Supergranule aggregation for constant heat flux-driven convection

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joint work with

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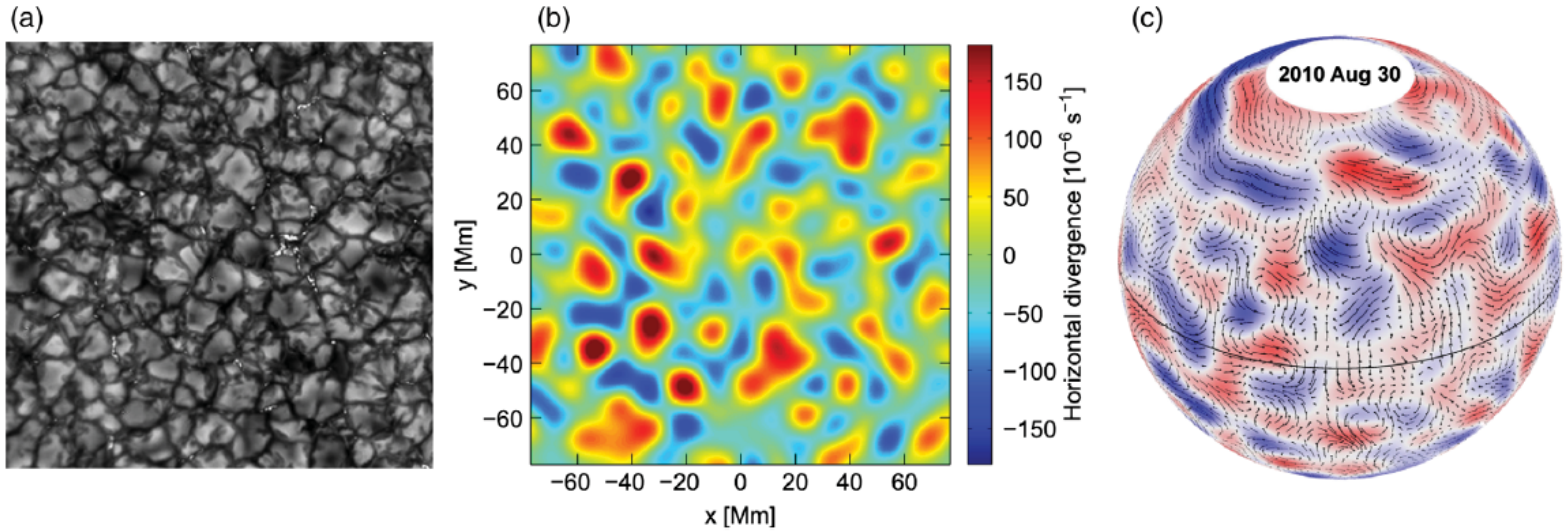
Solar Convection

JS and K.R. Sreenivasan, Rev. Mod. Phys. Rev. ***92****, 041001 (2020)*



	Granules	**Supergranules**	**Giant Cells (?)**
Scale	1,000 km	30,000 km	200,000 km
Lifetime	10 min	24 h	1 month
Velocity	3 km/s	500 m/s	10 m/s
Observation	Optical	Helioseismology or Granule Tracking	Helioseismology

Not consistently reproduced in simulations !

Rayleigh-Bénard convection model

$$\nabla \cdot \boldsymbol{u} = 0\,,$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla p + \sqrt{\frac{\mathrm{Pr}}{\mathrm{Ra}_{D,N}}}\nabla^2\boldsymbol{u} + T\boldsymbol{e}_z\,,$$

$$\frac{\partial T}{\partial t} + (\boldsymbol{u}\cdot\nabla)T = \frac{1}{\sqrt{\mathrm{Ra}_{D,N}\mathrm{Pr}}}\nabla^2 T\,,$$

$$L_x = L_y = 60H$$

$$\mathrm{Pr} = \frac{\nu}{\kappa} = 1 \qquad \mathrm{Ra}_D = \frac{g\alpha\Delta T H^3}{\nu\kappa} \qquad \mathrm{Ra}_N = \frac{g\alpha\beta H^4}{\nu\kappa}$$

D=Dirichlet
N=Neumann

4 sets of boundary conditions

Top and Bottom

$$\text{(ns)} \quad u_x = u_y = u_z = 0\,,$$

$$\text{(fs)} \quad \frac{\partial u_x}{\partial z} = \frac{\partial u_y}{\partial z} = 0 \quad \text{and} \quad u_z = 0$$

$$\text{(D)} \quad T(z=0) = T_{\text{bot}} \quad \text{and} \quad T(z=H) = T_{\text{top}}$$

$$\text{(N)} \quad \left.\frac{\partial T}{\partial z}\right|_{z=0} = \left.\frac{\partial T}{\partial z}\right|_{z=H} = -\beta\,,$$

Side planes are periodic

$$\mathrm{Ra}_N = \mathrm{Nu}_D \mathrm{Ra}_D$$

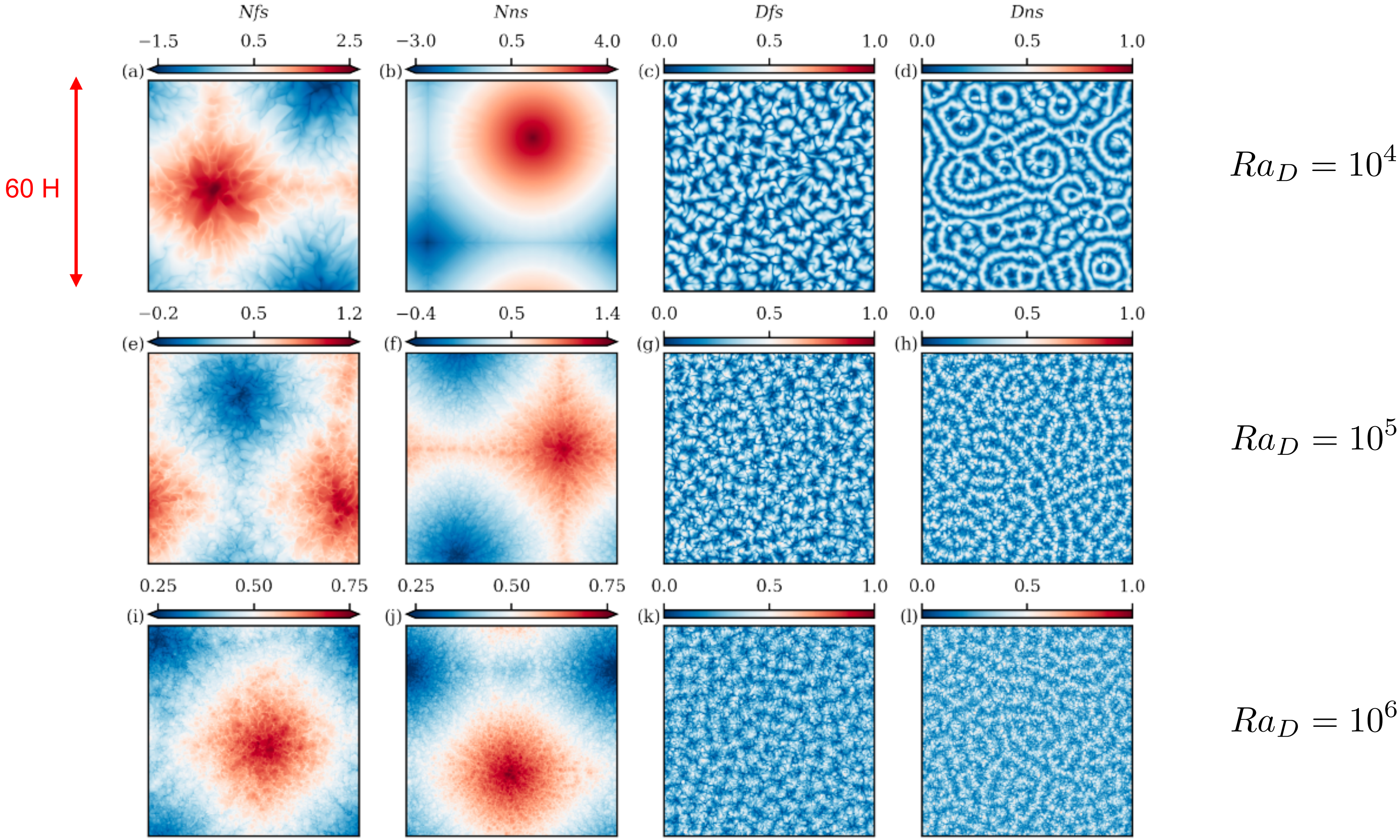
*See e.g. H. Johnston and C.R. Doering, Phys. Rev. Lett. **102**, 064501 (2009)*

$$\mathrm{Nu}_D = -\left\langle \left.\frac{\partial T}{\partial z}\right|_{z=0} \right\rangle_{A,t} = -\left\langle \left.\frac{\partial T}{\partial z}\right|_{z=1} \right\rangle_{A,t} \qquad \mathrm{Nu}_N = \frac{1}{\langle T(z=0)\rangle_{A,t} - \langle T(z=1)\rangle_{A,t}}$$

$$\mathrm{Re} = \sqrt{\frac{\mathrm{Ra}_{D,N}}{\mathrm{Pr}}\,\langle \boldsymbol{u}^2\rangle_{V,t}}$$

Comparison

$$T(x, y, z = 1 - \delta_T/2, t_0)$$



Temperature boundary conditions matter!

Onset of convection

On the solution of the Bénard problem with boundaries of finite conductivity

By D. T. J. Hurle, E. Jakeman and E. R. Pike
Royal Radar Establishment, Great Malvern, Worcestershire

Dirichlet ... Neumann

a \ κ_F/κ_S ...	0	0·5	1	2	∞
	Rayleigh number, $10^{-3}R$				
0	∞	∞	∞	∞	0·720
1	5·854	2·568	1·855	1·368	0·748
2	2·177	1·520	1·297	1·117	0·846
2·071	—	—	—	1·16412	—
2·397	—	—	1·267471	—	—
2·669	—	1·415093	—	—	—
3	1·711	1·435	1·323	1·224	1·057
3·116	1·707762	—	—	—	—
4	1·879	1·712	1·638	1·569	1·444
5	2·439	2·314	2·255	2·199	2·094
6	3·418	3·311	3·259	3·209	3·112
7	4·919	4·820	4·771	4·723	4·630
8	7·085	6·989	6·941	6·894	6·801
9	10·090	9·995	9·947	9·900	9·805
10	14·130	14·040	13·990	13·940	13·840
∞	∞	∞	∞	∞	∞

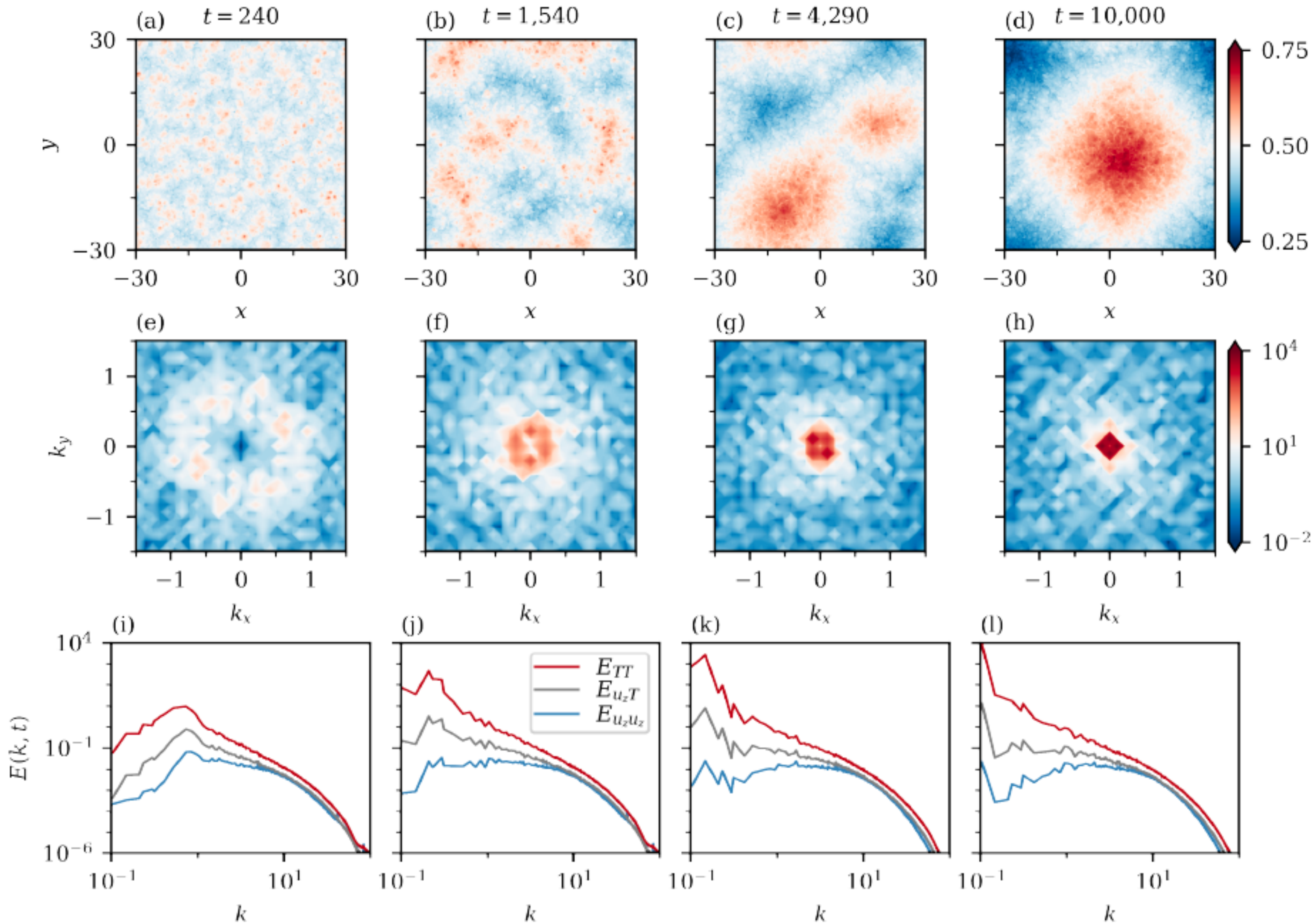
← No-slip

Free-slip
R=120

D.T.J. Hurle, E. Jakeman and E.R. Pike, Proc. Roy. Soc. Lond. A 296, 469 (1967)
E. M. Sparrow, R.J. Goldstein and V.K. Johnson, J. Fluid Mech. 18, 513 (1964)

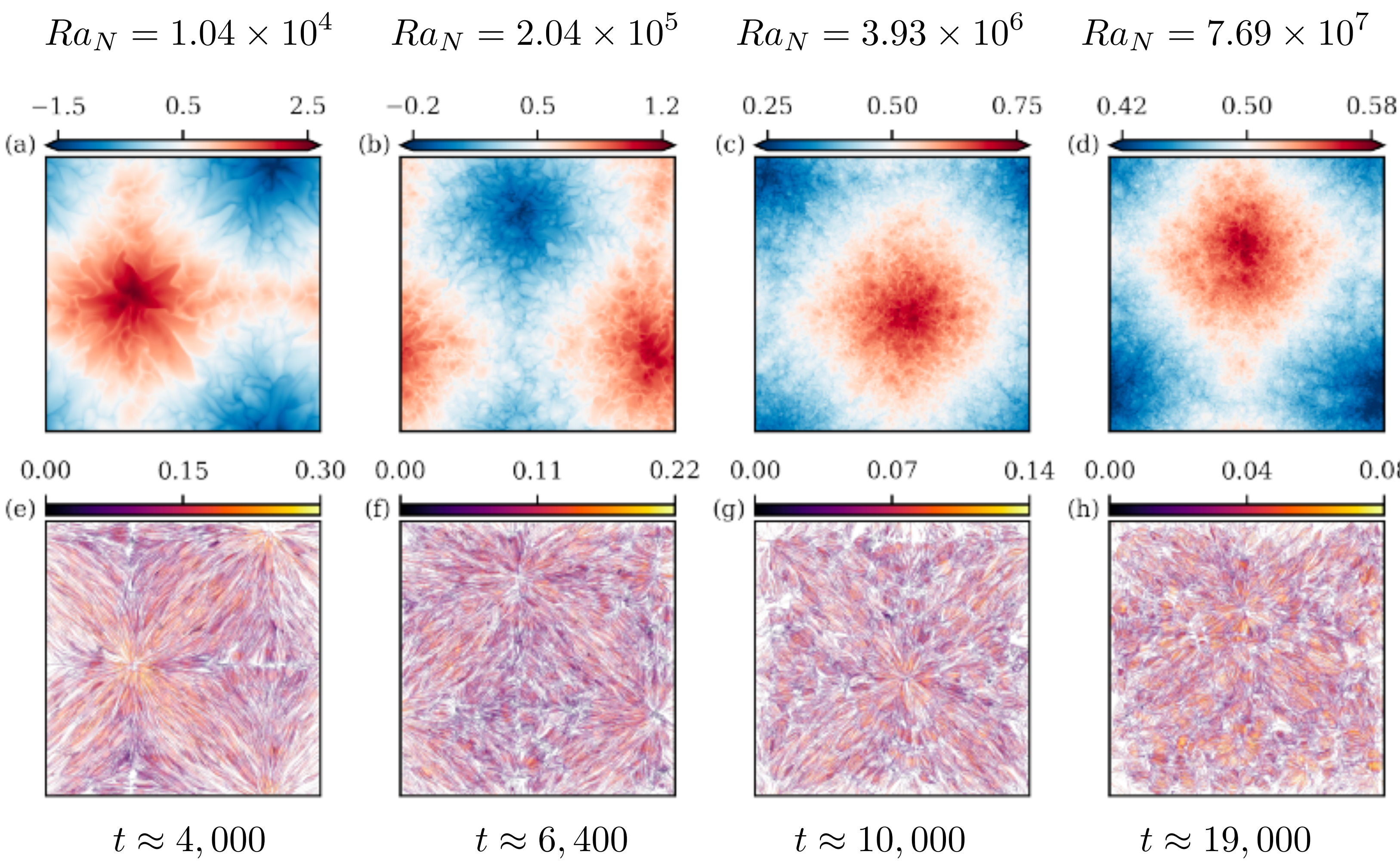
Long-term aggregation …

$$Ra_N = 3.93 \times 10^6$$



Gradual supergranule formation!

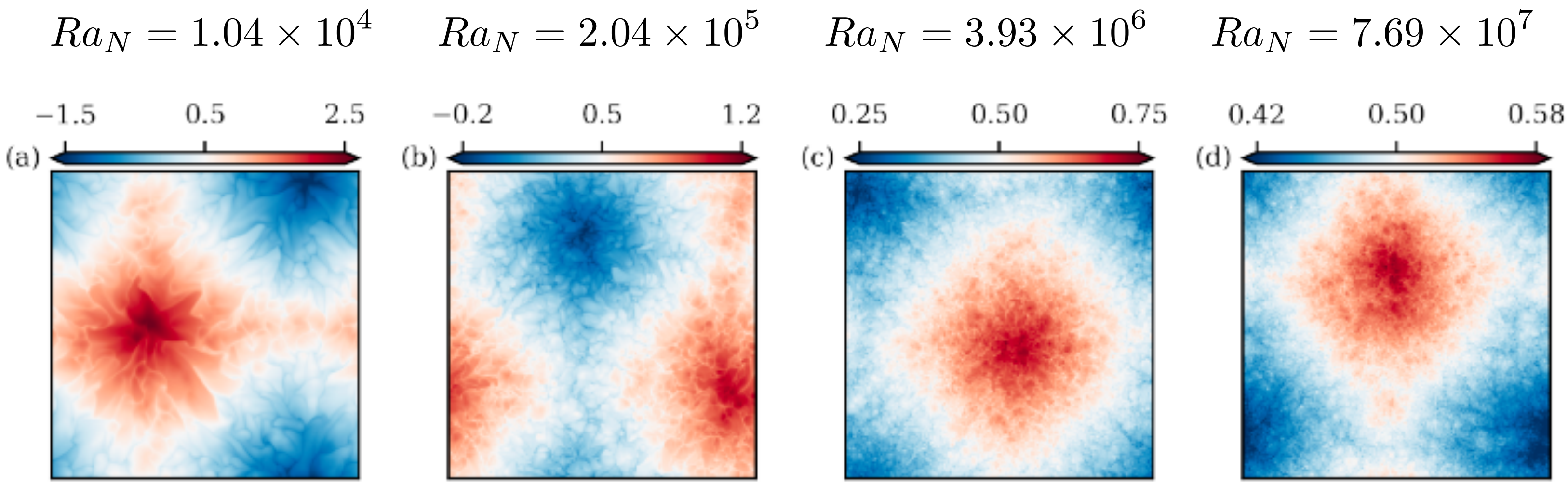
… for all analysed Rayleigh numbers

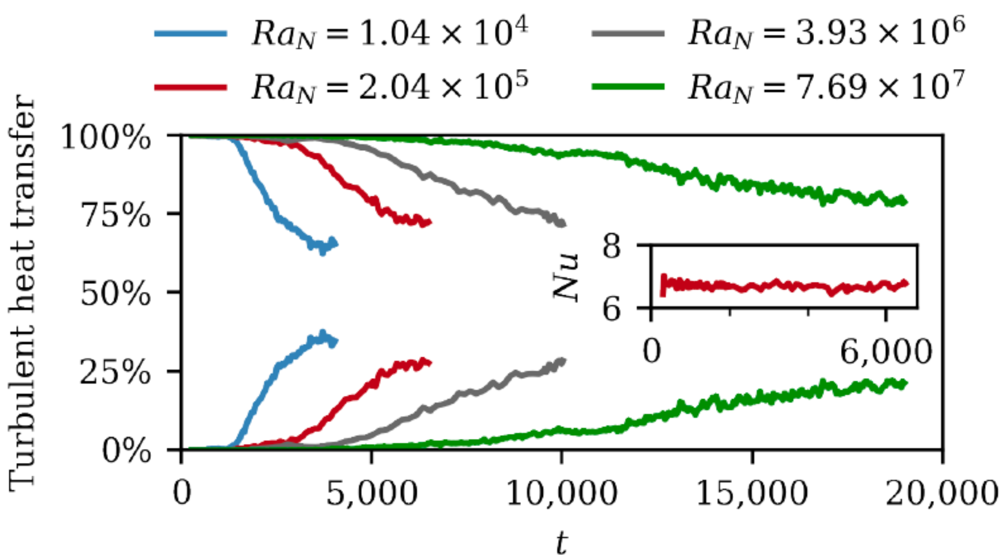


The larger Rayleigh number, the longer formation time

$z_0 = 1 - \delta_T/2$

Contribution to heat transfer





Nusselt number remains unchanged during aggregation

Weakly nonlinear regime in finite domain

J. Fluid Mech. (1980), *vol.* 101, *part* 4, *pp.* 759–782 759
Printed in Great Britain

Nonlinear Rayleigh–Bénard convection between poorly conducting boundaries

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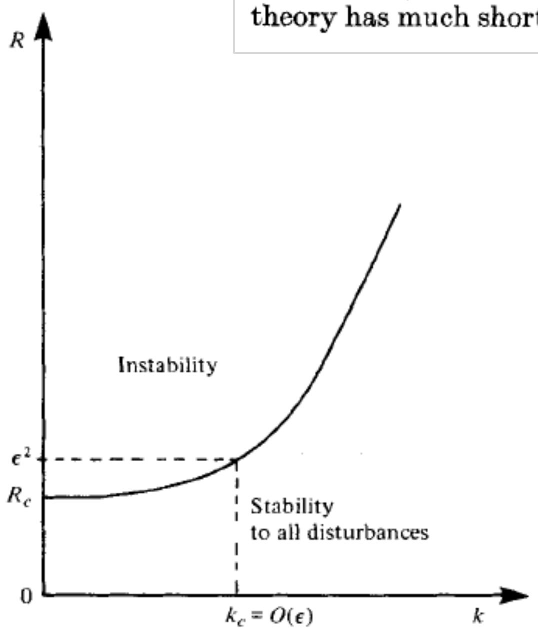
Department of Applied Mathematics and Theoretical Physics, University of Cambridge

wave numbers smaller than a given cut off. Stability analysis reveals, however, that each mode is unstable to one of longer wavelength than itself, so that any long box will eventually contain a single roll, even though the most rapidly growing mode on linear theory has much shorter wavelength.



2d convection in a box with length $L = 2\pi/k$

$$R = R_c + \mu^2\epsilon^2, \qquad \mu = O(1)$$

$$\partial/\partial x = \epsilon\partial/\partial X,$$

$$\frac{\partial}{\partial t} = \epsilon^4\frac{\partial}{\partial \tau},$$

Transform Boussinesq equations and sort by $\mathcal{O}(1), \mathcal{O}(\varepsilon^2)$

$$\theta = \theta_0(X, z, \tau) + \epsilon^2\theta_2(X, z, \tau) + \ldots$$

$$\phi = \phi_0(X, z, \tau) + \epsilon^2\phi_2(X, z, \tau) + \ldots$$

Steady solution and their stability

Intermediate summary

- Constant flux boundary conditions lead to the formation of a supergranule in an extended domain at $Pr = 1$
- Independent of velocity field boundary conditions
- Observed for all accessible Rayleigh numbers
- The larger Ra, the longer the aggregation time
- Onset of convection in an infinitely extended layer gives $k_c = 0$
- An expansion of this limit to a finite domain in the weakly nonlinear theory suggests a pair of convection rolls that grows up to wavelength $\lambda \sim L$

The convection flow does not forget where it comes from even though being fully turbulent!

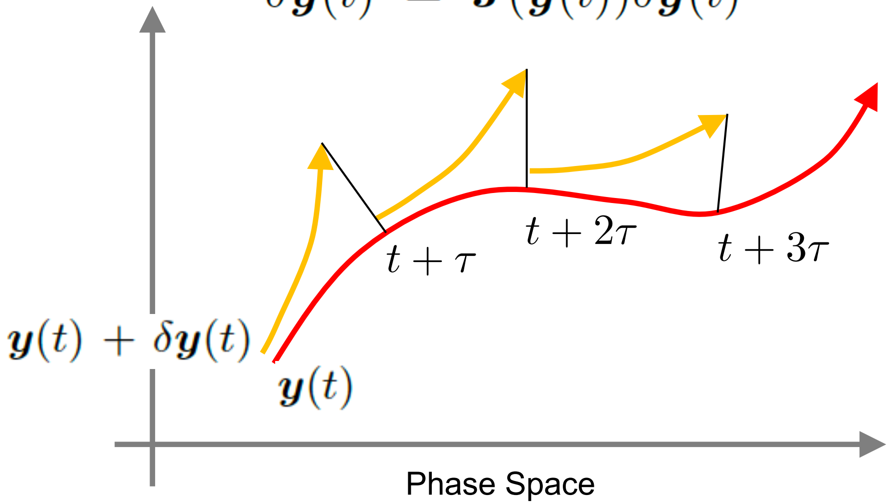
Can we quantify this in the turbulent case?

Lyapunov analysis

A. Pikovsky and A. Politi, *Lyapunov exponents – A tool to explore complex dynamics*, Cambridge University Press, 2016

$$\dot{\boldsymbol{y}}(t) = \boldsymbol{F}(\boldsymbol{y}, t)$$

$$\delta\dot{\boldsymbol{y}}(t) \;=\; \boldsymbol{J}(\boldsymbol{y}(t))\delta\boldsymbol{y}(t)$$



$$\boldsymbol{y}(t) = (\boldsymbol{u}(\boldsymbol{x}_k, t), T(\boldsymbol{x}_k, t)) \qquad k = 1, \ldots, N_e N^3$$

Leading Lyapunov exponent $\lambda_1(t) = \frac{d}{dt} \log\left(\frac{\|\delta\boldsymbol{y}_1(t)\|}{\|\delta\boldsymbol{y}_1(0)\|}\right)$

with $\|\delta\boldsymbol{y}_1(t)\| = \sqrt{\frac{1}{V}\int_V (\delta\boldsymbol{u}_1(t)^2 + \delta T_1(t)^2) dV}$

Number of degrees of freedom is 1.5×10^{10}

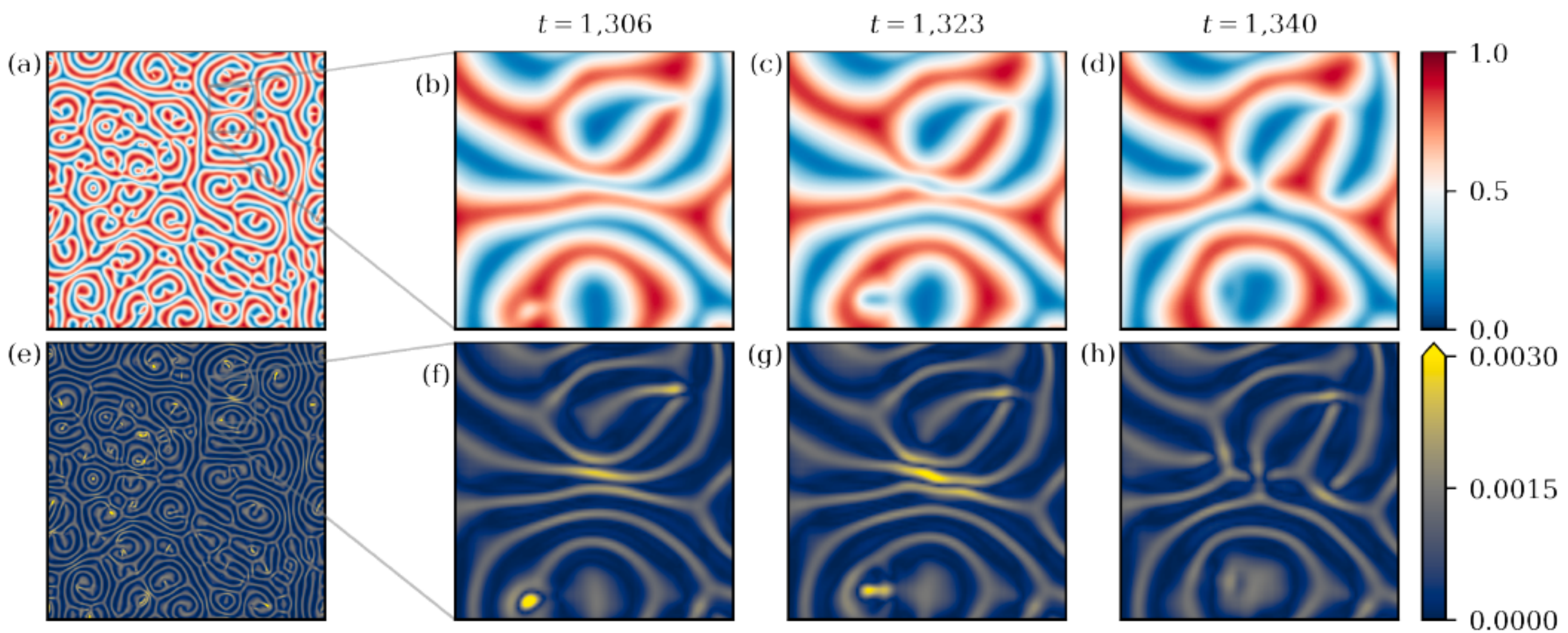
Leading Lyapunov vector

D.A. Egolf, I.M. Melnikov, W. Pesch and R.E. Ecke, Nature 404, 733 (2000)
J.D. Scheel and M.C. Cross, Phys. Rev. E 74, 066301 (2006)

$$\nabla \cdot \delta \boldsymbol{u} = 0$$

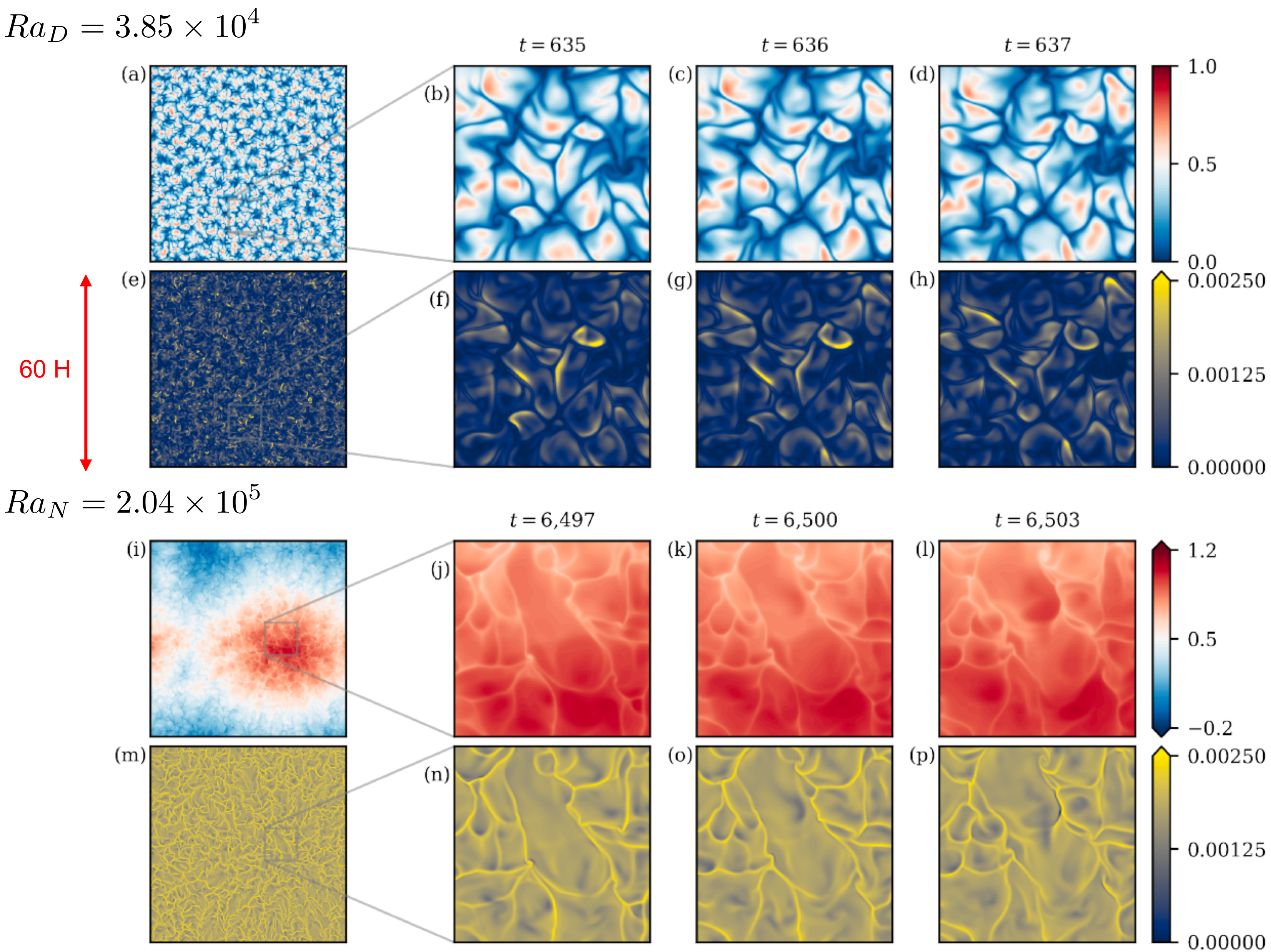
$$\frac{\partial \delta \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\delta \boldsymbol{u} + (\delta \boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla \delta p + \sqrt{\frac{\mathrm{Pr}}{\mathrm{Ra}_{D,N}}}\nabla^2 \delta \boldsymbol{u} + \delta T \boldsymbol{e}_z$$

$$\frac{\partial \delta T}{\partial t} + (\boldsymbol{u} \cdot \nabla)\delta T + (\delta \boldsymbol{u} \cdot \nabla)T = \frac{1}{\sqrt{\mathrm{Ra}_{D,N}\mathrm{Pr}}}\nabla^2 \delta T$$



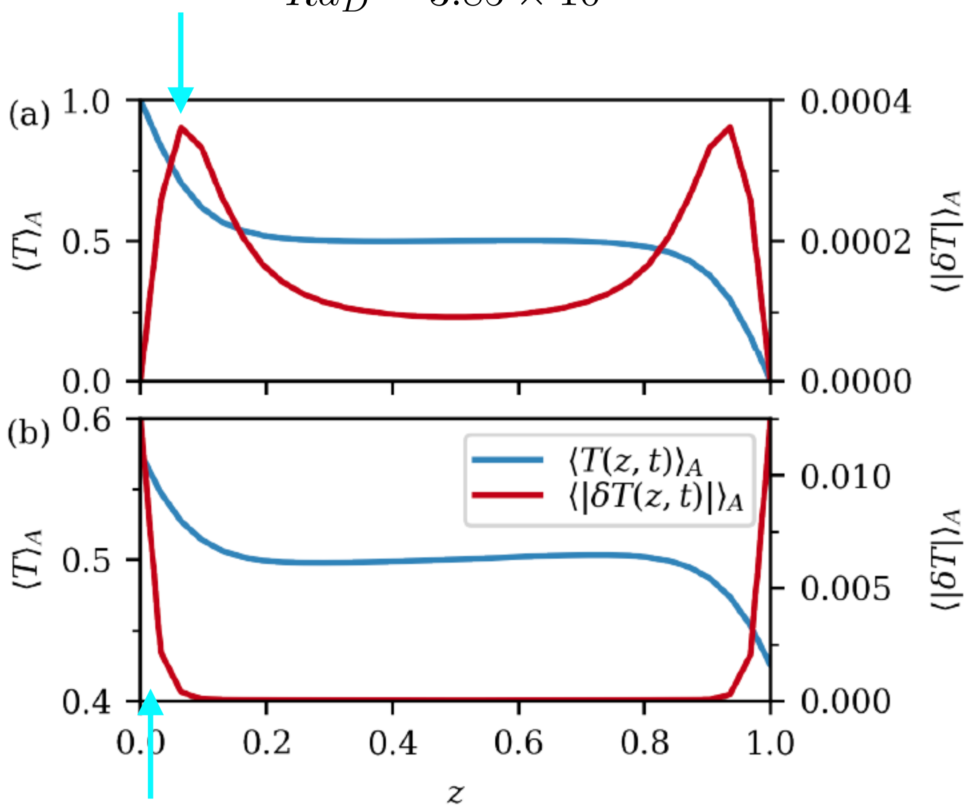
$\mathrm{Ra}_D = 5 \times 10^3$

Lyapunov vector in turbulent flow



Where to analyse Lyapunov vector?

$Ra_D = 3.85 \times 10^4$

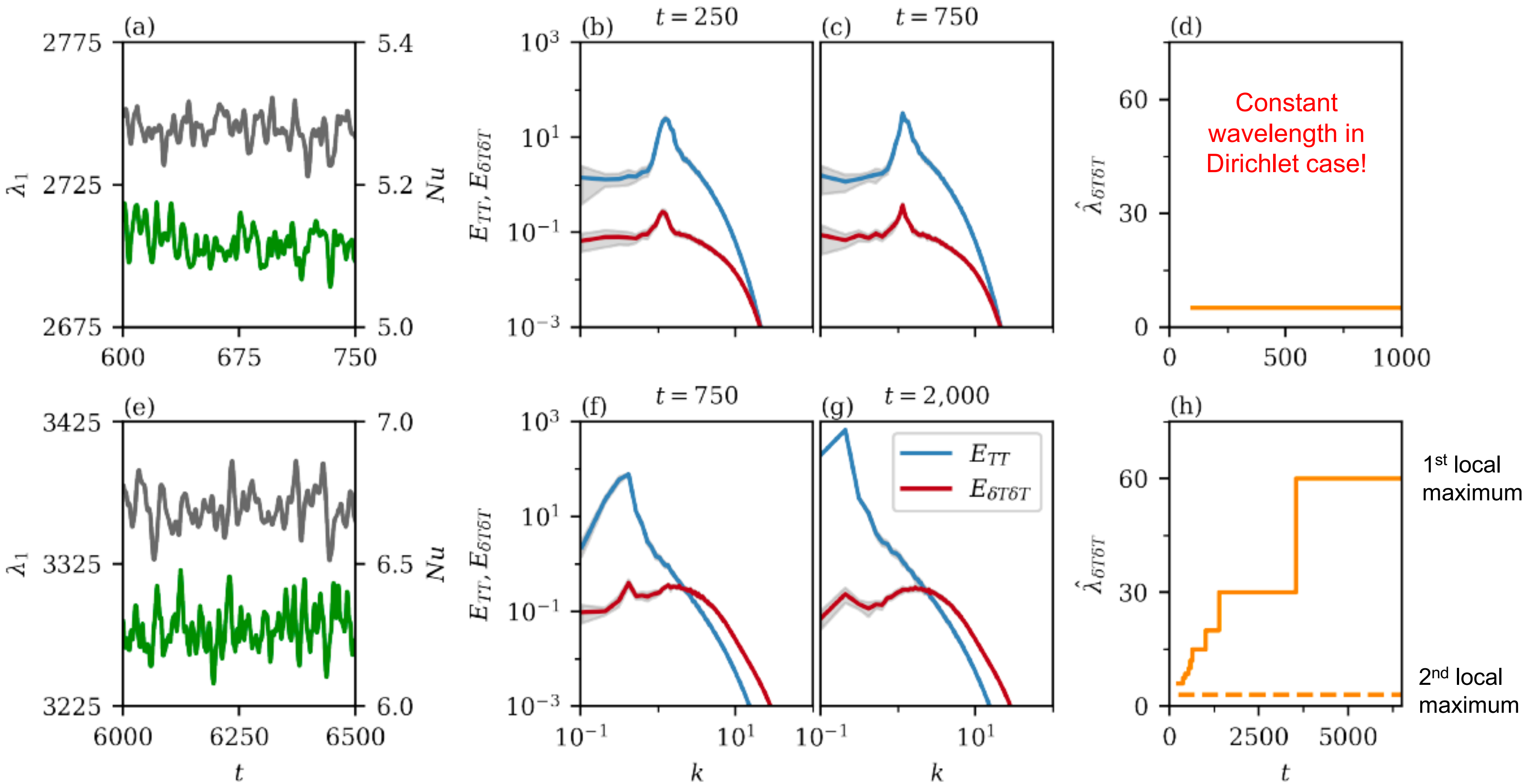


$Ra_N = 2.04 \times 10^5$

Take the plane with large (or largest) magnitude of temperature component of leading Lyapunov vector

Instability at increasingly larger scale

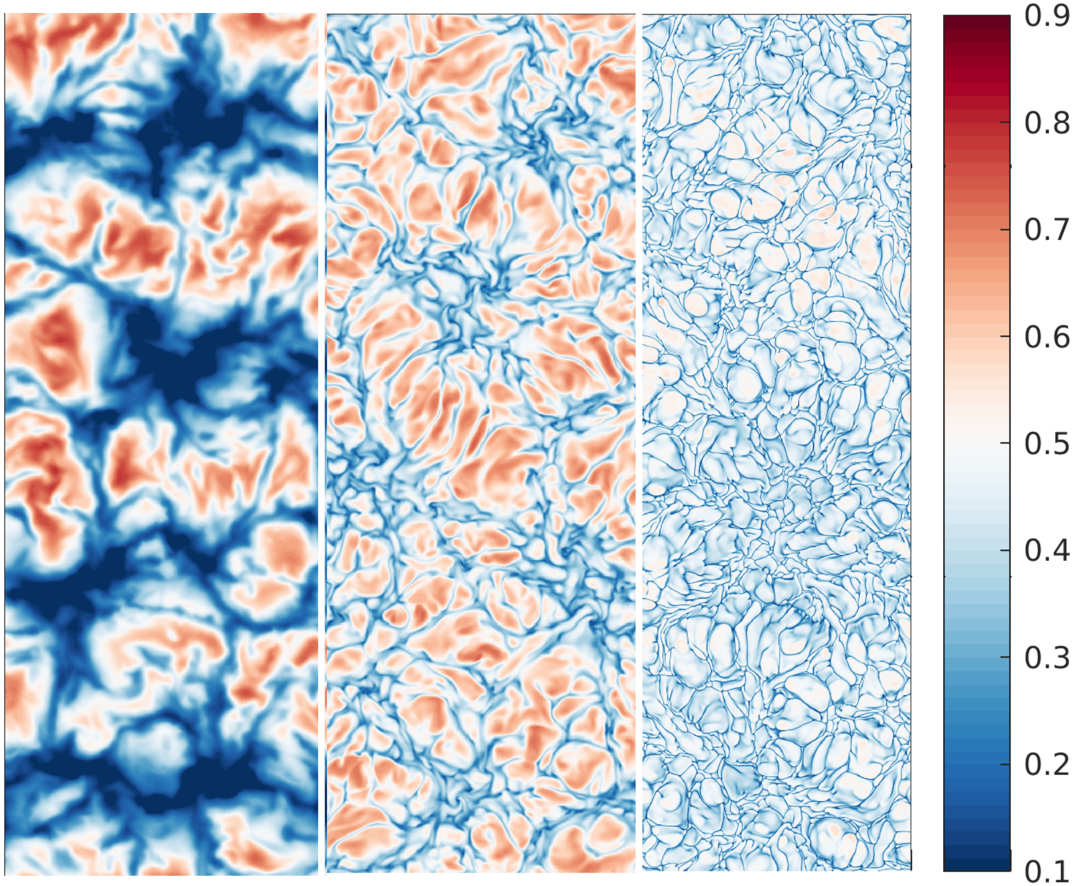
$Ra_D = 3.85 \times 10^4$



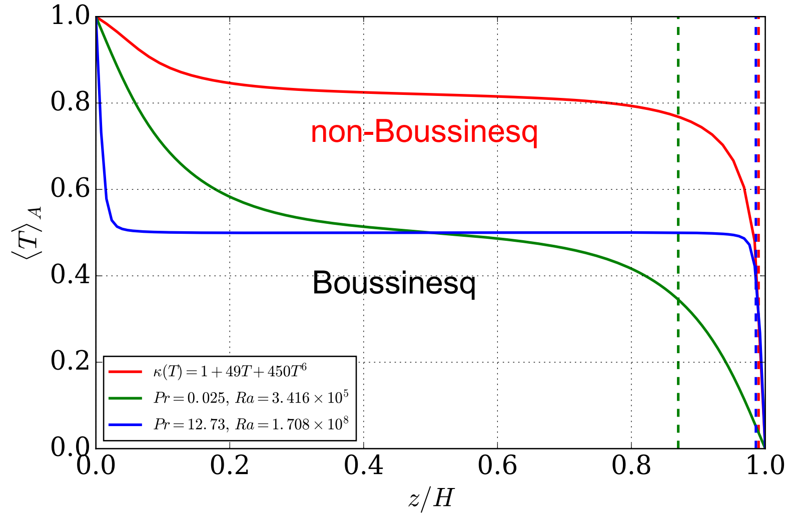
$Ra_N = 2.04 \times 10^5$

Leading Lyapunov vector analysis reveals tendency to develop instabilities with increasingly larger scales in the turbulent flow case

Towards solar convection



$$\kappa_{\mathrm{bot}} = 500\,\kappa_{\mathrm{top}}$$



$$\kappa \rightarrow \kappa(T)$$

Temperature-dependent thermal diffusivity

A. Pandey, JS and K.R. Sreenivasan, ApJ., in press (2021)

Summary and outlook

- Observed a supergranule aggregation in an extended RBC flow with constant flux boundary conditions independent of velocity boundary conditions
- Lyapunov vector analysis demonstrates that the trend to develop instabilities at increasingly larger scales (which is known from the weakly nonlinear regime) continues in the turbulent regime
- The aggregation proceeds gradually until the whole box is filled by a pair of convection cells
- An additional physical process (e.g. slow rotation) is required to stop this growth *(see e.g. Vasil, Julien and Featherstone, arXiv:2010.15383)*

P. P. Vieweg, J. D. Scheel, and JS, arXiv:2010.13383

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