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MELBOURNE

By Gary J Mooney

Large-Scale Entanglement on Physical Quantum Computers



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IBM Quantum Network Hub
at the University of Melbourne



Noisy intermediate Scale Quantum (NISQ) Computing

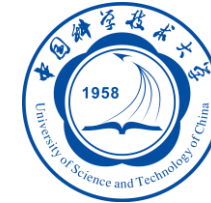
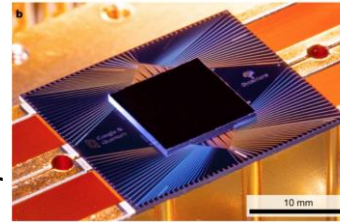
Superconducting Qubits



433q *Osprey* device



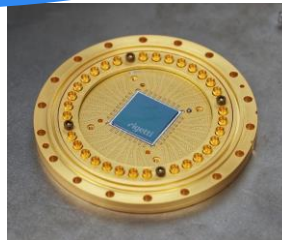
70q *Sycamore* processor



176q *Zuchongzhi-2* chip



84q *Ankaa-1* system

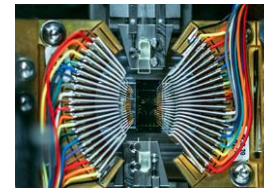


Photonic Qubits



XANADU

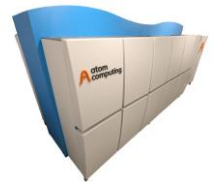
216q *Borealis* device



Neutral Atom



1,180q *Aquila* machine



Trapped Ion

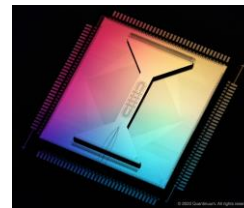


32q *Forte* device

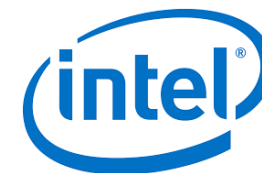


QUANTINUUM

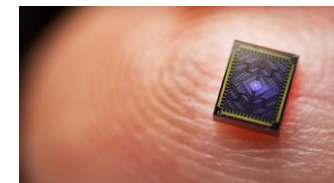
32q *H2* device



Quantum Dot



12q *Tunnel Falls* device



Benchmark devices: measure extent of quantumness

Overview

Benchmarking quantum devices

Forms of Multipartite Entanglement

- Bipartite and Genuine Multipartite entanglement

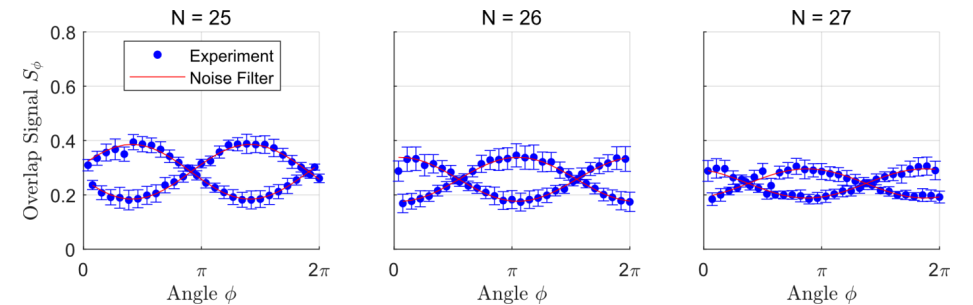
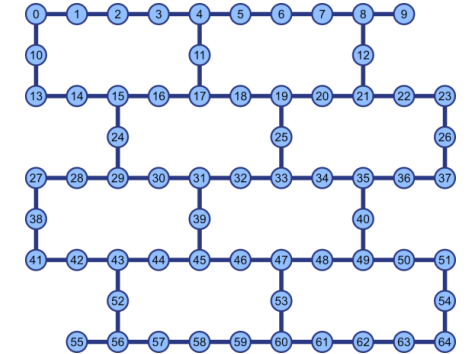
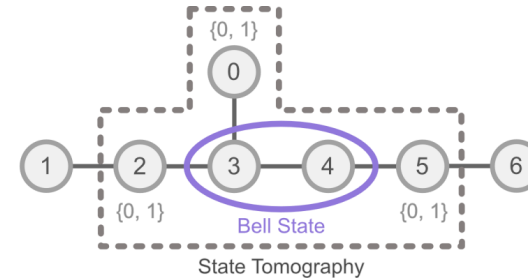
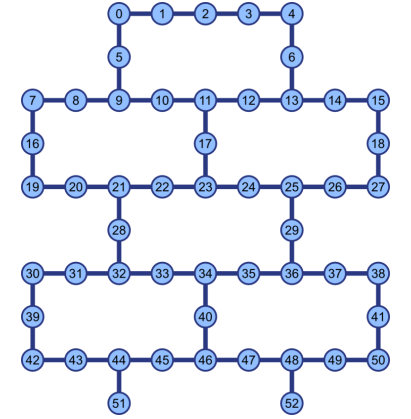
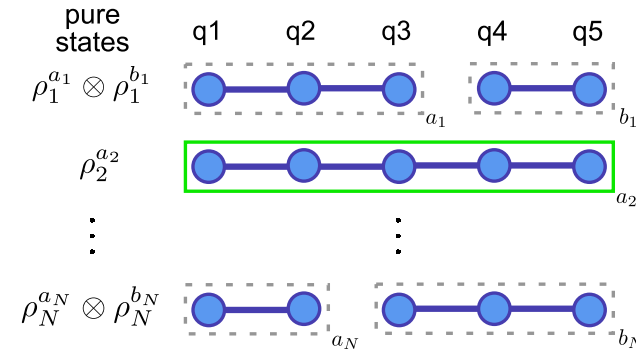
Detecting **Bipartite entanglement**

- By preparing Graph states on *IBM Quantum* devices

Detecting **Genuine multipartite entanglement**

- By preparing GHZ states
- GHZ decoherence rates

Bell state teleportation





Overview

Benchmarking quantum devices

Forms of Multipartite Entanglement

- Bipartite and Genuine Multipartite entanglement

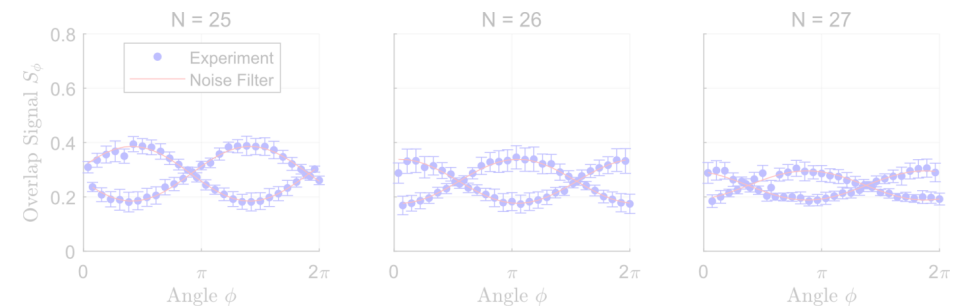
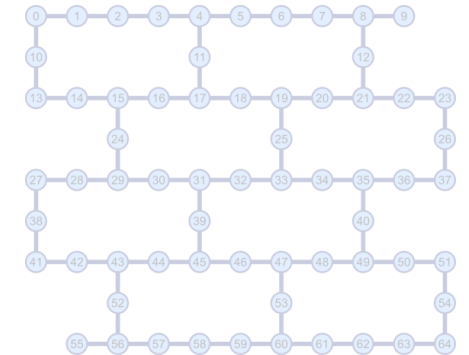
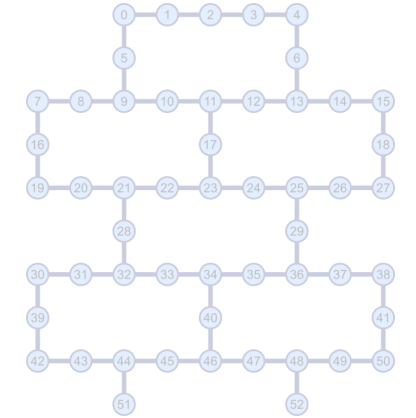
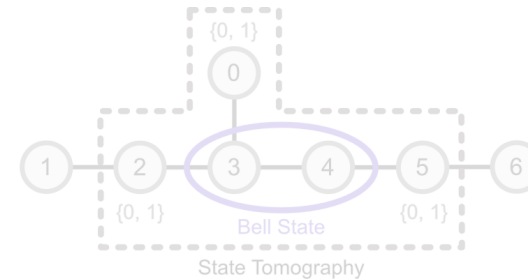
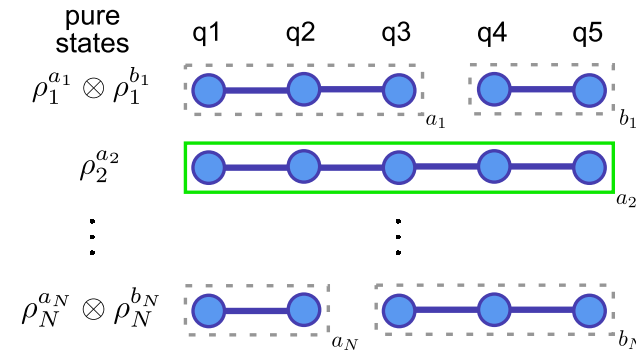
Detecting **Bipartite** entanglement

- By preparing Graph states on *IBM Quantum* devices

Detecting **Genuine** multipartite entanglement

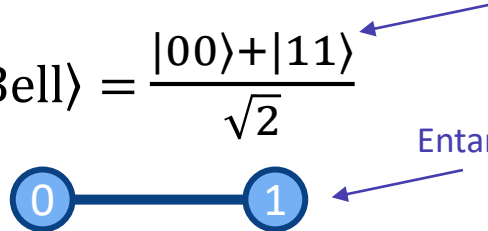
- By preparing GHZ states
- GHZ decoherence rates

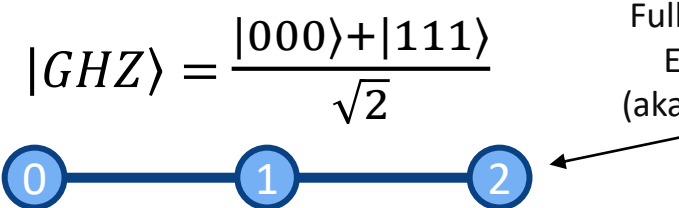
Bell state teleportation

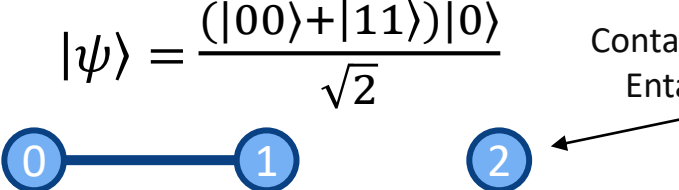


Pure State Entanglement - Definitions

- Entanglement
 - Non-classical correlations
 - State is not separable: $|\psi\rangle \neq |\phi_1\rangle \otimes |\phi_2\rangle$, for any $|\phi_1\rangle$ and $|\phi_2\rangle$
 - State is entangled
- Multiqubit Entanglement
 - A **multiqubit state** either:
 - “is fully bipartite entangled”
 - (aka “is entangled”)
 - “contains bipartite entanglement”

$$|\text{Bell}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$


$$|\text{GHZ}\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}$$


$$|\psi\rangle = \frac{(|00\rangle + |11\rangle)|0\rangle}{\sqrt{2}}$$


Bipartition $\{0\}$ and $\{1,2\}$: Entangled
 Bipartition $\{0,1\}$ and $\{2\}$: Separable

What is Mixed State Entanglement?

Real quantum device → Noise

- Quantum state is a **Mixed state**: probabilistic mixture of pure states
- More complicated than pure states

Mixed State

$$\rho = \sum_{i=1}^N p_i \rho_i$$

Pure states ρ_i
Probabilities p_i

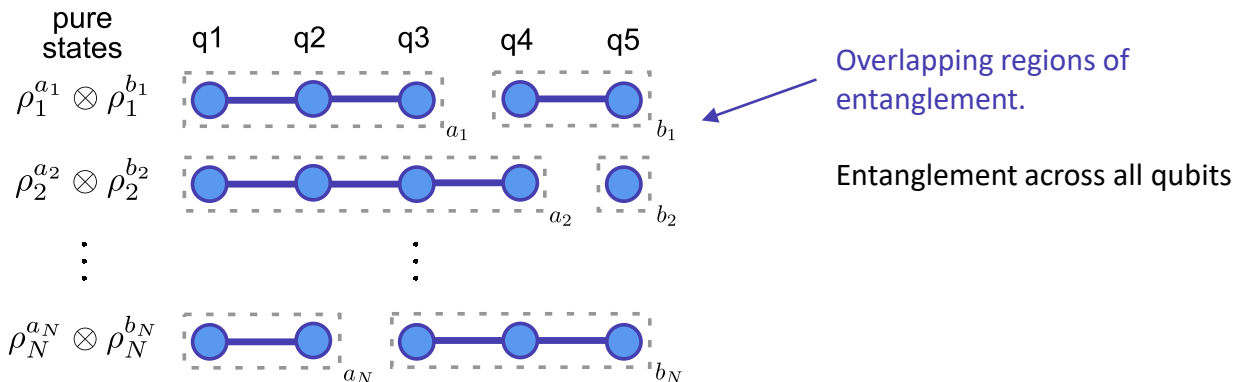
Density matrix ρ

$$\rho = \begin{pmatrix} a_{1,1} & \cdots & a_{1,2^N} \\ \vdots & \ddots & \vdots \\ a_{2^N,1} & \cdots & a_{2^N,2^N} \end{pmatrix}$$

Quantum Theory: Concepts and Methods, (1993)

Bipartite Entangled (or just Entangled)

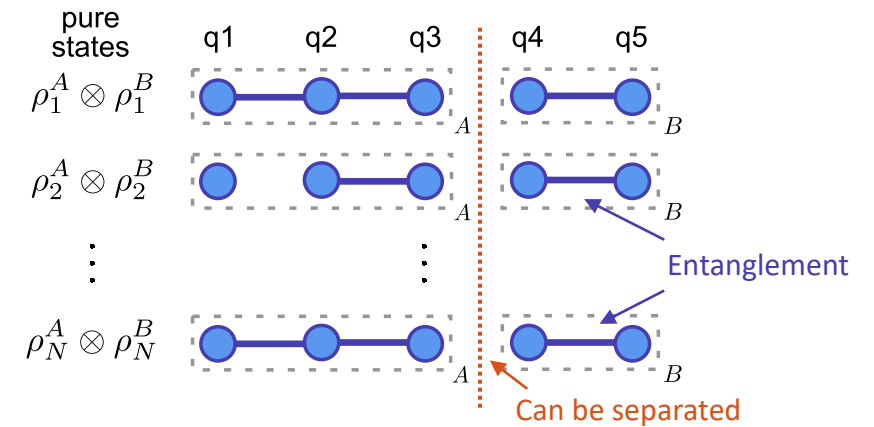
- State is **not separable**, i.e. $\rho \neq \rho^A \otimes \rho^B$, for all bipartitions A and B
- Although, individual pure states ρ_i might be separable



Mooney, Hill and Hollenberg, Sci. Rep. (2019)

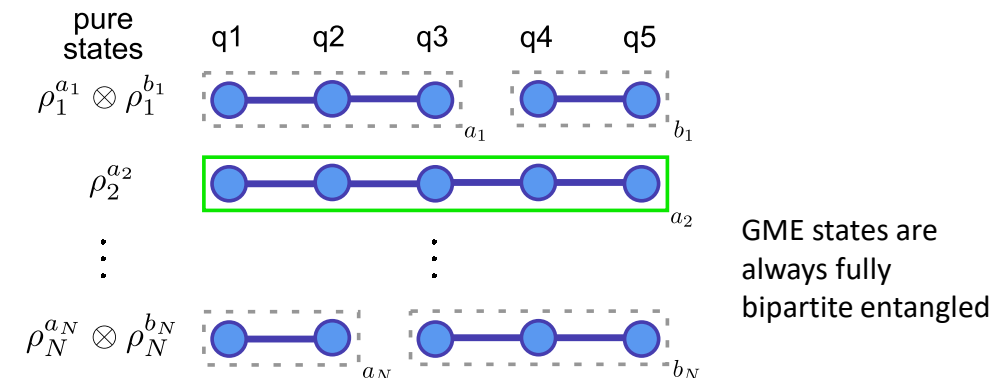
Separable

$$\rho = \rho^A \otimes \rho^B \quad (\text{fixed bipartition } A \text{ and } B)$$



Genuine Multipartite Entangled (GME)

- Stronger form of entanglement
- There is always a fully entangled pure state

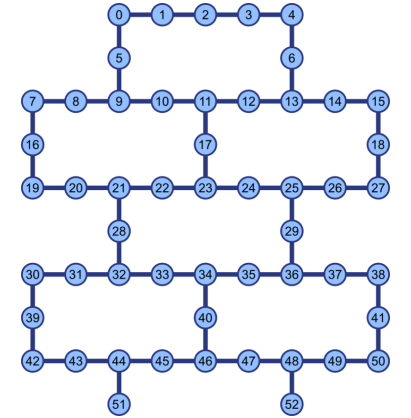
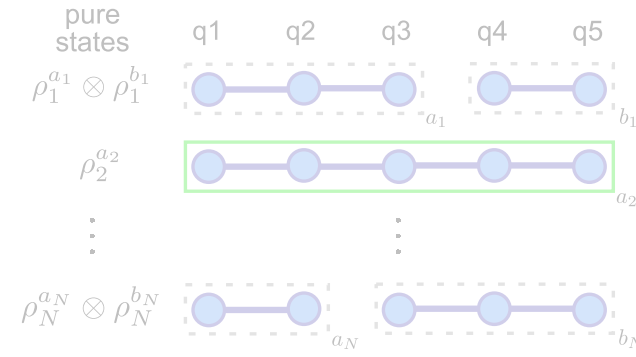




Overview

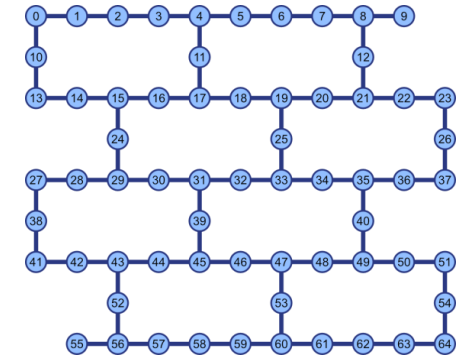
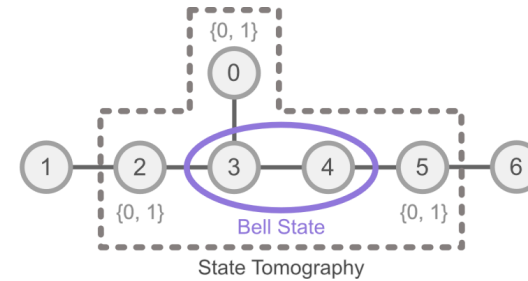
Benchmarking quantum devices

- Forms of Multipartite Entanglement
- Bipartite and Genuine Multipartite entanglement



Detecting Bipartite entanglement

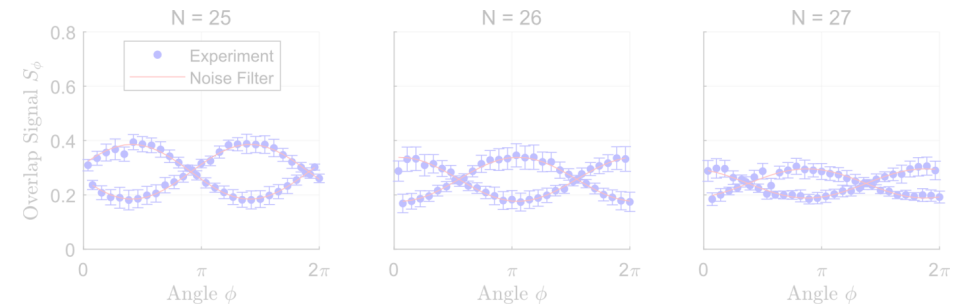
- By preparing Graph states on *IBM Quantum* devices



Detecting Genuine multipartite entanglement

- By preparing GHZ states
- GHZ decoherence rates

Bell state teleportation



Graph States

➤ Choose a quantum state to prepare

Graph State (Cluster State)

- Robust to noise Briegel and Raussendorf, Phys. Rev. Lett. (2001)
 - Requires $n/2$ local measurements to disentangle
- Low circuit depth
- Convenient for detecting bipartite entanglement

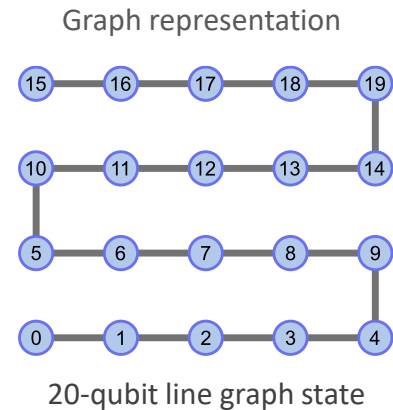
- Applications
 - One-way quantum computing Raussendorf and Briegel, Phys. Rev. Lett. (2001)
 - Fault-tolerant error correction Kitaev, arXiv. (1997)
 - Measurement-Based variational quantum eigensolver Raussendorf, Harrington, Goyal, New Journal of Physics. (2007)
 - Measurement-Based variational quantum eigensolver Ferguson, Phys. Rev. Lett. (2021)

How do we detect entanglement?

$$|G_n\rangle = \prod_{(\alpha, \beta) \in E} \text{CZ}_{\beta}^{\alpha} |+\rangle^{\otimes n}$$

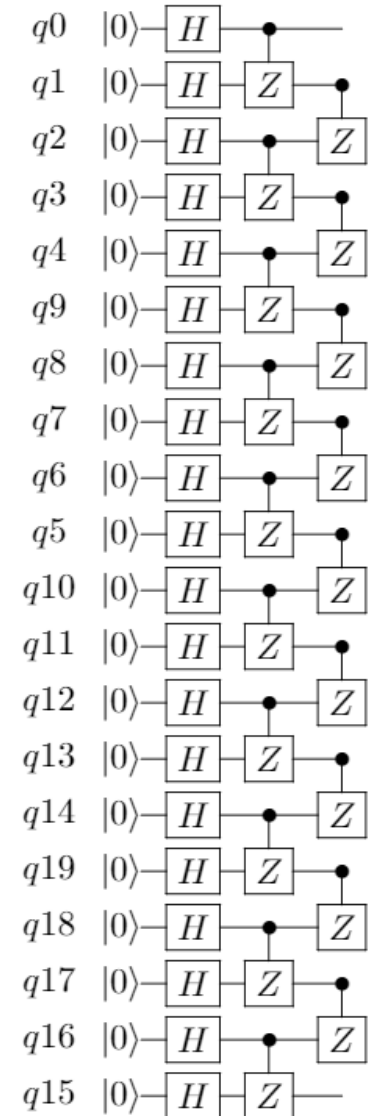
$E := (\text{edge set})$ ↑ Controlled-phase gate

Defined on a graph



Preparation

Circuit representation



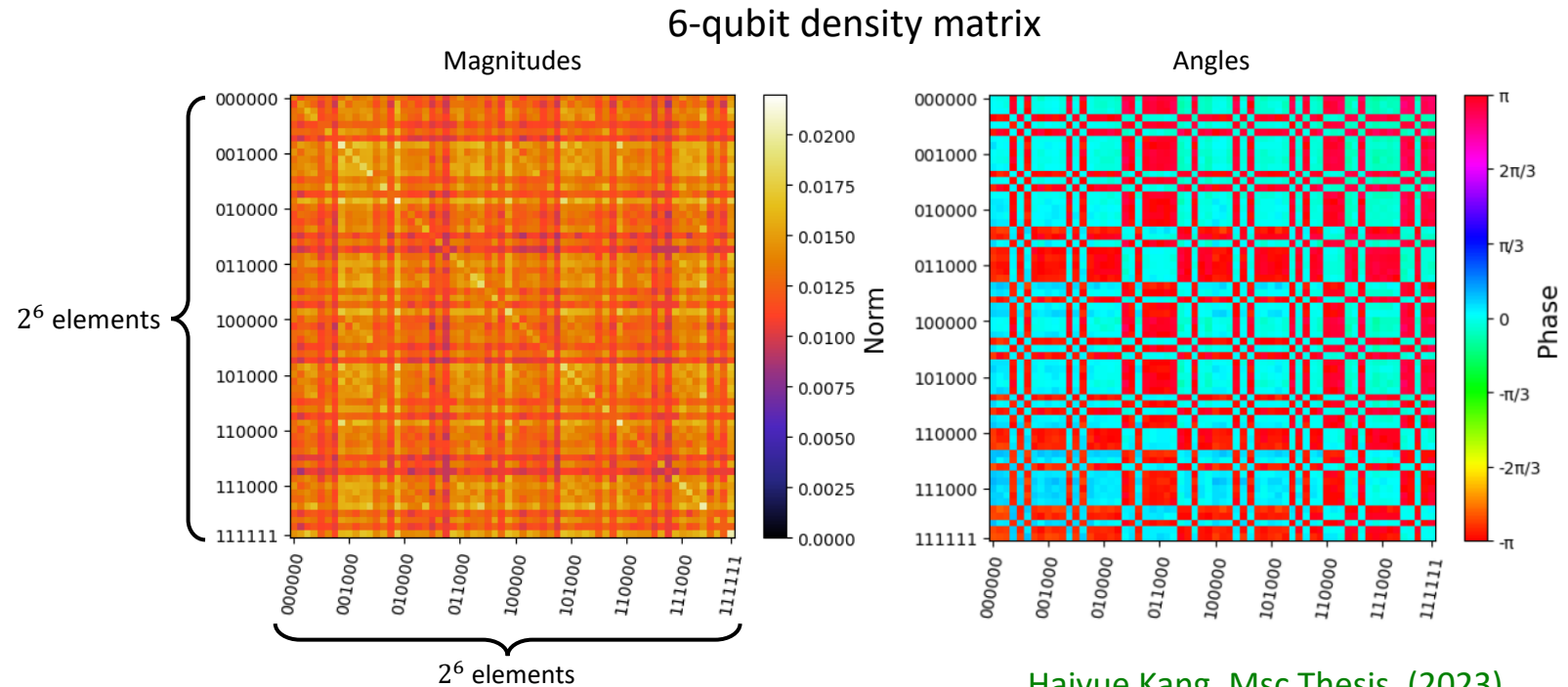
Full Quantum State Tomography

Quantum State Tomography (QST)

- Get a **density matrix**, encapsulating noise
 - $2^N \times 2^N$ complex matrix
- Analyse entanglement properties

Full quantum state information

- Can be overkill for measuring a particular property
- Use a detection strategy



Requires 3^N circuits

6 qubits: 729 circuits

10 qubits: ~59,000 circuits

20 qubits: ~3.5 Billion circuits!

Requires 3^N basis measurements

1 day \approx 15-30k circuits (approx)

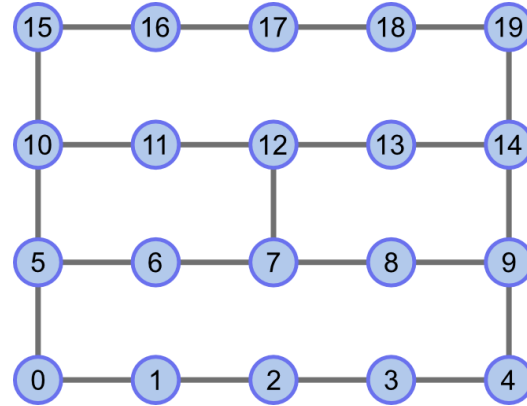
Detection Strategy – Show Non-Separability

Focus on bipartite entanglement

Detection Strategy

- State is **not separable**
 ⇒ State is at least **bipartite entangled**
1. Generate **entanglement graph**
 - Measure entanglement between qubit pairs
 2. Is entanglement graph **connected**?

Hardware Layout

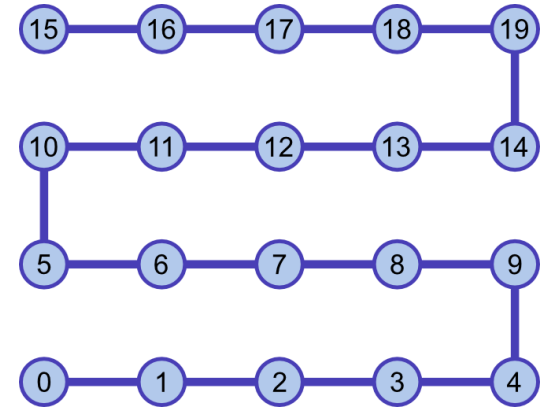


Generate Entanglement Graph

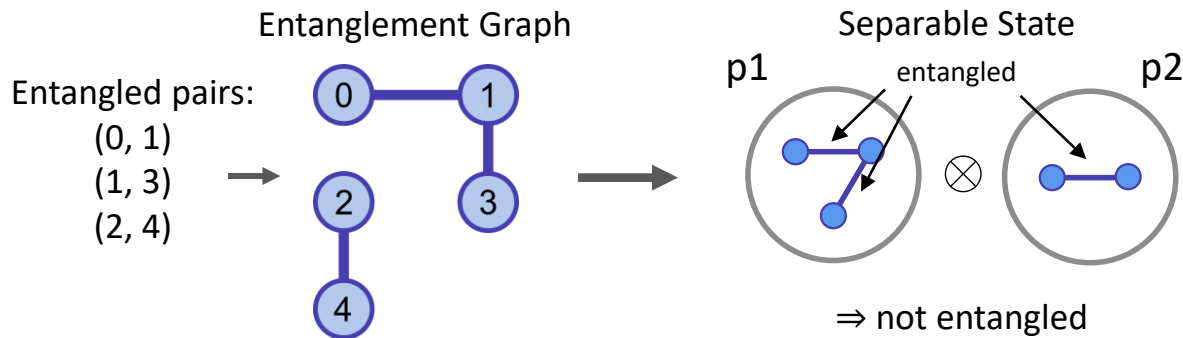


Measure entanglement between pairs

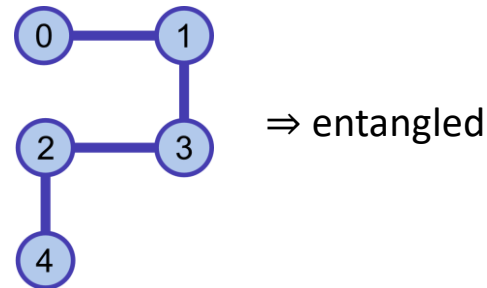
Entanglement Graph



Example: Separable State



So if **connected**, then **entangled**



On Quantum Devices
 To show:
 entanglement graph is
 connected

Detecting 2-Qubit Entanglement

Perform state tomography on qubit pair and their neighbours

Graph State Property:

Project neighbouring qubits to Z-basis states \rightarrow Bell state (up to local operations)

- Bell states produced from combinations of states $\{0, 1\}^{\#(\text{neighbours})}$:

$$\begin{aligned}
 H \otimes I |\Phi^+\rangle &= (|00\rangle + |01\rangle + |10\rangle - |11\rangle) / 2 \\
 H \otimes X |\Phi^+\rangle &= (|00\rangle + |01\rangle - |10\rangle + |11\rangle) / 2 \\
 XH \otimes I |\Phi^+\rangle &= (|00\rangle - |01\rangle + |10\rangle + |11\rangle) / 2 \\
 XH \otimes X |\Phi^+\rangle &= (|00\rangle - |01\rangle - |10\rangle - |11\rangle) / 2
 \end{aligned}$$

One of 4 Bell states (up to local operations)

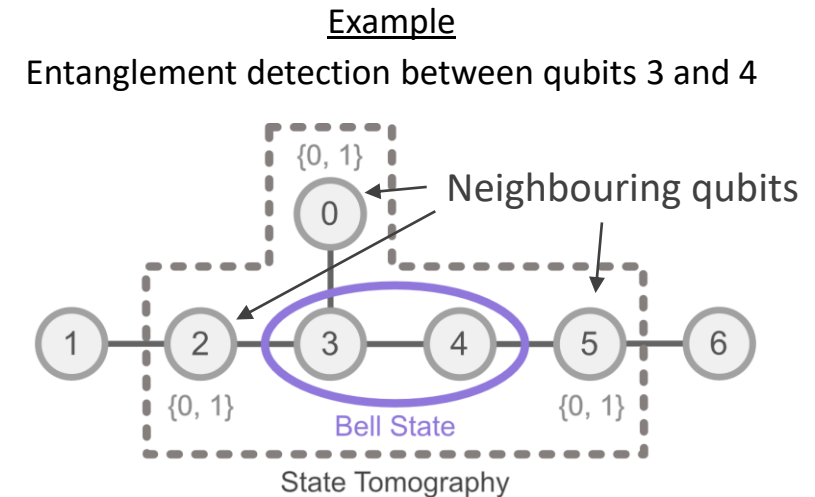
Calculate entanglement for each set of states $\{0, 1\}^{\#(\text{neighbours})}$

Negativity of partial transpose

- Calculate \rightarrow Sum over magnitudes of negative eigenvalues of $\rho_{3,4}^{T_4}$
- For **Two Qubits**: Non-zero negativity is **necessary** and **sufficient**
 - Negativity is non-zero \leftrightarrow Entanglement Horodecki, Horodecki and Horodecki, Phys. Lett. A. (1996)

Measure negativity for all neighbour states: $\{0, 1\}^{\#(\text{neighbours})}$

\rightarrow Extent of entanglement is the largest negativity (among the combinations)



Make physical
Michelot, J. Optim. Theory Appl. (1986)

Negativity

$$\mathcal{N}(\rho_{3,4}^{T_4}) = \sum_{\lambda_i < 0} |\lambda_i|$$

Negative eigenvalues of $\rho_{3,4}^{T_4}$

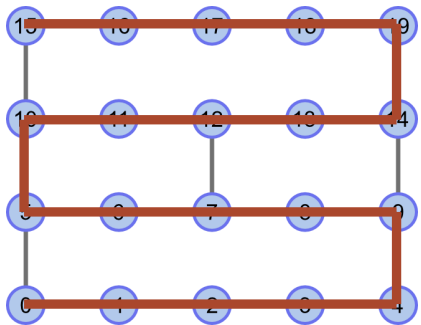


Results: Negativities on an *IBM Quantum* device

Apply these techniques on an IBM Quantum computer: the **20-qubit *ibmq_poughkeepsie*** device

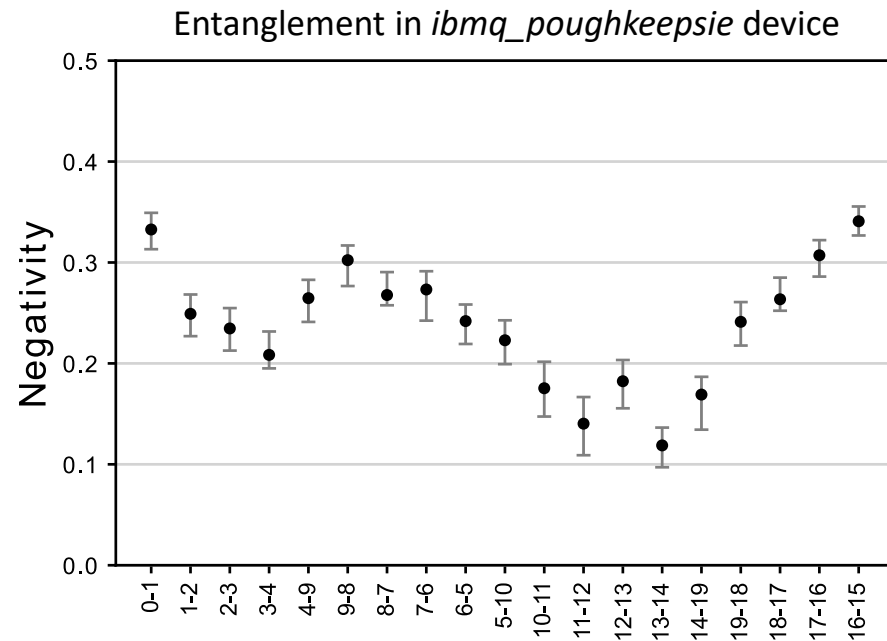
- Embed a line graph state
- Generate entanglement graph

Hardware Layout
20-qubit *ibmq_poughkeepsie* device



Error rates:
Readout: 3.8%, $\sigma = 1.6\%$
CNOT: 2.3%, $\sigma = 0.8\%$

Decoherence times:
• $T_1 = \sim 100 \mu s$ (relaxation)
• $T_2 = \sim 100 \mu s$ (dephasing)



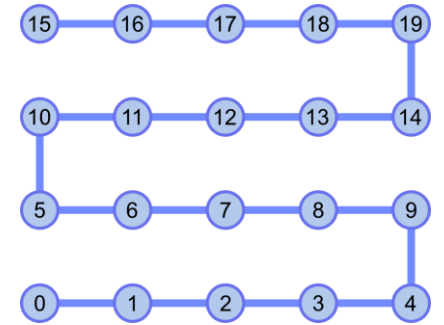
Mooney, Hill and Hollenberg, *Sci. Rep.* (2019)

Maximally entangled

Higher Entanglement

No entanglement

Entanglement Graph
(all qubits connected)



State is bipartite entangled

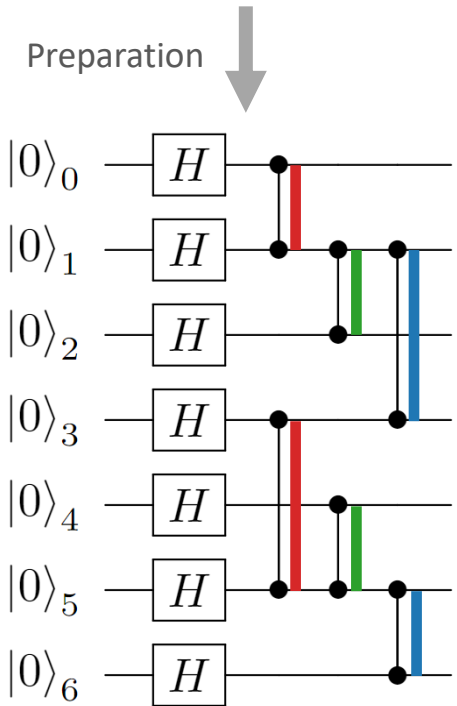
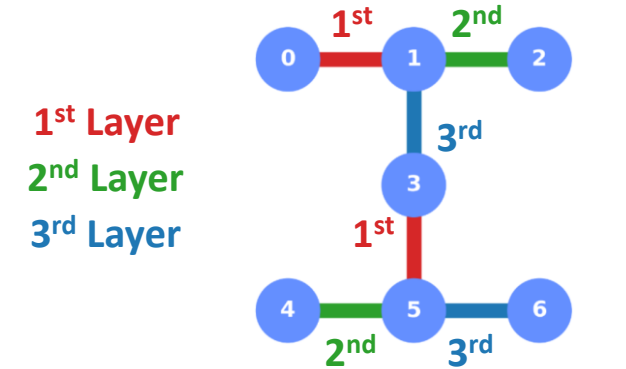
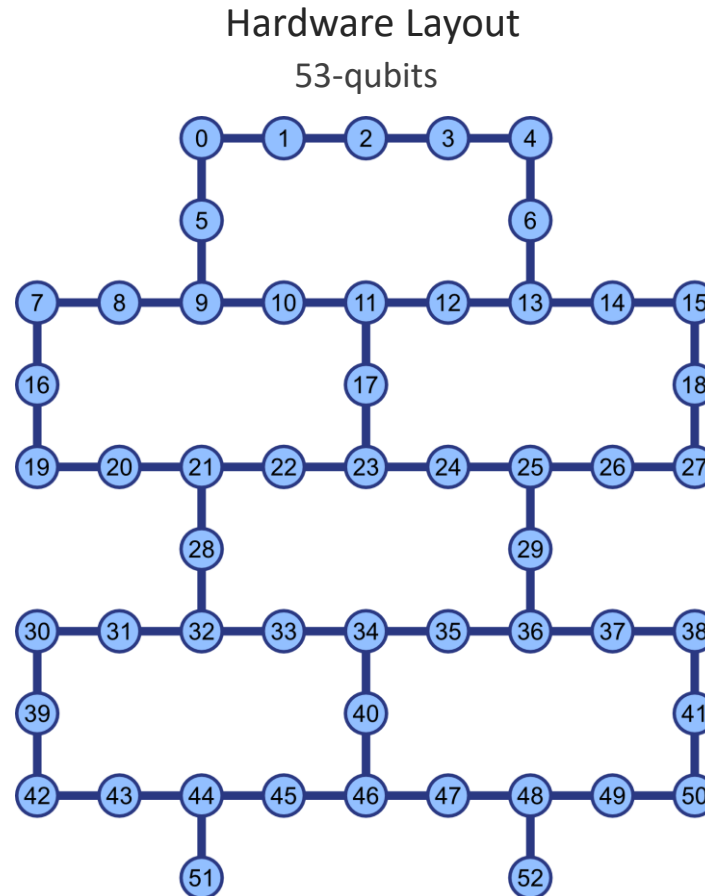
Scale up to larger devices

- Whole device is bipartite entangled
- At the time → largest quantum entangled state (on a universal quantum computer)
- Previous record was 16 qubits [Wang et. al. npj Quantum Information \(2018\)](#)

Native Graph State

Native graph state: match the hardware layout

- Benchmark qubit pairs
- More connected cycles
 - Requires more separable pairs to break connectedness
- Still constant circuit depth
 - Depth = largest neighbour count of qubits



3 Layers of 2-qubit gates

Constant depth → largest neighbour count of qubits



Quantum Readout-Error Mitigation (QREM)

- Readout assignment errors:
 - Obscures quantum data
 - State **appears less entangled** than it is

Calibrate readout-errors with stochastic matrix A :

$$A\vec{p} = \vec{p}_{\text{noisy}}$$

$$\rightarrow \vec{p} = A^{-1}\vec{p}_{\text{noisy}}$$

$$A = \bigotimes_{i=1}^N A_i$$

$$A_i = \begin{pmatrix} p_i(0|0) & p_i(0|1) \\ p_i(1|0) & p_i(1|1) \end{pmatrix}$$

Calibration matrix for qubit i

Requires only 2 measurements $\{|00 \dots 0\rangle, |11 \dots 1\rangle\}$

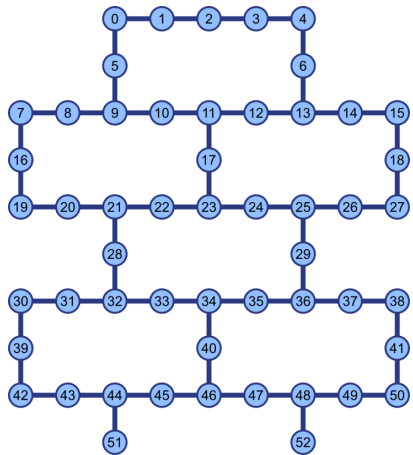
$$A^{-1} = \bigotimes_{i=1}^N A_i^{-1}$$

- Efficient application
 - Apply each qubit calibration matrix separately, zeroing small probabilities [Mooney, White, Hill and Hollenberg, J. Phys. Commun \(2021\)](#)
 - M3 python package [Nation et al. PRX Quantum \(2021\)](#)
- Make physical [Smolin et al. Phys. Rev. Lett. \(2012\)](#)



Results: Newer IBM Quantum devices

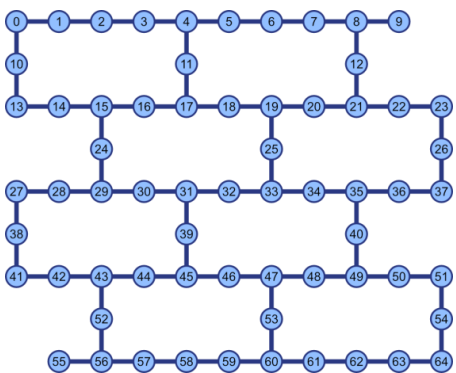
53-qubit *ibmq_rochester* device



- Error Rates:
- Readout: 12.6%, $\sigma = 9.3\%$
 - CNOT: 4.6%, $\sigma = 2.4\%$

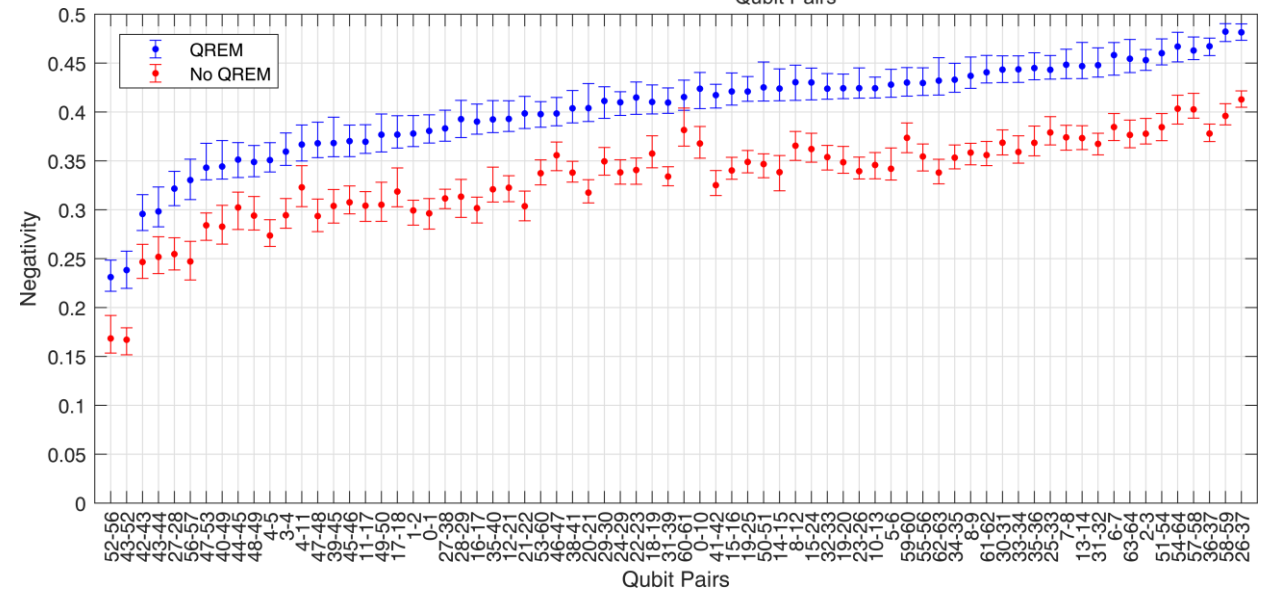
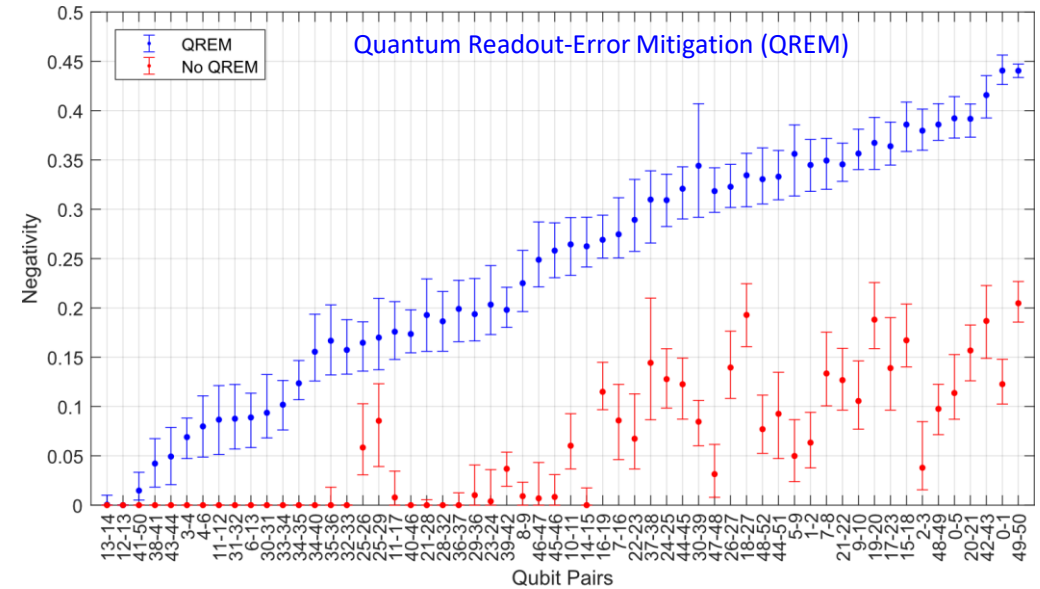
- Decoherence Times:
- $T_1 = \sim 53 \mu s$ (relaxation)
 - $T_2 = \sim 53 \mu s$ (dephasing)

65-qubit *ibmq_manhattan* device



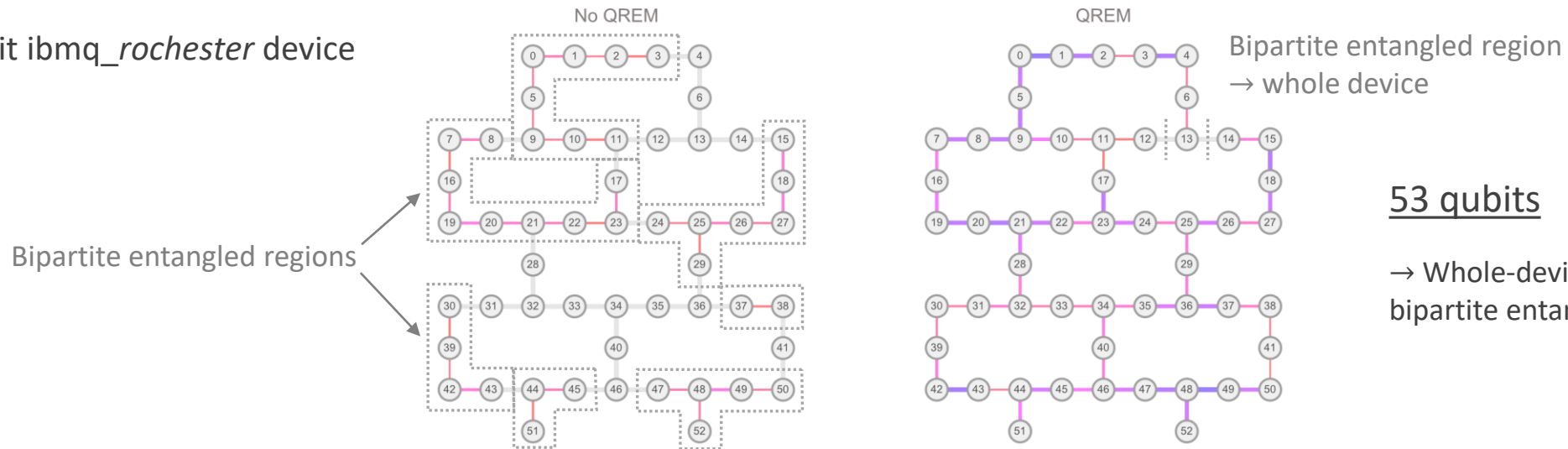
- Error Rates:
- Readout: 2.1%, $\sigma = 1.5\%$
 - CNOT: 1.5%, $\sigma = 0.6\%$

- Decoherence Times:
- $T_1 = \sim 60 \mu s$ (relaxation)
 - $T_2 = \sim 78 \mu s$ (dephasing)



Results: Entanglement Graphs

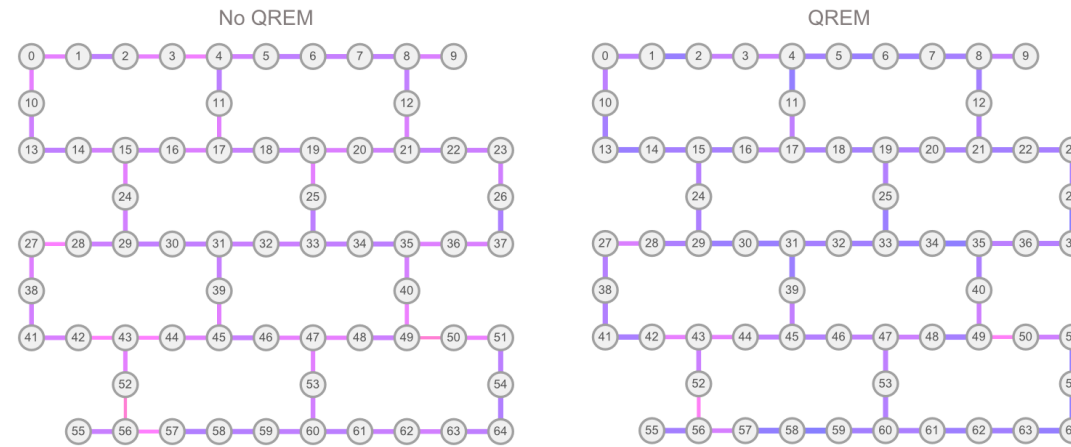
53-qubit *ibmq_rochester* device



53 qubits

→ Whole-device
bipartite entanglement

65-qubit *ibmq_manhattan* device



65 qubits

→ Whole-device
bipartite entanglement

Scale up to larger devices



Negativity

Let's Look at the Required Resources

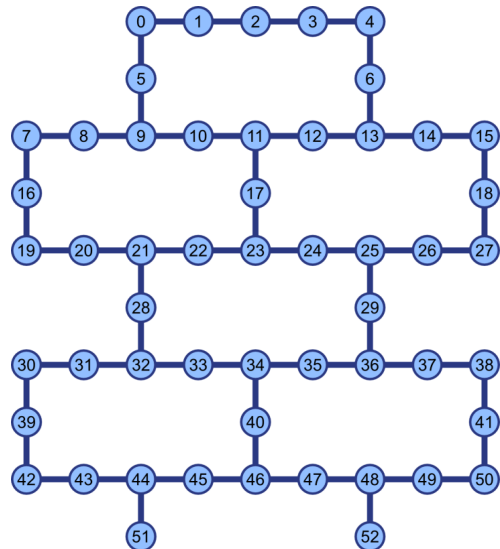
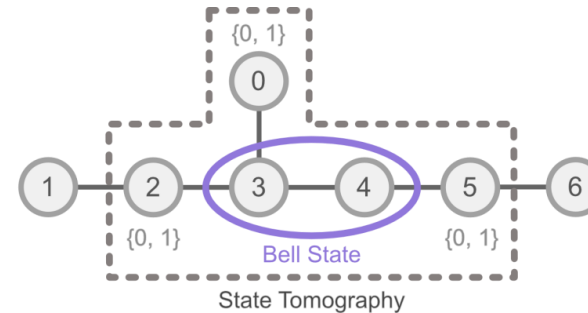
Quantum State Tomography on Each Pair

Requires 3^N

$N = 4$ qubits $\rightarrow 3^4 = 81$ circuits ($\sim 350k$ shots*)

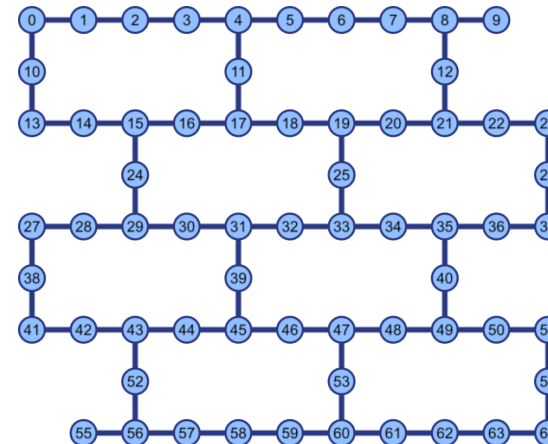
$N = 5$ qubits $\rightarrow 3^5 = 243$ circuits (~ 1 Million shots*)

* at ~ 4000 shots each



53-qubit Rochester:

- 58 qubit pairs
 - $\sim 11,000$ circuits
 - Or ~ 44 Million shots



65-qubit Manhattan:

- 72 qubit pairs
 - $\sim 14,000$ circuits!
 - Or ~ 56 Million shots!

Could take a **whole day** to complete

- Was barely executing within device calibration cycles

We need to go higher

Target: 127-qubit *ibm_washington* device

127-qubit Washington:

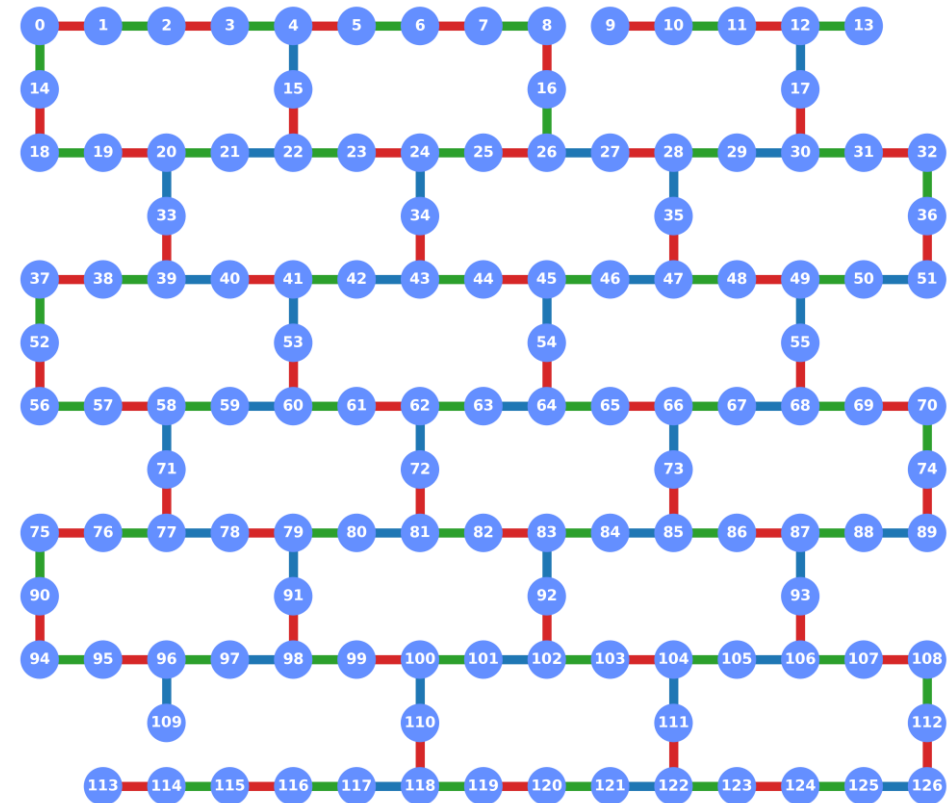
- 142 pairs of qubits
 - ~28,000 circuits
 - Or ~112 Million shots!
 - Error bars: x8 more

How to reduce this?

Two Optimisation Techniques

1. Neighbour-state bucketing
2. Parallel Quantum State Tomography

127-qubit *ibm_washington* device



Error Rates:

- Readout: 2.3%, $\sigma = 2.8\%$
- CNOT: 0.8%

Decoherence Times:

- $T_1 = \sim 240 \mu s$ (relaxation)
- $T_2 = \sim 142 \mu s$ (dephasing)

Optimisation 1: Neighbour-State Bucketing

Currently

- Tomography on qubit pair and neighbours (4 or 5 qubits)
- 5 qubits → requires $3^5 = 243$ circuits per pair

However

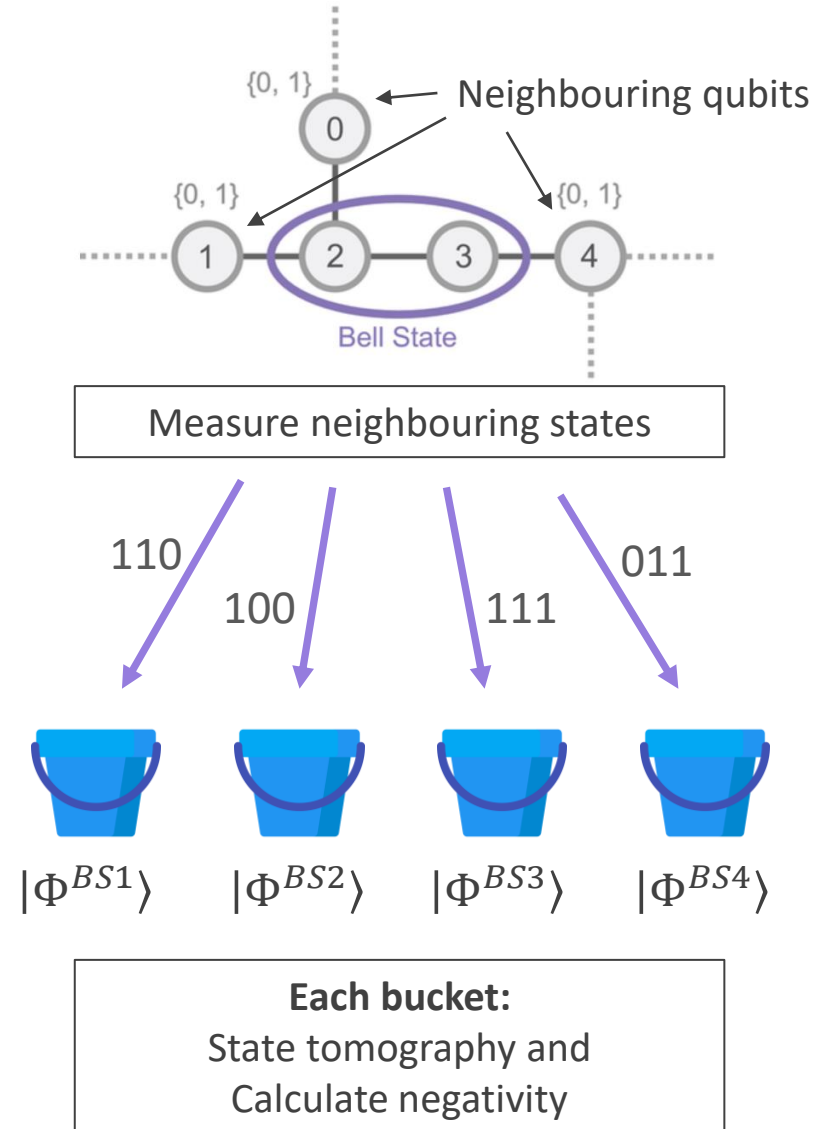
- Projecting neighbours only ever results in 1 of 4 Bell state variations

1. $|\Phi^{BS1}\rangle \equiv H \otimes I |\Phi^+\rangle = (|00\rangle + |01\rangle + |10\rangle - |11\rangle) / 2$
2. $|\Phi^{BS2}\rangle \equiv H \otimes X |\Phi^+\rangle = (|00\rangle + |01\rangle - |10\rangle + |11\rangle) / 2$
3. $|\Phi^{BS3}\rangle \equiv XH \otimes I |\Phi^+\rangle = (|00\rangle - |01\rangle + |10\rangle + |11\rangle) / 2$
4. $|\Phi^{BS4}\rangle \equiv XH \otimes X |\Phi^+\rangle = (|00\rangle - |01\rangle - |10\rangle - |11\rangle) / 2$

- Bucket based on obtained Bell state
- Perform 2-qubit tomography
- Requires only $3^2 = 9$ circuits per pair

• 127-qubit Washington:

- From $\sim 28,000$ circuits
 - down to **1,278 circuits!**
 - A factor of 22 saving





Optimisation 2: Parallel Quantum State Tomography

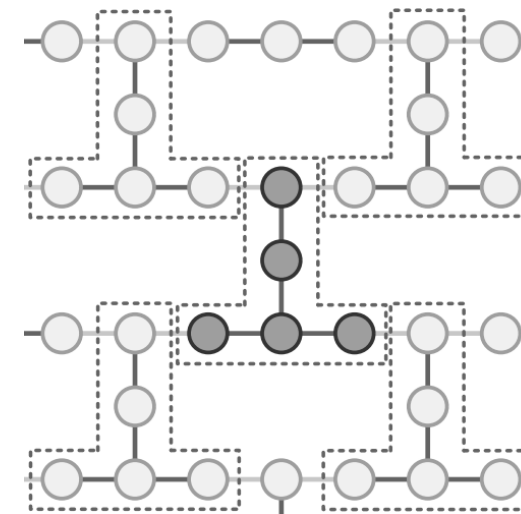
Currently

- Tomography on each qubit pair separately

Instead

- Perform in parallel on non-overlapping sets of qubit-pairs and their neighbours
- Only need up to **8 rounds** for IBM Quantum heavy-hexagonal layouts
 - Using layout-agnostic greedy algorithm
- Can reduce to **4 rounds** when sharing neighbours
- **127-qubit Washington:**
 - Originally ~28,000 circuits
 - Then 1,278 circuits (neighbour-state bucketing)
 - down to **constant 36 circuits!** (Parallel-QST)
- 36 circuits for all heavy hex layouts

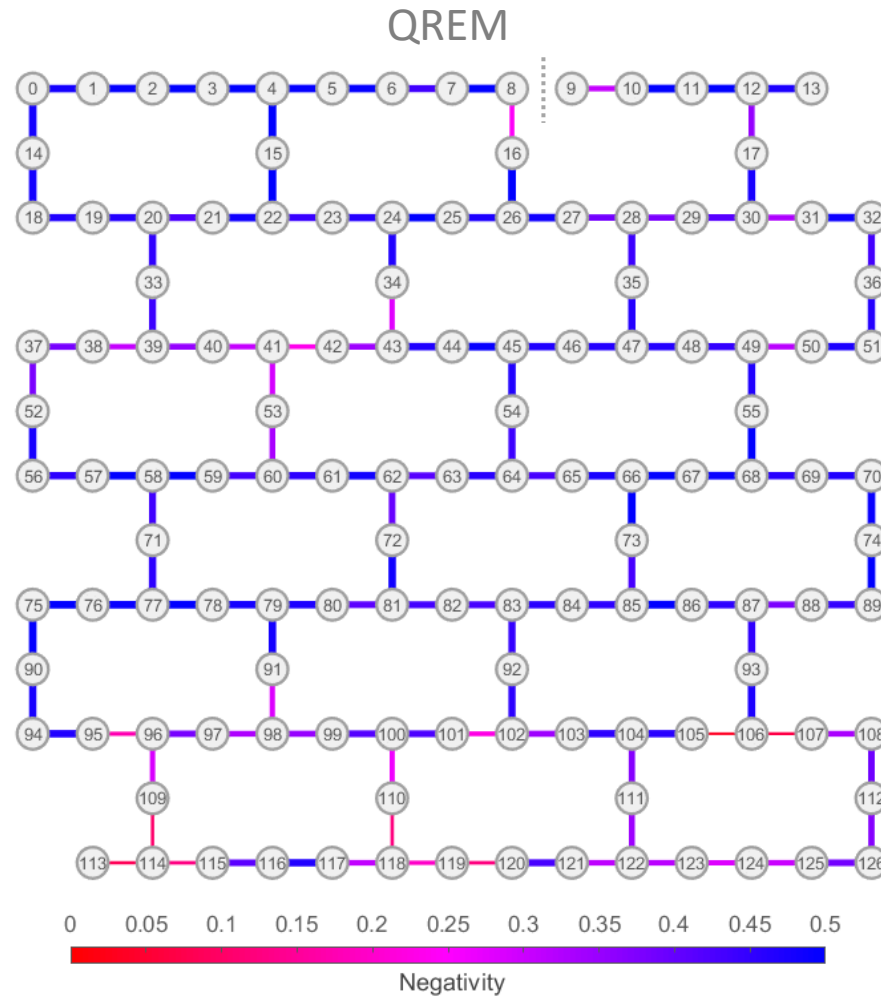
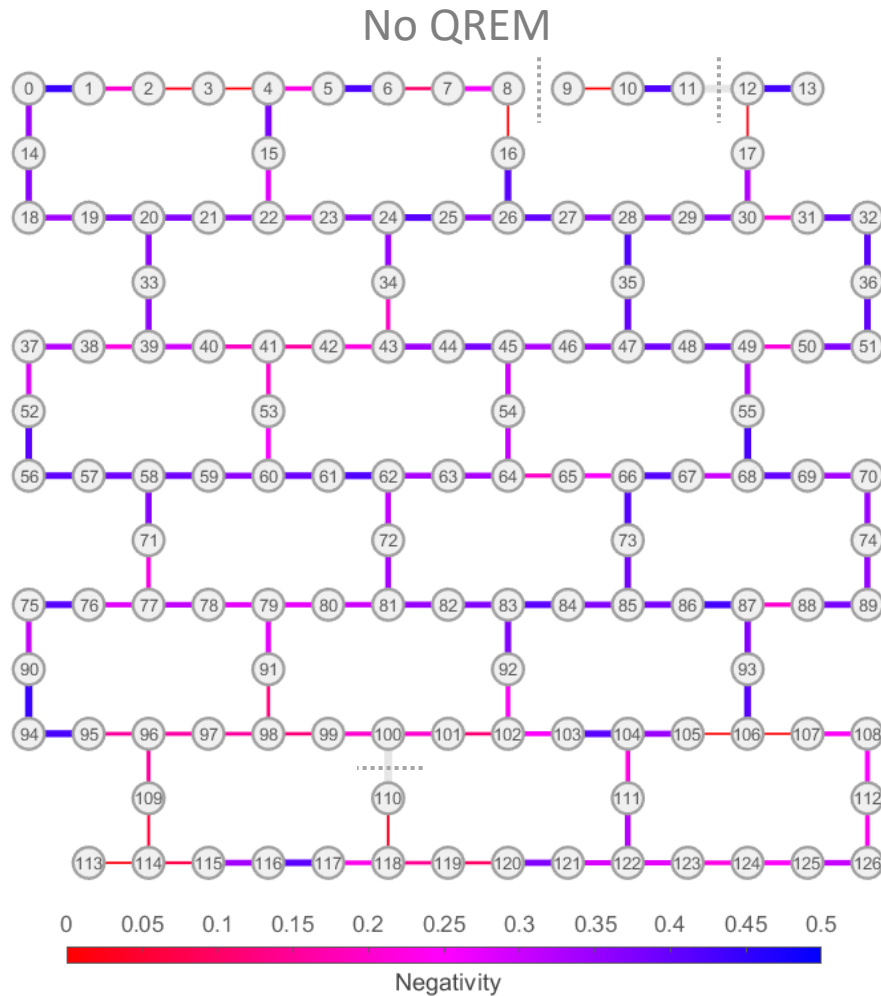
Hardware Layout



non-overlapping regions
performed in parallel

Results: 127-qubit *ibm_washington* Device

127-qubit *ibm_washington* device



127 qubits

→ Whole-device
bipartite entanglement

John Fidel Kam *et al.*,
(paper in production)

Latest: 433-qubit *ibm_seattle* Device

Device (*ibm_seattle*):

- 19 inactive qubits
- 433 → 414 active qubits

Error rates (mean):

- Readout: 7.6%, $\sigma = 7.4\%$
- CNOT: 2.9%, $\sigma = 3.4\%$

Decoherence times:

- $T_1 = \sim 90 \mu\text{s}$ (relaxation)
- $T_2 = \sim 60 \mu\text{s}$ (dephasing)

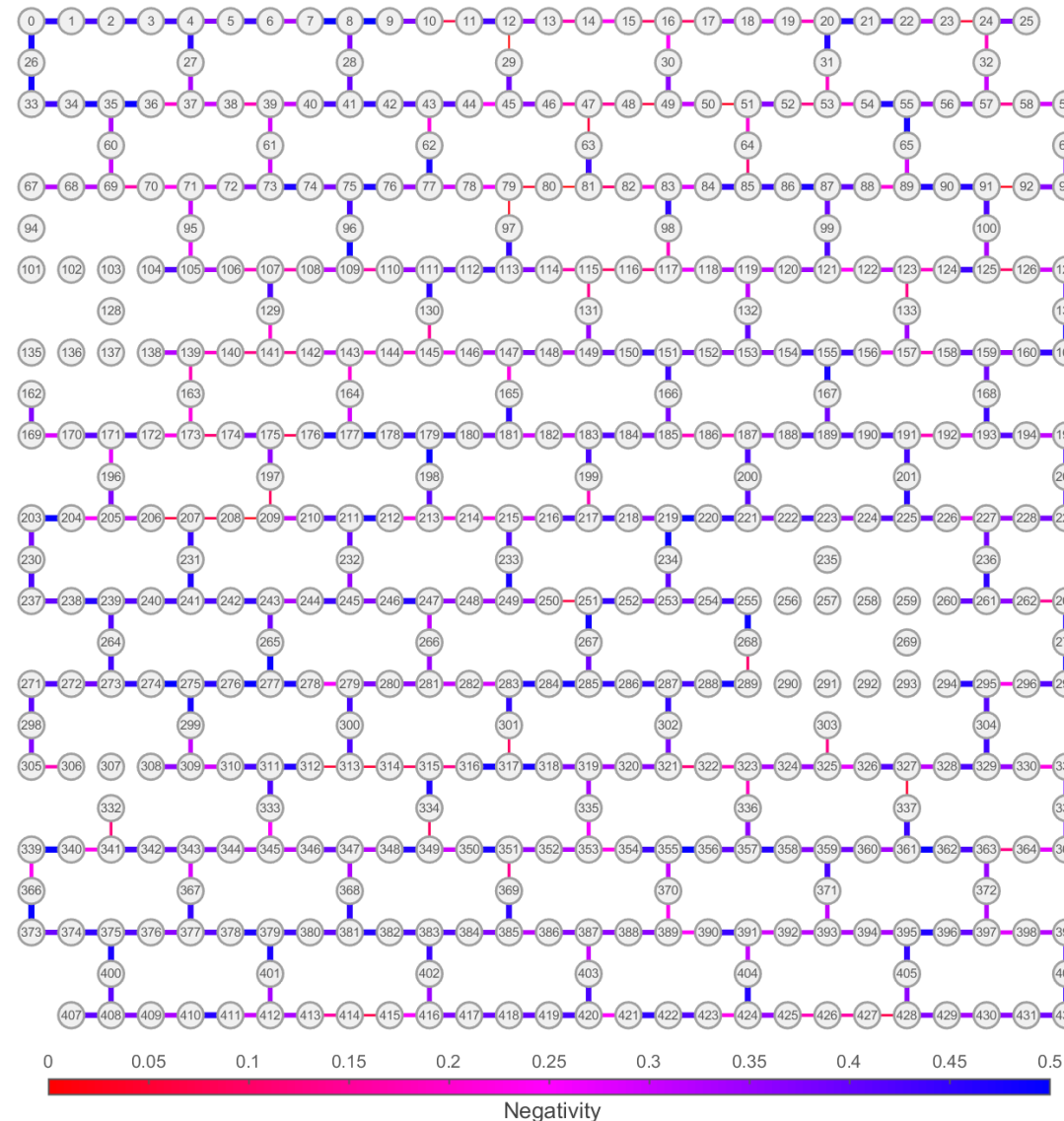
Experiment:

Originally $\sim 100,000$ circuits

➤ down to **36 circuits!**

Result:

- Graph state is bipartite entangled
- All active qubits



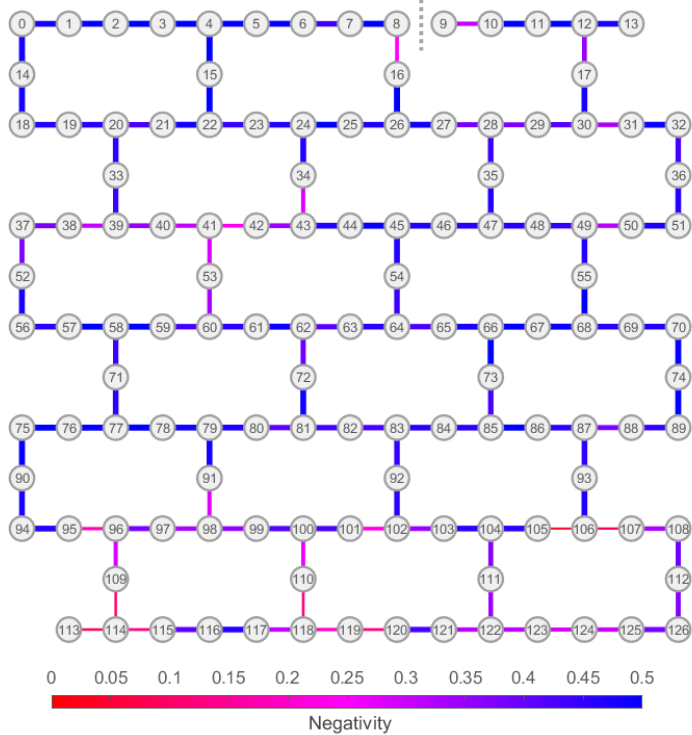
John Fidel Kam *et al.*,
(paper in production)



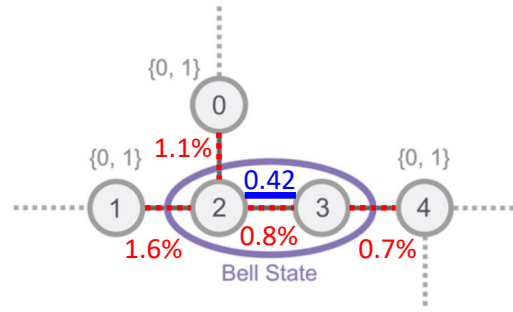
Relation to physical device

Benchmark: how does it relate to device?

127-qubit *ibm_washington*



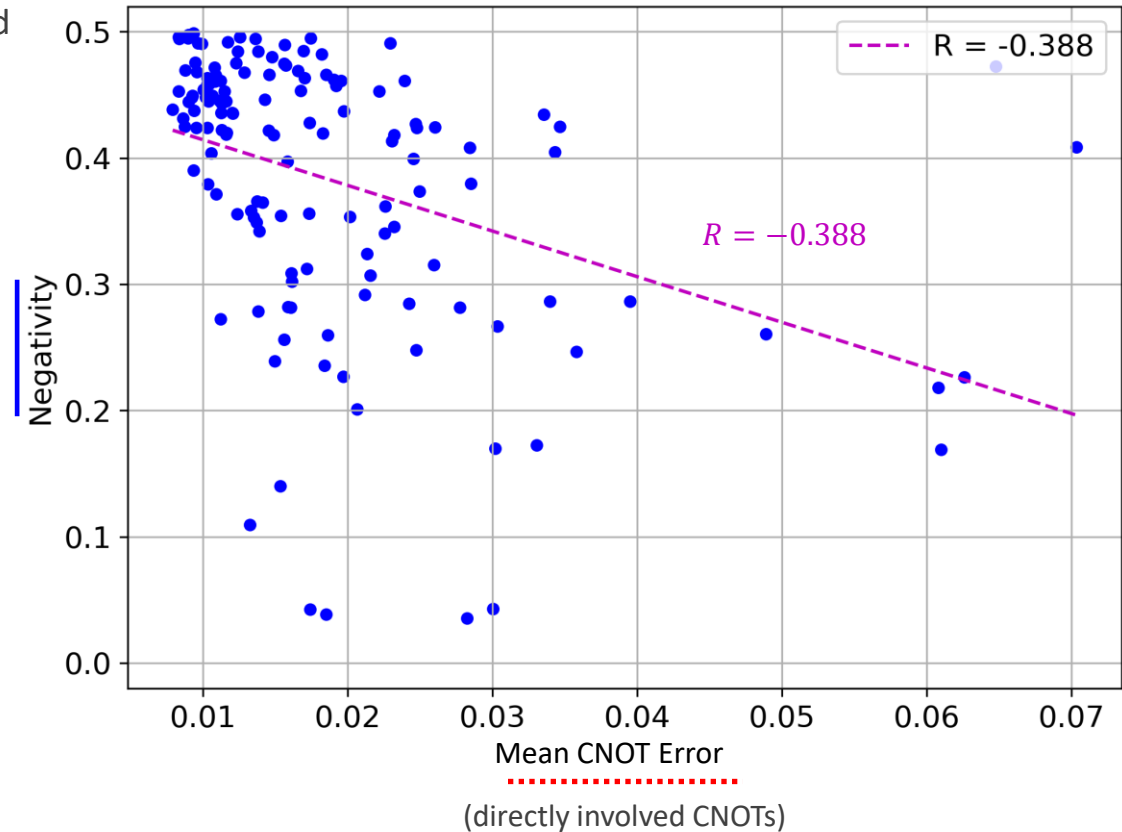
Errors of all CNOTs directly involved



Compare with

Negativity of qubit pair

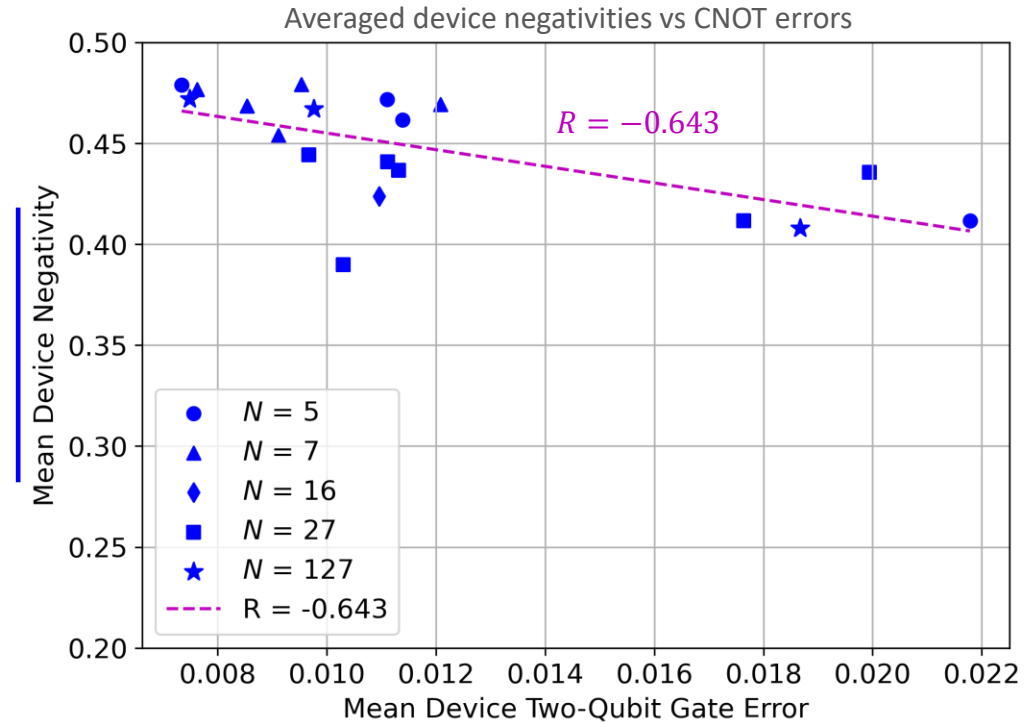
Individual couplings for 127-qubit *ibm_washington*



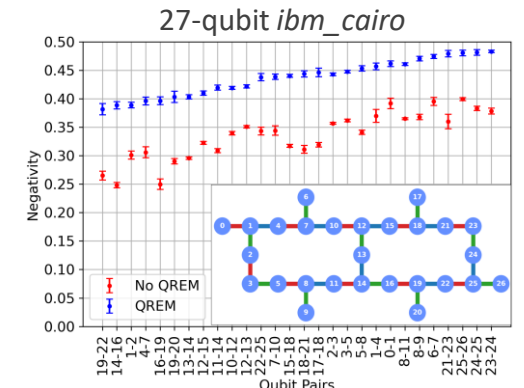
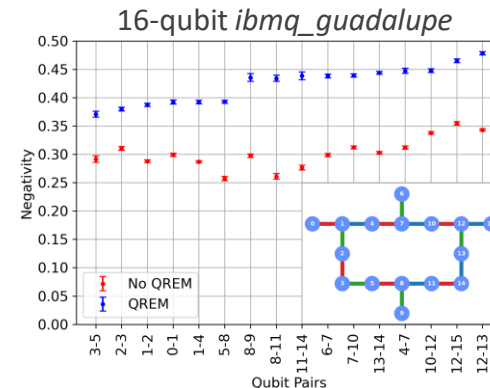
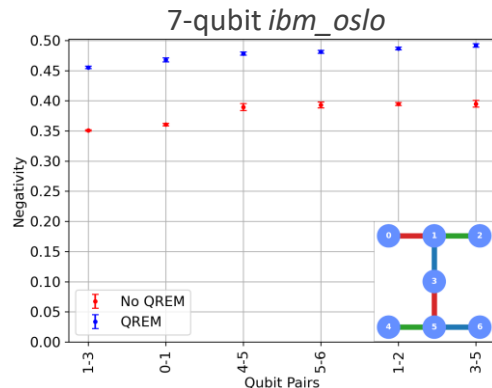
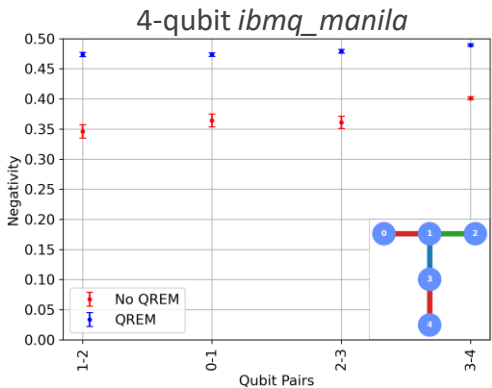
All Device Errors vs. Entanglement

Device Negativities (readout-error mitigated)

| Device | Qubits | QV | Mean \mathcal{N} | Whole-Device |
|-------------------|--------|-----|--------------------|--------------|
| <i>lima</i> | 5 | 8 | 0.470 ± 0.011 | ✓ |
| <i>belem</i> | 5 | 16 | 0.427 ± 0.010 | ✓ |
| <i>quito</i> | 5 | 16 | 0.486 ± 0.010 | ✓ |
| <i>manila</i> | 5 | 32 | 0.487 ± 0.003 | ✓ |
| <i>jakarta</i> | 7 | 16 | 0.482 ± 0.007 | ✓ |
| <i>oslo</i> | 7 | 32 | 0.488 ± 0.010 | ✓ |
| <i>nairobi</i> | 7 | 32 | 0.488 ± 0.004 | ✓ |
| <i>lagos</i> | 7 | 32 | 0.466 ± 0.008 | ✓ |
| <i>perth</i> | 7 | 32 | 0.482 ± 0.011 | ✓ |
| <i>guadalupe</i> | 16 | 32 | 0.447 ± 0.032 | ✓ |
| <i>toronto</i> | 27 | 32 | 0.403 ± 0.075 | ✓ |
| <i>geneva</i> | 27 | 32 | 0.461 ± 0.089 | ✓ |
| <i>hanoi</i> | 27 | 64 | 0.467 ± 0.026 | ✓ |
| <i>auckland</i> | 27 | 64 | 0.437 ± 0.060 | ✓ |
| <i>cairo</i> | 27 | 64 | 0.455 ± 0.026 | ✓ |
| <i>mumbai</i> | 27 | 128 | 0.460 ± 0.078 | ✓ |
| <i>montreal</i> | 27 | 128 | 0.424 ± 0.055 | ✓ |
| <i>kolkata</i> | 27 | 128 | 0.407 ± 0.134 | ✓ |
| <i>washington</i> | 127 | 64 | 0.403 ± 0.125 | ✓ |
| <i>Seattle</i> | 433 | - | 0.340 | ✓ (active) |



Quantum Volume





Overview

Benchmarking quantum devices

Forms of Multipartite Entanglement

- Bipartite and Genuine Multipartite entanglement

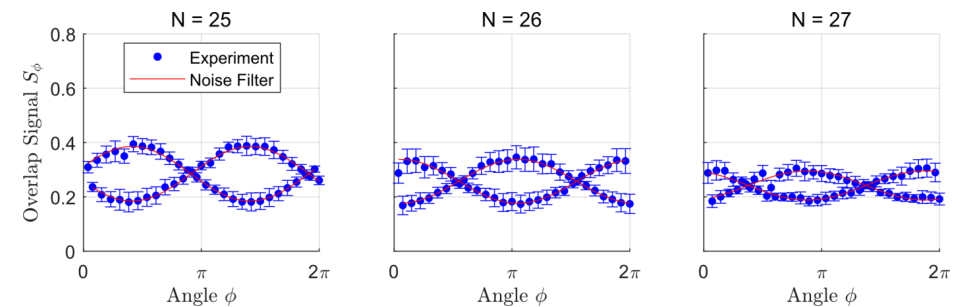
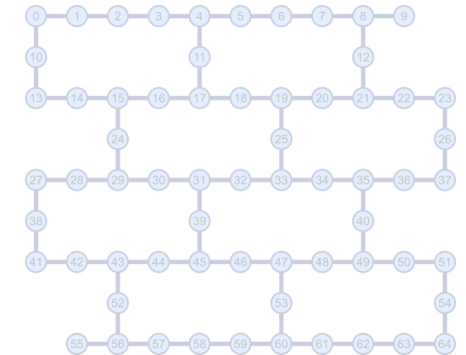
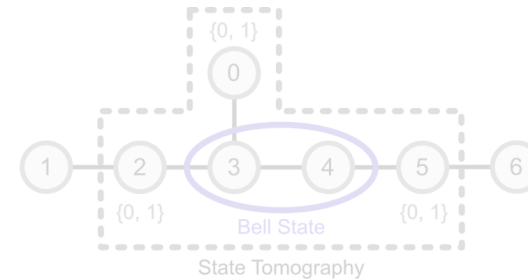
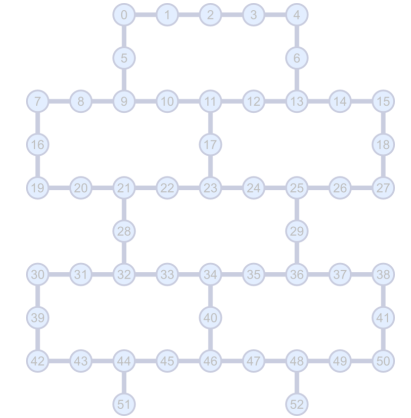
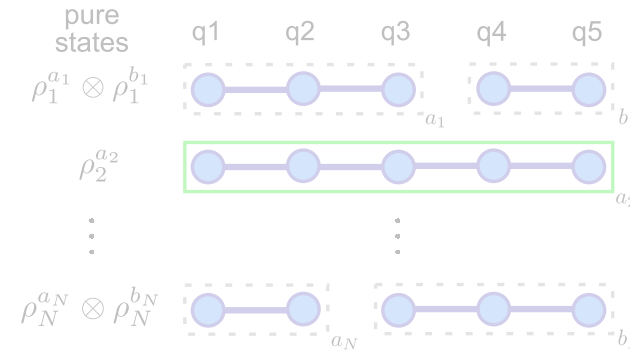
Detecting **Bipartite** entanglement

- By preparing Graph states on *IBM Quantum* devices

Detecting **Genuine multipartite** entanglement

- By preparing GHZ states
- GHZ decoherence rates

Bell state teleportation





GHZ States

- GHZ State → generalisation of Bell state to more qubits

$$|Bell\rangle = \frac{|0\rangle^{\otimes 2} + |1\rangle^{\otimes 2}}{\sqrt{2}} \rightarrow |GHZ\rangle_N = \frac{|0\rangle^{\otimes N} + |1\rangle^{\otimes N}}{\sqrt{2}}$$

- Conveniently sparse density matrix
- Sensitive to noise
 - Disentangled after only 1 local measurement

$$\frac{|000\rangle + |111\rangle}{\sqrt{2}} \rightarrow \{|000\rangle, |111\rangle\}$$

Density matrix (2 qubits)

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

Coherence

Population

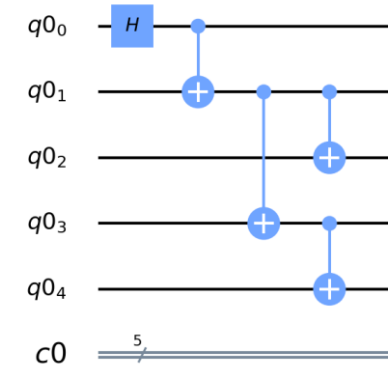
GHZ State density matrix (4-qubit)

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} / 2$$

Graph State density matrix (4-qubit)

$$\begin{pmatrix} 1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 \\ -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 \\ -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 \end{pmatrix} / 8$$

Preparation



$$\text{Hadamard} \rightarrow \frac{(|0\rangle + |1\rangle)|00\dots 0\rangle}{\sqrt{2}}$$

Grow the state with CNOTs

$$\text{CNOT} \rightarrow \frac{(|00\rangle + |11\rangle)|0\dots 0\rangle}{\sqrt{2}}$$

$$\text{CNOT} \rightarrow \frac{(|000\rangle + |111\rangle)|\dots 0\rangle}{\sqrt{2}}$$

All-to-all connectivity:

$$(\text{CNOT Depth}) \approx \log_2 N$$

Cruz et al., Adv. Quantum Technol (2019)

IBM Quantum Heavy Hex Layout:

$$(\text{CNOT Depth}) \approx \sqrt{2N}$$

Yang et al., IEEE J. Emerg. Sel. Topics in Circuits and Systems (2022)



Genuine Multipartite Entanglement (GME) in GHZ states

GHZ Fidelity (> 0.5) \rightarrow GME

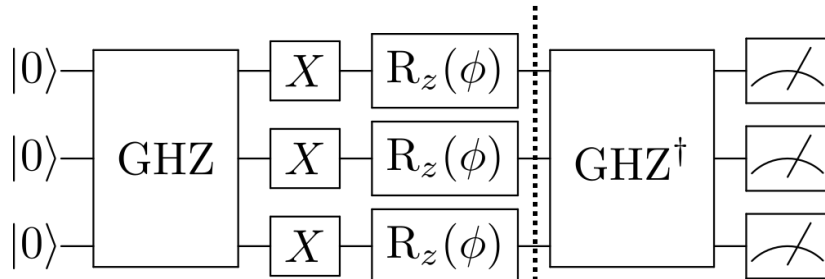
Fidelity = (Population)/2 + (Coherence)/2

- Population: Occupancies of $|00 \dots 0\rangle$ and $|11 \dots 1\rangle$
- Coherence: Multiple Quantum Coherences (MQC) *Wei et al., Phys. Rev. A (2020)*

Ideal GHZ State

| | | |
|--|--|---|
| State Vector | Density Matrix | |
| $\frac{ 00 \dots 0\rangle + 11 \dots 1\rangle}{\sqrt{2}}$ | $\begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$ | <div style="display: flex; justify-content: space-between; align-items: center;"> Coherence Population </div> |

Multiple Quantum Coherences (MQC)



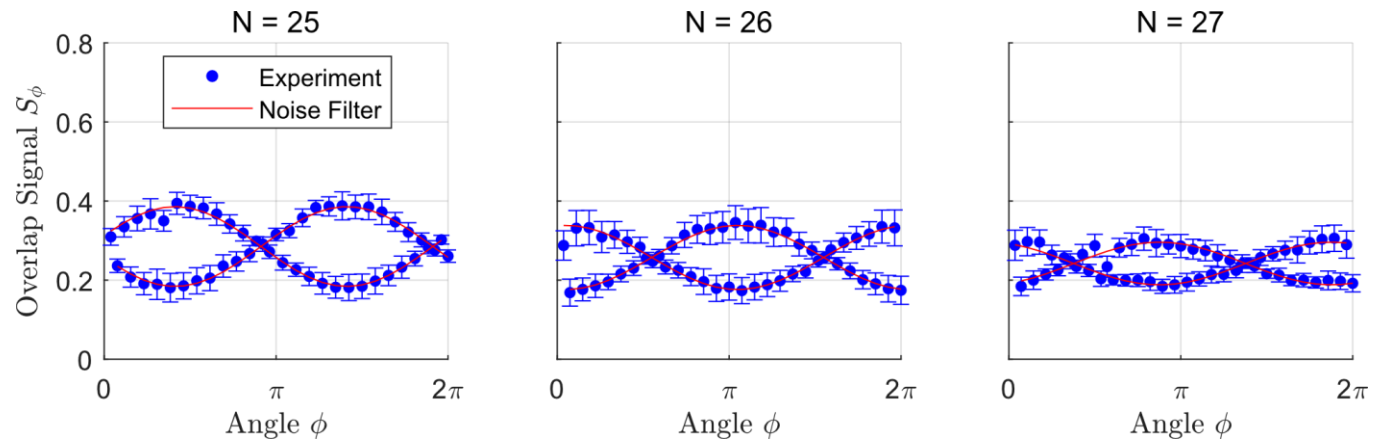
Hahn Echo: Refocussing π -pulse
(Reduce qubit phase errors)

Ideal state: phase of $N\phi$

$$\frac{|00 \dots 0\rangle + e^{iN\phi}|11 \dots 1\rangle}{\sqrt{2}}$$

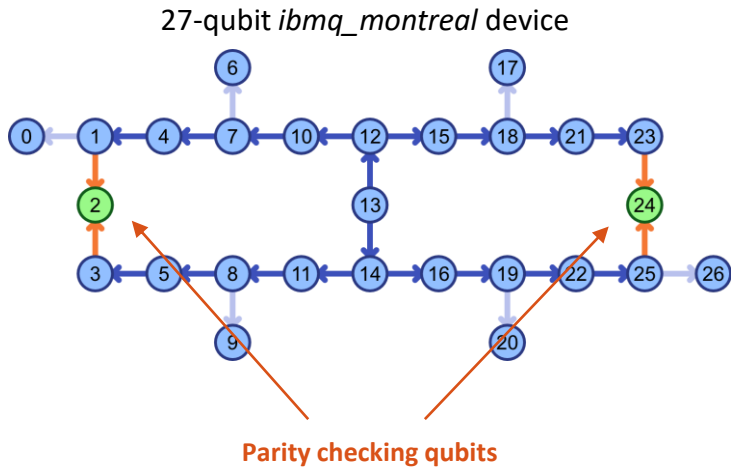
Overlap Signal:
Occupancy of $|00 \dots 0\rangle$

Coherence = $2\sqrt{I_N}$
 I_N : Amplitude of overlap signals



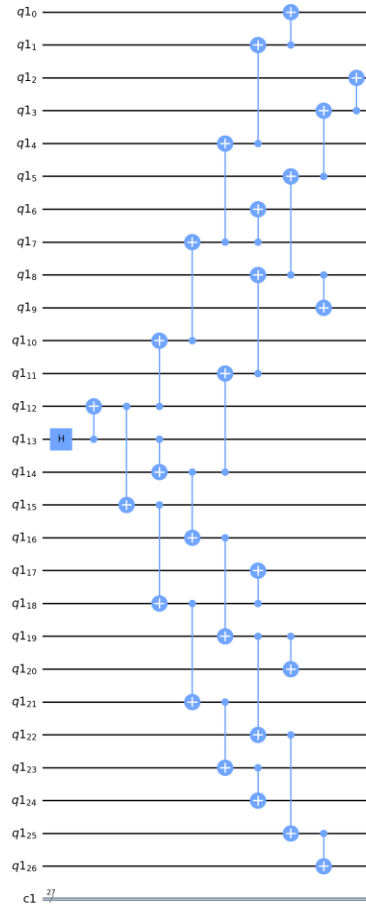
Results: On the 27-qubit *ibmq_montreal* device

- Prepare on device
- Add parity verification
→ effects on fidelity
→ discard when measure 1

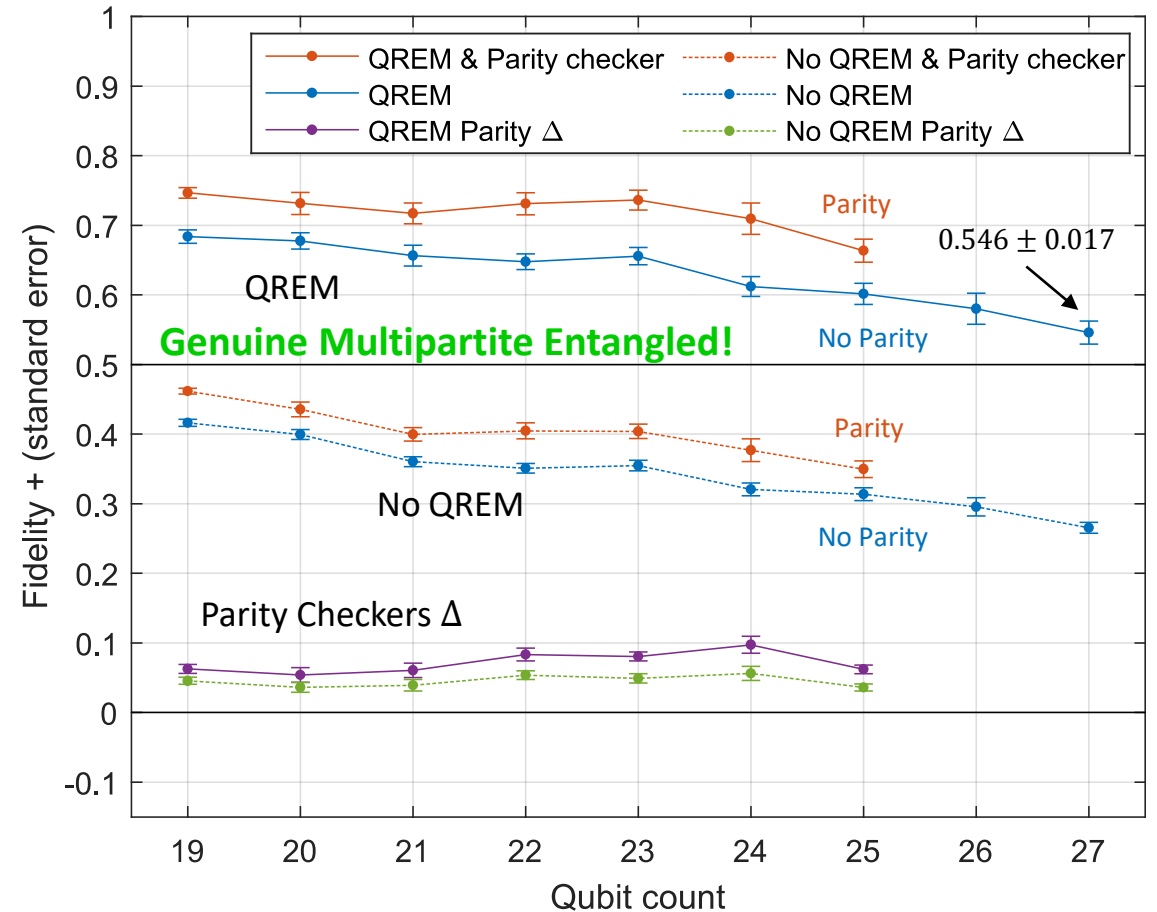


- 27-qubit GME, whole device!
- Next: Scale to larger devices

GHZ state circuit



Fidelity with 2 parity checker qubits (q2 and q24)



QREM : Quantum Readout-Error Mitigation

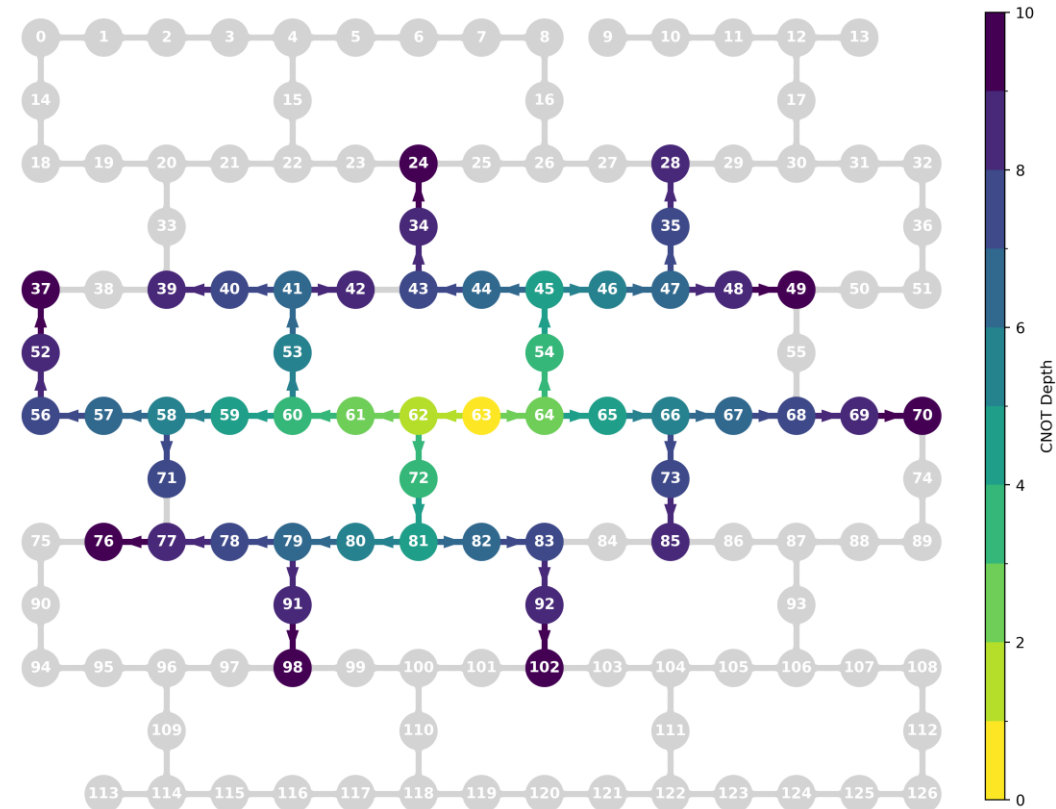
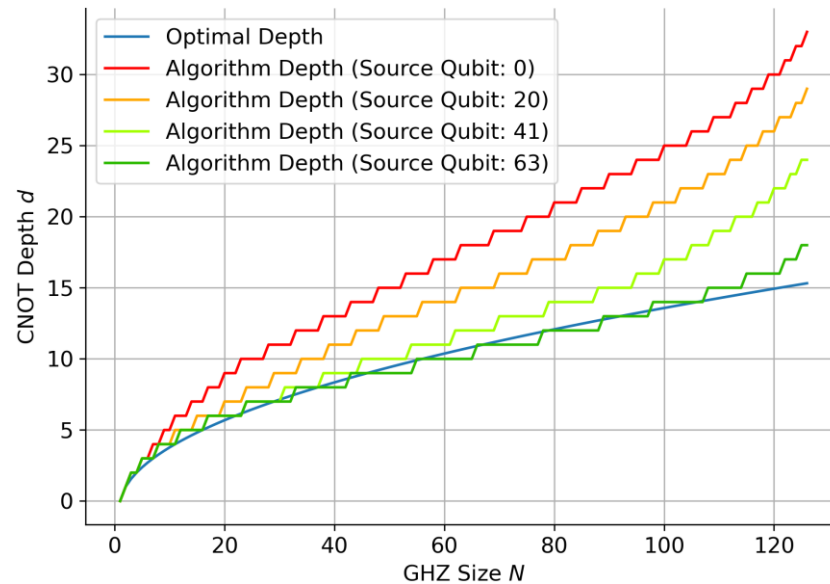
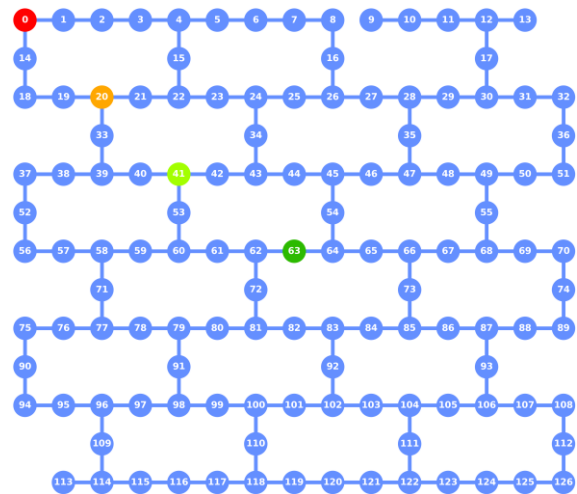


GHZ Embedding Algorithm

GHZ State embedding algorithm

1. Select a qubit
 - a) Breadth first search
 - b) Preference CNOTs with low error
2. Repeat 1. for each qubit and choose the best embedding

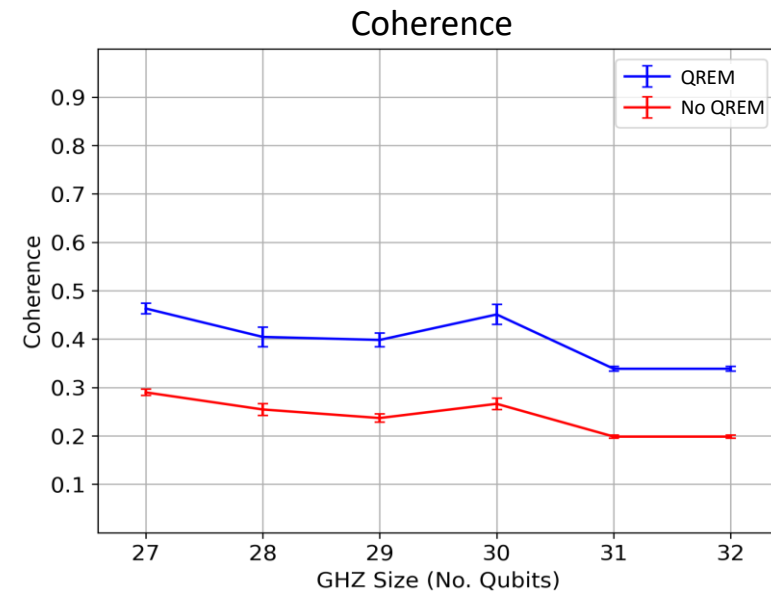
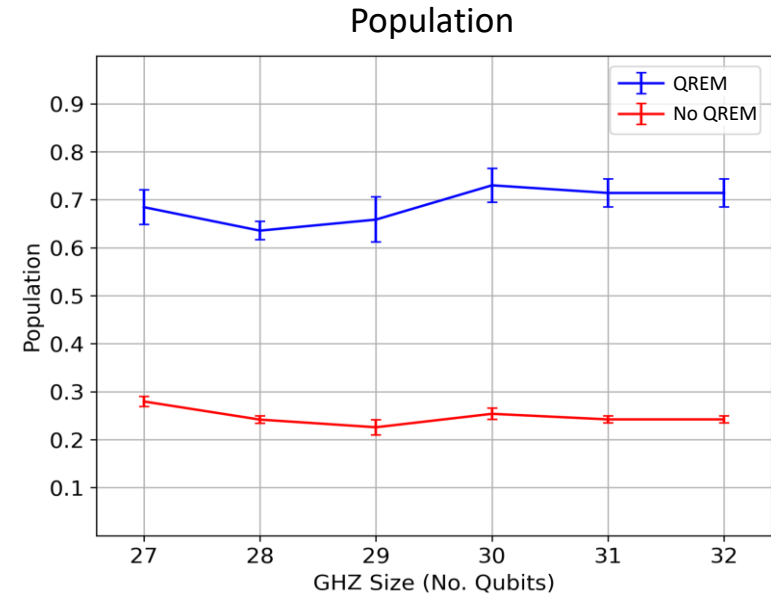
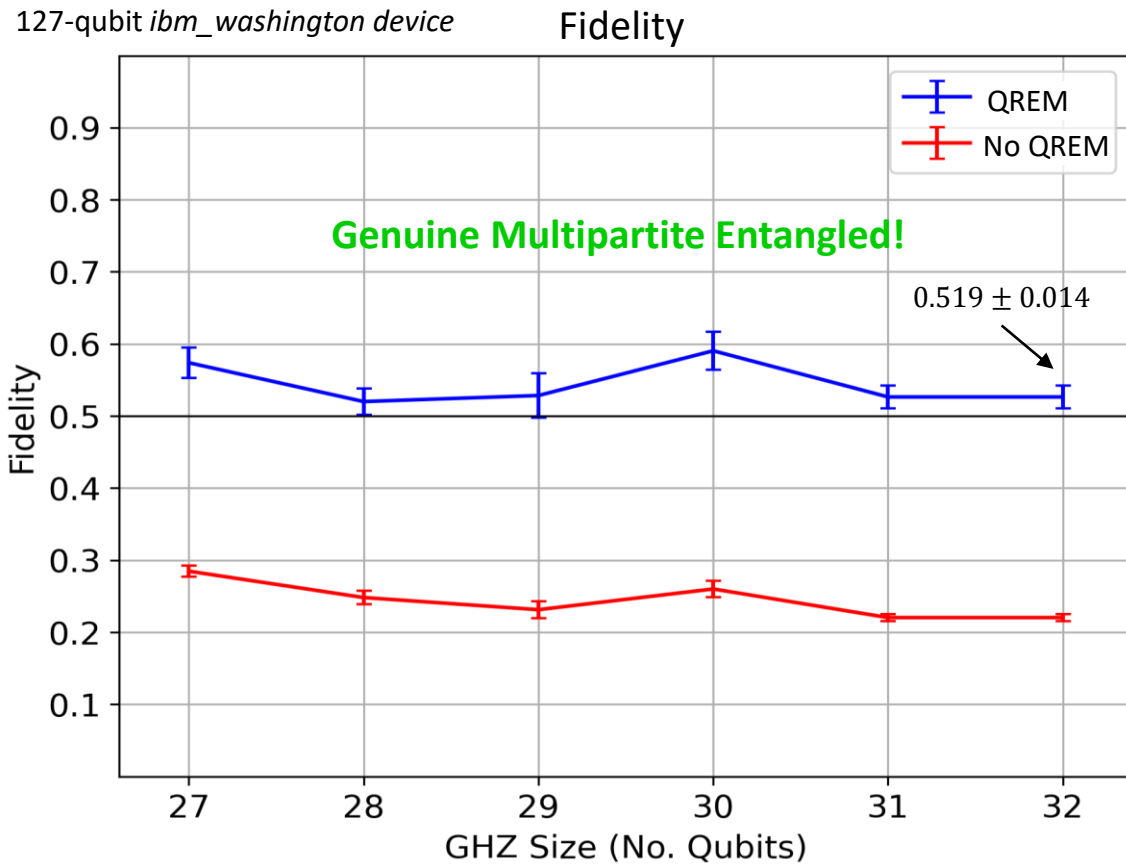
127-qubit *Ibm_Washington* device





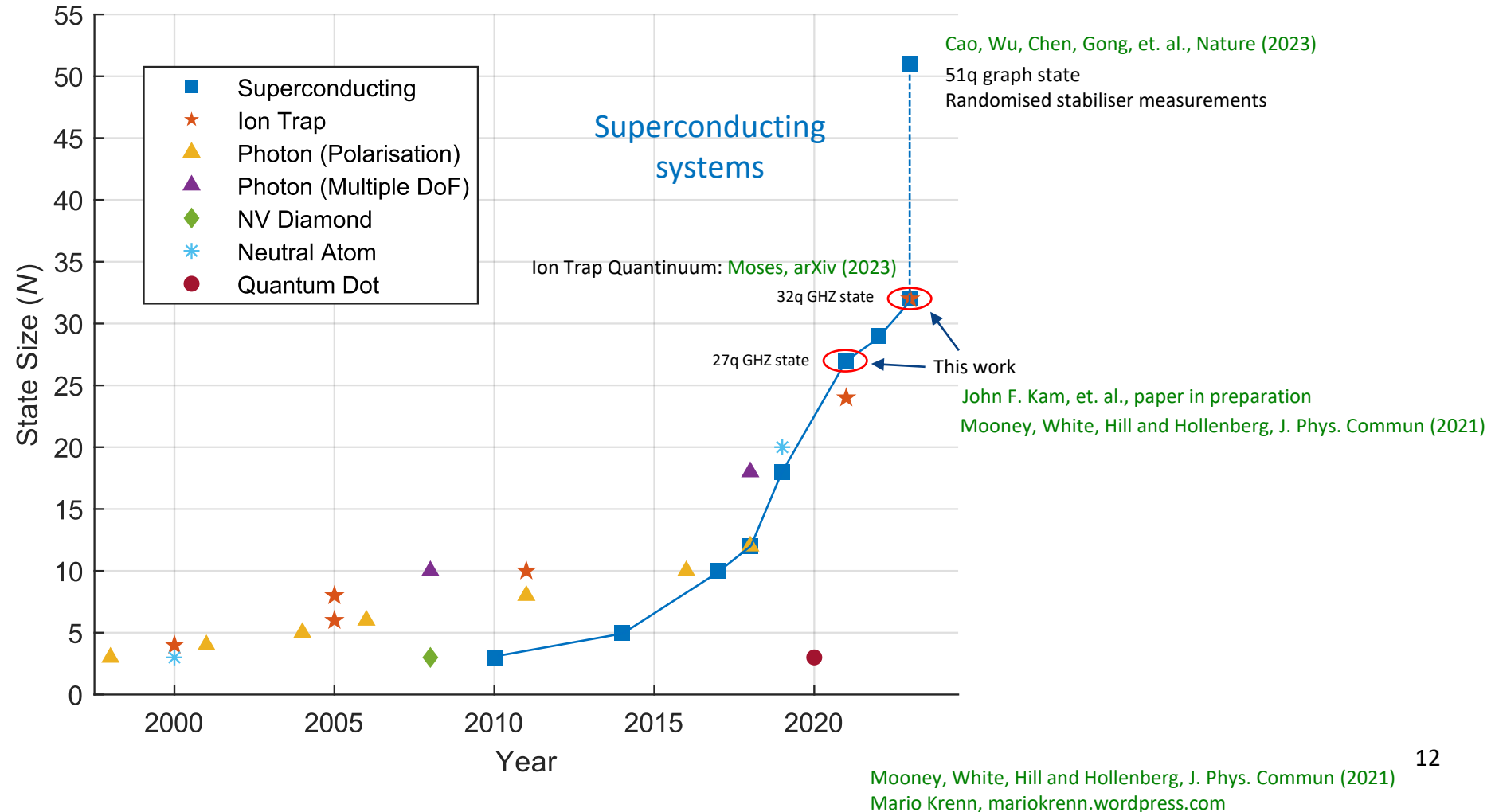
Results: *ibm_washington*

The 32-qubit GHZ is genuine multipartite entangled!



History of Genuine Multipartite Entanglement

Experimentally prepared states shown to exhibit GME

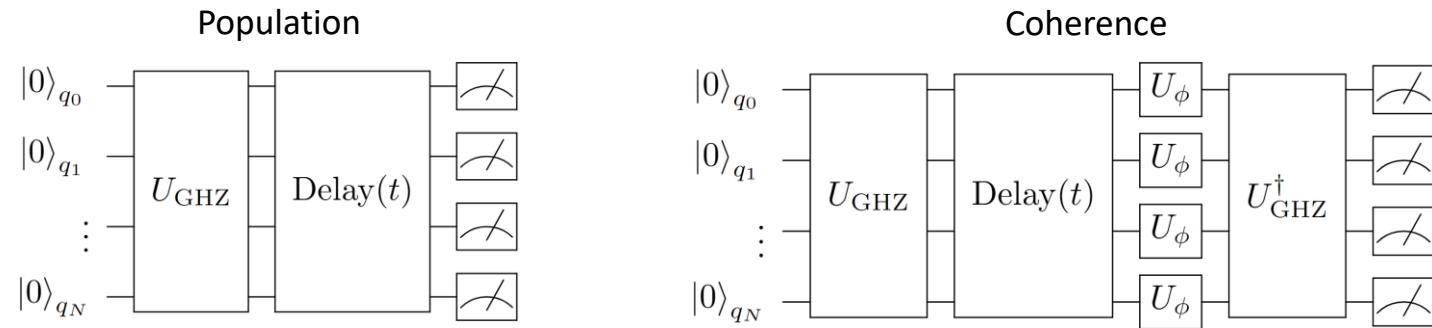


Preserving the GHZ State Fidelity

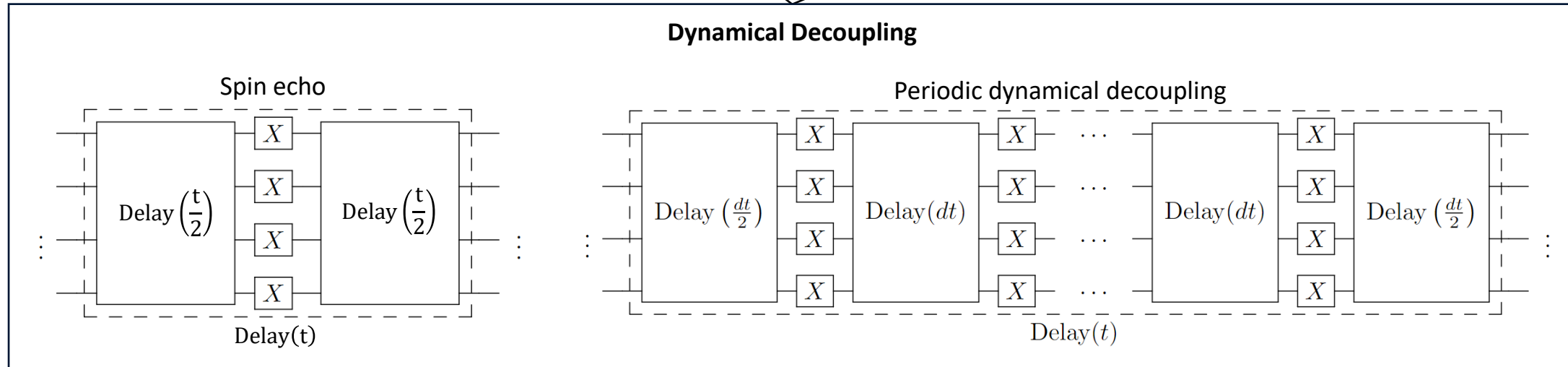
Wait after preparation

- **Spin Echo**
 - One π pulses
- **Dynamical Decoupling**
 - A series of pulses
- **Periodic Dynamical Decoupling**
 - A series of π pulses

Free Delay Circuits



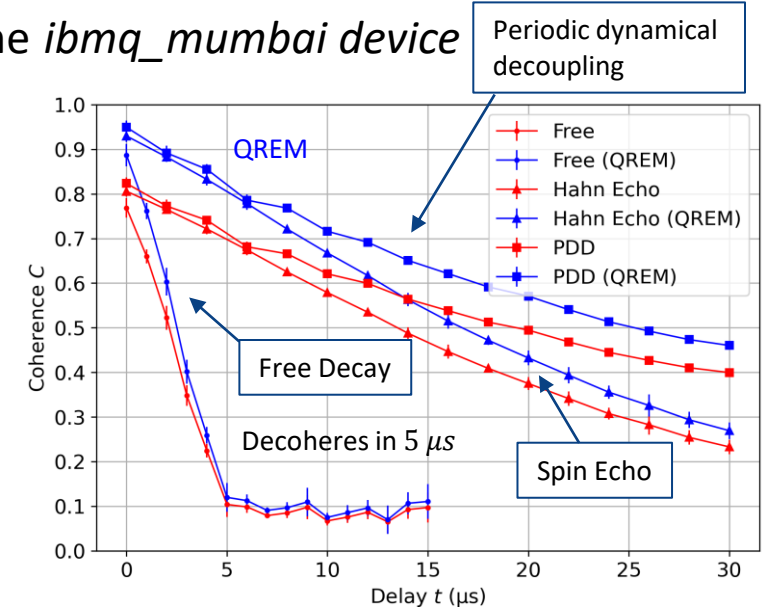
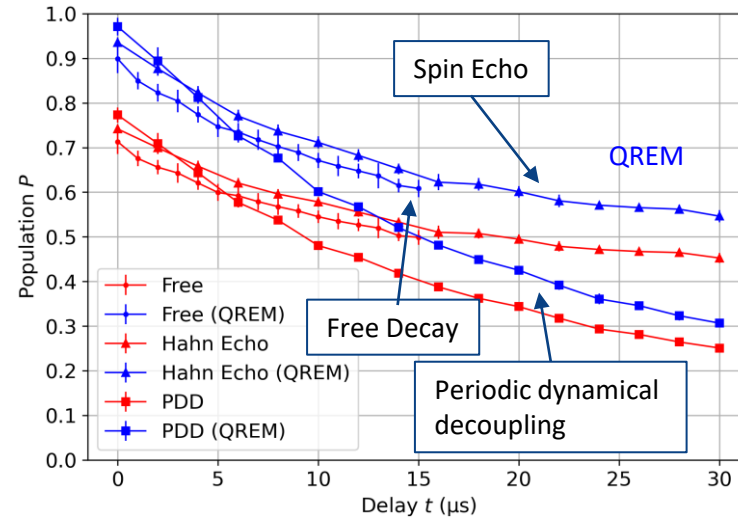
Dynamical Decoupling



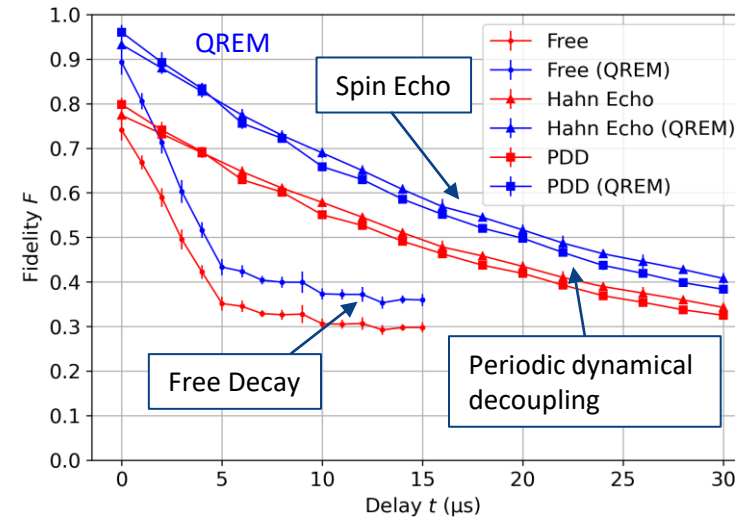
Results: GHZ State Decay Curves

- Decay curves for 7-qubit state
- Measured **Population**
 - No real improvement
 - PDD gates introduce errors
- Measured **Coherence**
 - Big improvement
 - Idle phase are being cancelled
- Measured **Fidelity**
 - **Big Improvement**

7-qubit GHZ state on the *ibmq_mumbai* device



Point of reference:
(CNOT gate time) $\approx 400 \text{ ns}$





Superdecoherence?

- Plot GHZ decoherence vs qubit count
- **Superdecoherence**
 - Qubit decoherence rates scale with system size
 - When qubits are coupled to the same reservoir

$$C^{(N)}(t) = C_0^{(N)} e^{-\alpha^{(N)}t}$$

α : GHZ decoherence rate

- **Previous work**

- An Ion trap system $\rightarrow N^2$ GHZ decoherence rate scaling

- **Superdecoherence**

- Up to 6-qubit GHZ states

Monz, et. al., Phys Rev Lett (2011)

- IBM Quantum device $\rightarrow N$ GHZ decoherence rate scaling

- **No Superdecoherence**

- Up to 8-qubit GHZ states Ozaeta and McMahon, Quantum Sci. Technol. (2019)

16-qubit *ibmq_melbourne*



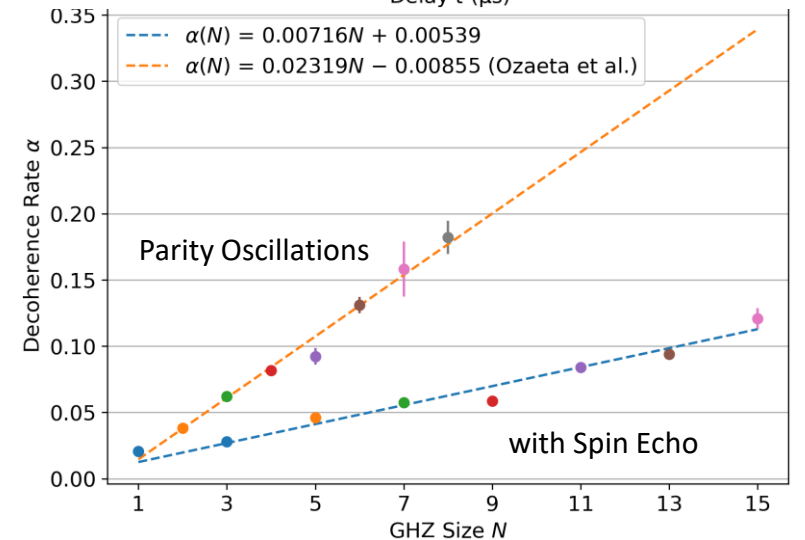
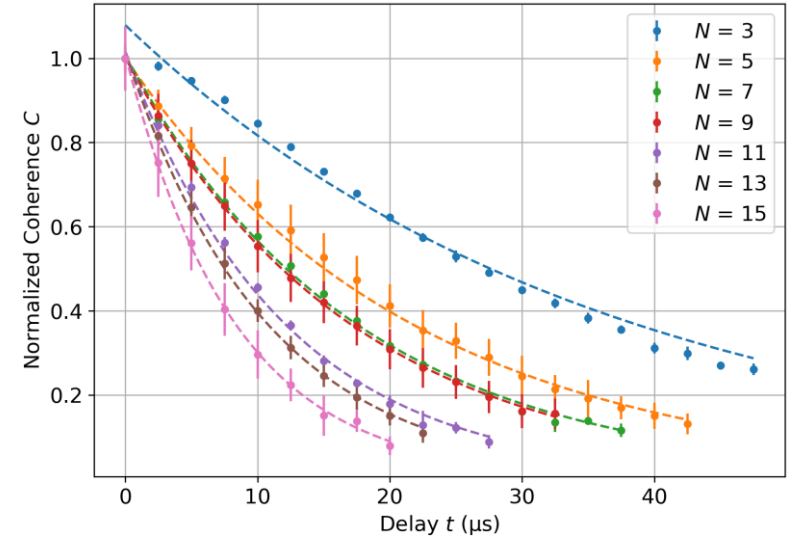
27-qubit *ibm_hanoi*



- Test on current IBM Quantum device

(CNOT gate time) ≈ 400 ns

27-qubit *ibm_hanoi* device



John F. Kam, et. al., paper in preparation



Overview

Benchmarking quantum devices

Forms of Multipartite Entanglement

- Bipartite and Genuine Multipartite entanglement

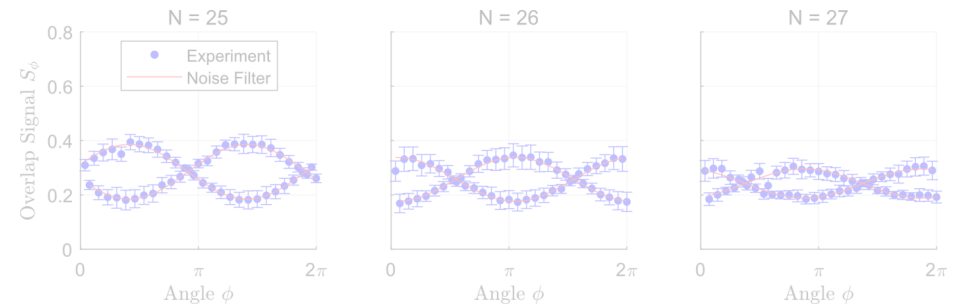
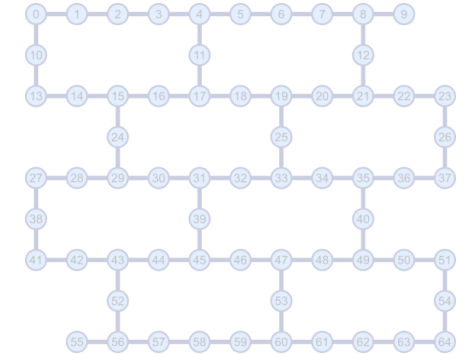
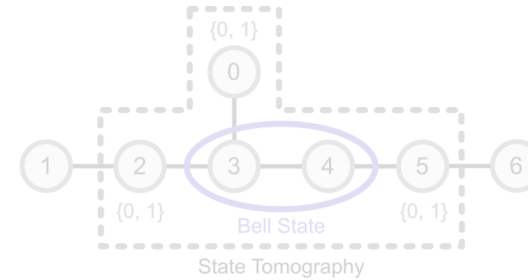
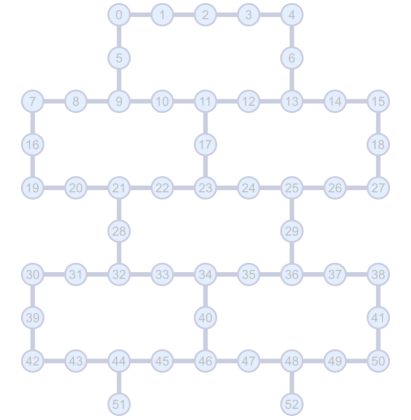
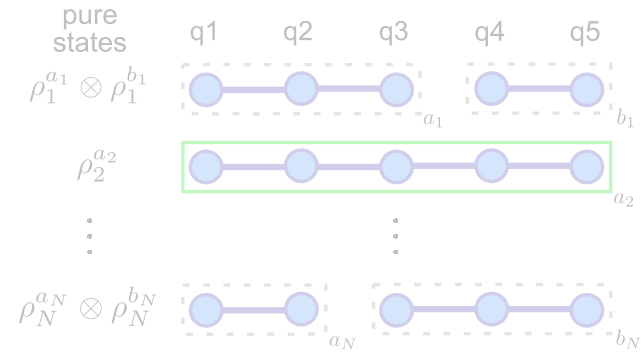
Detecting **Bipartite** entanglement

- By preparing Graph states on *IBM Quantum* devices

Detecting **Genuine** multipartite entanglement

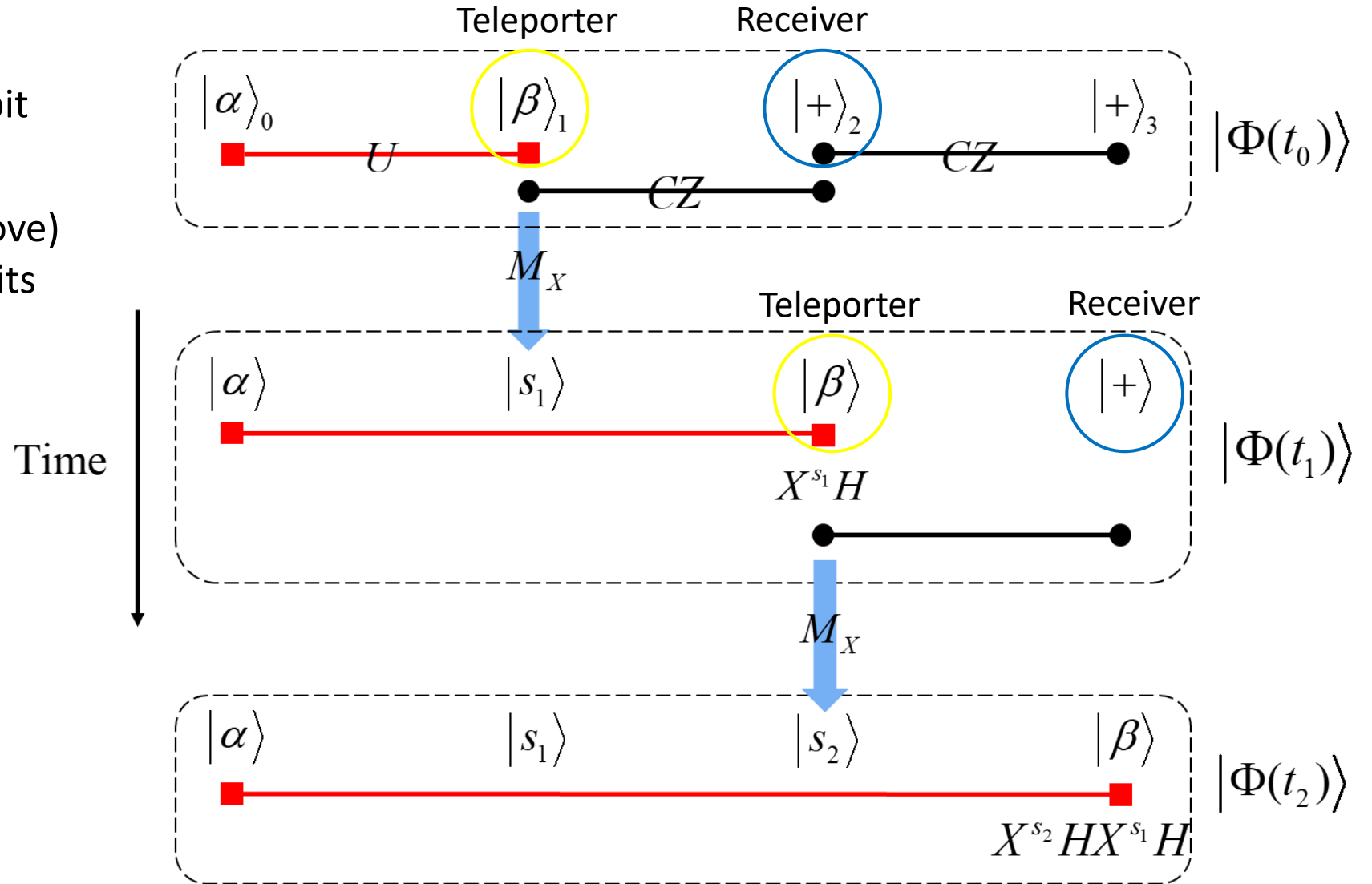
- By preparing GHZ states
- GHZ decoherence rates

Bell state teleportation



One-Way Quantum Computation

- Start with 2 entangled qubit states $|\alpha\rangle_0$ and $|\beta\rangle_1$
- Challenge: **Teleport** (or move) $|\beta\rangle_1$ across a chain of qubits
 - Keeping $|\alpha\rangle_0$ and $|\beta\rangle_1$ entangled
- Compare **3 approaches**
 - Swap gates
 - Teleportation: Dynamic circuits
 - Teleportation: Post-selection

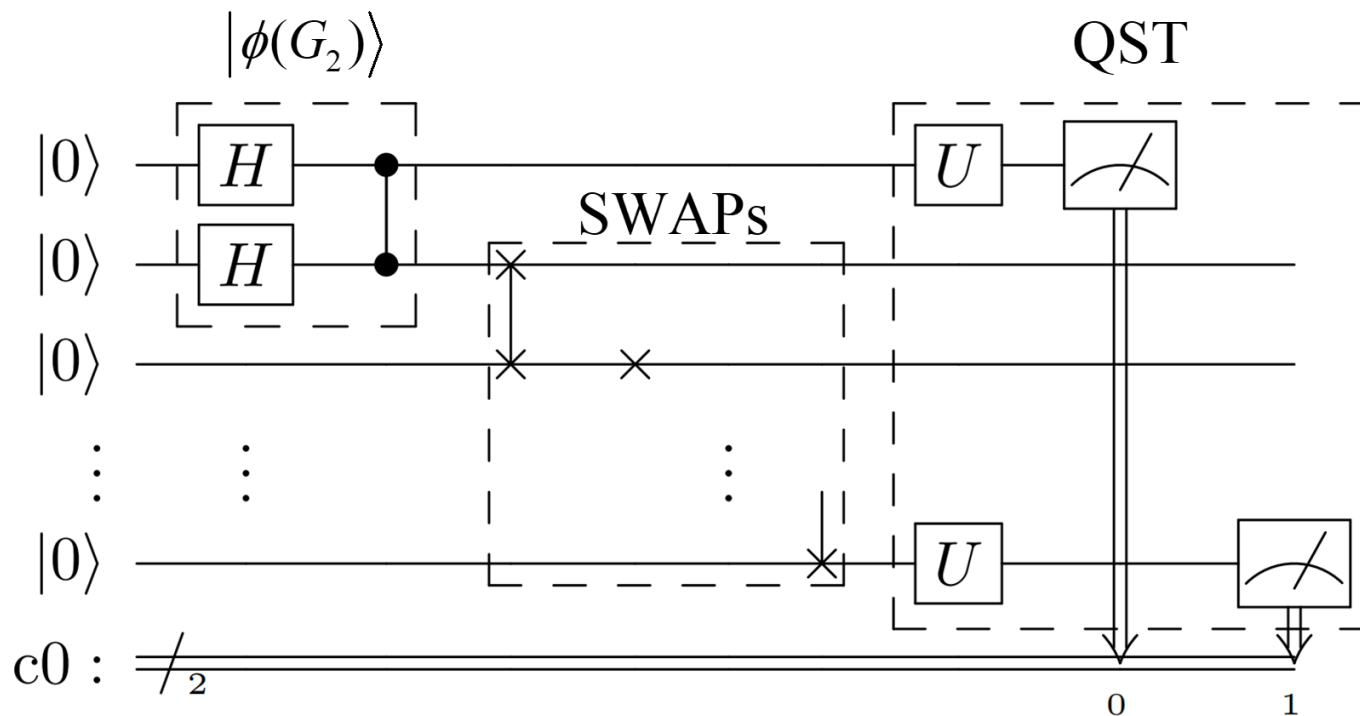




Using SWAP gates

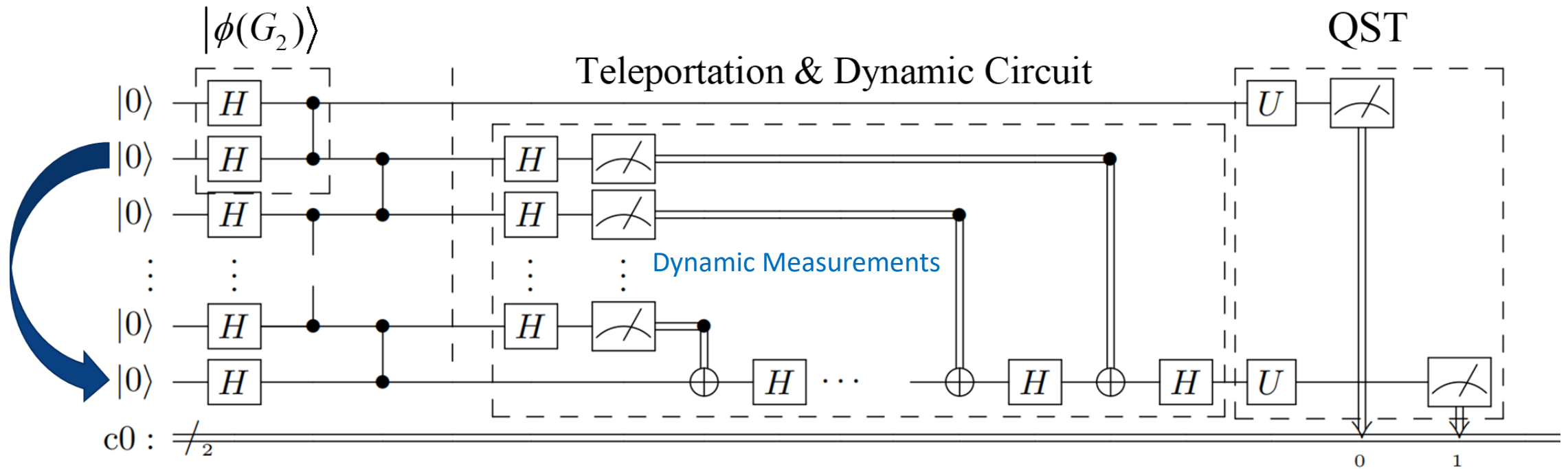
- Start with 2-qubit Bell state
- Use SWAP gates to transport the state along a chain
- Circuit depth scales with chain length

$$SWAP_{12} |q_1 q_2\rangle = |q_2 q_1\rangle$$



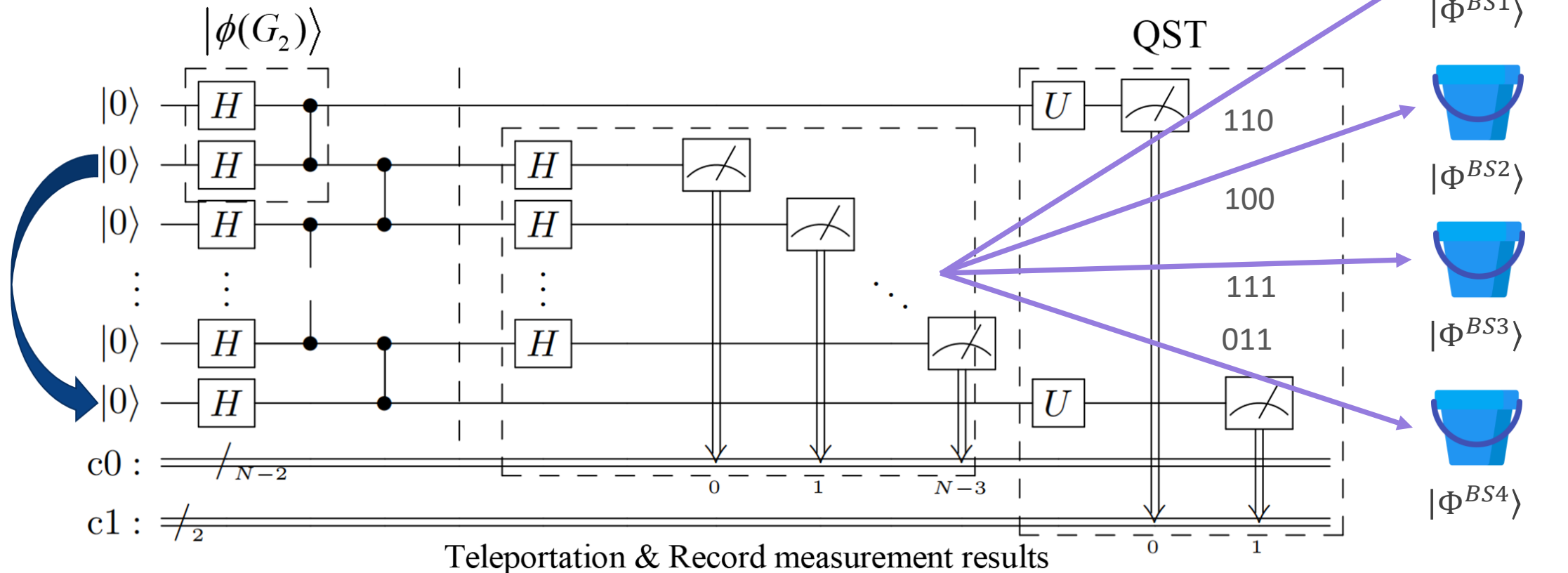
Teleportation: Dynamic Circuit

- Corrections are applied after each measurement
- Depth grows with qubit-measurement count



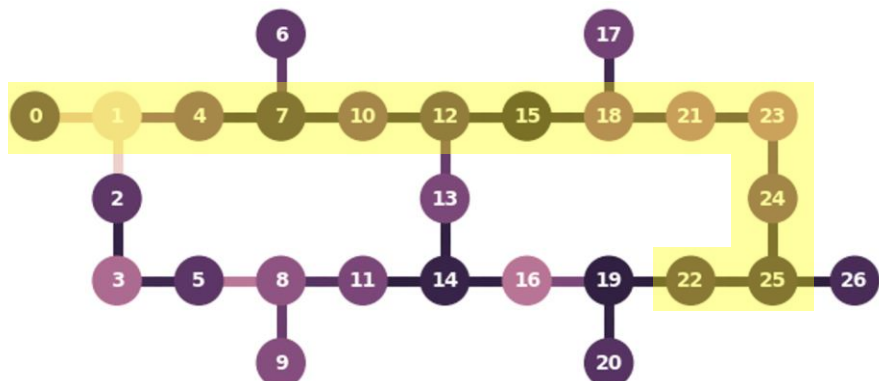
Teleportation: Post-Selection Circuit

- Bucket the teleported qubits into four possible Bell state variations
- Only 4 combinations → repeat until success

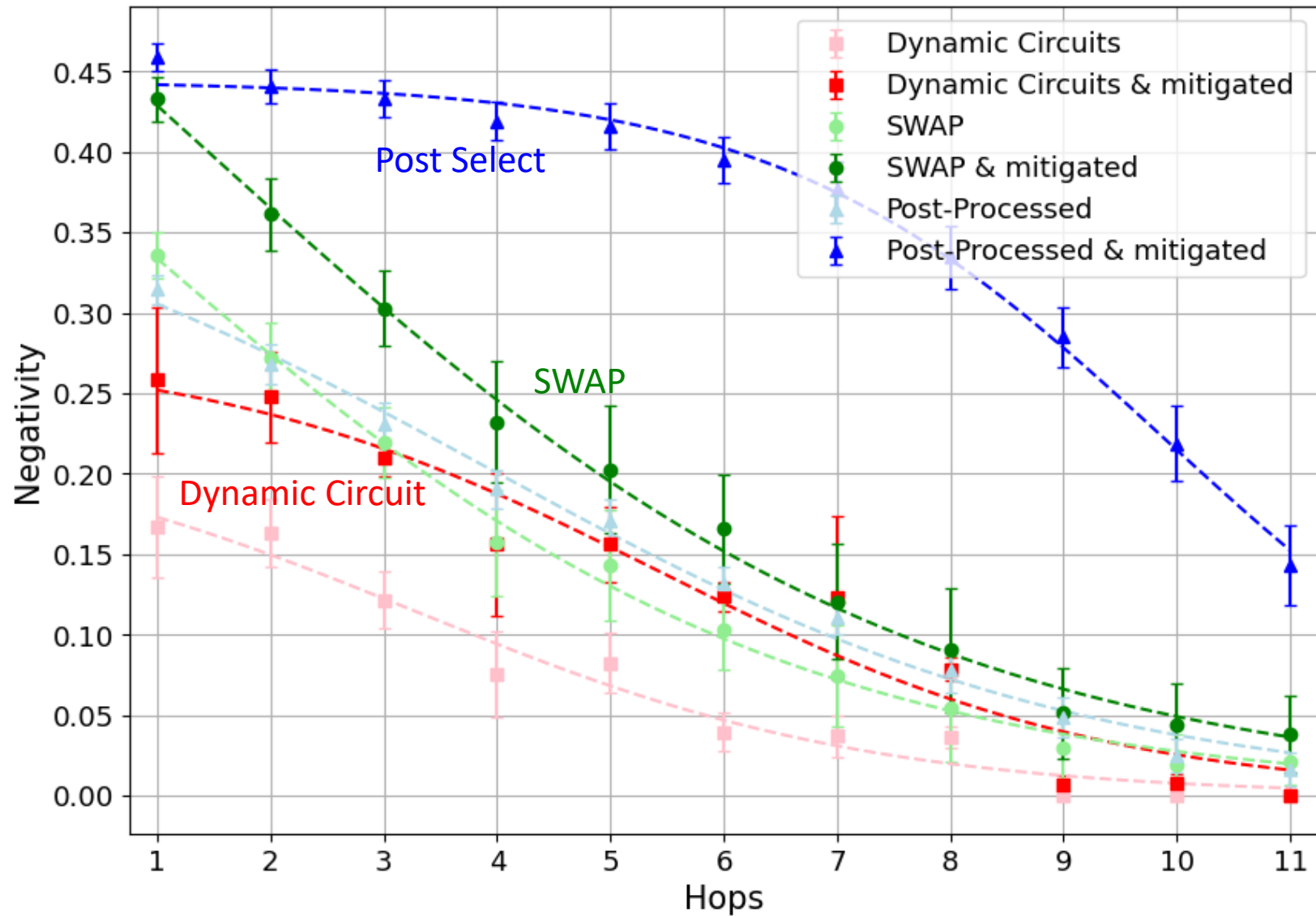




Comparisons



27-qubit *ibmq_Mumbai* device





Summary



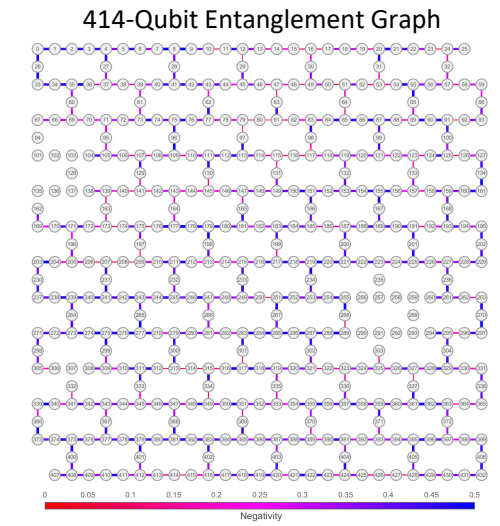
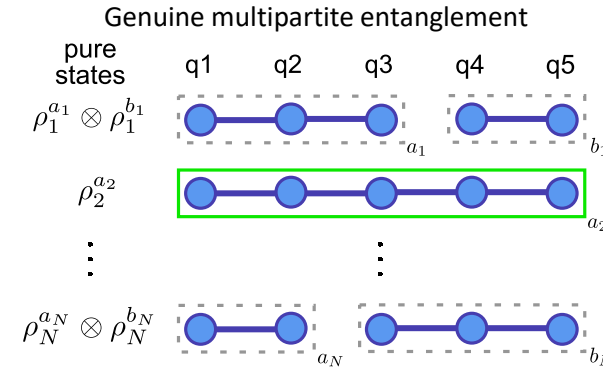
IBM Quantum Network Hub
at the University of Melbourne

Forms of Multipartite Entanglement

- Bipartite entanglement
- Genuine multipartite entanglement

$$\rho = \sum_{i=1}^N p_i \rho_i$$

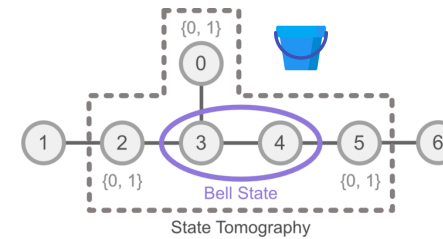
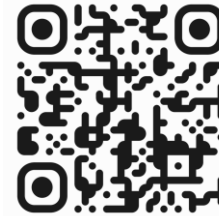
Pure states ρ_i
Probabilities p_i



Bipartite entanglement in graph states

- Constant 36-circuit algorithm
- Whole-device entanglement on up to **414-qubits**
- Negativity correlated with CNOT fidelities

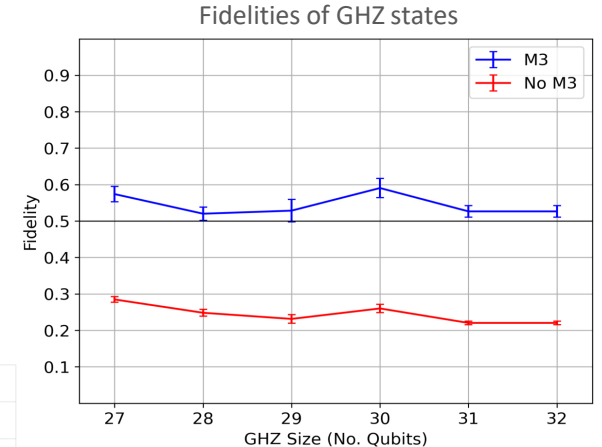
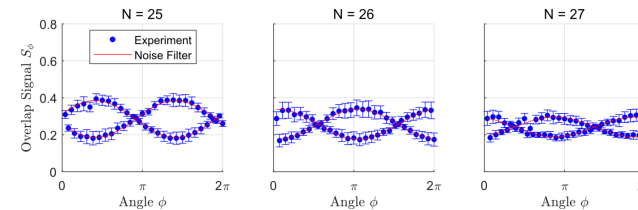
Mooney, Hill and Hollenberg, *Sci. Rep.* (2019)
 Mooney, White, Hill and Hollenberg, *Adv. Quantum Technol* (2021)
 John F Kam et. al. paper in preparation



Genuine multipartite entanglement in GHZ state

- GME across **32** qubits on *ibm_washington* device
- Looked at GHZ decoherence times
 - No superdecoherence

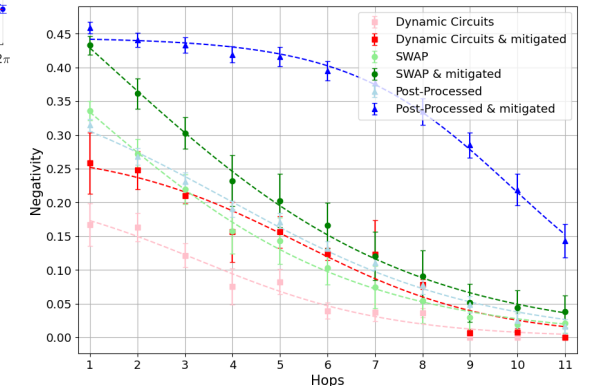
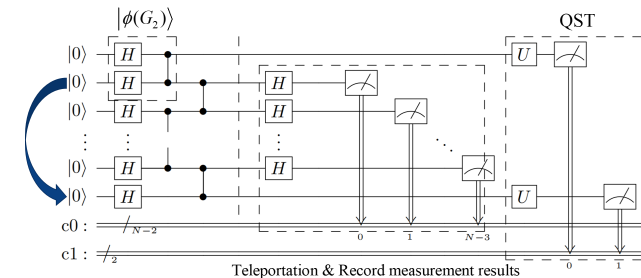
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Bell state teleportation

- Post-selected teleportation easily hopped over 11 qubits

Haiyue Kang, et. al. paper in preparation





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Thank you

Contributors



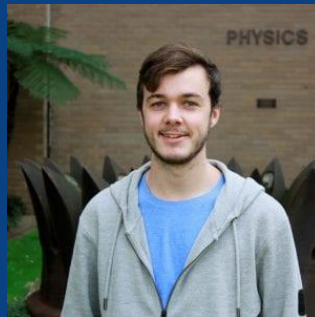
Dr Gary J Mooney



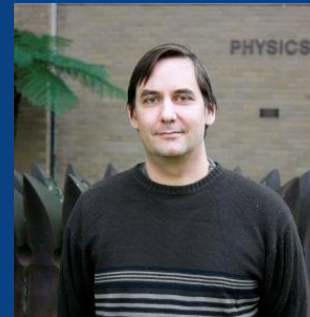
John F Kam



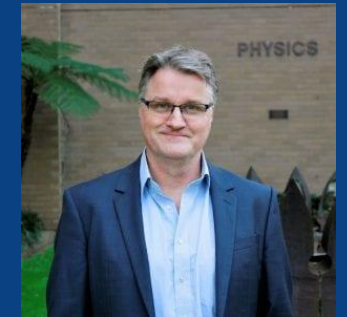
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