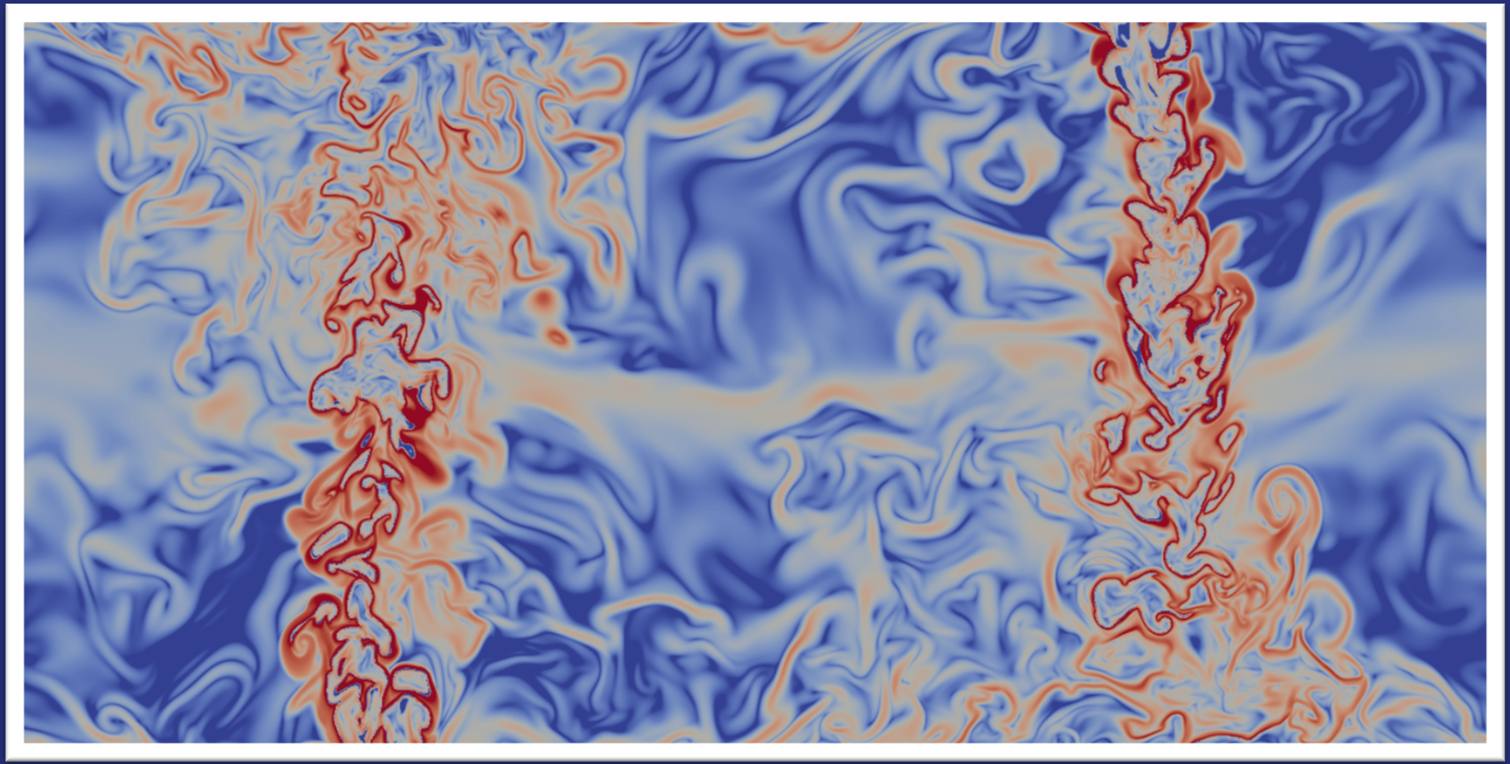
Mixing by plumes in boxes



Megan Davies Wykes
Department of Engineering, University of Cambridge

Jamie Partridge, Charlie Hogg
DAMTP, University of Cambridge

Graham Hughes, John Craske
Imperial College London

What is the mixing efficiency?

“mixing efficiency... seeks to provide a number quantifying the ability of a particular turbulent mixing event in dissipating [mechanical energy] preferentially diffusively rather than viscously.”

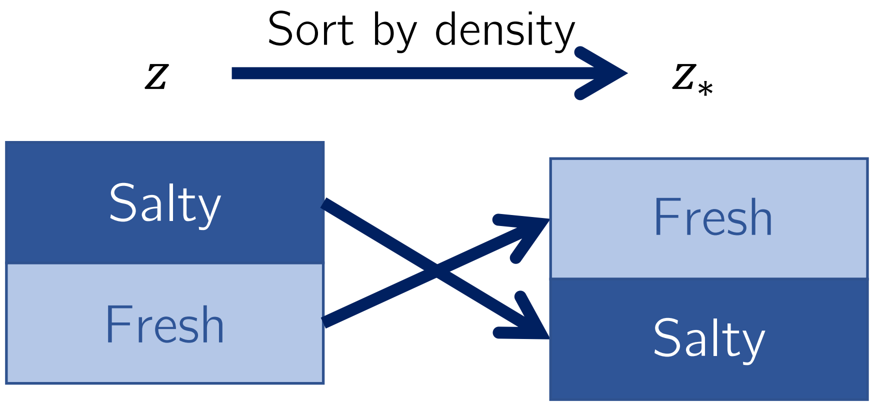
Tailleux, JFM 2009

$$\eta = \frac{\text{Energy used in mixing}}{\text{Energy available for mixing}}$$

Energy used in mixing = increase in BPE

$$E_p = \int_V \rho g\, z\, dz = \int_V \rho\, g\, z_*\, dz + \int_V \rho g\, (z - z_*)\, dz$$

Background Potential Energy (first term on the right) — Available Potential Energy (second term on the right)



BPE is **minimum PE** reachable by adiabatic rearrangement of fluid parcels.

Winters, Lombard, Riley & D'Asaro, *J Fluid Mech,* 289 (1995)

Peltier & Caulfield *Ann Rev of Fluid Mech* 35 (2003)

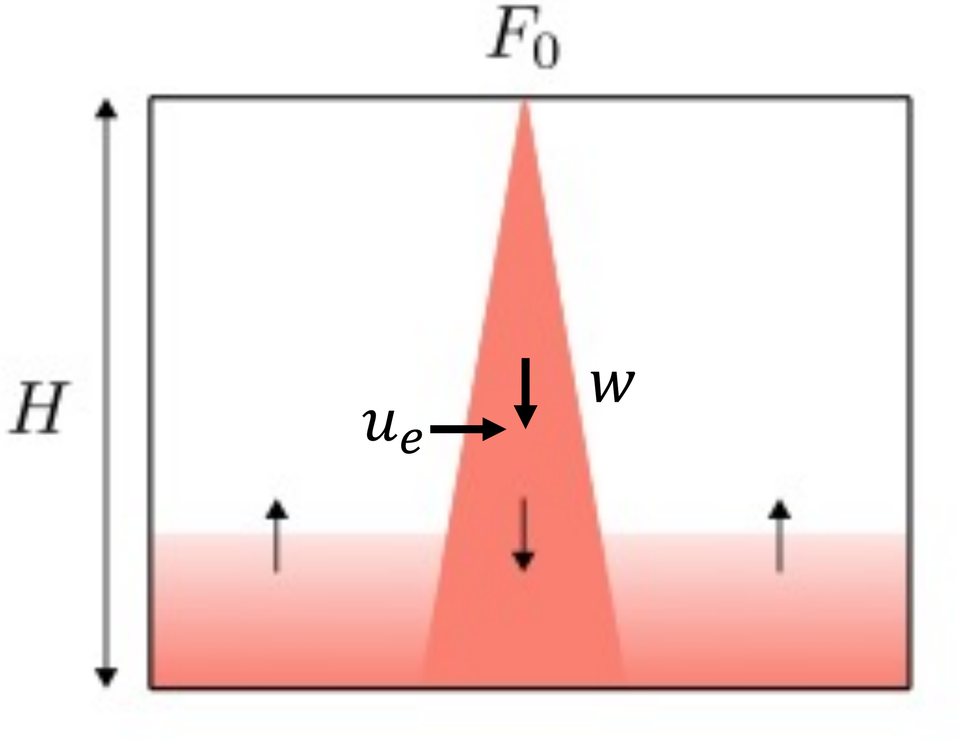
Mixing increases the background potential energy (BPE).

Available potential energy (APE) is present
if the system is gravitationally unstable.

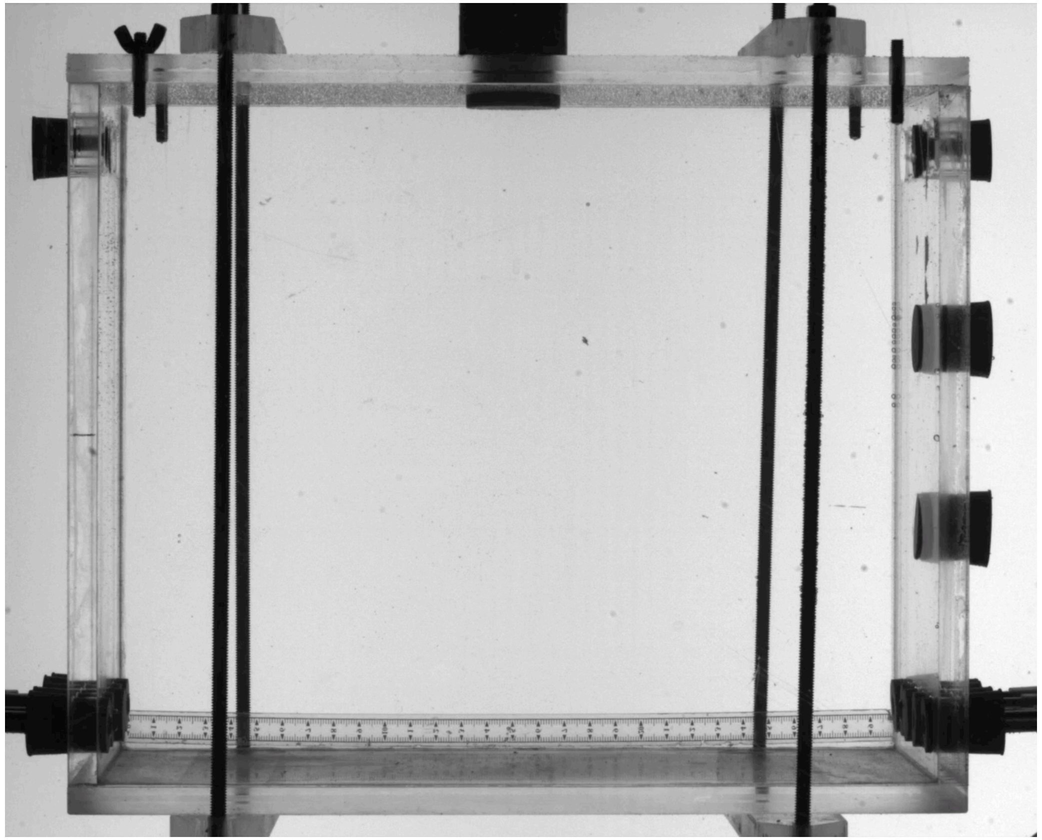
$$\text{Total available energy: } \int_V \Big[\underbrace{\rho g(z - z_*)}_{E_a} + \underbrace{\tfrac{1}{2}\rho v^2}_{E_k}\Big]\, dV$$

$$\eta = \frac{\Delta E_b}{|\Delta E_a + \Delta E_k|}$$

Filling box



$$u_e = \alpha w$$



Video: Jamie Partridge

Germeles (1975) Forced plumes and mixing of liquids in tanks. *J Fluid Mech* 71 (601–623)

Filling box: asymptotic state

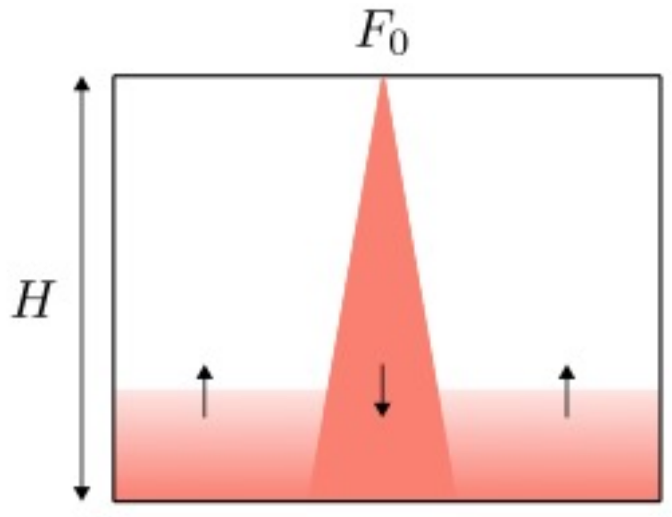
$$\frac{\partial \rho(z,t)}{\partial t} = \frac{\hat{\rho} F_0}{gV}$$

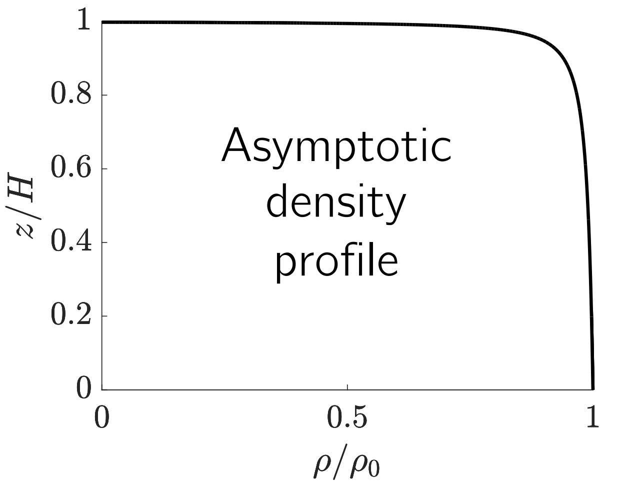
$\hat{\rho}$ = reference density

V = box volume

$$\frac{dE_b}{dt} = g \int_V \frac{\partial \rho}{\partial t} z\, dV - \hat{\rho} F_0 z_0 = \frac{\hat{\rho} F_0 H}{2}$$

$$\frac{dE_a}{dt} = \hat{\rho} F_0 H \longrightarrow \eta_i = \frac{1}{2}$$





Germeles model

ζ = height

τ = time

δ = buoyancy

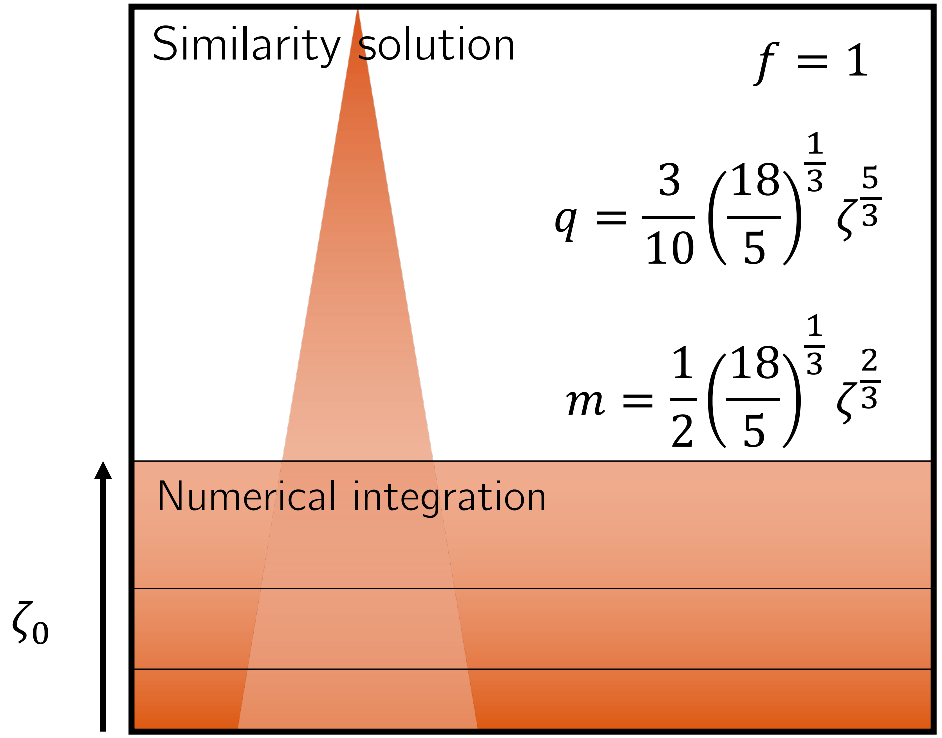
m = momentum flux

q = volume flux

f = buoyancy flux

$$\frac{dq}{d\zeta} = m \qquad \frac{dm^2}{d\zeta} = \frac{qf}{m^2}$$

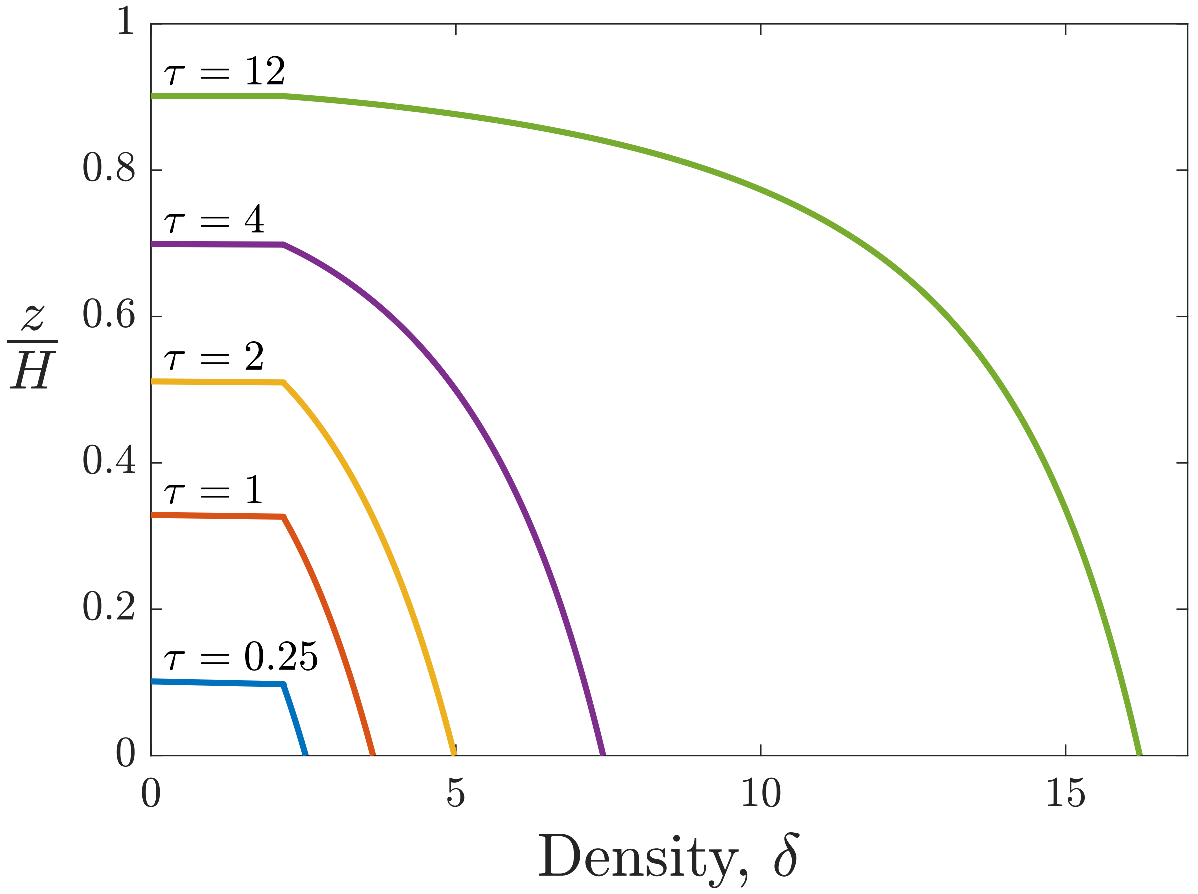
$$\frac{df}{d\zeta} = q\frac{\partial\delta}{\partial\zeta} \qquad \frac{\partial\delta}{\partial\tau} = q\frac{\partial\delta}{\partial\zeta}$$

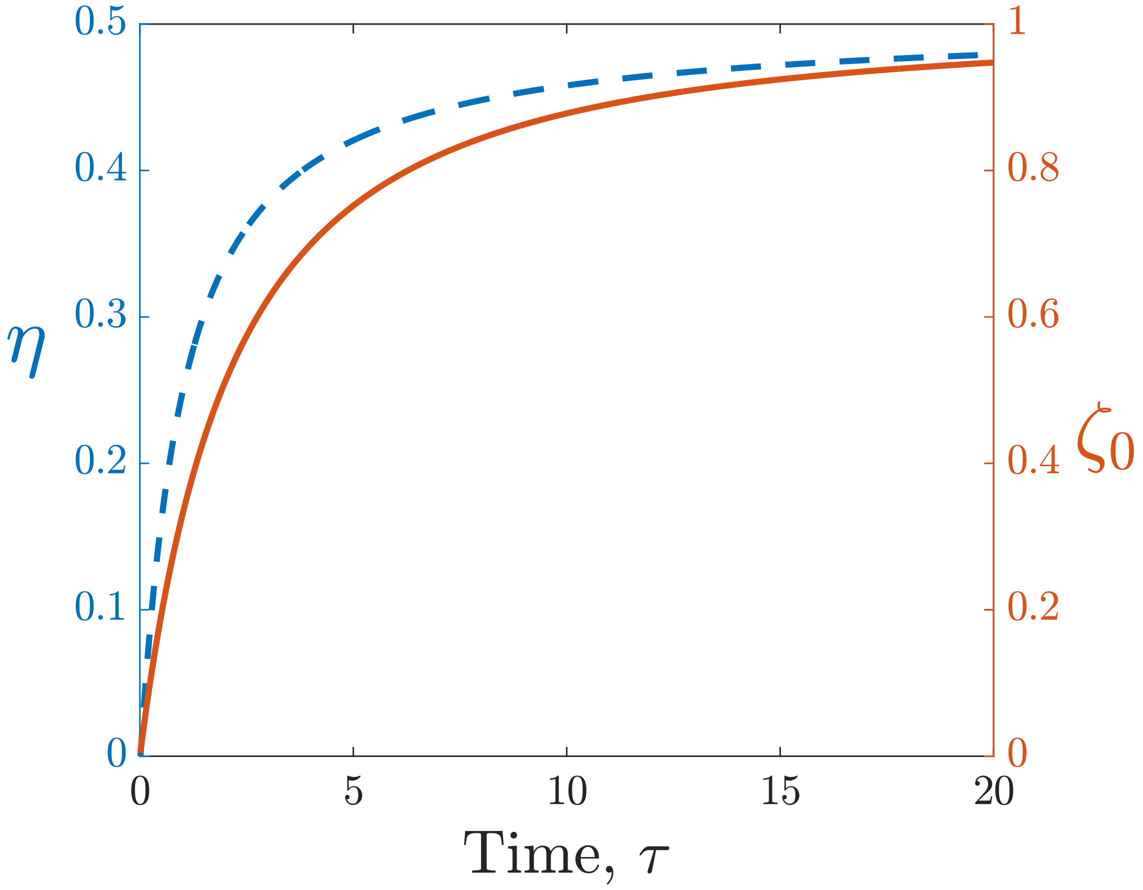


Germeles (1975) Forced plumes and mixing of liquids in tanks. *J Fluid Mech* 71 (601–623)

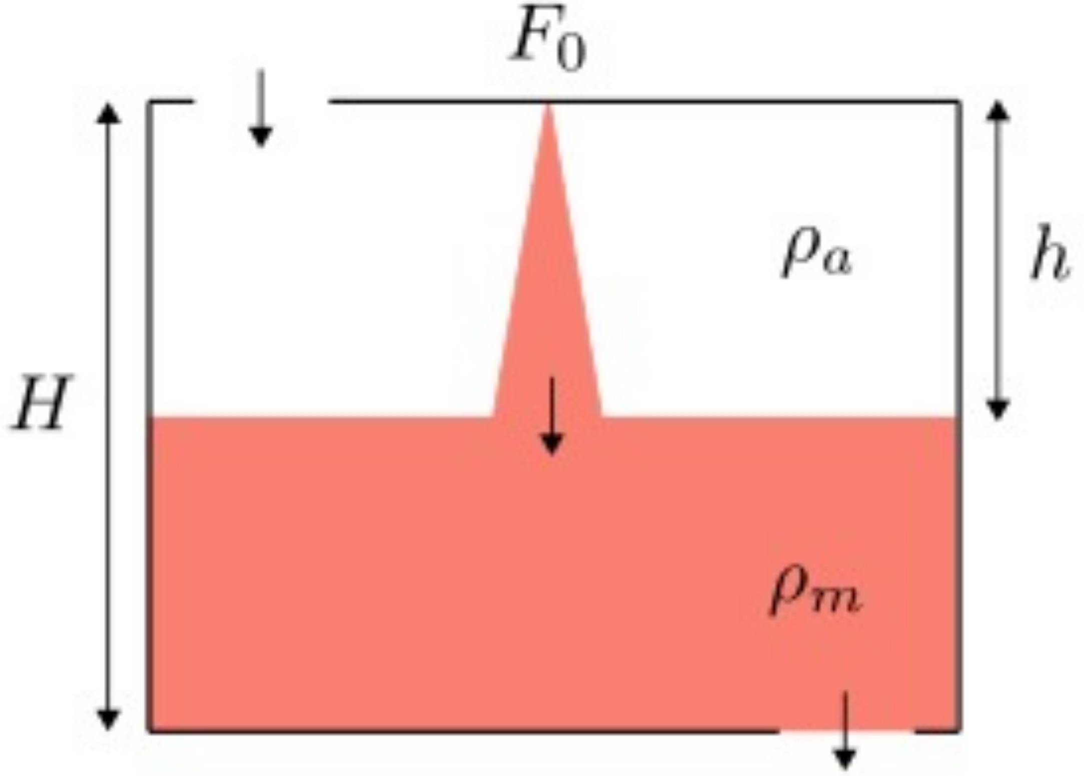
Worster & Huppert (1983) Time-dependent density profiles in a filling box. *J Fluid Mech* 132 (457–466)

Filling box: transient

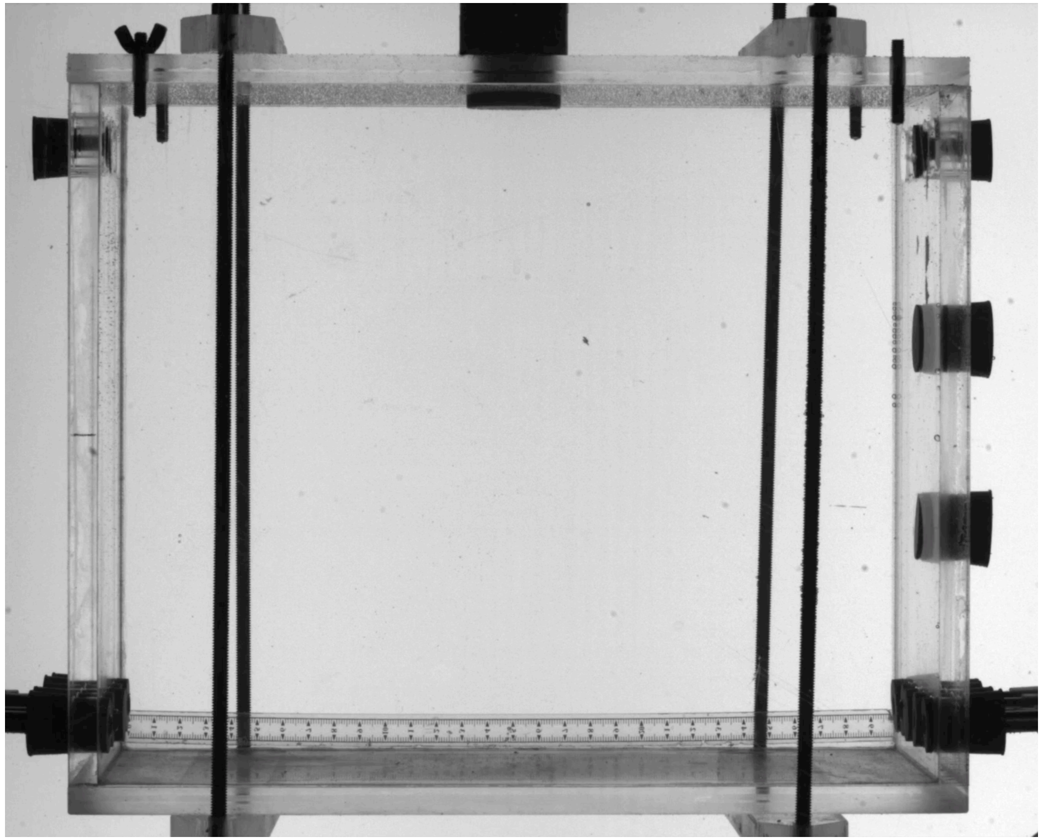




Emptying-filling box



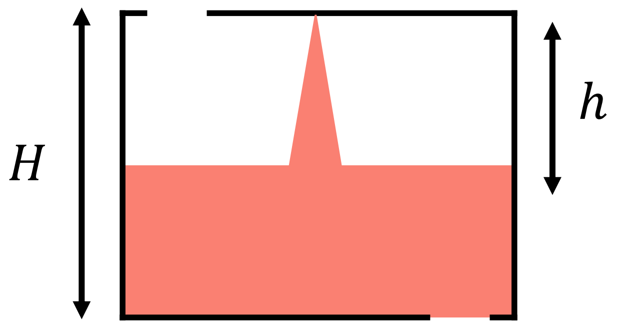
Linden, Lane-Serff & Smeed (1990) Emptying filling boxes: the fluid mechanics of natural ventilation. *J Fluid Mech* 212 (309-335)



Video: Jamie Partridge

Emptying-filling box: steady state

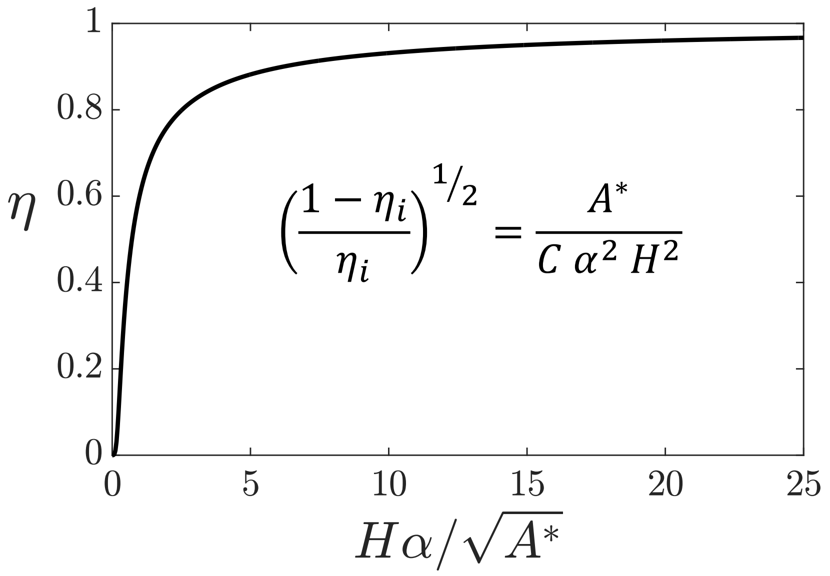
In steady state: $$\left(\frac{\xi}{1-\xi}\right)^{1/2} = \frac{A^*}{C\,\alpha^2\,H^2}$$



$$\xi = \frac{h}{H} \qquad C = \text{constant} \qquad A^* = \text{rescaled opening area}$$

$$\frac{dE_b}{dt} = (H-h)\hat{\rho}F_0 \qquad \frac{dE_a}{dt} = \hat{\rho}F_0 H$$

$$\longrightarrow \quad \eta_i = 1-\xi$$

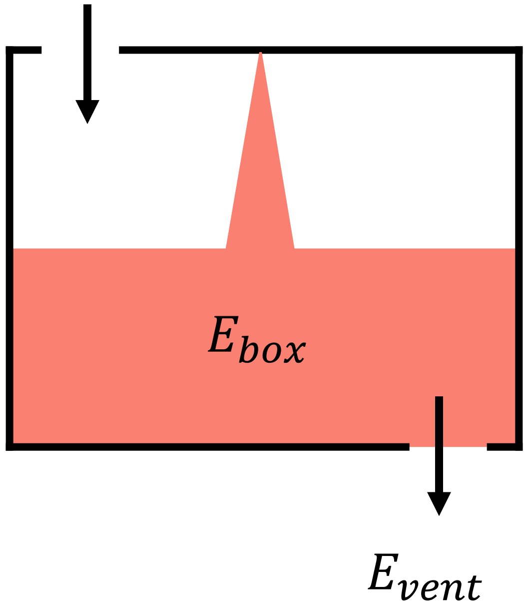


Germeles model with ventilation

$$Q_v = A^* \left(\int_0^H g' \, dz \right)^{\frac{1}{2}}$$

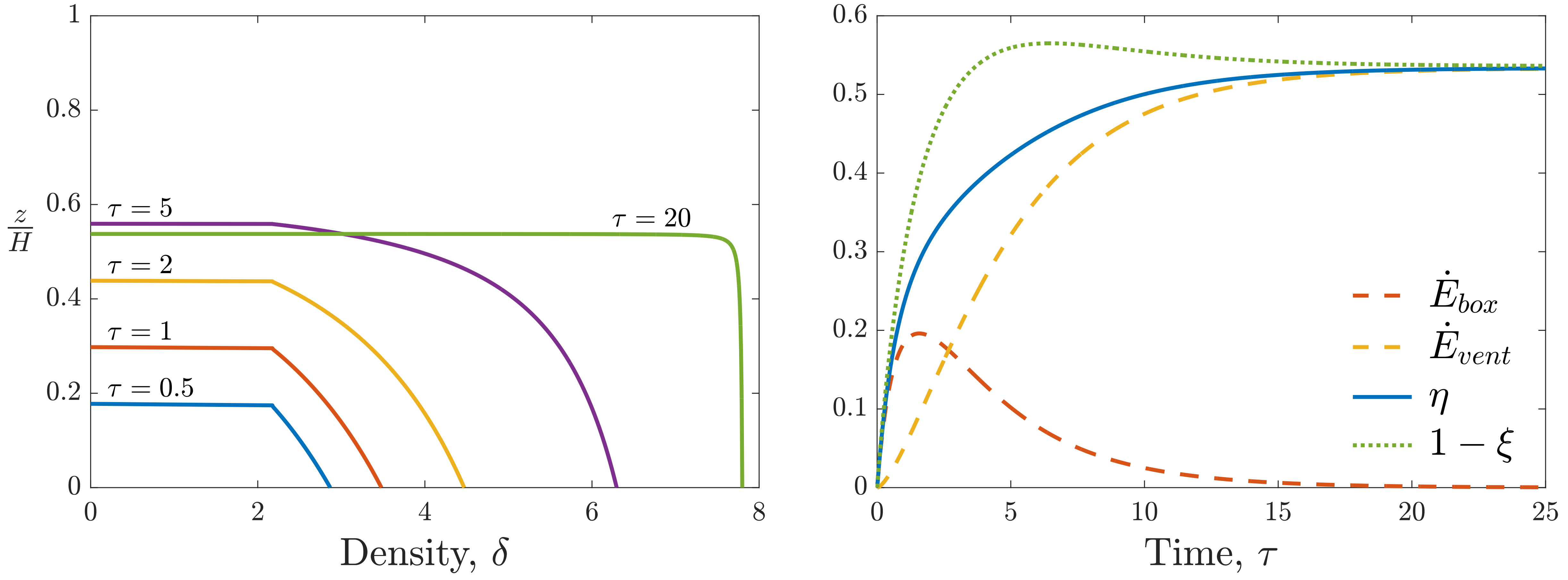
E_{box} = Increase in PE of density profile in box

E_{vent} = PE lost through ventilation



Sandbach & Lane-Serff (2011) Transient buoyancy-driven ventilation: Part 1. Modelling advection. *Build Environ* 46 (1578–1588)

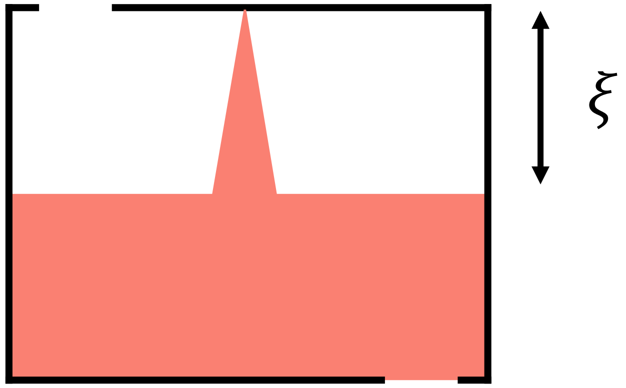
Emptying-filling box: transient



Mixing efficiency of closed vs open systems

Filling box in asymptotic state:

$$\eta_i = \frac{1}{2}$$

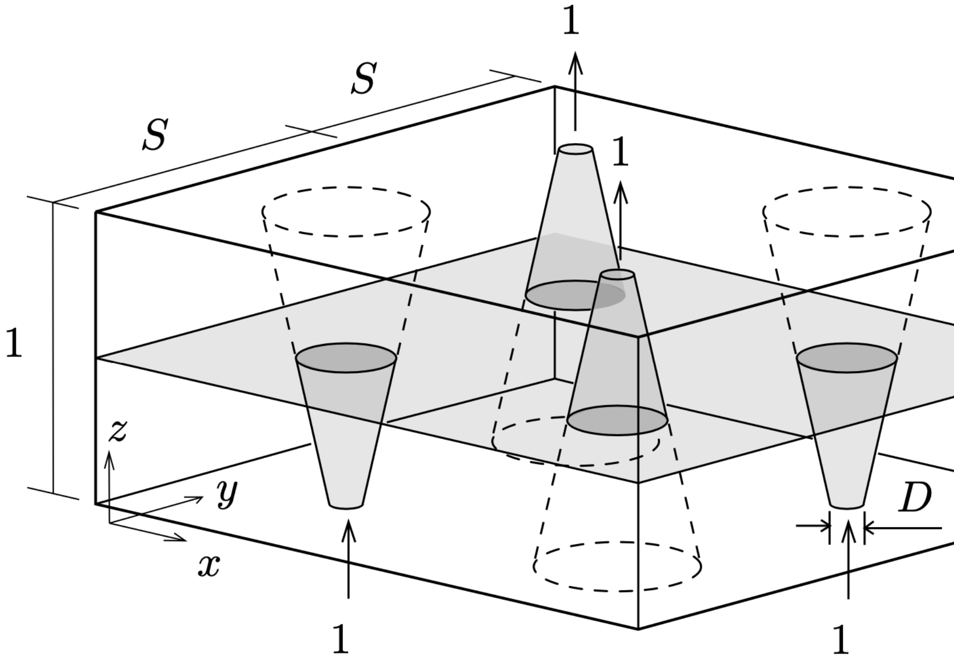


Emptying-filling box in steady state:

$$\eta_i = 1 - \xi$$

Davies Wykes, Hogg, Partridge & Hughes (2019) Energetics of mixing for the filling box and the emptying-filling box. *Environ Fluid Mech* **19**, 819–831

DNS of equal and opposite plumes in a box



$$Pr = 0.7$$

$$Re = 4185$$

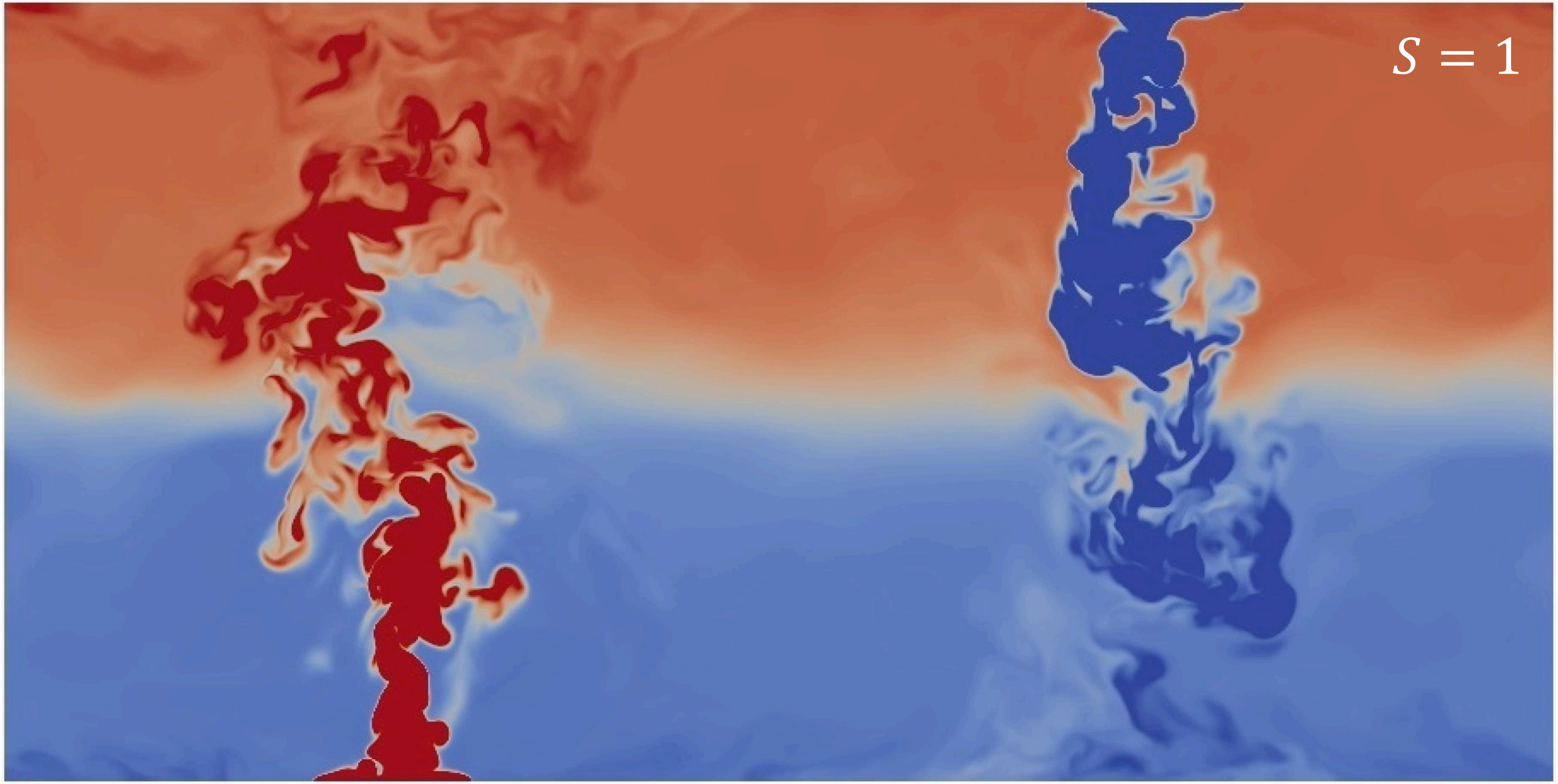
$$Ra = 1.24 \times 10^7$$

$$D = 1/5$$

S	$Nx \times Ny \times Nz$
1/2	$768 \times 768 \times 768$
7/12	$896 \times 896 \times 768$
2/3	$1024 \times 1024 \times 768$
1	$1536 \times 1536 \times 768$
4/3	$2048 \times 2048 \times 768$

Craske & Davies Wykes (2020). The entrainment and energetics of turbulent plumes in a confined space. *J of Fluid Mech*, **883**, A2

Plumes create a two-layer stratification



Local available potential energy

$$E_a = \int_V b\,(z_* - z)\,dz \longrightarrow E_a = b(z_* - z) - \int_0^{z-z_*} b_*(z - \hat{z}, t)\,d\hat{z} \geq 0$$

Holliday & McIntyre 1981, JFM

$$E_b = -bz - E_a$$

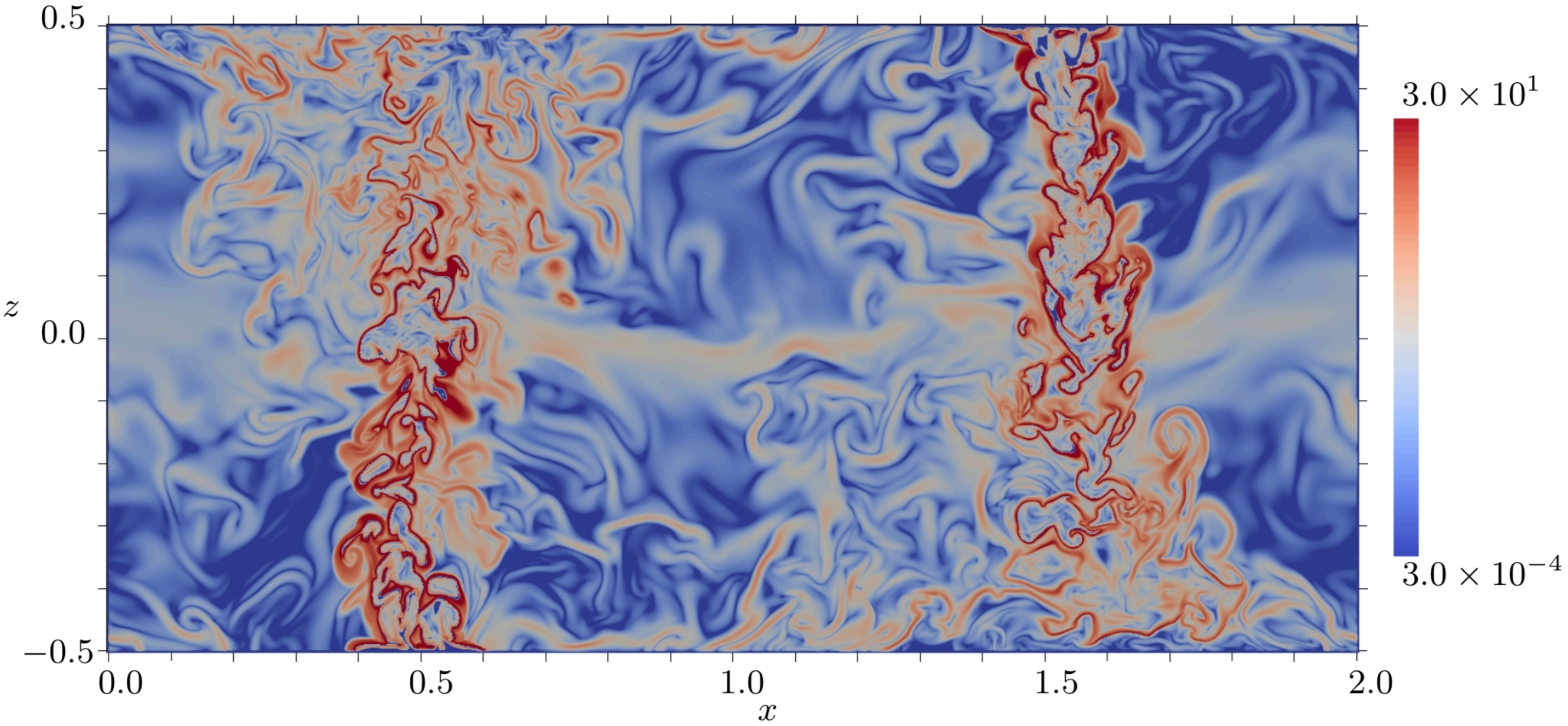
$$\frac{\partial E_k}{\partial t} + \nabla \cdot (\boldsymbol{u}\,E_k + \ \dots) = -\Phi_z - \varepsilon$$

$$\frac{\partial E_a}{\partial t} + \nabla \cdot (\boldsymbol{u}\,E_a + \ \dots) = \Phi_z - \Phi_d + \ \dots$$

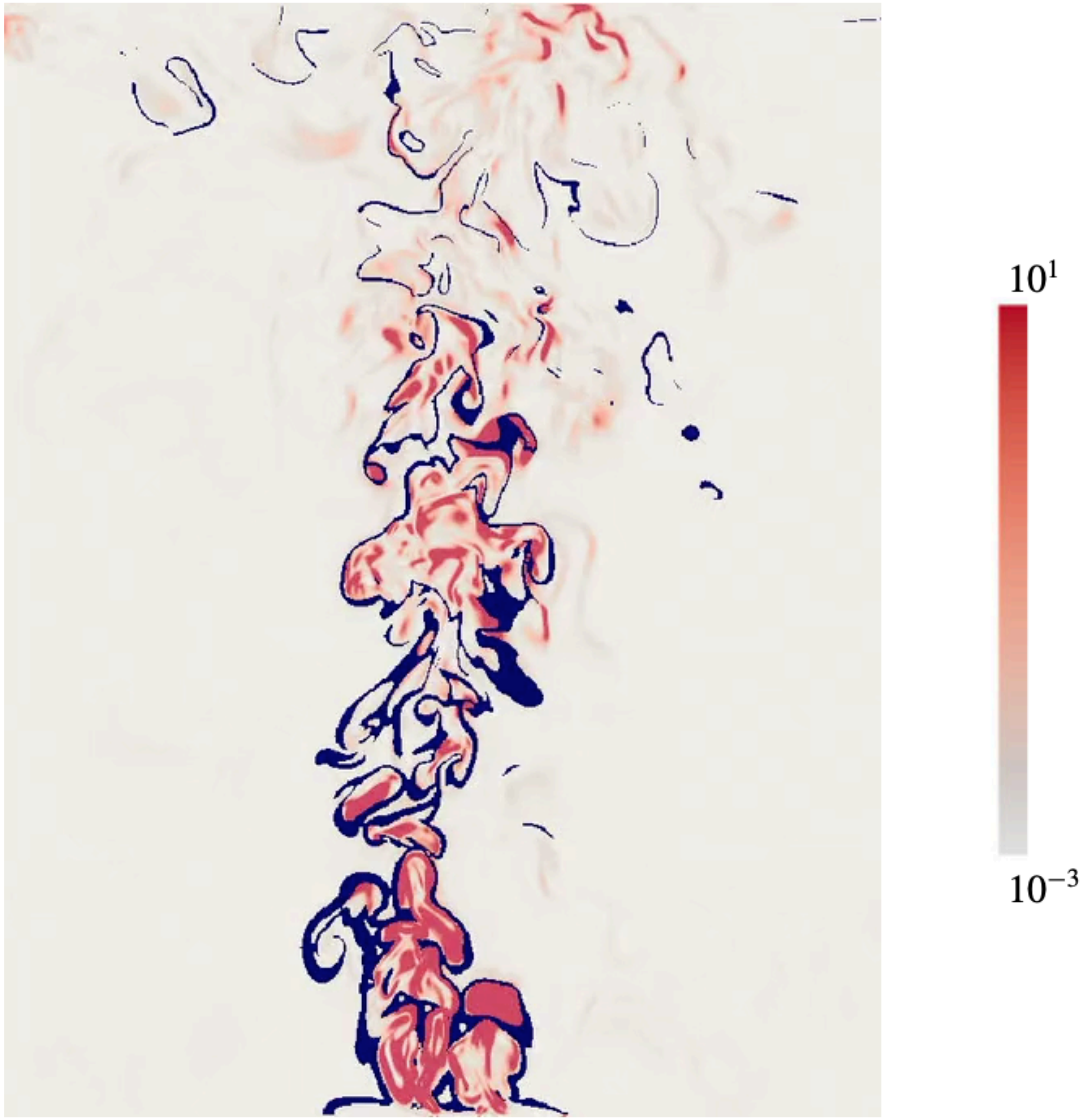
$$\frac{\partial E_b}{\partial t} + \nabla \cdot (\boldsymbol{u}\,E_b + \ \dots) = \Phi_d + \ \dots$$

$$\Phi_z = w(b_* - b)$$

$$\Phi_d = \frac{(\nabla b)^2}{Pe}\frac{dz_*}{db} \qquad \varepsilon = \frac{\left(\partial_j u_i\right)^2}{Re}$$



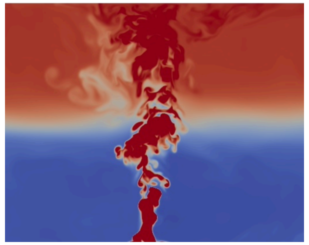
BPE Production, $\Phi_d = \frac{(\nabla b)^2}{Pe}\frac{dz_*}{db}$

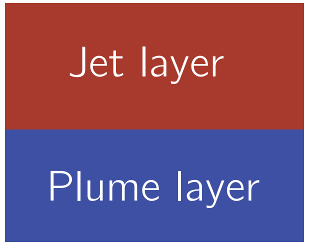


Dissipation, ε

BPE Production,
$\Phi_d > 3$

Energetics within the layers



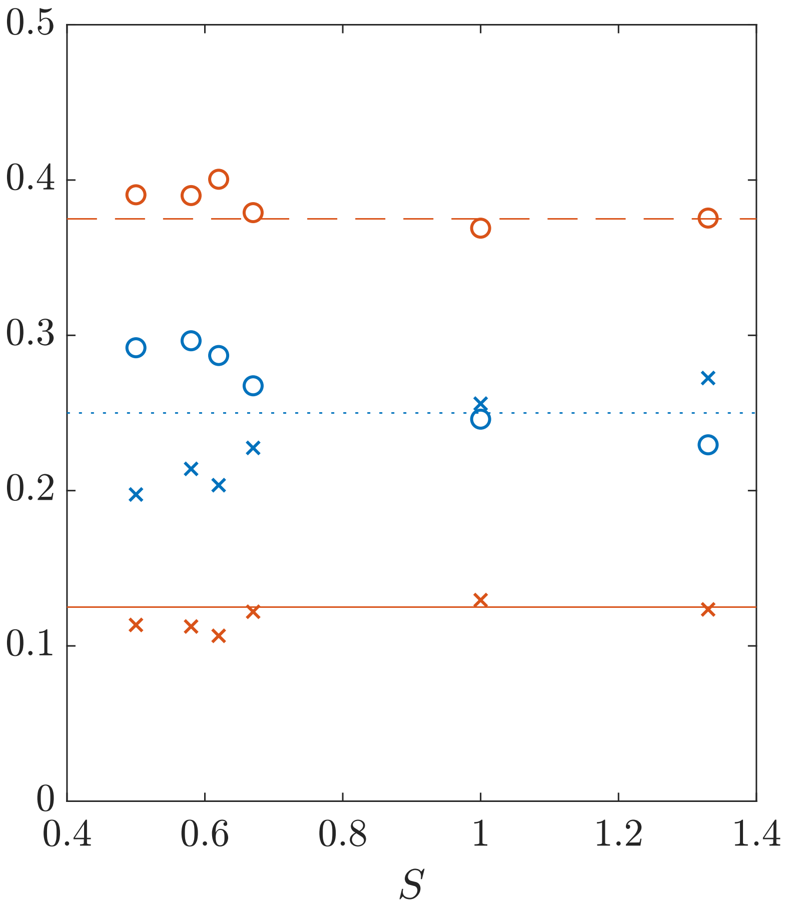


× Jet layer KE dissipation $\langle wE_k \rangle = \dfrac{\gamma M_m^2}{2Q_m} = \dfrac{3\gamma}{8}\zeta = \dfrac{1}{4}$

○ Plume layer KE dissipation $\left(1 - \dfrac{3\gamma}{8}\right)\zeta = \dfrac{1}{4}$

× Jet layer BPE Production $\langle wE_a \rangle = (1-\zeta)\dfrac{\langle|\Phi_z|\rangle}{2}\Big|_{z=0} = \dfrac{1}{8}$

○ Plume layer BPE Production $1 - \zeta - (1-\zeta)\dfrac{\langle|\Phi_z|\rangle}{2}\Big|_{z=0} = \dfrac{3}{8}$



Conclusions

- Mixing efficiency in a closed system can be limited when in an open system it is not.
- Energetics are independent of entrainment, even though entrainment determines the buoyancy profile and volume flux.

Davies Wykes, Hogg, Partridge & Hughes (2019) Energetics of mixing for the filling box and the emptying-filling box. *Environ Fluid Mech*, **19**, 819–831

Craske & Davies Wykes (2020) The entrainment and energetics of turbulent plumes in a confined space. *J of Fluid Mech*, **883**, A2