Force Noise

$$\chi^2(\theta) = \left\langle \left\| \xi_f \mathbf{F}(\mathbf{r}) + \nabla U(\xi_r(\mathbf{r}); \theta) \right\|^2 \right\rangle_{\mathbf{r}}$$

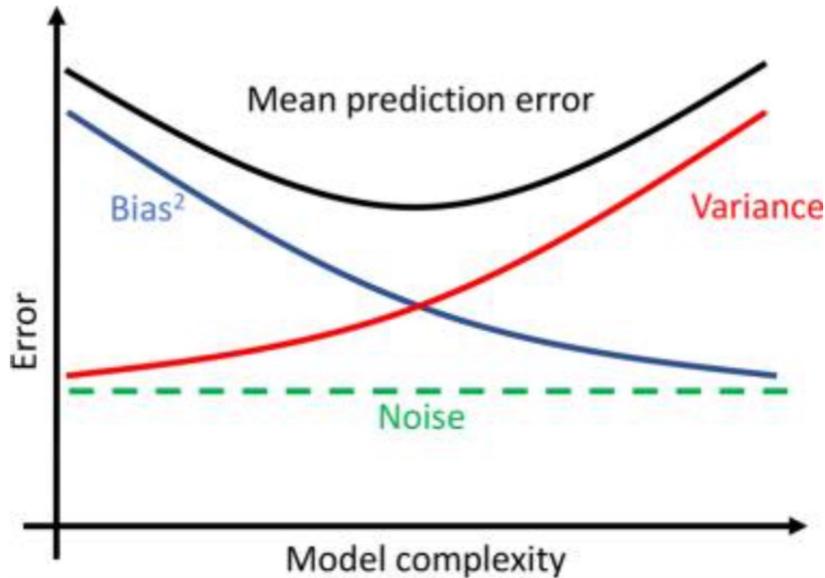


# Optimization: CG Force Noise

$$\chi^2(\theta) = \left\langle \left\| \xi_f \mathbf{F}(\mathbf{r}) + \nabla U(\xi_r(\mathbf{r}); \theta) \right\|^2 \right\rangle_{\mathbf{r}}$$



$$L(\theta) = \left\| \xi_f \mathbf{F}(\mathbf{r}) + \nabla U(\xi_r(\mathbf{r}); \theta) \right\|^2$$



# Optimization: CG Force Noise

$$\chi^2(\theta) = \left\langle \left\| \xi_f \mathbf{F}(\mathbf{r}) + \nabla U(\xi_r(\mathbf{r}); \theta) \right\|^2 \right\rangle_{\mathbf{r}}$$

$$L(\theta) = \left\| \xi_f \mathbf{F}(\mathbf{r}) + \nabla U(\xi_r(\mathbf{r}); \theta) \right\|^2$$

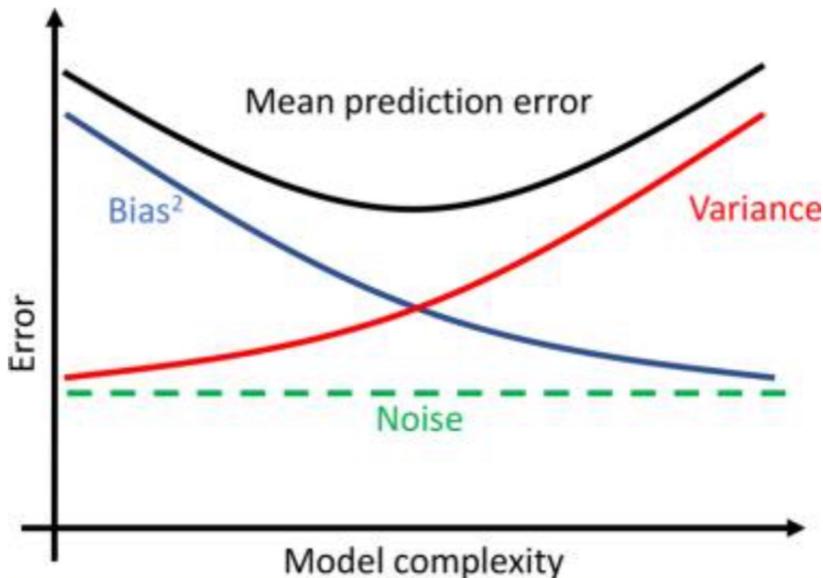
$$\mathbb{E}[L(\theta)] = \text{Parameter Bias}^2 + \text{Parameter variance} + \text{CG Bias}$$

$$\mathbf{f} = \left\langle \xi_f \mathbf{F}(\mathbf{r}) \right\rangle_{\mathbf{r}|R}$$

$$\text{Parameter Bias}^2 = \left\| \mathbf{f} - \mathbb{E}[-\nabla U(\xi_r \mathbf{r}; \theta)] \right\|^2$$

$$\text{Parameter Var} = \mathbb{E} \left[ \left\| \mathbb{E}[-\nabla U(\xi_r \mathbf{r})] + \nabla U(\xi_r \mathbf{r}; \theta) \right\|^2 \right]$$

$$\text{CG Force Noise} = \left\langle \left\| \xi_f \mathbf{F}(\mathbf{r}) - \mathbf{f} \right\|^2 \right\rangle_{\mathbf{r}}$$

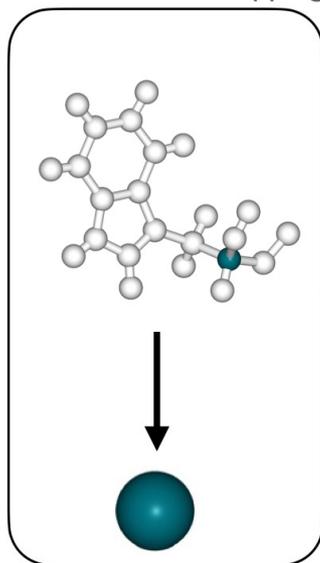


# Force Aggregation Strategies

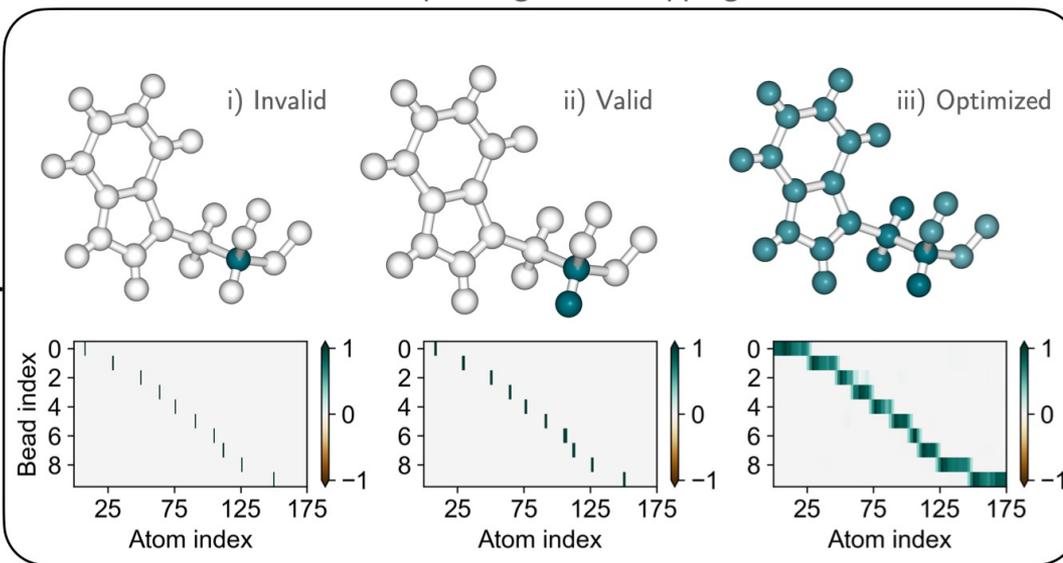
$$\xi_r \neq \xi_f$$

$$\text{CG Force Noise} = \left\langle \left\| \xi_f \mathbf{F}(\mathbf{r}) - \mathbf{f} \right\|^2 \right\rangle_r$$

Coordinate Mapping



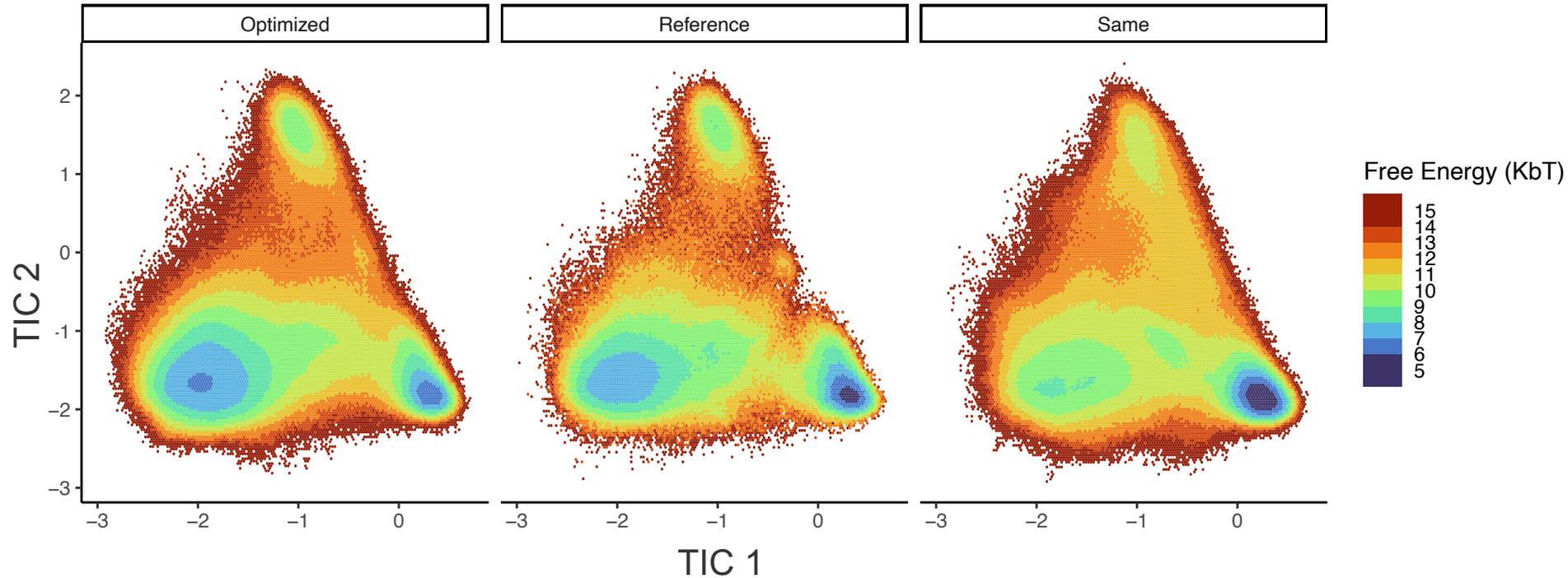
Corresponding Force Mappings



# Force Aggregation Strategies

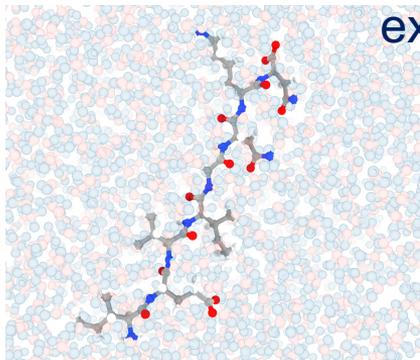
$$\xi_r \neq \xi_f$$

$$\xi_r = \xi_f$$

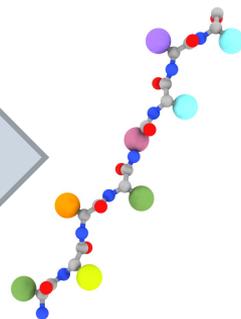


## INPUT

1100 different octapeptide sequences in explicit solvent

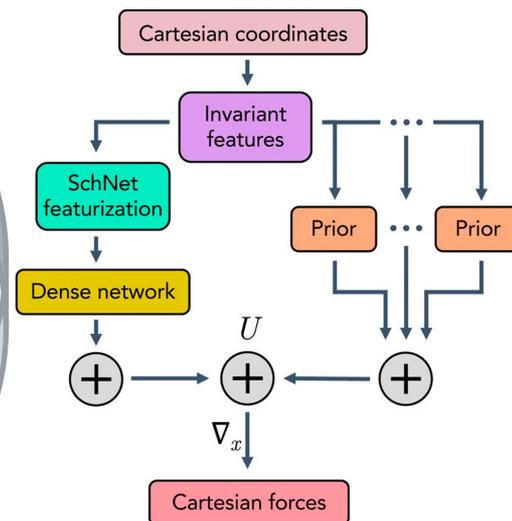
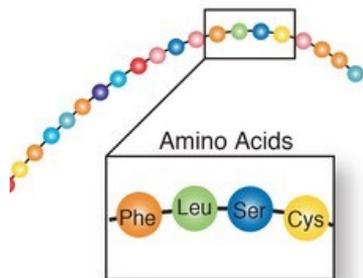


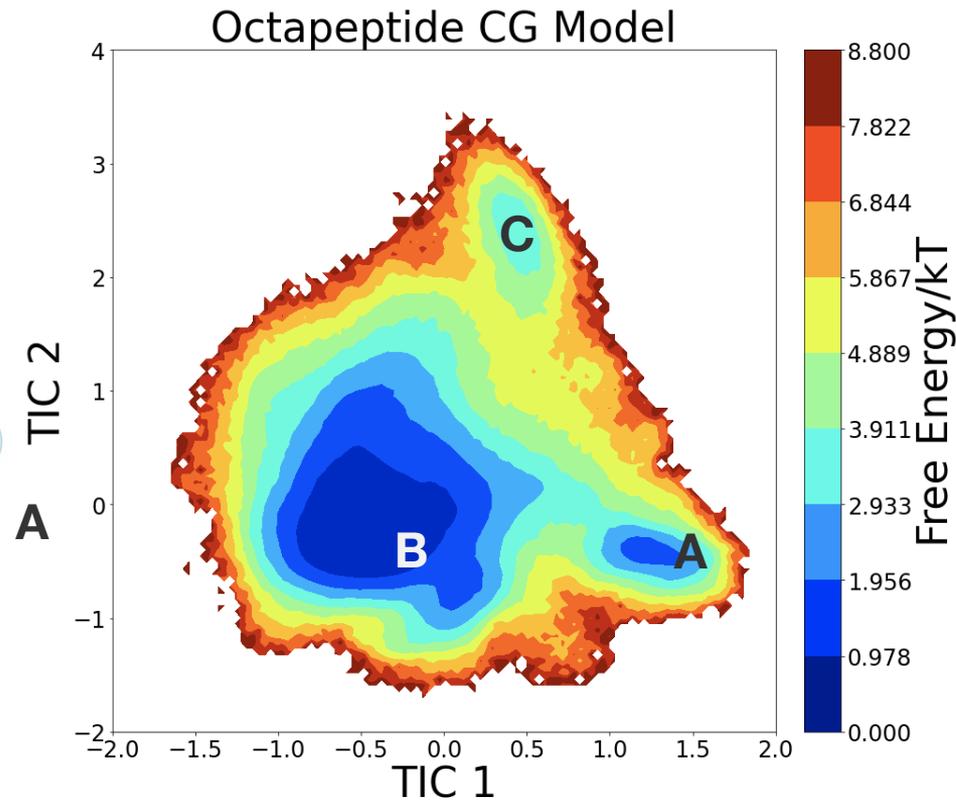
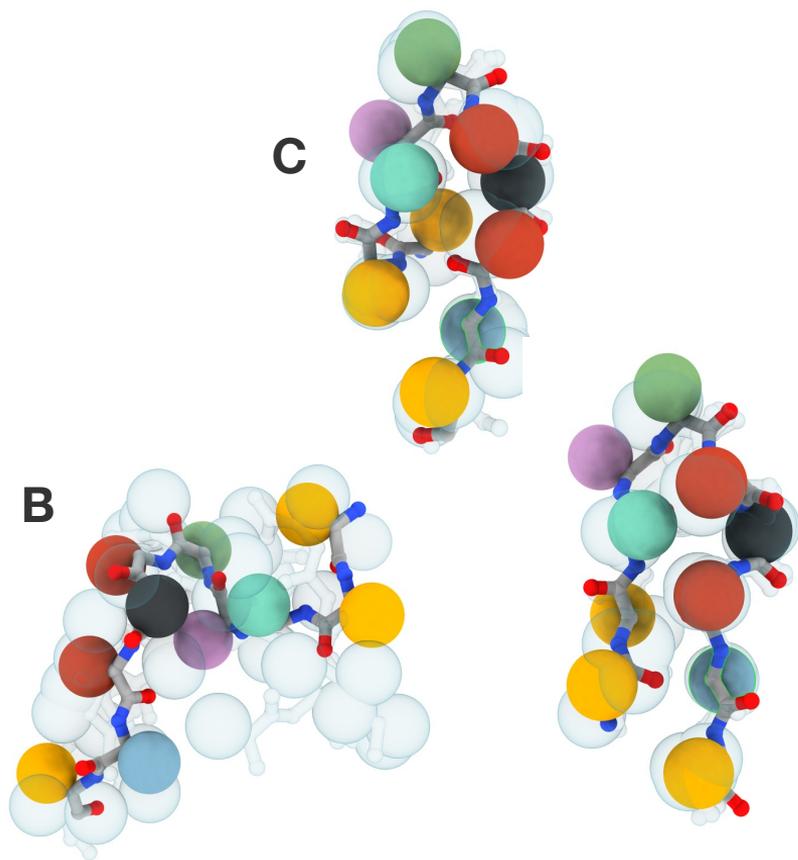
CG mapping



## OUTPUT

Learn a “universal” CG force-field



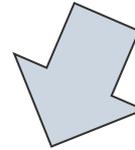
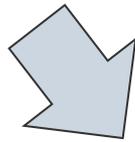
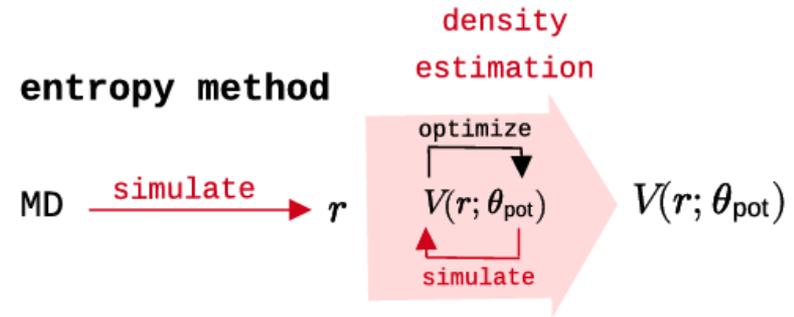


# Force-matching without forces

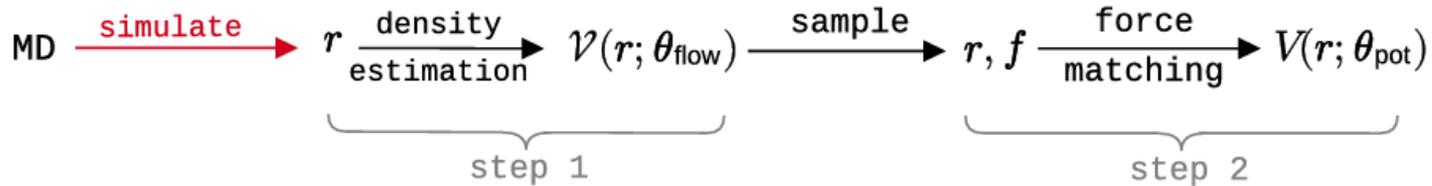
## force matching



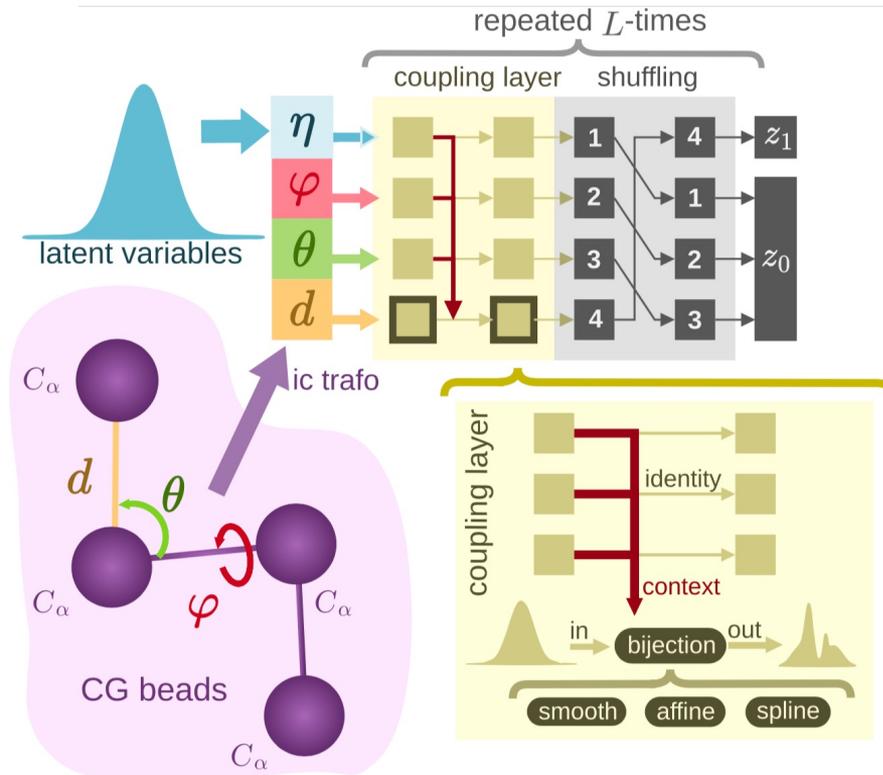
## relative entropy method



## force matching without forces



# Force-matching without forces



Flow density  $p$  corresponds to a (noisy) coarse-grained potential

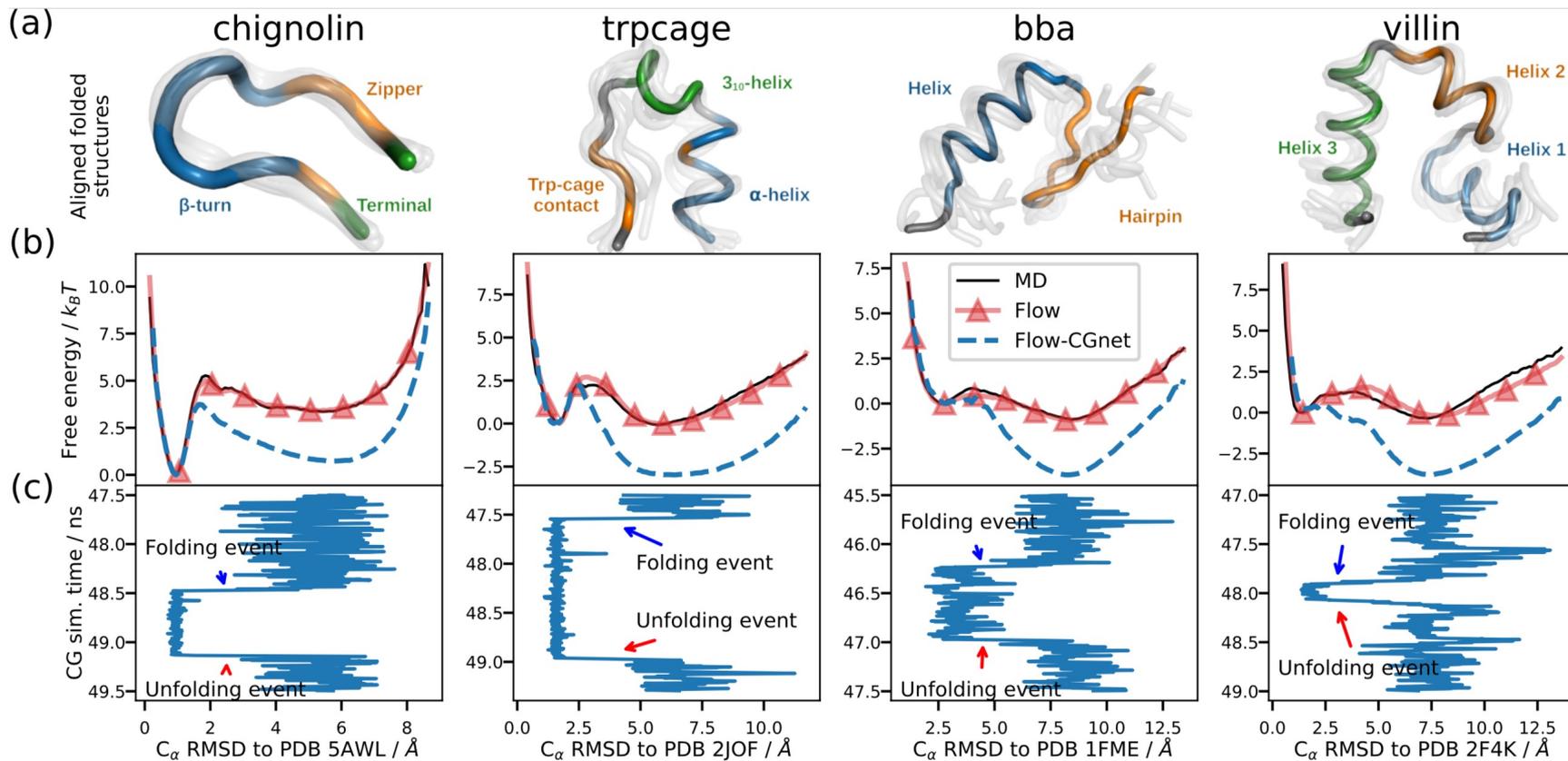
$$\mathcal{V}(\mathbf{r}, \boldsymbol{\eta}; \boldsymbol{\theta}) = -\log p(\mathbf{r}, \boldsymbol{\eta}; \boldsymbol{\theta}),$$

Noisy forces

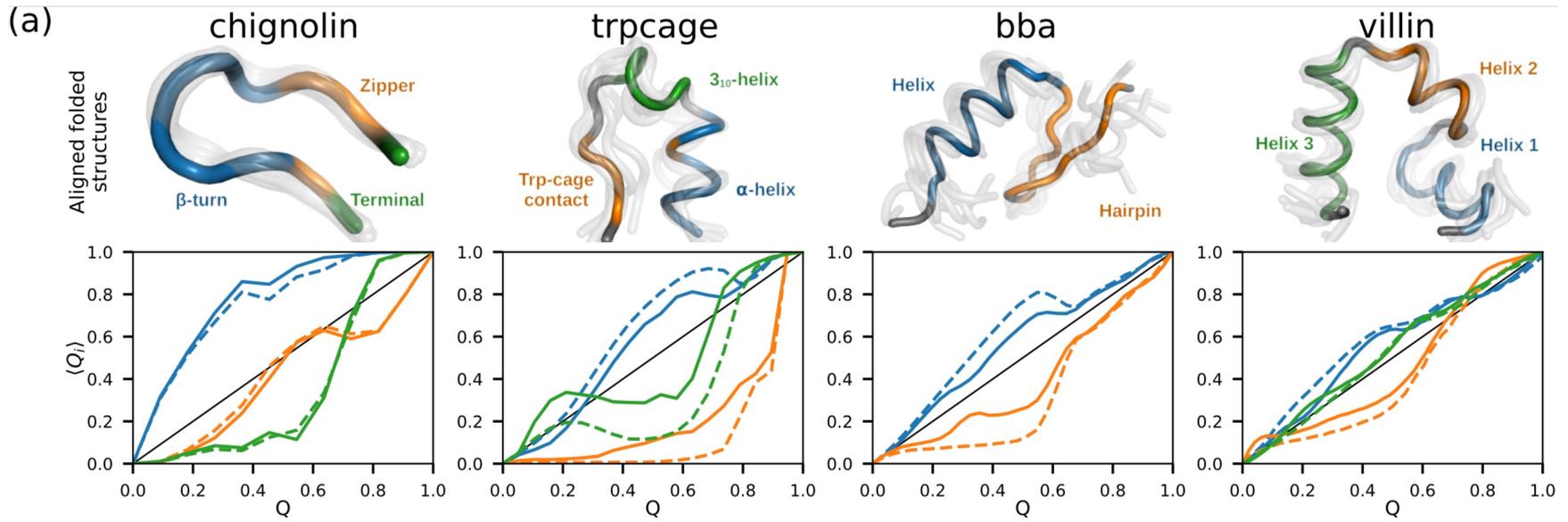
$$\mathbf{f}(\mathbf{r}, \boldsymbol{\eta}; \boldsymbol{\theta}) = \nabla_{\mathbf{r}} \mathcal{V}(\mathbf{r}, \boldsymbol{\eta}; \boldsymbol{\theta}),$$

→ Match these forces on average

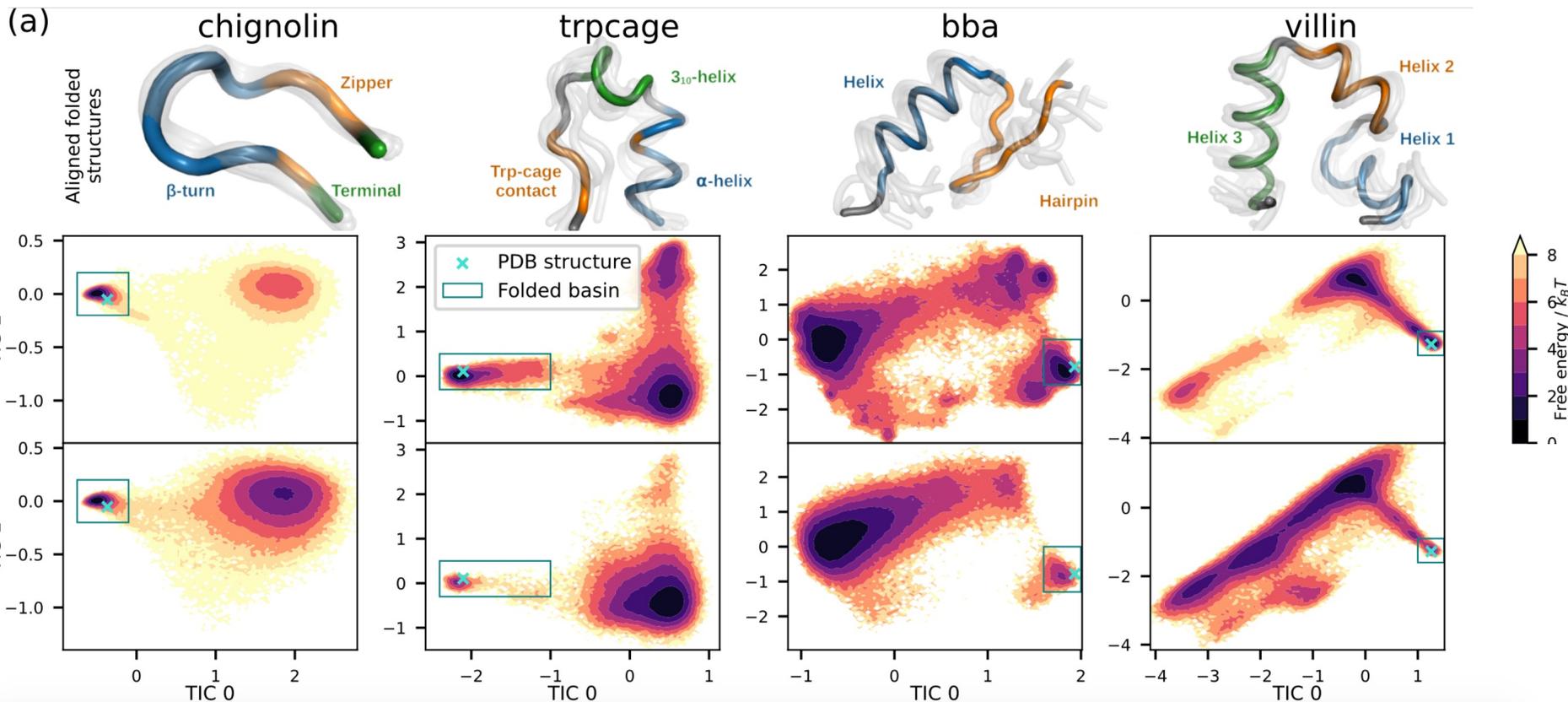
# Force-matching without forces



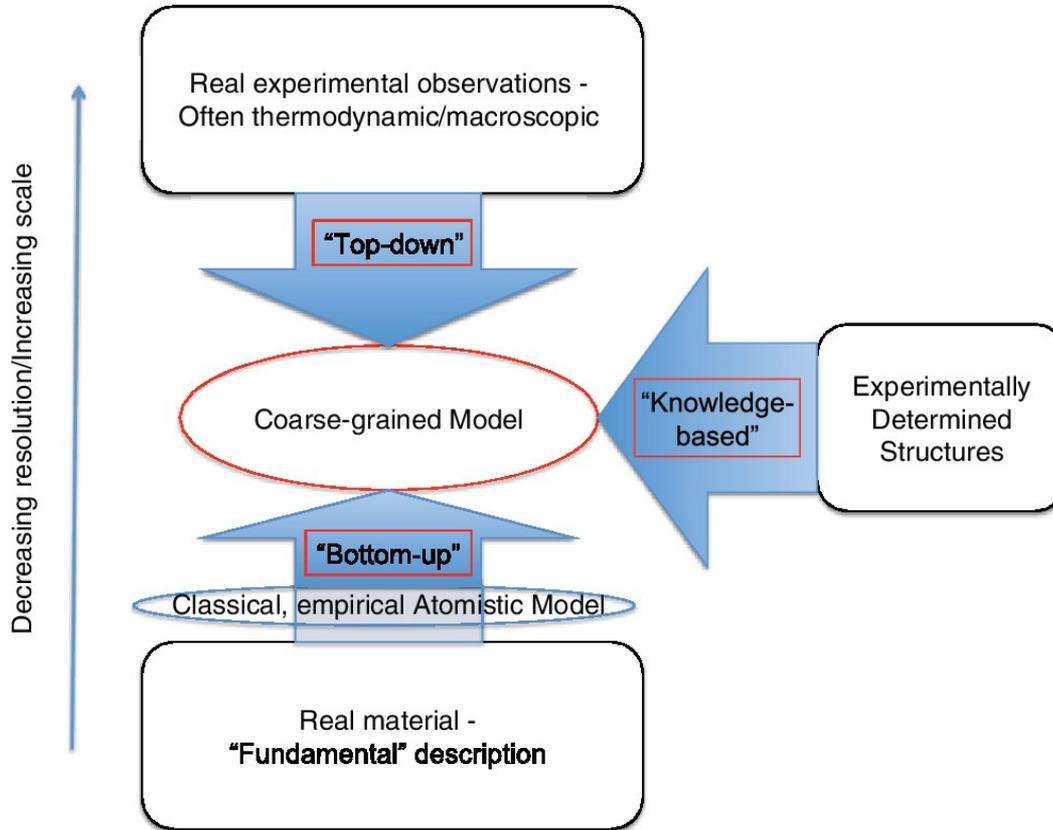
# Force-matching without forces



# Force-matching without forces



# CG models: which properties to preserve?

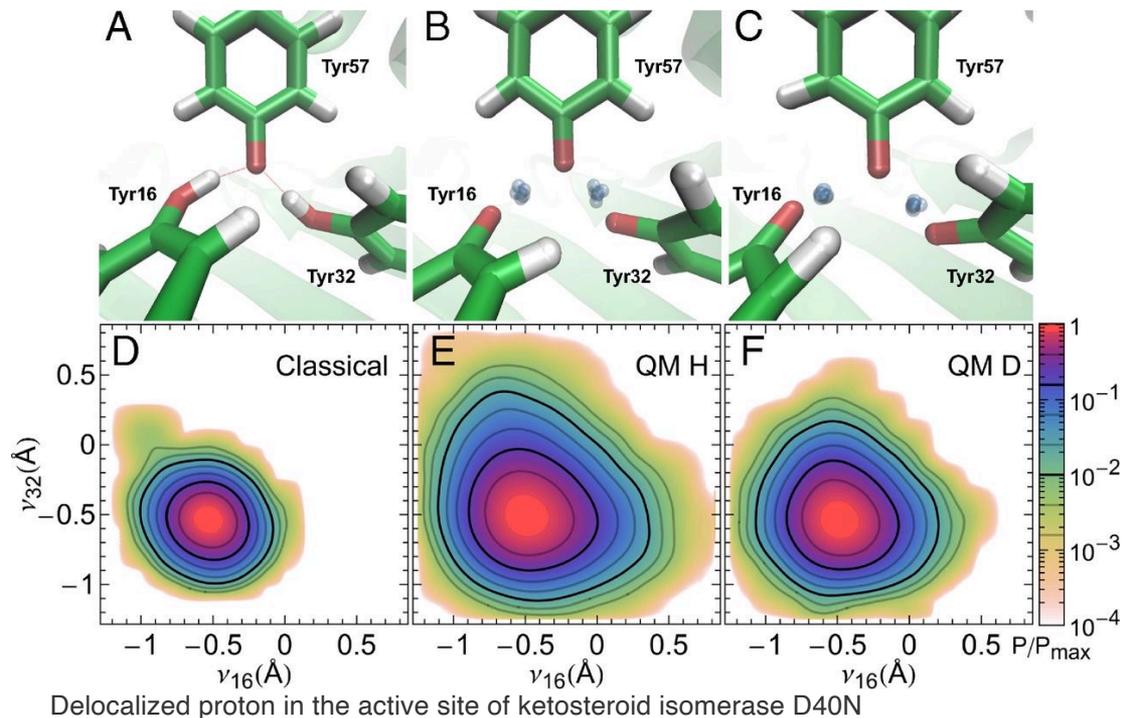


1. Learning CG models from all-atom simulations
2. **Incorporating nuclear quantum effects into all-atom simulation**
3. Learning CG models from experimental data

**Classical picture:** nuclei move on a potential energy surface according to the laws of classical physics

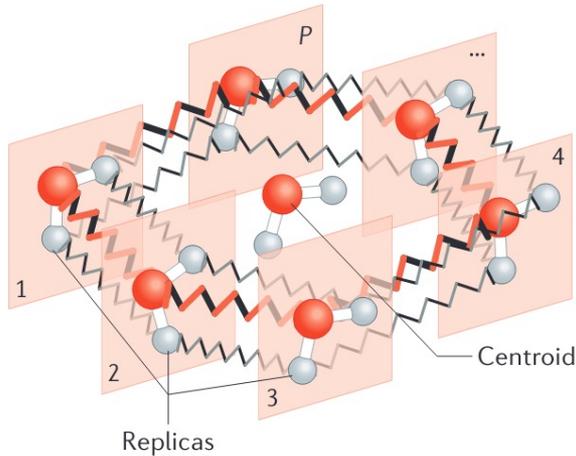
**Quantum picture:** nuclei obey Schrödinger equation

**Effects present only in quantum picture:** tunnelling, isotope effects ...



Delocalized proton in the active site of ketosteroid isomerase D40N

# Quantum-classical isomorphism and path integral molecular dynamics



Sample the quantum canonical distribution by moving to an extended phase space:

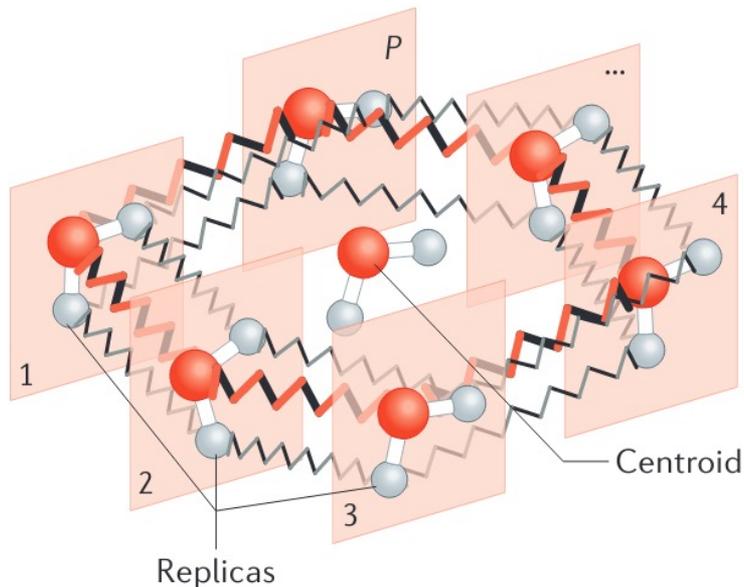
$$H(\mathbf{q}, \mathbf{p}) = \sum_{k=1}^P \left[ \frac{\mathbf{p}_k^2}{2m'} + \frac{1}{2} m \omega_P^2 (\mathbf{q}_{k+1} - \mathbf{q}_k)^2 + \frac{1}{P} U(\mathbf{q}_k) \right]_{\mathbf{q}_{P+1} = \mathbf{q}_1}$$
$$\omega_P = \frac{\sqrt{P}}{\beta \hbar} \quad P \rightarrow \infty$$

Can we coarse-grain the ring polymer back to a centroid?

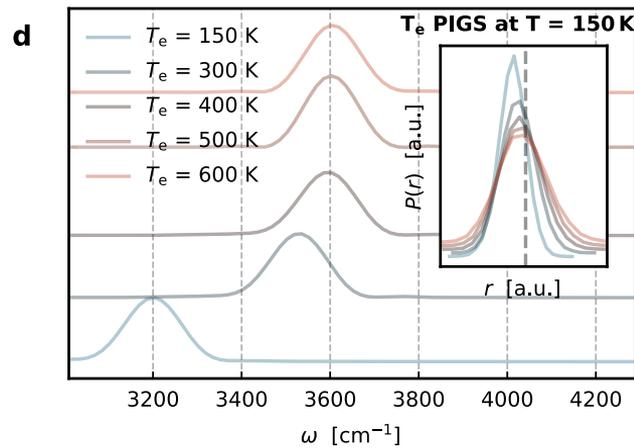
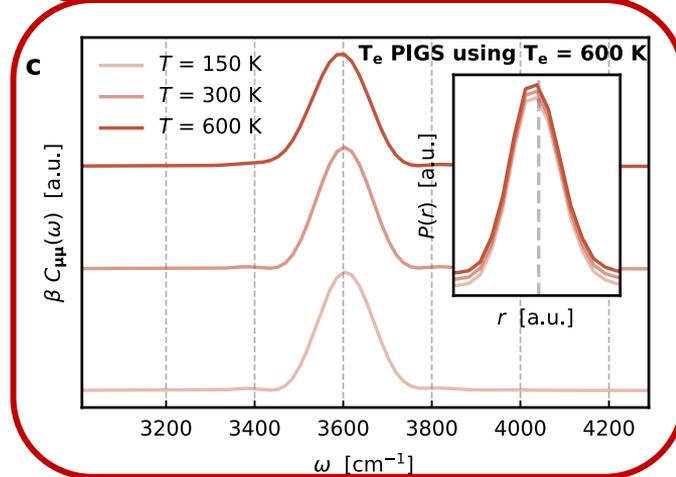
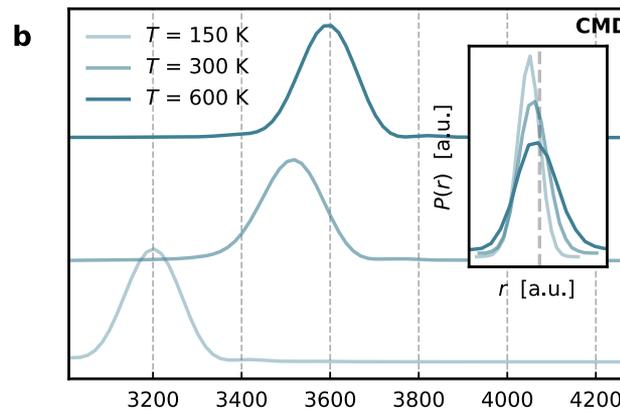
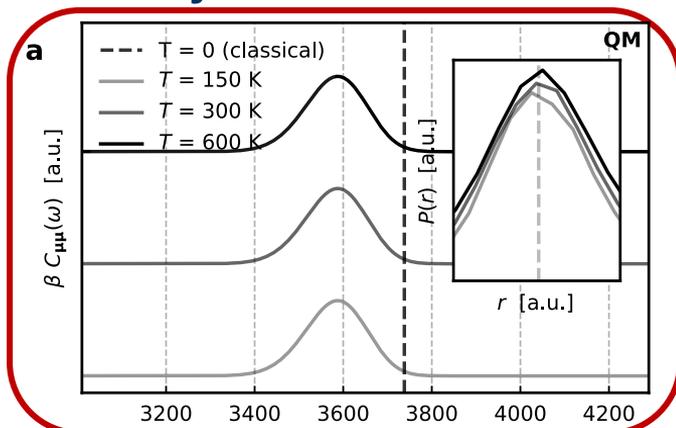
# Quantum-classical isomorphism and path integral molecular dynamics

Can we coarse-grain the ring polymer back to a centroid?

centroid Path Integral coarse-Grained simulation  
**PIGs**



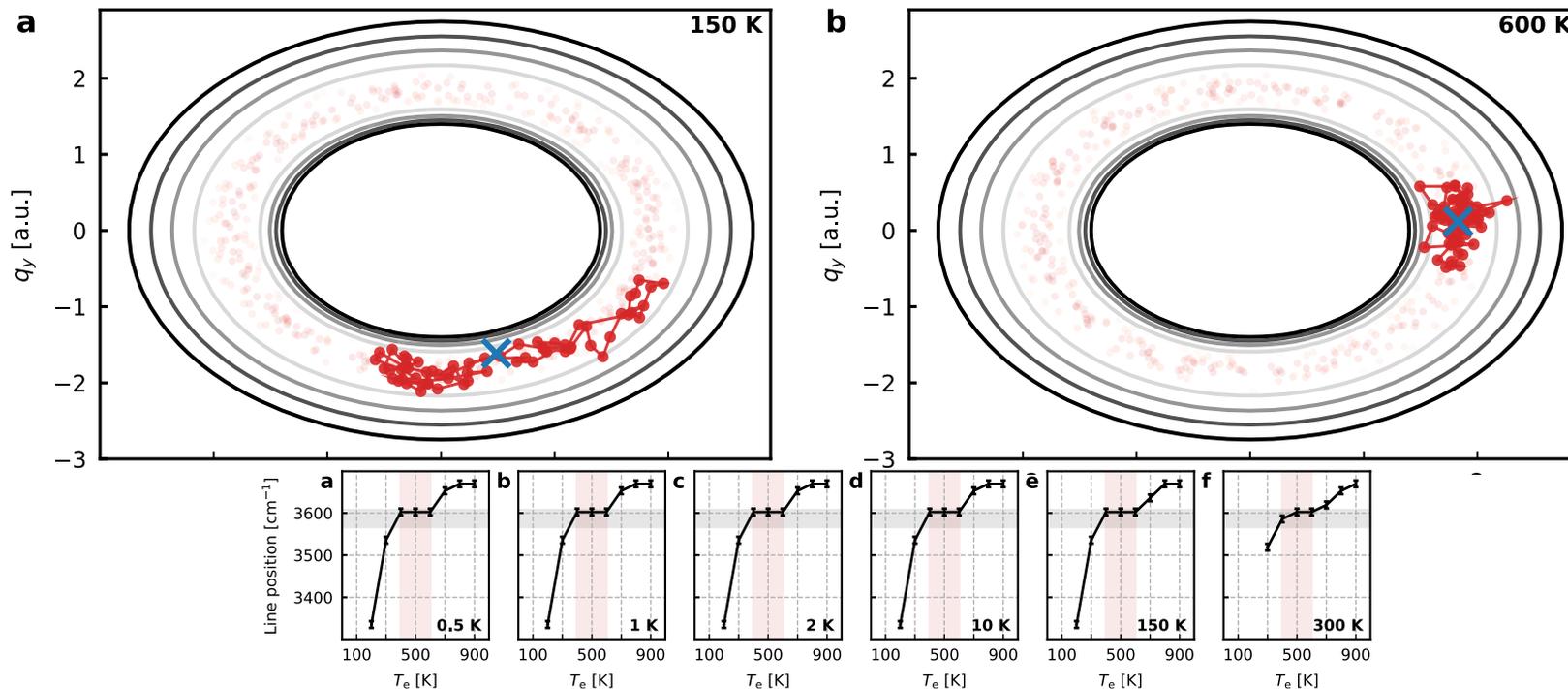
# Benchmark system: OH bond



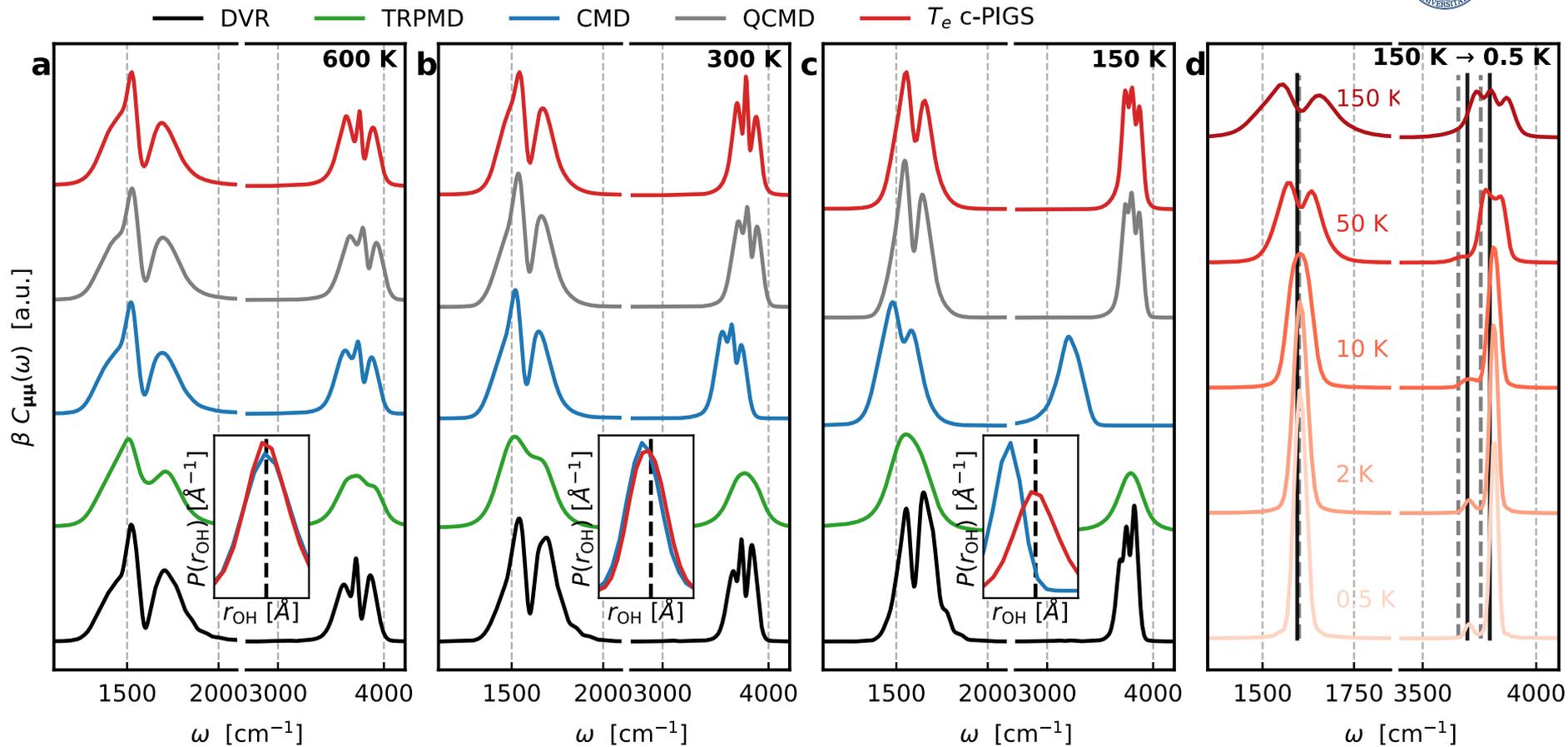
# Quantum-classical isomorphism and path integral molecular dynamics

Temperature elevation approximation

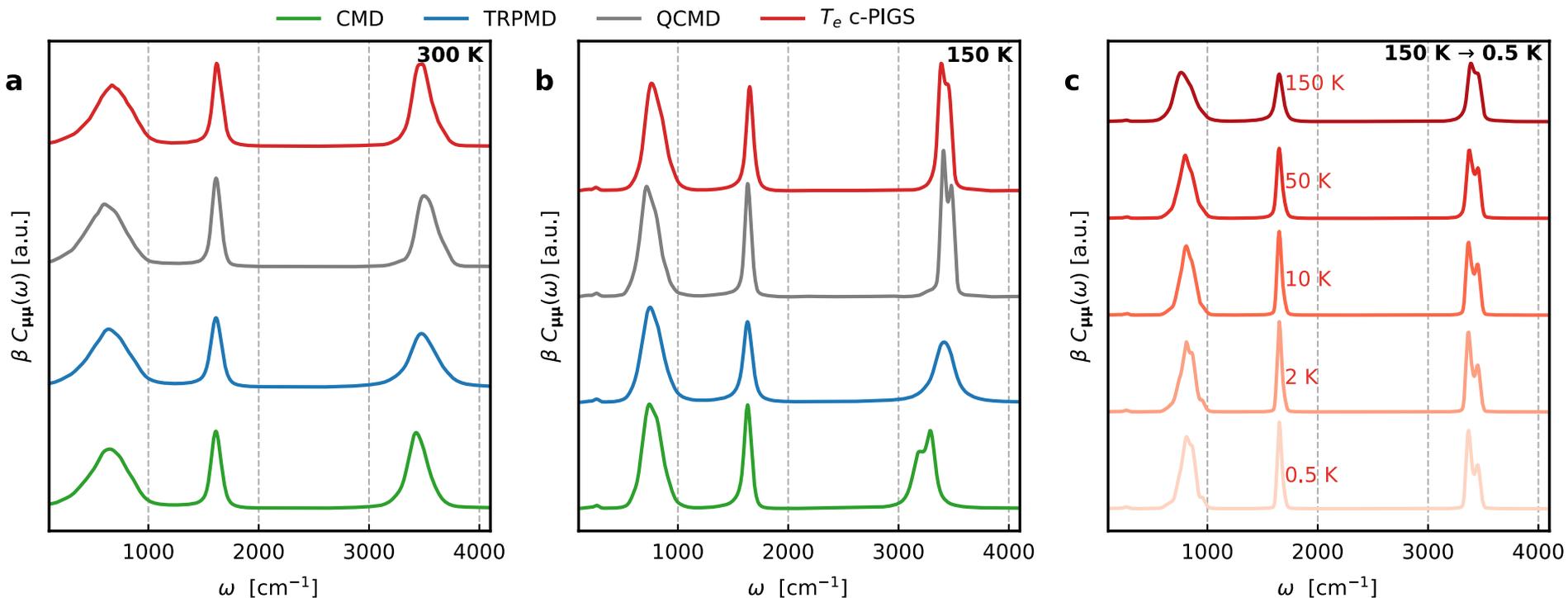
$T_e$



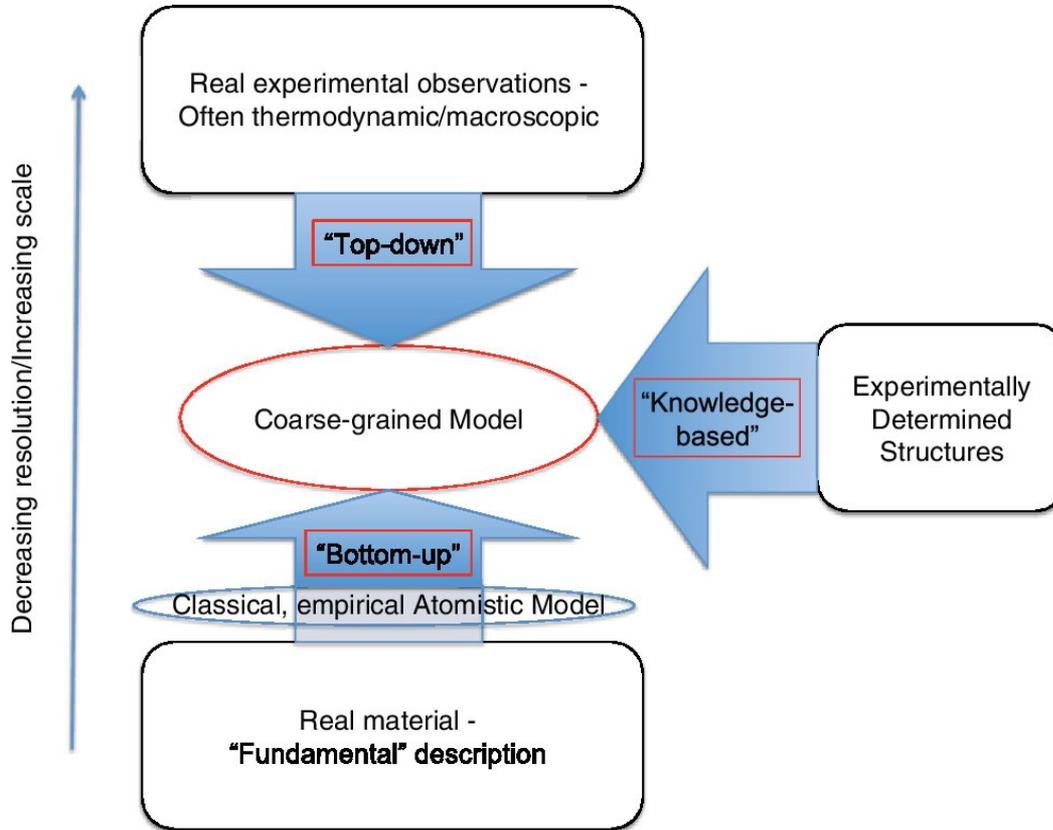
# Water molecule



# Bulk water



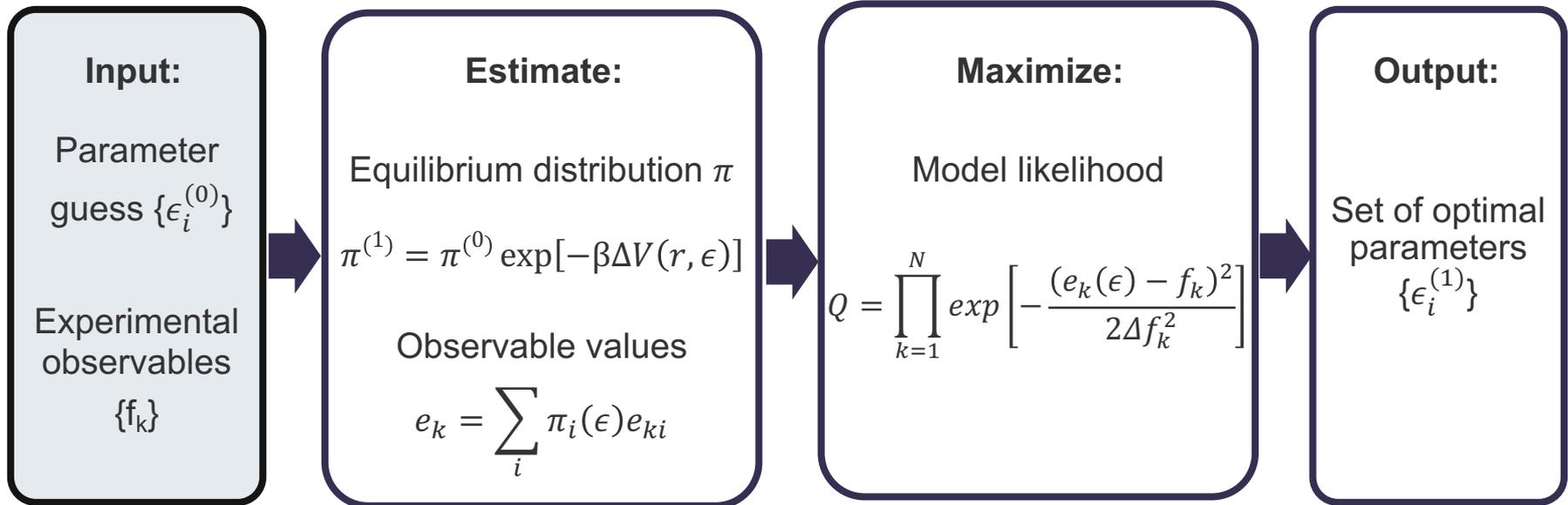
# CG models: which properties to preserve?



1. Learning CG models from all-atom simulations
2. Incorporating nuclear quantum effects into all-atom simulation
3. **Learning CG models from experimental data**

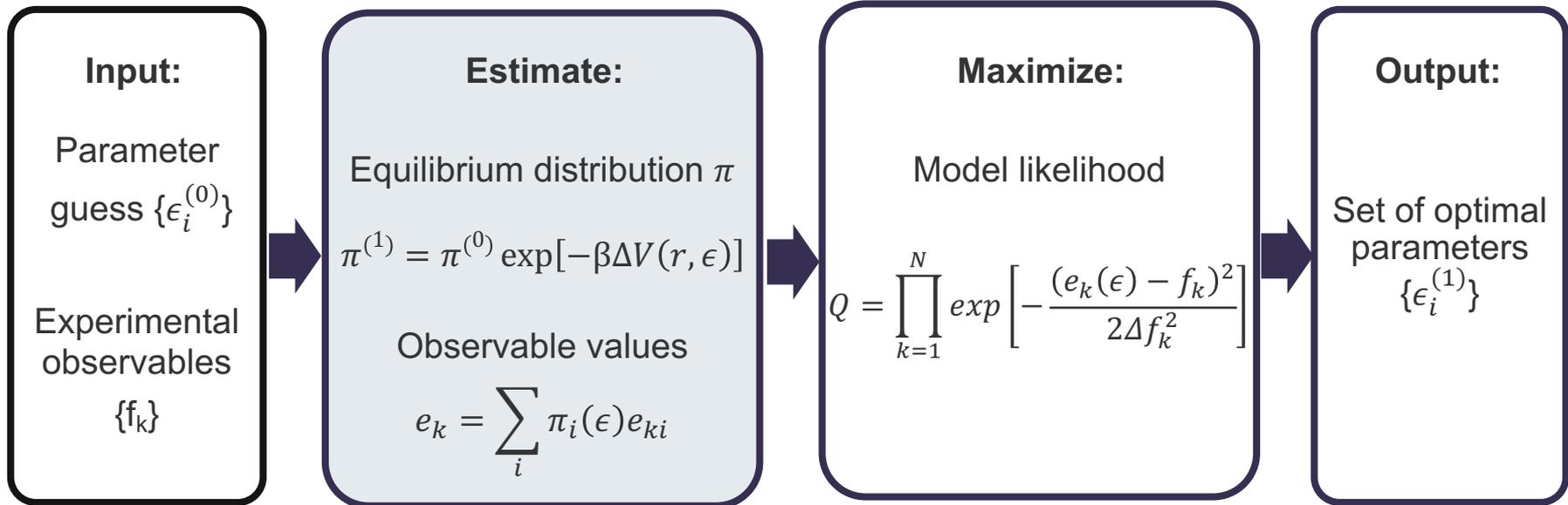
# Observable-driven Design of Effective Molecular Models (ODEM)

$$V = V_{bond}(r, k_b) + V_{angle}(r, k_a) + V_{dihedral}(r, k_d) + V_{nonbonded}(r, \epsilon)$$



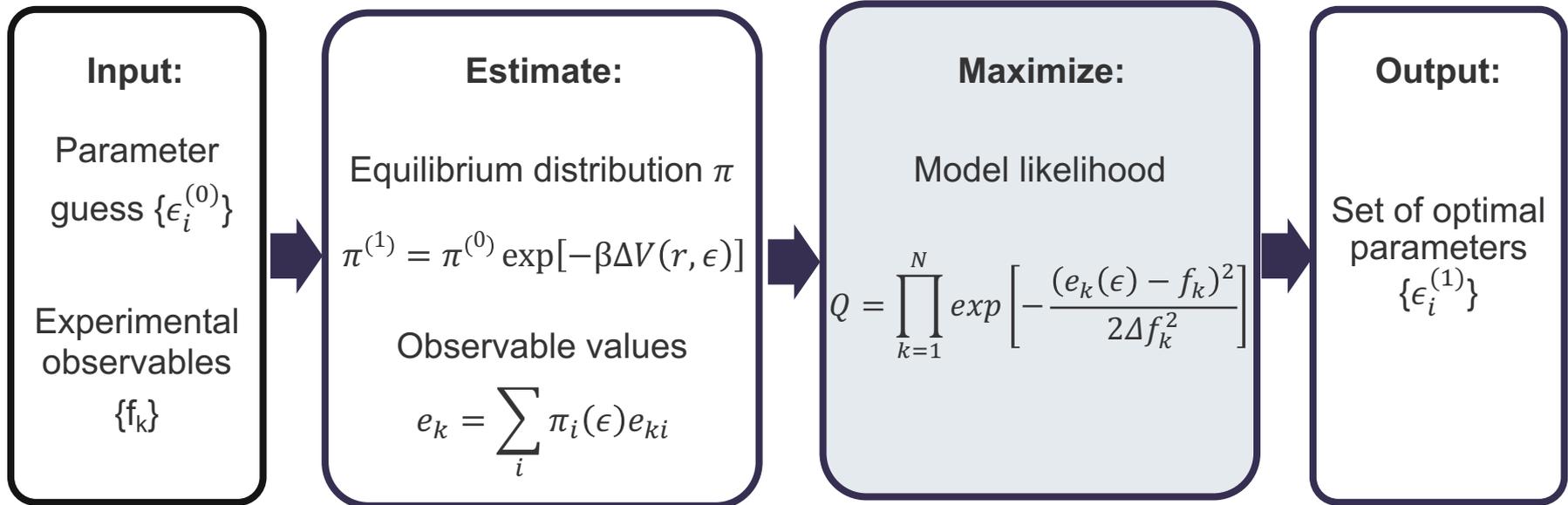
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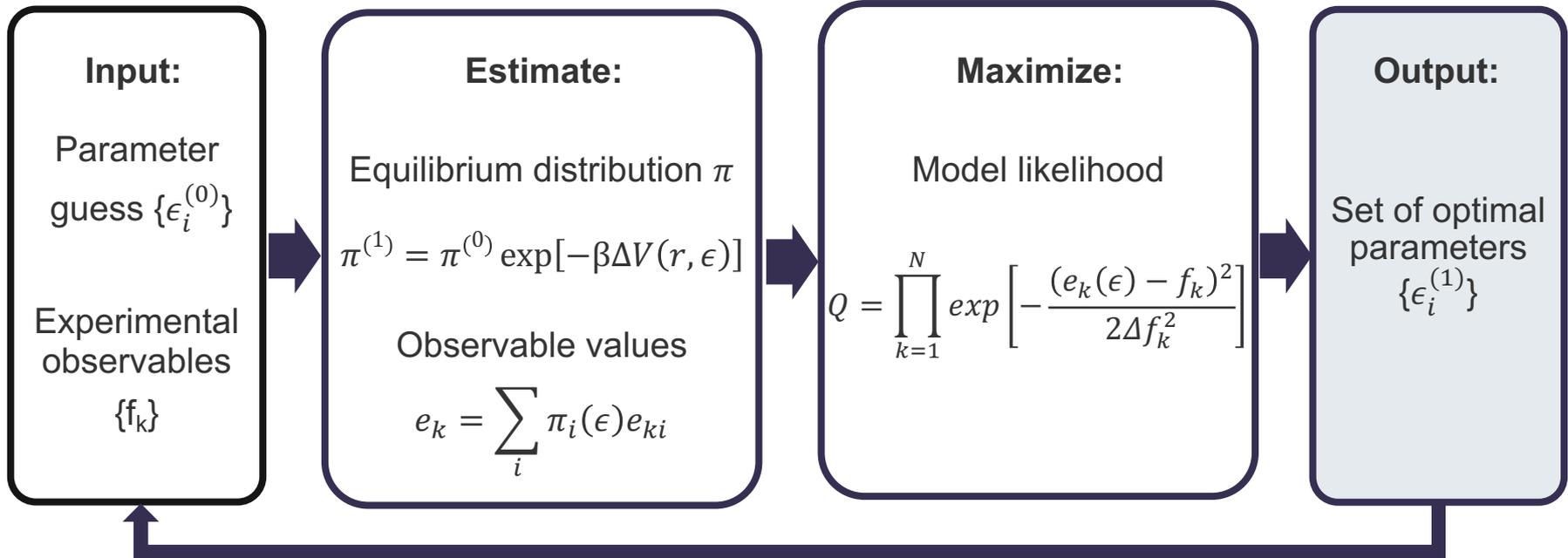
# Observable-driven Design of Effective Molecular Models (ODEM)

$$V = V_{bond}(r, k_b) + V_{angle}(r, k_a) + V_{dihedral}(r, k_d) + V_{nonbonded}(r, \epsilon)$$

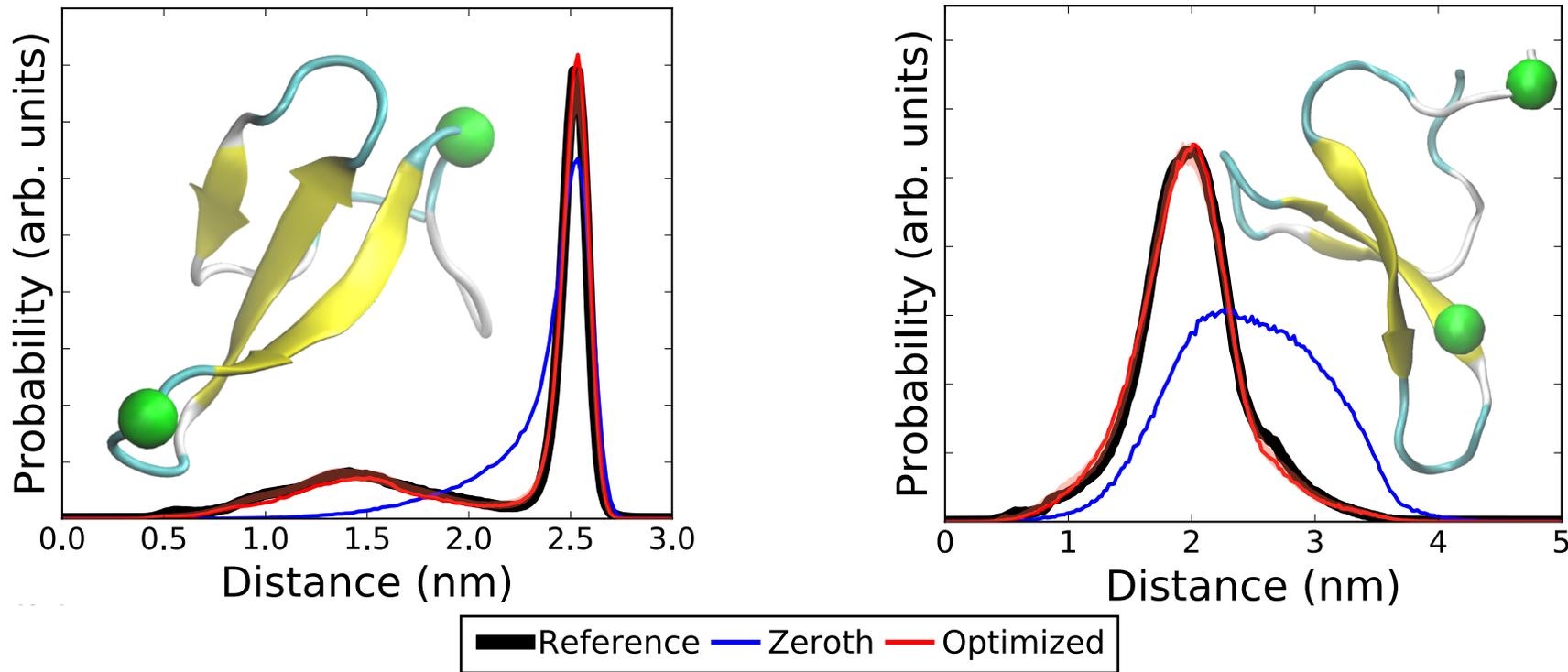


# Observable-driven Design of Effective Molecular Models (ODEM)

$$V = V_{bond}(r, k_b) + V_{angle}(r, k_a) + V_{dihedral}(r, k_d) + V_{nonbonded}(r, \epsilon)$$



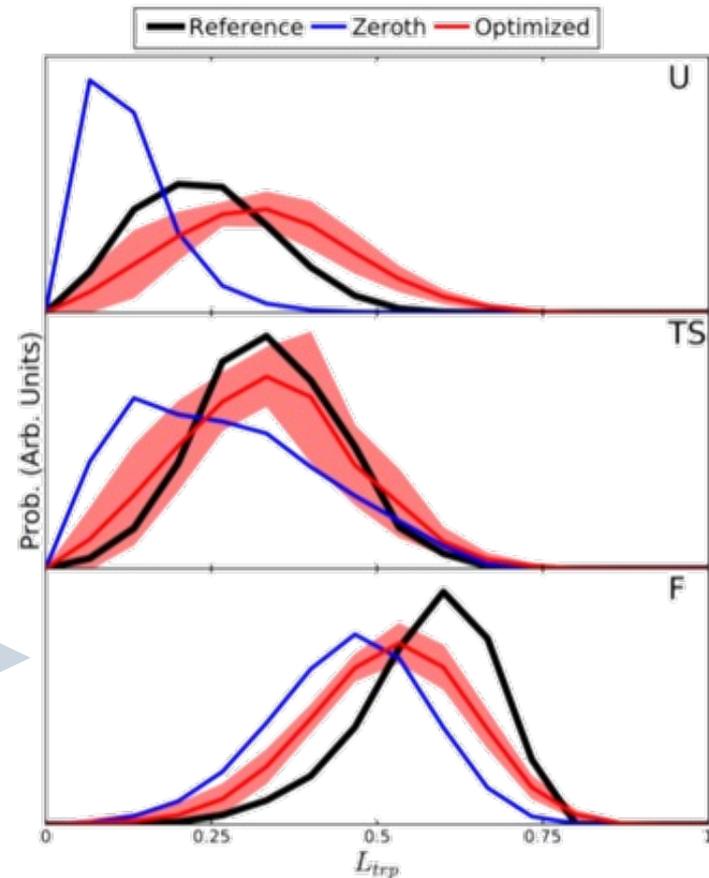
# CG model of FIP35 from "synthetic FRET"



# Model parametrized with FRET reproduces TRP fluorescence

Model trained on synthetic FRET data can better reproduce folding mechanism and TRP fluorescence

Histograms for the simulated Trp fluorescence measurements for unfolded (U), transition (TS), and folded (F) states:



# Model parametrization with NMR data

- **Test system:**

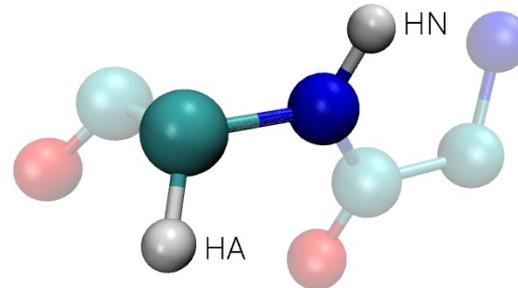
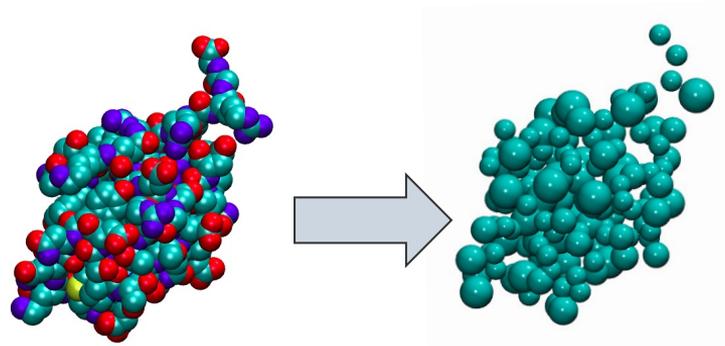
Ubiquitin, structure-based  
CaC $\beta$  model.

- **Adjustable parameters:**

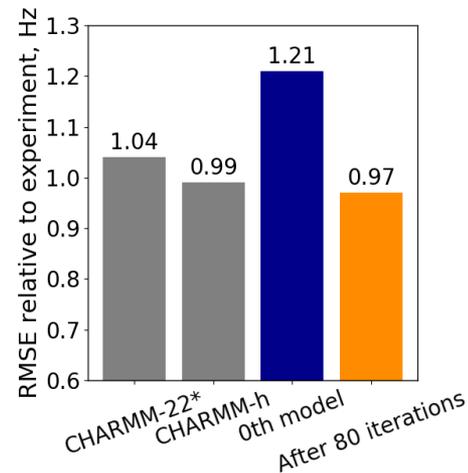
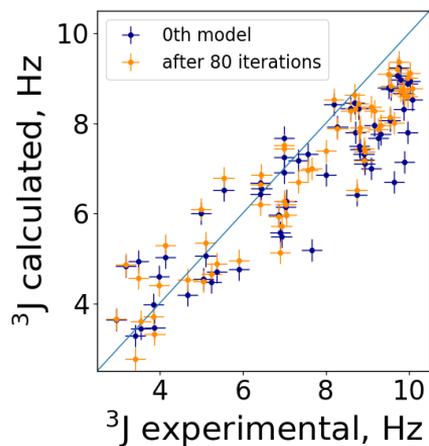
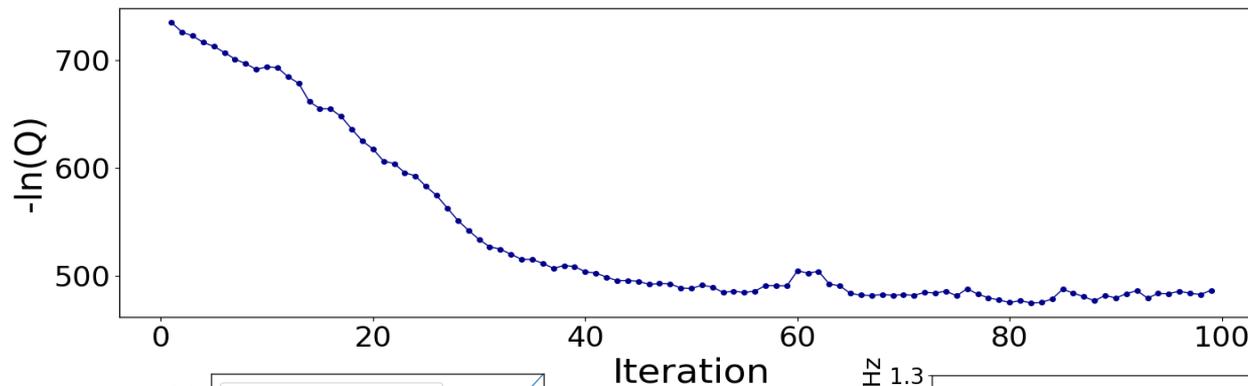
Strength of native interactions  
(219 parameters)

- **Experimental observables:**

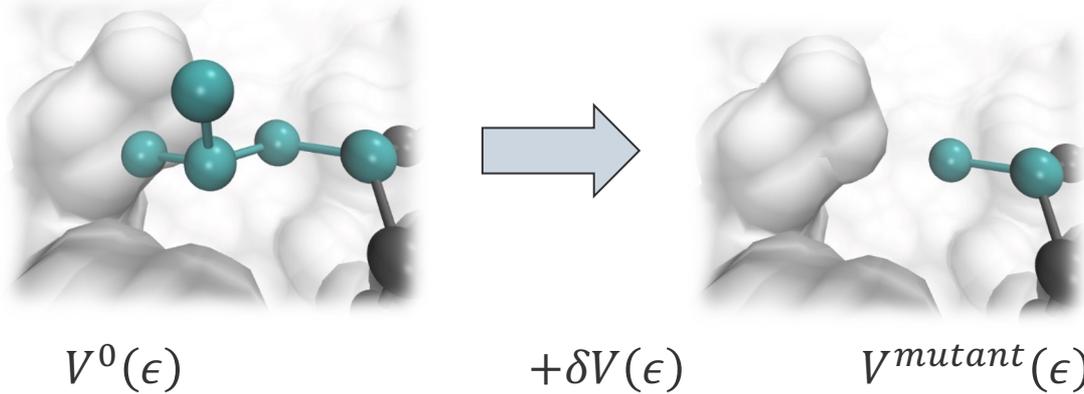
Scalar spin-spin  
coupling constants  
( $^3J$  (HA-HN), N = 63)



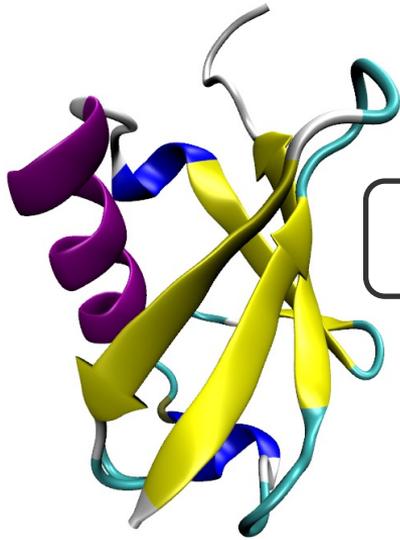
# Model parametrization with NMR data



$$\Delta\Delta G = \Delta G_{folding}^{mutant} - \Delta G_{folding}^{wild-type}$$



$$\beta\Delta\Delta G = \ln \frac{\langle \exp(-\beta\delta V(\epsilon)) \rangle^{unfolded}}{\langle \exp(-\beta\delta V(\epsilon)) \rangle^{folded}}$$



1UBQ  
76 residues



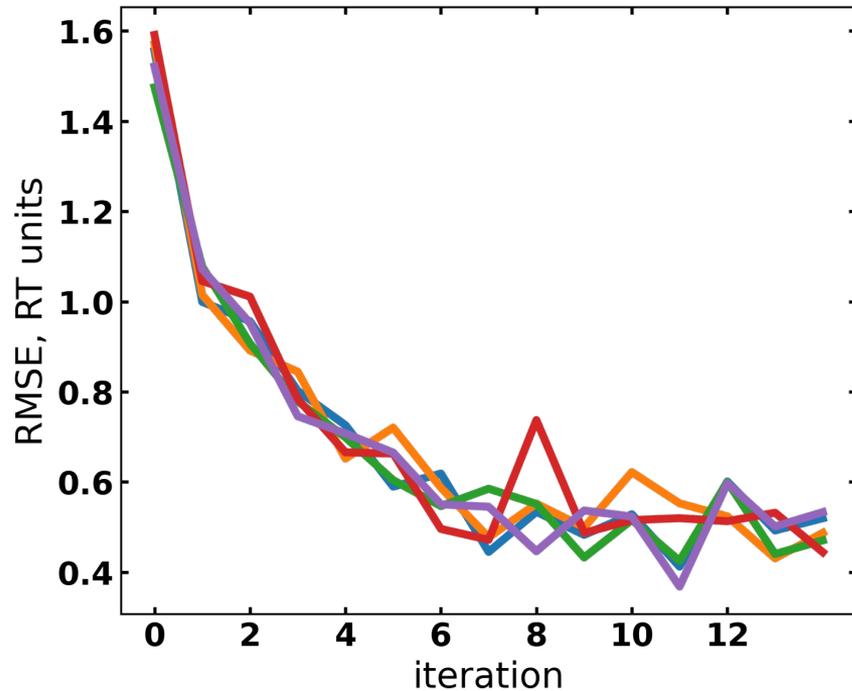
1PGB  
56 residues

## **Adjustable parameters:**

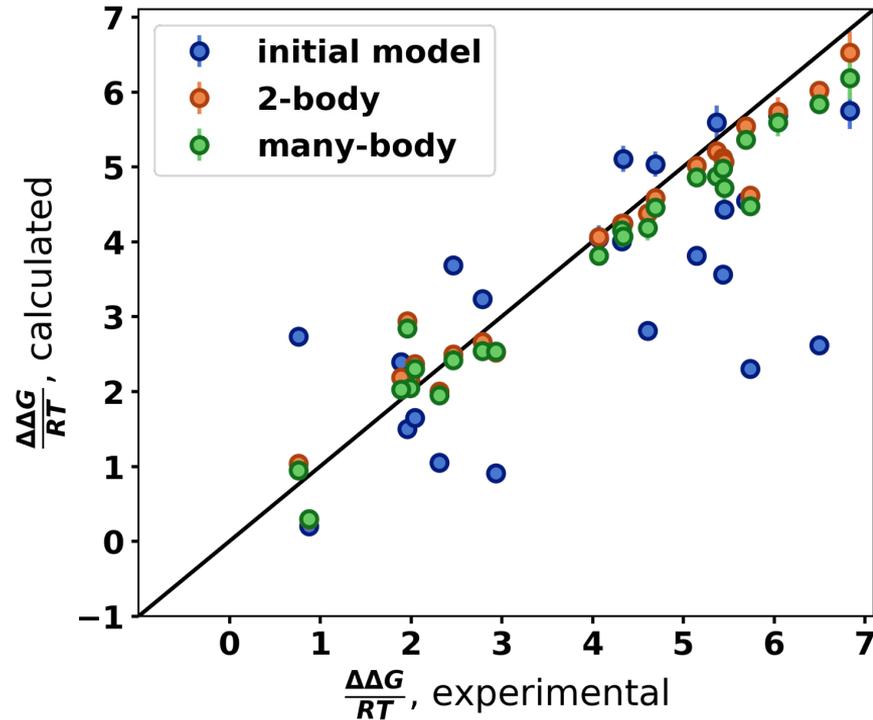
Strength of both native and nonnative interactions

**Representation:**  $\text{C}\alpha$ -model

# Coarse-grained modelling with mutational data

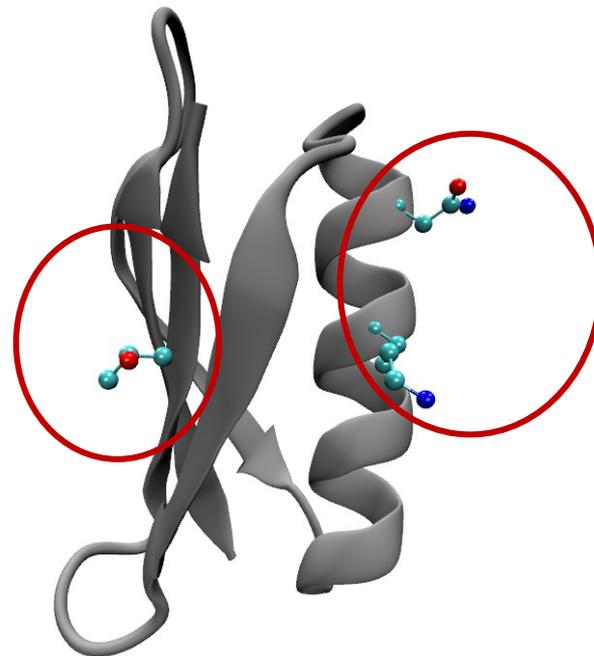
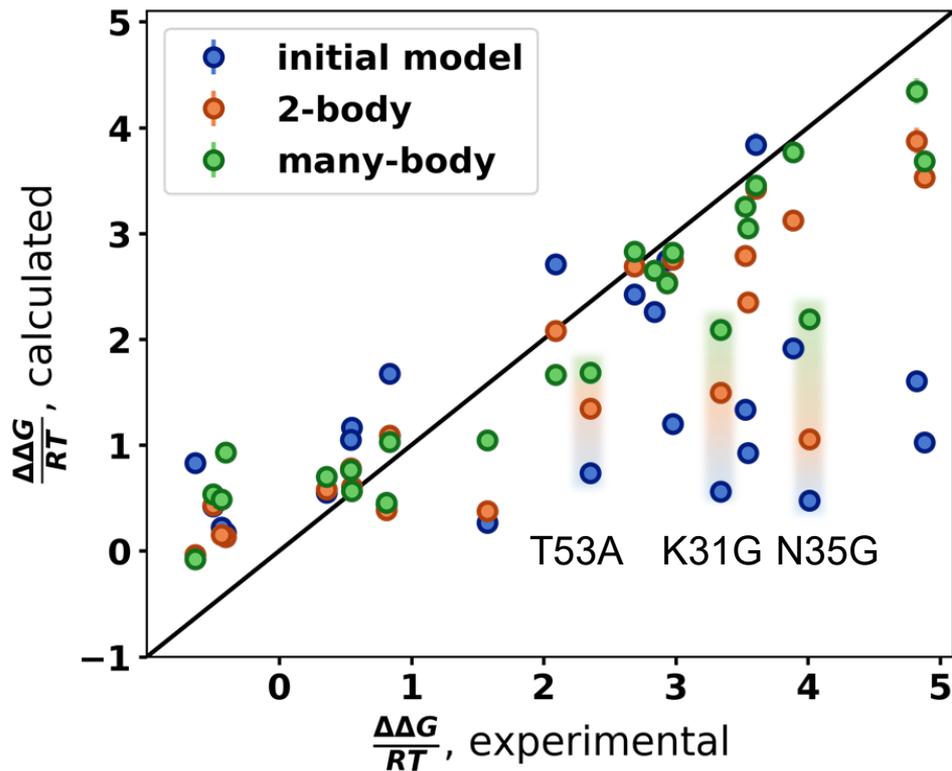


Root mean square error:  
change during the optimization



Correlation between calculated  
and experimental data

# Coarse-grained modelling with mutational data: results for protein G



# Protein G: Multibody terms are needed!

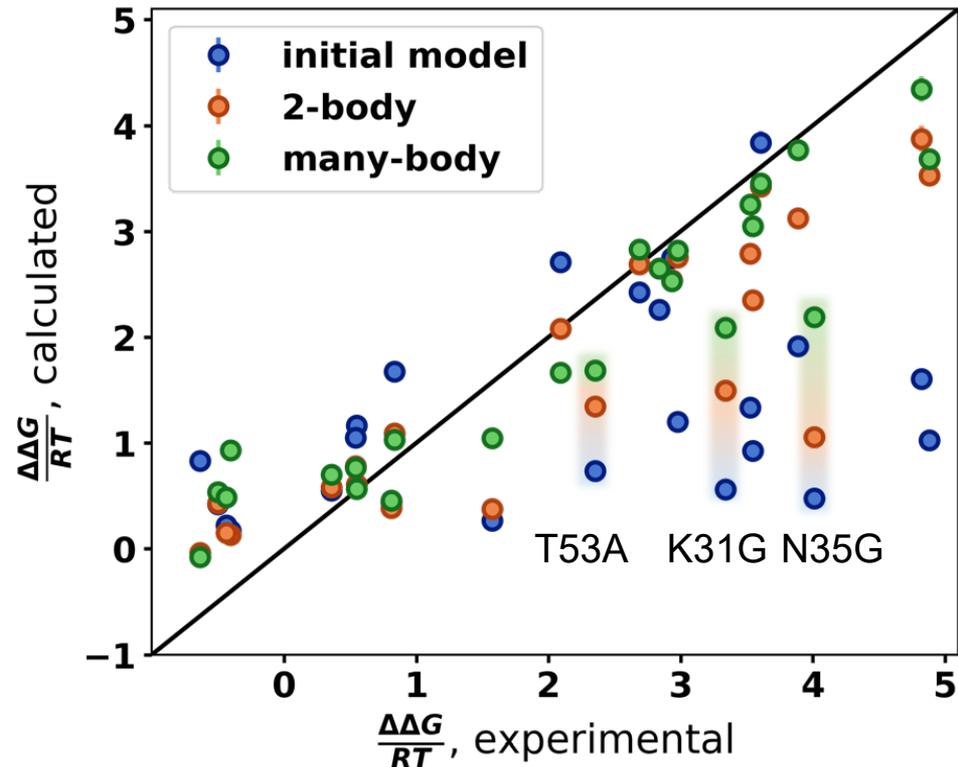
$$V = V_{2\text{-body}} + \frac{\lambda_{\text{burial}}}{2} \sum_{\mu=1}^3 \sum_{i=1}^N \gamma_{\text{burial}}(a_i, \mu) [\tanh(\eta(\rho_i - \rho_{\text{min}}^{\mu})) + \tanh(\eta(\rho_{\text{max}}^{\mu} - \rho_i))]$$

$\eta, \lambda_{\text{burial}}, \rho_{\text{max}}, \rho_{\text{min}}$  - fixed parameters

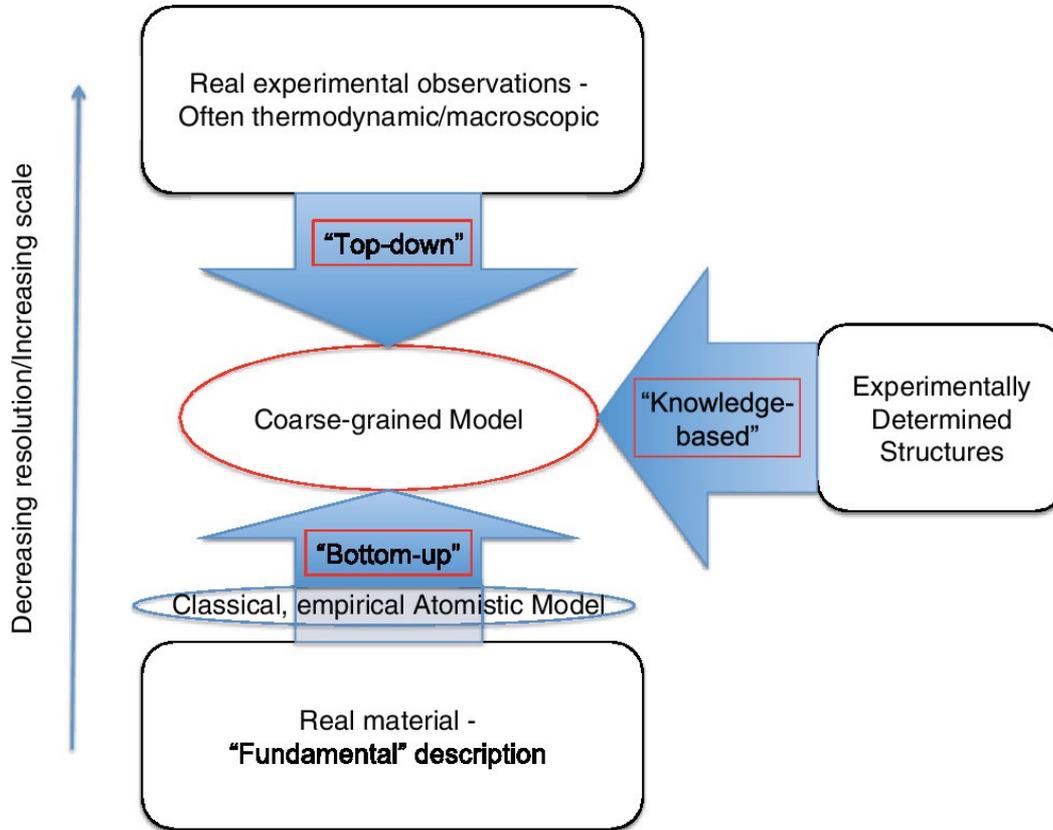
$\gamma_{\text{burial}}(a_i, \mu)$  - learnable parameters

$\rho$  - local density

Additional multibody terms produce better agreement with experimental data



# Future direction



**Ultimate goal:**  
combine  
top-down and bottom-up  
approaches to construct  
accurate and transferable yet  
computationally inexpensive  
coarse-grained force fields

# Cecilia Clementi's research group

<https://www.physik.fu-berlin.de/en/einrichtungen/ag/ag-clementi/>



## In collaboration with:

Frank Noé  
Gianni de Fabritiis  
Venkat Kapil

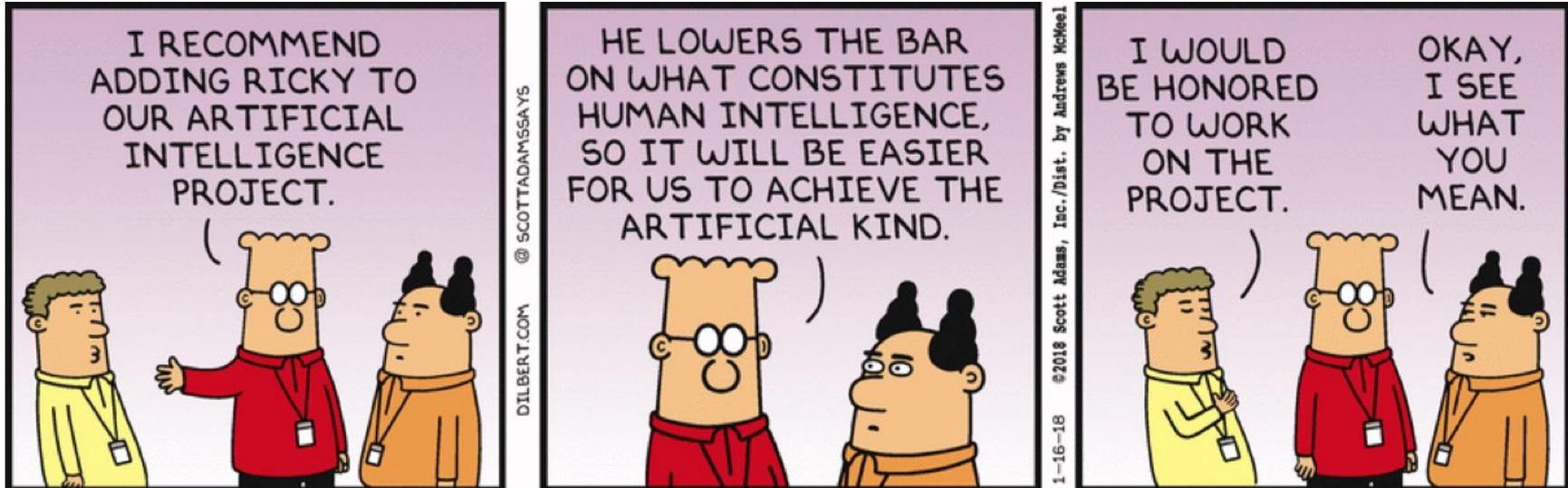
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Nick Charron  
Iryna Zaporozhets  
Tim Hempel  
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Dr. Eugen Hruska (Emory)  
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Dr. Feliks Nüske (U. Paderborn)  
Dr. Lorenzo Boninsegna (UCLA)  
Dr. Fernando Yrazu (Rice)  
Dr. Jordane Preto (U Alberta)  
Dr. Mary Rohrdanz (MD Anderson)  
Dr. Wenwei Zheng (U Arizona)  
Dr. Amarda Shehu (GMU)  
Dr. Payel Das (IBM)  
Dr. Silvina Matysiak (U. Maryland)  
Dr. Brad Lambeth (Shell)

# Thank you for your attention!



## Questions?