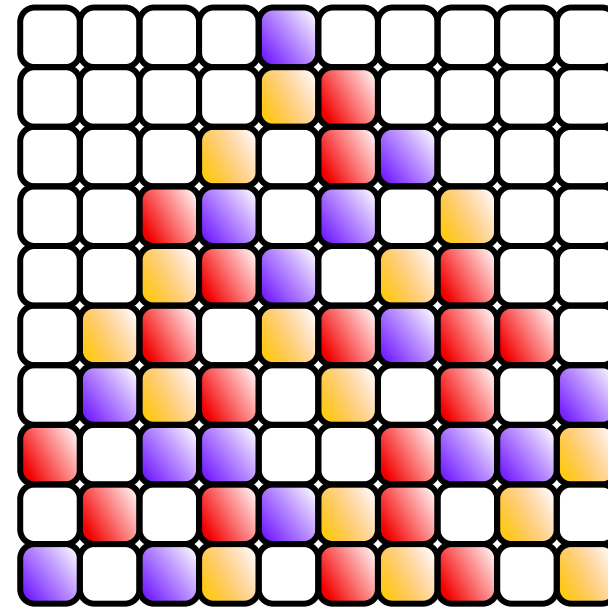
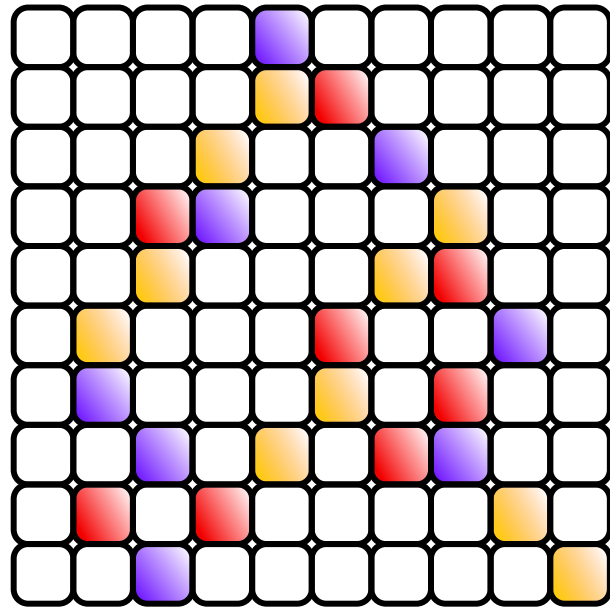


# Noise, complexity, and information dynamics in quantum circuits



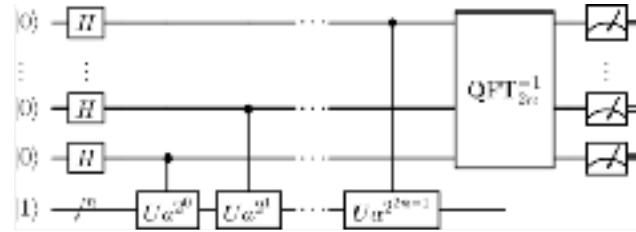
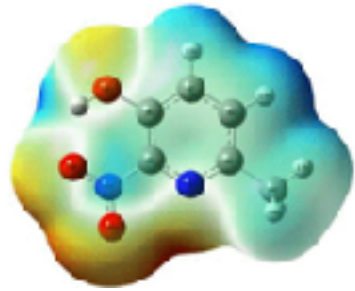
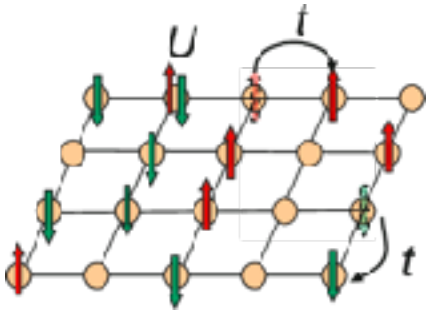
TS, Yao PRL 131, 160402 (2023)

TS, Yao forthcoming (2023)

Thomas Schuster  
IPAM, November 2023

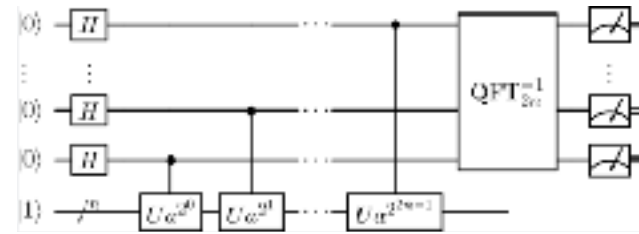
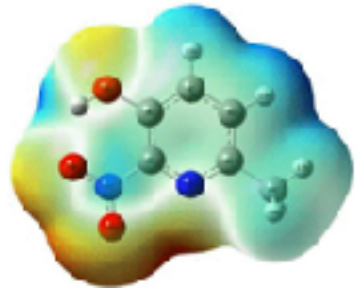
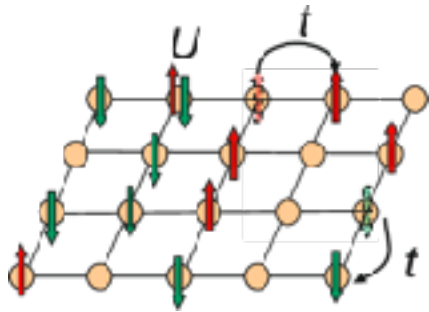
# Many-body quantum science

Coherent large-scale quantum systems may achieve large advantages in computation, physics, chemistry, sensing, cryptography, ...

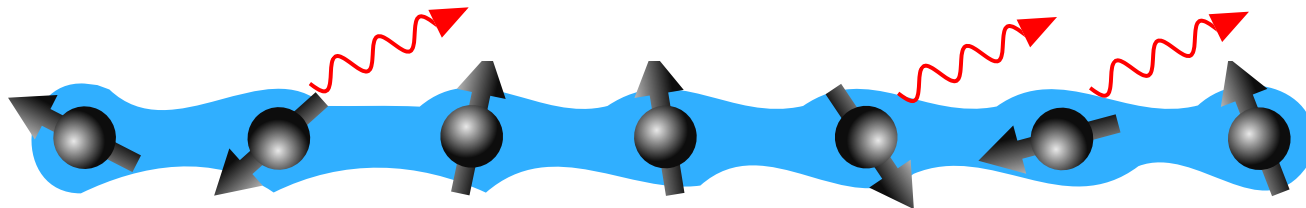


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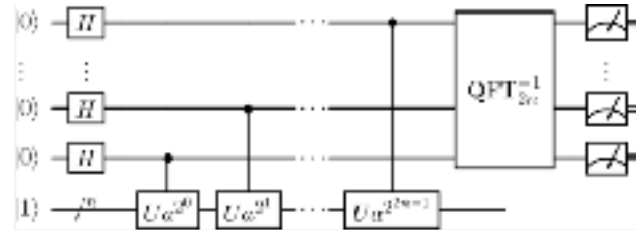
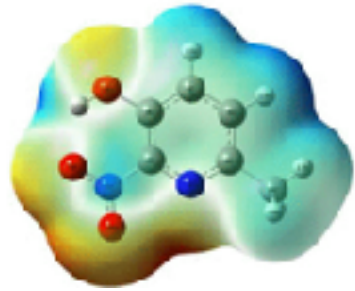
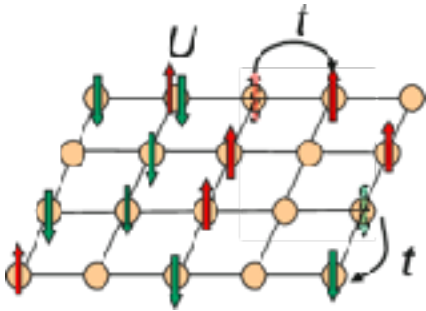


However, near-term quantum devices are impacted by noise

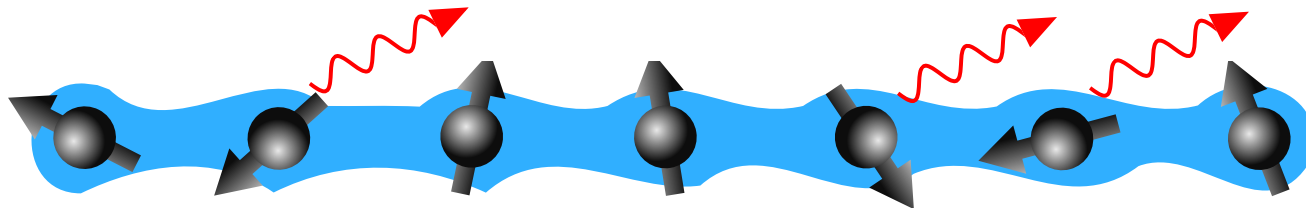


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Quantum error correction can address this, but requires additional capabilities/overhead

# Noisy many-body quantum science

What can we do with noisy quantum devices?

# Noisy many-body quantum science

What can we do with noisy quantum devices?

Physics experiments

$$|\psi_{QSL}\rangle = \left| \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right\rangle + \left| \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right\rangle + \dots$$

Harvard/MIT (2022)

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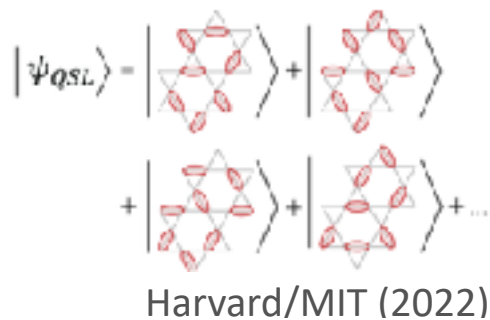
## NISQ algorithms



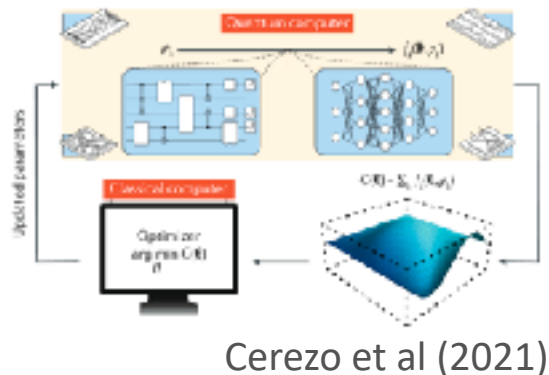
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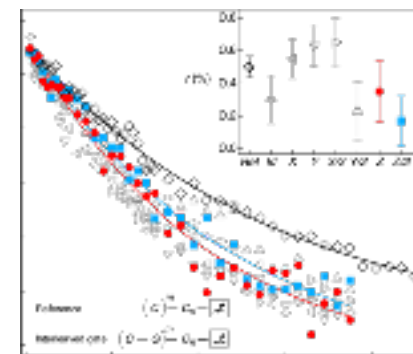
## Physics experiments



## NISQ algorithms



## Benchmarking noise



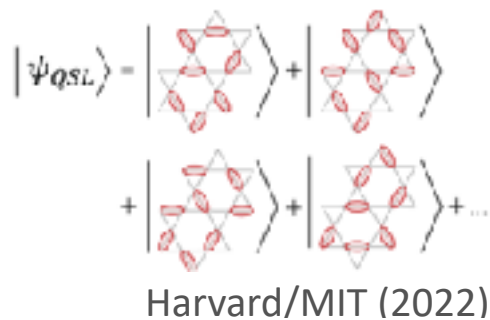
Marcus group (2015)



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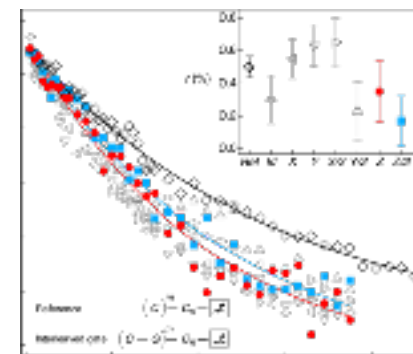
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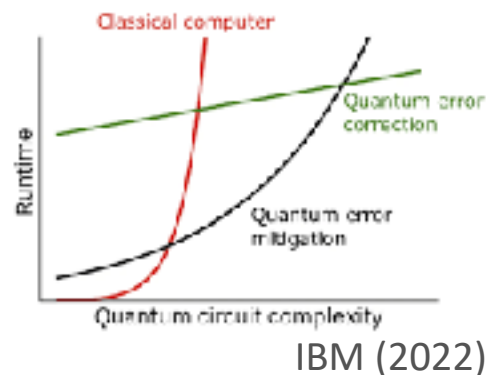
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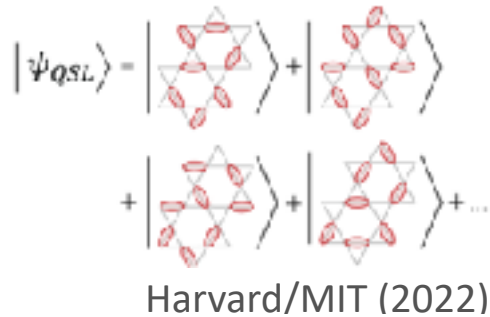
## Error mitigation



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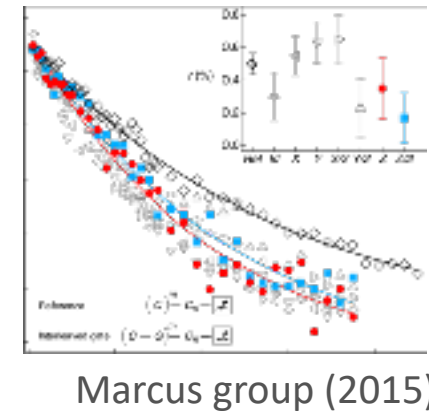
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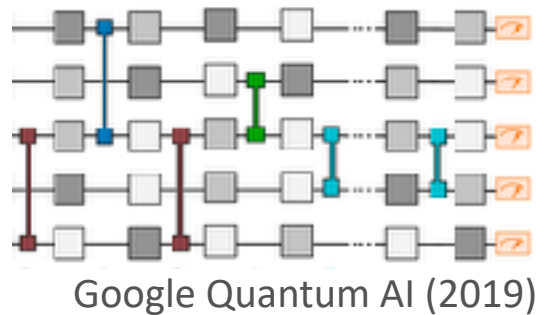
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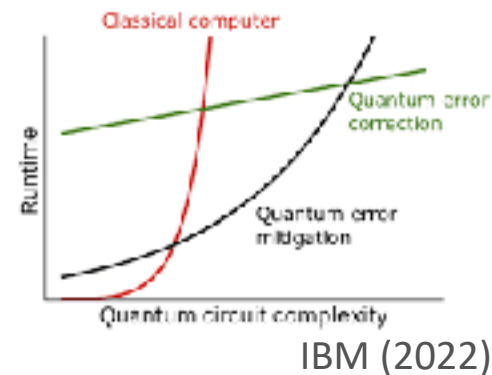
Benchmarking noise



Complexity



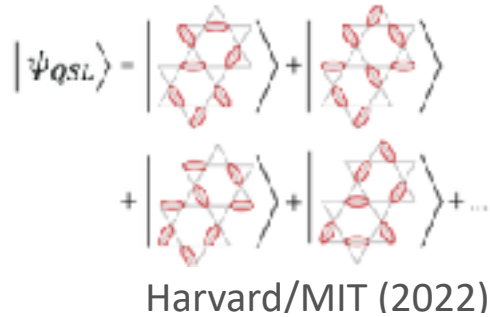
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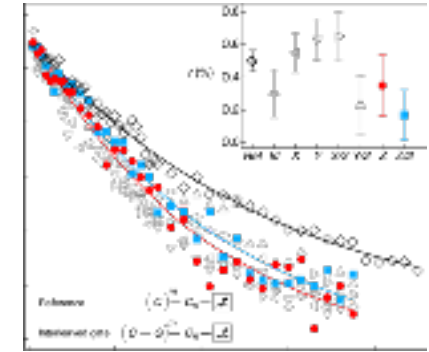
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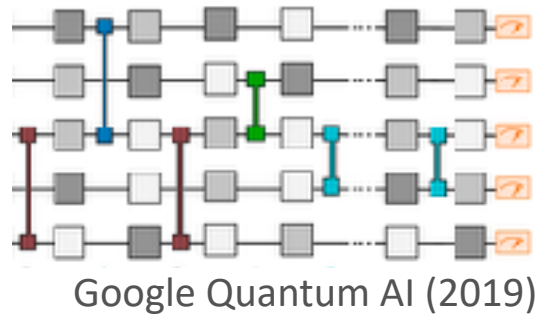
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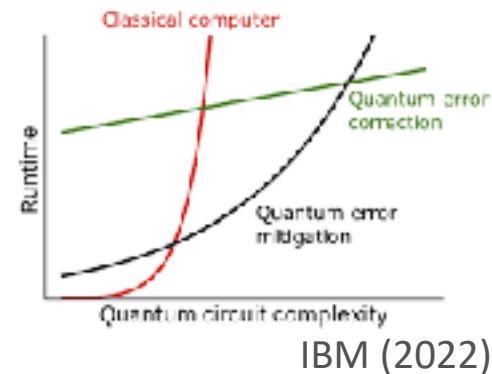
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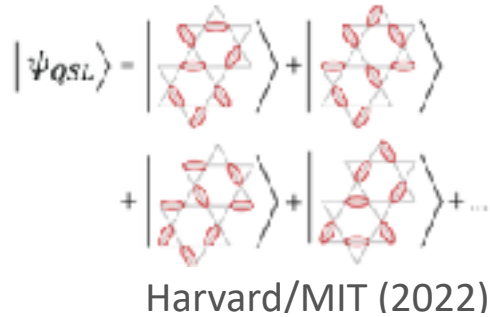
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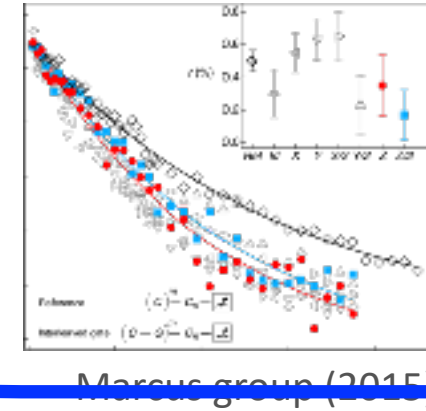
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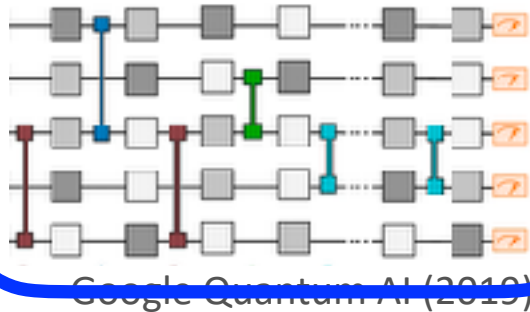
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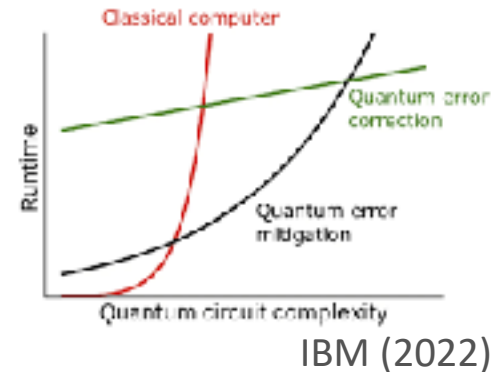
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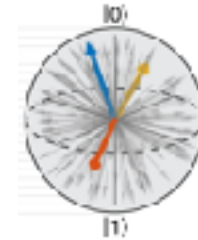
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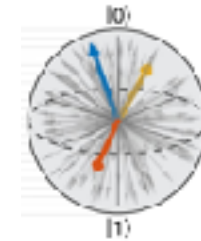


Endres group (2021)

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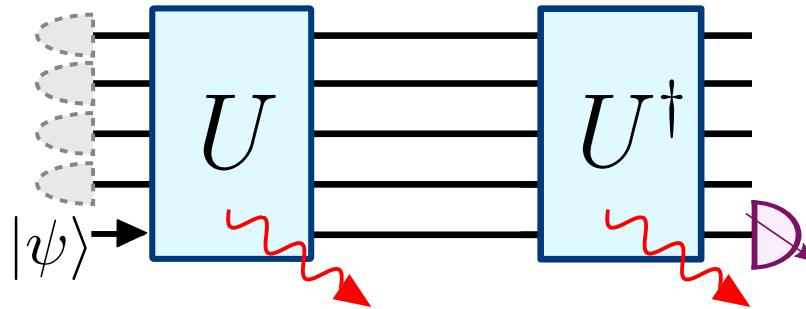
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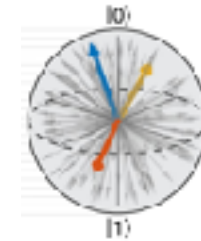
**Loschmidt echo:** thought expt  $\sim$ 1870's, NMR  $\sim$ 1980's, single-particle theory  $\sim$ 2000's, NISQ expts  $\sim$ 2010s Pines, Pastawski, Peres, Jalabert, Zurek, Cappellaro, Sanchez,...



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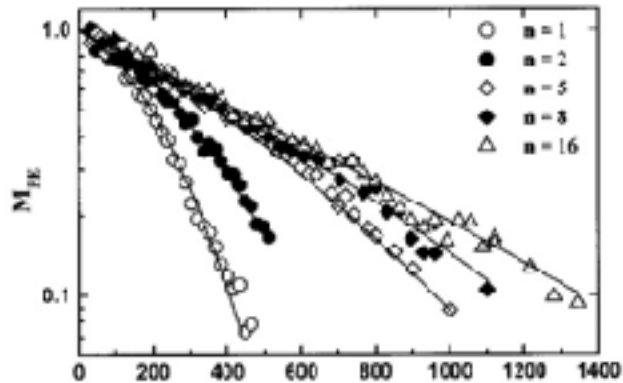
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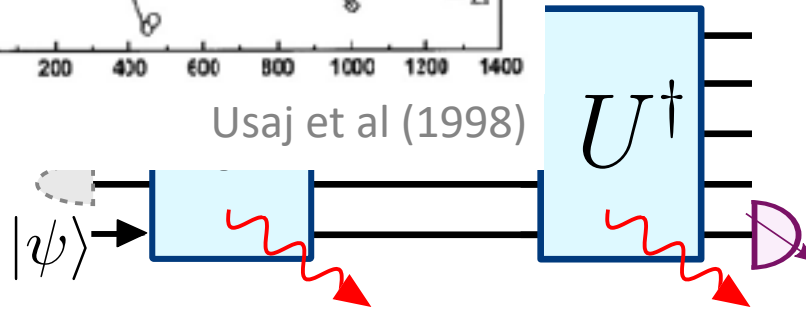
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Usaj et al (1998)

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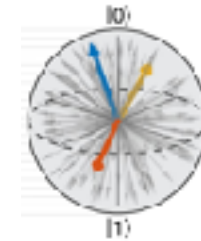




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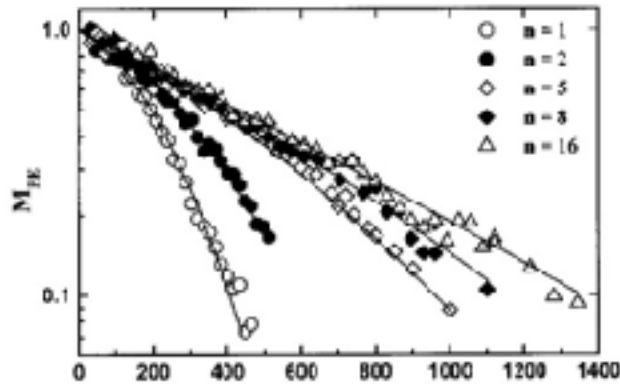
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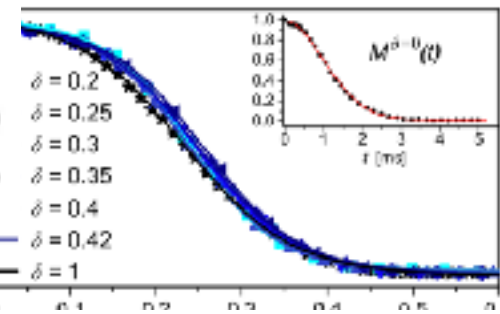
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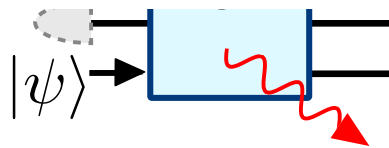


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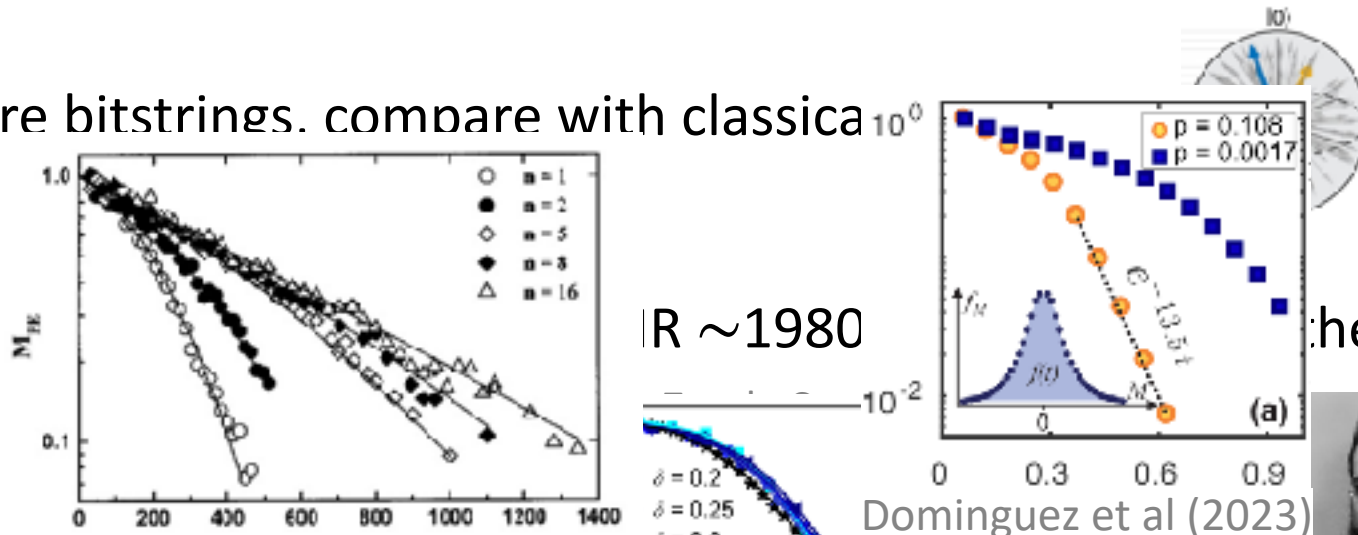


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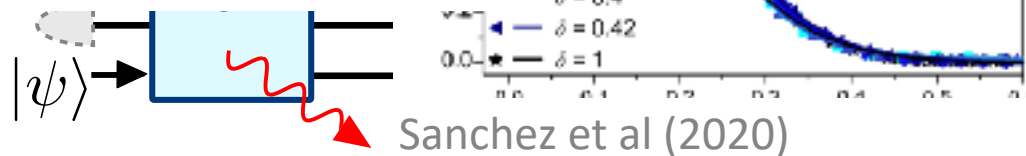


Endres group (2021)

R ~1980

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Dominguez et al (2023)



Sanchez et al (2020)

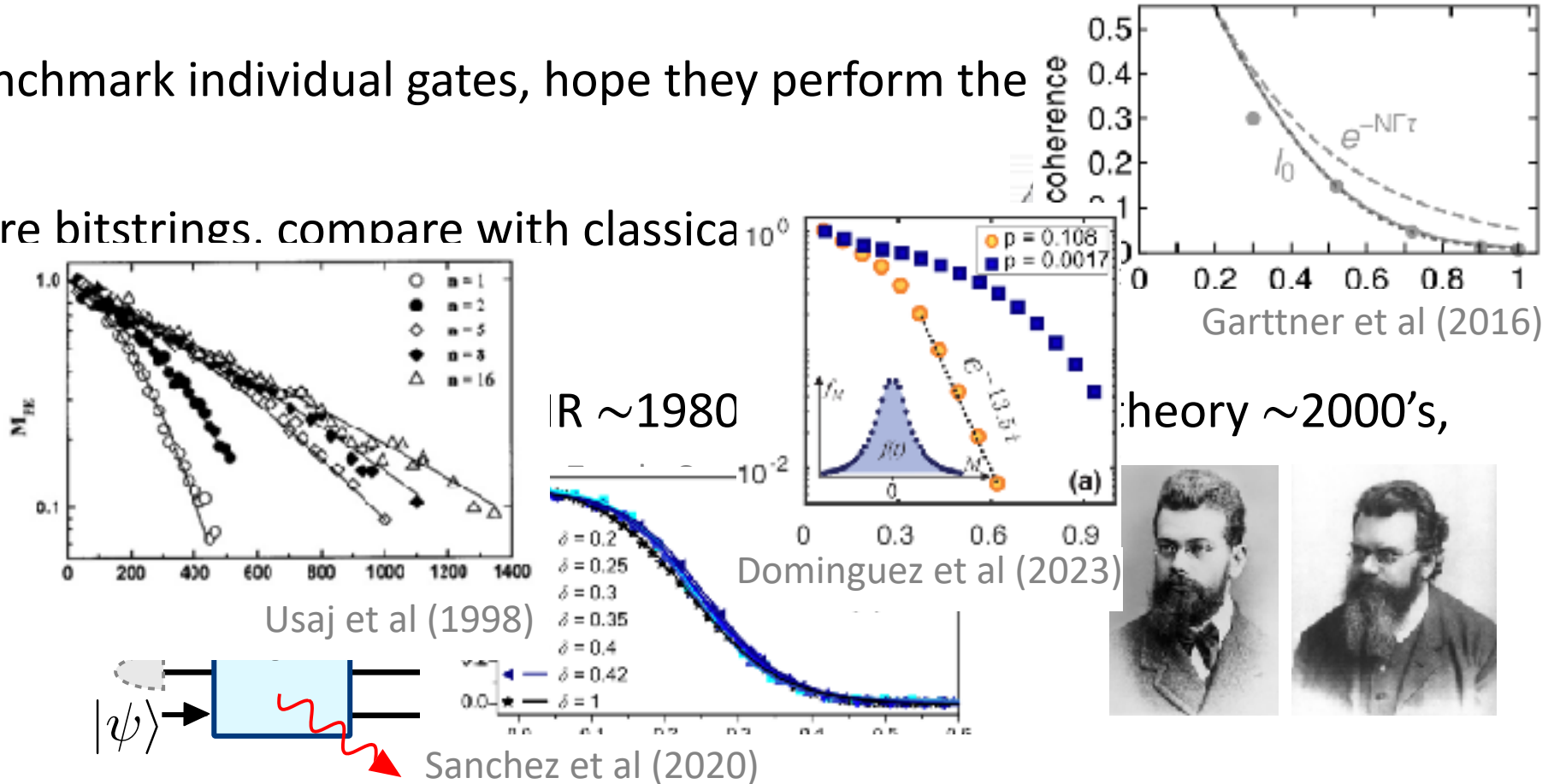


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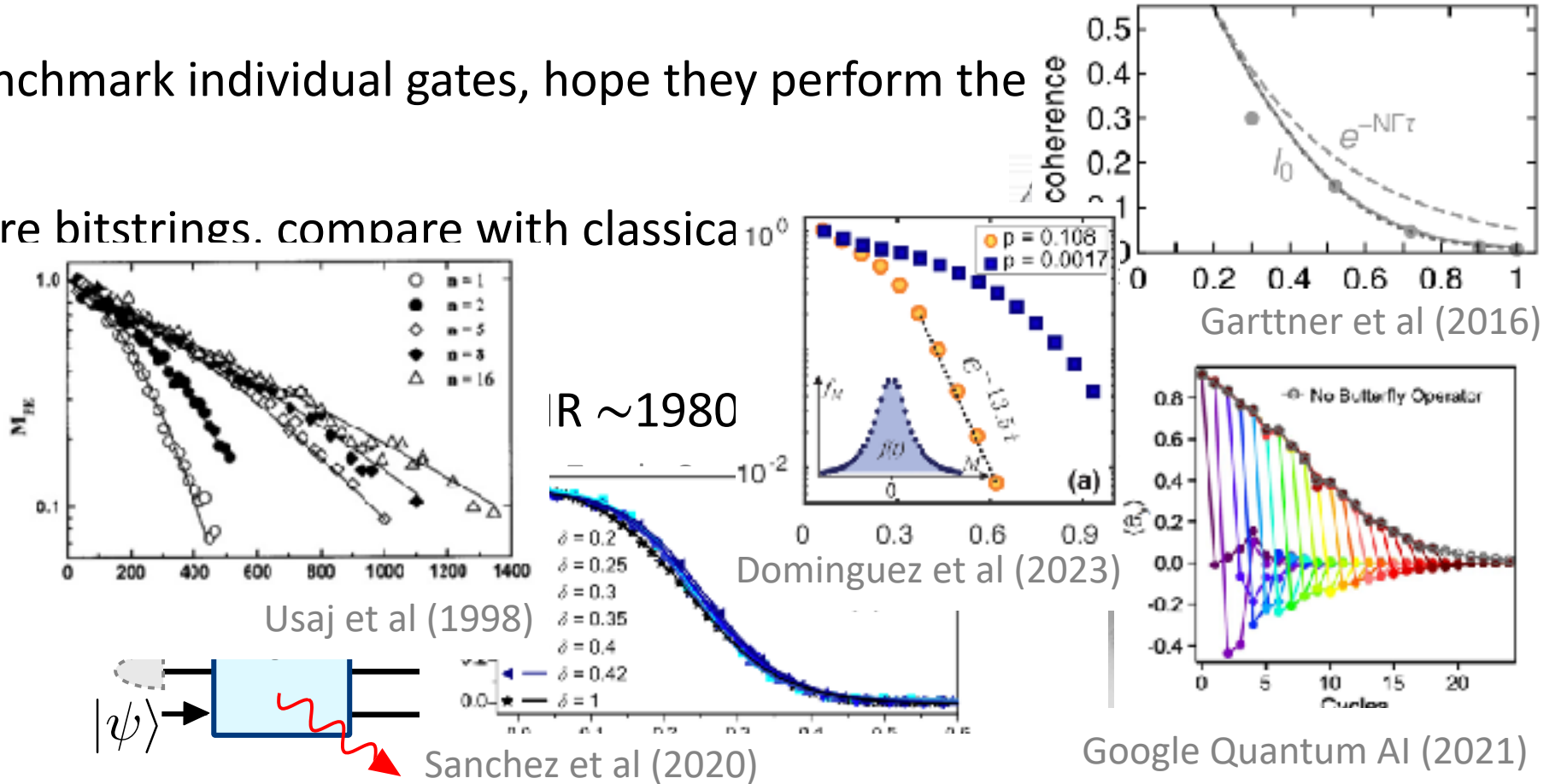
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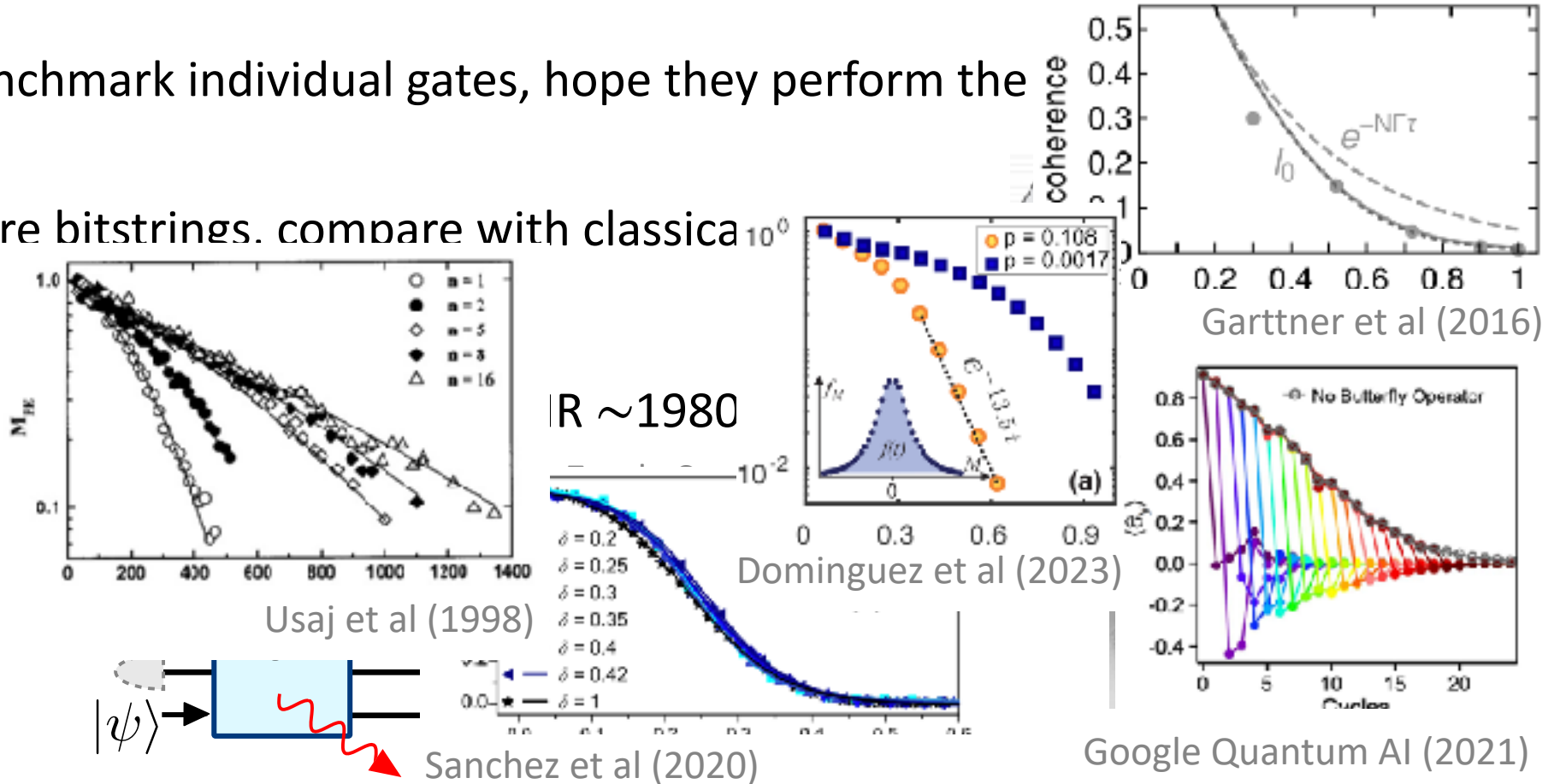


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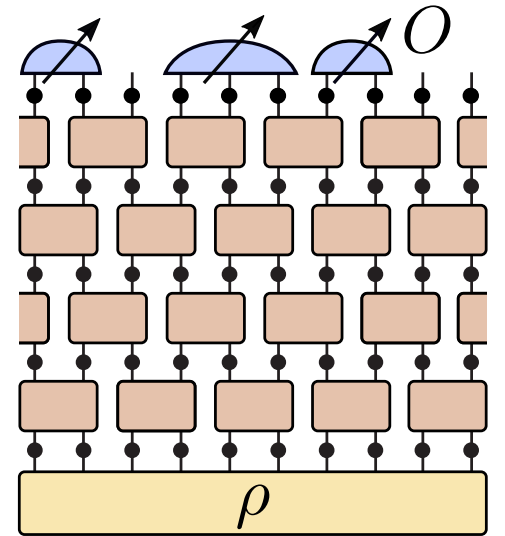
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Experiments show the Loschmidt echo decay *depends on U itself*

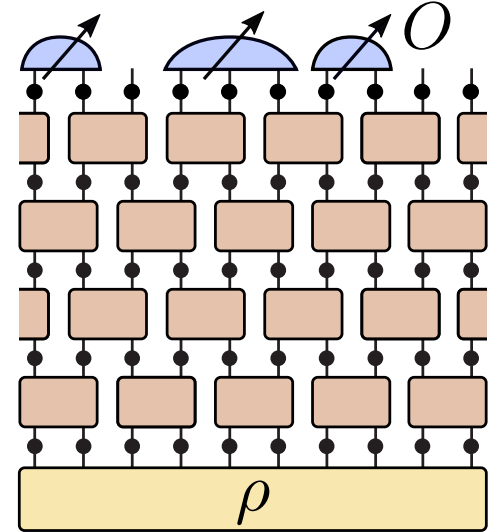
How can we understand this in many-body quantum systems?

# How does noise impact complexity?



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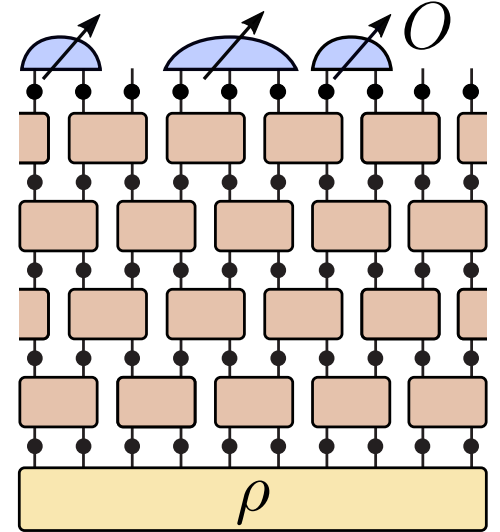
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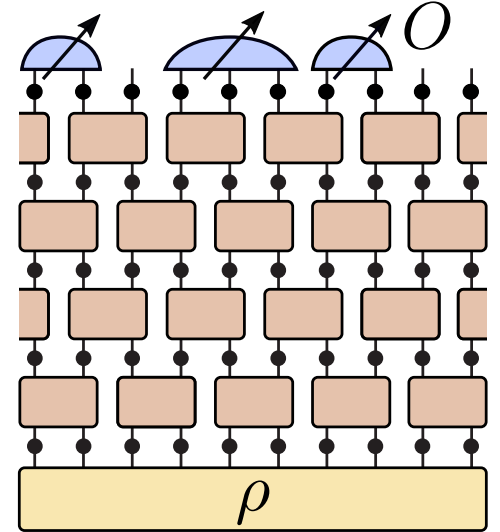
Aharonov, Ben-Or, Impagliazzo, Nisan (1996)





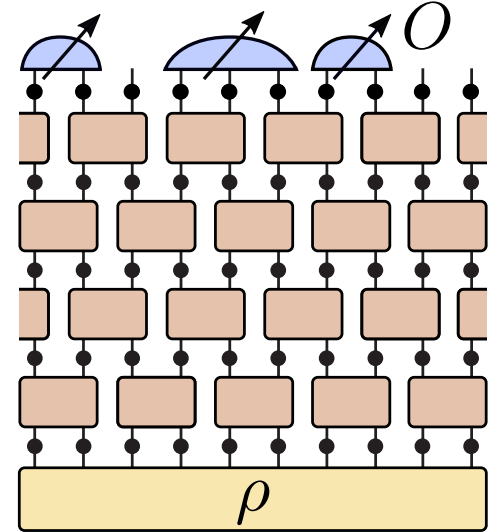
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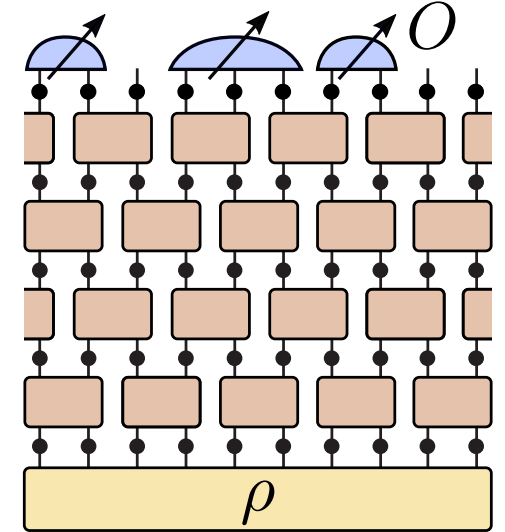
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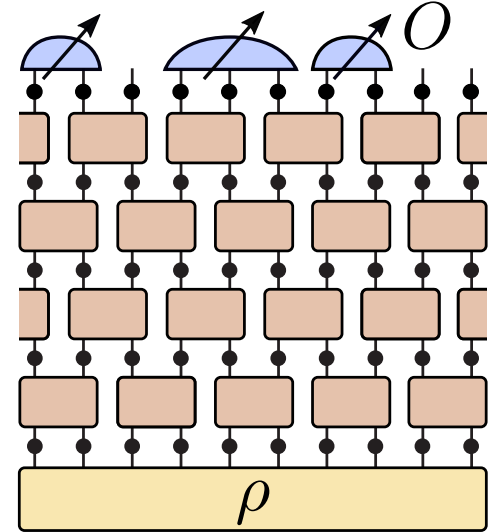


How does noise impact the circuits *that we are actually performing?*

- Recent results show that **random noisy circuits** are classically simulable (in scaling with  $n$ )  
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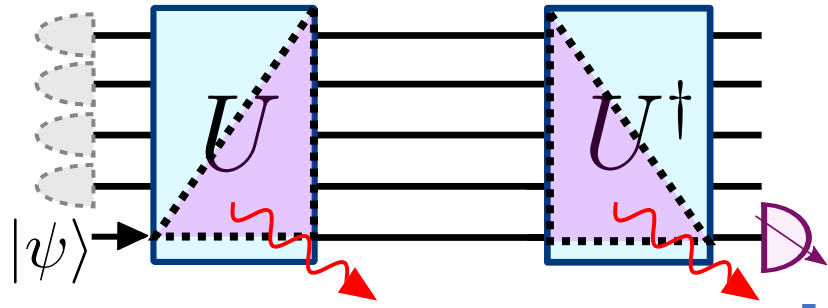
Does this extend to more general classes of quantum circuits?

**This talk:**

**Noise**  $\Leftrightarrow$  **Information dynamics**  $\Leftrightarrow$  **Complexity**

# Operator growth in open quantum systems

TS, Yao PRL 131, 160402 (2023)



**Noise**



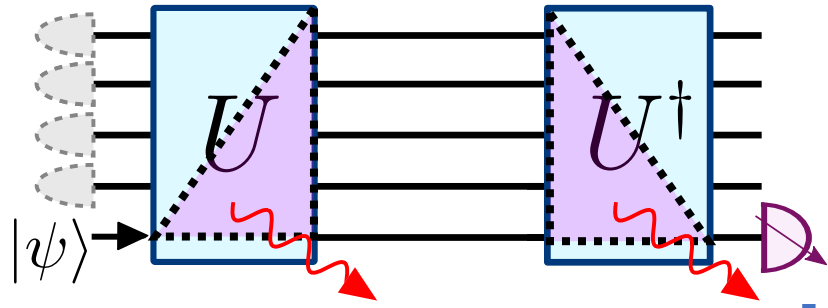
**Information dynamics**



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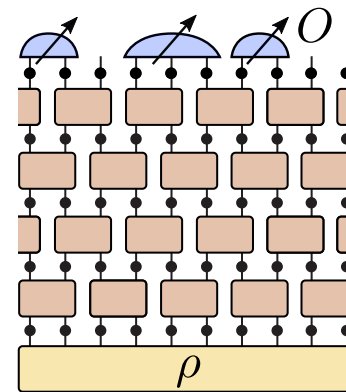
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**Information dynamics**



**Complexity**

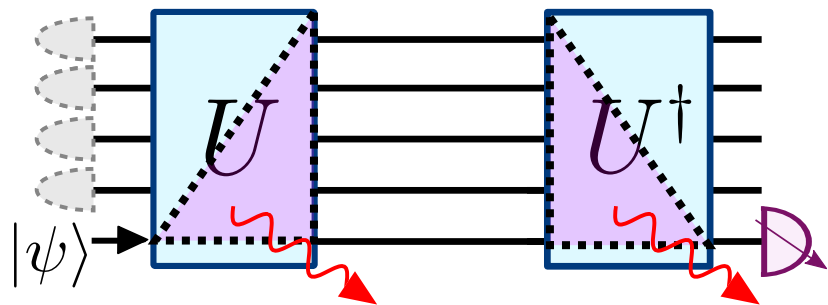


TS, Yao forthcoming (2023)

A polynomial-time classical algorithm for almost any noisy quantum circuit

# Operator growth in open quantum systems

TS, Yao PRL 131, 160402 (2023)



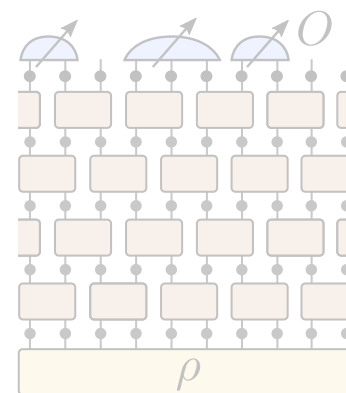
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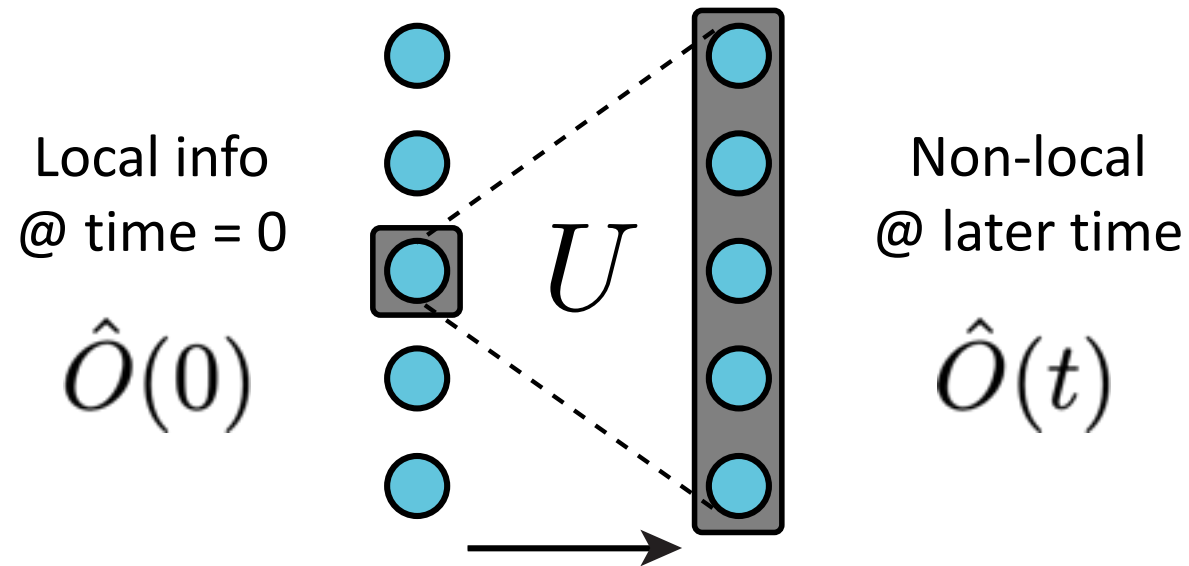
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# Quantum information dynamics

**Operator growth:** How does local information evolve in time?

e.g.  $U = e^{-iHt}$



Linked to recent developments in quantum gravity, quantum chaos, quantum state transfer, tensor network algorithms, quantum thermalization...

# Zooming in... **Operator size distributions**

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Define the **size** (a.k.a. “weight”) of a Pauli string = its number of non-identity elements

$$\hat{R} = \boxed{1} \otimes \boxed{1} \otimes \boxed{X} \otimes \boxed{1} \otimes \boxed{Y} \otimes \boxed{Z} \otimes \boxed{X} \otimes \boxed{X} \otimes \boxed{1} \otimes \boxed{Y} \quad \text{Size} = 6$$

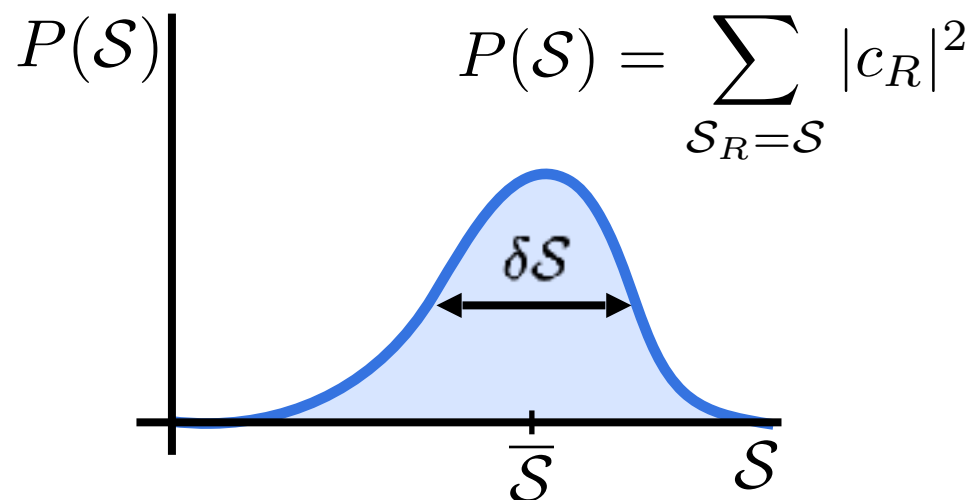
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Time-evolved operators have a **size distribution**:

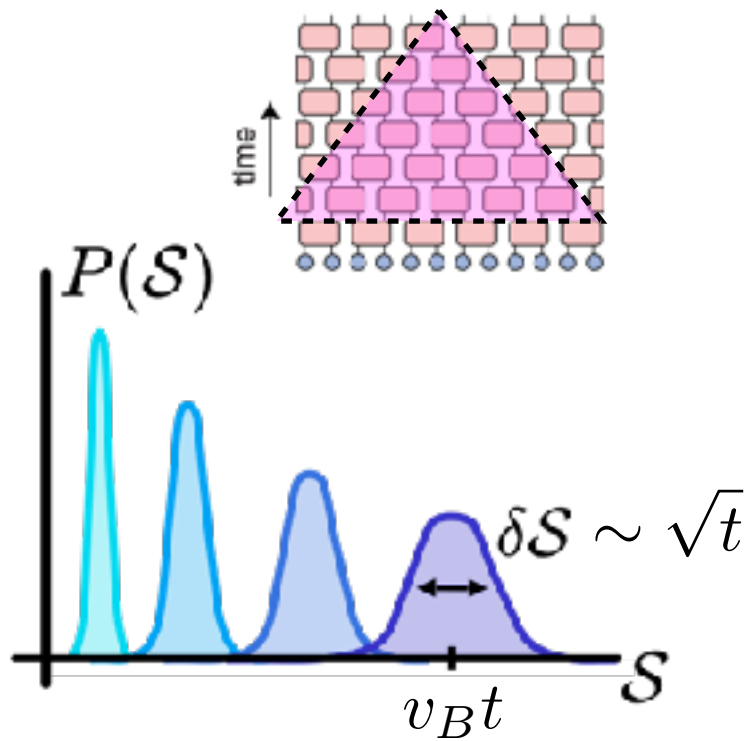
$$\hat{Q}(t) = \sum_R c_R \hat{R}$$



# Universal classes of quantum information dynamics

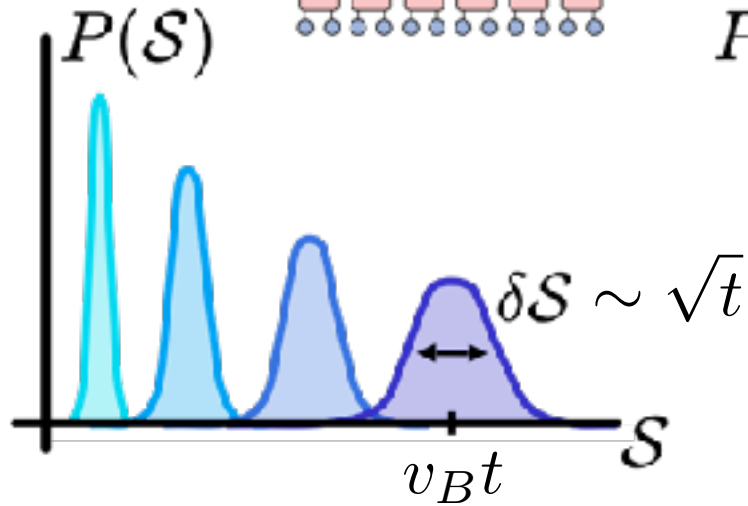
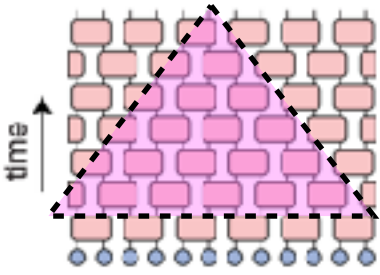
# Universal classes of quantum information dynamics

## 1D circuits

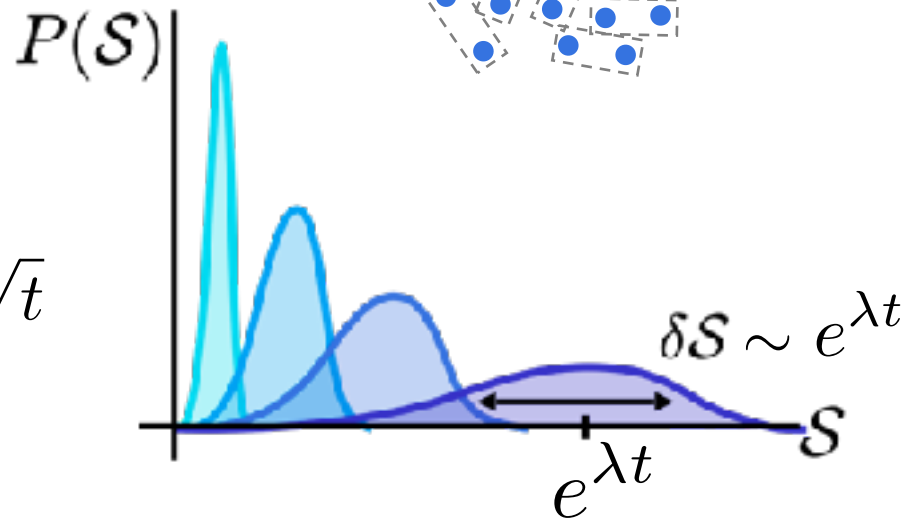
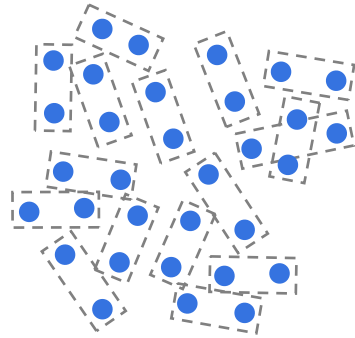


# Universal classes of quantum information dynamics

1D circuits

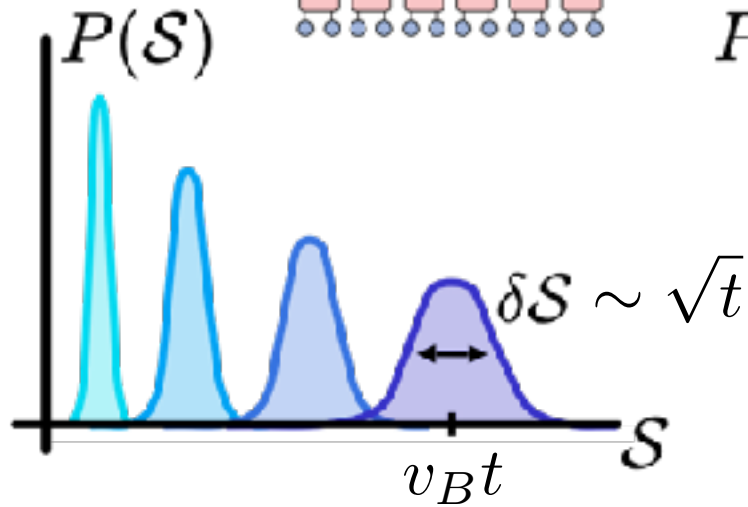
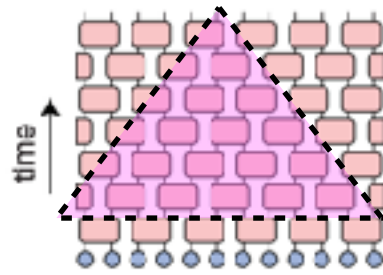


All-to-all systems

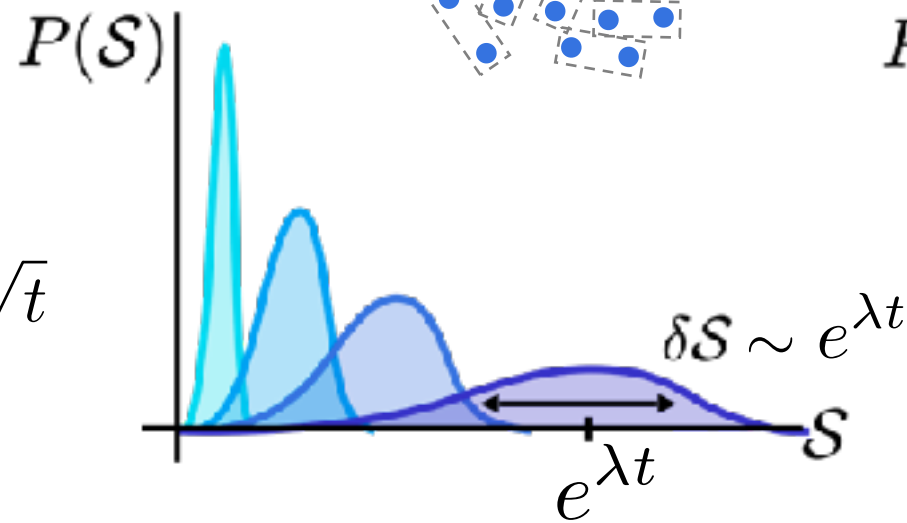
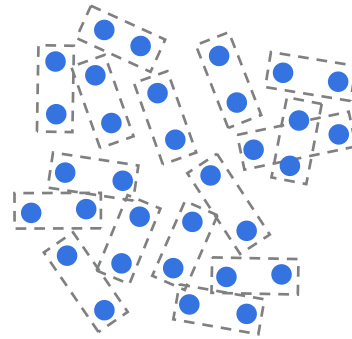


# Universal classes of quantum information dynamics

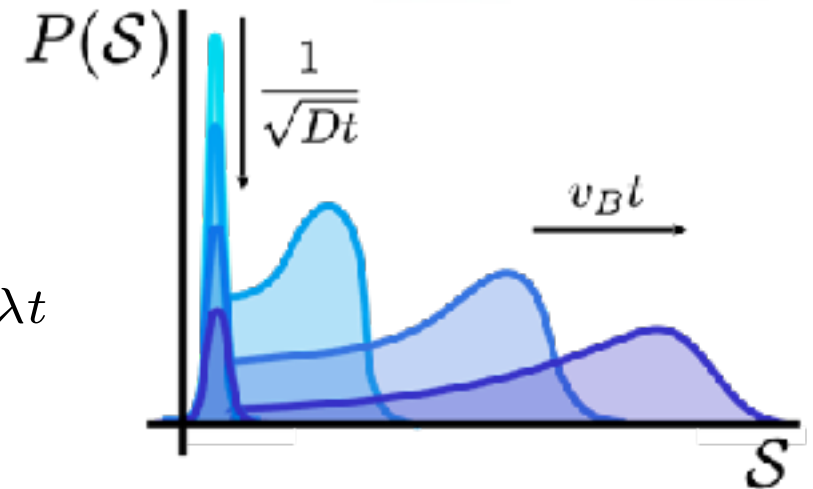
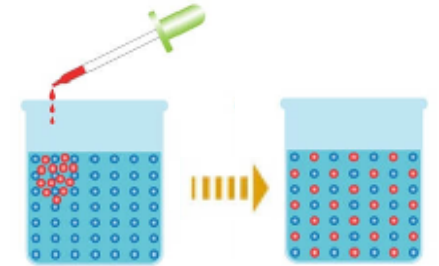
1D circuits



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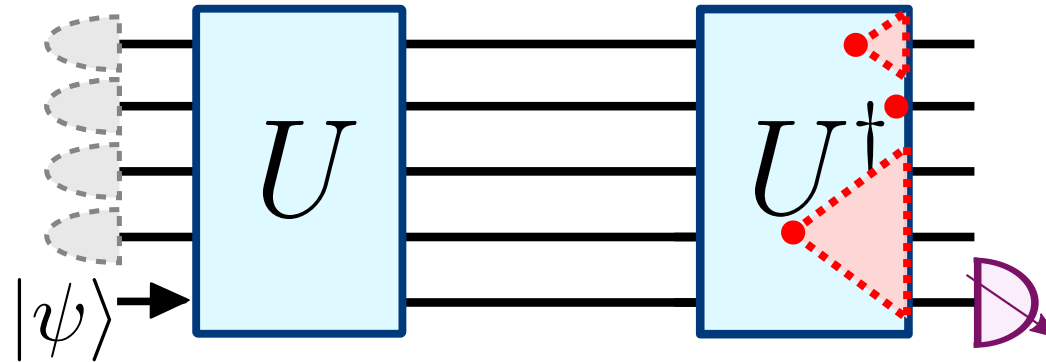
Conserved quantity





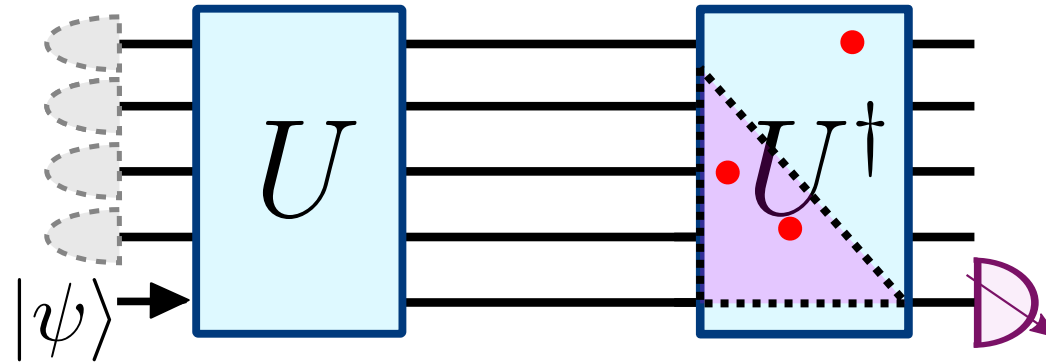
# How does noise propagate in many-body quantum systems?

**Intuition:** noise propagates according to the circuit's operator growth dynamics



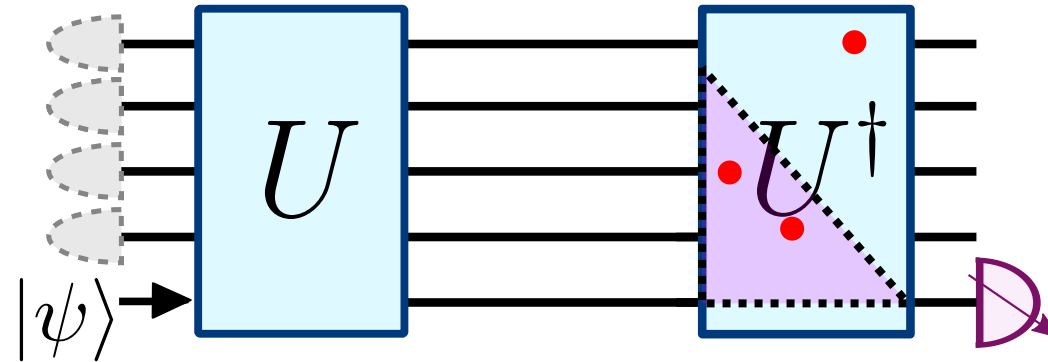
# How does noise propagate in many-body quantum systems?

**Intuition:** highly non-local operators  
are more sensitive to noise



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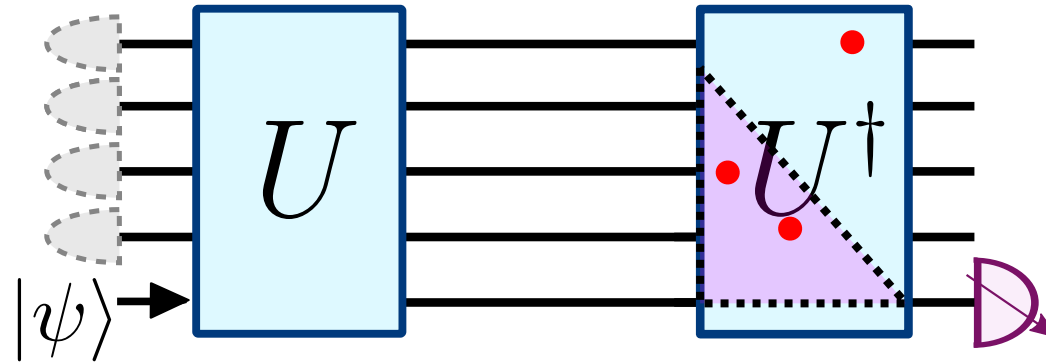
**Intuition:** highly non-local operators  
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$$\hat{M}_t = \underbrace{X}_{\text{local}} + \underbrace{Y \otimes \mathbb{1} \otimes Z}_{\text{2-body}} + \dots + \underbrace{Y \otimes Z \otimes \mathbb{1} \otimes X \otimes Z}_{\text{4-body}}$$

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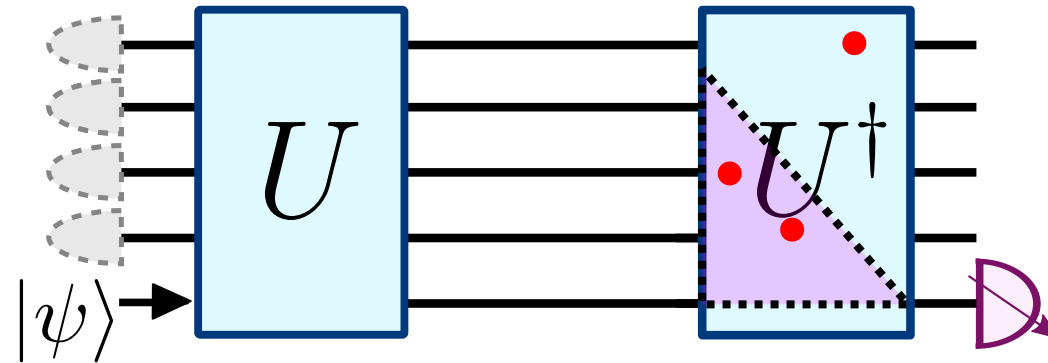


$$\hat{M}_t = X + Y \otimes \mathbb{1} \otimes Z + \dots + Y \otimes Z \otimes \mathbb{1} \otimes X \otimes Z$$

Red brackets and arrows are drawn under the equation to highlight the non-local nature of the terms. The first term  $X$  is bracketed and has an arrow pointing down. The second term  $Y \otimes \mathbb{1} \otimes Z$  is bracketed and has an arrow pointing down. The last term  $Y \otimes Z \otimes \mathbb{1} \otimes X \otimes Z$  is bracketed and has an arrow pointing down. The arrows indicate that these terms involve multiple qubits, illustrating how noise can propagate across the system.

# How does noise propagate in many-body quantum systems?

**More precise:** Noise decays Pauli strings at a rate proportional to their size  $\sim \gamma \mathcal{S}$



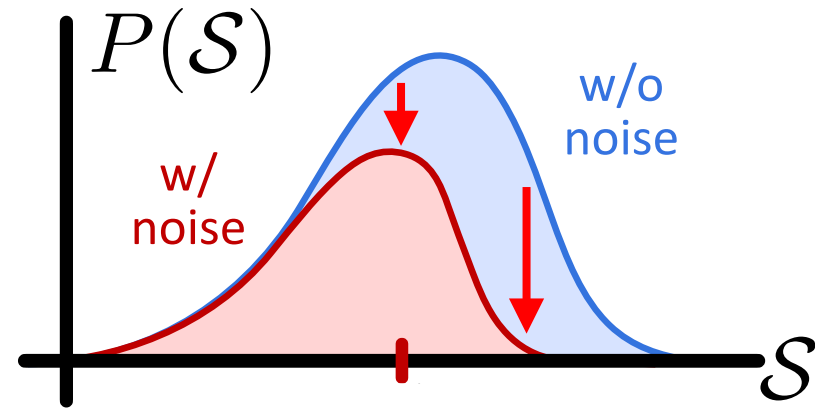
$$\hat{M}_t = X + Y \otimes \mathbb{1} \otimes Z + \dots + Y \otimes Z \otimes \mathbb{1} \otimes X \otimes Z$$

Red brackets and wavy arrows are drawn under the equation to indicate the size of the Pauli strings. The first term  $X$  has a bracket of length 1. The second term  $Y \otimes \mathbb{1} \otimes Z$  has a bracket of length 3. The final term  $Y \otimes Z \otimes \mathbb{1} \otimes X \otimes Z$  has a bracket of length 5. Wavy red arrows point downwards from each bracket.

(exact for single-qubit decoherence;  
otherwise, approximation for *high-size components* under *ergodic dynamics*)

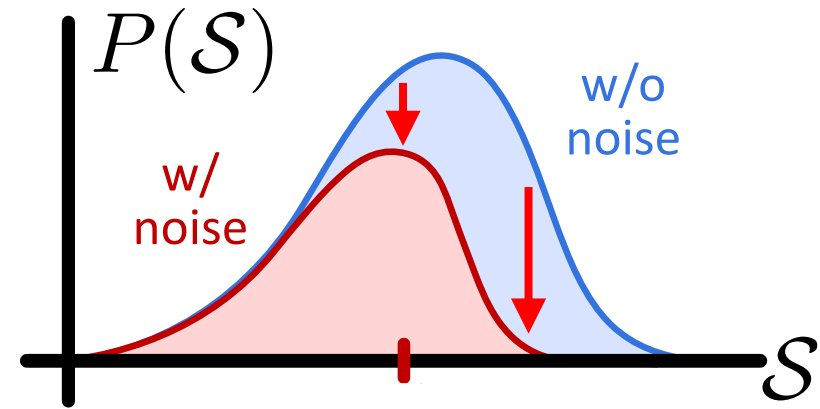
# Noise $\sim$ Size

Two effects of noise on size distribution:



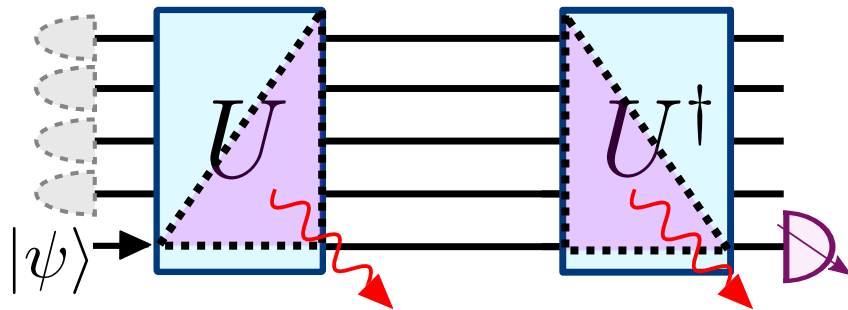
# Noise $\sim$ Size

Two effects of noise on size distribution:



(1) Decrease normalization

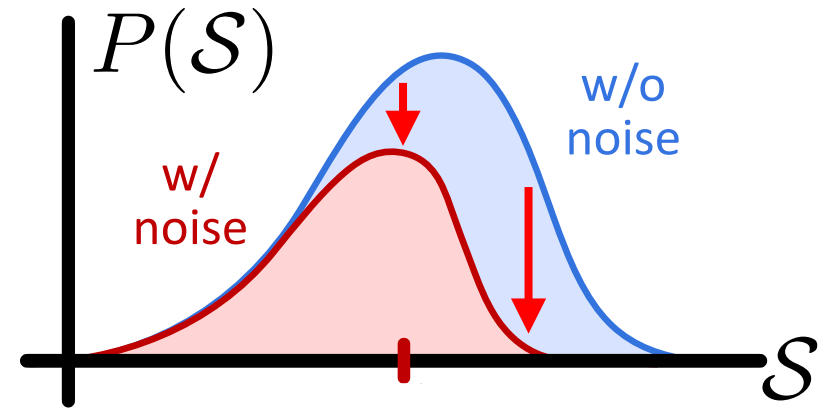
$$\partial_t \mathcal{N} = -\gamma \bar{\mathcal{S}} \mathcal{N}$$



equal to Loschmidt echo!

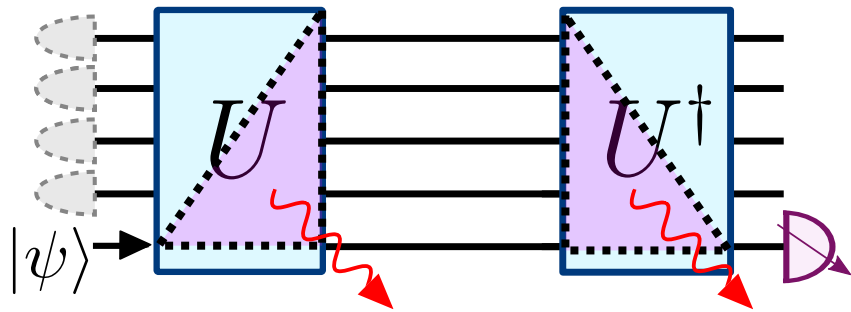
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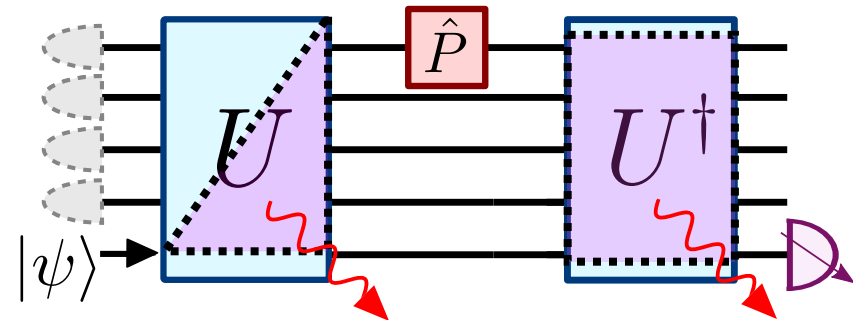
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(2) Shift “shape” towards small sizes

$$\partial_t \bar{\mathcal{S}} = (\text{unitary}) - \gamma \delta \mathcal{S}^2$$



equal to average OTOCs!

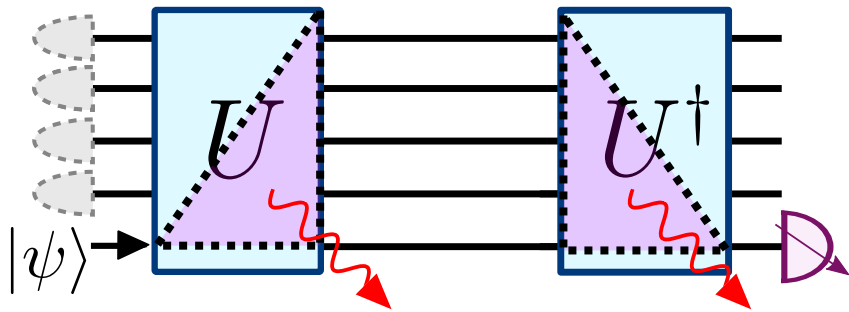


# Noise $\sim$ Size

Interplay between operator growth and noise determines the Loschmidt echo

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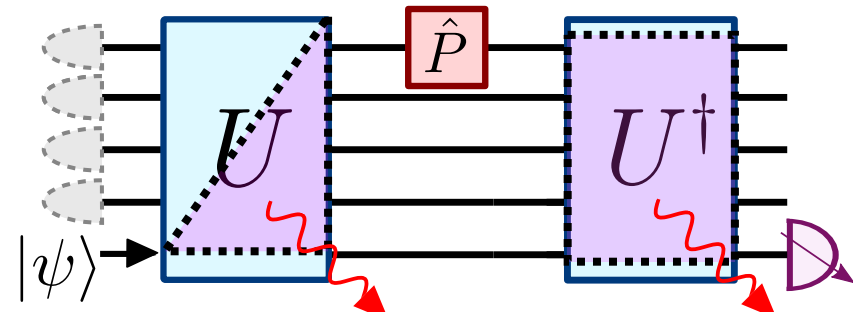
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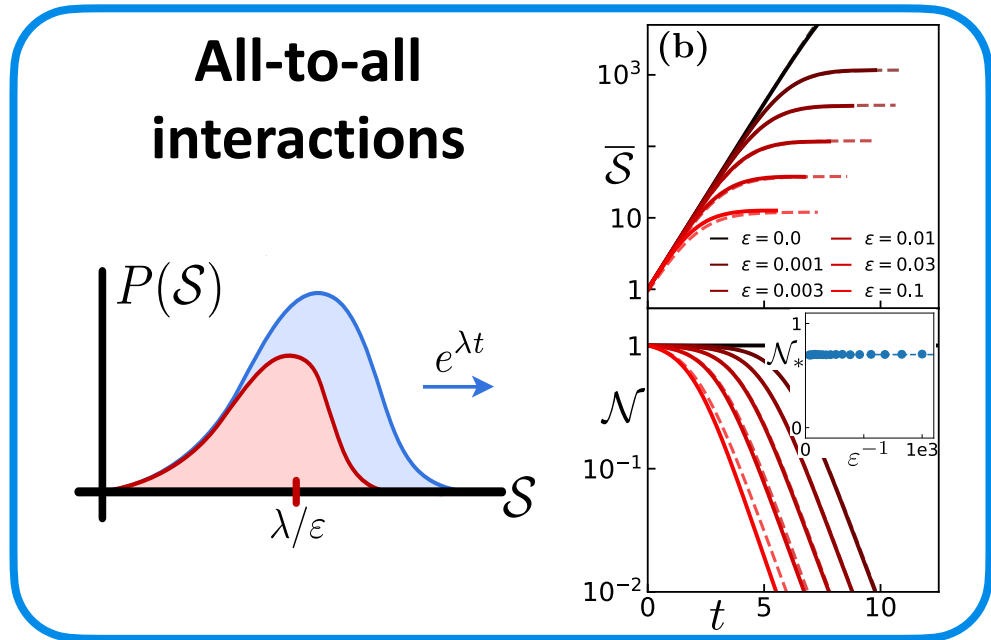
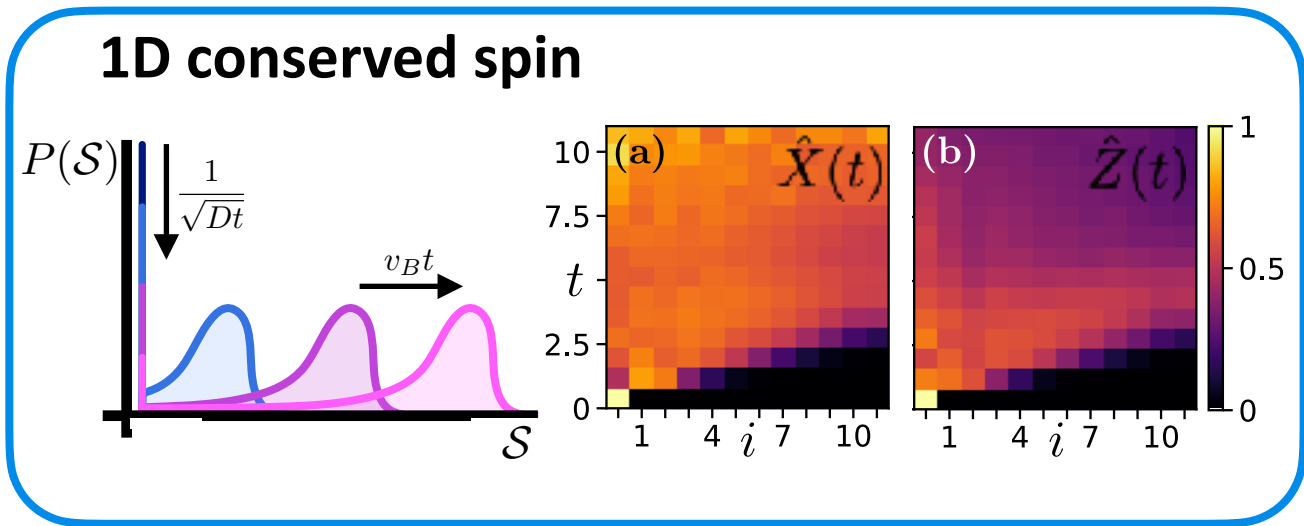
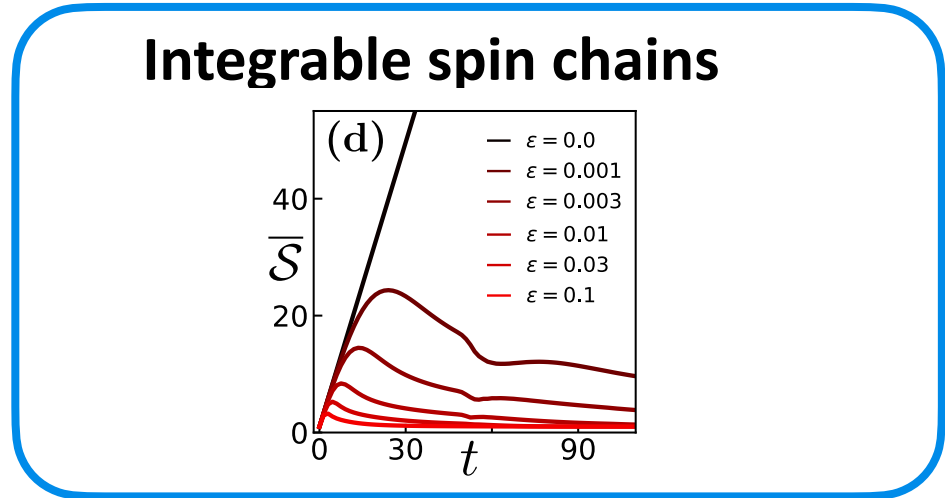
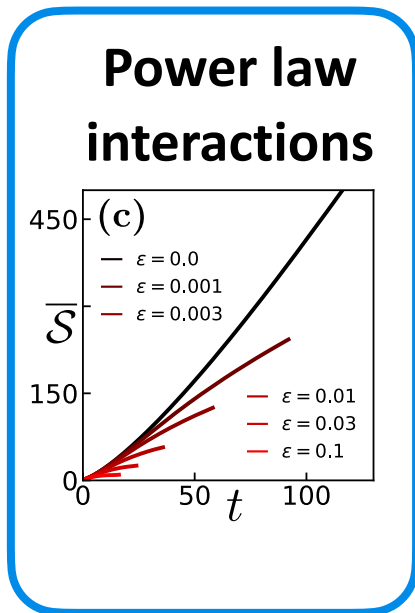
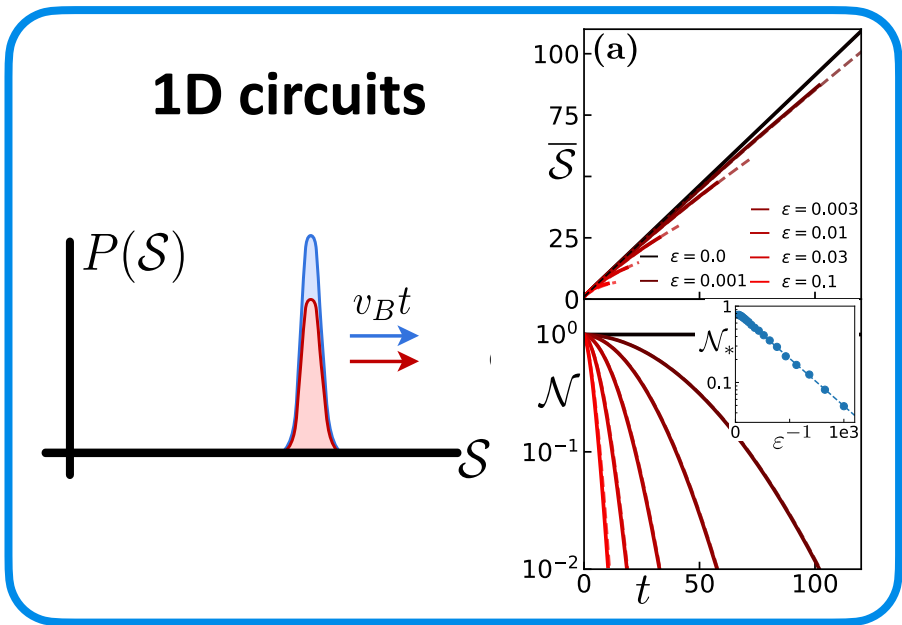
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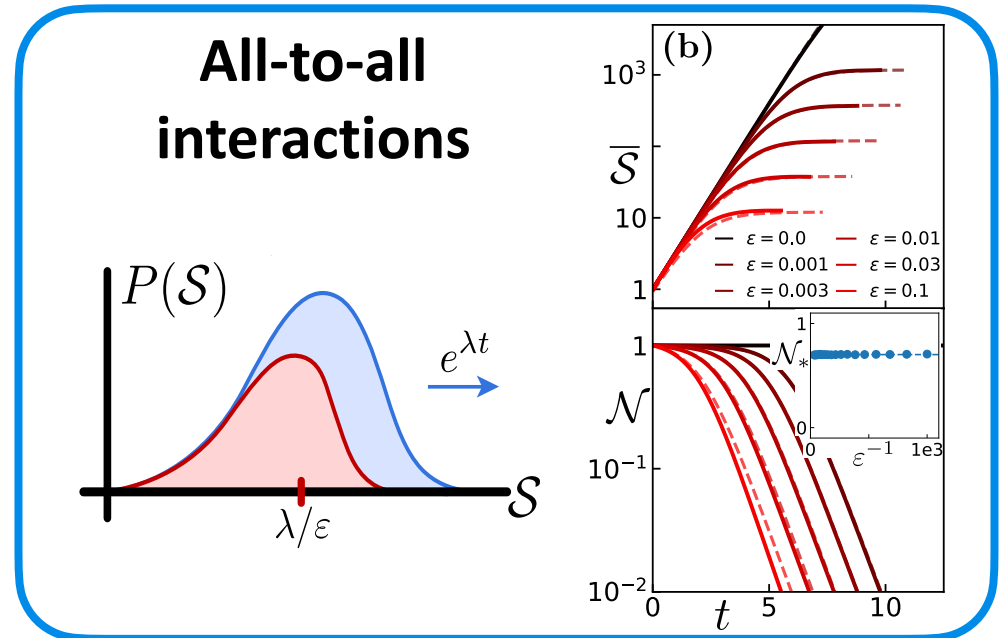
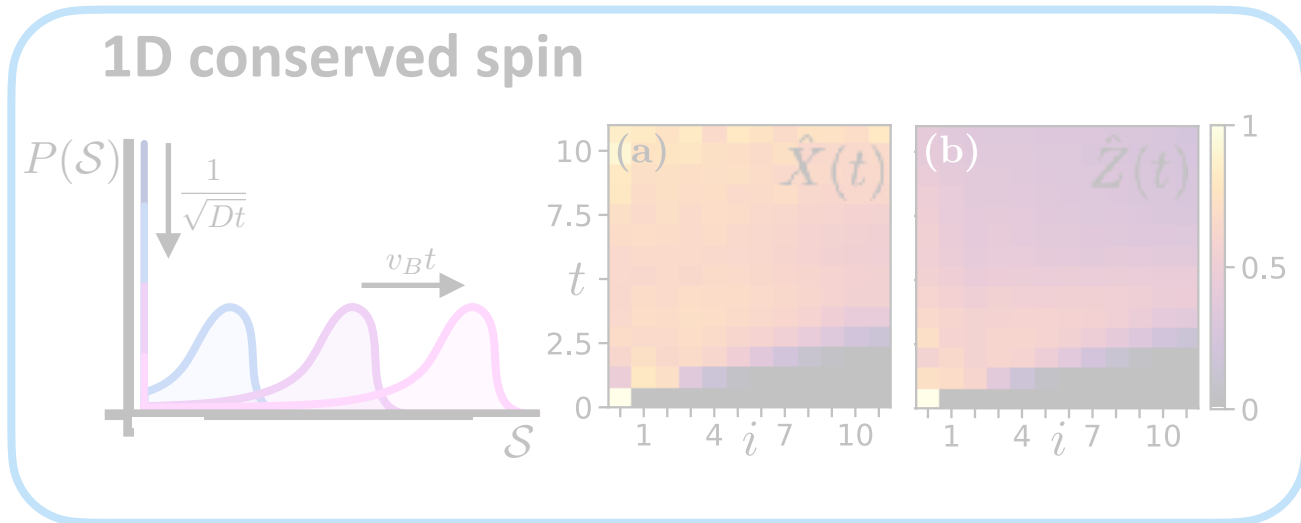
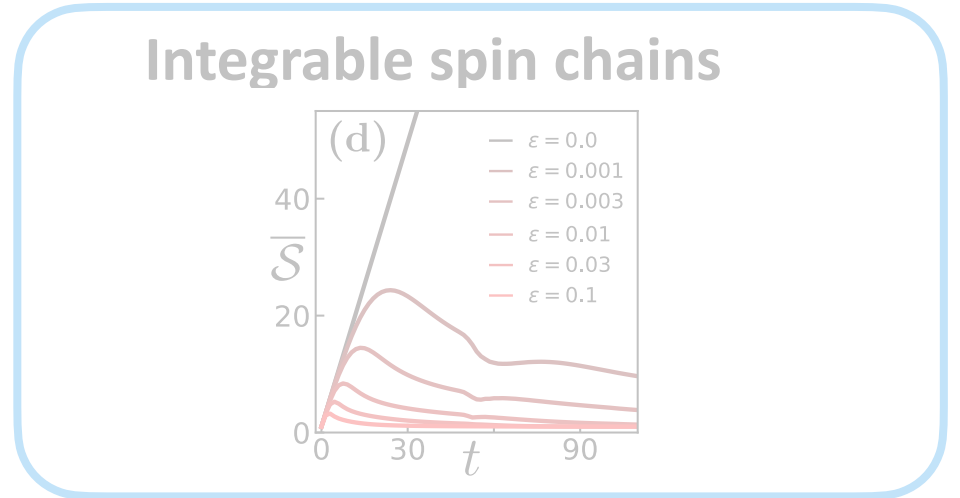
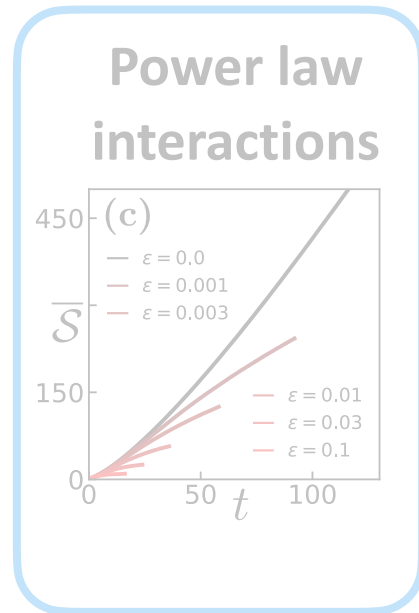
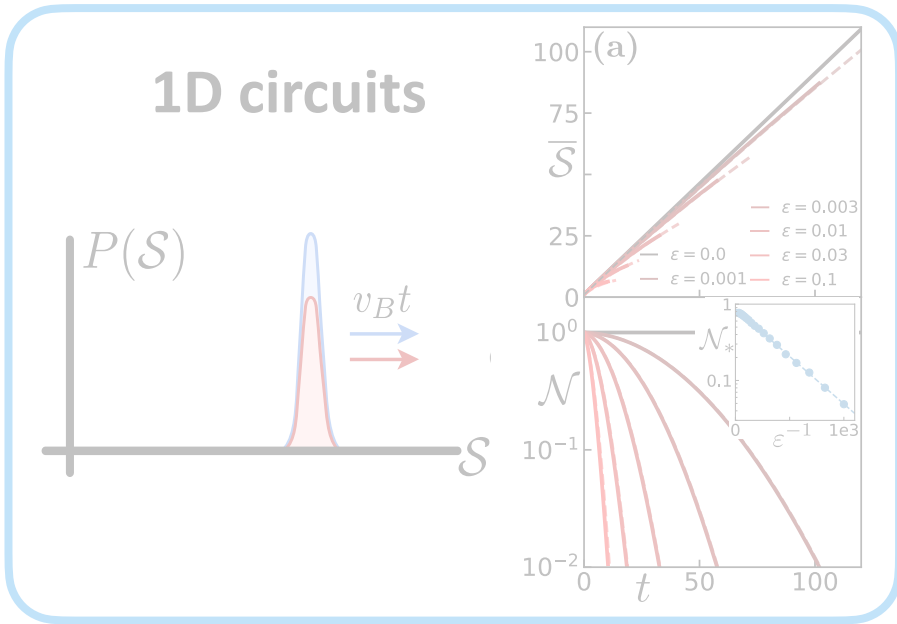


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# Universal classes of **noisy** quantum information dynamics

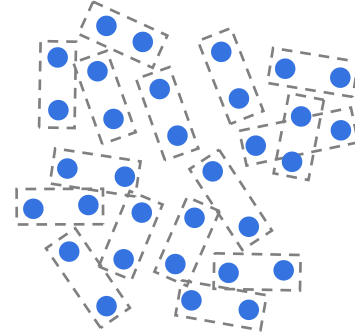


# Universal classes of **noisy** quantum information dynamics



# Example: All-to-all dynamics

**Unitary:** Size grows exponentially in time. Distribution is broad.



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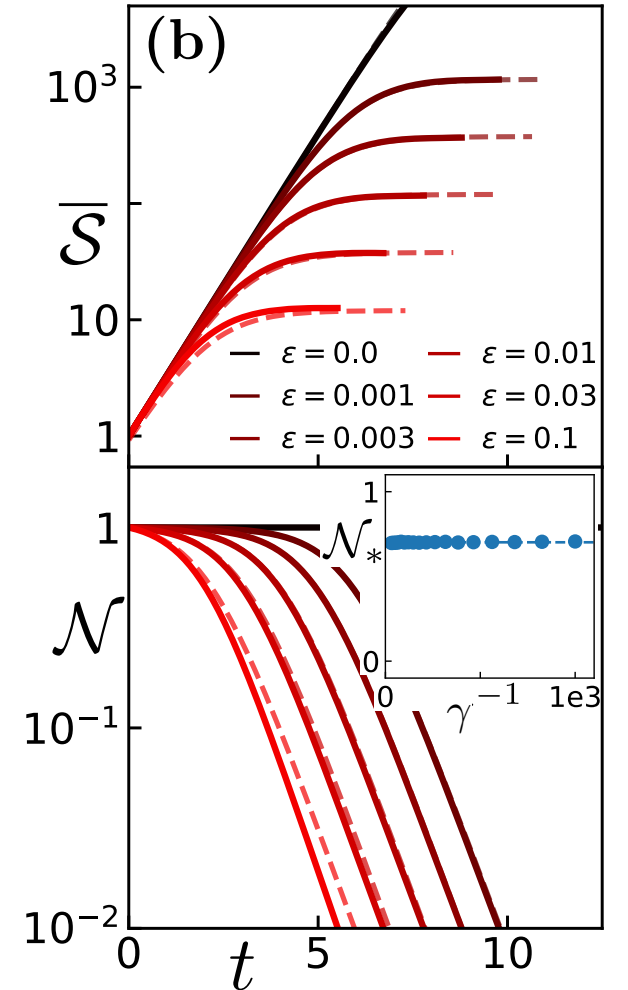
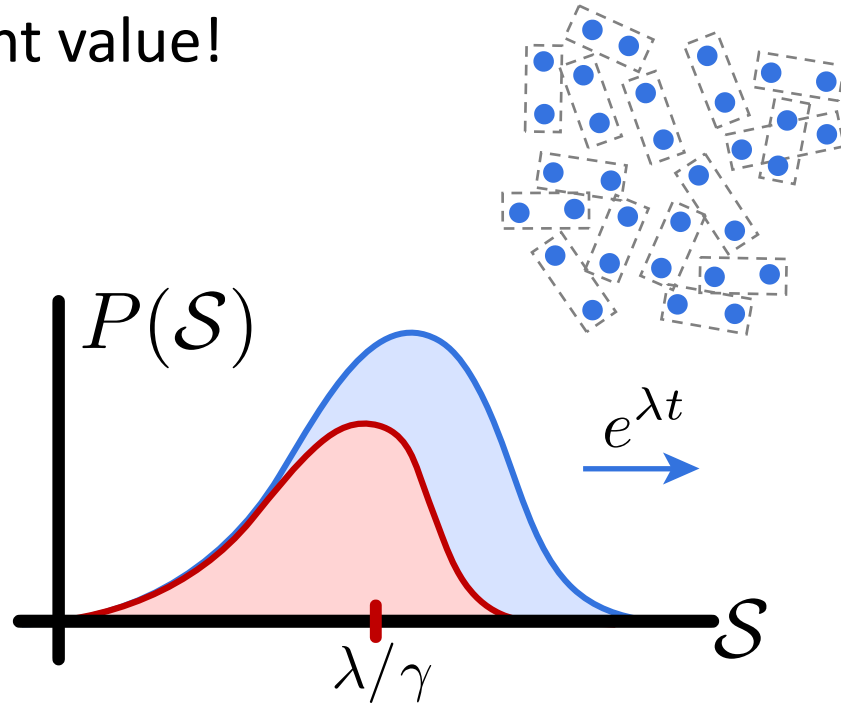
**Unitary:** Size grows exponentially in time. Distribution is broad.

**Noisy:** Size *plateaus* to constant value!

$$\partial_t \bar{\mathcal{S}} \approx \lambda \bar{\mathcal{S}} - \gamma \delta \mathcal{S}^2$$

↓  $\delta \mathcal{S} \sim \mathcal{S}$

$$\mathcal{S}_{\text{sat}} = \frac{\lambda}{\gamma}$$



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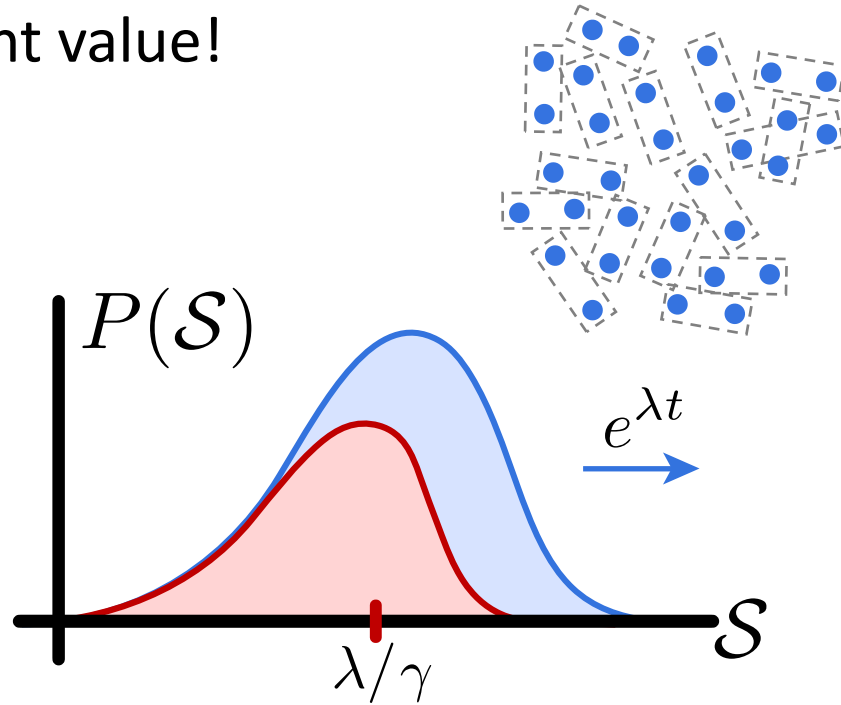
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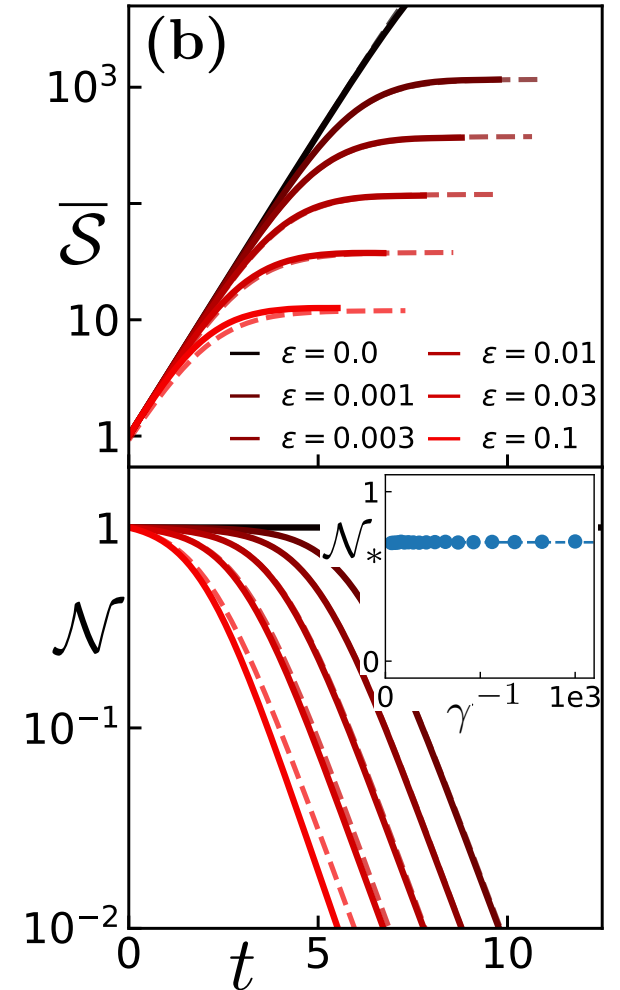
$$\mathcal{S}_{\text{sat}} = \frac{\lambda}{\gamma}$$



⇒ Loschmidt echo decay is *independent* of noise rate!

Echoes seminal results in single-particle quantum chaos

Jalabert, Pastawski (2001)

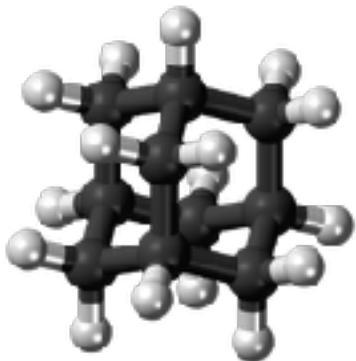
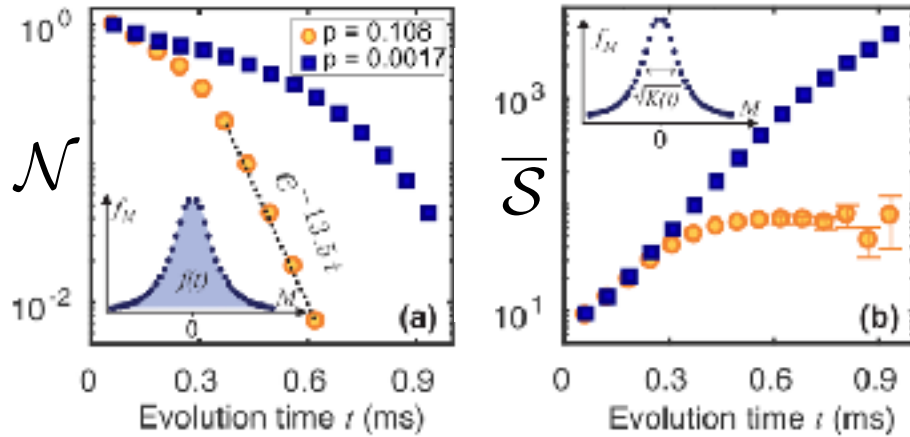


$$\mathcal{N}_t = e^{-\gamma \int dt \mathcal{S}_t}$$

# Recent NMR experiments

NMR expts w/  $\sim 1000$  spins

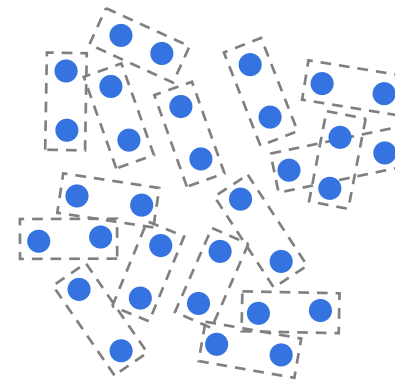
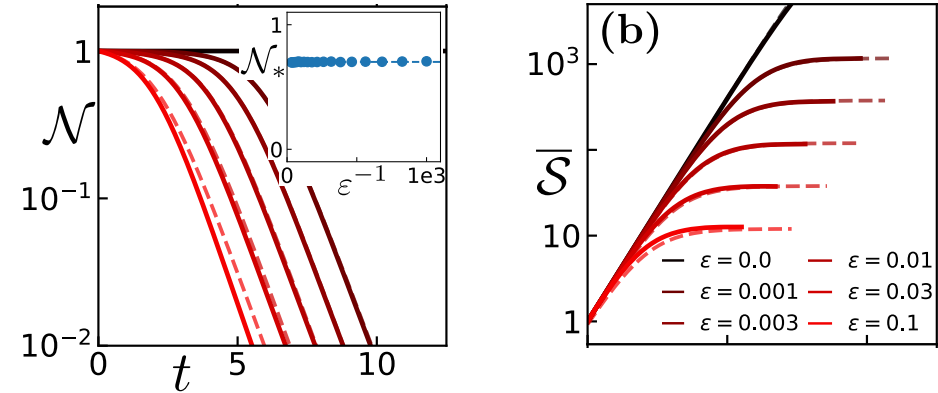
Domínguez et al. (2021)



Adamantane lattice  
192 nearest neighbors

All-to-all random circuits

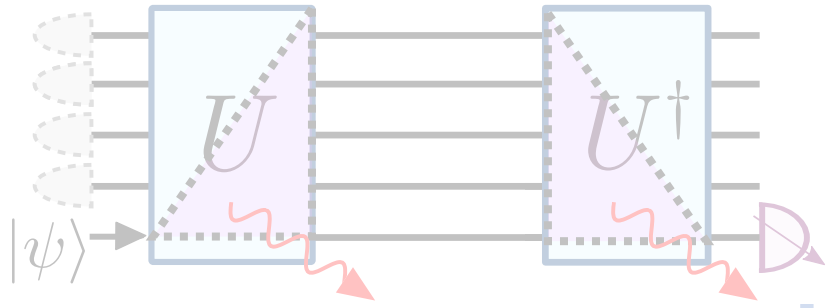
TS, Yao, arXiv:2208.12272 (2022)



All-to-all  
random circuit

# Operator growth in open quantum systems

TS, Yao PRL 131, 160402 (2023)



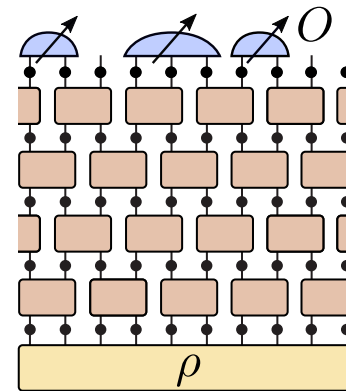
**Noise**



**Information dynamics**



**Complexity**



TS, Yao forthcoming (2023)

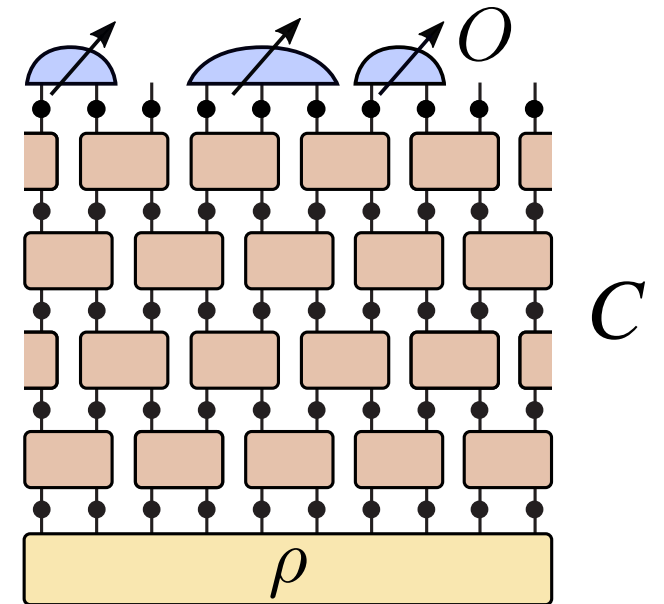
A polynomial-time classical algorithm for almost any noisy quantum circuit



# A classical algorithm for almost any noisy quantum circuit

We provide an efficient classical algorithm for “almost any” noisy quantum circuit

**Task:** Compute expectation values  $\text{tr}(C\{\rho\}O)$



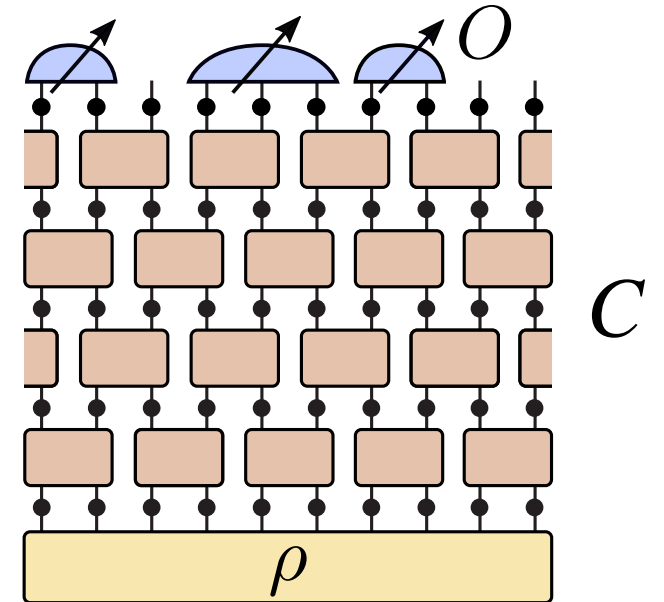
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“Almost any”: only require success *with low average error* over input states drawn from a complete basis

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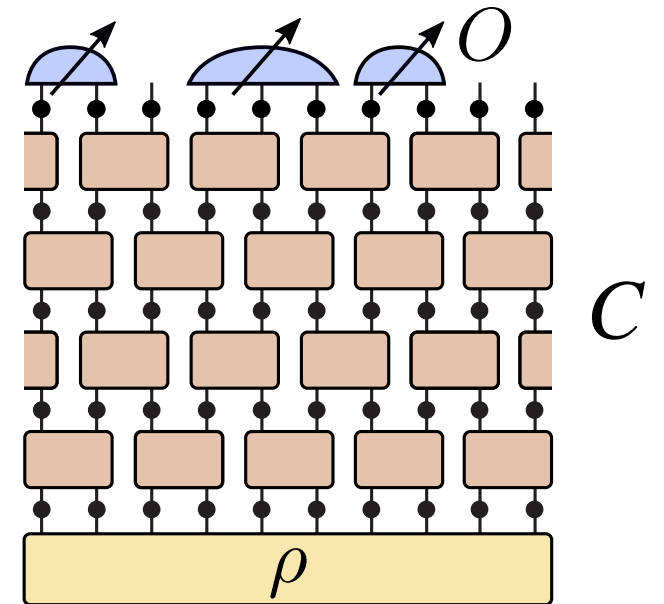
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**Our result:** A classical algorithm to compute expectation values with root-mean-square error  $\varepsilon \cdot \|O\|_F$  in time

$$n^{\mathcal{O}\left(\frac{1}{\gamma} \log(\sqrt{T}/\varepsilon)\right)}$$



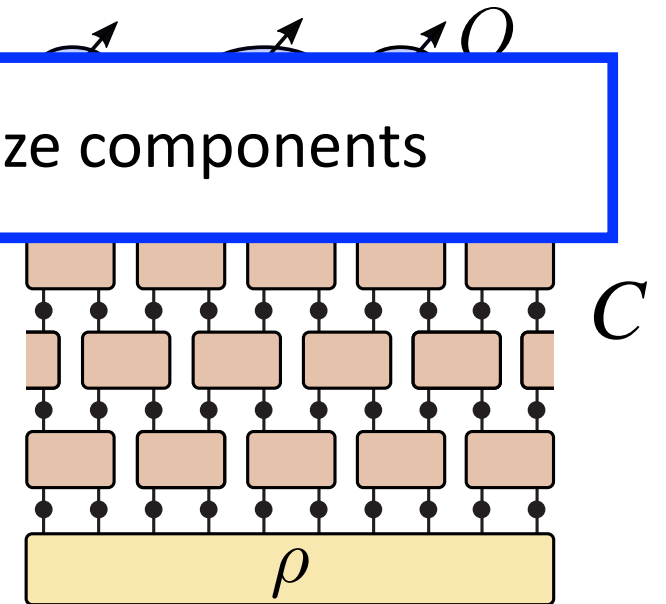
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**Idea:** Simulate in Heisenberg picture, keep only low-size components

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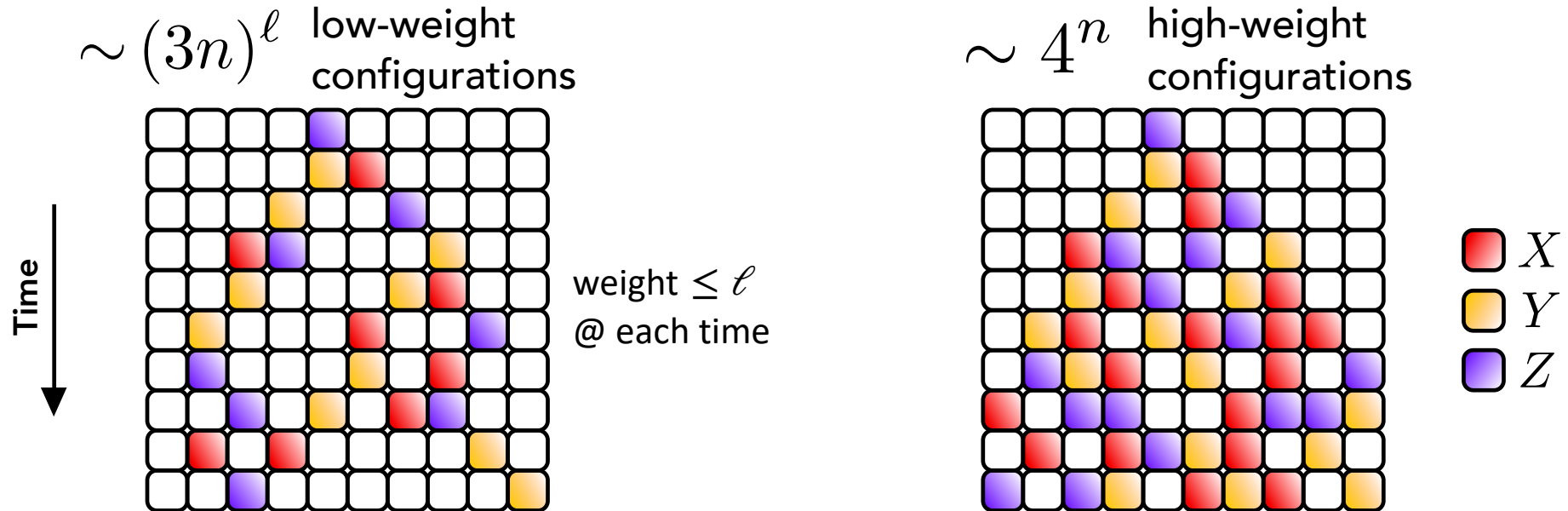
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# Sensitivity to noise $\Leftrightarrow$ Complexity

Decompose operator in Pauli string basis at each time step

Close correlation between “paths” of Pauli strings that are hard to simulate, and those that are strongly damped by noise

[Aharonov, Gao, Landau, Liu, Vazirani (2023)]

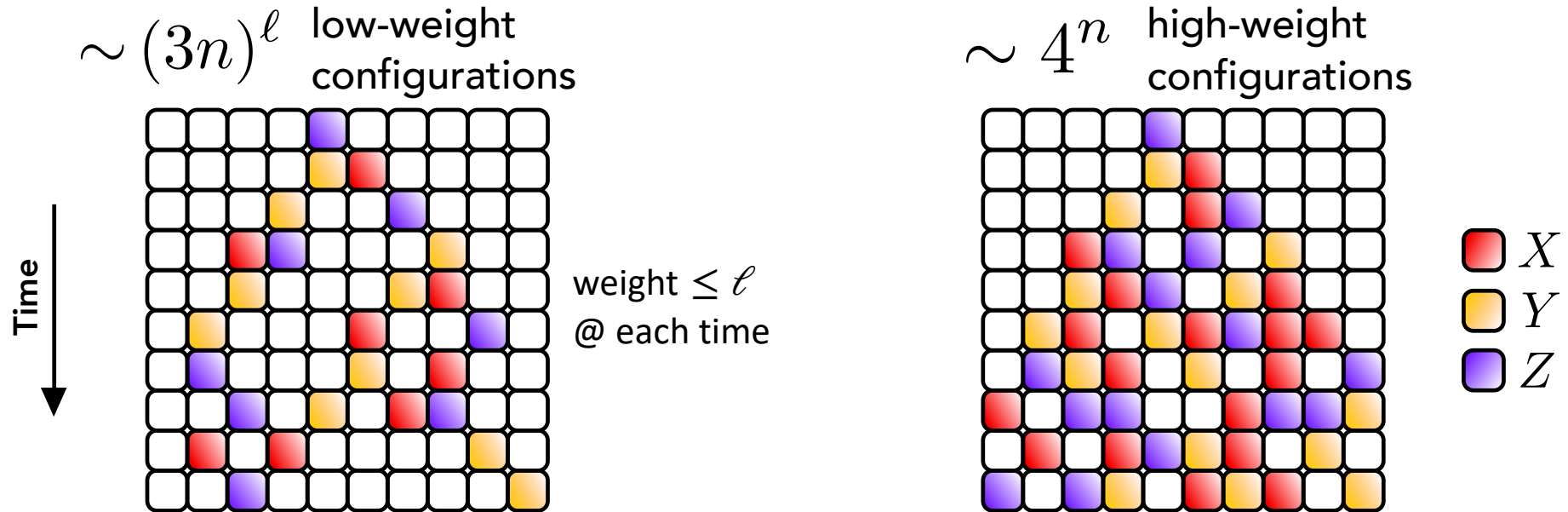


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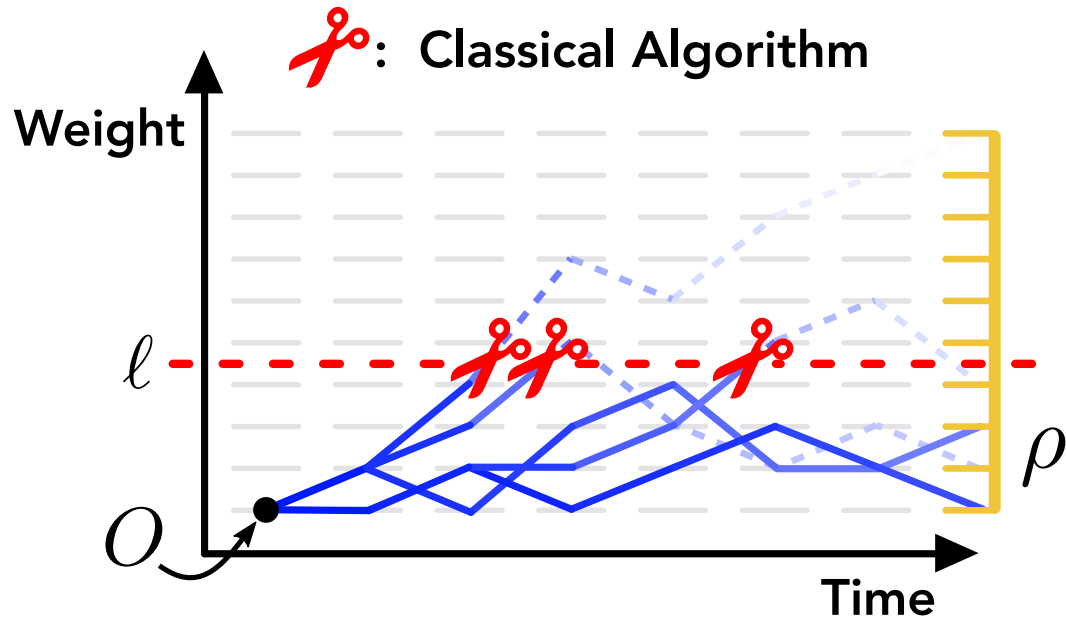
To bound contribution from high weights, previous works used that in random circuits, Pauli paths on average do not coherently interfere

# A polynomial-time classical algorithm

Builds upon recent algorithms for noisy random circuit sampling, but with modifications in the algorithm + proof techniques

[Aharonov, Gao, Landau, Liu, Vazirani (2023)]

**Algorithm:** At each time step, truncate any component of  $O(t)$  with size above a threshold  $\ell$

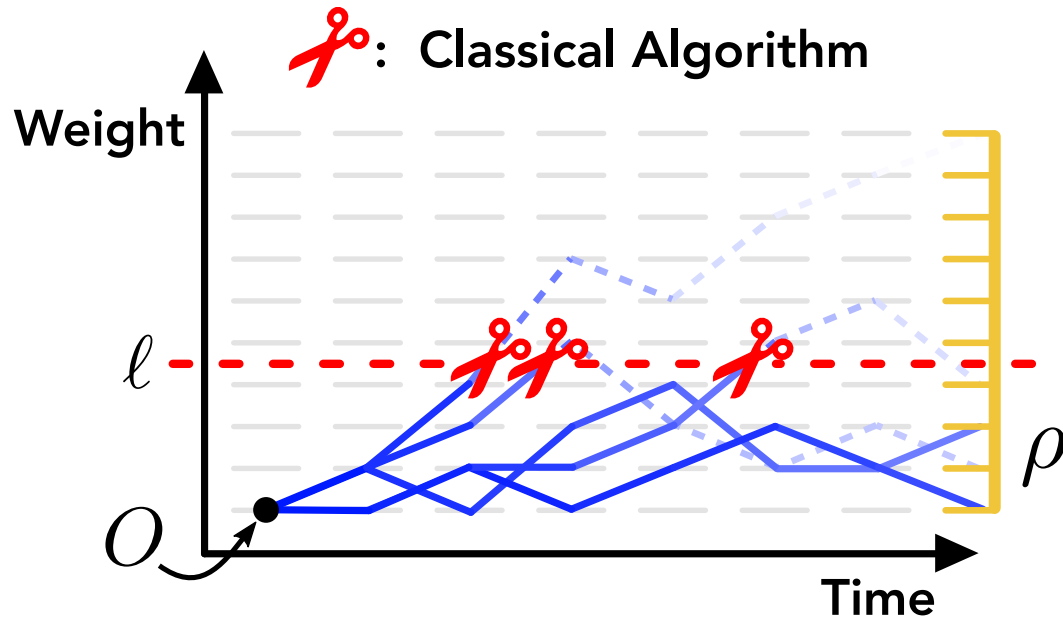


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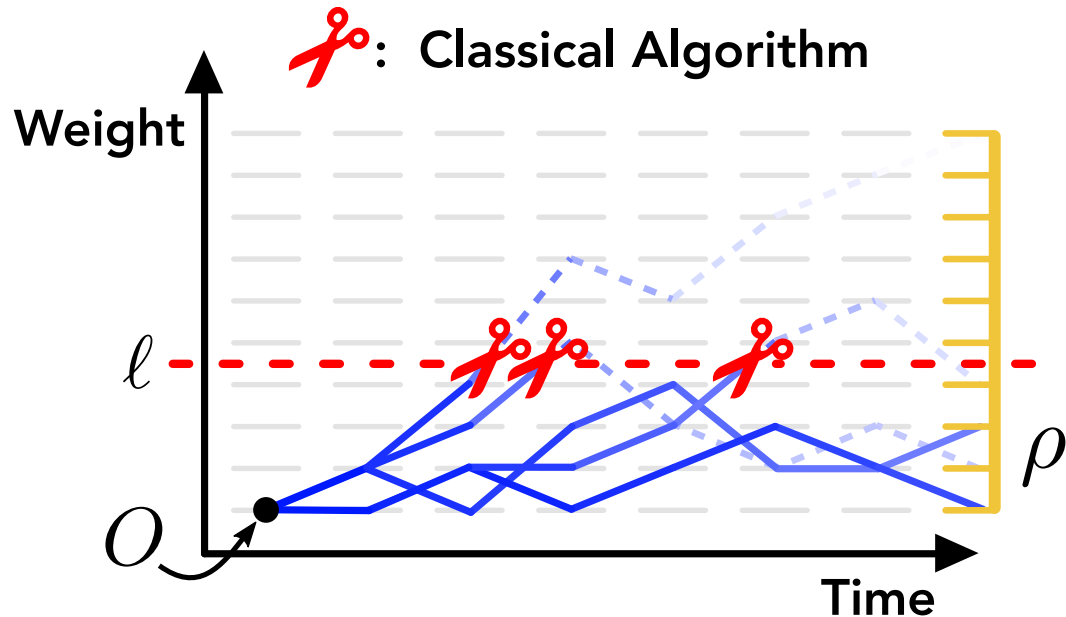
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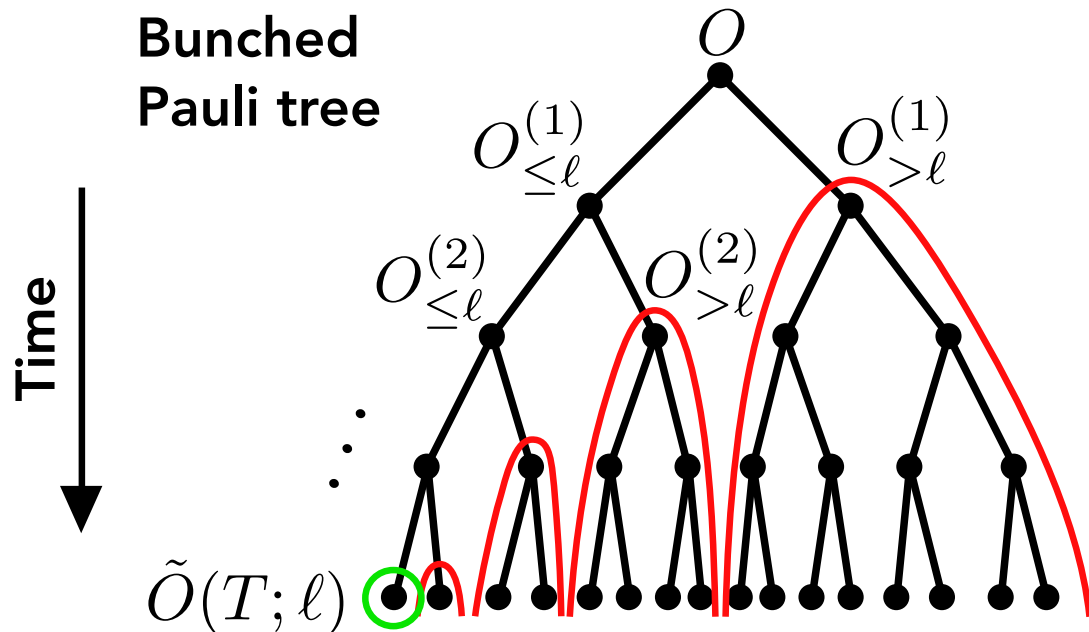
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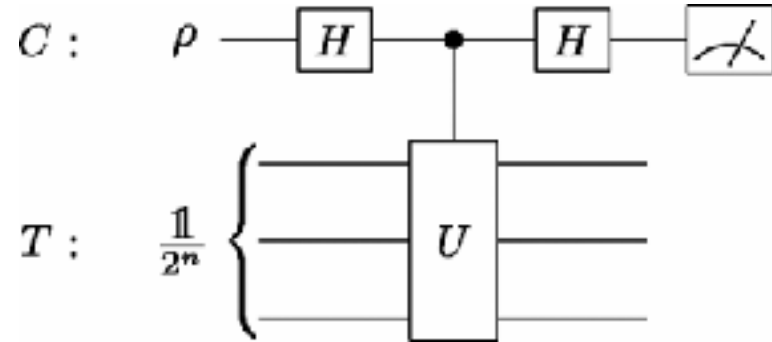
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# Simple extensions

**Corollary 1:** Can replace ensemble of input states with a single *highly mixed* state:

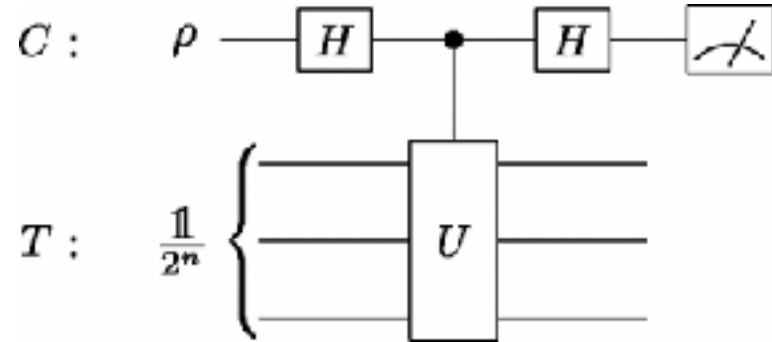
Noisy DQC1  $\subseteq$  BPP



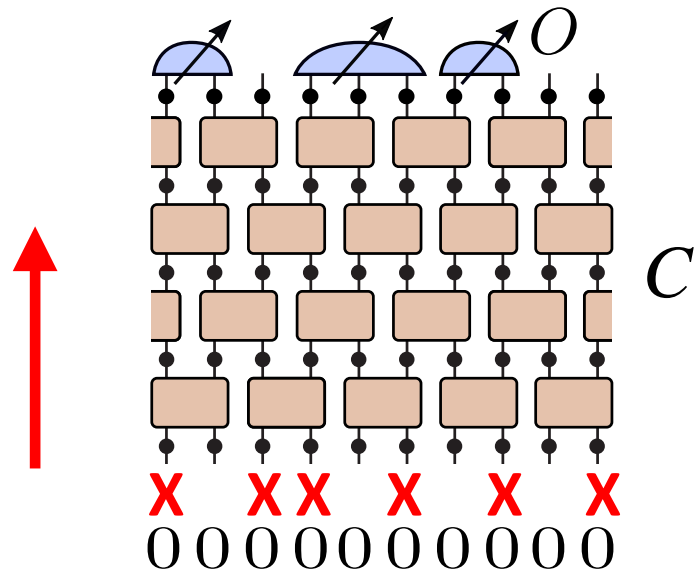
# Simple extensions

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Noisy DQC1  $\subseteq$  BPP



**Corollary 2:** Ensemble of input states  $\rightarrow$  ensemble of circuits with *spatial disorder*



“Push” randomness from initial state into circuit

Encompasses quantum simulation of disordered spin models

# Implications for NISQ experiments?

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But designing NISQ circuits that take advantage of these noise budgets is challenging!

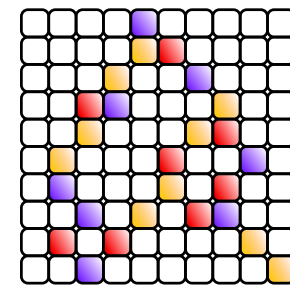
see e.g. White, Refael, Pollman, Rakovsky, von Keyserlingk, Ye, Machado, Nahum, Zhou...  
also IBM (2023) + Kedadzchi et al, Anand et al, Begusic et al, Rudolph et al, ...

# Implications for NISQ experiments?

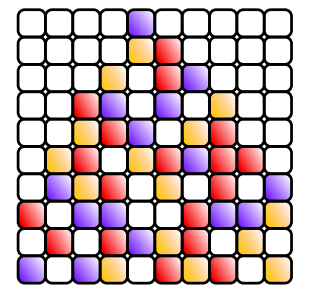
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**Classically simulable**  
Insensitive to noise



**Potentially complex**  
Sensitive to noise



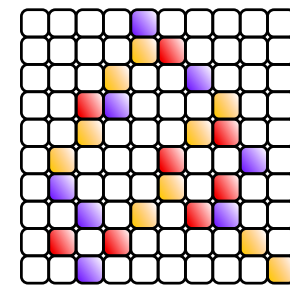
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Our bounds are weak for noise rates on leading quantum devices

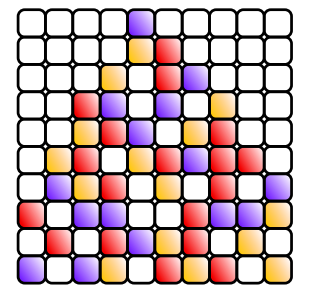
But designing NISQ circuits that take advantage of these noise budgets is challenging!

see e.g. White, Refael, Pollman, Rakovsky, von Keyserlingk, Ye, Machado, Nahum, Zhou...  
also IBM (2023) + Kdezchi et al, Anand et al, Begusic et al, Rudolph et al, ...

Our result provides a simple “test” for if a circuit is classically simulable



Classically simulable  
Insensitive to noise



Potentially complex  
Sensitive to noise

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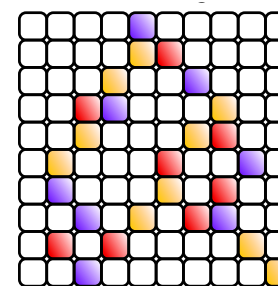
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**Corollary 3:** Complex quantum circuits **must be** highly sensitive to noise:

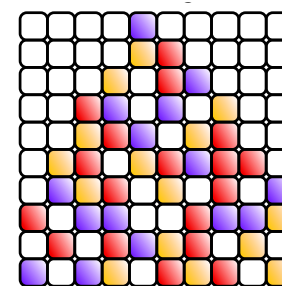
Any quantum experiment\* with classical complexity  $\chi(n, \epsilon)$ , *must only succeed* for

$$\gamma \leq \mathcal{O} \left( \frac{\log(n) \log(\sqrt{T^*}/\epsilon)}{\log(\chi(n, \epsilon))} \right)$$

e.g. if  $\chi \sim \exp(n)$ ,  
require  $\gamma \lesssim 1/n$



Classically simulable  
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\*For computing expectation values with small rms error

Quantum information dynamics provides a natural language for understanding noise and complexity in NISQ experiments

## Future Directions

- Benchmarking Loschmidt echo predictions with NMR experiments; connection to noise-induced transitions Google Quantum AI (2023), Ware et al (2023)
- “Experimental tests” for efficient classical simulations
- Dephasing noise, related e.g. to Clifford + T gate simulations
- What is the sensitivity to noise of physical many-body dynamics?  
Can we identify classes of dynamics that are / are not sensitive?  
White, Refael, Pollman, Rakovsky, von Keyserlingk, Ye, Machado, Nahum, Zhou...

TS, Yao PRL 131, 160402 (2023)

TS, Yao forthcoming (2023)

Norman Yao

