Transport and Mixing in Complex and Turbulent Flows

Institute for Pure & Applied Mathematics *January 11 - 14, 2021*

Extension of QSQH theory to all velocity components

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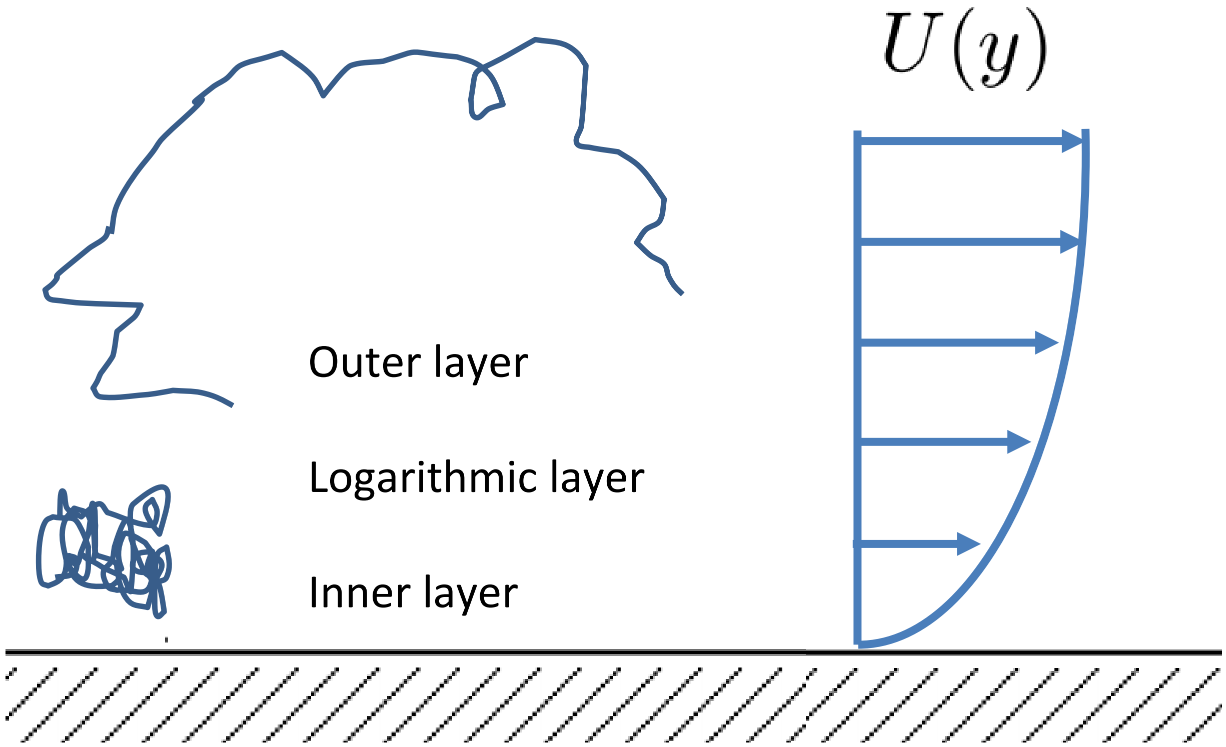
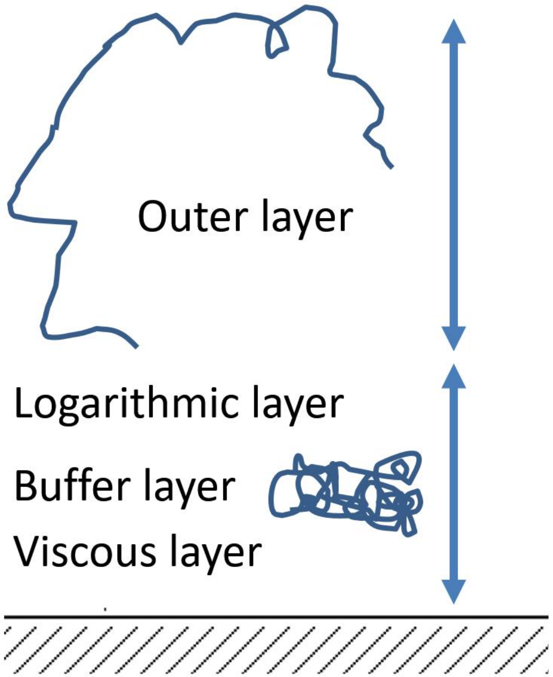
The context: near-wall turbulence, large and small structures, modulation, Re-effects, …



Old recipe that survived a century: near the wall in wall units flow statistics are independent of Re, dp/dx, …

$$u_\tau = \sqrt{\bar{\tau}/\rho},$$

$$u^+ = u/u_\tau,\ t^+ = \frac{t u_\tau^2}{\nu},\ x^+ = \frac{x u_\tau}{\nu},\ y^+ = \ldots,\ z^+ = \ldots$$

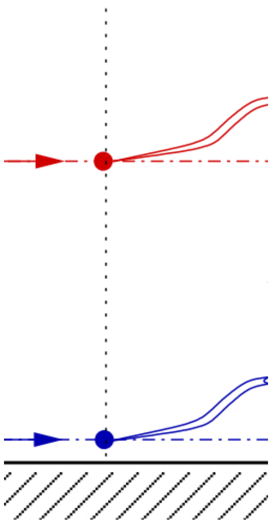


$$u^+ = u^+(t^+, x^+, y^+, z^+, Re)$$

$$u^+ = u^*(t^+, x^+, y^+, z^+)$$

"Classic universality"

Large-scale influence violates classic universality



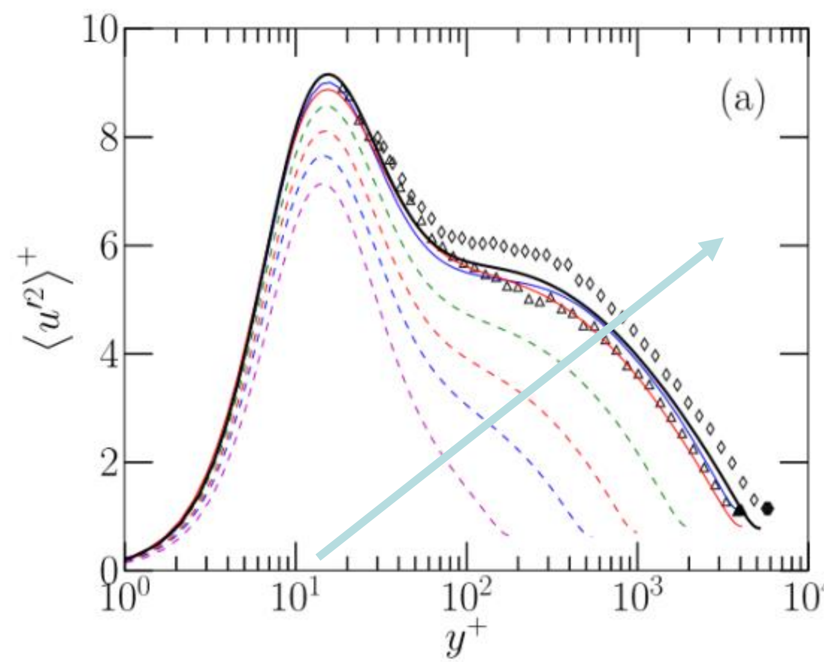
Filter

$$u'_{OL} = u'_L(y_o)$$

Marusic et al. 2010
Mathis et al. 2011…

$$u'^{+} = \alpha(y^{+})u'^{+}_{OL} + [1 + \beta(y^{+})u'^{+}_{OL}]u^{*}(t^{+}, x^{+}, y^{+}, z^{+})$$

~~$$u^{+} = u^{*}(t^{+}, x^{+}, y^{+}, z^{+})$$~~



Lee & Moser, arXiv:1410.7809, 2014

Main idea: quasi-steady-quasi-homogeneous (QSQH) universality

$$u = \sqrt{\frac{\overline{\tau}}{\rho}} u^* \neq \bar{u}_\tau u^* \left(\frac{t\bar{u}_\tau^2}{\nu}, \frac{x\bar{u}_\tau}{\nu}, \frac{y\bar{u}_\tau}{\nu}, \frac{z\bar{u}_\tau}{\nu} \right)$$

$$\tau = \overline{\tau} + \tau' \quad \Rightarrow \quad \tau = \tau_L + \tau_S'$$

$$u = u_{\tau_L}(t, x, z) \tilde{u} \left(\frac{t u_{\tau_L}^2}{\nu}, \frac{x u_{\tau_L}}{\nu}, \frac{y u_{\tau_L}}{\nu}, \frac{z u_{\tau_L}}{\nu} \right)$$

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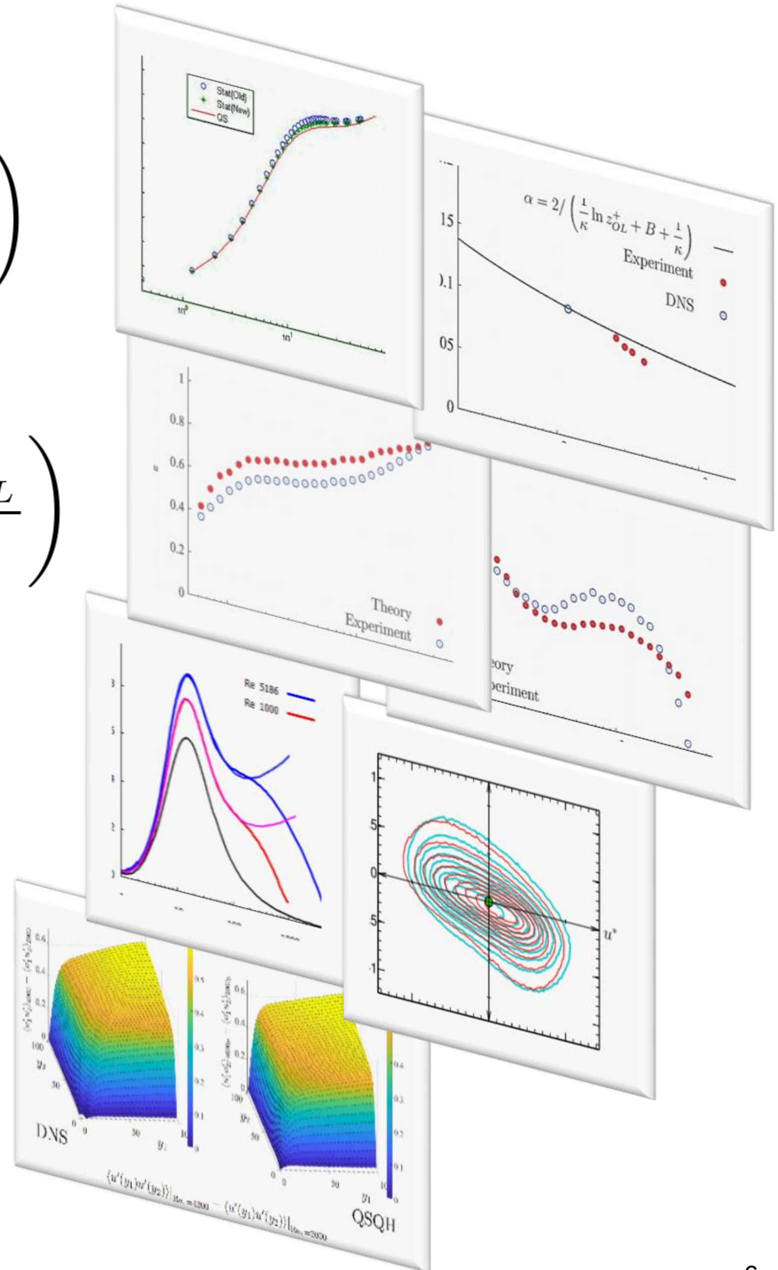
$$\tau = \overline{\tau} + \tau' \quad \Rightarrow \quad \tau = \tau_L + \tau'_S$$

$$u = u_{\tau_L}(t, x, z)\tilde{u} \left(\frac{tu_{\tau_L}^2}{\nu}, \frac{xu_{\tau_L}}{\nu}, \frac{yu_{\tau_L}}{\nu}, \frac{zu_{\tau_L}}{\nu} \right)$$



Chernyshenko, Marusic, Mathis 2012 arXiv:1203.3714

UCLA | Mathematics of Turbulence Program | IPAM

Workshop IV: Turbulence in Engineering Applications | 19 November 2014

Relationship between large-scale structures and near-wall turbulence: theory and implications

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Acknowledgements:

Michael Leschziner, Emile Touber, Chi Zhang, Lionel Agostini, Ivan Marusic, Romain Mathis, Ravish Karve

Rigour is achieved by specifying filter properties instead of specifying the filter

QSQH theory describes a statistical ensemble

$$u = u_{\tau_L}(t,x,z)\tilde{u}\left(\frac{tu_{\tau_L}^2}{\nu}, \frac{xu_{\tau_L}}{\nu}, \frac{yu_{\tau_L}}{\nu}, \frac{zu_{\tau_L}}{\nu}\right)$$

with $u_{\tau_L}(t,x,z)$ and $\tilde{u}(\theta,\xi,\eta,\zeta)$ statistically independent,

averaging $\langle f \rangle$ and large-scale filtering $\mathrm{L}f$ defined for any $f(t,x,y,z)$,

u_{τ_L} being large scale: $u_{\tau_L} = \mathrm{L}g$ for some g,

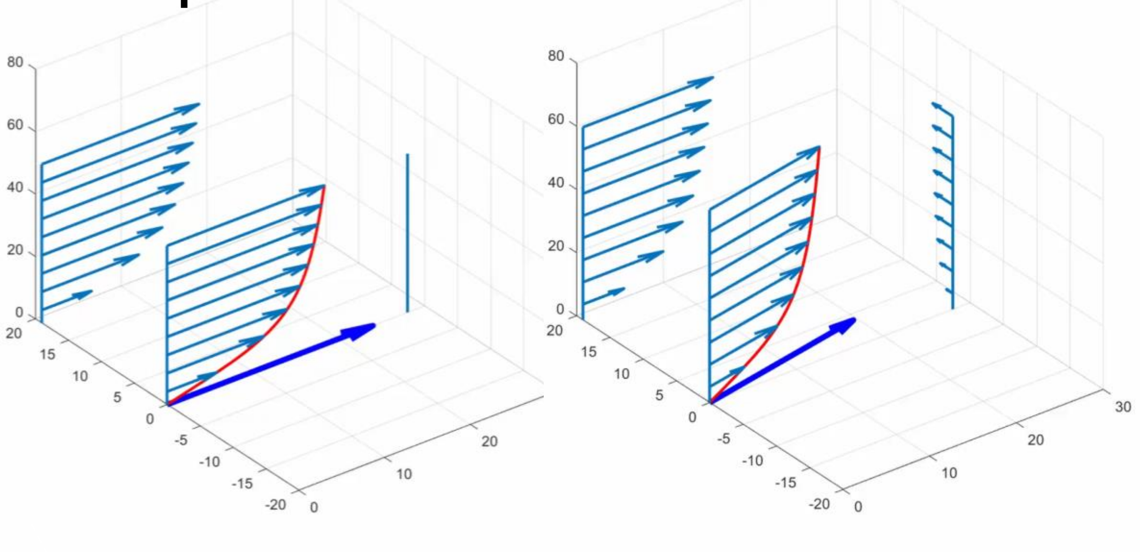
and L having the properties:

1. Linearity: $\mathrm{L}(af + bg) = a\mathrm{L}f + b\mathrm{L}g$
2. Invariance of averages: $\mathrm{L}\langle f \rangle = \langle f \rangle$
3. Projection property: $\mathrm{LL}f = \mathrm{L}f$
4. Commuting with averaging: $\langle \mathrm{L}f \rangle = \mathrm{L}\langle f \rangle$
5. Scale-separation property:

$$\mathrm{L}f(t,x,y,z,\mathrm{L}g_1,\ldots,\mathrm{L}g_n) = \langle f(t,x,y,z,\xi_1,\ldots,\xi_n)\rangle_{\xi_1=\mathrm{L}g_1,\ldots,\xi_n=\mathrm{L}g_n}$$

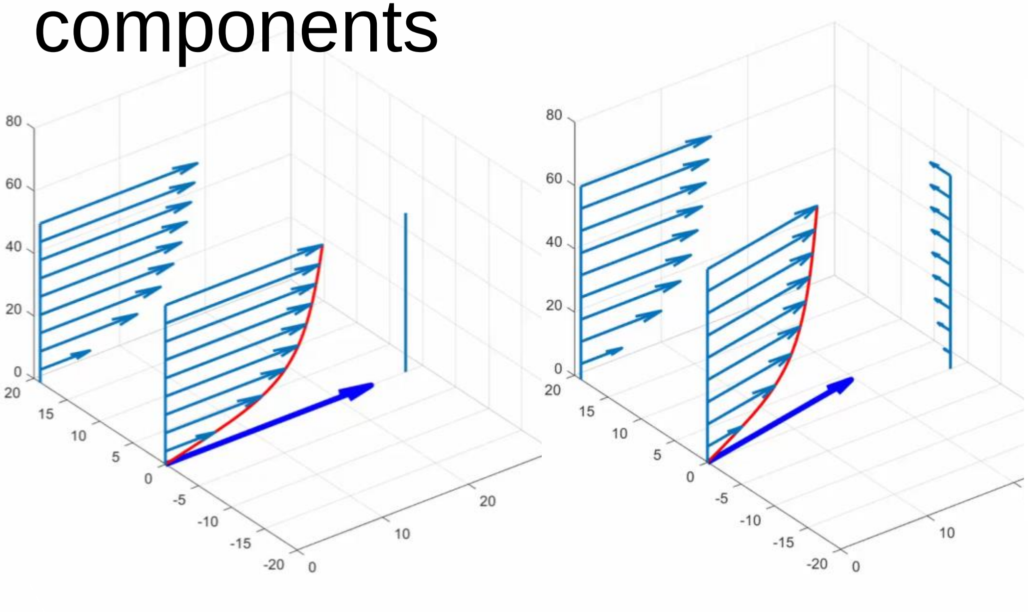
Plus 2 physical assumptions: A) there is such L that real flows satisfy the above at least approximately, and B): statistics of $\tilde{u}(\theta,\xi,\eta,\zeta)$ are independent of outer parameters.

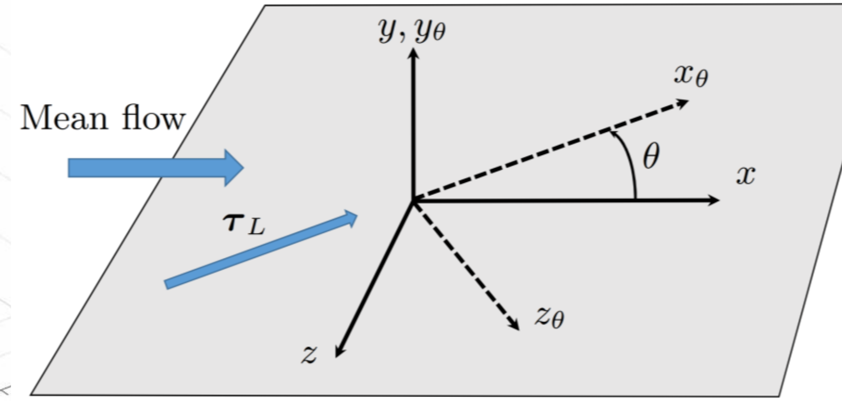
Extension of the QSQH theory to all velocity components



$$u = u_{\tau_L}(t, x, z)\tilde{u}\left(\frac{tu_{\tau_L}^2}{\nu}, \frac{xu_{\tau_L}}{\nu}, \frac{yu_{\tau_L}}{\nu}, \frac{zu_{\tau_L}}{\nu}\right)$$

Extension of the QSQH theory to all velocity components





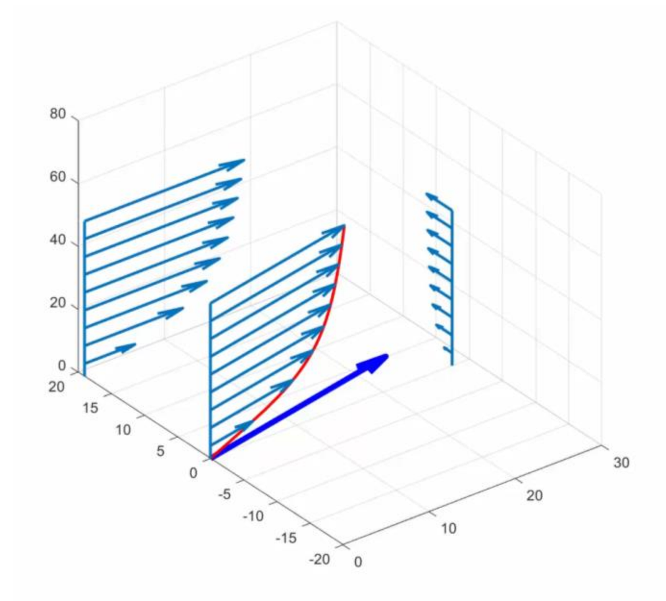
$$u_\theta(t, x_\theta, y_\theta, z_\theta) = u_{\tau_L}\tilde{u}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z})$$

$$v_\theta(t, x_\theta, y_\theta, z_\theta) = u_{\tau_L}\tilde{v}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z})$$

$$w_\theta(t, x_\theta, y_\theta, z_\theta) = u_{\tau_L}\tilde{w}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z})$$

$$\tilde{t} = t u_{\tau_L}^2, \quad \tilde{x} = x_\theta u_{\tau_L}, \quad \tilde{y} = y_\theta u_{\tau_L}, \quad \tilde{z} = z_\theta u_{\tau_L}$$

$$P[u_{\tau_L}, \theta, \tilde{u}, \tilde{v}, \tilde{w}, Re] = P_\mathrm{o}[u_{\tau_L}, \theta, Re] P_\mathrm{u}[\tilde{u}, \tilde{v}, \tilde{w}]$$



A wish list, and the benefits of workshops

VIRTUAL POSTER SESSION FOR IPAM’S WORSHOP, TRANSPORT AND MIXING IN COMPLEX AND TURBULENT FLOWS

“**Far-field boundary conditions for simulating a ‘patch’ of wall-bounded turbulent flow**,”
presented by Sean P. Carney (University of California, Los Angeles)

$$u_\theta(t, x_\theta, y_\theta, z_\theta) = u_{\tau_L} \tilde{u}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z})$$

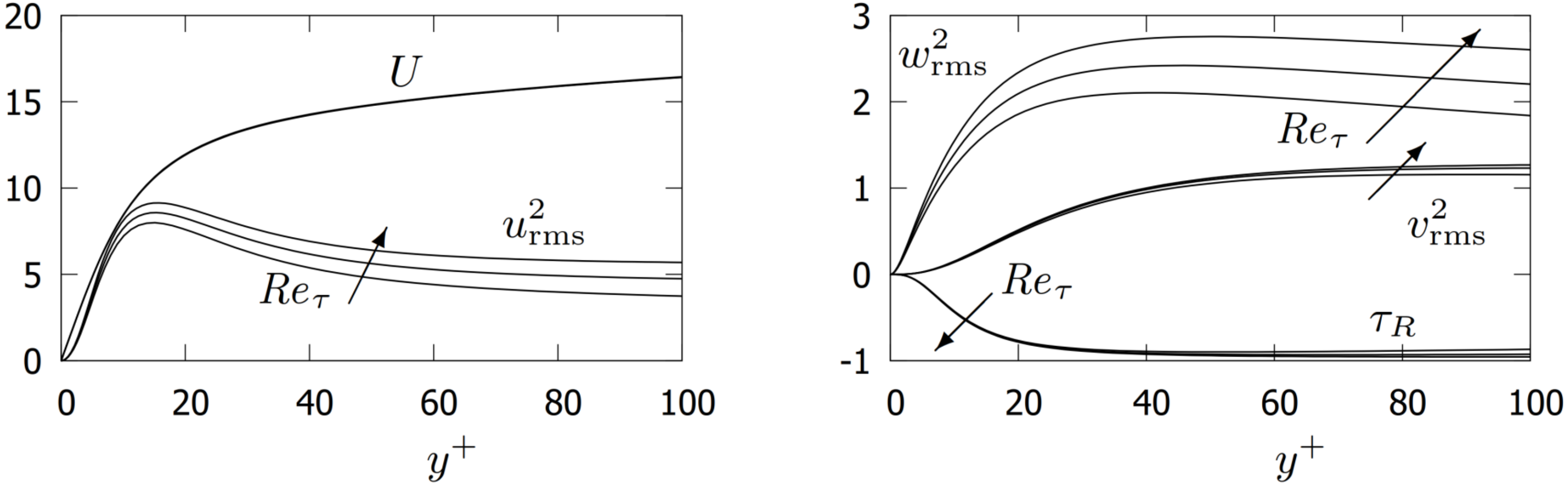
$$v_\theta(t, x_\theta, y_\theta, z_\theta) = u_{\tau_L} \tilde{v}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z})$$

$$w_\theta(t, x_\theta, y_\theta, z_\theta) = u_{\tau_L} \tilde{w}(\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z})$$

$$\tilde{t} = t u_{\tau_L}^2, \quad \tilde{x} = x_\theta u_{\tau_L}, \quad \tilde{y} = y_\theta u_{\tau_L}, \quad \tilde{z} = z_\theta u_{\tau_L}$$

$$P[u_{\tau_L}, \theta, \tilde{u}, \tilde{v}, \tilde{w}, Re] = P_{\mathrm{o}}[u_{\tau_L}, \theta, Re] P_{\mathrm{u}}[\tilde{u}, \tilde{v}, \tilde{w}]$$

Why only streamwise and spanwise rms?



DNS data for $Re_\tau = 950$, 2000 and 5200 showing that u^2_{rms} and w^2_{rms} vary with Re_τ significantly more than U, v^2_{rms} and $\tau_R = \langle u'v' \rangle$.

At Re attainable the amplitude of the fluctuations of large-scale motions is very small

(Zhang & Chernyshenko 2016), $\left\langle u'^2_{\tau_L} \right\rangle = 0.004364$ for $Re_\tau = 1000$

$$U(y) = \tilde{U}(y) + \frac{1}{2}\left\langle u'^2_{\tau_L} \right\rangle \frac{d}{dy} y^2 \frac{d\tilde{U}}{dy} - \frac{1}{2}\left\langle \theta^2 \right\rangle \tilde{U}(y) + \ldots.$$

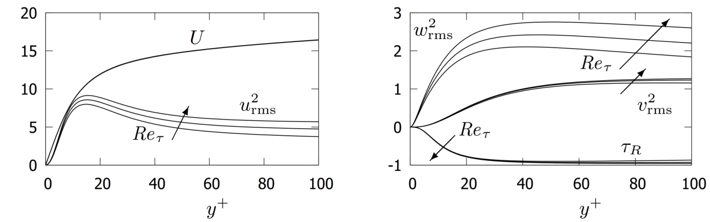
$$u^2_{\mathrm{rms}}(y) = \left\langle u'^2_{\tau_L} \right\rangle \left(\frac{dy\tilde{U}}{dy} \right)^2 + \tilde{u}^2_{\mathrm{rms}}(y) + \frac{\left\langle u'^2_{\tau_L} \right\rangle}{2} \frac{d^2 y^2 \tilde{u}^2_{\mathrm{rms}}}{dy^2} + \left\langle \theta^2 \right\rangle \left(\tilde{w}^2_{\mathrm{rms}} - \tilde{u}^2_{\mathrm{rms}} \right) + \ldots$$

$$v^2_{\mathrm{rms}}(y) = \tilde{v}^2_{\mathrm{rms}}(y) + \frac{1}{2}\left\langle u'^2_{\tau_L} \right\rangle \frac{d^2 y^2 \tilde{v}^2_{\mathrm{rms}}}{dy^2} + \ldots,$$

$$w^2_{\mathrm{rms}}(y) = \left\langle \theta^2 \right\rangle \tilde{U}^2 + \tilde{w}^2_{\mathrm{rms}}(y) + \frac{1}{2}\left\langle u'^2_{\tau_L} \right\rangle \frac{d^2 y^2 \tilde{w}^2_{\mathrm{rms}}}{dy^2} + \left\langle \theta^2 \right\rangle \left(\tilde{u}^2_{\mathrm{rms}} - \tilde{w}^2_{\mathrm{rms}} \right) + .$$

$$\left\langle u'v' \right\rangle (y) = \tilde{\tau}_R(y) + \frac{1}{2}\left\langle u'^2_{\tau_L} \right\rangle \frac{d^2 y^2 \tilde{\tau}_R}{dy^2} - \frac{1}{2}\left\langle \theta^2 \right\rangle \tilde{\tau}_R + \ldots.$$

A few terms are large



$$\Delta U(y) = \frac{1}{2}\Delta\langle u'^2_{\tau L}\rangle \frac{d}{dy} y^2 \frac{d\tilde{U}}{dy} - \frac{1}{2}\Delta\langle\theta^2\rangle \tilde{U}(y) + \ldots,$$

$$\Delta u^2_{\rm rms} = \Delta\langle u'^2_{\tau L}\rangle \left(\frac{dy\tilde{U}}{dy}\right)^2 + \frac{1}{2}\Delta\langle u'^2_{\tau L}\rangle \frac{d^2 y^2 \tilde{u}^2_{\rm rms}}{dy^2} + \Delta\langle\theta^2\rangle\left(\tilde{w}^2_{\rm rms} - \tilde{u}^2_{\rm rms}\right) + \ldots,$$

$$\Delta v^2_{\rm rms} = \frac{\Delta\langle u'^2_{\tau L}\rangle}{2} \frac{d^2 y^2 \tilde{v}^2_{\rm rms}}{dy^2} + \ldots,$$

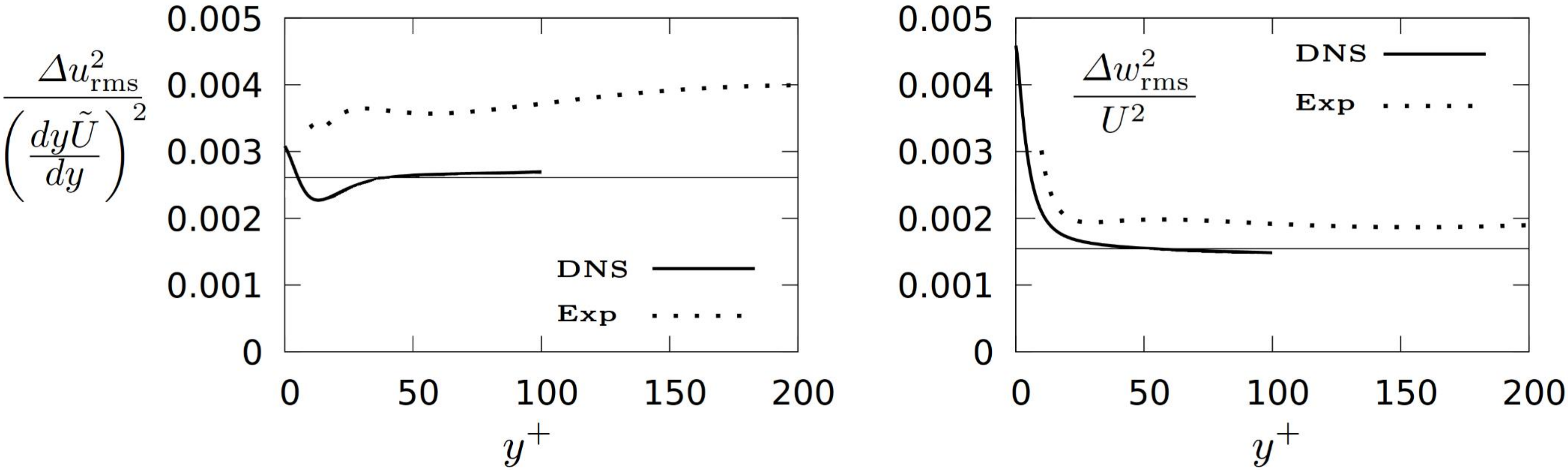
$$\Delta w^2_{\rm rms} = \frac{\Delta\langle u'^2_{\tau L}\rangle}{2} \frac{d^2 y^2 \tilde{w}^2_{\rm rms}}{dy^2} + \Delta\langle\theta^2\rangle\left(\tilde{U}^2 + \tilde{u}^2_{\rm rms} - \tilde{w}^2_{\rm rms}\right) + \ldots,$$

$$\Delta\langle u'v'\rangle = \frac{\Delta\langle u'^2_{\tau L}\rangle}{2} \frac{d^2 y^2 \tilde{\tau}_R}{dy^2} - \frac{1}{2}\langle\theta^2\rangle \tilde{\tau}_R + \ldots.$$

$$y^+ = 40,\ (dy\tilde{U}/dy)^2 \approx 285,\quad u^2_{\rm rms} \approx 6,\quad \tilde{U}^2 \approx 203,\quad w^2_{\rm rms} \approx 2.4$$

$$\Delta u^2_{\rm rms} \approx \Delta\langle u'^2_{\tau L}\rangle \left(\frac{dy\tilde{U}}{dy}\right)^2,\ \Delta w^2_{\rm rms} \approx \Delta\langle\theta^2\rangle \tilde{U}^2,\ \Delta v^2_{\rm rms} \approx \Delta\langle u'v'\rangle \approx 0.$$

Comparisons are satisfactory



The DNS data are for a plane channel with $Re_\tau = 2000$ and 5200 (Lee & Moser (2015))

The experimental data are for a zero pressure gradient boundary layer with $Re_\tau = 3000$ and 10500 (Baidya *et al.* 2012)

QSQH resolved a paradox: how can dp/dx change the kappa but not the log?

As $Re \to \infty$, dU/dy depends only on u_τ and y.
By Π theorem, $dU/dy = u_\tau / y$.
Integration gives

$$U = \frac{u_\tau}{\varkappa} \log y + B u_\tau$$

with 'log' and with constant $\varkappa$.

$$U(y) = \left\langle u_{\tau_L} \widetilde{U}\left(y u_{\tau_L}\right) \right\rangle ,$$

$$\tilde{U}(\tilde{y}) = \frac{1}{\varkappa} \log \tilde{y} + B$$

$$U^+(y^+) = \frac{\langle u_{\tau_L}^+ \cos\theta \rangle}{\tilde{\kappa}} \log y^+ + \left\langle \left(\frac{1}{\tilde{\kappa}} \log u_{\tau_L}^+ + \tilde{B} \right) u_{\tau_L}^+ \cos\theta \right\rangle$$

$$\frac{\Delta\kappa}{\kappa} \approx -\frac{\Delta u_{\mathrm{rms}}^2}{2\left(\frac{dy\tilde{U}}{dy}\right)^2} + \frac{\Delta w_{\mathrm{rms}}^2}{2\tilde{U}^2}$$

$$\frac{\Delta B}{B} \approx -\left(1 - \frac{1}{B}\right) \frac{\Delta u_{\mathrm{rms}}^2}{2\left(\frac{dy\tilde{U}}{dy}\right)^2} - \frac{\Delta w_{\mathrm{rms}}^2}{2\tilde{U}^2} .$$

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$$\frac{\Delta\kappa}{\kappa} \approx -\frac{\Delta u^2_{\mathrm{rms}}}{2\left(\frac{dy\tilde{U}}{dy}\right)^2} + \frac{\Delta w^2_{\mathrm{rms}}}{2\tilde{U}^2}$$

< 1%<current experimental uncertainty

$$\frac{\Delta B}{B} \approx -\left(1 - \frac{1}{B}\right) \frac{\Delta u^2_{\mathrm{rms}}}{2\left(\frac{dy\tilde{U}}{dy}\right)^2} - \frac{\Delta w^2_{\mathrm{rms}}}{2\tilde{U}^2}.$$

Summary

- QSQH theory:
  - Postulates that near-wall turbulence adjusts to the large-scale component of wall friction
  - Achieves rigour by postulating the required properties of large-scale filter instead of specifying the filter
  - Generates nontrivial results
  - Compares satisfactory in the viscous and buffer layer
- QSQH theory is extended to all velocity components
- Taking into account the fluctuations of the direction of large-scale motion makes a difference, in particular for spanwise velocity
- QSQH theory implies the dependence of the log-law constants on the large-scale motions while retaining the log law.
- For more see Sergei Chernyshenko, “Extension of QSQH theory of scale interaction in near-wall turbulence to all velocity components”, arXiv:2002.05585 **[physics.flu-dyn]**