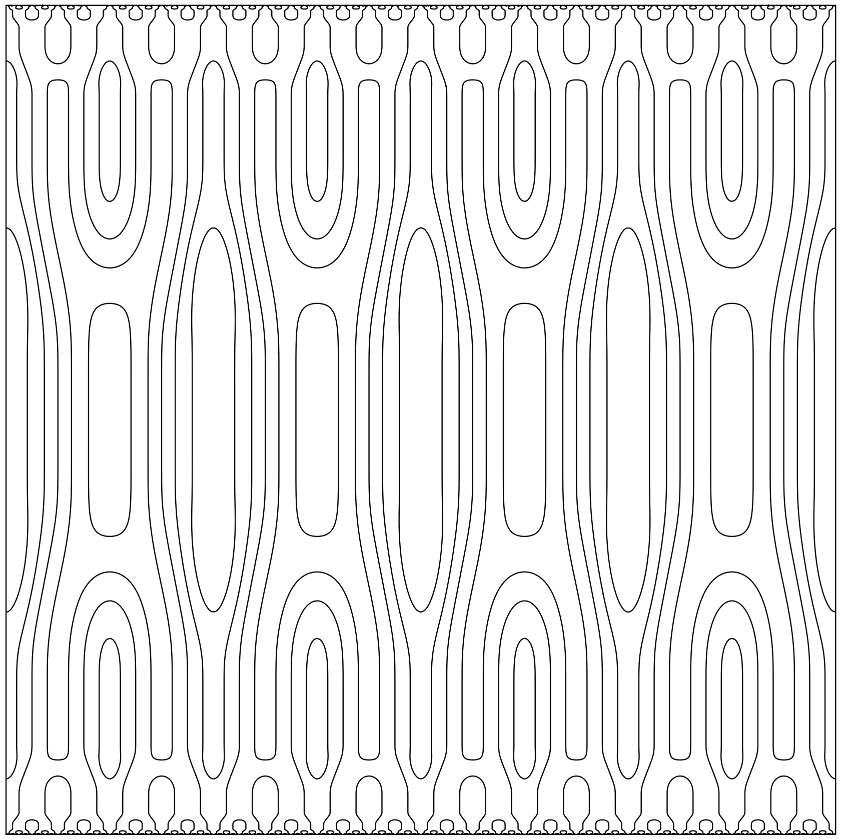
Branching patterns in the optimal design of heat transport



Ian Tobasco

U of Illinois at Chicago

IPAM Workshop (*virtual!*)
Jan. 11-14, 2021

A. Souza, I. Tobasco & C. R. Doering, JFM 889 (2020)
I. Tobasco & C. R. Doering, CPAM 72 (2019)
C. R. Doering & I. Tobasco, PRL 118 (2017)

+ more *in prep*

Two problems

$T = 0$

$$\partial_t T + \mathbf{u} \cdot \nabla T = \Delta T$$

transport
between parallel walls

$T = 1$

$$\partial_t T + \mathbf{u} \cdot \nabla T = \Delta T + 1$$

cool
a heated disc

$T = 0$

P. Hassanzadeh, G. P. Chini, C. R. Doering, JFM 751 (2014)

F. Marcotte, C. R. Doering, J-L Thiffeault, W. R. Young, SIAM J Appl Math 78 (2018)

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$$T = 0$$

Goal: Transport/cool optimally…

$$\max_{\mathbf{u}(x,t)} \left\langle |\nabla T|^2 \right\rangle \qquad \min_{\mathbf{u}(x,t)} \left\langle |\nabla T|^2 \right\rangle$$

…amongst a given class of velocities

Two problems

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$$\max_{\mathbf{u}(x,t)} \left\langle |\nabla T|^2 \right\rangle \qquad \min_{\mathbf{u}(x,t)} \left\langle |\nabla T|^2 \right\rangle$$

…subject to $\left\langle |\nabla \mathbf{u}|^2 \right\rangle^{1/2} = Pe$

Enstrophy-constrained optimization

natural from the point of view of a power budget

$$
\begin{cases}
\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = \nabla p + \nu \Delta \mathbf{u} + \mathbf{f} & \text{in } \Omega \\
\nabla \cdot \mathbf{u} = 0 & \text{in } \Omega \\
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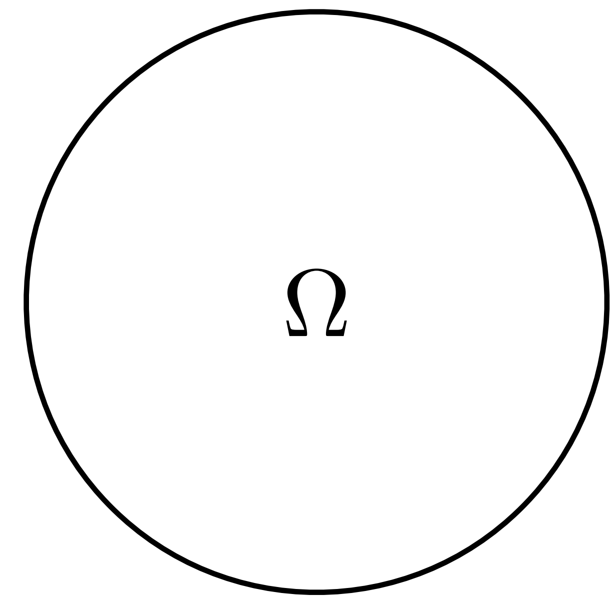
\end{cases}
$$



Enstrophy-constrained optimization

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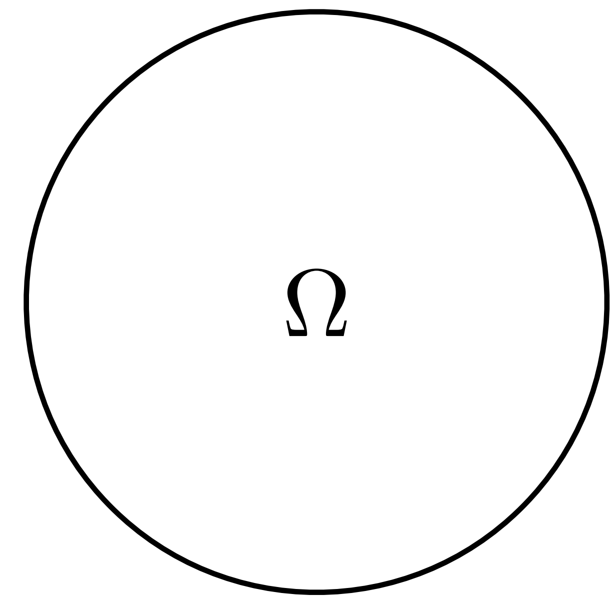
$$\begin{cases} \partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = \nabla p + \nu\Delta\mathbf{u} + \mathbf{f} & \text{in } \Omega \\ \nabla\cdot\mathbf{u} = 0 & \text{in } \Omega \\ \mathbf{u} = 0 & \text{at } \partial\Omega \end{cases}$$



$$\nu\left\langle \left|\nabla\mathbf{u}\right|^2 \right\rangle = \langle \mathbf{f}\cdot\mathbf{u}\rangle$$

Energy dissipation balances power input to sustain flow

THM: Optimal scaling laws

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$$\partial_t T + \mathbf{u} \cdot \nabla T = \Delta T$$

transport
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between parallel walls

$$T = 1$$

$$\max_{\mathbf{u}(x,t)} \left\langle |\nabla T|^2 \right\rangle \sim Pe^{2/3}$$

$$\left\langle |\nabla \mathbf{u}|^2 \right\rangle^{1/2} = Pe$$

$$\mathbf{u} = 0 \text{ at } \partial\Omega$$

$$\partial_t T + \mathbf{u} \cdot \nabla T = \Delta T + 1$$

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a heated disc

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cool
a heated disc

$$T = 0$$

$$\min_{\mathbf{u}(x,t)} \left\langle |\nabla T|^2 \right\rangle \sim Pe^{-2/3}$$

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$$\left\langle |\nabla \mathbf{u}|^2 \right\rangle^{1/2} = Pe$$

$$\mathbf{u} = 0 \text{ at } \partial\Omega$$

Both scaling laws hold <u>up to log corrections</u> for $Pe \gg 1$.

Focus on the <u>heated disc</u>

"warmup" — energy-constrained transport

$$\partial_t T + \mathbf{u} \cdot \nabla T = \Delta T + 1$$

cool
a heated disc

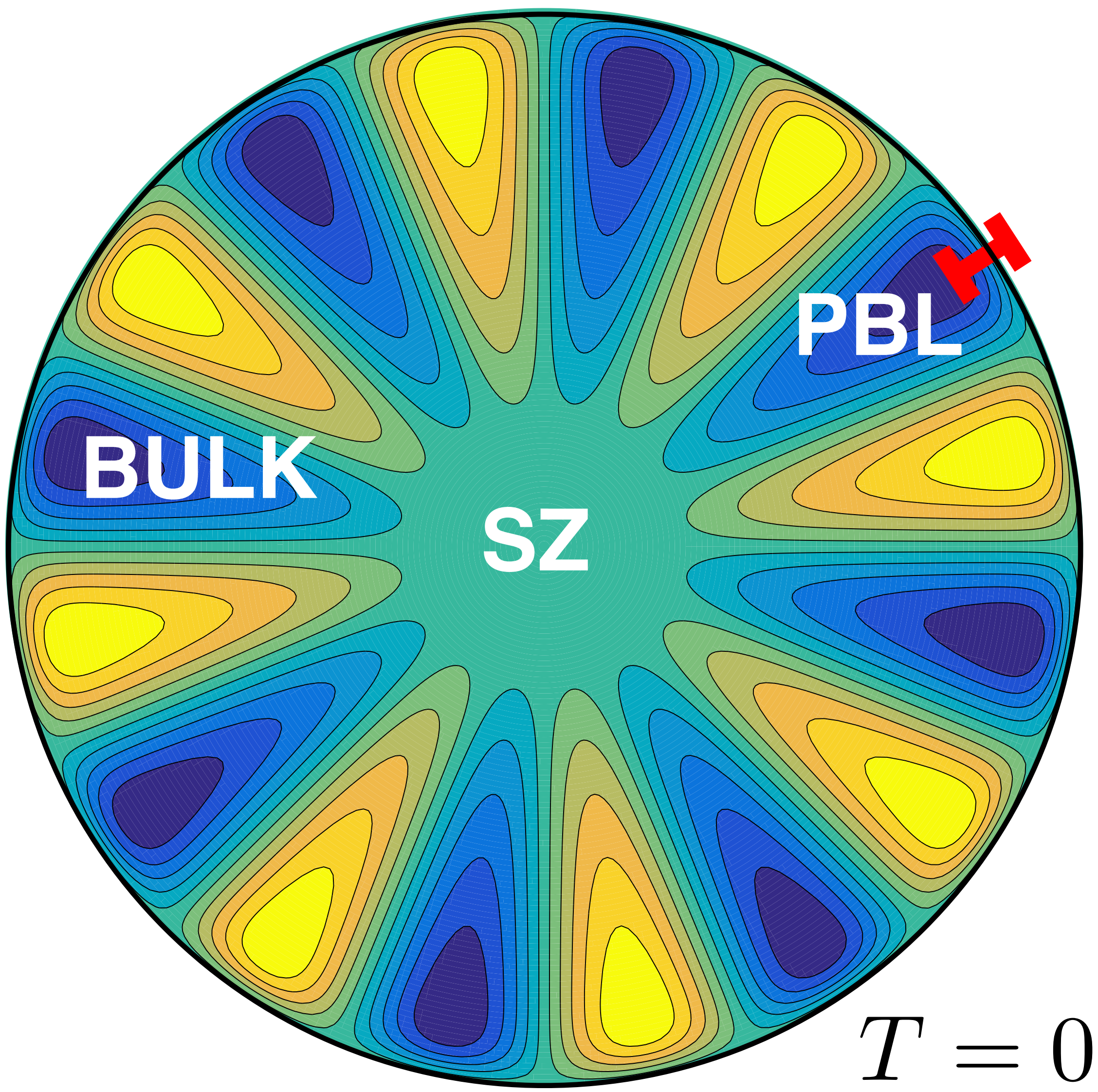
$T = 0$

$$\min_{\substack{\mathbf{u}(x,t) \\ \left\langle |\mathbf{u}|^2 \right\rangle^{1/2} = Pe \\ \mathbf{u}\cdot\hat{\mathbf{r}}=0 \text{ at } \partial\Omega}} \left\langle |\nabla T|^2 \right\rangle$$

F. Marcotte, C. R. Doering, J-L Thiffeault, W. R. Young, SIAM J Appl Math 78 (2018)

Focus on the heated disc

"warmup" — energy-constrained transport

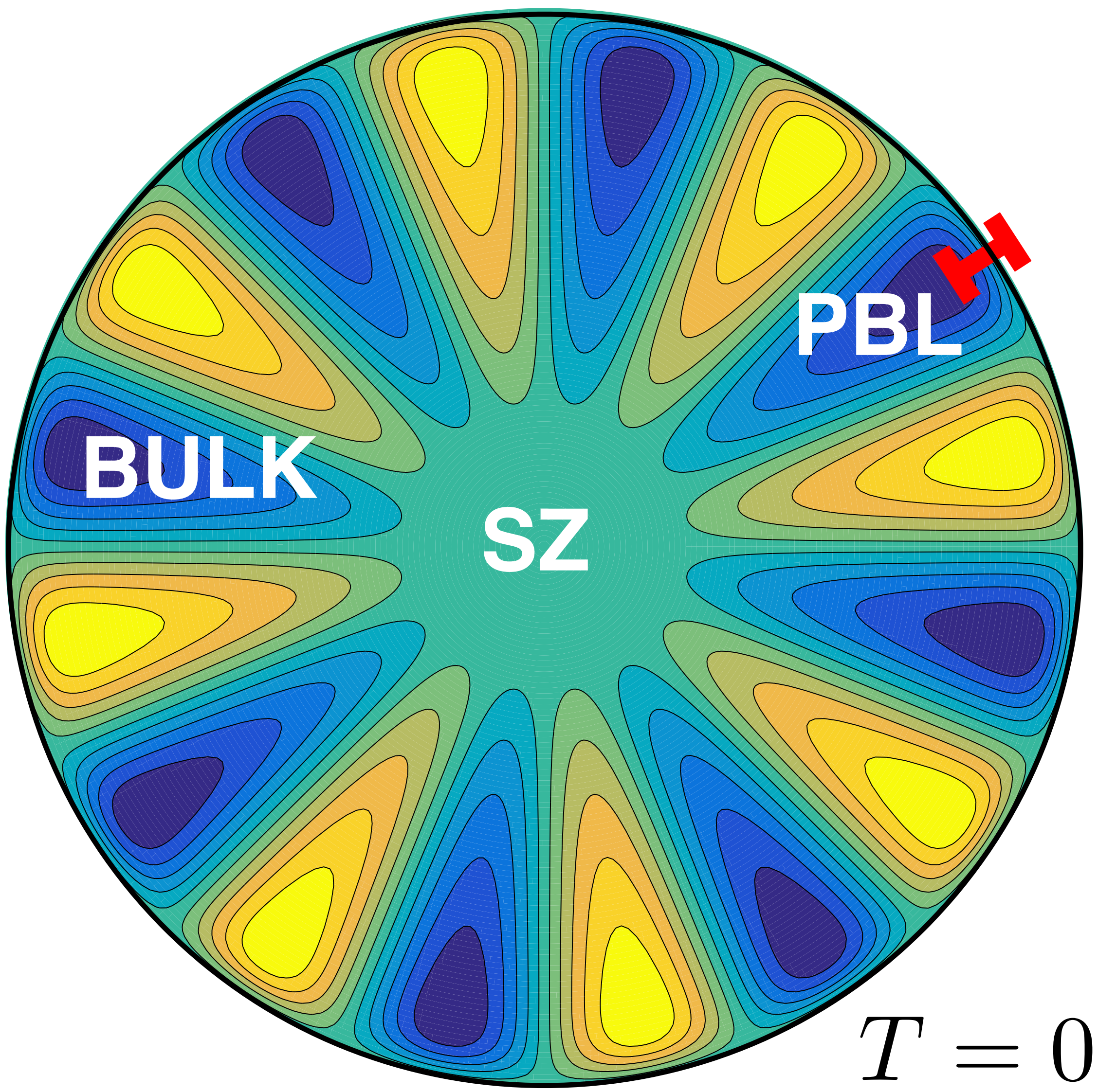


$$\min_{\substack{\mathbf{u}(x,t) \\ \left\langle|\mathbf{u}|^2\right\rangle^{1/2}=Pe \\ \mathbf{u}\cdot\hat{\mathbf{r}}=0 \text{ at } \partial\Omega}} \left\langle|\nabla T|^2\right\rangle \leq C' \frac{1}{Pe}$$

F. Marcotte, C. R. Doering, J-L Thiffeault, W. R. Young, SIAM J Appl Math 78 (2018)

Focus on the heated disc

"warmup" — energy-constrained transport



$$? \le \min_{\substack{\mathbf{u}(x,t) \\ \langle |\mathbf{u}|^2 \rangle^{1/2} = Pe \\ \mathbf{u}\cdot\hat{\mathbf{r}}=0 \text{ at } \partial\Omega}} \left\langle |\nabla T|^2 \right\rangle \le C' \frac{1}{Pe}$$

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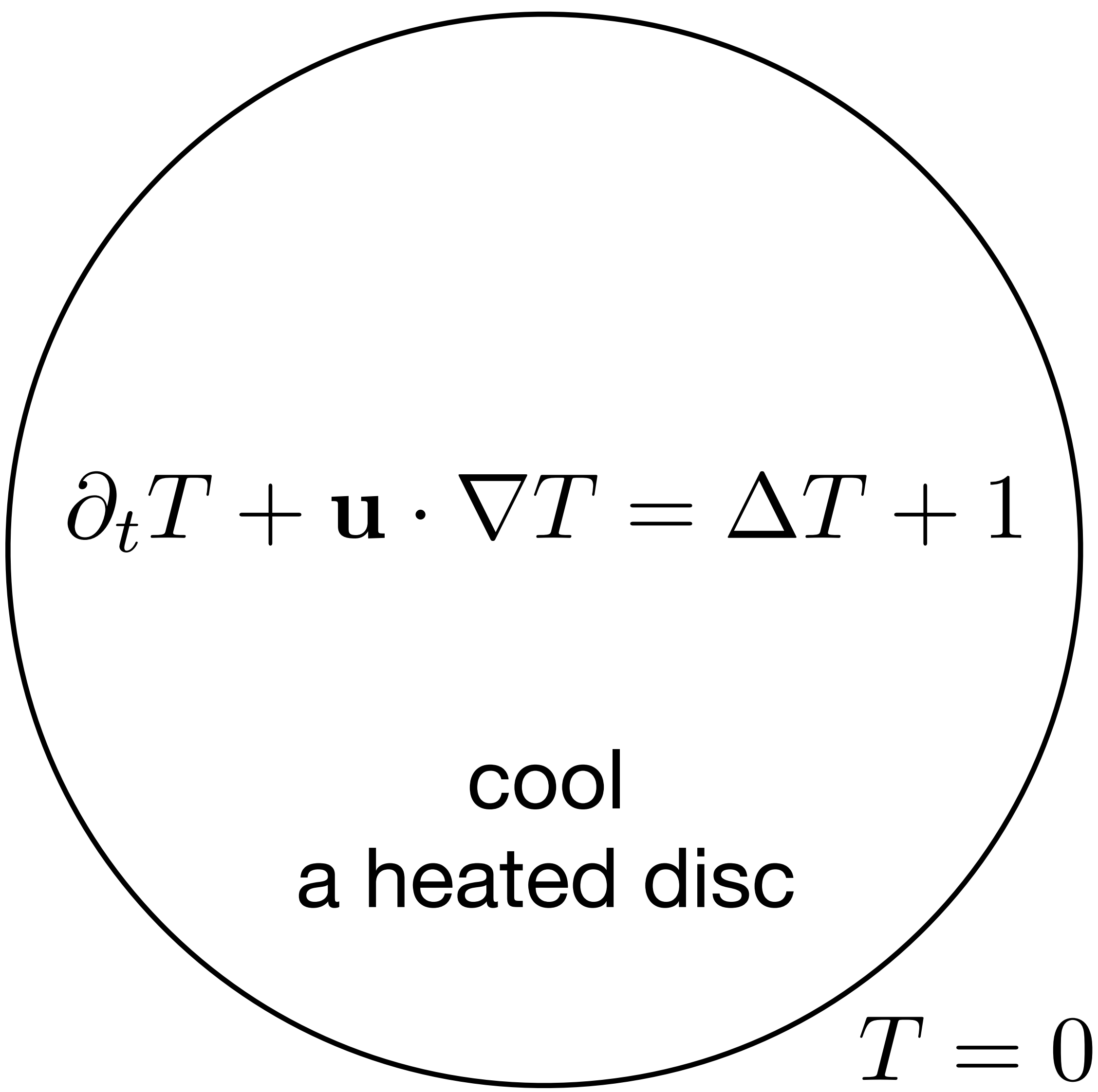
Focus on the heated disc

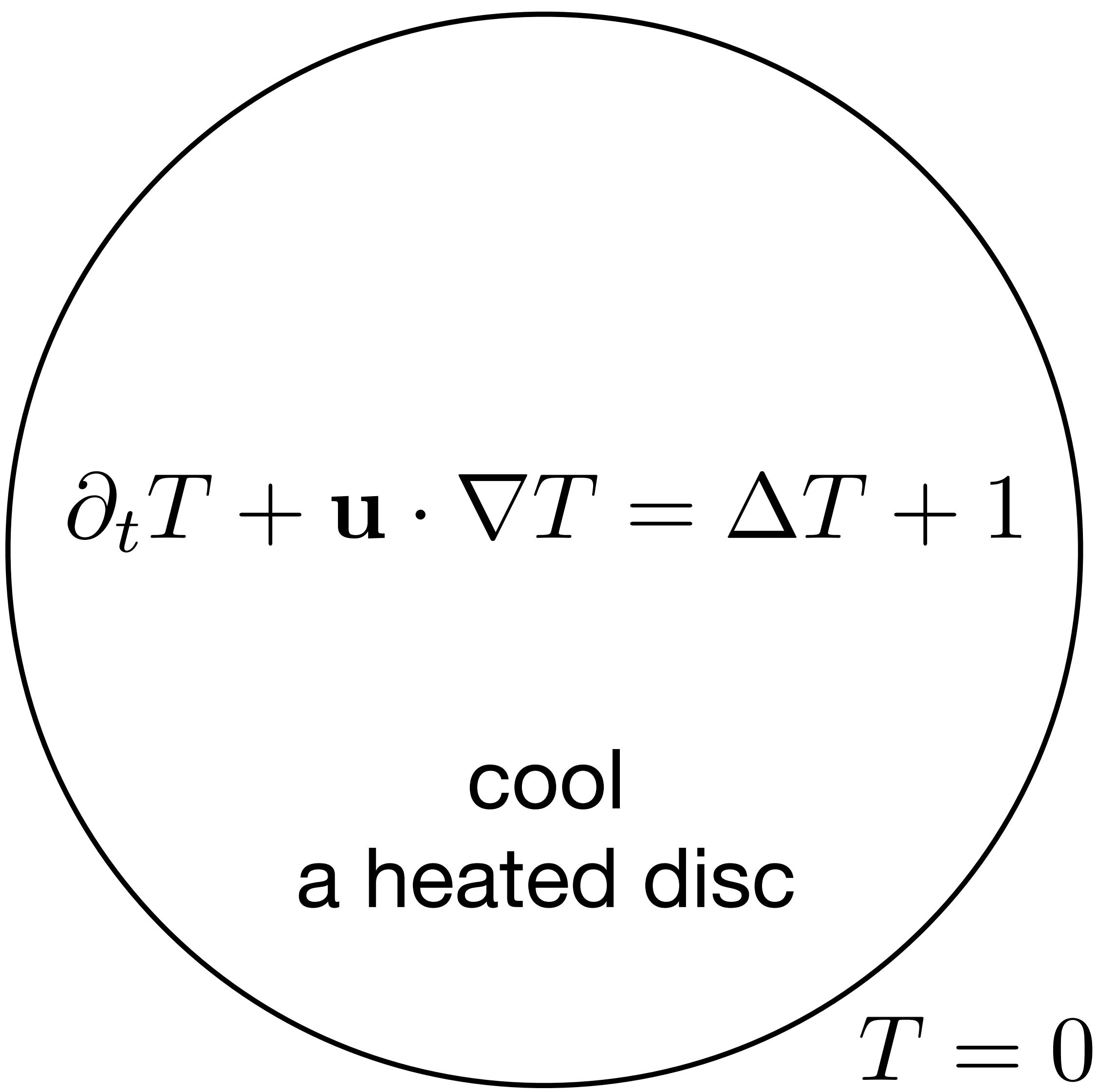
back to enstrophy-constrained transport



$$C \frac{1}{Pe^{2/3}} \leq \min_{\substack{\mathbf{u}(x,t) \\ \left\langle |\nabla \mathbf{u}|^2 \right\rangle^{1/2} = Pe \\ \mathbf{u} = 0 \text{ at } \partial\Omega}} \left\langle |\nabla T|^2 \right\rangle \leq C' \frac{\log^{4/3} Pe}{Pe^{2/3}}$$

Focus on the heated disc

back to enstrophy-constrained transport



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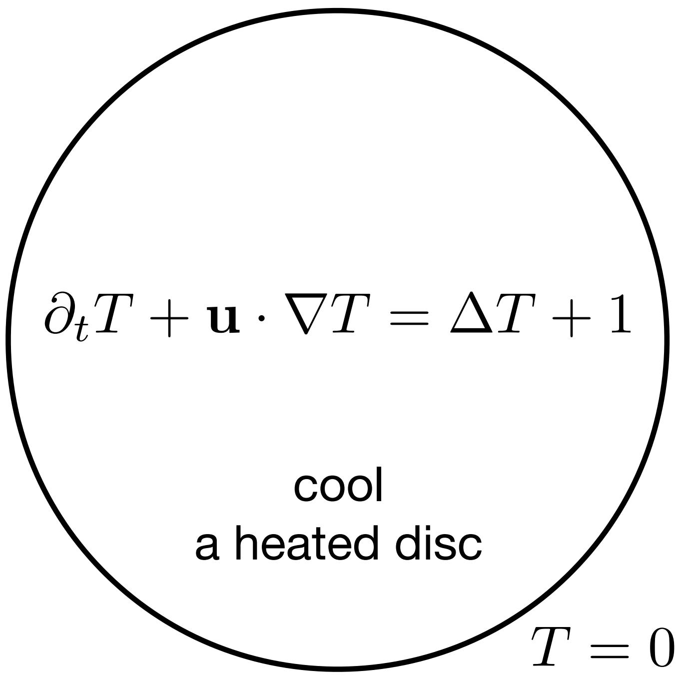
How do optimal flows behave?

Variational bounds on cooling



$$\left\langle |\nabla T|^2 \right\rangle \leq \left\langle |\nabla \eta|^2 + |\nabla \Delta^{-1} \left((\partial_t + \mathbf{u} \cdot \nabla)\eta - 1 \right)|^2 \right\rangle$$

if $\eta = 0 \text{ at } \partial\Omega$

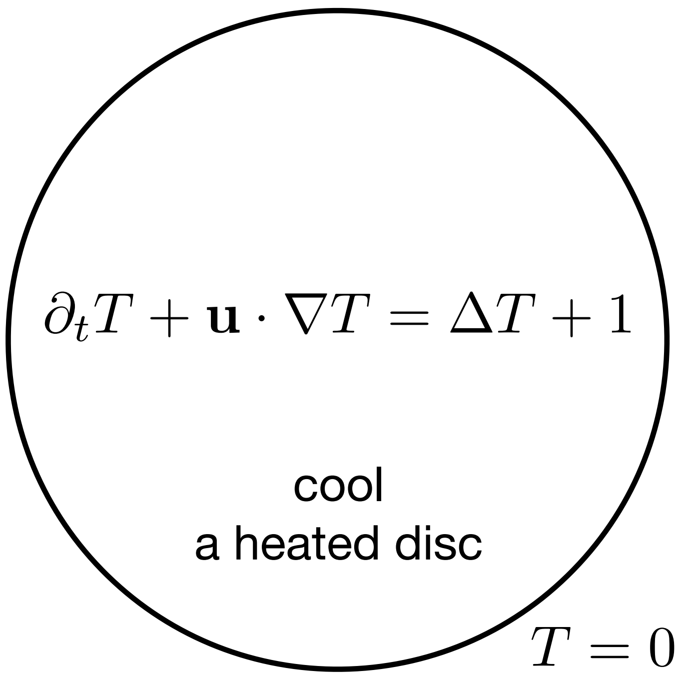
$$\left\langle |\nabla T|^2 \right\rangle \geq \left\langle 2\xi - |\nabla \xi|^2 - |\nabla \Delta^{-1} (\partial_t + \mathbf{u} \cdot \nabla)\xi|^2 \right\rangle$$

if $\xi = 0 \text{ at } \partial\Omega$

(sharp in case of steady flows)

Variational bounds on cooling

proof by “symmetrization”

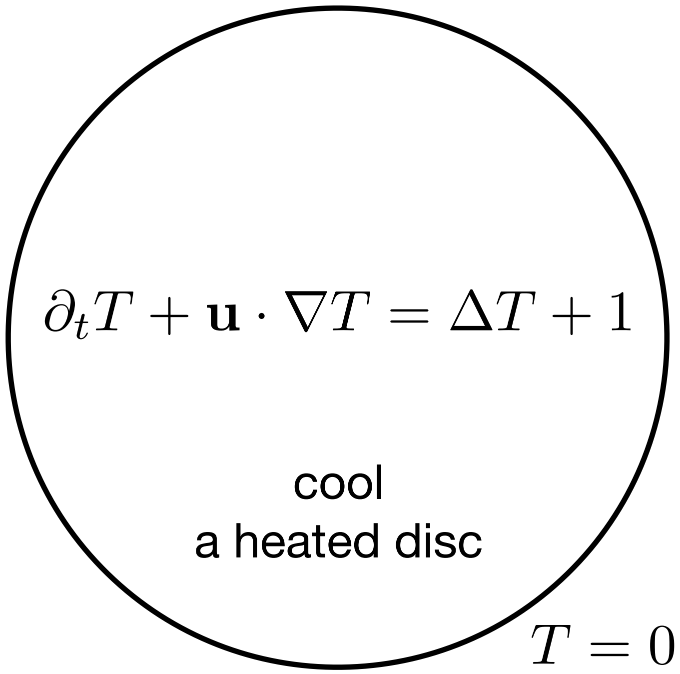


$$\begin{cases} \partial_t T_+ + \mathbf{u}\cdot\nabla T_+ = \Delta T_+ + 1 \\ -\partial_t T_- - \mathbf{u}\cdot\nabla T_- = \Delta T_- + 1 \end{cases}$$

□

Variational bounds on cooling

proof by "symmetrization"



$$\begin{cases} \partial_t T_+ + \mathbf{u} \cdot \nabla T_+ = \Delta T_+ + 1 \\ -\partial_t T_- - \mathbf{u} \cdot \nabla T_- = \Delta T_- + 1 \end{cases}$$

$$\eta = \frac{1}{2}(T_+ - T_-) \qquad \xi = \frac{1}{2}(T_+ + T_-)$$

$$\begin{cases} (\partial_t + \mathbf{u} \cdot \nabla)\eta = \Delta \xi + 1 \\ (\partial_t + \mathbf{u} \cdot \nabla)\xi = \Delta \eta \end{cases}$$

□

Variational bounds on cooling

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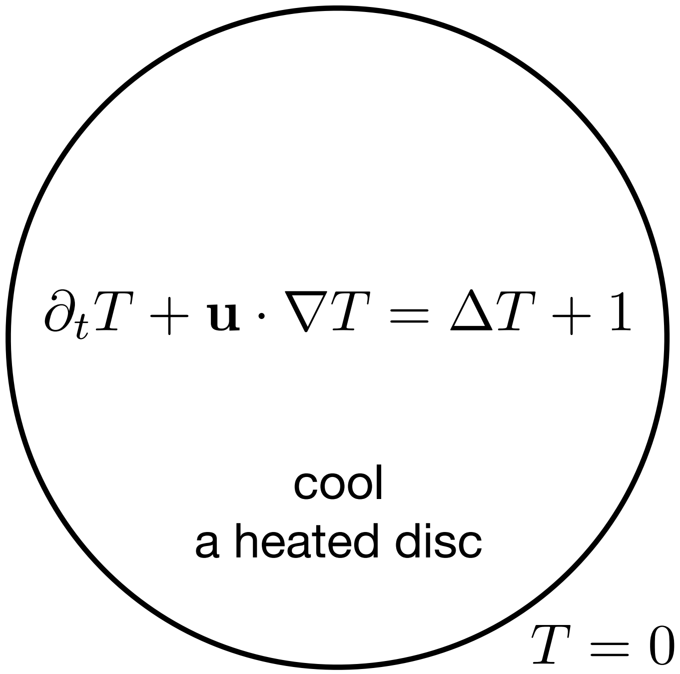

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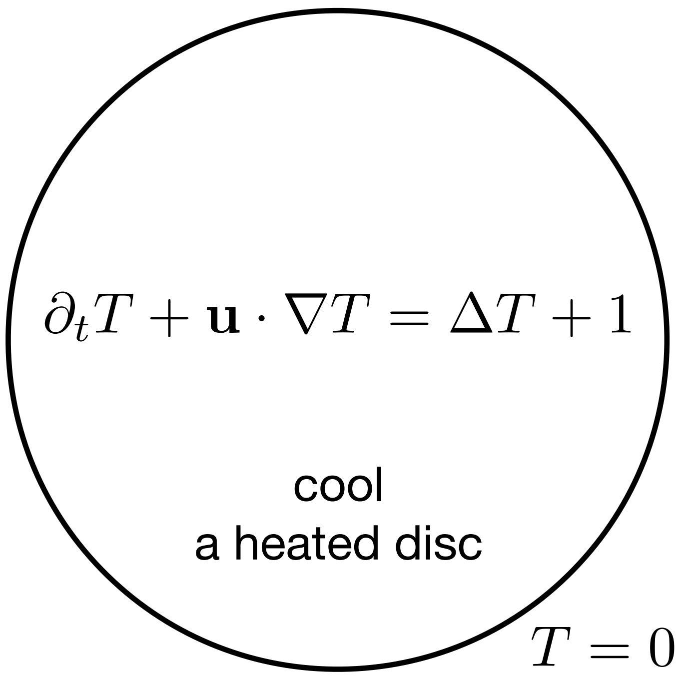
$$\begin{cases} (\partial_t + \mathbf{u} \cdot \nabla)\eta = \Delta \xi + 1 \\ (\partial_t + \mathbf{u} \cdot \nabla)\xi = \Delta \eta \end{cases}$$

$$\langle \nabla \eta \cdot \nabla \xi \rangle = 0 \quad \Longrightarrow \quad \langle |\nabla T|^2 \rangle = \langle |\nabla T_+|^2 \rangle = \langle |\nabla \eta|^2 + |\nabla \xi|^2 \rangle$$

$\square$

Variational bounds on cooling

proof by "symmetrization"



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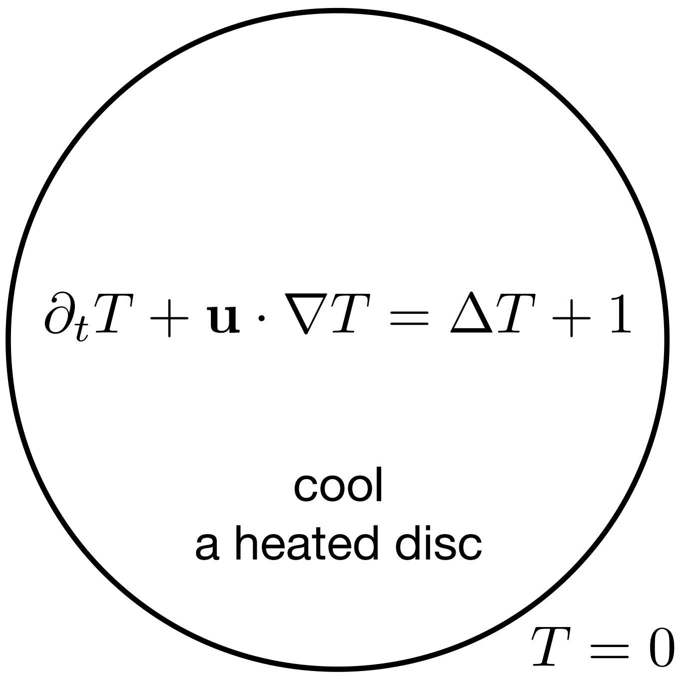
$$\langle \nabla\eta\cdot\nabla\xi \rangle = 0 \quad \Longrightarrow \quad \langle |\nabla T|^2 \rangle = \langle |\nabla T_+|^2 \rangle = \langle |\nabla\eta|^2 + |\nabla\xi|^2 \rangle$$

$$\therefore \quad \begin{aligned} \langle |\nabla T|^2 \rangle &\le \langle |\nabla\eta|^2 + |\nabla\Delta^{-1}\left((\partial_t + \mathbf{u}\cdot\nabla)\eta - 1\right)|^2 \rangle \\ \langle |\nabla T|^2 \rangle &\ge \langle 2\xi - |\nabla\xi|^2 - |\nabla\Delta^{-1}(\partial_t + \mathbf{u}\cdot\nabla)\xi|^2 \rangle \end{aligned}$$

$\square$

Lower bound on cooling

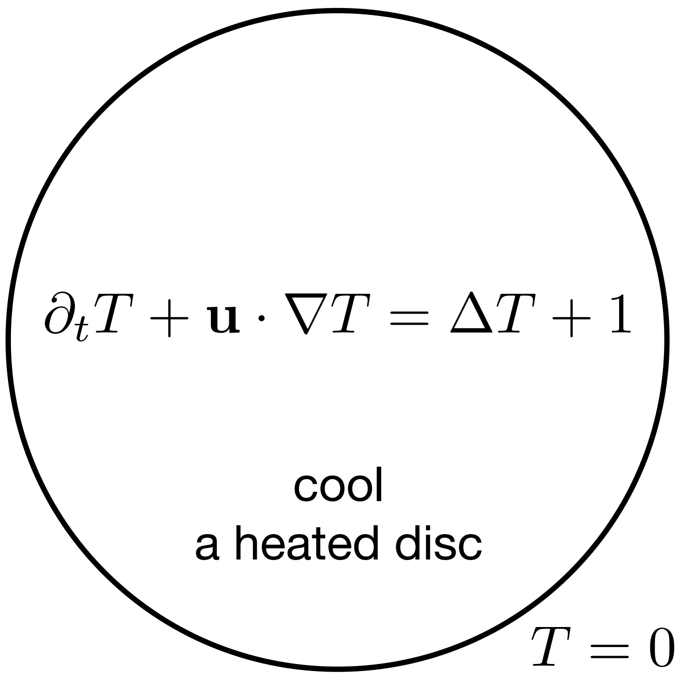


$$\left\langle |\nabla T|^2 \right\rangle \geq \left\langle 2\xi - |\nabla \xi|^2 - |\nabla \Delta^{-1}(\partial_t + \mathbf{u} \cdot \nabla)\xi|^2 \right\rangle$$

A test function with a boundary layer yields

$$\langle |\nabla T|^2 \rangle \gtrsim \langle |\nabla \mathbf{u}|^2 \rangle^{-1/3}$$

Lower bound on cooling



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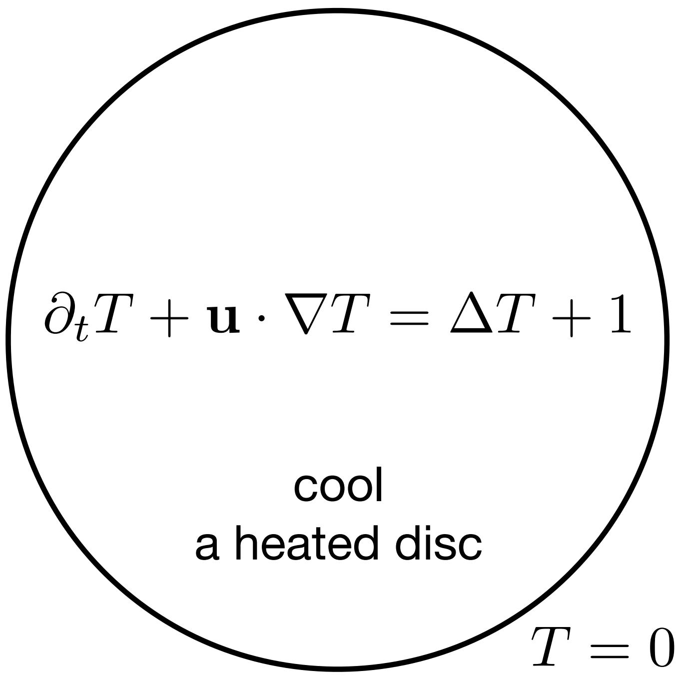
Very reminiscent of the background method…

Lower bound on cooling



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Very reminiscent of the background method…

…regardless of how you prove it, is it optimal?

Search for steady optimal flows

$$\min_{\substack{\mathbf{u}(x) \\ \left(\fint_\Omega |\nabla \mathbf{u}|^2\right)^{1/2} = Pe \\ \mathbf{u}=0 \text{ at } \partial\Omega}} \fint_\Omega |\nabla T|^2 = \min_{\substack{\mathbf{u}(x),\eta(x) \\ \mathbf{u}=0 \text{ at } \partial\Omega \\ \eta=0 \text{ at } \partial\Omega}} \fint_\Omega |\nabla \Delta^{-1} (\mathbf{u}\cdot\nabla\eta - 1)|^2 + \frac{1}{Pe^2} \fint_\Omega |\nabla \mathbf{u}|^2 \fint_\Omega |\nabla \eta|^2$$

non-convex
lower order term,
prefers patterns

regularizing
higher order terms,
sets a lengthscale

Search for steady optimal flows

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Together, the terms compete to determine optimal flows.

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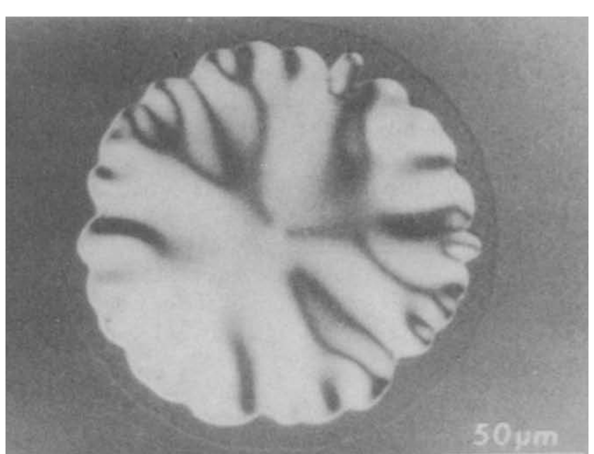
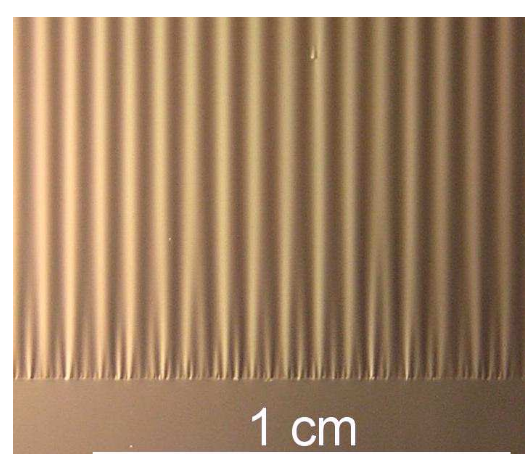
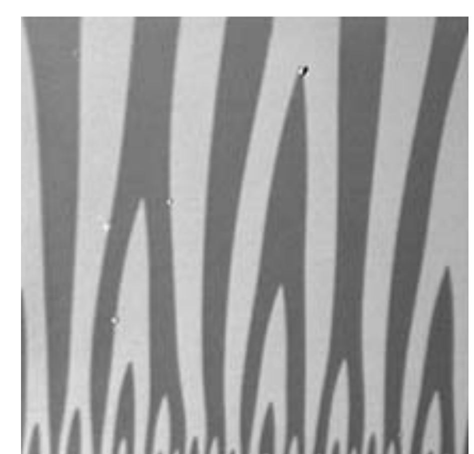
Reminds of problems in elasticity, micromagnetics, …



Ortiz & Gioia JMPS 1994

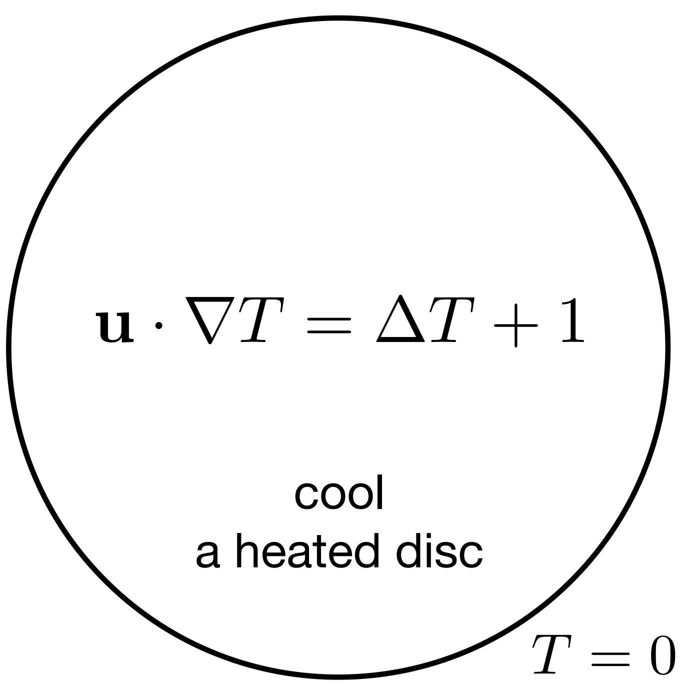


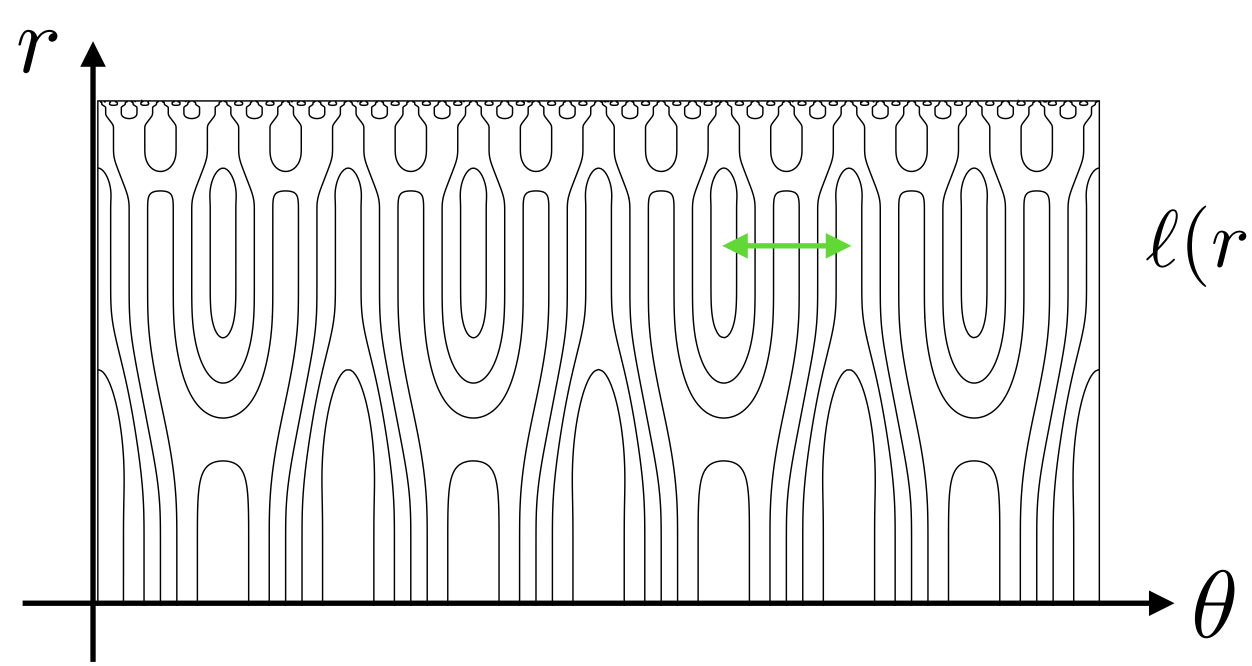
Huang et al PRL 2010


Hubert & Schafer 1998

Building a branching ansatz

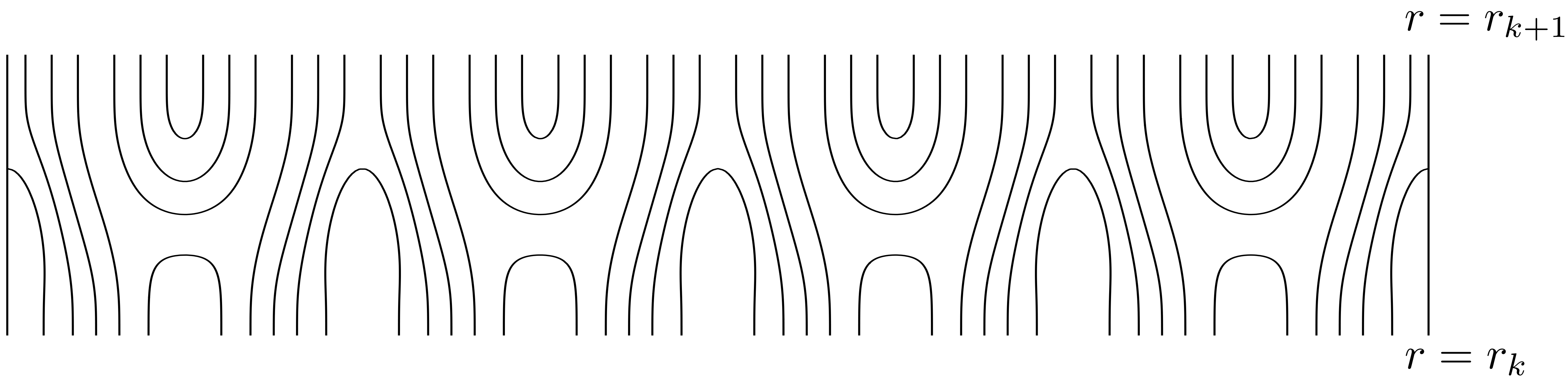
$$\min_{\substack{\mathbf{u}(x),\eta(x) \\ \mathbf{u}=0 \text{ at } \partial\Omega \\ \eta=0 \text{ at } \partial\Omega}} \fint_\Omega |\nabla\Delta^{-1}(\mathbf{u}\cdot\nabla\eta - 1)|^2 + \frac{1}{Pe^2}\fint_\Omega |\nabla \mathbf{u}|^2 \fint_\Omega |\nabla\eta|^2$$





A transition layer

$$\min_{\substack{\mathbf{u}(x),\eta(x) \\ \mathbf{u}=0 \text{ at } \partial\Omega \\ \eta=0 \text{ at } \partial\Omega}} \fint_\Omega \left|\nabla\Delta^{-1}\left(\mathbf{u}\cdot\nabla\eta - 1\right)\right|^2 + \frac{1}{Pe^2}\fint_\Omega |\nabla\mathbf{u}|^2 \fint_\Omega |\nabla\eta|^2$$



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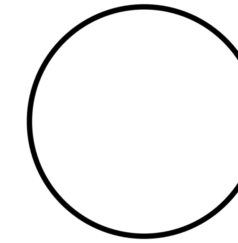

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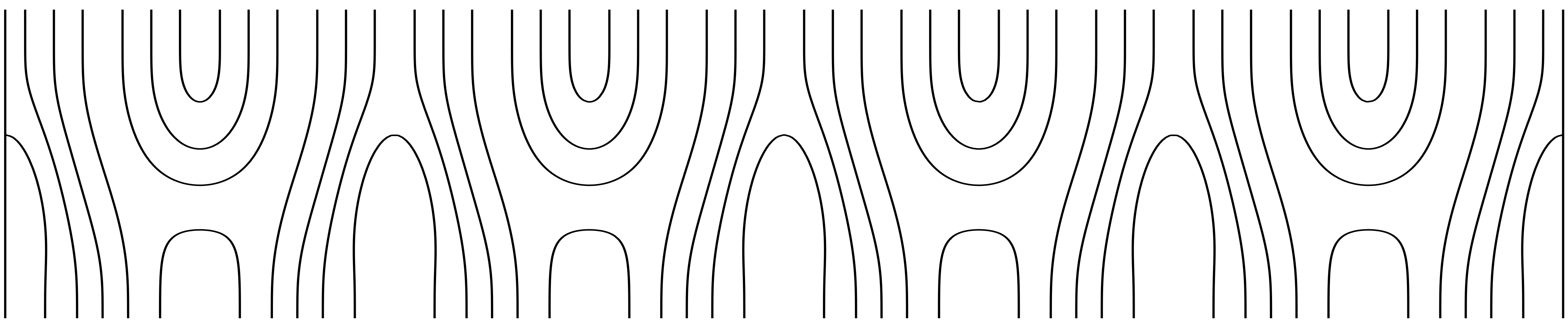
$r = r_{k+1}$

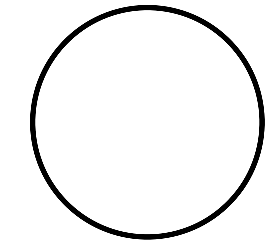


$r = r_k$

$$\fint_\Omega \left|\nabla\Delta^{-1}\left(\mathbf{u}\cdot\nabla\eta - 1\right)\right|^2$$

$$= \fint_{r=0}^{r=R} \left|\overline{\mathbf{u}\cdot\hat{\mathbf{r}}\eta} - \frac{r}{2}\right|^2 r\,dr + \mathcal{Q}\{\mathbf{u}\eta\}$$

A transition layer



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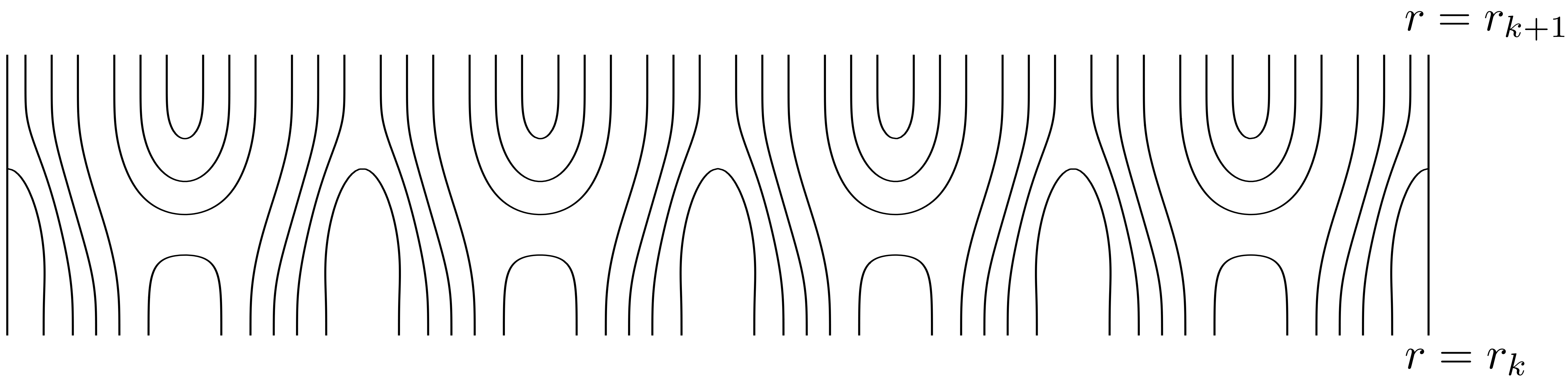

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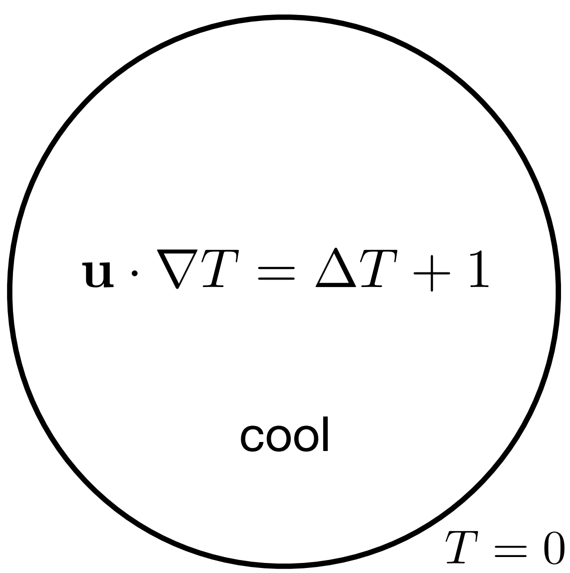
$$= \fint_{r=0}^{r=R} \left|\overline{\mathbf{u}\cdot\hat{\mathbf{r}}\eta} - \frac{r}{2}\right|^2 r\,dr + \mathcal{Q}\{\mathbf{u}\eta\}$$

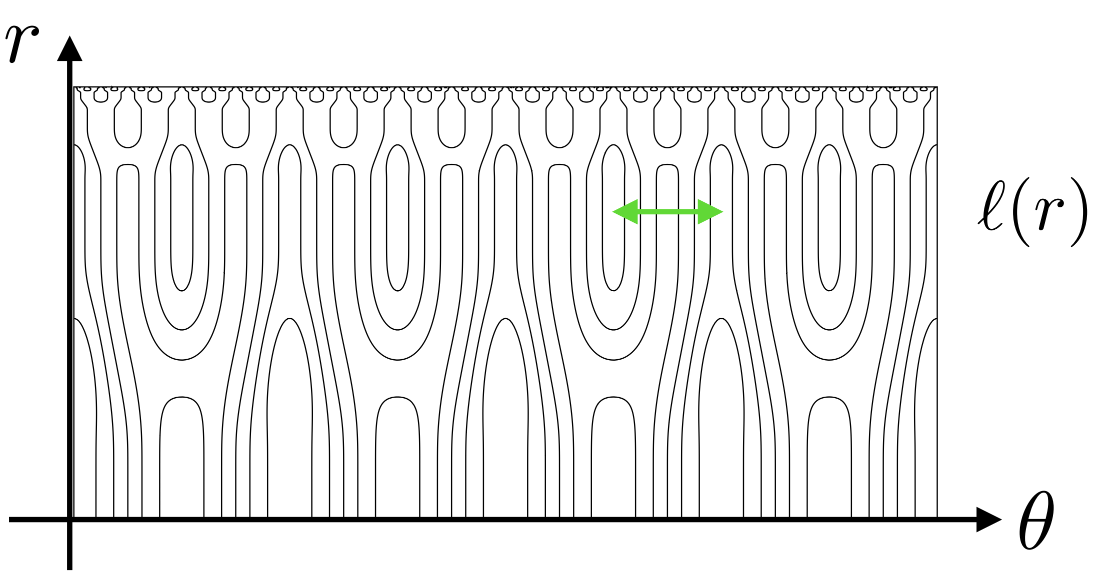
$$\boxed{\overline{\mathbf{u}\cdot\hat{\mathbf{r}}\eta} \approx \frac{r}{2}}$$

Optimal branching

$$\min_{\substack{\mathbf{u}(x),\eta(x) \\ \mathbf{u}=0 \text{ at } \partial\Omega \\ \eta=0 \text{ at } \partial\Omega}} \fint_\Omega |\nabla\Delta^{-1}(\mathbf{u}\cdot\nabla\eta - 1)|^2 + \frac{1}{Pe^2}\fint_\Omega |\nabla\mathbf{u}|^2 \fint_\Omega |\nabla\eta|^2$$

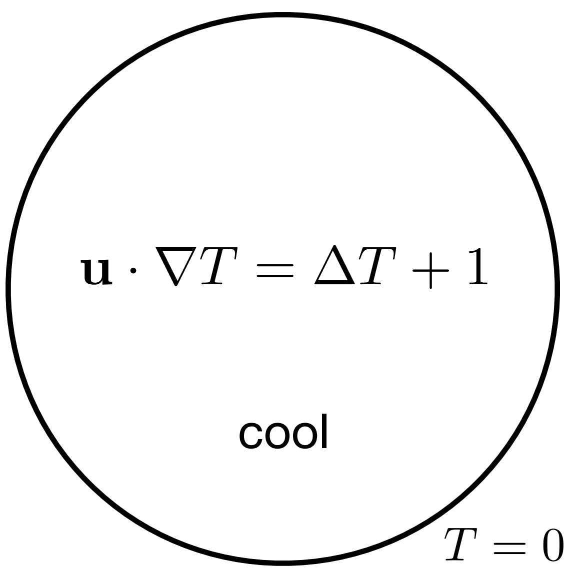


$$\min_{\ell(r)} \int_{r_{\text{core}}}^{r_{\text{b.l.}}} |\ell'|^2 r^5\, dr + \left(\frac{1}{Pe^2}\int \frac{1}{\ell^2}\, dr\right)^2$$



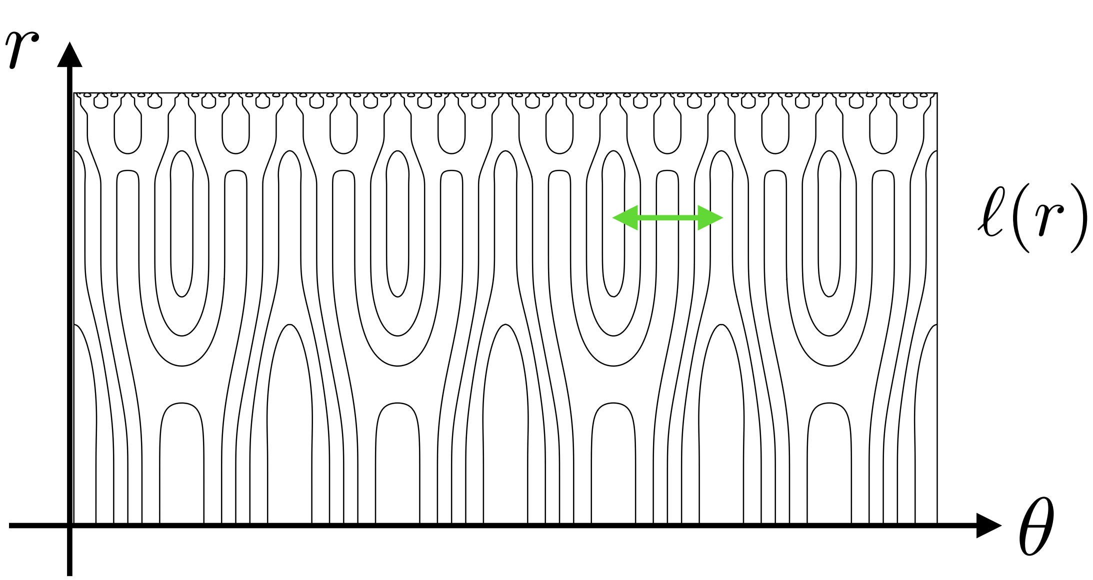
Optimal branching

$$\min_{\substack{\mathbf{u}(x),\eta(x)\\ \mathbf{u}=0 \text{ at } \partial\Omega\\ \eta=0 \text{ at } \partial\Omega}} \fint_\Omega |\nabla\Delta^{-1}(\mathbf{u}\cdot\nabla\eta - 1)|^2 + \frac{1}{Pe^2}\fint_\Omega |\nabla\mathbf{u}|^2 \fint_\Omega |\nabla\eta|^2$$



$$\min_{\ell(r)} \int_{r_{\text{core}}}^{r_{\text{b.l.}}} |\ell'|^2 r^5\, dr + \left(\frac{1}{Pe^2}\int \frac{1}{\ell^2}\, dr\right)^2$$

$$\ell(r) \sim \frac{1}{Pe^{1/3}} \log^{1/6}\left(\frac{(1-r)^{1/2}}{Pe^2 r^{3/4}}\right)$$



Optimal branching

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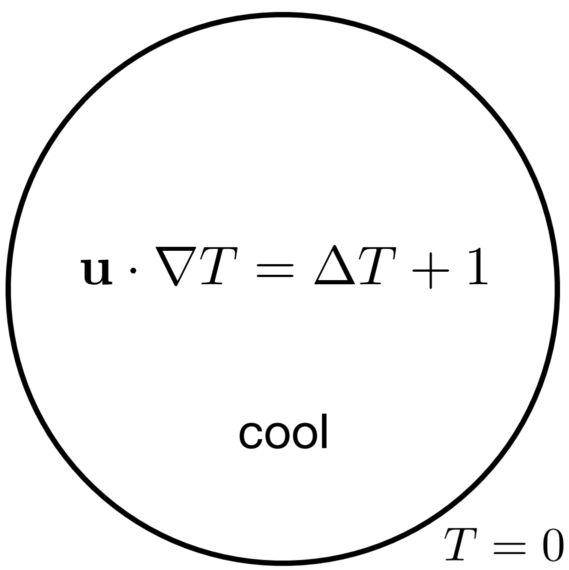

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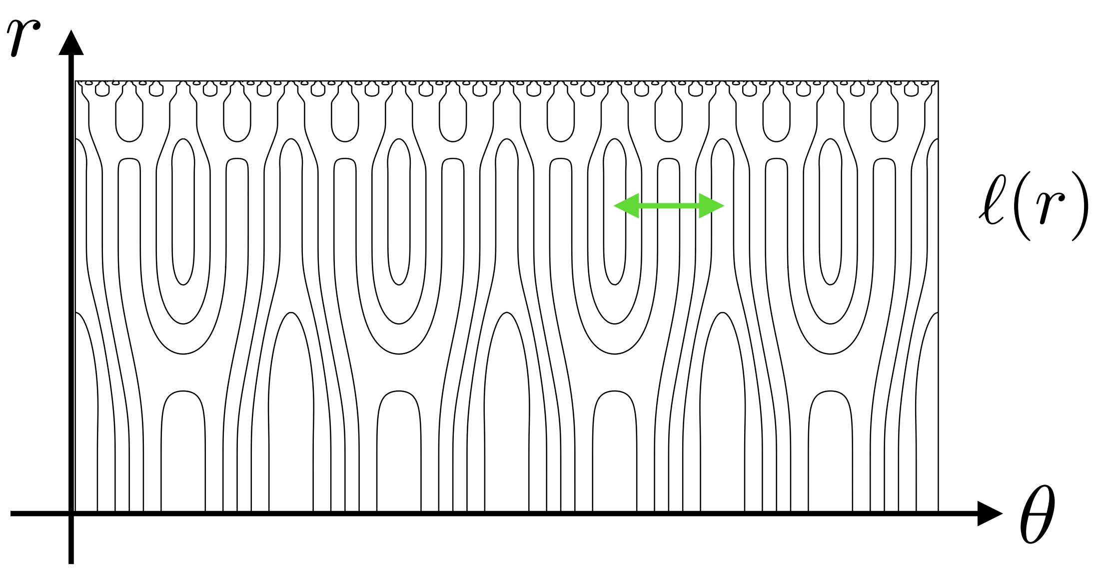


$$\fint_\Omega |\nabla T|^2 \lesssim \frac{\log^{4/3} Pe}{Pe^{2/3}}$$

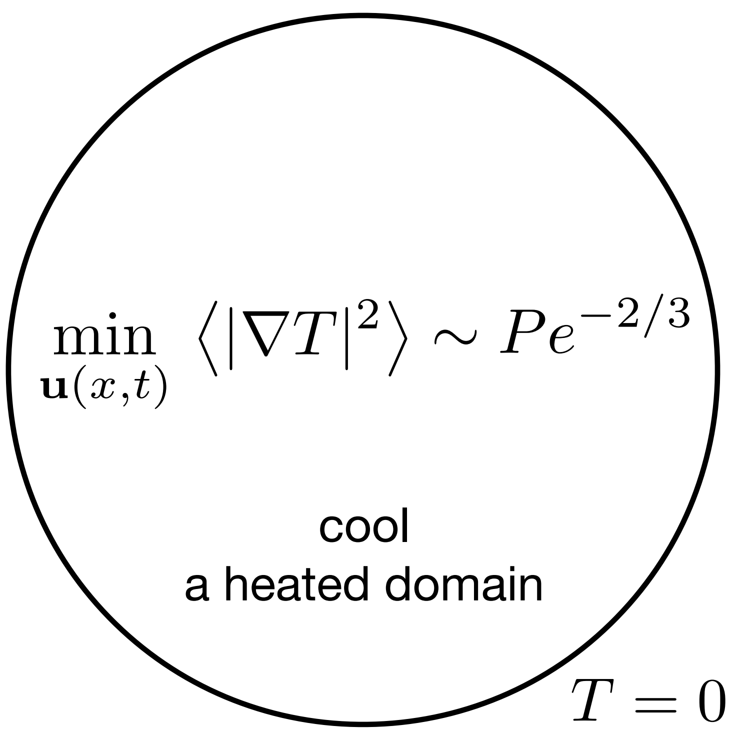
Takeaways/Questions

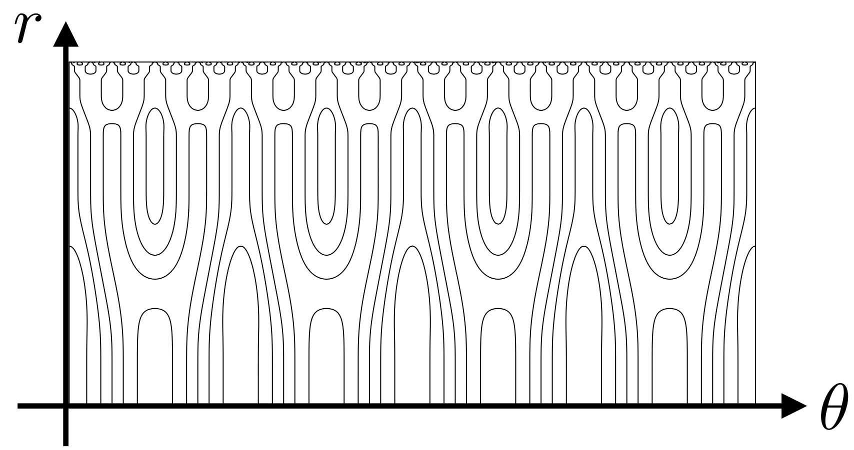
$$T = 0$$

$$\max_{\mathbf{u}(x,t)} \left\langle |\nabla T|^2 \right\rangle \sim Pe^{2/3}$$

transport
between walls

$$T = 1$$





$$\ell(r) \sim \frac{1}{Pe^{1/3}} \log^{1/6}\left(\frac{(1-r)^{1/2}}{Pe^2 r^{3/4}}\right)$$

Possible log corrections?

Does time-dependence play a role?

Missing bound in energy-constrained cooling?

A. Souza, I. Tobasco & C. R. Doering, JFM 889 (2020)
I. Tobasco & C. R. Doering, CPAM 72 (2019)
C. R. Doering & I. Tobasco, PRL 118 (2017)

enstrophy-constrained optimal cooling *in prep*

email: itobasco@uic.edu

Takeaways/Questions

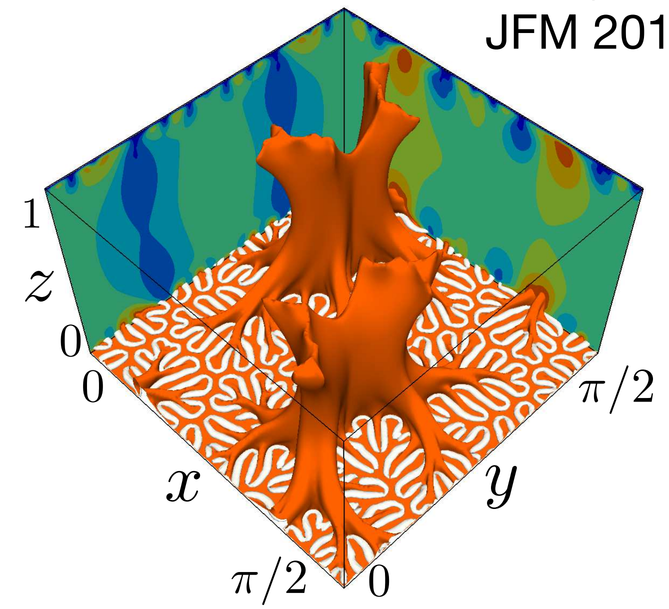
$$T = 0$$

$$\max_{\mathbf{u}(x,t)} \langle |\nabla T|^2 \rangle \sim Pe^{2/3}$$

transport
between walls

$$T = 1$$

Shingo, Kawahara & Shimizu
JFM 2018 $\langle |\nabla T|^2 \rangle \sim Pe^{2/3}$

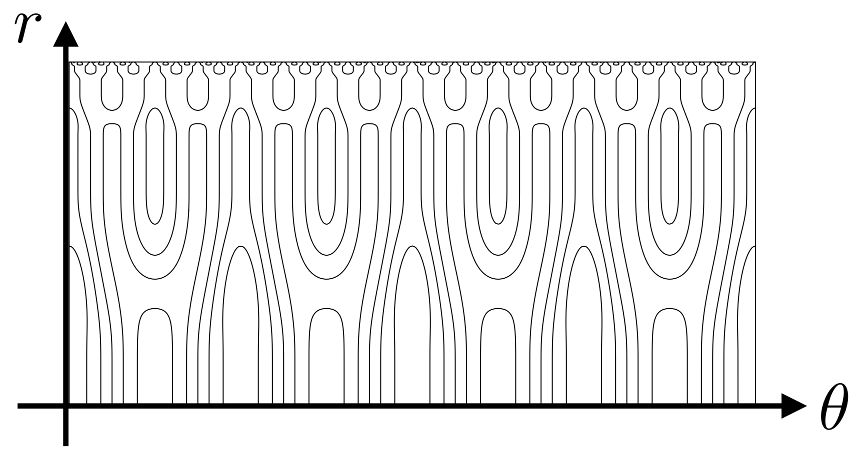


Possible log corrections?

$$\min_{\mathbf{u}(x,t)} \langle |\nabla T|^2 \rangle \sim Pe^{-2/3}$$

cool
a heated domain

$$T = 0$$



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