# Classical Verification of Quantum Learning

Matthias C. Caro

Freie Universität Berlin

October 17, 2023 – IPAM Workshop “Mathematical Aspects of Quantum Learning”

Based on arXiv:2306.04843 + ongoing work

# My collaborators

Marcel Hinsche

Marios Ioannou

Alexander Nietner

Ryan Sweke

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Marcel Hinsche


Marios Ioannou


Alexander Nietner


Ryan Sweke


Jens Eisert

# Motivation

What this talk is about

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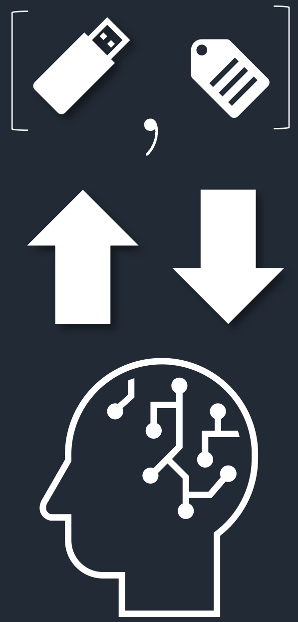

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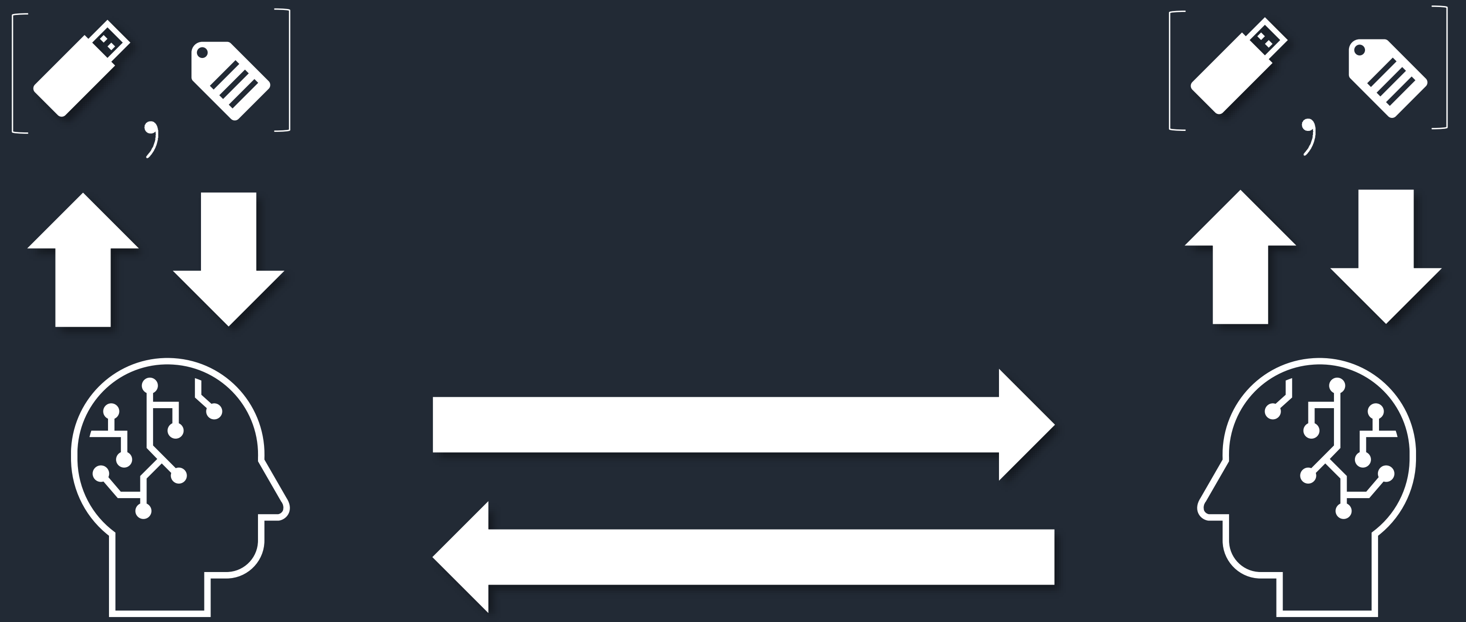

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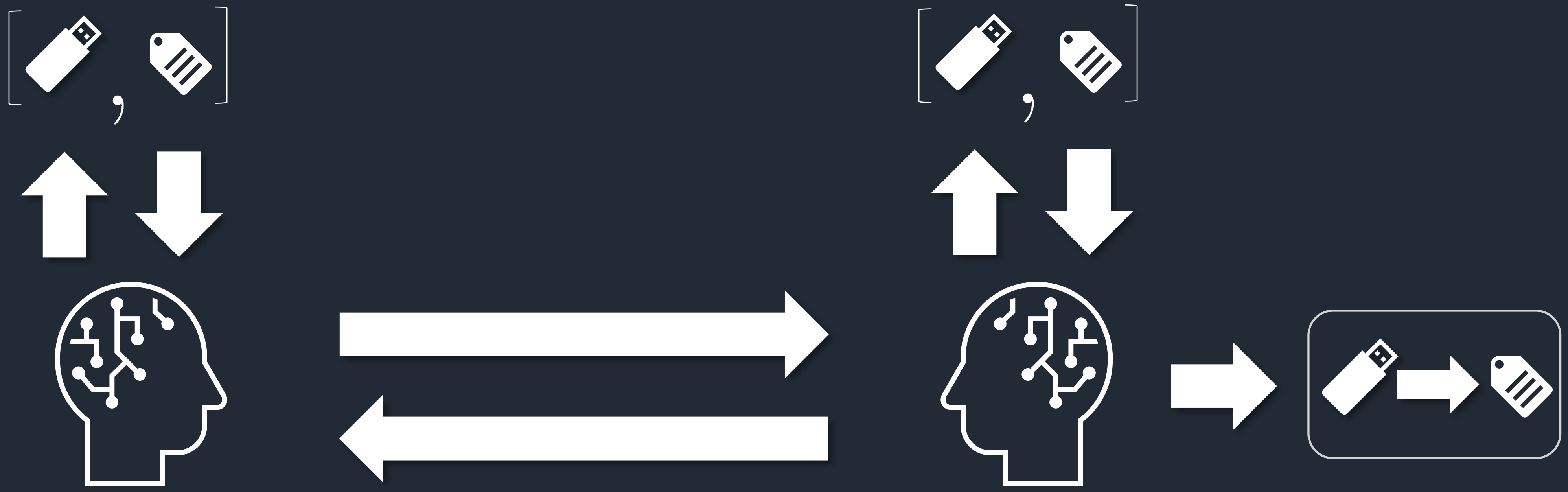


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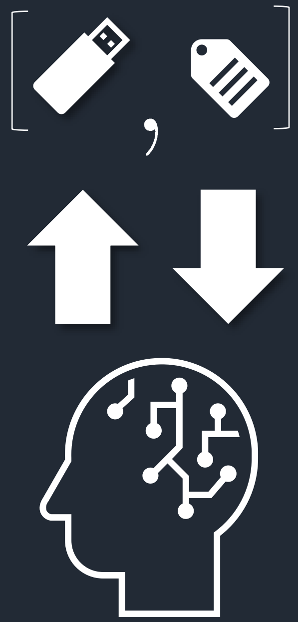

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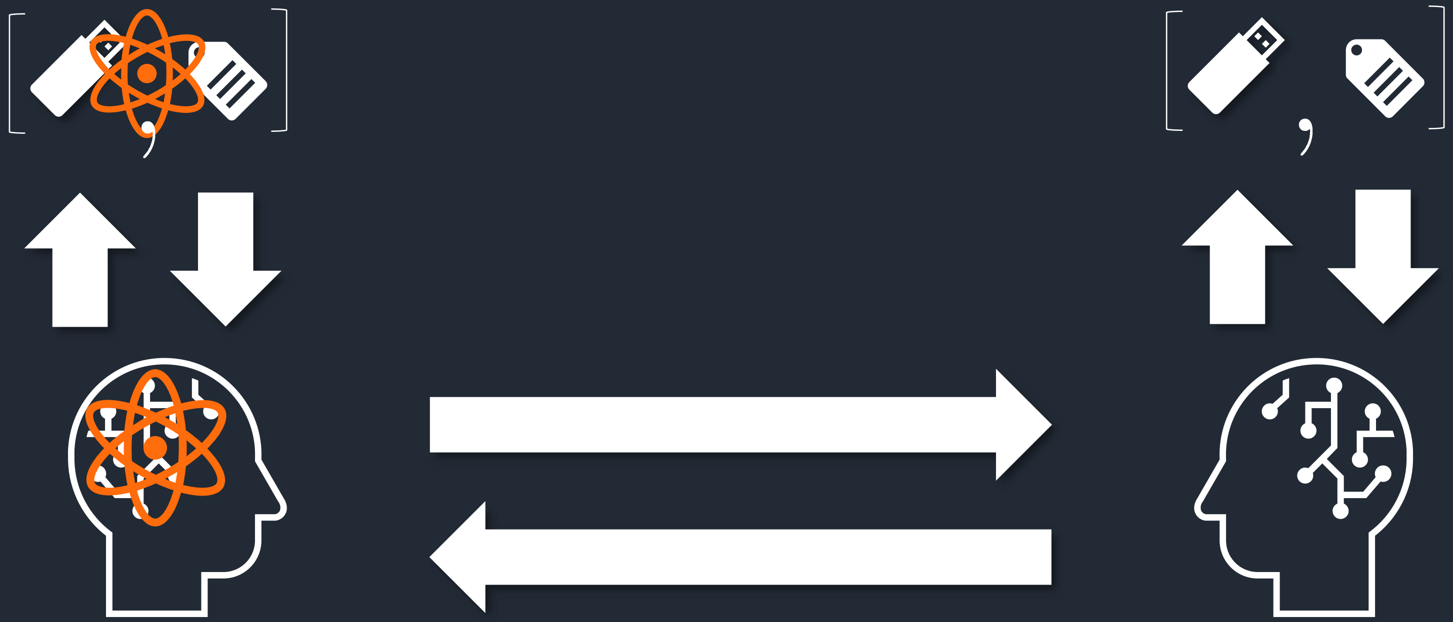

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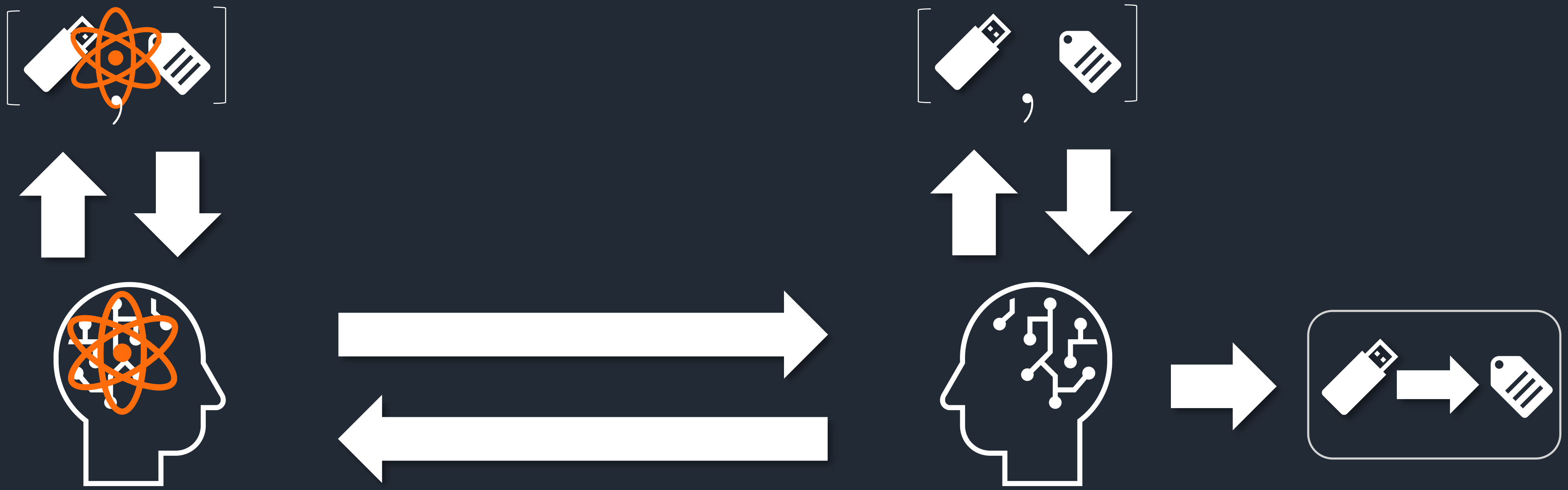

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# Guiding Question

**Question:**

Is there an (agnostic) learning problem that simultaneously...

1. ...is intractable for classical learners but efficiently solvable by quantum learners using quantum data,
2. ...and can be efficiently verifiably delegated from a classical verifier to an untrusted quantum prover?

# Outline

# Quantum Computational Learning Theory – An Intro

How to learn classical hypotheses from quantum data

# Classical Learning Theory Crash Course

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Ingredients for $\alpha$-agnostic learning:

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$$\mathbb{P}_{(x,y)\sim\mathcal{D}}[m(x) \neq y] \leq \alpha \cdot \min_{b\in\mathcal{B}} \mathbb{P}_{(x,y)\sim\mathcal{D}}[b(x) \neq y] + \varepsilon.$$

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For a distribution $\mathcal{D} = (\mathcal{U}_n, \varphi)$, with $\varphi(z) = \mathbb{E}_{(x,y)\sim\mathcal{D}}[y|x = z]$, and a function $f: \mathcal{X}_n \to \{0,1\}$, if we write $\mathrm{g} = (-1)^f = 1 - 2f$ and $\phi = 1 - 2\varphi$, then we get:

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$$\mathbb{P}_{(x,y)\sim\mathcal{D}}[f(x) \neq y] = \frac{1 - \sum_{s\in\mathcal{X}_n} \hat{g}(s)\, \hat{\phi}(s)}{2}.$$

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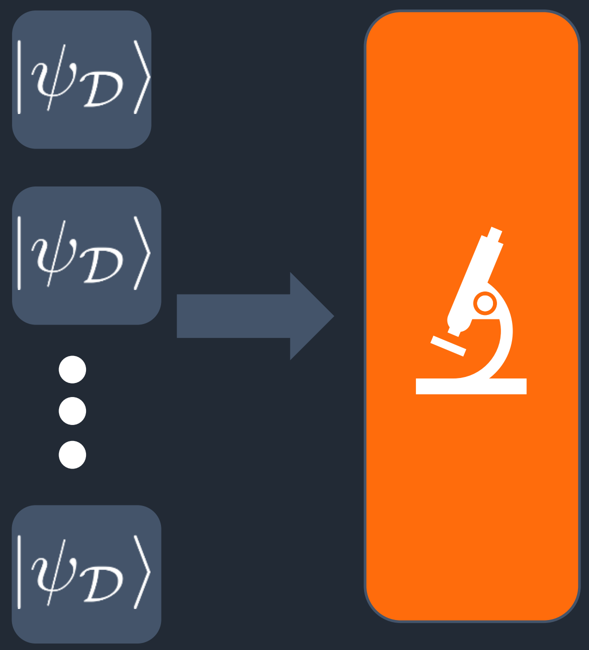

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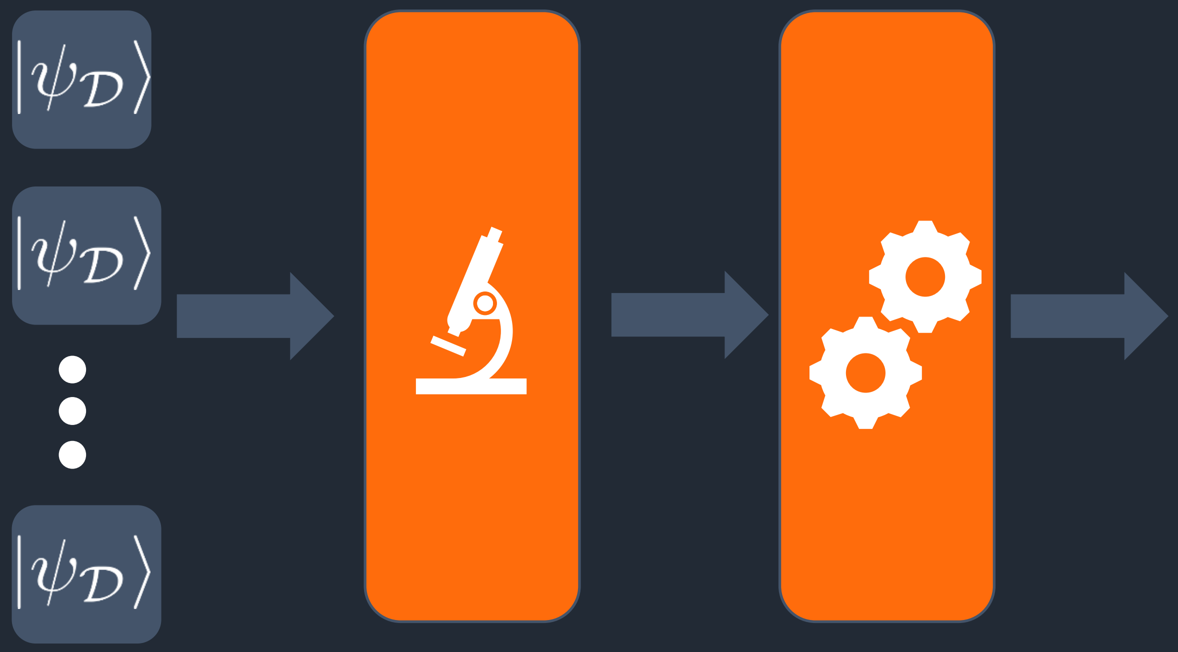

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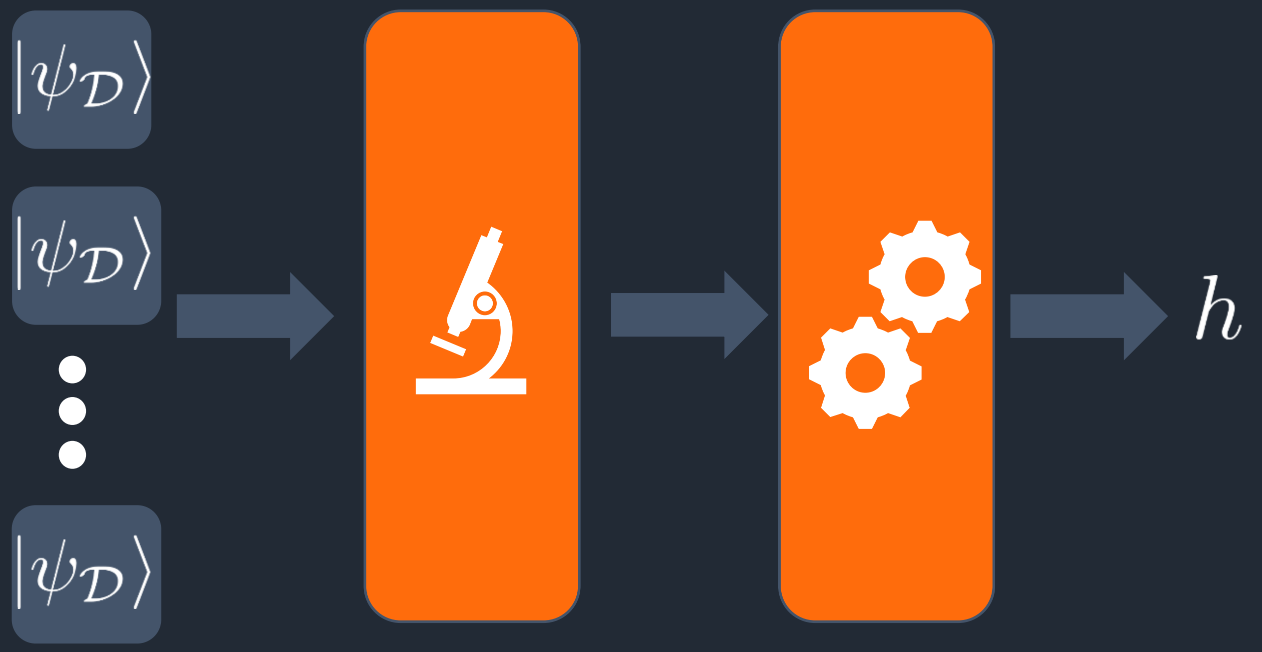

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# A New Data Oracle for Agnostic Quantum Learning

How we redefine quantum data for agnostic learning

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2. Measuring $\rho_{\mathcal{D}}$ in the computational basis = Sampling from $\mathcal{D}$.

# Quantum Fourier Sampling from MoS

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Quantum Fourier sampling central to learning from $|\psi_{(\mathcal{U}_n,f)}\rangle$

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For the next results, all $\varepsilon$ are $\varepsilon > 2^{-\left(\frac{n}{2}-2\right)}$.

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- We can sample from a distribution that is close to $\left\{\left(\hat{\phi}(s)\right)^2\right\}_s$ up to error $\frac{1}{2^n} < \frac{\varepsilon^2}{16}$.

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  With high probability, $\mathcal{O}\left(\frac{1}{\tau^2}\right)$ samples suffice to approximate the probability distribution to accuracy $\tau$.

[13] A. Dvoretzky, J. Kiefer, and J. Wolfowitz, *Ann. Math. Stat. 27.3, 642-669 (1956)*
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Using $\tilde{\mathcal{O}}\left(\frac{1}{\varepsilon^4}\right)$ copies of $\rho_{\mathcal{D}}$, we can efficiently produce a bit string $s \in \{0,1\}^n$ such that

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Note: This is (at least) LPN-hard from classical examples!

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Using $\tilde{\mathcal{O}}\left(\frac{k^4}{\varepsilon^4}\right)$ copies of $\rho_{\mathcal{D}}$, we can efficiently produce a randomized hypothesis $h: \{0,1\}^n \to \{0,1\}$ such that

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# Guiding Question

**Question:**

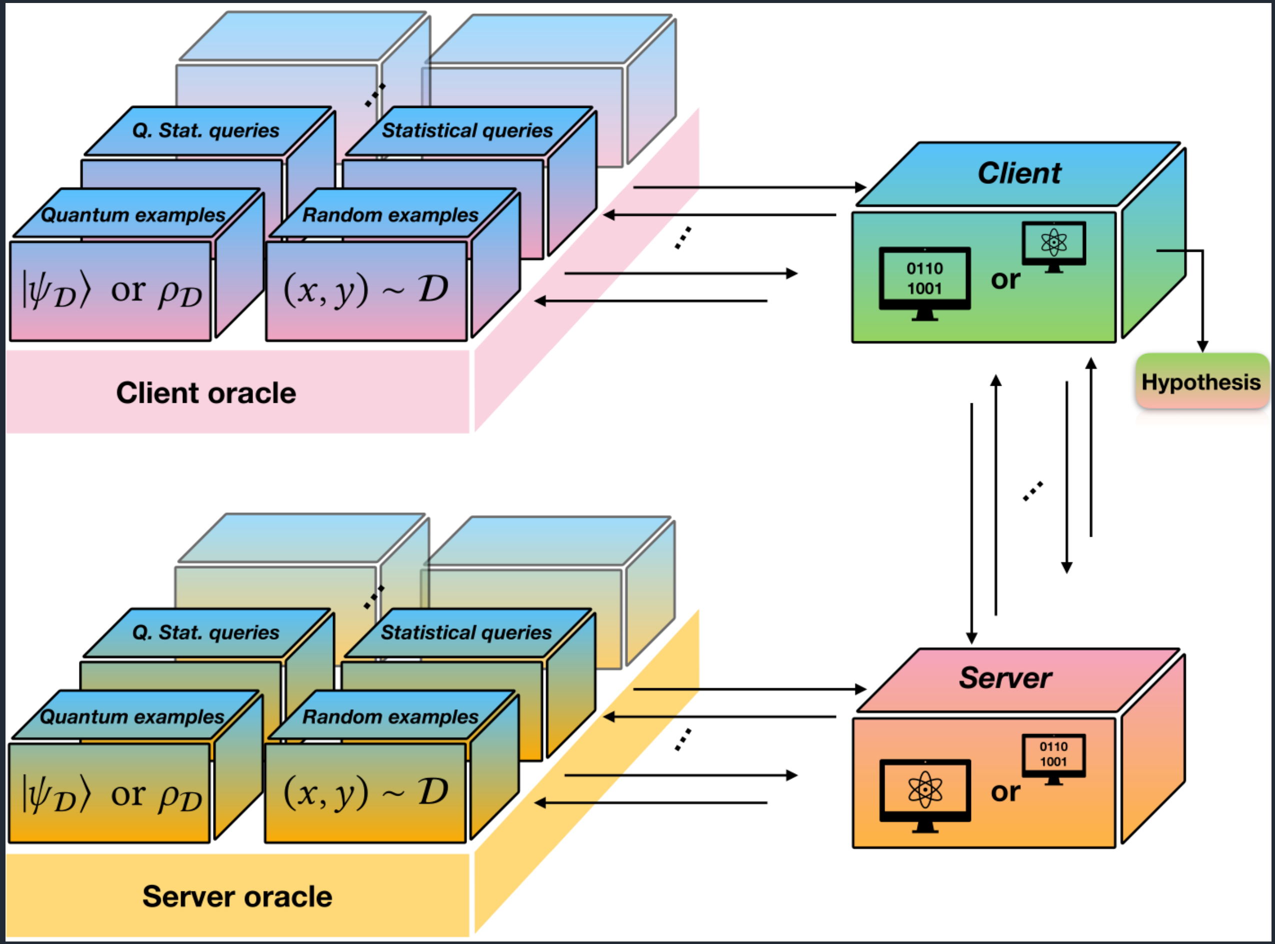
Is there an (agnostic) learning problem that simultaneously...

1. ...is intractable for classical learners but efficiently solvable by quantum learners using quantum data,
2. ...and can be efficiently verifiably delegated from a classical verifier to an untrusted quantum prover?

# Interactive Classical Verification of Quantum Learning

How classical clients can verifiably delegate learning to quantum servers

# Verifying Quantum Learners



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➢Classical agnostic learning is hard in these settings!

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➢ Can we classically verify this quantum subroutine?

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Let $\vartheta > 2^{-\left(\frac{n}{2}-3\right)}$ and $\varepsilon > 2\sqrt{b^2 - a^2}$. There is a classical-quantum pair $(V, P)$ that, for any $\mathcal{D} \in \mathfrak{D}_{\mathcal{U}_n;\geq\vartheta} \cap \mathfrak{D}_{\mathcal{U}_n;[a^2,b^2]}$, achieves:

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4. If $V$ interacts with any $P'$, then $V$ accepts and outputs $\left\|\tilde{\phi} - \hat{\phi}\right\|_1 > \varepsilon$ only with small probability.

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4. If the total accumulated Fourier weight of the estimates is large enough, $V$ is happy. Otherwise, $V$ rejects.

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# Interactive Verification of Quantum Agnostic Learning

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4. If $V$ interacts with any $P'$, then $V$ accepts and outputs a hypothesis $h$ s.t. $\mathbb{P}_{(x,y)\sim\mathcal{D}}[h(x) \neq y] > \min_{\tilde{f}\text{ Fourier}-\text{k}-\text{sparse}} \mathbb{P}_{(x,y)\sim\mathcal{D}}\left[\tilde{f}(\boldsymbol{x}) \neq y\right] + \varepsilon$ with low prob.

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**Proof idea (sketchy; inspired by [15]):**

[15] S. Mutreja and J. Shafer, *COLT 2023*

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**Proof idea (sketchy; inspired by [15]):**

Classical-quantum pair $(V, P)$

- ➢Classical distinguisher between random noisy parities and fully uniform distribution
  - ➢Violating a known $\Omega(n)$ sample complexity lower bound ⚡

[15] S. Mutreja and J. Shafer, *COLT 2023*

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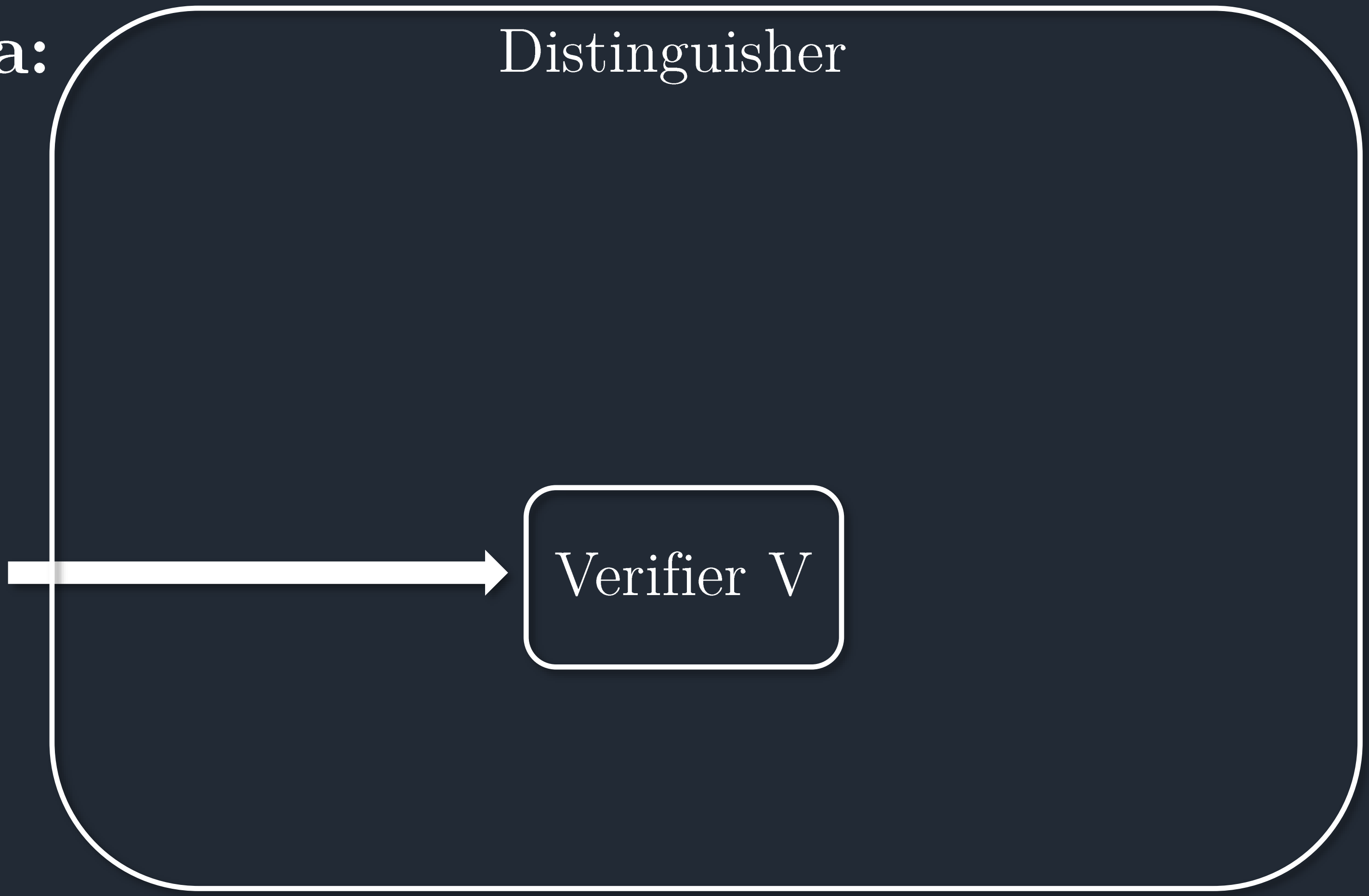

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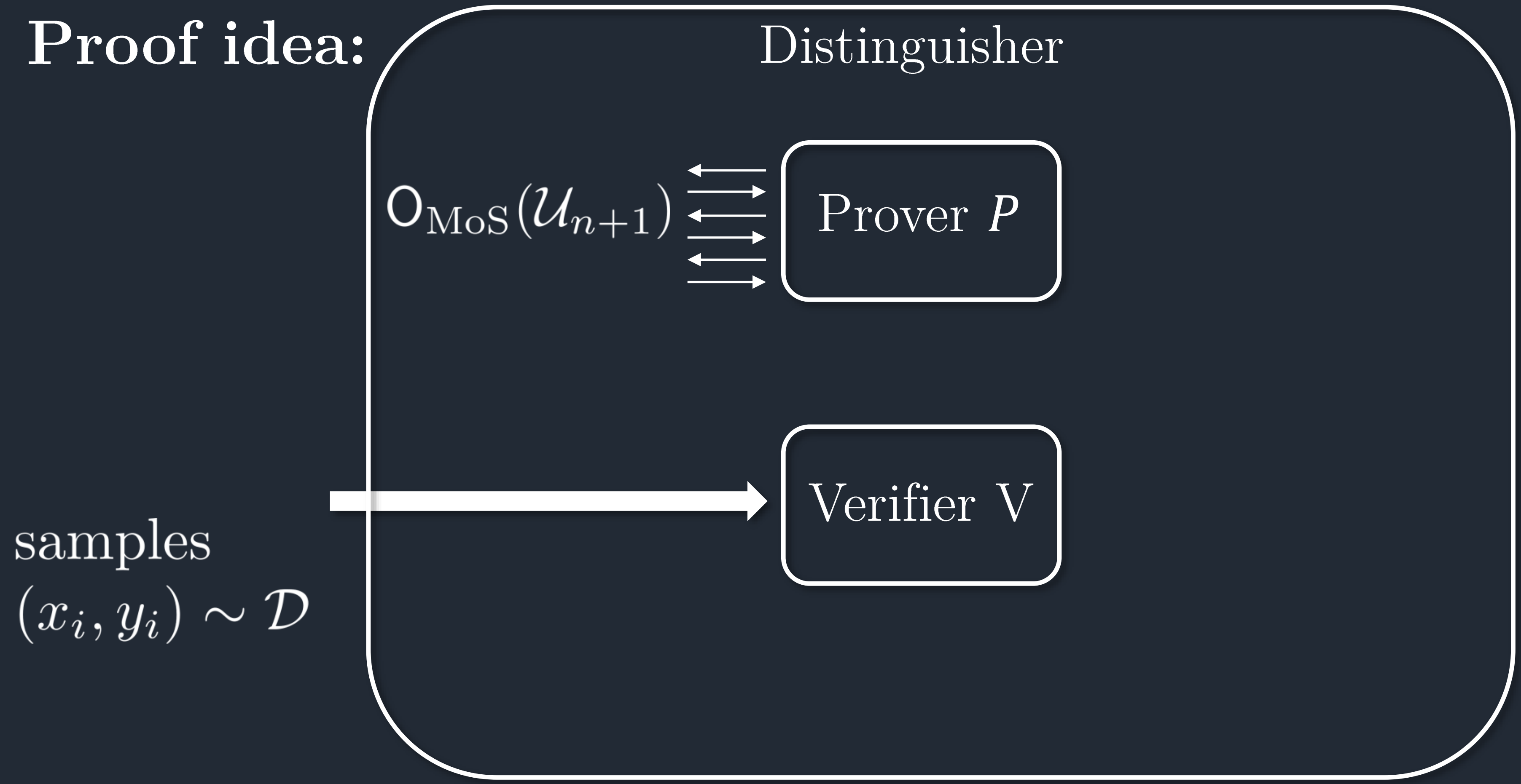

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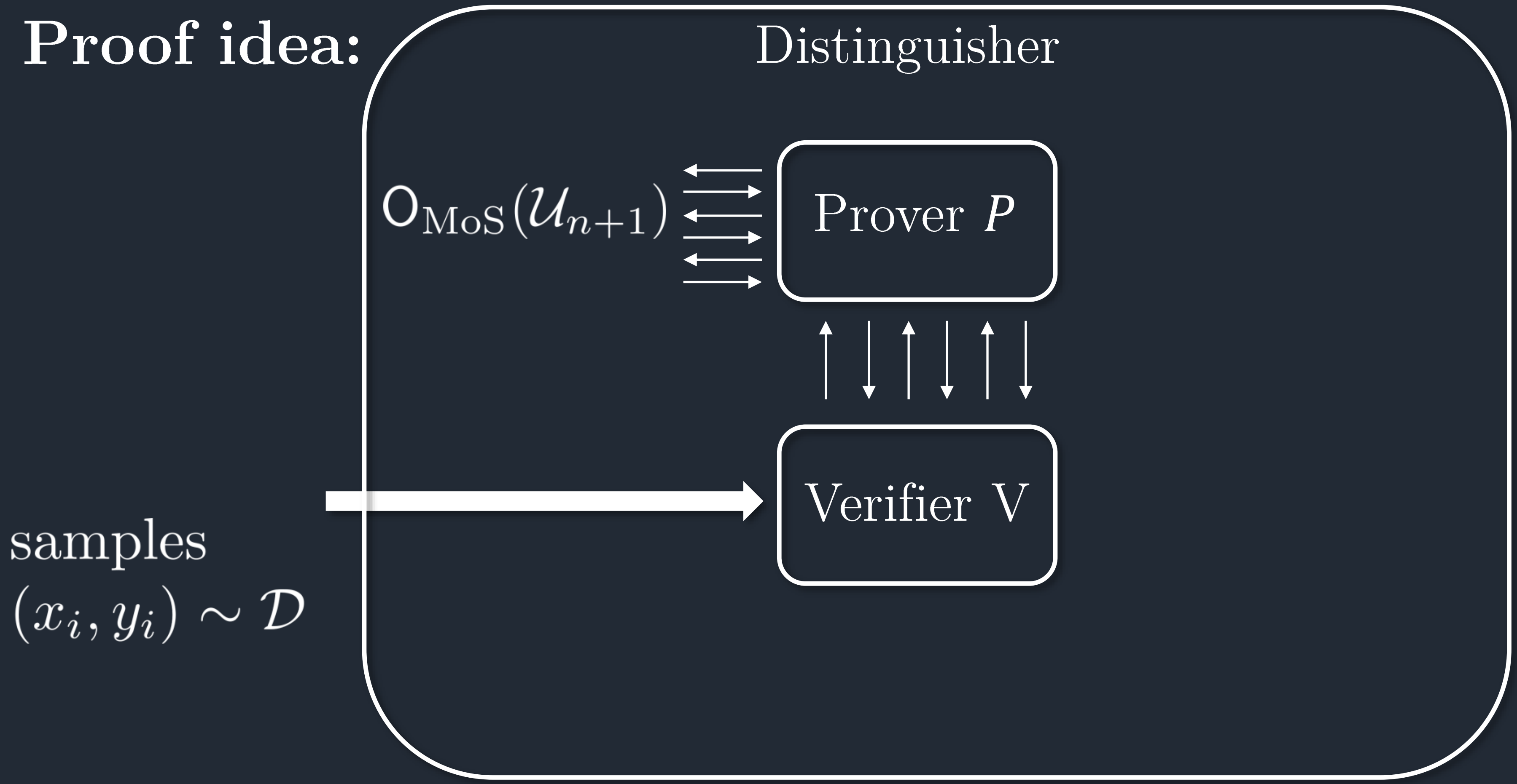

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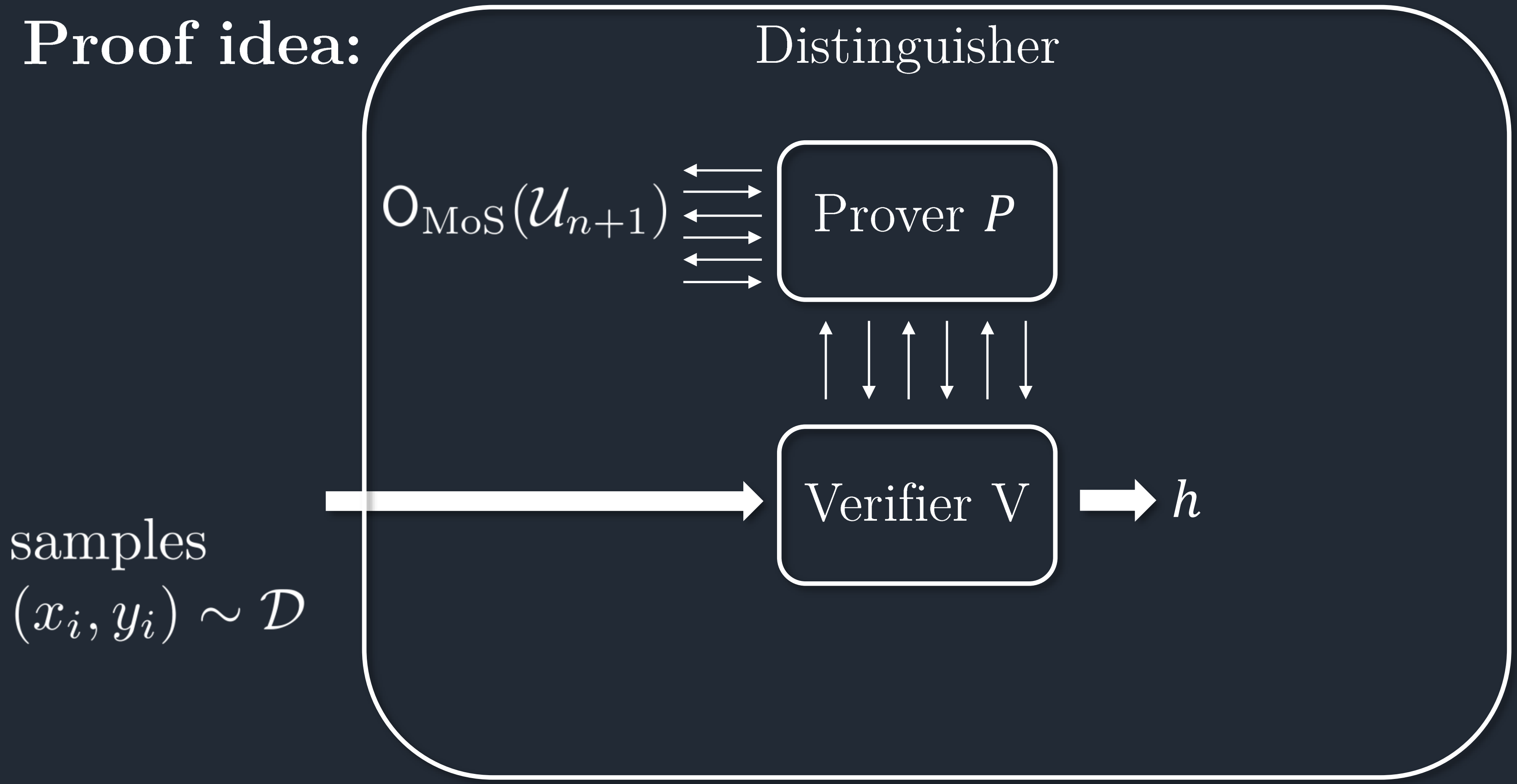

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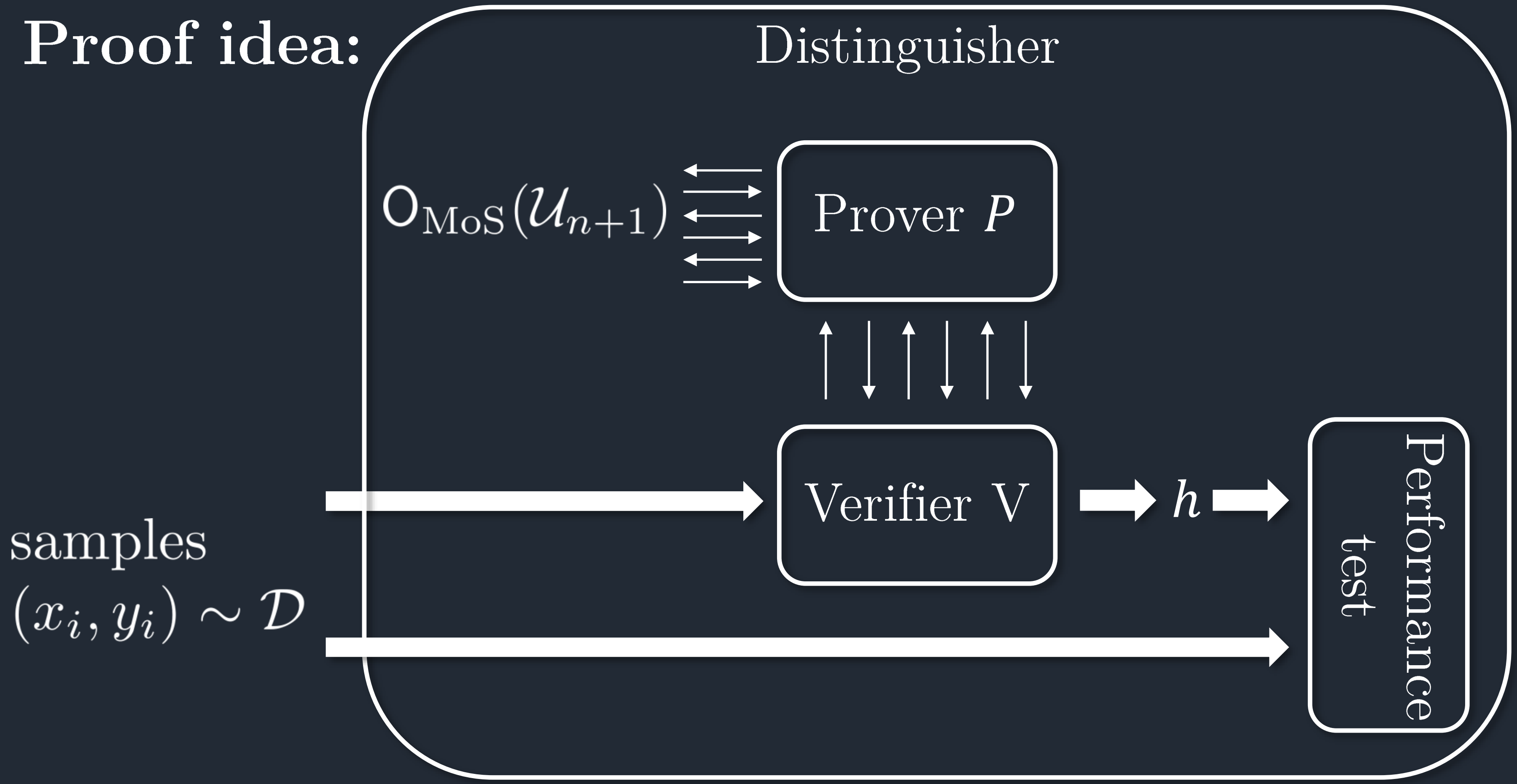

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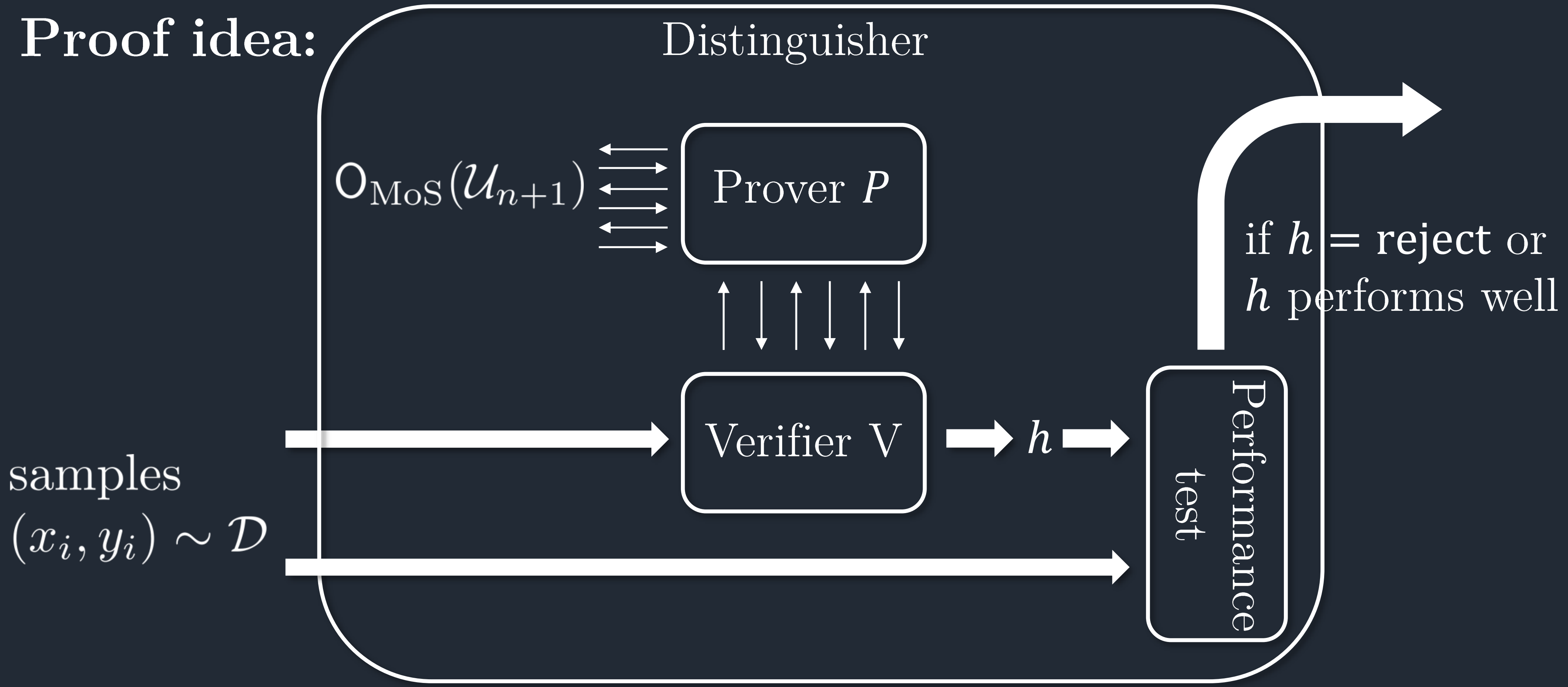

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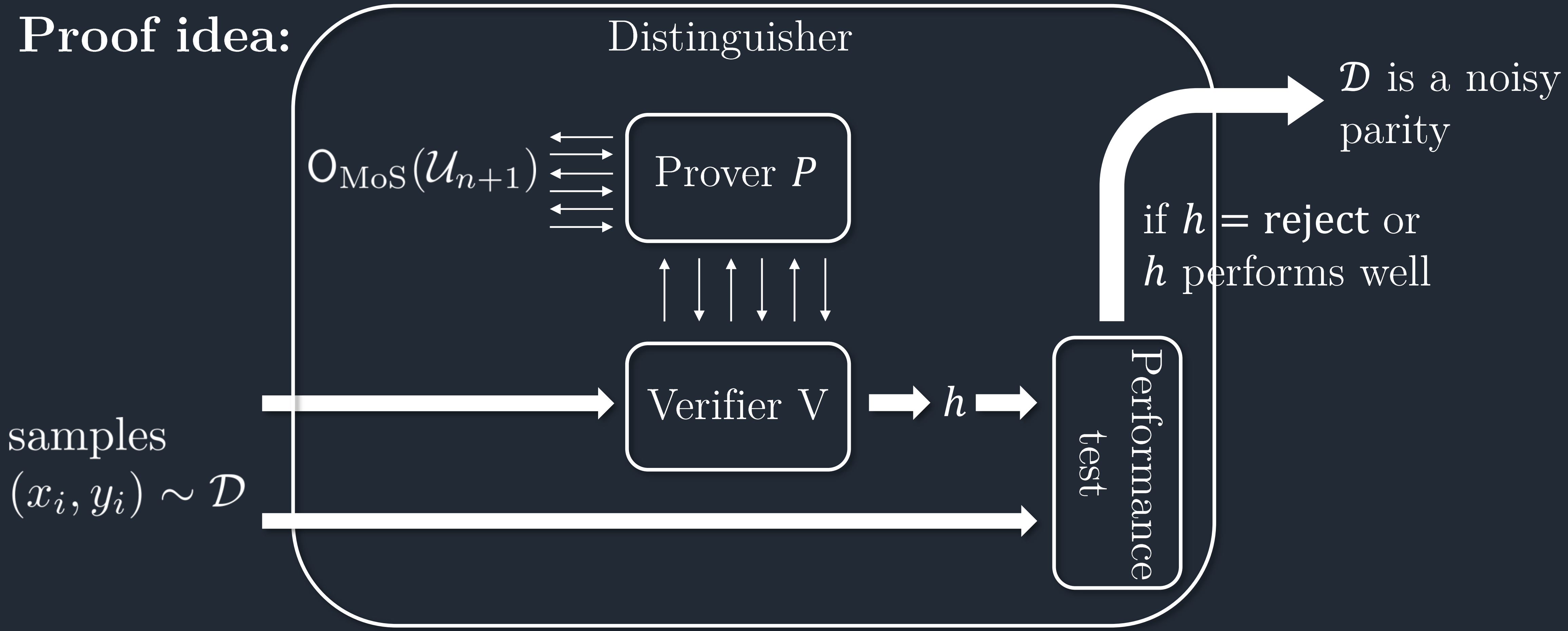

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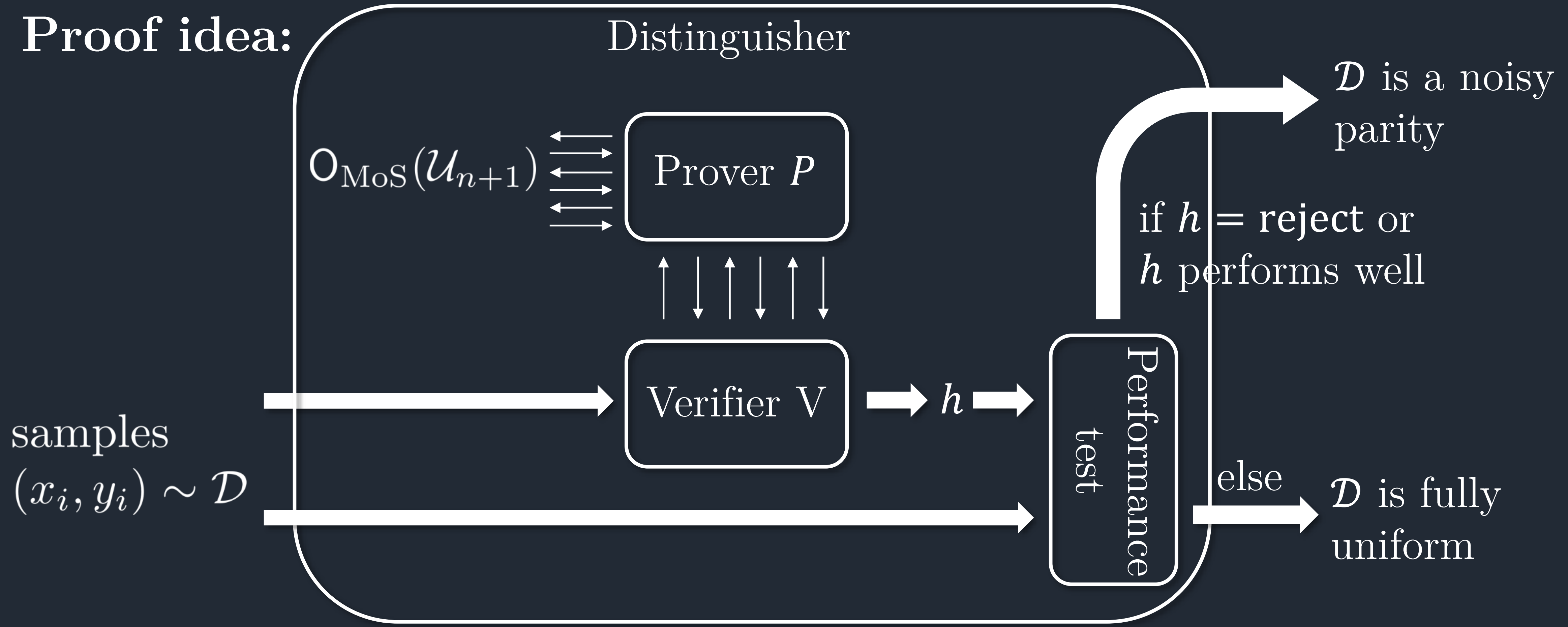

# Limitations on the Power of Quantum Data

Why quantum data is not all-powerful

# “Usefulness” of MoS for Distribution-Independent Agnostic Learning

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- This equals the classical sample complexity up to log-factors.
- This is (almost) the same limitation as for superposition examples [16].

[16] S. Arunachalam and R. de Wolf; *JMLR 19.1, 2879–2878 (2018)*

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   - This step uses the specific distributions $\mathcal{D}_a$ and involves a detailed eigenvalue analysis for the resulting MoS states.

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The sample complexity of a verifier interacting with an MoS prover for distribution-independent agnostic learning is at most quadratically better than what the verifier can achieve alone.

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[15] showed that interacting with a classical prover can also lead to an at most quadratic sample complexity improvement.

[15] S. Mutreja and J. Shafer, *COLT 2023*

# Conclusion and Outlook

What we talked about and where to go from here

# Guiding Question

**Question:**

Is there an (agnostic) learning problem that simultaneously...

1. ...is intractable for classical learners but efficiently solvable by quantum learners using quantum data,
2. ...and can be efficiently verifiably delegated from a classical verifier to an untrusted quantum prover?

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| | Oracle type | Problem type | | | |
|---|---|---|---|---|---|
| | | Fourier sampling | Heavy Fourier coefficient estimation | 1-agnostic parity learning | 2-agnostic Fourier-sparse learning |
| Functional | superposition examples = mixture-of-superpositions | ✓ | ✓ | ✓ | ✓ |
| Functional | superposition QSQ = mixture-of-superpositions QSQ | Probably ✗ | ✓ | ✓ | ✓ |
| Distributional | superposition examples | ? | ? | ? | ? |
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- NISQ-friendly variants of our framework, e.g., for variational quantum machine learning or for NISQ verifiers?

# Your Questions