High Rayleigh Number Convection in a Slender Cylinder for Prandtl Number of 1

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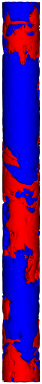

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- Experiments for heat transport at very high Rayleigh number $Nu \propto Ra^{\beta}$:
  - Brno, Chicago, Eugene, Trieste, Göttingen, Grenoble
  - Range from about $Ra = 10^6 - 10^{15}$, $0.5 < \Gamma < 1$
  - Results for a sharp transition in scaling exponent β are mixed.
- DNS for heat transport at very high Rayleigh number $Nu \propto Ra^{\beta}$:
  - Stevens, et. al (2011): Highest $Ra = 2 \times 10^{12}$ for $\Gamma = 1/2, 1/4$.
  - No transition in scaling exponent β.

Dimensionless Boussinesq equations

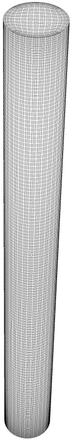
$$
\begin{aligned}
(\partial_t + \mathbf{u} \cdot \nabla)\,\mathbf{u} &= -\nabla P + \sqrt{\frac{Pr}{Ra}}\nabla^2 \mathbf{u} + T\hat{z} \\
(\partial_t + \mathbf{u} \cdot \nabla)\,T &= \frac{1}{\sqrt{Ra\,Pr}}\nabla^2 T \\
\nabla \cdot \mathbf{u} &= 0
\end{aligned}
$$

- $\mathbf{u} = (u_x, u_y, u_z)$ = velocity field, T = temperature, P = pressure field
- Ra=Rayleigh number=$\left(\alpha g H^3/\kappa\nu\right)\Delta T$, Pr=Prandtl number=ν/κ
- Γ = diameter/depth for cylinders
- ν=kinematic viscosity,κ=therm diffusivity, α=therm exp coeff, g=gravity
- Variables rescaled with free-fall velocity $U_f = \sqrt{g\alpha(\Delta T)H}$, temperature ΔT, and height H

Fine resolution with Nek5000—Paul Fischer ANL

$\Gamma = 0.1$, $Pr = 1$

Ra	N	N_e	$N_{e,z}$	$N_e N^3$	Runtime (t_f)
1×10^{11}	9	192,000	300	1×10^8	136
1×10^{12}	12	537,600	400	7×10^8	58
1×10^{13}	13	925,200	500	2×10^9	24
1×10^{14}	11	17,145,600	1200	2×10^{10}	14
1×10^{15}	11	17,145,600	1200	2×10^{10}	7



N = polynomial order
N_e=total number of elements
$N_{e,z}$=number of elements in z direction

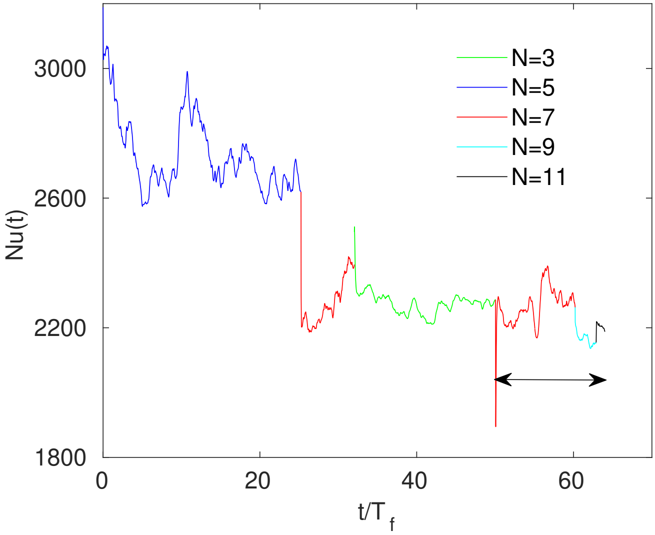
Largest runs done on Mira with 524,288 MPI tasks

See JS, Emran & Schumacher, New J. Phys. (2013) for more info.

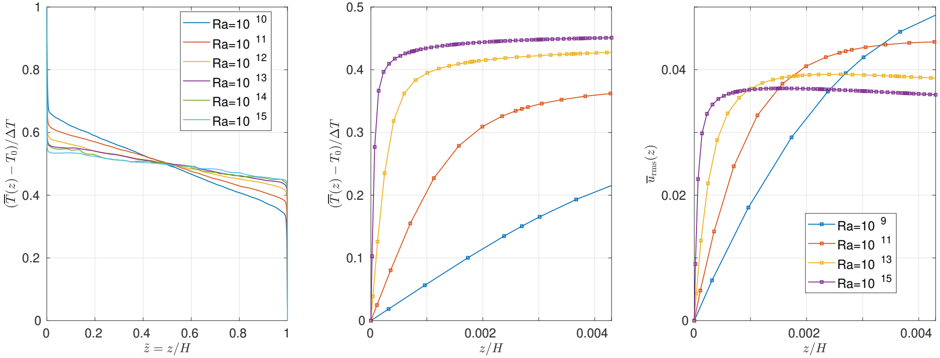
Convergence of Nusselt number

$Nu = |\langle dT/dz \rangle_{A,t}|/A$

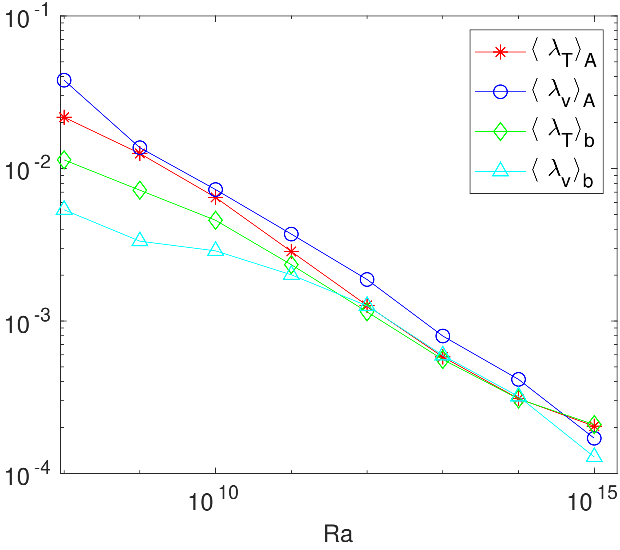
$Ra = 1 \times 10^{14}$



Temperature and Velocity profiles



Boundary Layer thicknesses

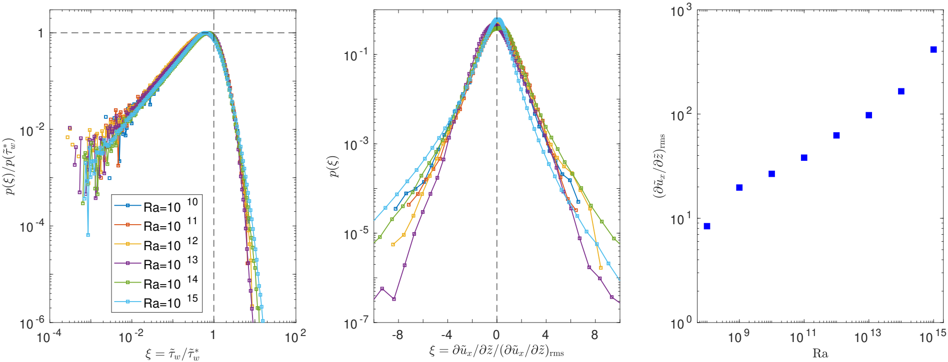


area, $r < 0.05$

bulk, $r < 0.03$

Wall shear stress and velocity gradients

$$\tau_w(x, y, z = 0) = \sqrt{\frac{Pr}{Ra}\left[\left(\frac{\partial u_x}{\partial z}\right)^2 + \left(\frac{\partial u_y}{\partial z}\right)^2\right]}$$



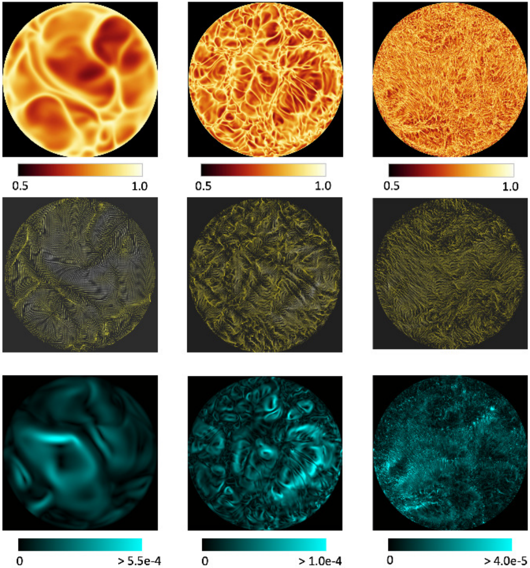
Boundary Layer structure

$Ra = 1 \times 10^{11}$ $Ra = 1 \times 10^{13}$ $Ra = 1 \times 10^{15}$

Temp →

Streamlines →

Wall shear stress →



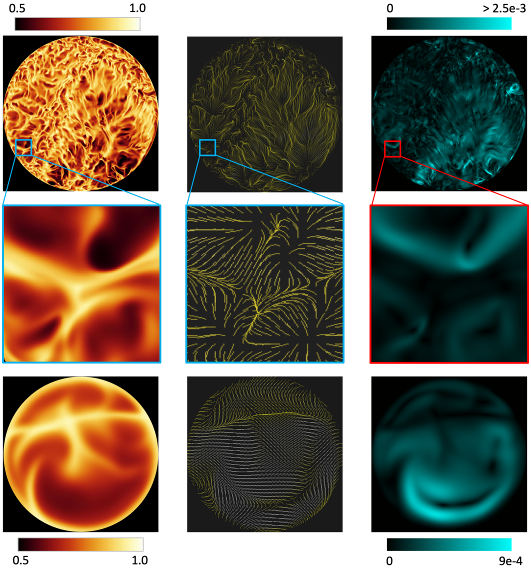
Different Aspect Ratios, $Ra = 1 \times 10^{10}$

Temperature Streamlines Wall shear stress

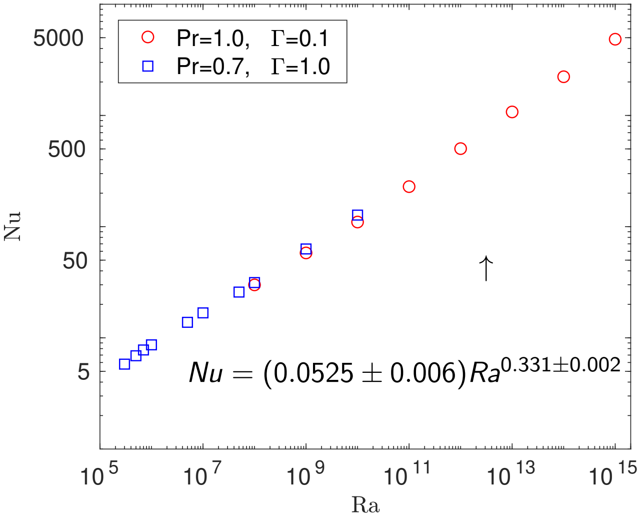
$\Gamma = 1.0$

$\Gamma = 1.0$
Zoom

$\Gamma = 0.1$

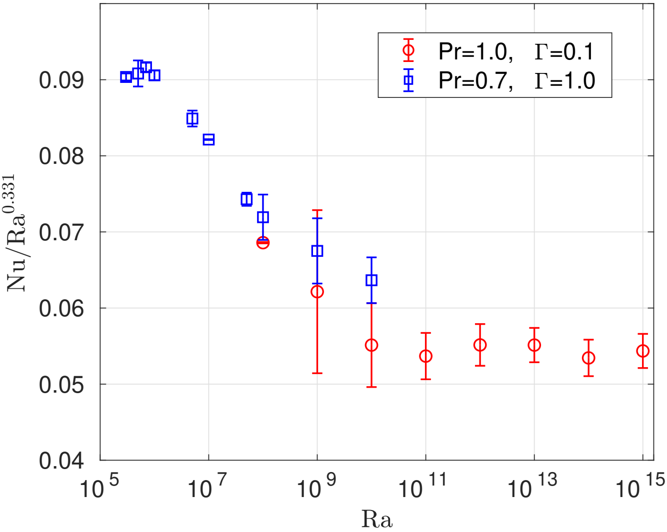


Nusselt Number



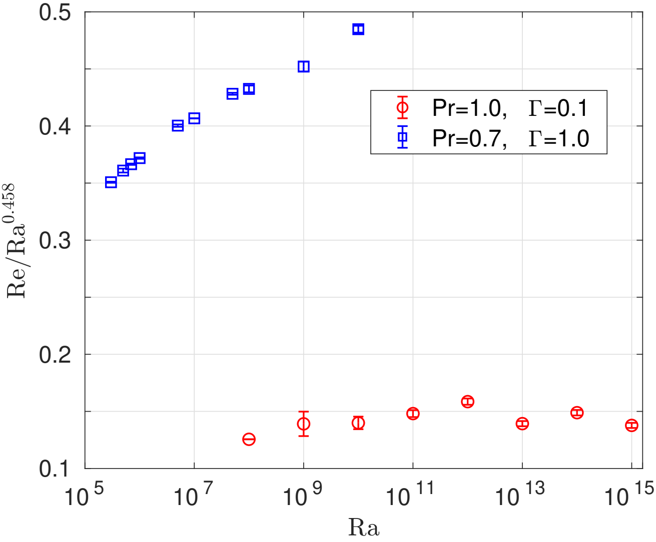
Good agreement w/ Malkus (exponent $= 1/3$) & Spiegel (prefactor $= 0.07$)

Nusselt Number, compensated plot



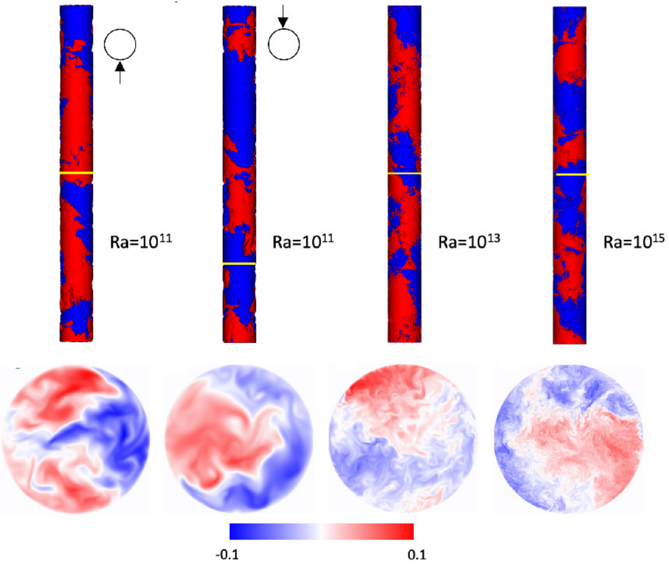
Reynolds Number, compensated plot

$\mathrm{Re} = \langle u_x^2 + u_y^2 + u_z^2 \rangle_{V,t}^{1/2} \left(\frac{\mathrm{Ra}}{\mathrm{Pr}}\right)^{1/2}$



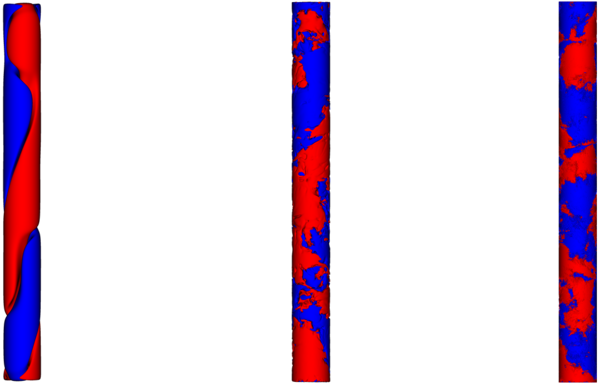
Flow states

$\Gamma = 0.1$, $Pr = 1$, Plots of u_z



Movies

$Pr = 1.0, \Gamma = 0.1$



$Ra = 1 \times 10^8$ $\quad$ $Ra = 1 \times 10^{11}$ $\quad$ $Ra = 1 \times 10^{15}$

Conclusions

- Well resolved, very high Rayleigh number simulations for $\Gamma = 0.1$ and $Pr = 1.0$ have been performed.
- Scaling laws have been determined for heat ($\propto Ra^{0.331}$) and momentum transport ($\propto Ra^{0.458}$) up to $Ra = 10^{15}$.
- Aspect ratio matters for momentum transport, but not for heat transport.
- Heat transport scaling is consistent with the $1/3$-power law of Malkus.

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