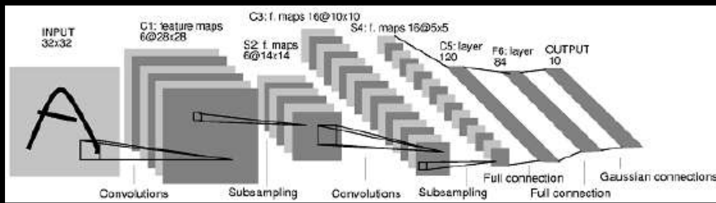
Learning geometry

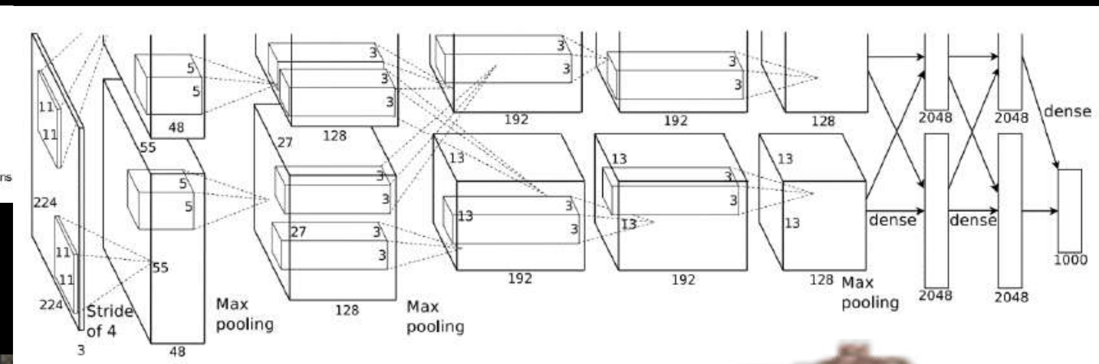
Ron Kimmel

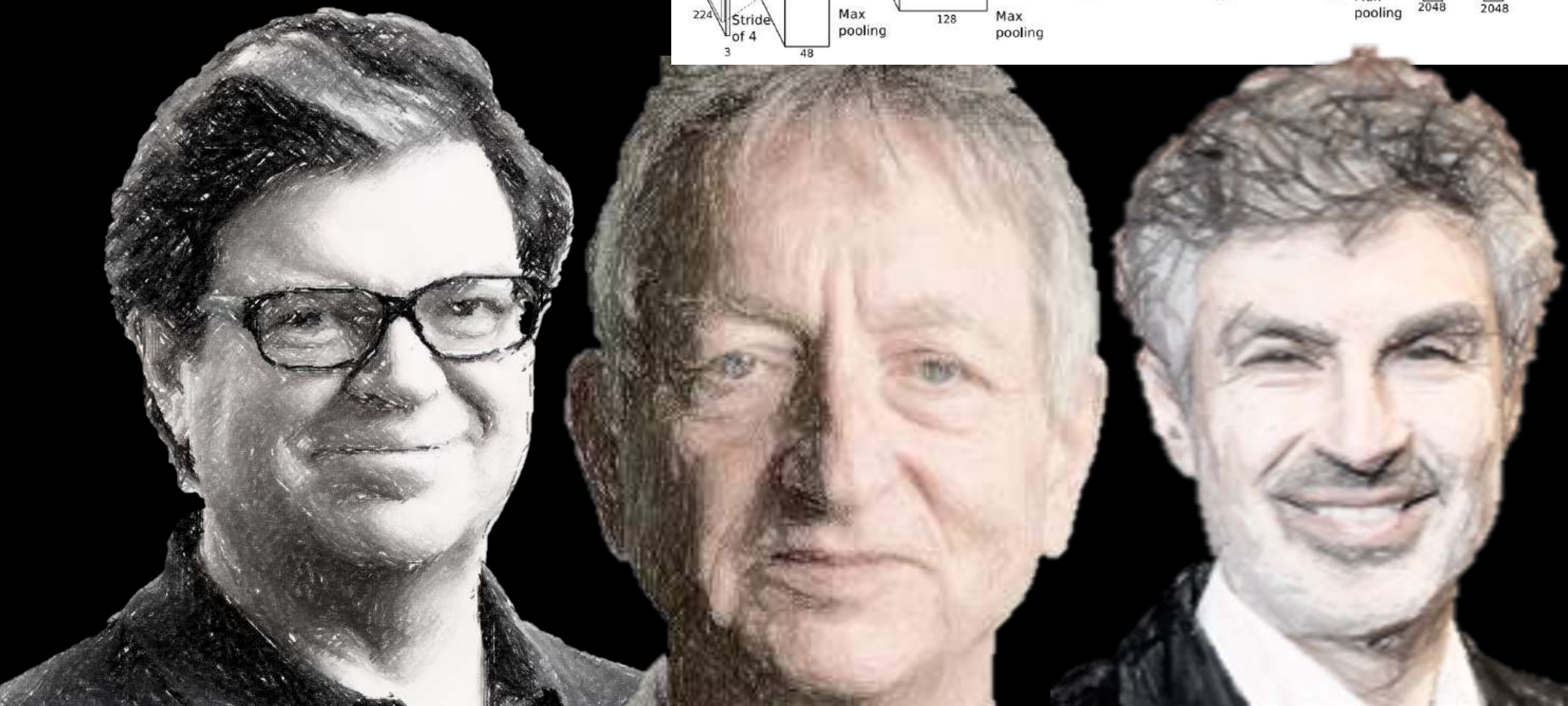
Geometric Image Processing Lab.

Technion – Israel Institute of Technology

IPAM Workshop on deep learning &
combinatorial optimization
UCLA Feb. 22, 2021







Yann LeCunn Geoff Hinton Yoshua Bengio

How could we
use algebra to
describe geometry?

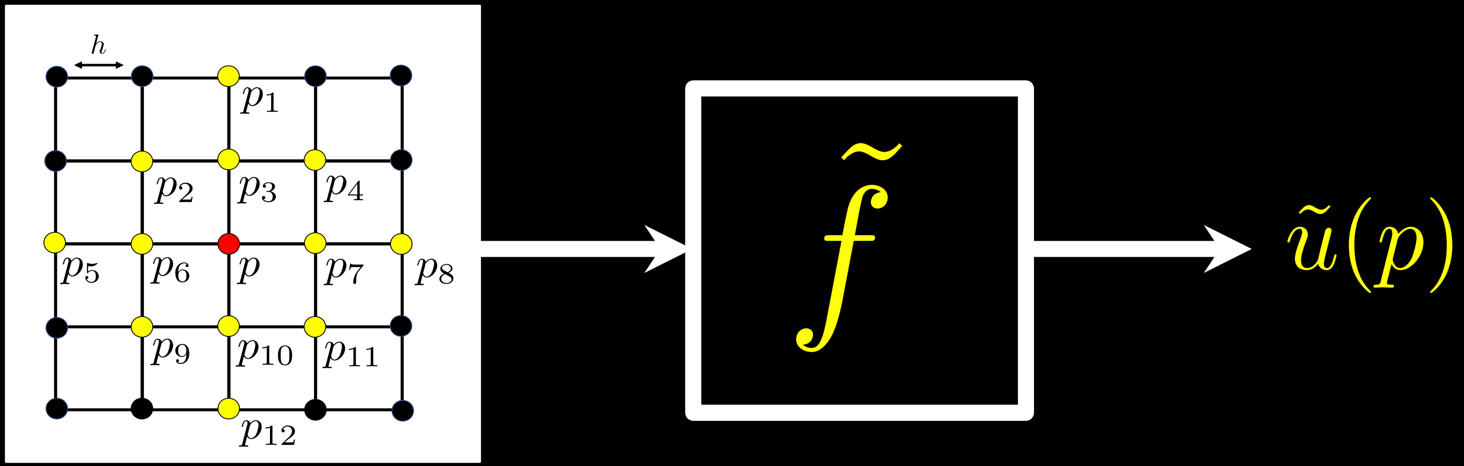
Deep Eikonal solvers

$$|\nabla u(x)| = 1, \quad x \in \Omega \setminus \Gamma$$

$$u(x) = 0, \quad x \in \Gamma$$

Local numerical approximations using neural networks

- Easy to extend the support
- Approximation is learned from analytically known solutions

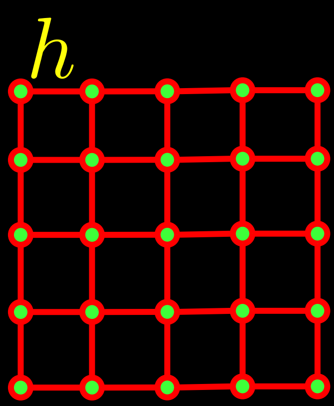


Lichtenstein, Pai, K., SSVM 2019 - Best Student Paper Award

On the evolution of accuracy/complexity

- Fast Marching (quasi-linear complexity) $\sim O(n)$

Regular grids



1995 $O(h)$

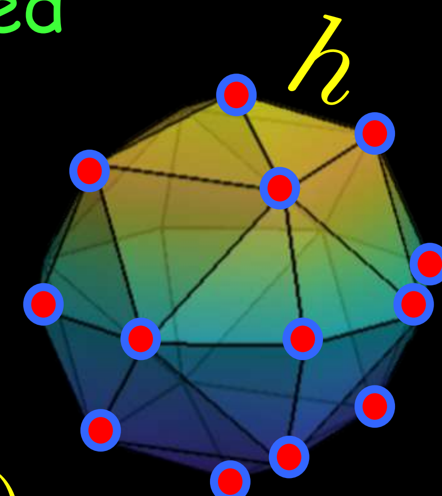
↓

1999 $O(h^2)$

↓

2011 $O(h^3)$

Triangulated domains

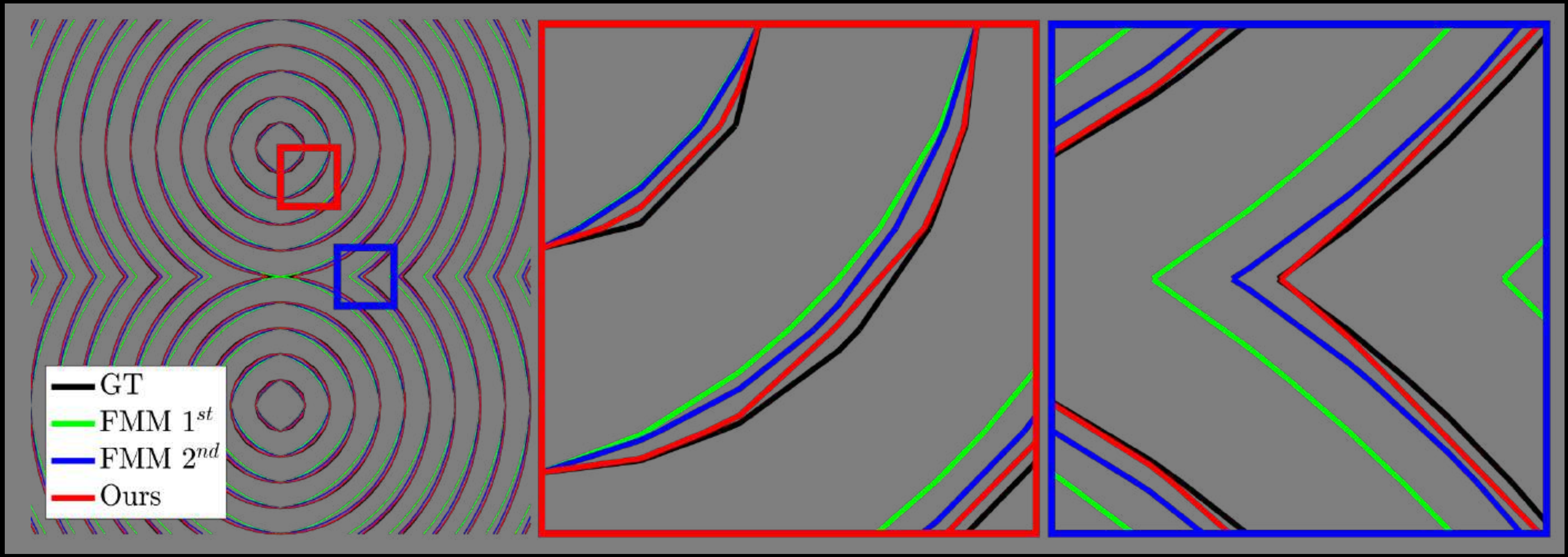


1998 $O(h)$

2005 $O(h^2)$ ← $O(n^2)$

Surazhsky et al. implementation of MMP's `exact geodesic' 1987 (~2 decades!)

Results for Cartesian grids



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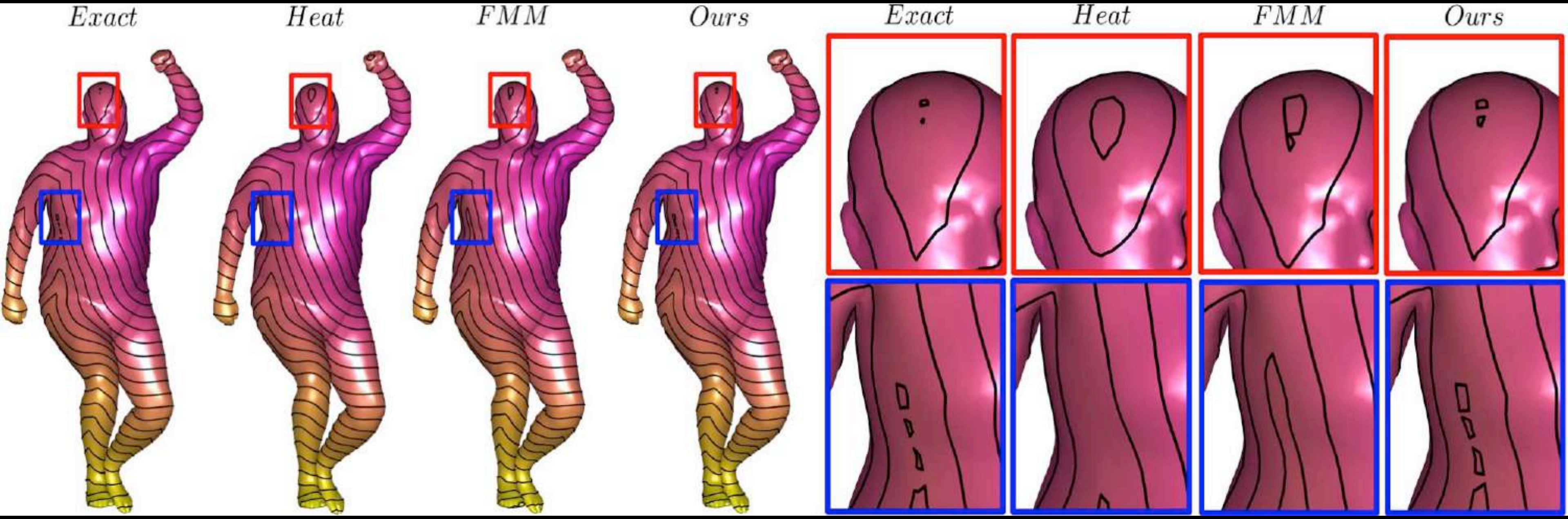
Inter-dataset generalization



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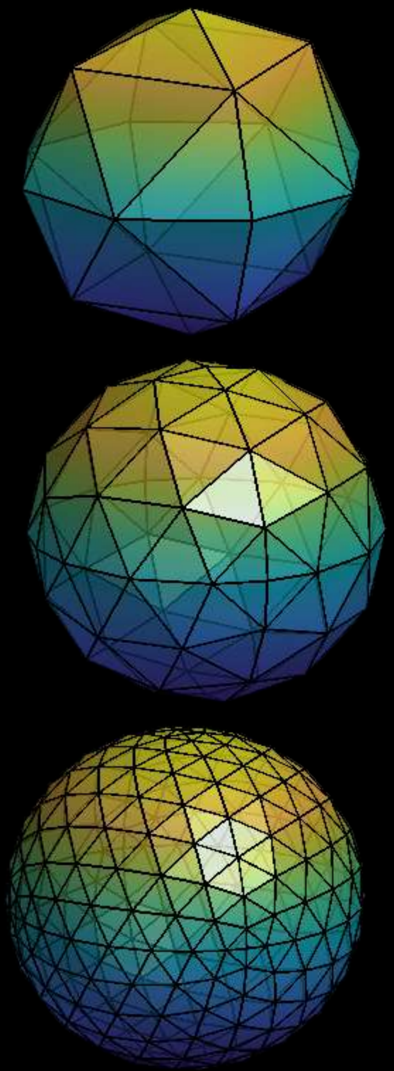
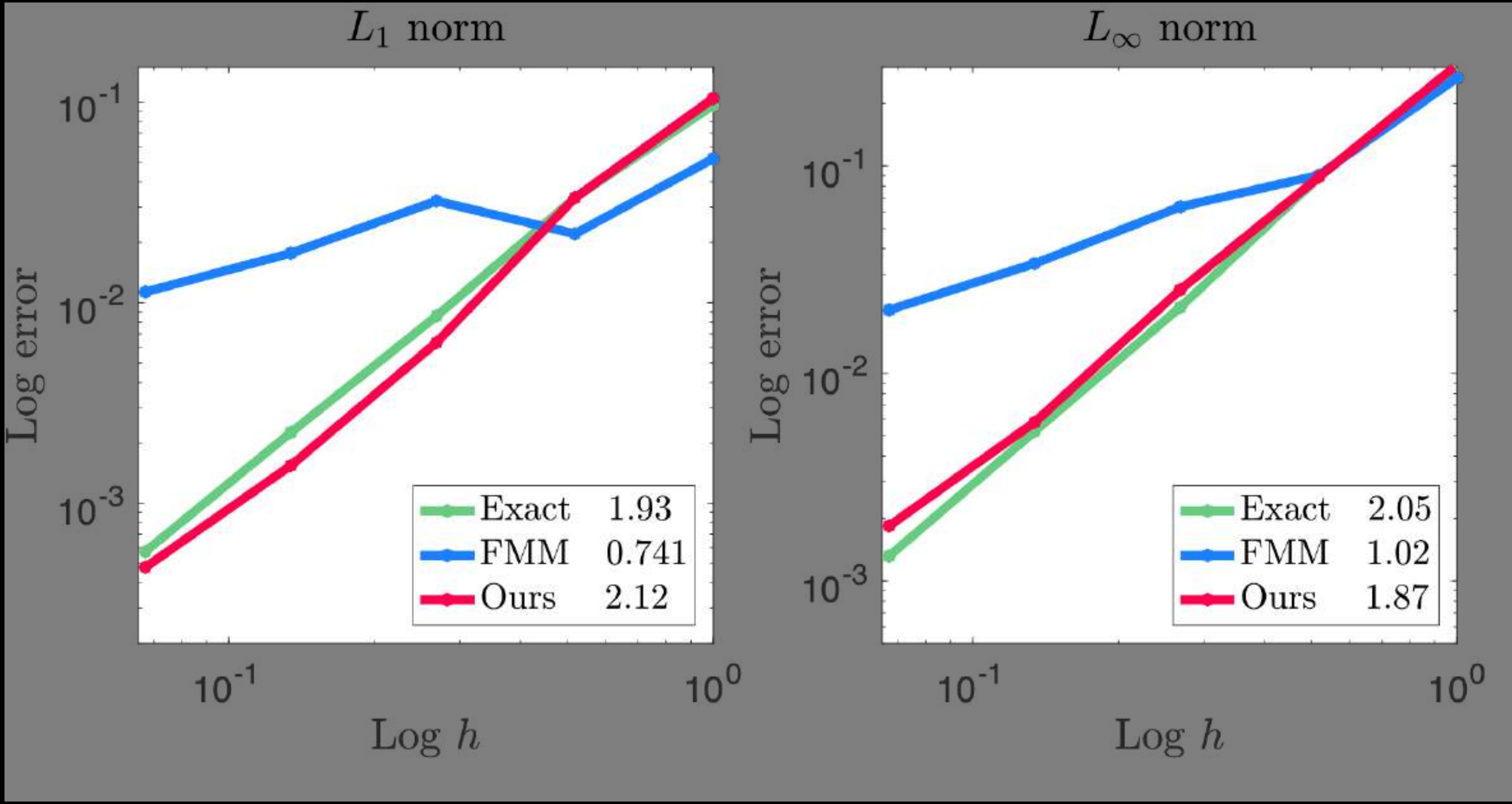
Order of accuracy





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Momenet

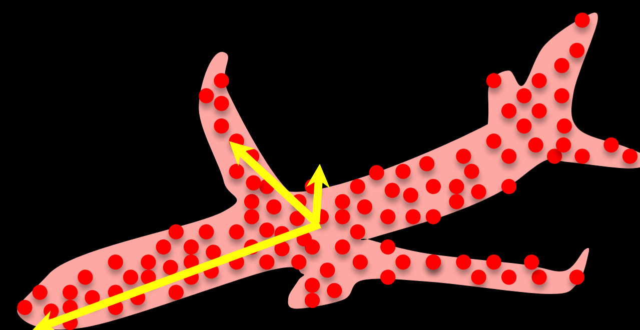
Algebraic invariants	3D moment as invariants	High order NN	Deep NN	PointNet	Momenet
1903	1980	1987	2010	2017	2019

Geometric moments

Given a set of points $P \subset \mathbb{R}^3$ and $p_i = (x_i, y_i, z_i)^T \in P$

1st moments $\bar{p} = (\bar{x}, \bar{y}, \bar{z})^T = \frac{1}{n}\sum_{i=1}^{n} p_i$

2nd moments $\frac{1}{n}\sum_{i=1}^{n} p_i p_i^T$



Young. The algebra of invariants, 1903

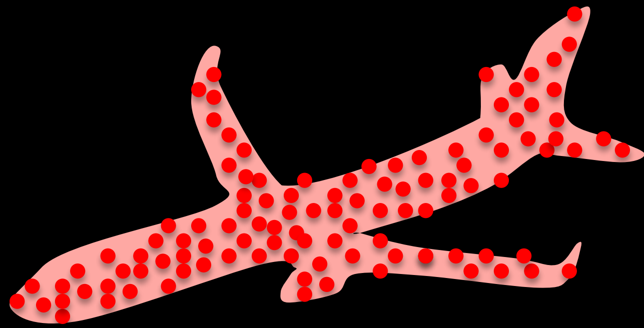
Hall. Three-dimensional moment invariants. PAMI, 1980

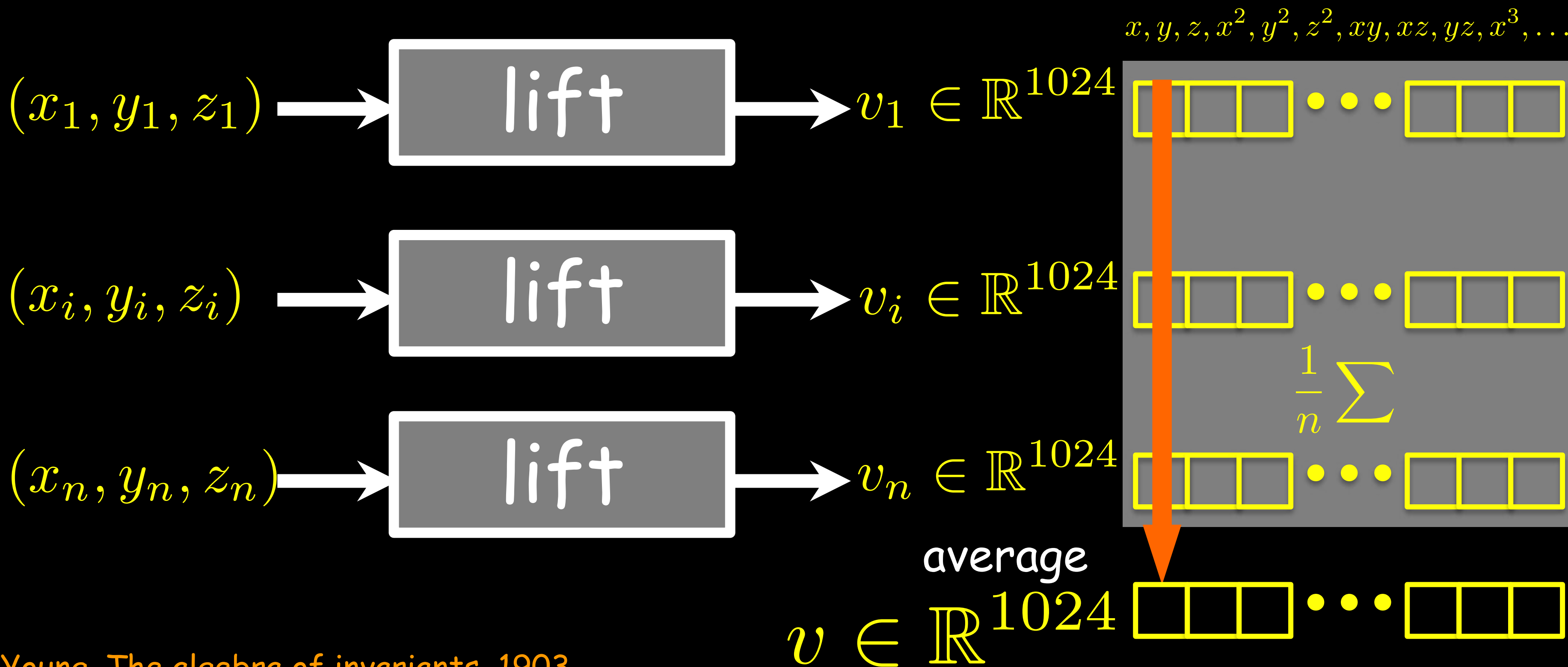
Maxwell, Learning, invariance, and generalization in high-order neural networks. Applied optics, 1987

Su, Mo, Guibas. Pointnet: Deep learning on point sets. CVPR'17

Joseph-Rivlin, Zvirin, K., GMDL workshop, ICCV'19

Moments





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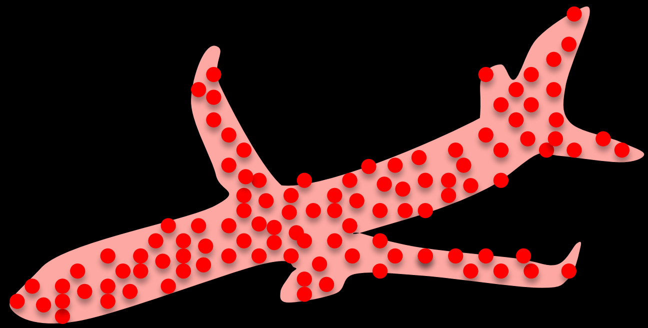
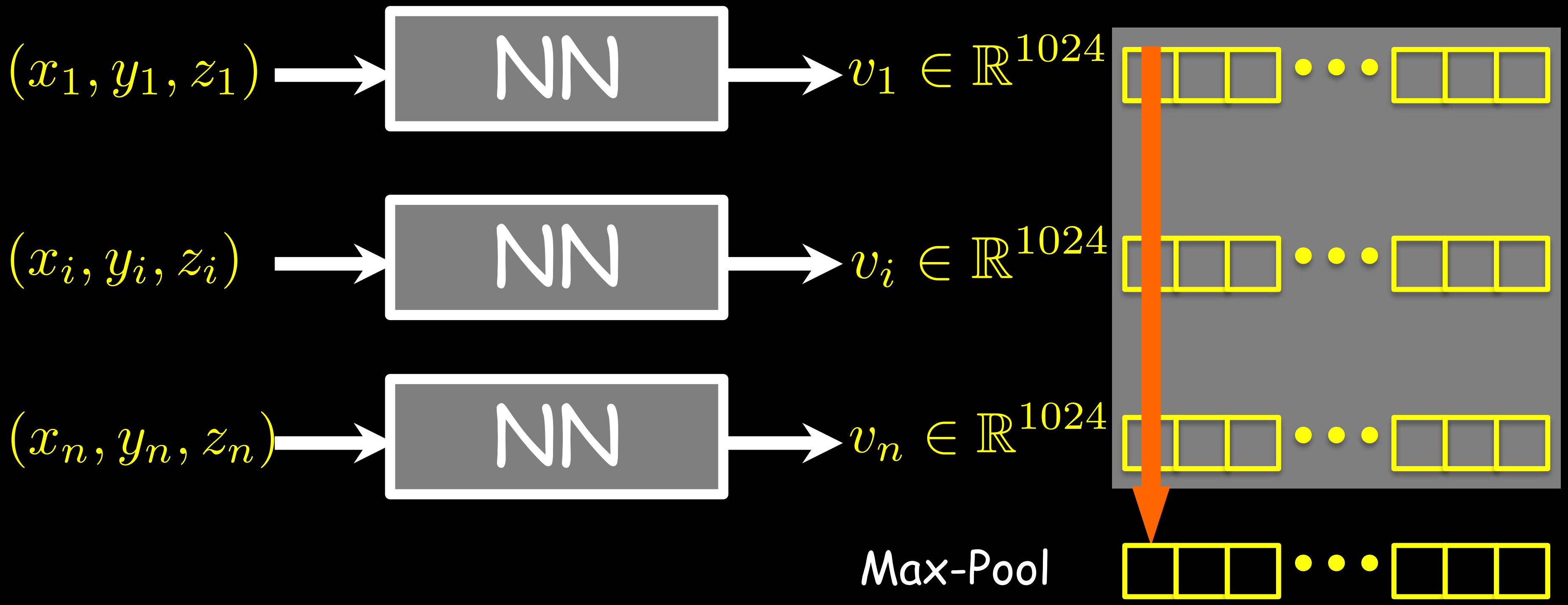
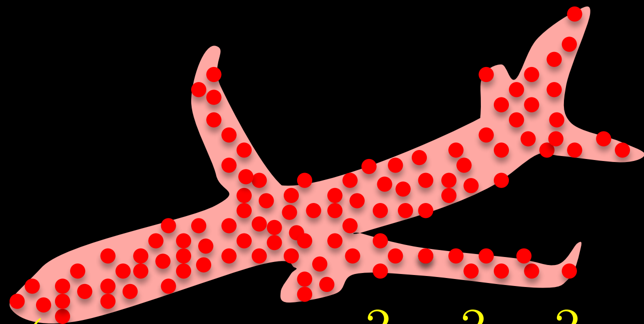


PointNet



Su, Mo, Guibas. CVPR'17

Momenet



$(x_1, y_1, z_1, x_1^2, y_1^2, z_1^2, x_1 y_1, x_1 z_1, y_1 z_1)$

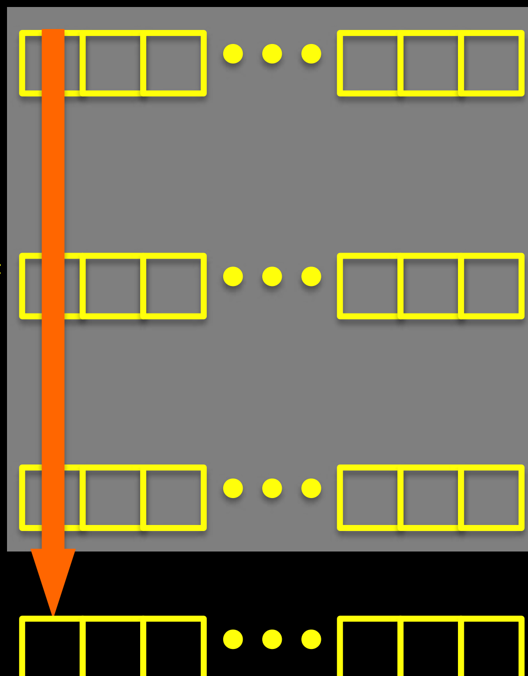
$(x_1, y_1, z_1) \rightarrow$ NN $\rightarrow v_1 \in \mathbb{R}^{1024}$

$(x_i, y_i, z_i, x_i^2, y_i^2, z_i^2, x_i y_i, x_i z_i, y_i z_i)$

$(x_i, y_i, z_i) \rightarrow$ NN $\rightarrow v_i \in \mathbb{R}^{1024}$

$(x_n, y_n, z_n, x_n^2, y_n^2, z_n^2, x_n y_n, x_n z_n, y_n z_n)$

$(x_n, y_n, z_n) \rightarrow$ NN $\rightarrow v_n \in \mathbb{R}^{1024}$



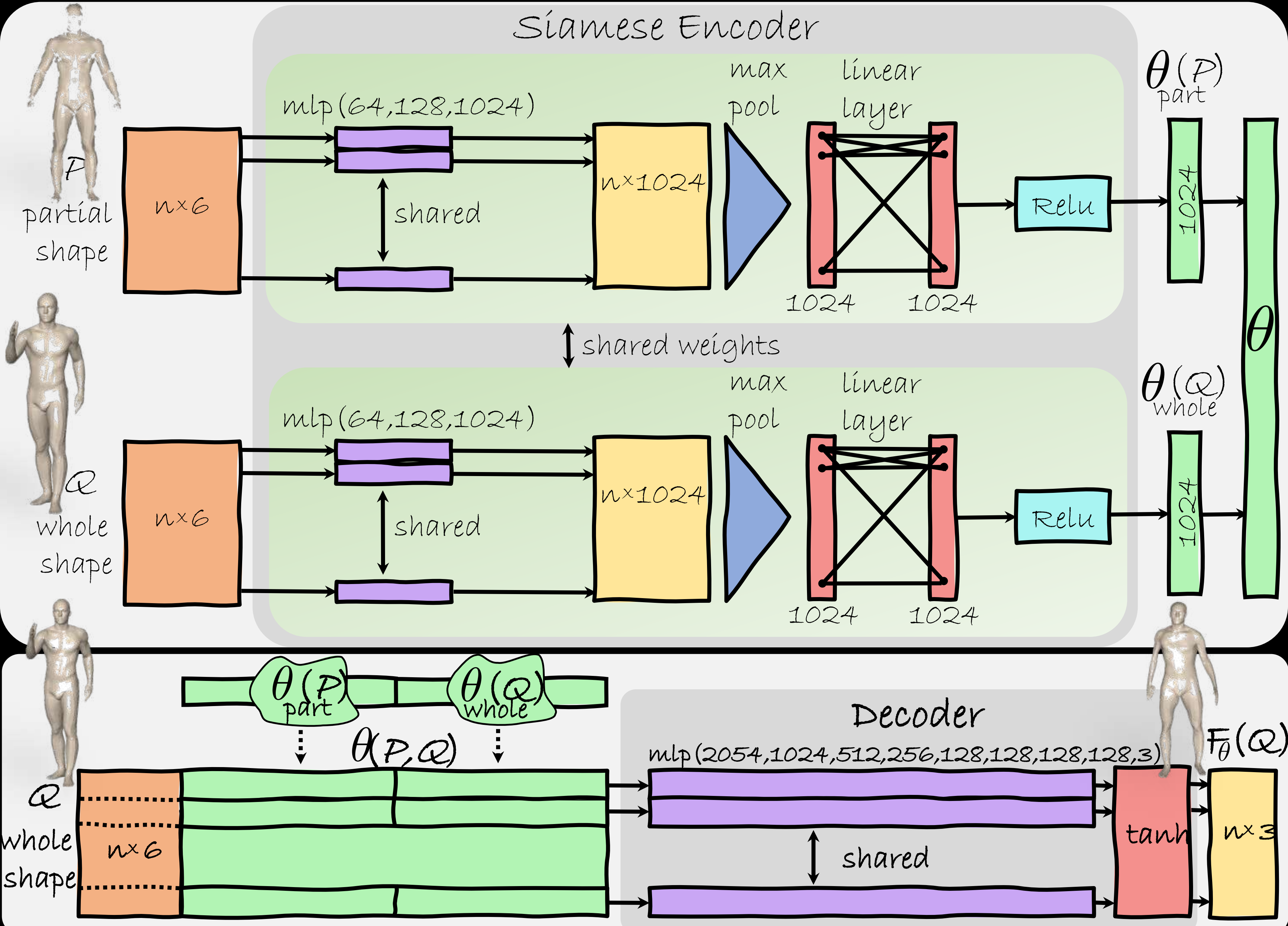
Max-Pool

Joseph-Rivlin, Zvirin, K., Flavor the moments, GMDL workshop, ICCV'19

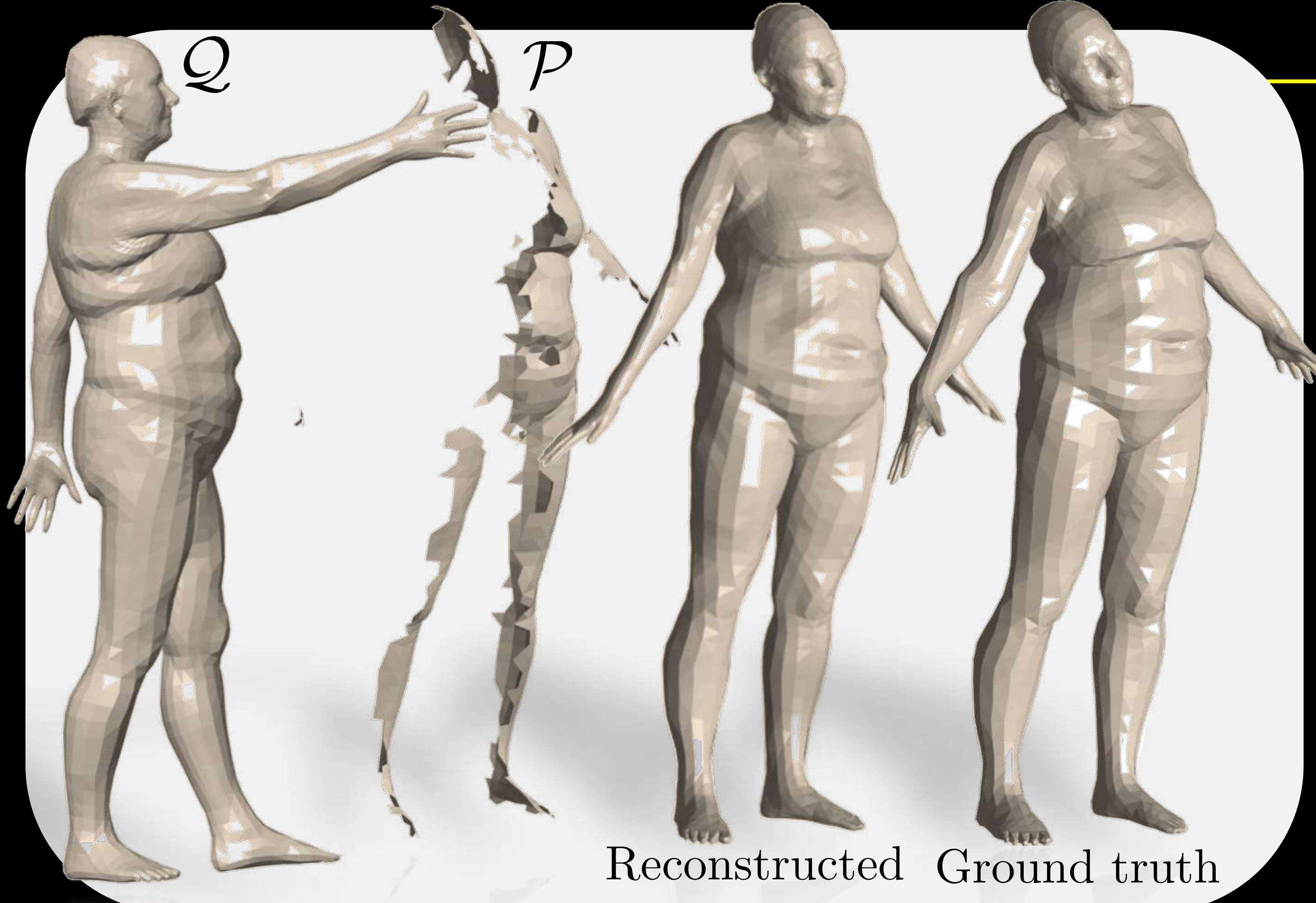
Results on ModelNet40

	Memory (MB)	Inference Time (msec)	Overall Accuracy (%)
PointNet	40	5.6	89.2
PointNet(baseline)	20	5.1	87.9
Momen^{e}t	**20**	**5.1**	**89.6**
PointNet++	12	10.4	90.7
DGCNN	21	17.3	92.2
PCNN	17	54.1	92.3
Momen^{e}t (+kNN)	**21**	**9.6**	**92.4**

Wu, et al.. 3D shapenets. CVPR2015.
Wang et al. . Dynamic graph CNN. Arxiv 2018.
Atzmon, Maron, & Lipman. Point convolutional NN. ACM T Graph. 2018
Joseph-Rivlin, Zvirin, K., Flavor the moments, GMDL workshop, ICCV'19

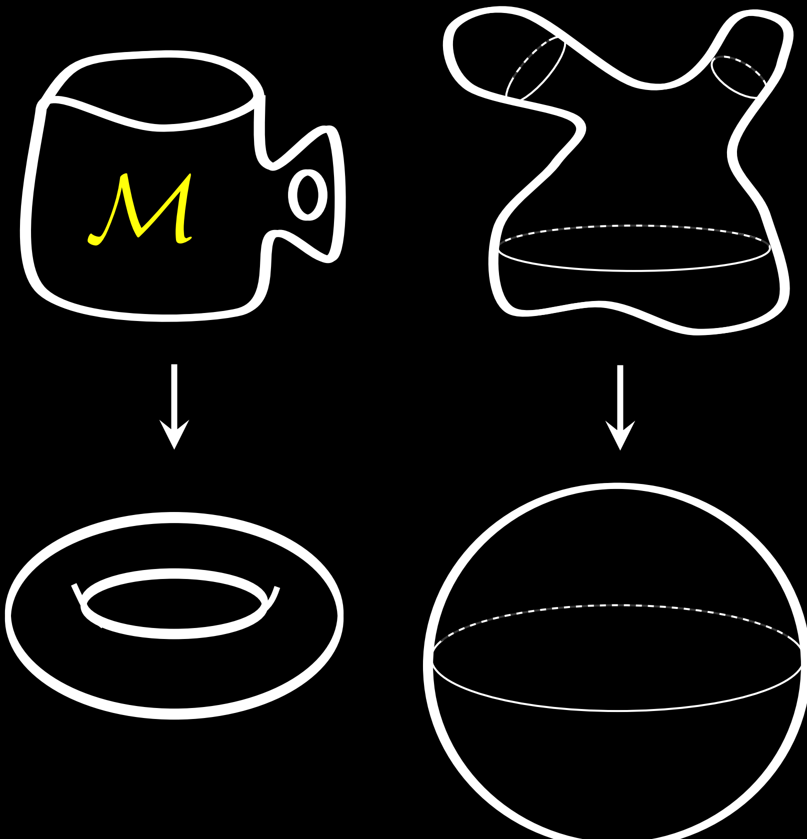


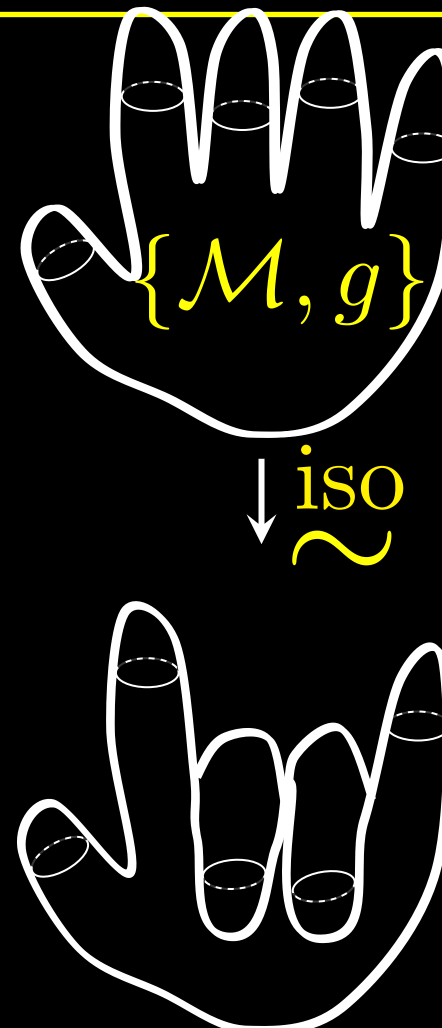
Halimi et al. ECCV 2020



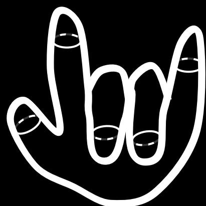
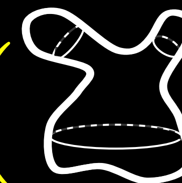
$\mathcal{Q}$
$\mathcal{P}$
Reconstructed
Ground truth

Manifold vs. Riemannian Manifold





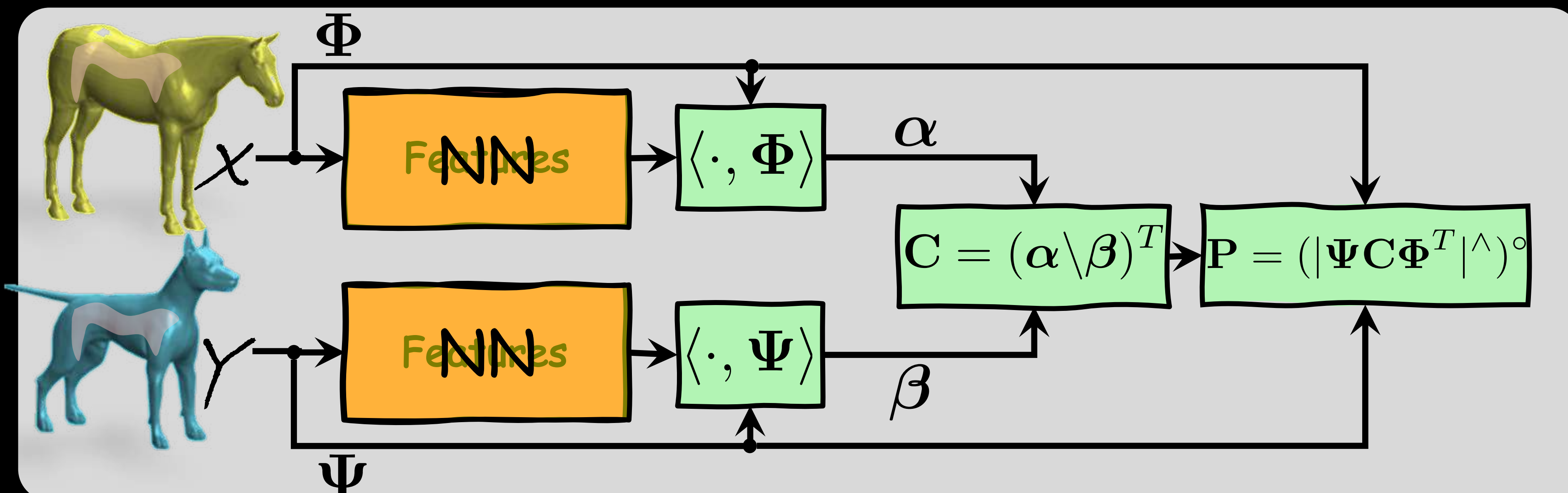
Shapes as metric spaces

$d_{GH}($  $,$  $) < d_{GH}($  $,$  $)$

$MDS, GMDS, SGMDS, GDD, PCA, RPCA,$
$F\text{-}Maps, FM\text{-}Net, GMDS\text{-}Net, SF\text{-}Maps$

Functional Maps

$$\mathcal{Y} \approx P\mathcal{X}$$



Ovsjanikov et al. 2012

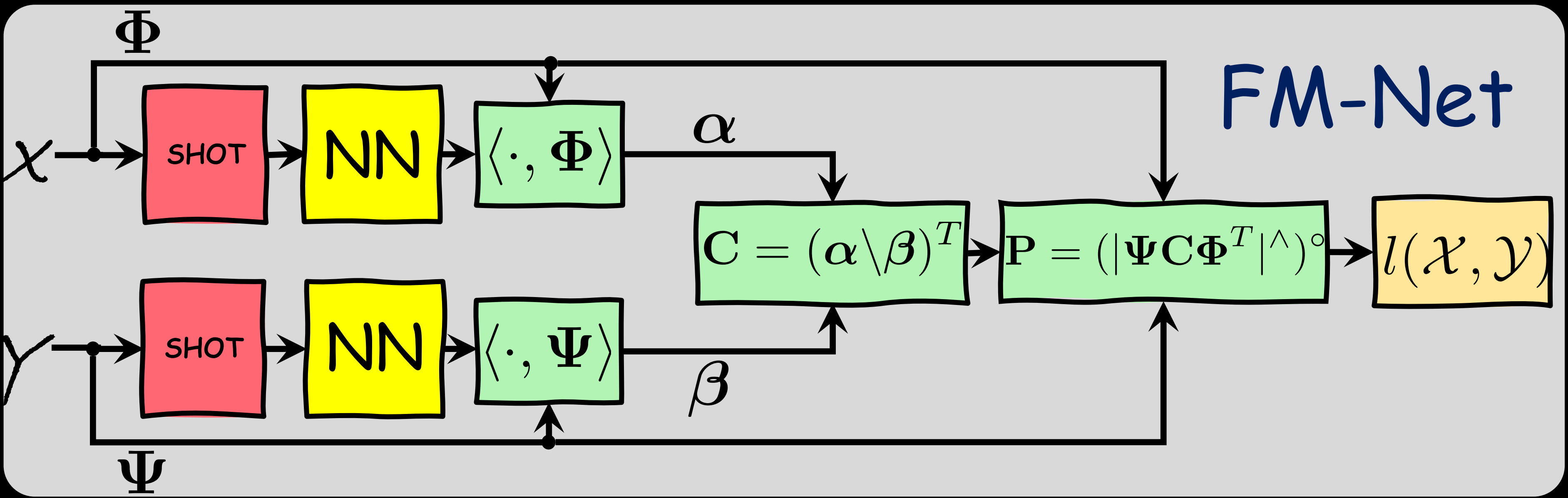
Litany, Remez, Rodola, A Bronstein, M Bronstein ICCV'17

Halimi, Litany, Rodolà, Bronstein, K. CVPR'19

Roufosse, Sharma, Ovsjanikov, ICCV'19

Functional Maps-Net

$$l_{sup}(\mathcal{X},\mathcal{Y}) = \sum_{i\in\mathcal{X}}\sum_{j\in\mathcal{Y}} p_{ij} d^2_{\mathcal{Y}}(j,\pi^*(i))$$



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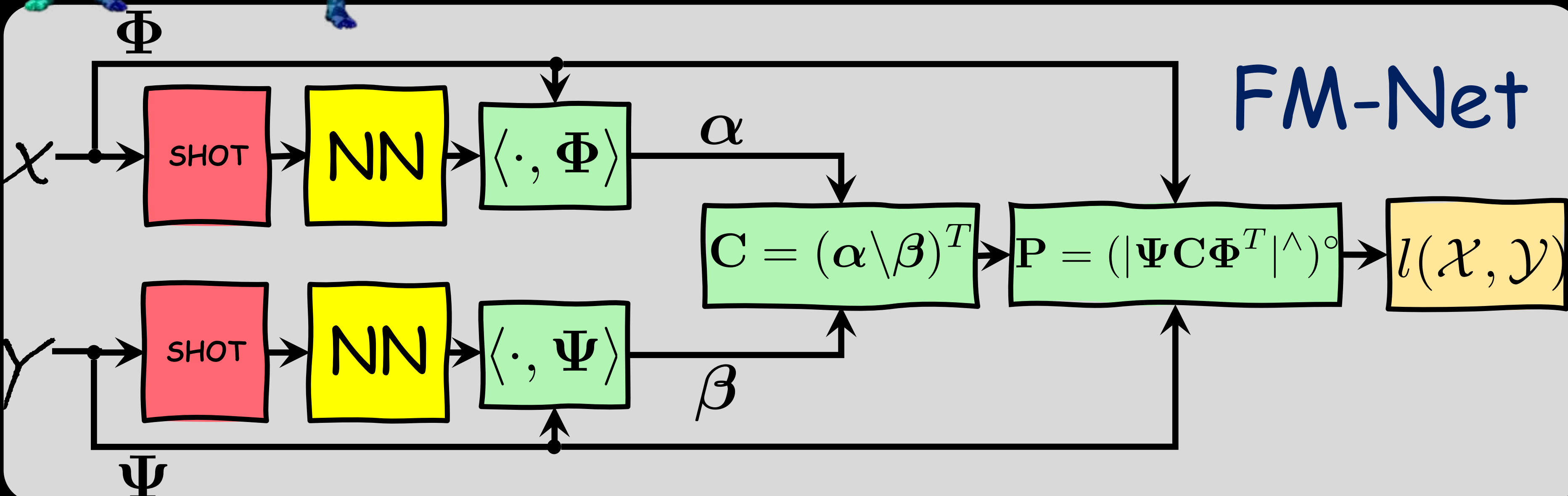
Unsupervised Functional Maps-Net

$$l_{uns}(\mathcal{X},\mathcal{Y}) = \|D_{\mathcal{X}} - P D_{\mathcal{Y}} P^T\|$$



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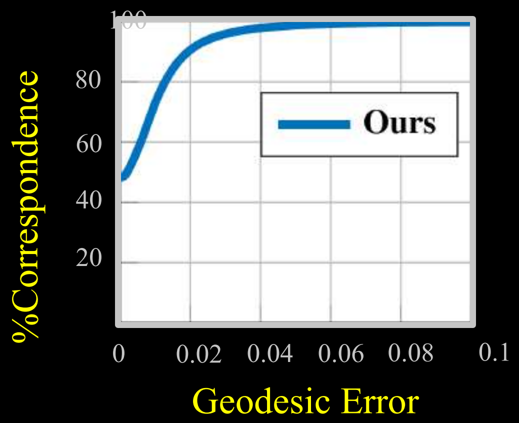
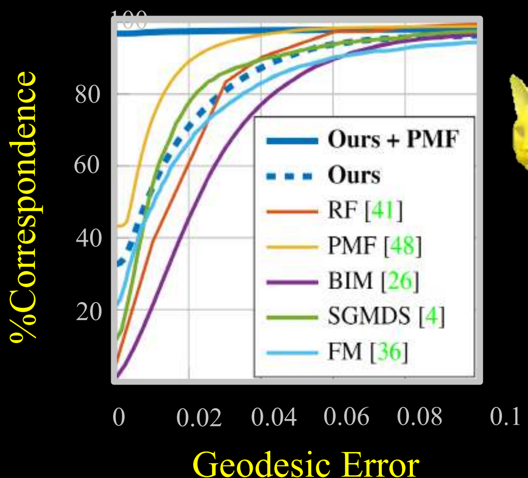
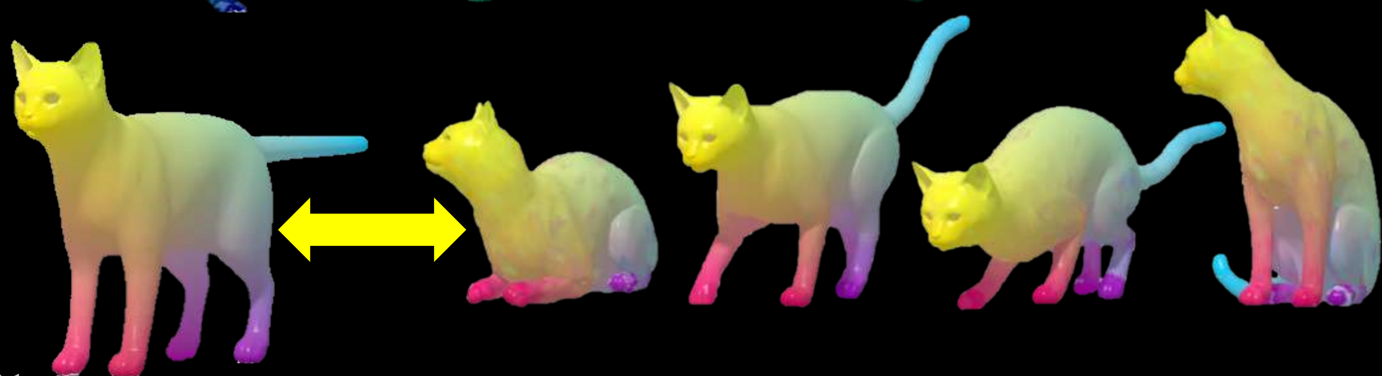
Generalization







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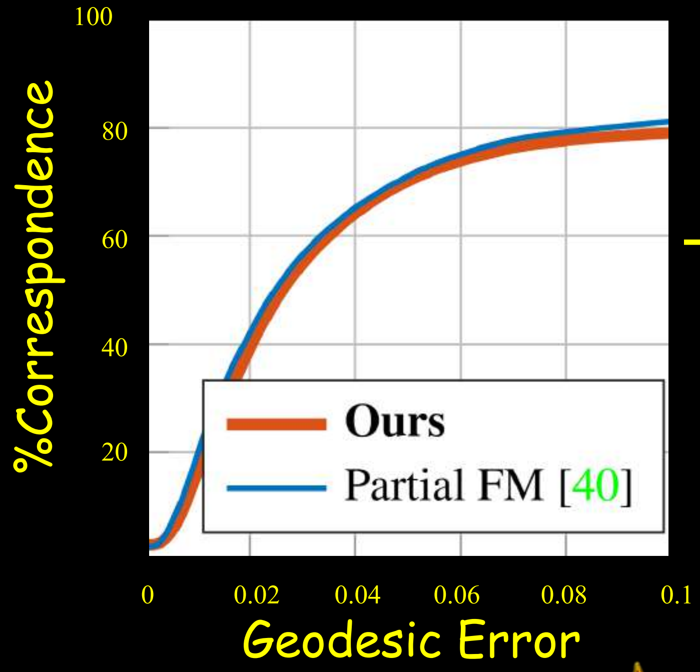
Partial Correspondence



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Single-pair correspondences

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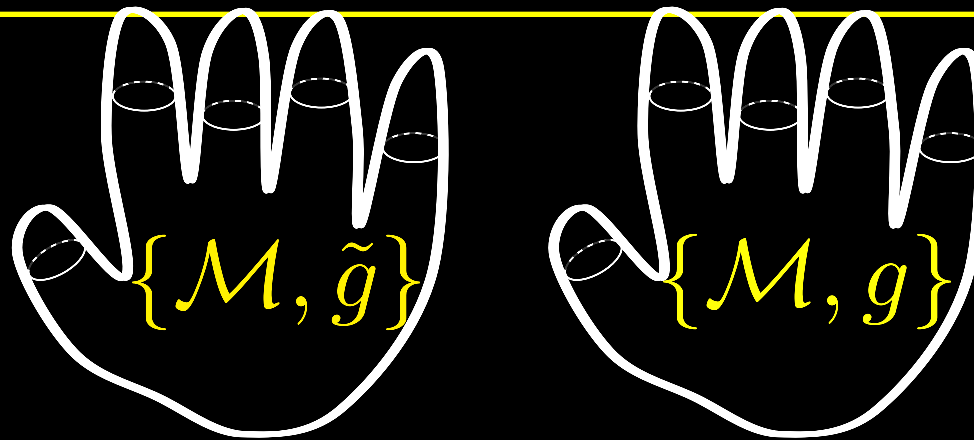
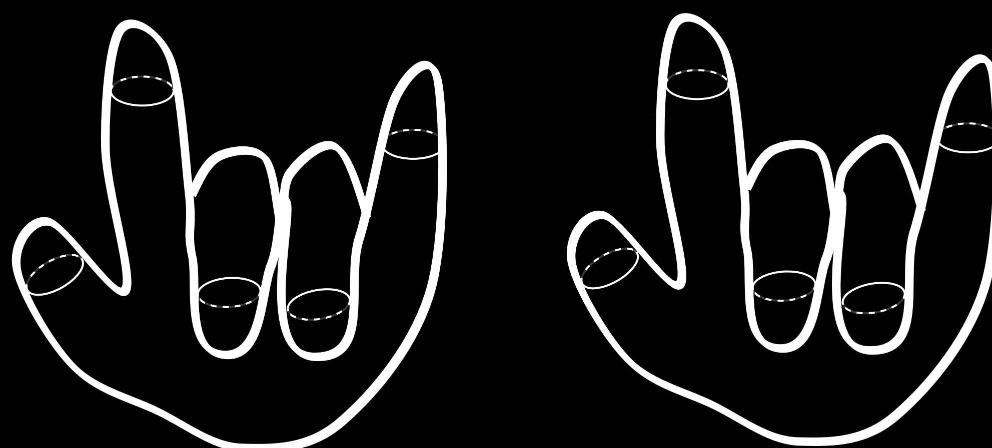
Manifold vs. Riemannian Manifolds

$$\Delta_g \psi_i = \lambda_i \psi_i$$

$$\Delta_{\tilde{g}} \tilde{\psi}_i = \tilde{\lambda}_i \tilde{\psi}_i$$

$$C_{ij} = \langle \psi_i, \tilde{\psi}_j \rangle_{\tilde{g}}$$





Surface Laplacian

$$\Delta_{\tilde{g}} \equiv -\frac{1}{\sqrt{\tilde{g}}}\partial_i \sqrt{\tilde{g}}\,\tilde{g}^{ij}\partial_j$$

$$\tilde{g}_{ij} =$$  $$= |\kappa_1 \kappa_2| \langle S_i, S_j \rangle$$

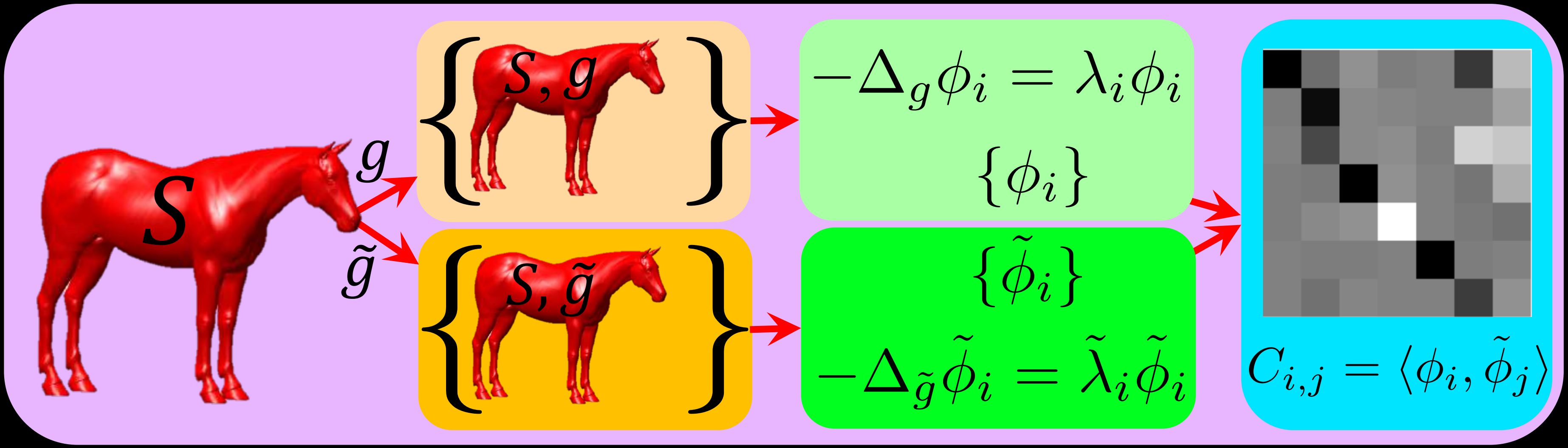
Eigenfunctions

$$g_{ij} = \langle S_{\omega_i}, S_{\omega_j} \rangle \qquad -\Delta_g \psi_i = \lambda_i \psi_i$$

$$\tilde{g}_{ij} = |K| g_{ij} \qquad -\Delta_{\tilde{g}} \tilde{\psi}_i = \tilde{\lambda}_i \tilde{\psi}_i$$

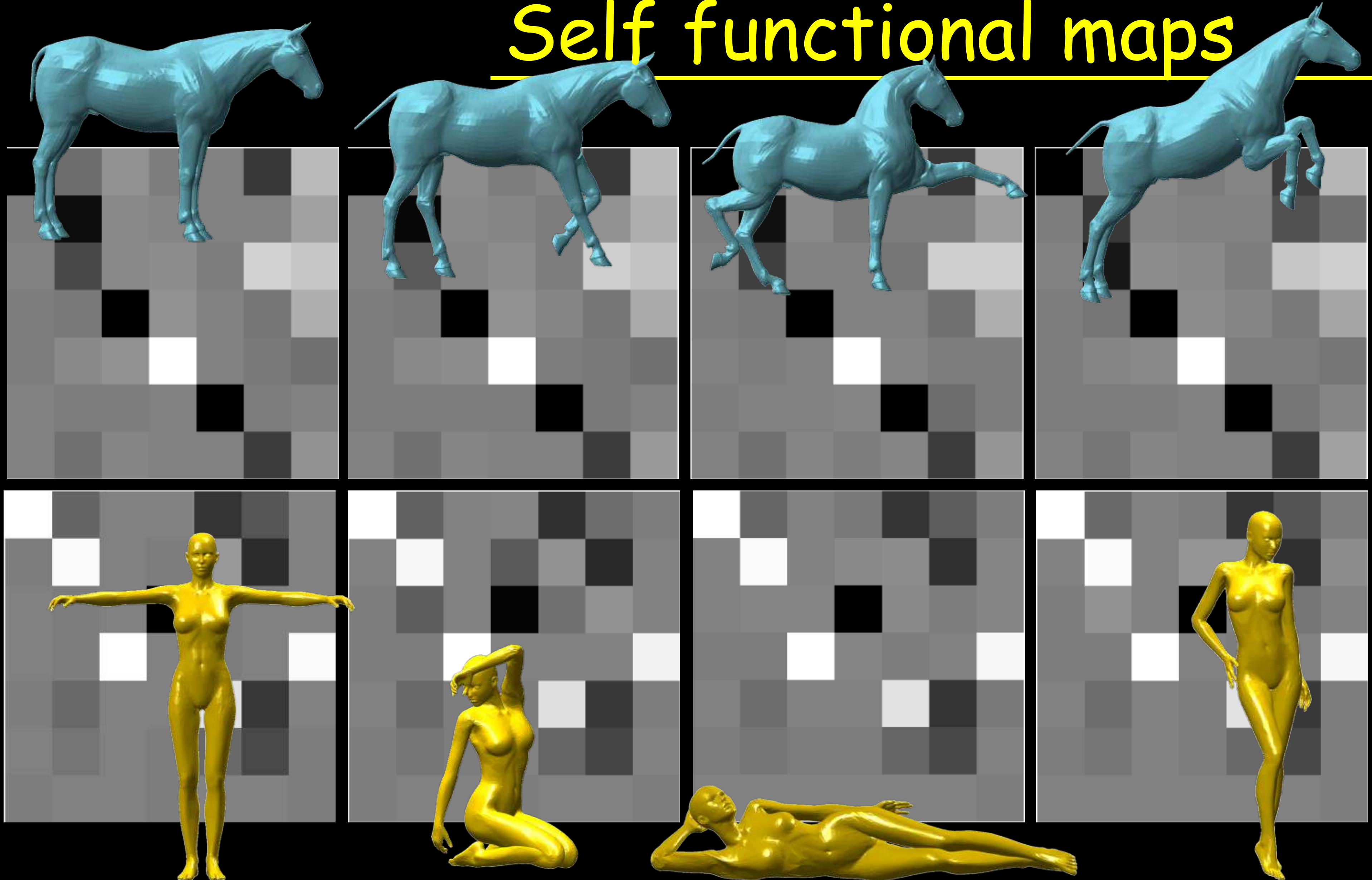
Halimi & K. 2018

Self functional maps

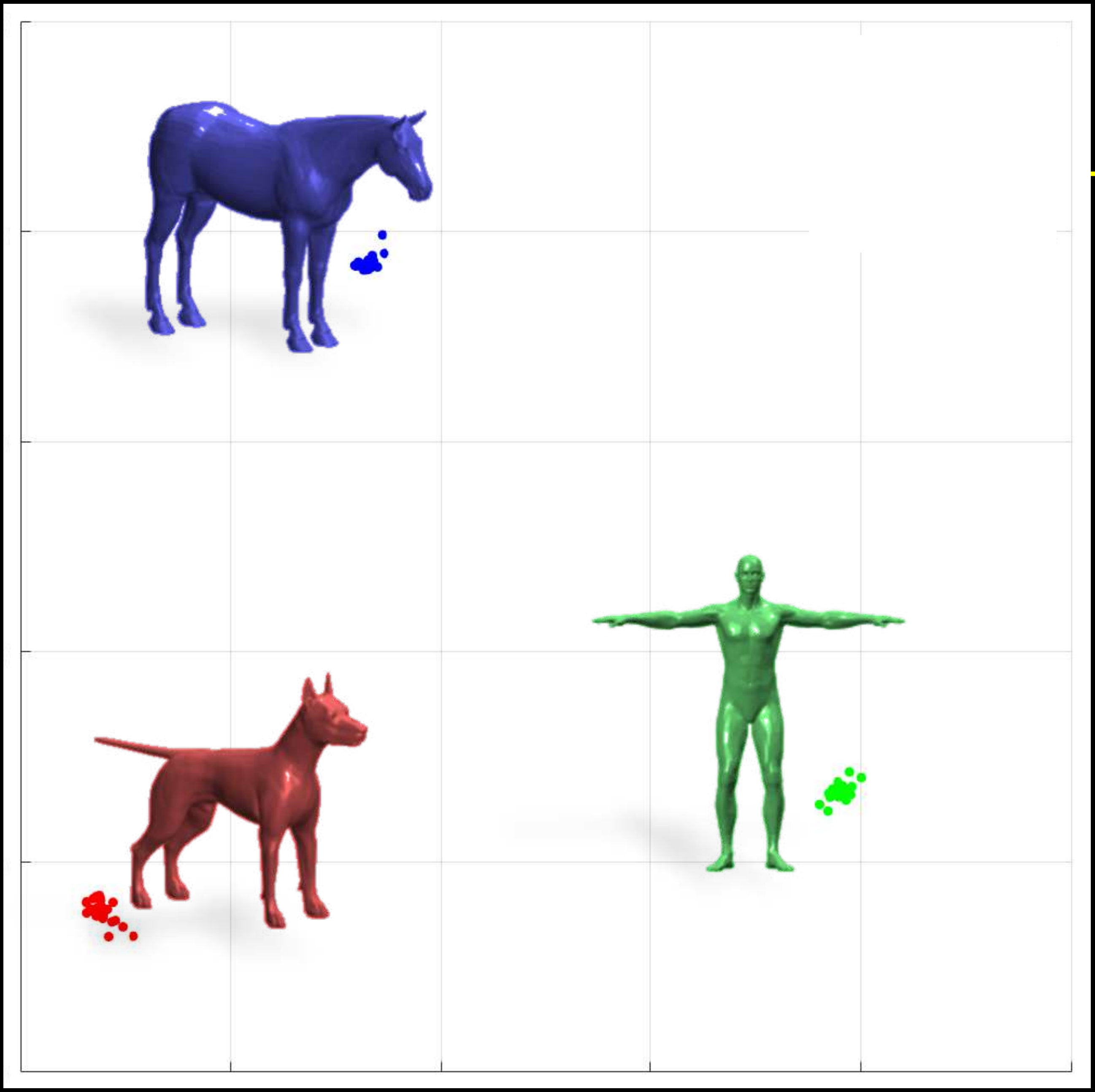


Halimi & K. 2018

Self functional maps



Halimi & K. 2018



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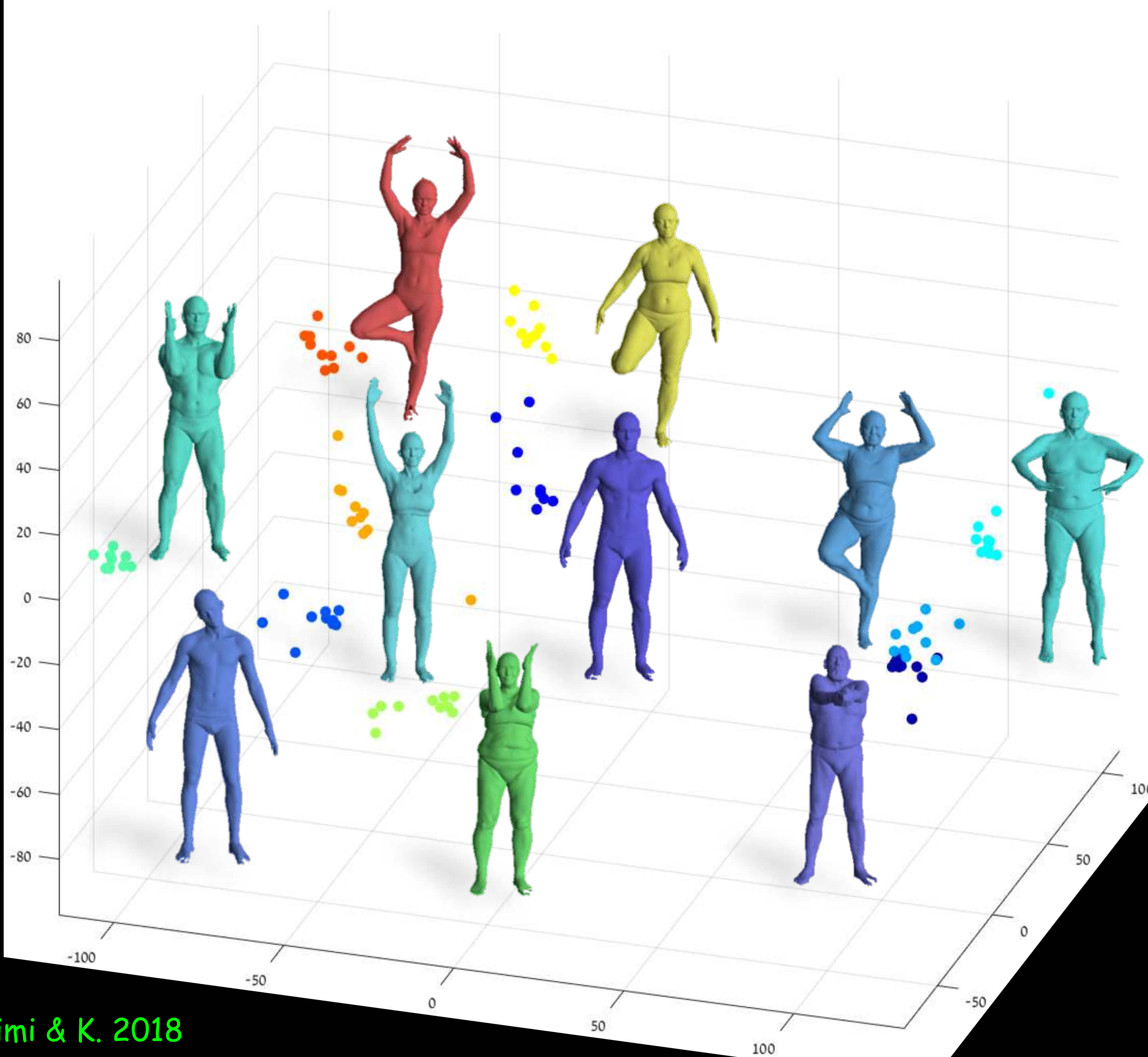

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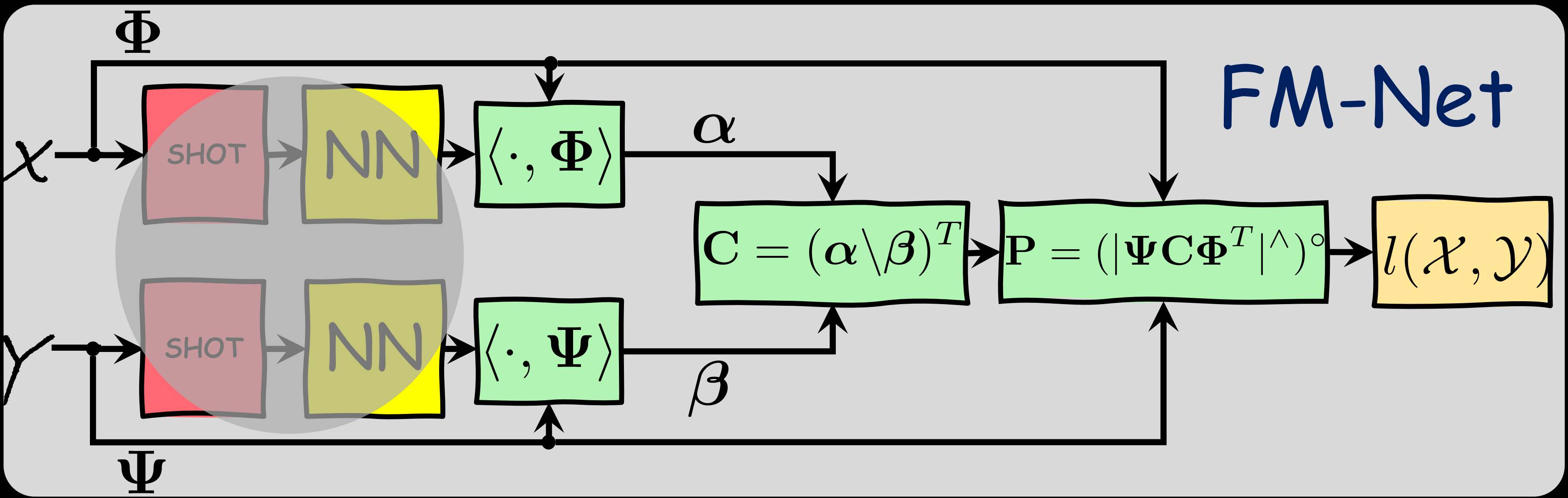
Unsupervised Functional Maps-Net

$$l_{uns}(\mathcal{X},\mathcal{Y}) = \|D_{\mathcal{X}} - P D_{\mathcal{Y}} P^T\|$$



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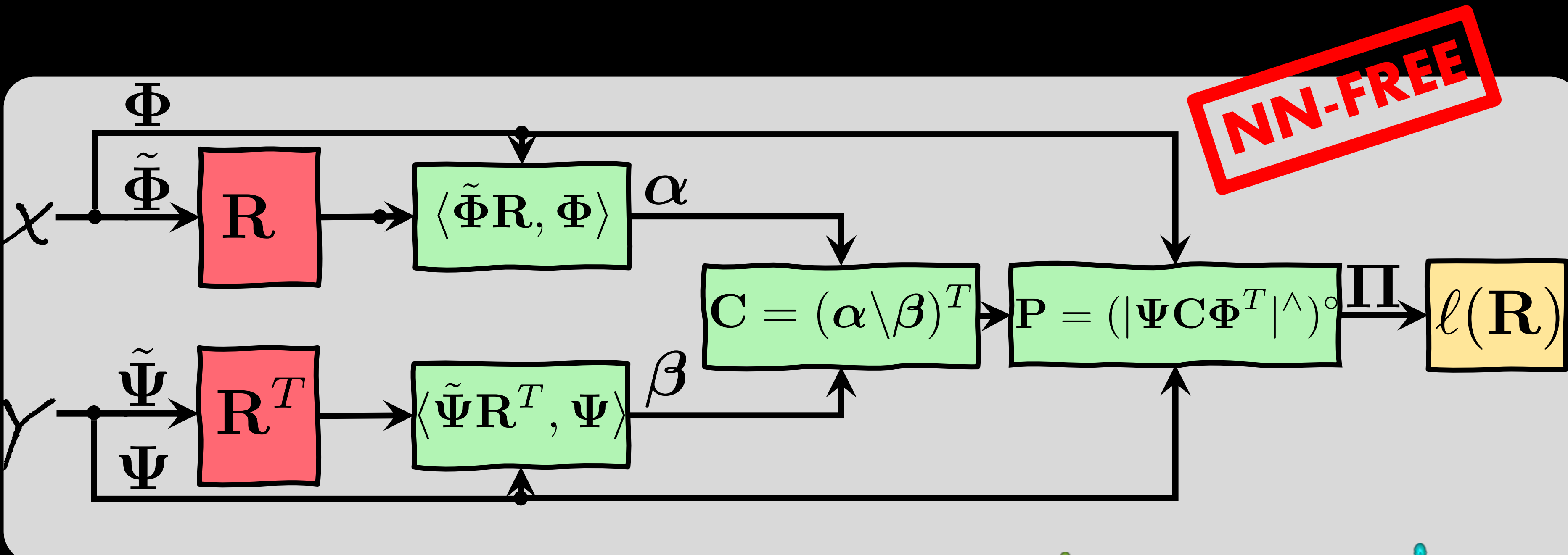
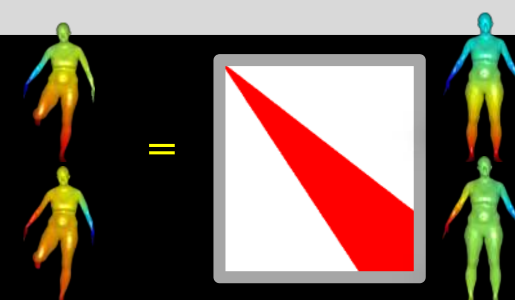
Halimi, Litany, Rodola, A Bronstein, K. CVPR'19

Roufosse, Sharma, Ovsjanikov, ICCV'19

Aligning scale-invariant LBO eigenfunctions

$$l_{uns}(\mathcal{X},\mathcal{Y}) = \|D_{\mathcal{X}} - P D_{\mathcal{Y}} P^T\|$$

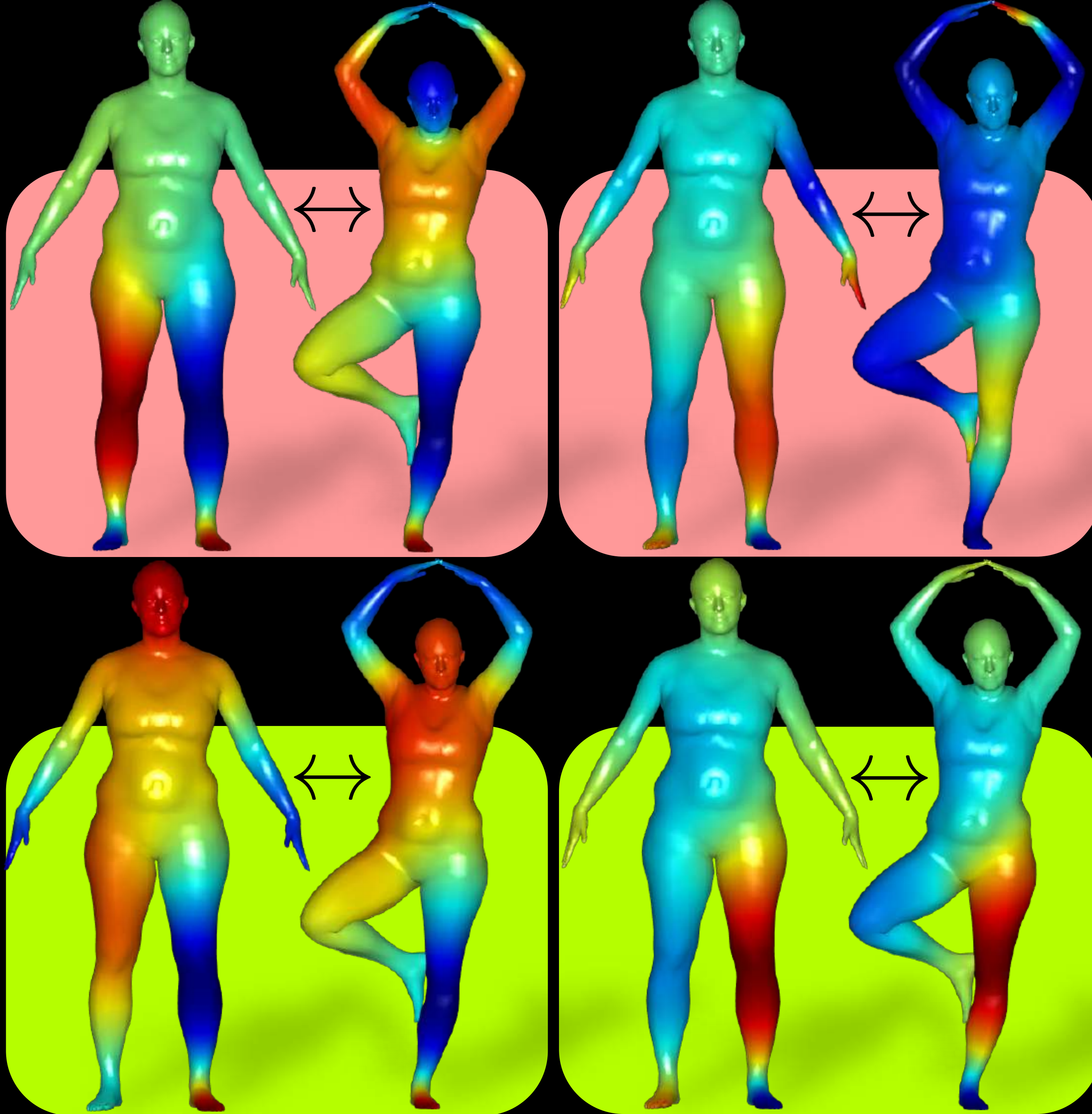




Bracha, Halimi, K. 3DOR'20

Before

After



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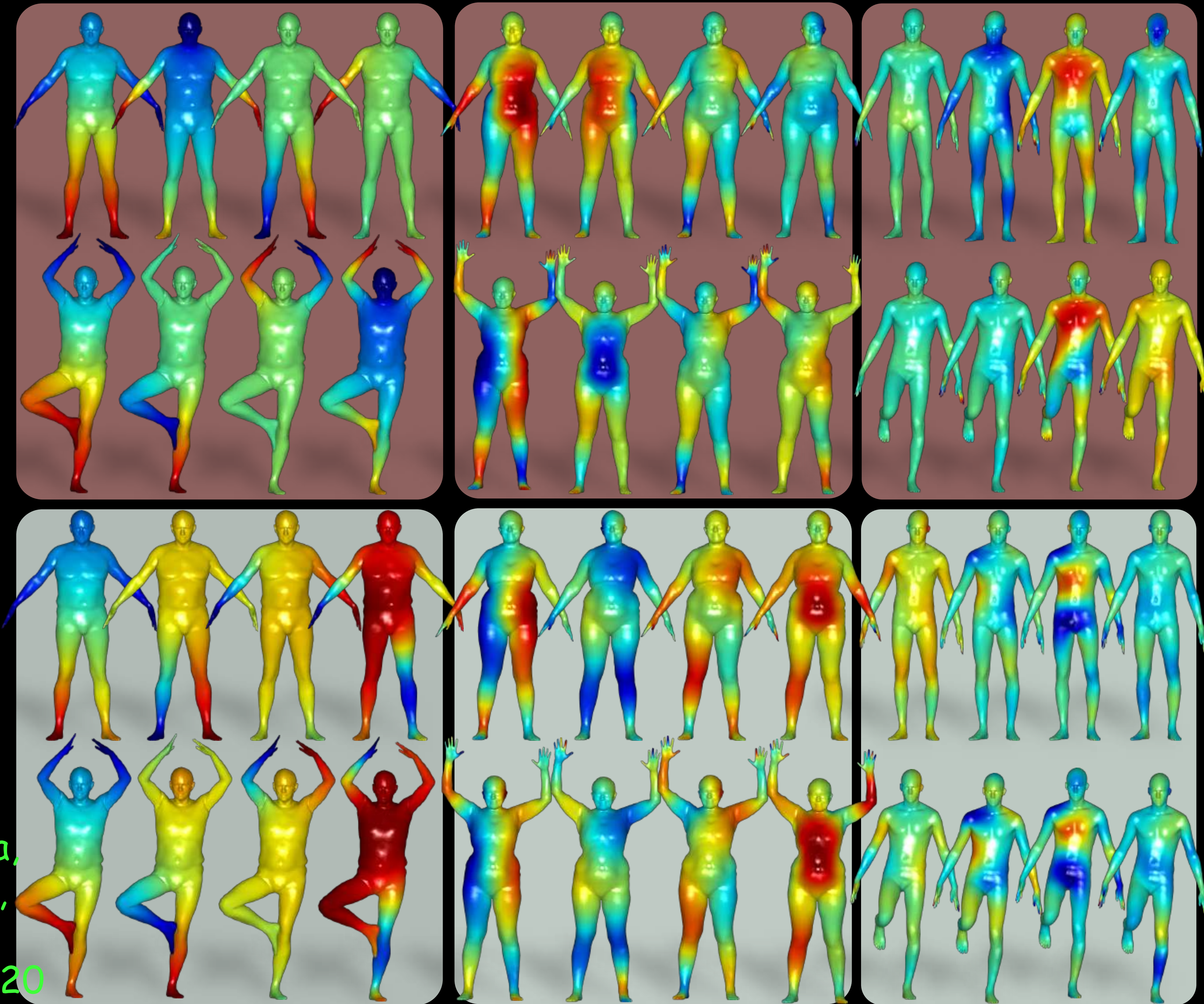
Before

After



Bracha, Halimi, K. 3DOR'20

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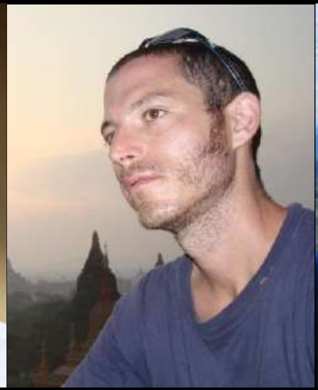
Artificial Intelligence Algorithms to Assess Hormonal Status From Tissue Microarrays in Patients With Breast Cancer

Gil Shamai

Yoav Binenbaum



Ron Slossberg

Irit Duek

Ziv Gil

Ron Kimmel

Technion
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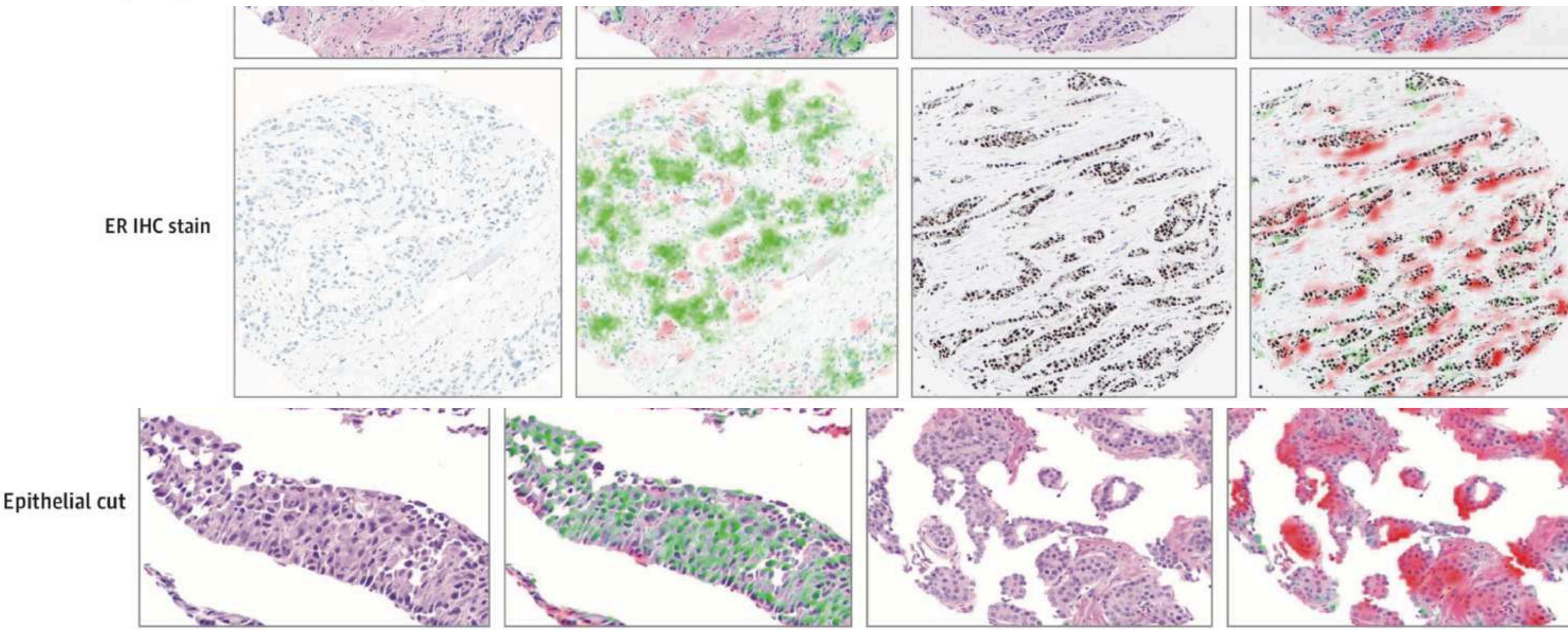
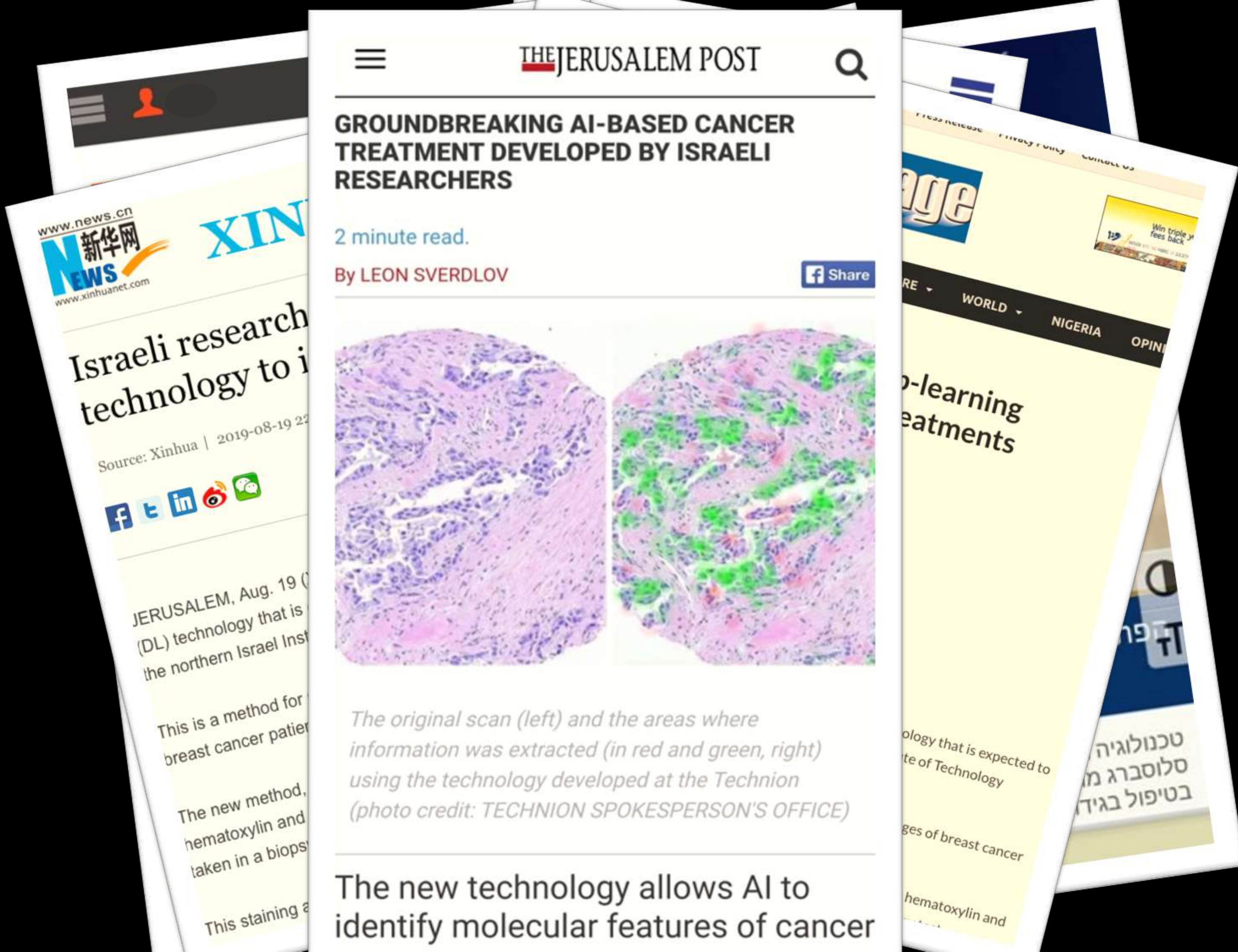
Artificial Intelligence Algorithms to Assess Hormonal Status From Tissue Microarrays in Patients With Breast Cancer

Gil Shamai, MSc; Yoav Binenbaum, MD, PhD; Ron Slossberg, MSc; Irit Duek, MD; Ziv Gil, MD, PhD; Ron Kimmel, DSc



Shamai, Binenbaum, Slossberg, Duek, Gil, K, JAMA'19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