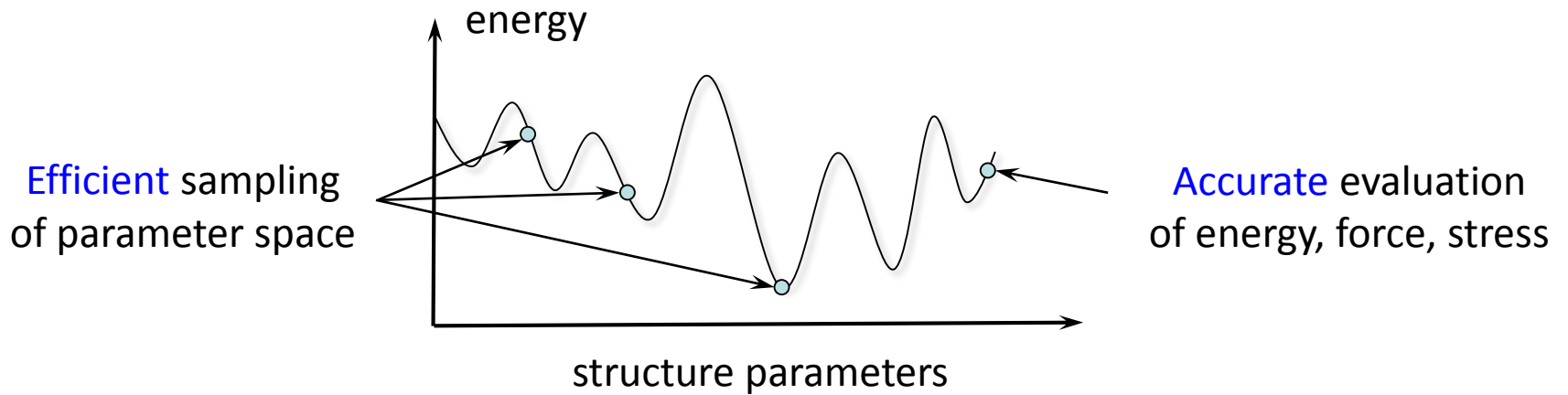


# Construction of standardized neural network-based interatomic models for structure prediction acceleration

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Binghamton University  
State University of New York

|                            |                            |                            |                                                                      |                            |                            |                            |
|----------------------------|----------------------------|----------------------------|----------------------------------------------------------------------|----------------------------|----------------------------|----------------------------|
| <sup>3</sup><br><b>Li</b>  | <sup>4</sup><br><b>Be</b>  | <sup>5</sup><br><b>B</b>   | <b>MAISE library<br/>of multicomponent<br/>neural network models</b> |                            |                            |                            |
| <sup>11</sup><br><b>Na</b> | <sup>12</sup><br><b>Mg</b> | <sup>13</sup><br><b>Al</b> |                                                                      |                            |                            |                            |
| <sup>19</sup><br><b>K</b>  | <sup>20</sup><br><b>Ca</b> | <sup>21</sup><br><b>Sc</b> | <sup>22-27</sup><br><b>3d</b>                                        | <sup>28</sup><br><b>Ni</b> | <sup>29</sup><br><b>Cu</b> | <sup>30</sup><br><b>Zn</b> |
| <sup>37</sup><br><b>Rb</b> | <sup>38</sup><br><b>Sr</b> | <sup>39</sup><br><b>Y</b>  | <sup>40-45</sup><br><b>4d</b>                                        | <sup>46</sup><br><b>Pd</b> | <sup>47</sup><br><b>Ag</b> | <sup>48</sup><br><b>Cd</b> |
| <sup>55</sup><br><b>Cs</b> | <sup>56</sup><br><b>Ba</b> | <sup>57</sup><br><b>La</b> | <sup>72-77</sup><br><b>5d</b>                                        | <sup>78</sup><br><b>Pt</b> | <sup>79</sup><br><b>Au</b> | <sup>80</sup><br><b>Hg</b> |

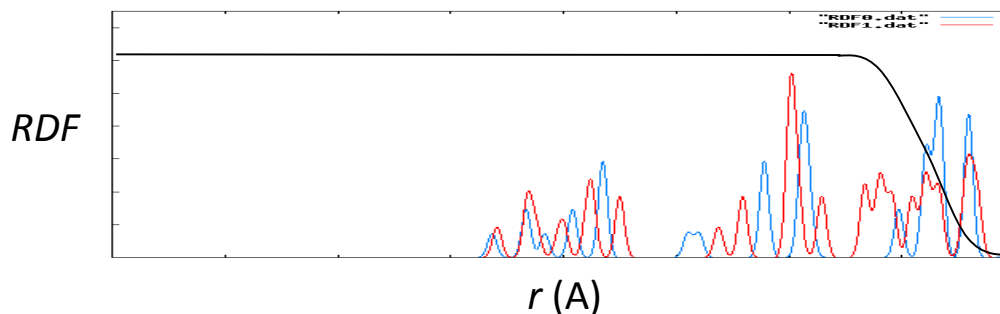
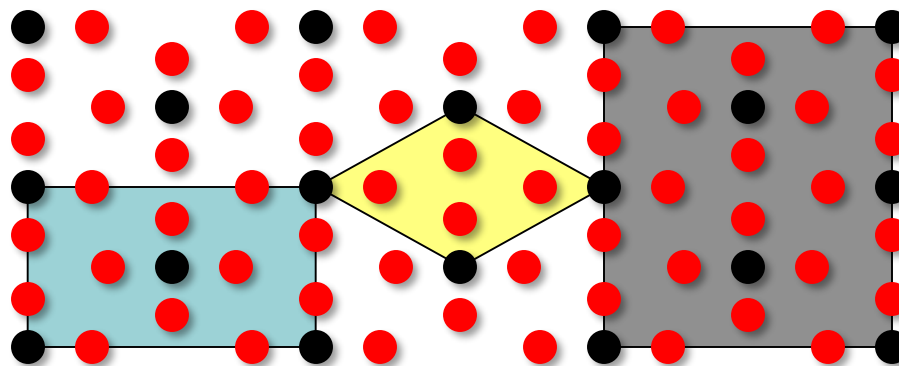


## Structure prediction

search strategies  
confirmed predictions  
need for acceleration

## Interaction description with neural networks

advantages and challenges  
hierarchical (stratified) construction  
performance and application



$$C_1 \times C_2 = \int_0^{R_{cut}} \sum_{ij} RDF_{1,ij}(r) RDF_{2,ij}(r) dr,$$

### [Duplicate elimination in ES](#)

Kolmogorov *et al.* PRL 105, 217003 (2010)

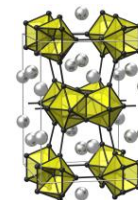
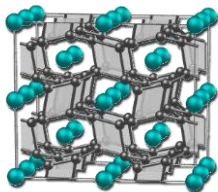
### [Atomic environment analysis](#)

Choi *et al.* PRL 108, 127204 (2012)

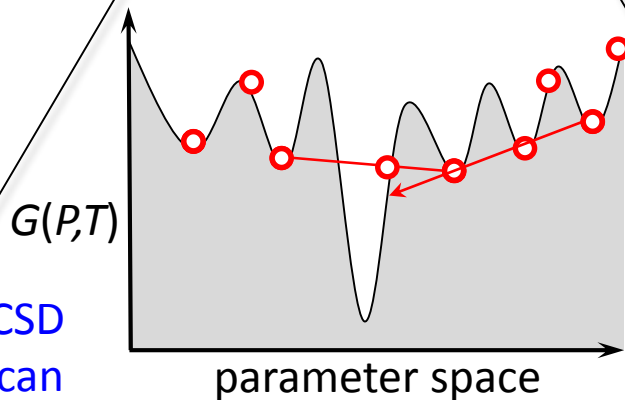
### [Distortion analysis](#)

Kolmogorov *et al.* PRL 109, 075501 (2012)

## Unconstrained search



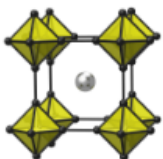
random Monte-Carlo evolutionary



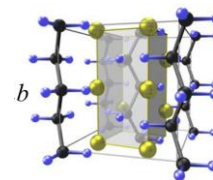
ICSD scan

trend analysis

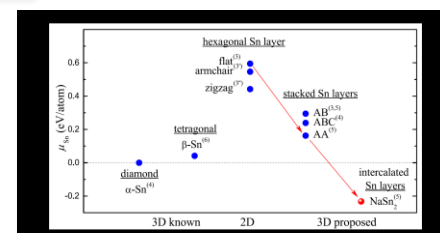
## Constrained search



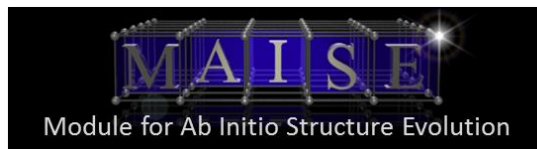
## 'Rational' design



|    |    |                  |    |    |    |    |    |    |    |    |    |    |
|----|----|------------------|----|----|----|----|----|----|----|----|----|----|
| Li | Be | 41 M-B systems   |    |    |    |    |    |    |    |    |    | B  |
| Na | Mg | 12,000 compounds |    |    |    |    |    |    |    |    |    | Al |
| K  | Ca | Sc               | Ti | V  | Cr | Mn | Fe | Co | Ni | Cu | Zn | Ga |
| Rb | Sr | Y                | Zr | Nb | Mo | Tc | Ru | Rh | Pd | Ag | Cd | In |
| Cs | Ba | La               | Hf | Ta | W  | Re | Os | Ir | Pt | Au | Hg | Tl |



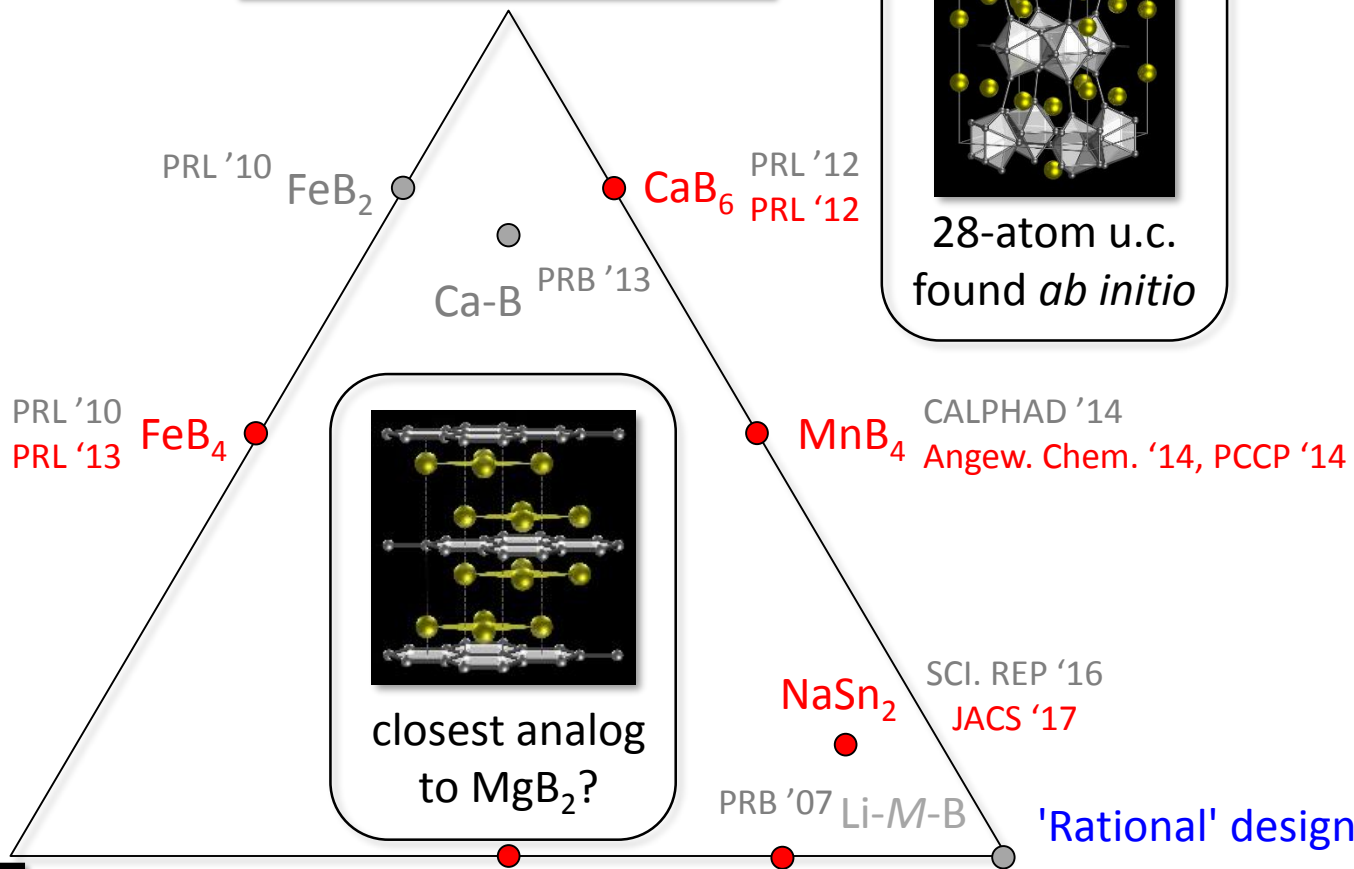
## Unconstrained search



predictions  
confirmations

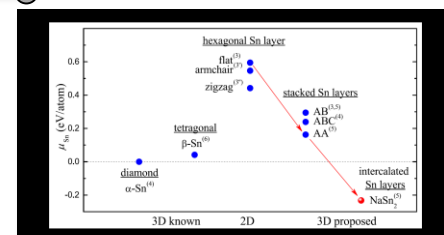
28-atom u.c.  
found *ab initio*

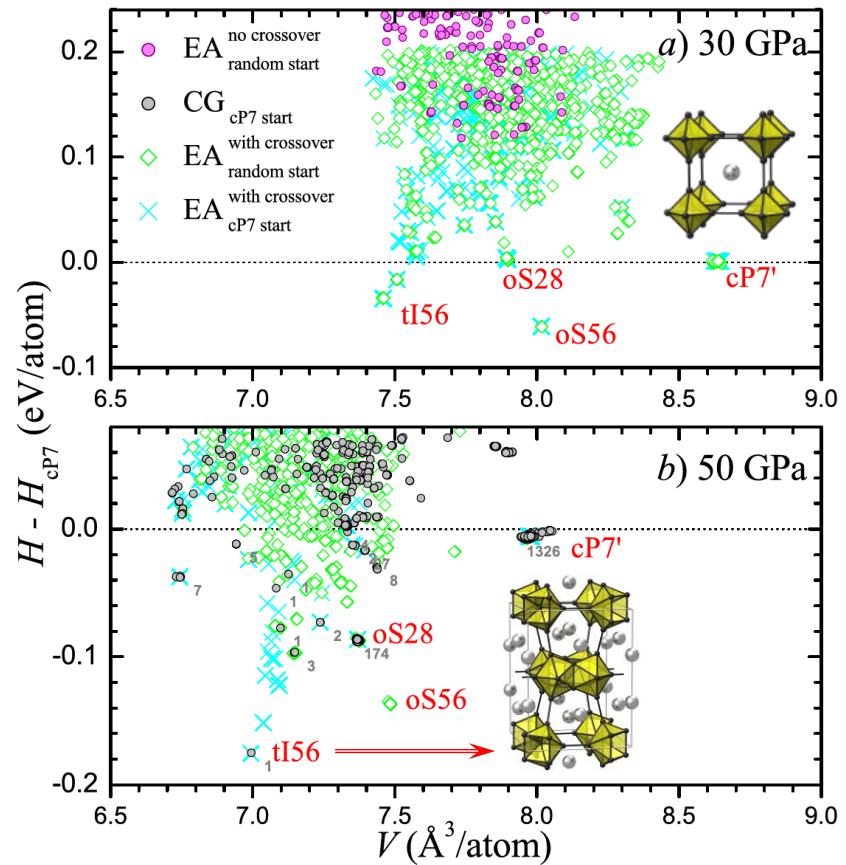
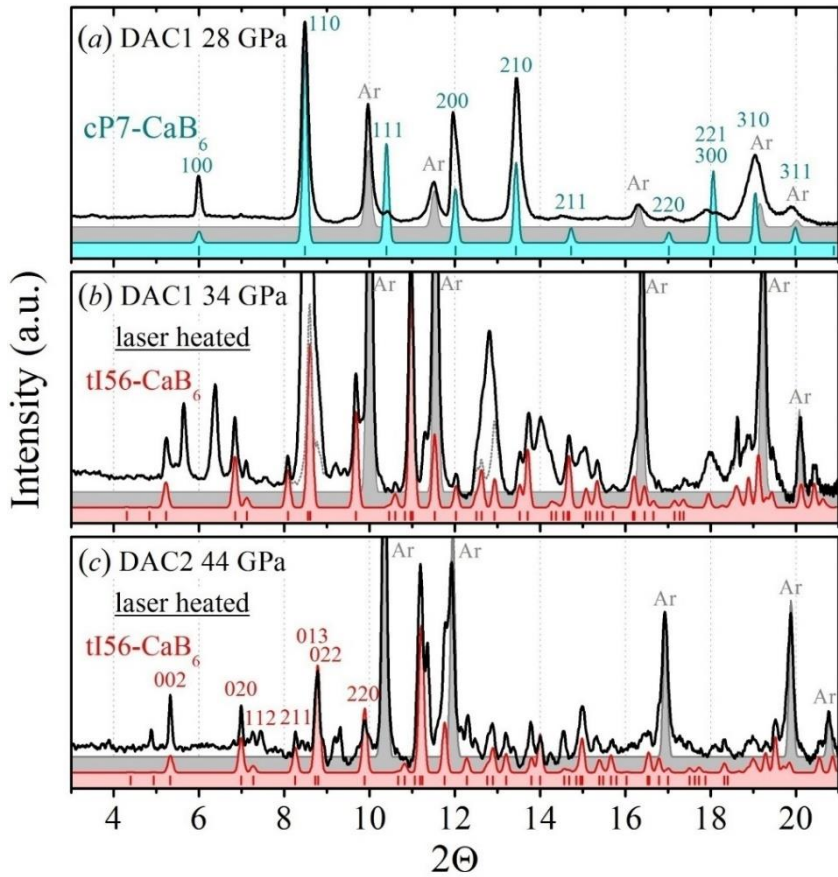
confirmed BCS  
superconductor



## ICSD screening

|    |    |                  |    |    |    |    |    |    |    |    |    |    |  |  |  |  |  |    |
|----|----|------------------|----|----|----|----|----|----|----|----|----|----|--|--|--|--|--|----|
| Li | Be | 41 M-B systems   |    |    |    |    |    |    |    |    |    |    |  |  |  |  |  | B  |
| Na | Mg | 12,000 compounds |    |    |    |    |    |    |    |    |    |    |  |  |  |  |  | Al |
| K  | Ca | Sc               | Ti | V  | Cr | Mn | Fe | Co | Ni | Cu | Zn | Ga |  |  |  |  |  |    |
| Rb | Sr | Y                | Zr | Nb | Mo | Tc | Ru | Rh | Pd | Ag | Cd | In |  |  |  |  |  |    |
| Cs | Ba | La               | Hf | Ta | W  | Re | Os | Ir | Pt | Au | Hg | Tl |  |  |  |  |  |    |





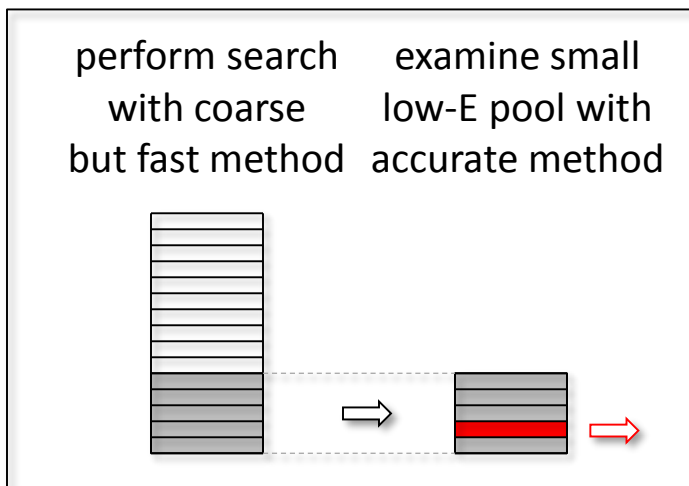
|                       |                                    | From scratch                | Supercell initialization   | Known lattice parameters |
|-----------------------|------------------------------------|-----------------------------|----------------------------|--------------------------|
| 28 $\text{CaB}_6$ [1] | With crossover (evolutionary)      | $\diamond \sim 10,000$ runs | $\times \sim 500$ runs     | $\sim 200$ runs          |
|                       | Without crossover (Monte-Carlo)    | $\bullet$ failed            |                            | $\sim 200$ runs          |
|                       | Conjugate gradient (deterministic) |                             | $\bullet \sim 10,000$ runs |                          |
| 28 $\gamma$ -B [2]    | Conjugate gradient (deterministic) |                             |                            | $\sim 200$ runs          |
| 44 Li [3]             | Conjugate gradient (deterministic) |                             |                            | $\sim 200$ runs          |

[1] Kolmogorov et al., PRL 105, 217003 (2012)

[2] Oganov et al., Nature 457, 863 (2009)

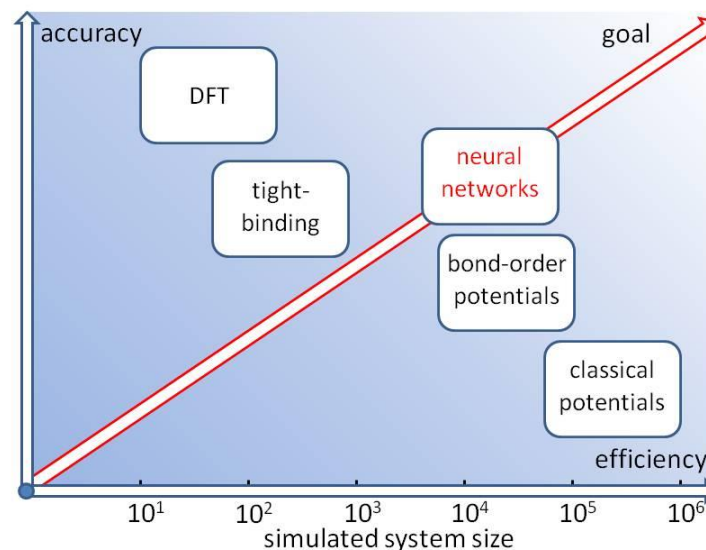
[3] Marques et al., PRL 106, 095502 (2011)

## Basic acceleration strategy

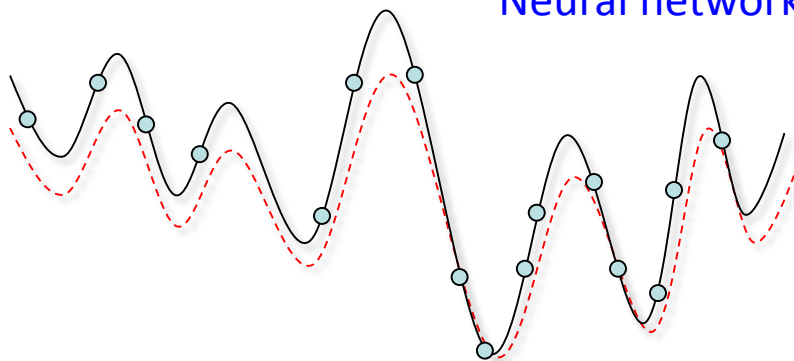


Farrow, Chow, Woodley, PCCP 16, 21119 (2014)  
Wu, *et al.*, K.M. Ho: JPCM 26, 035402 (2014)

## Interaction description methods



## Neural network interpolator of DFT data



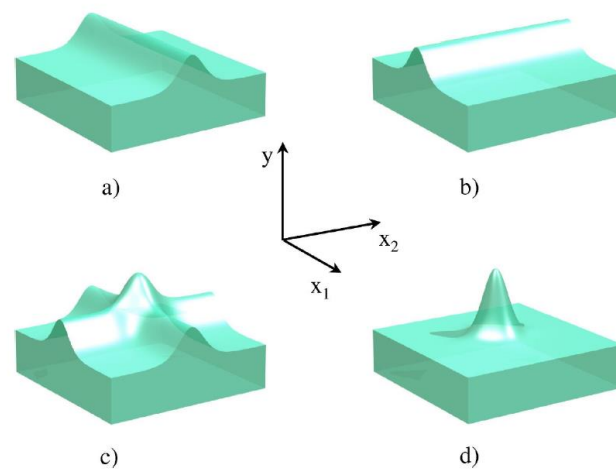
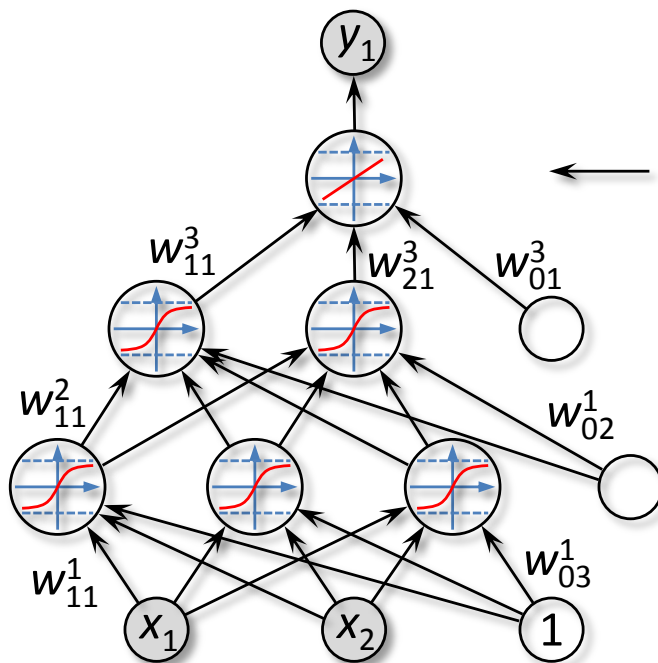
|                            |                        |
|----------------------------|------------------------|
| generate DFT database      | 50,000 CPU h / system  |
| train neural network       | 1,000 CPU h / system   |
| expected/observed speed-up | $\times 10^3$ - $10^4$ |

## DFT reliability for predicting ground states (PBE/LDA, T = 0 K)

metal-metal compound stability  
metal-boron compound stability

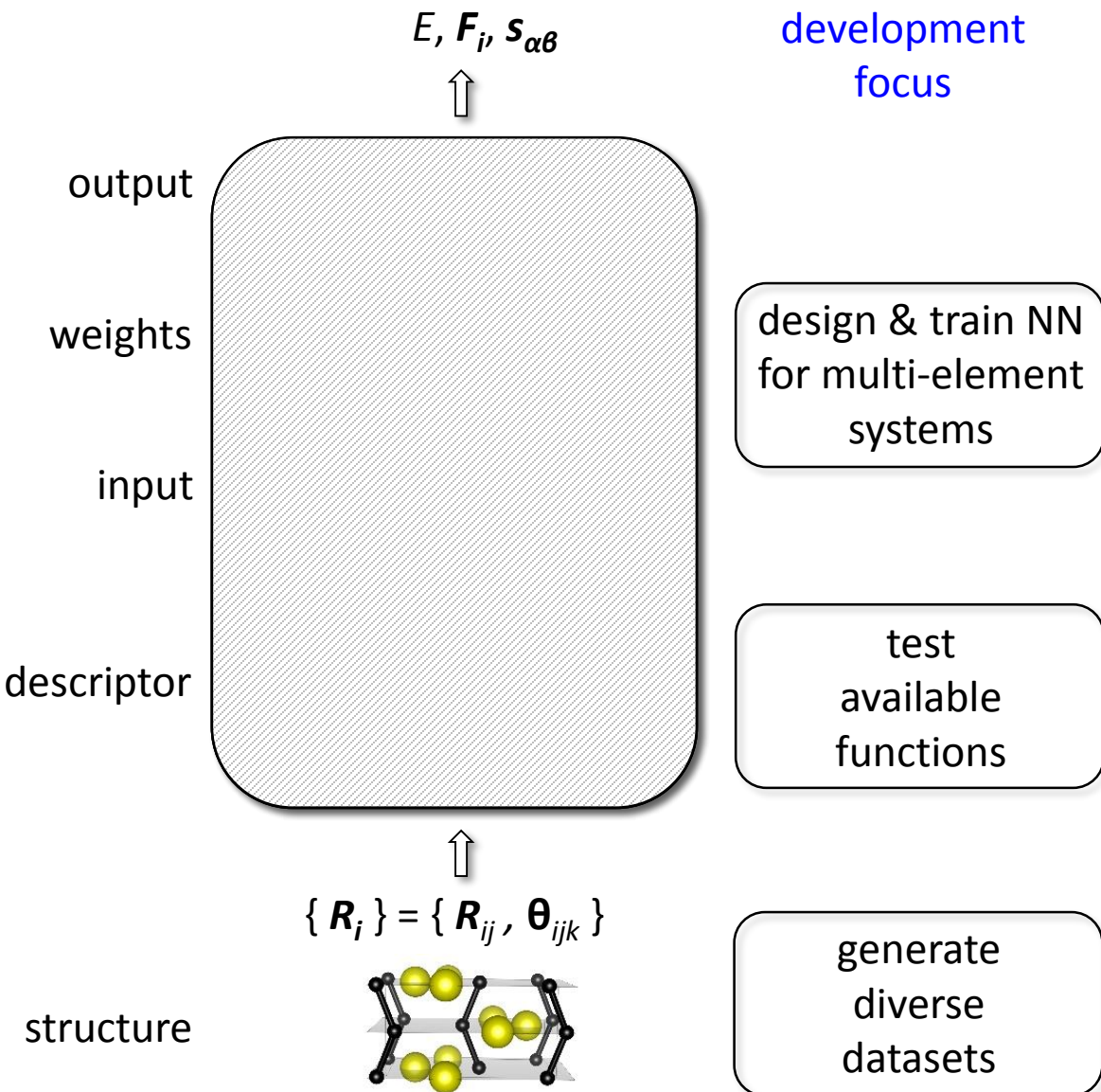
97% agreement with experiment, Curtarolo 2005  
83% agreement with experiment, Kolmogorov 2014

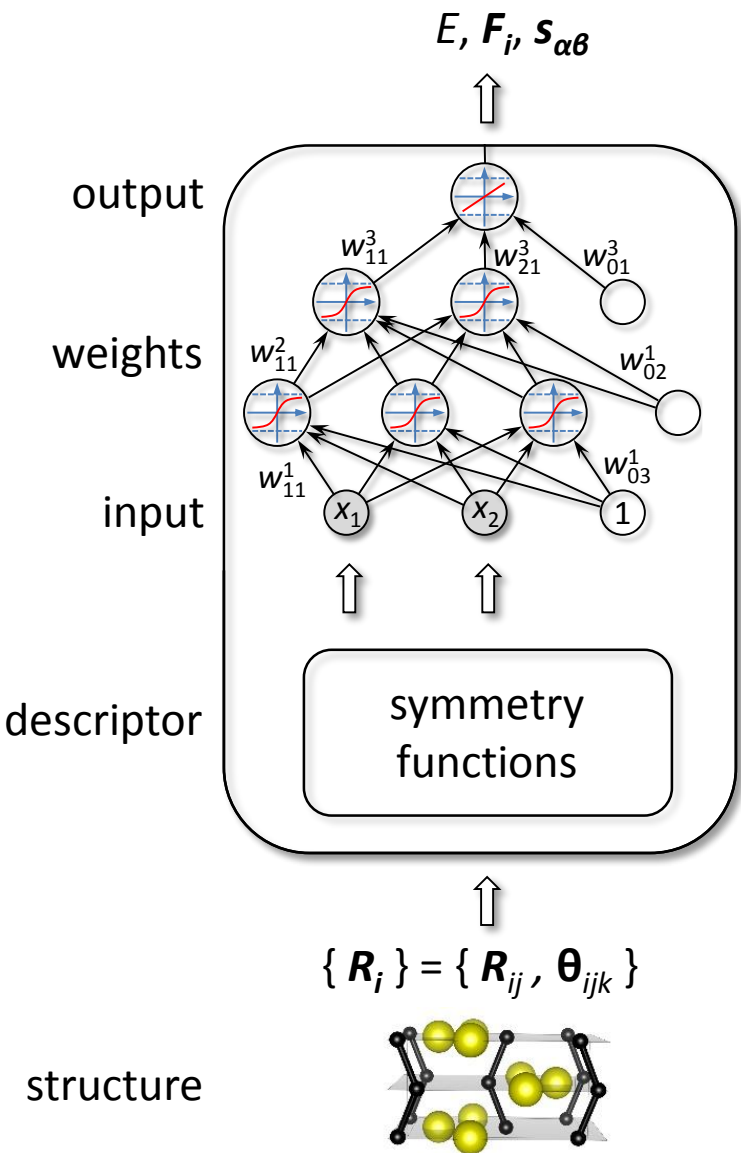
example of a peak approximation with a multi-layer neural network  
the position, shape, and height controlled by weights



possible to represent a continuous multivariable function  
by superposition of functions of one variable







development focus

design & train NN for multi-element systems

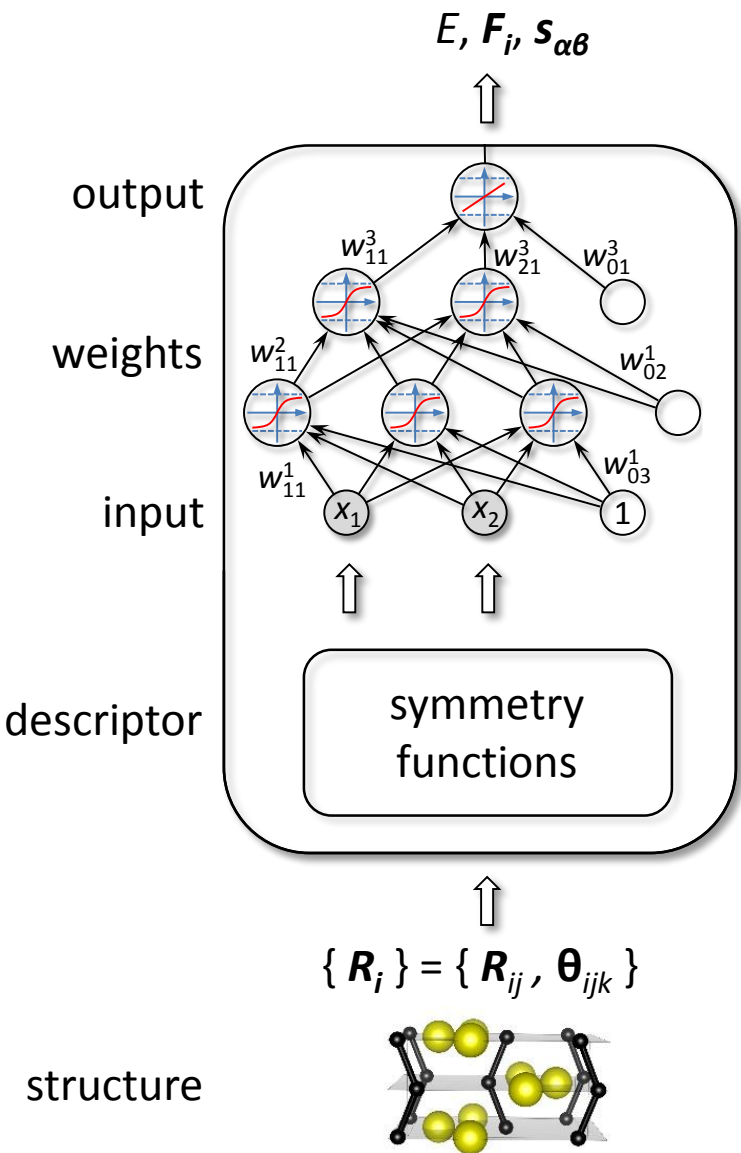
test available functions

generate diverse datasets

data generation protocols

traditional MD-based vs proposed evolutionary

sampling of relevant space  
automated generation  
built-in data diversification



development focus

design & train NN for multi-element systems

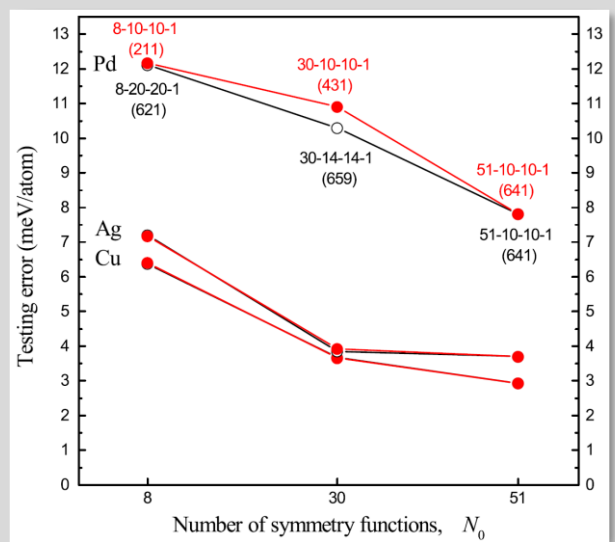
test available functions

generate diverse datasets

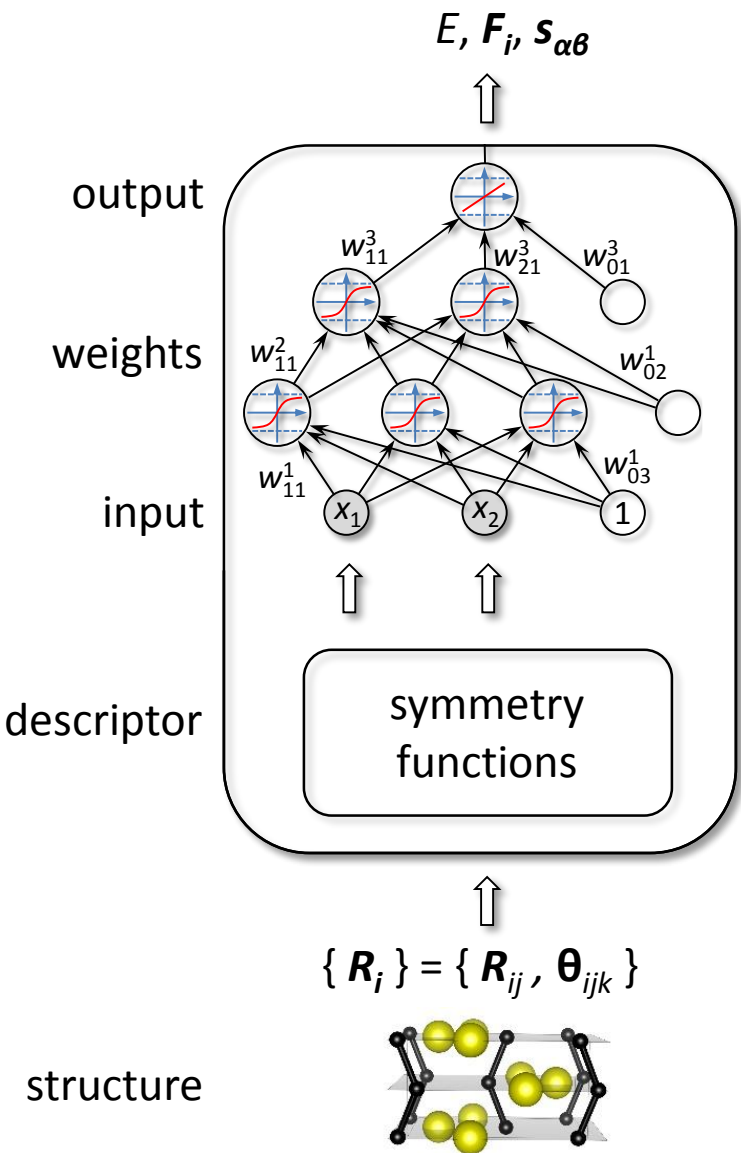
descriptor choice

Bartok et al. PRB 87, 184115 (2013)  
Behler Parrinello PRL 98, 146401 (2007)

test & use of PB descriptor



good for metallic bonding up to 3-body terms  
30 or 51-function sets

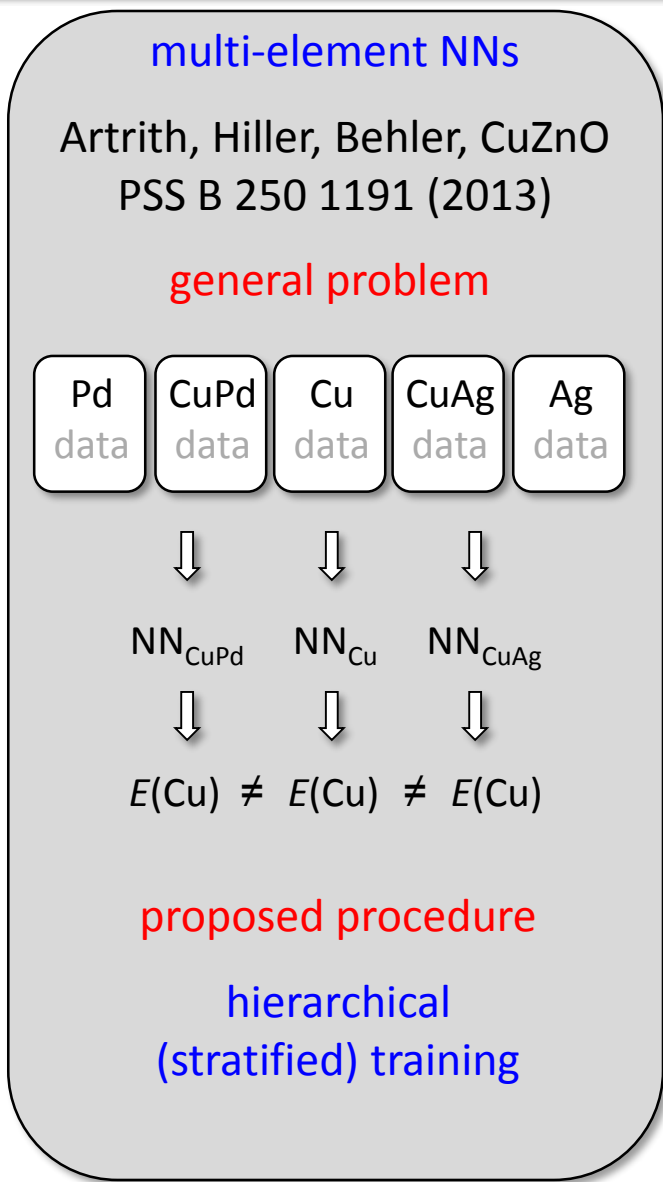


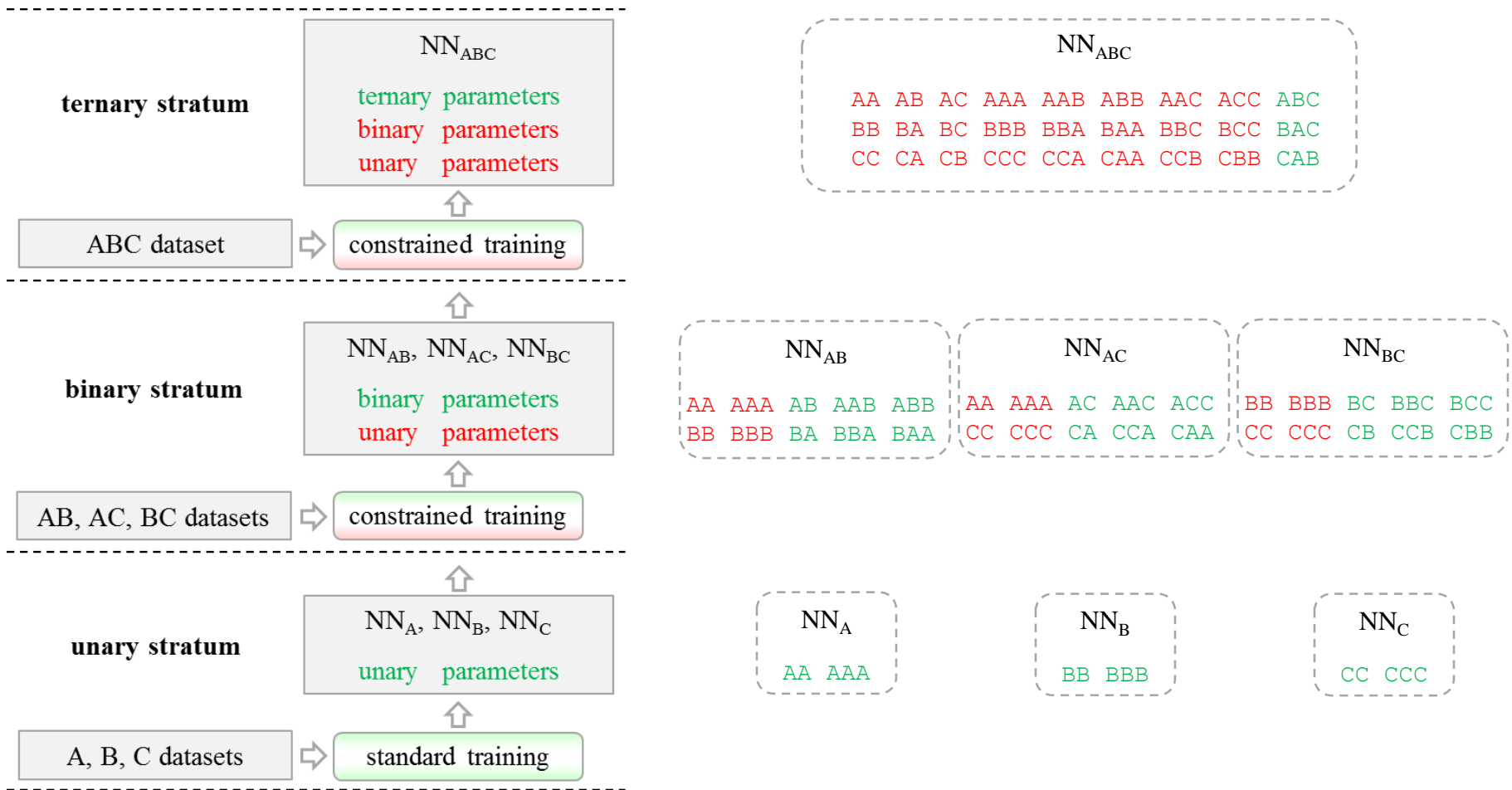
development focus

design & train NN for multi-element systems

test available functions

generate diverse datasets





## Training stratification: key points

has been used previously for classical and tight-binding models

offers training speed-up and standardization of NN models

could lead to drop in accuracy due to constraints?

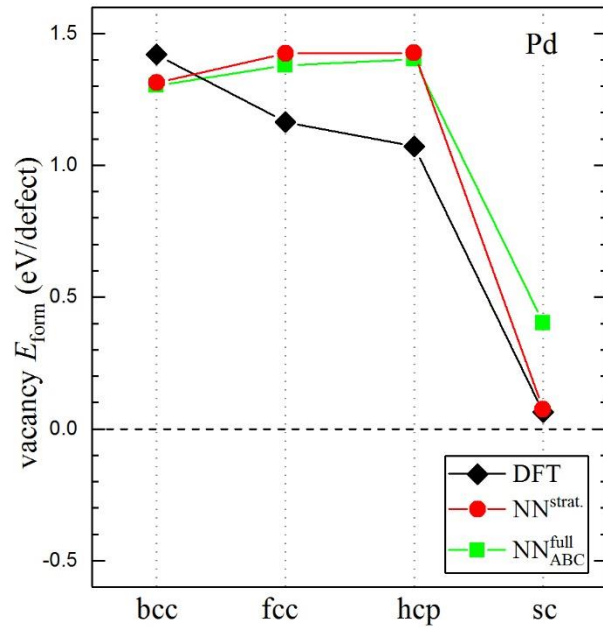
DFT: PAW-PBE,  $E_{\text{cut}} = 500$  eV

57,000 energy data

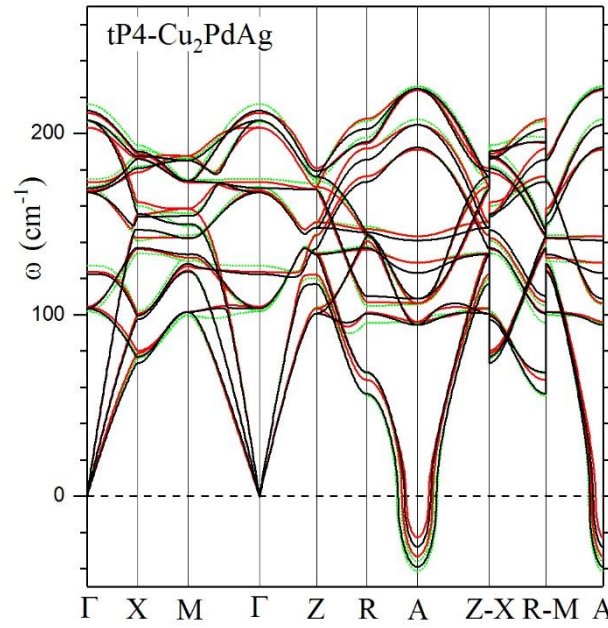
NNs: 30/82/145-10-10 MLP

5,073 parameters

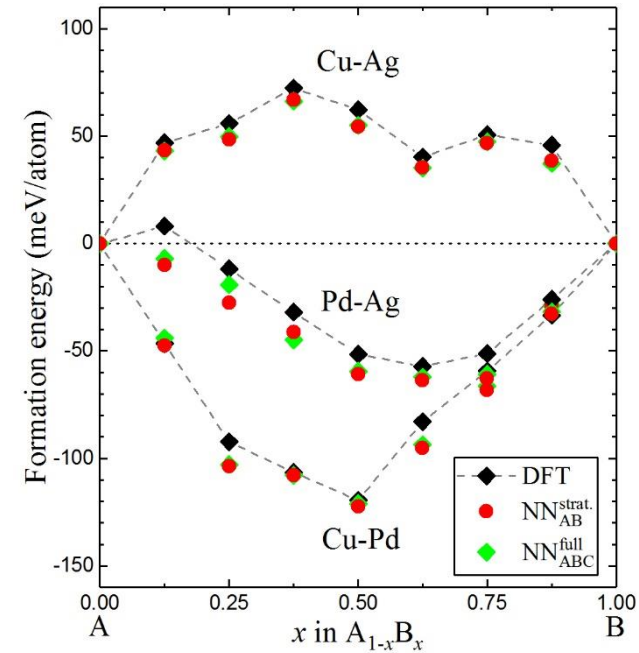
vacancy formation



phonon dispersion



formation energy



NNs vs DFT

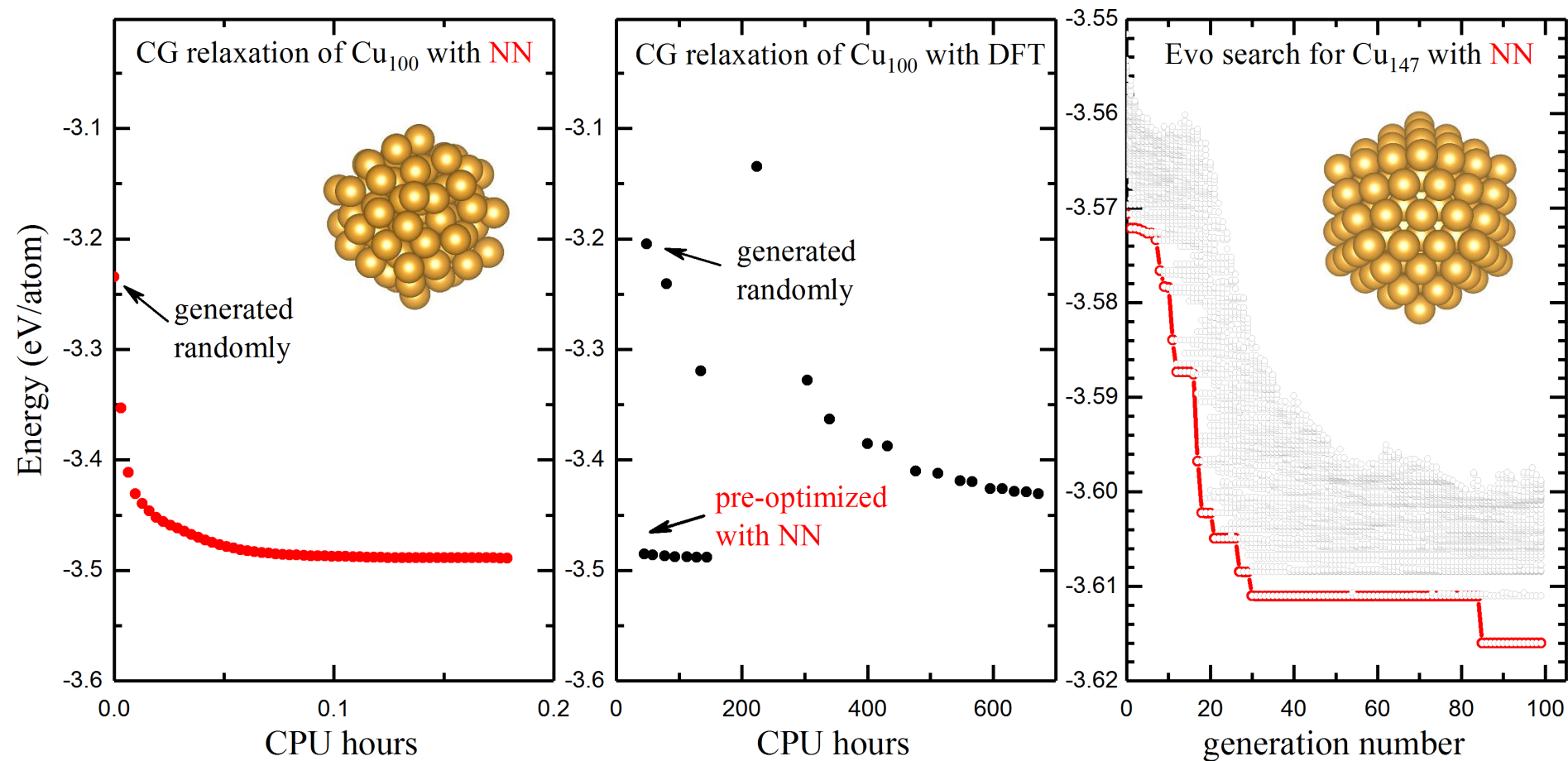
within ~0.3 eV/defect

captures softening

within 10 meV/atom

stratified vs full NN training:

no decrease in accuracy due to constraints

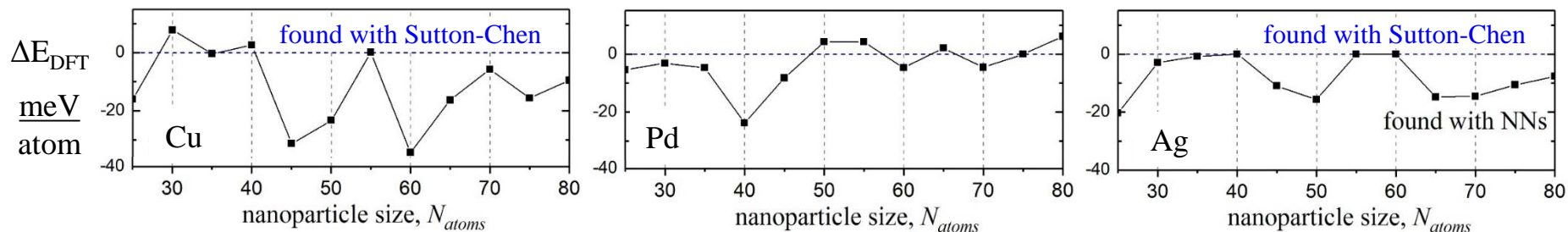


Cu<sub>100</sub> relaxation with NN takes 10 mins on a single core

NPs pre-optimized with NN are a better start for DFT optimization

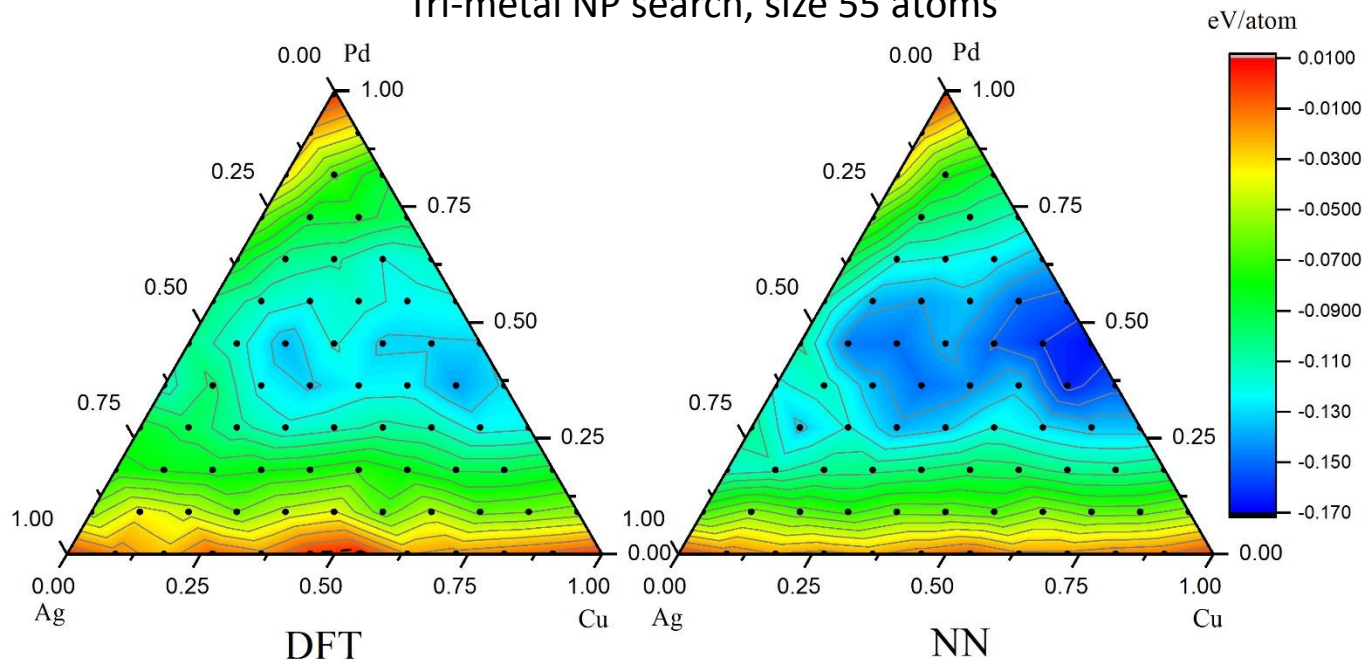
Evolutionary search with NNs converges to correct g.s.

Single-element NP search, sizes 25-80 atoms compared to Doye & Wales, New J. Chem. 733 (1998)



more stable configurations found, confirmed with DFT

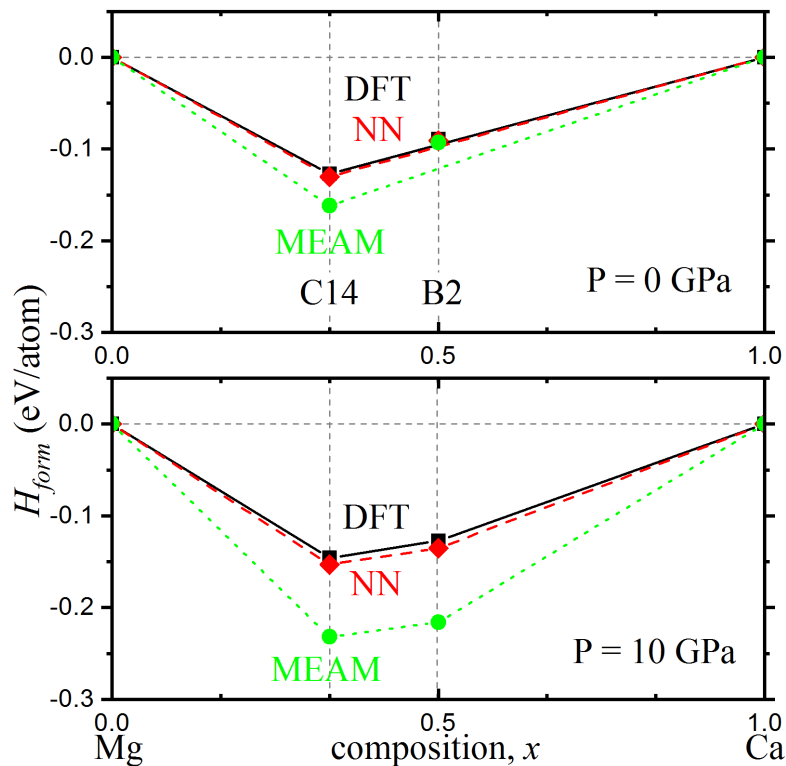
Tri-metal NP search, size 55 atoms



full composition range explored with ES+NN  
stability islands agree in DFT and NN treatments

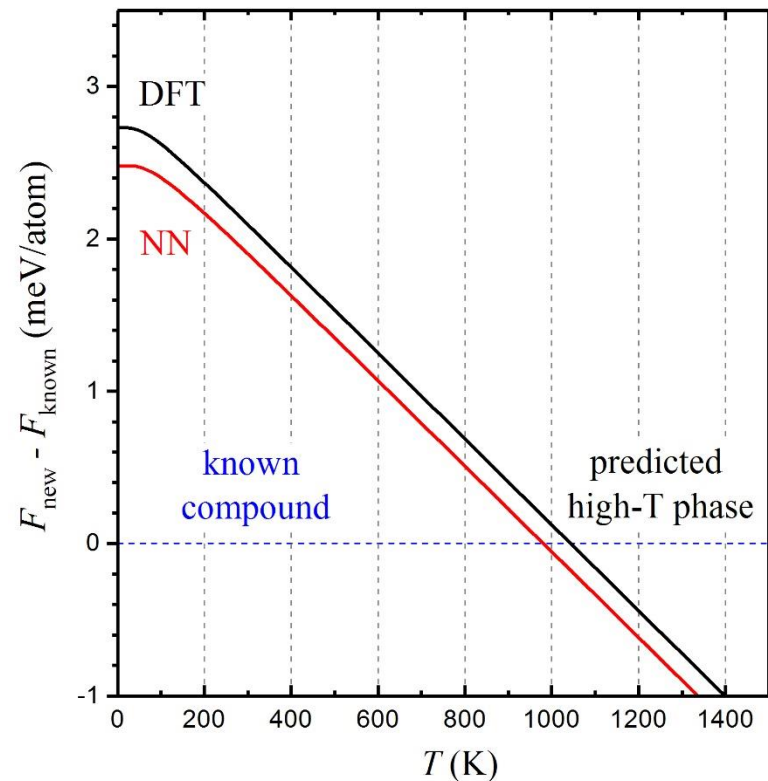


NN vs DFT and MEAM  
for formation enthalpy  $H(P) = E + PV$



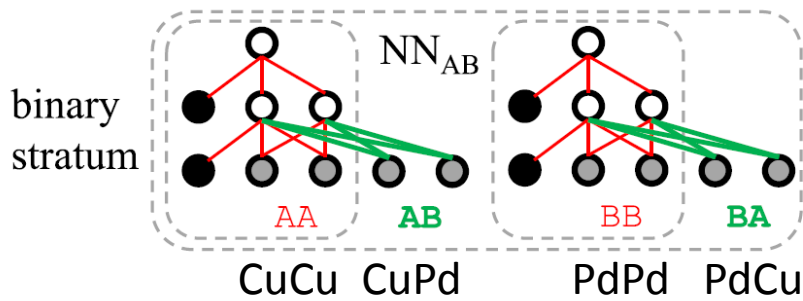
NN reproduces convex hull within 10 meV/atom  
MEAM overbinds compounds by 100 meV/atom

NN vs DFT  
for free energy at high T  $F(T) = E - TS_{phon}$



$\sim 10^4$ -fold acceleration of high-T analysis  
new high-T Mg-Ca phases predicted

architecture of a  
simplified 16-10-1 NN



are there correlations between  
CuCu and CuPd weights  
connecting to same neurons?

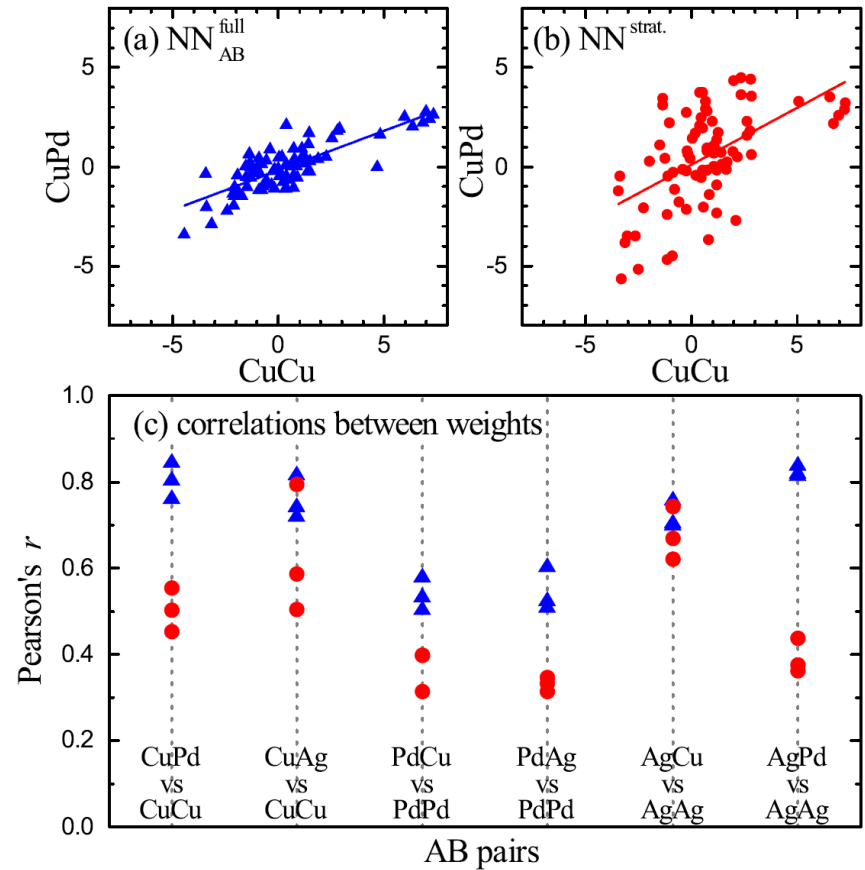
more correlation for 'full' NN

still little insight into 'inner workings'

weight correlations

'full' NN

stratified NN

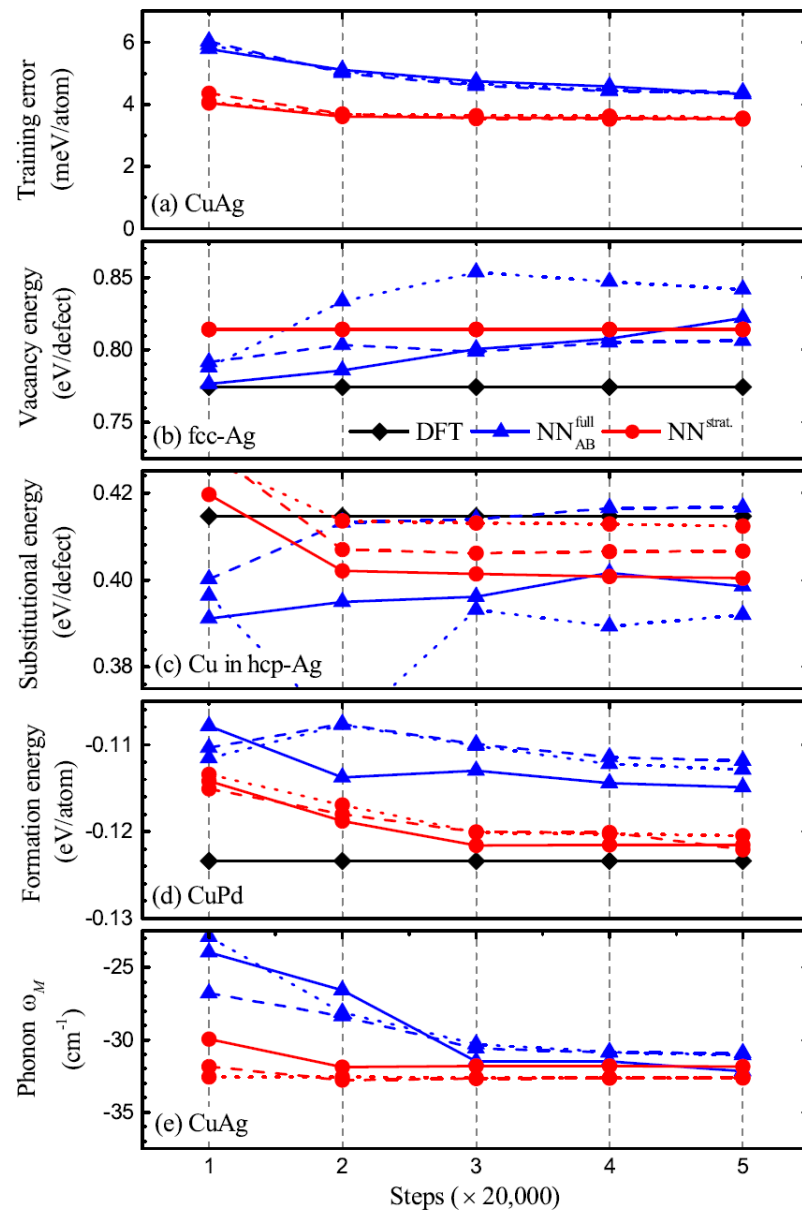


| Data set | Stratified NN         |                |                   | Full NN               |                |                   |
|----------|-----------------------|----------------|-------------------|-----------------------|----------------|-------------------|
|          | $\Delta E$<br>eV/atom | No. of<br>data | No. of<br>weights | $\Delta E$<br>eV/atom | No. of<br>data | No. of<br>weights |
| Cu       | 0.5172                | 8551           | 431               | –                     | –              | 431               |
| Pd       | 0.6262                | 8487           | 431               | –                     | –              | 431               |
| Ag       | 0.4481                | 8533           | 431               | –                     | –              | 431               |
| CuPd     | 0.5975                | 7623           | 1040              | 0.5829                | 24661          | 1902              |
| CuAg     | 0.5391                | 7617           | 1040              | 0.6601                | 24701          | 1902              |
| PdAg     | 0.5989                | 7601           | 1040              | 0.6833                | 24621          | 1902              |
| CuPdAg   | 0.2170                | 8917           | 660               | 0.6227                | 57329          | 5073              |

total energy as target only; forces/stresses possible

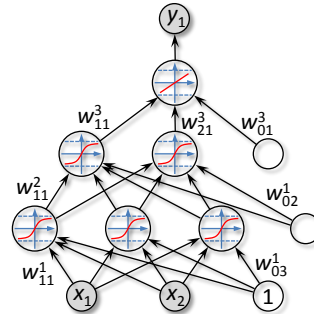
100,000 optimization steps with BFGS/CG

performance insensitivity to weight initialization



# Stratified construction of neural network based interatomic models for multicomponent materials

S. Hajinazar, J. Shao, and A.N. Kolmogorov, PRB 95, 014114 (2017)



|          |          |          |                                                                      |          |          |          |  |  |
|----------|----------|----------|----------------------------------------------------------------------|----------|----------|----------|--|--|
| 3<br>Li  | 4<br>Be  | 5<br>B   | <b>MAISE library<br/>of multicomponent<br/>neural network models</b> |          |          |          |  |  |
| 11<br>Na | 12<br>Mg | 13<br>Al |                                                                      |          |          |          |  |  |
| 19<br>K  | 20<br>Ca | 21<br>Sc | 22-27<br>3d                                                          | 28<br>Ni | 29<br>Cu | 30<br>Zn |  |  |
| 37<br>Rb | 38<br>Sr | 39<br>Y  | 40-45<br>4d                                                          | 46<br>Pd | 47<br>Ag | 48<br>Cd |  |  |
| 55<br>Cs | 56<br>Ba | 57<br>La | 72-77<br>5d                                                          | 78<br>Pt | 79<br>Au | 80<br>Hg |  |  |

## Method development

evolutionary data generation  
stratified construction of models

sampling of relevant space  
no loss of accuracy

## NN model development

metallic: 13 unaries, 7 binaries, 2 ternaries  
covalent: in progress

3-10 meV/atom  
noticeably lower accuracy

## NN model application

robust for evolutionary optimization  
accurate for high-T stability analysis

new low-energy NPs predicted  
new high-T phases predicted

## MAISE development

evolutionary optimization, MD  
to be released as open source

crystals surfaces nanoparticles  
collaborations welcome

## Acknowledgments

S. Hajinazar, E. Sandoval (BU)  
J. Shao (BU), A. Romero (WVU)

A Jephcoat (Okayama U.)  
N & L Dubrovinskii (Bayreuth)

## Support

TAE (Binghamton)

NSF-DMR

