# DO QUANTUM COMPUTERS HAVE APPLICATIONS IN MACHINE LEARNING AND COMBINATORIAL OPTIMIZATION?

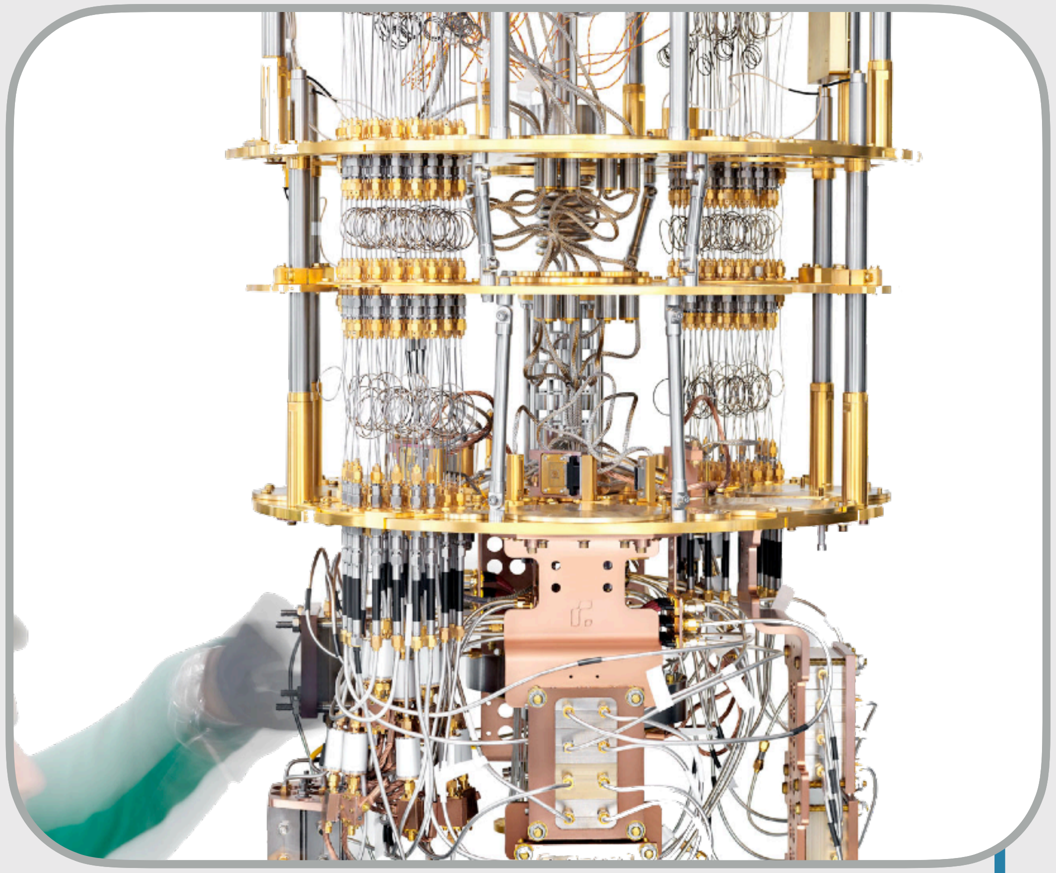

## JENS EISERT, FU BERLIN


IPAM Workshop II: Mathematical Aspects of Quantum Learning, University of California, LA, October 2023

With R Sweke, M Hinsche, D Hangleiter, M Ioannou, Y Quek, J Haferkamp, A Nietner, J-P Seifert and others

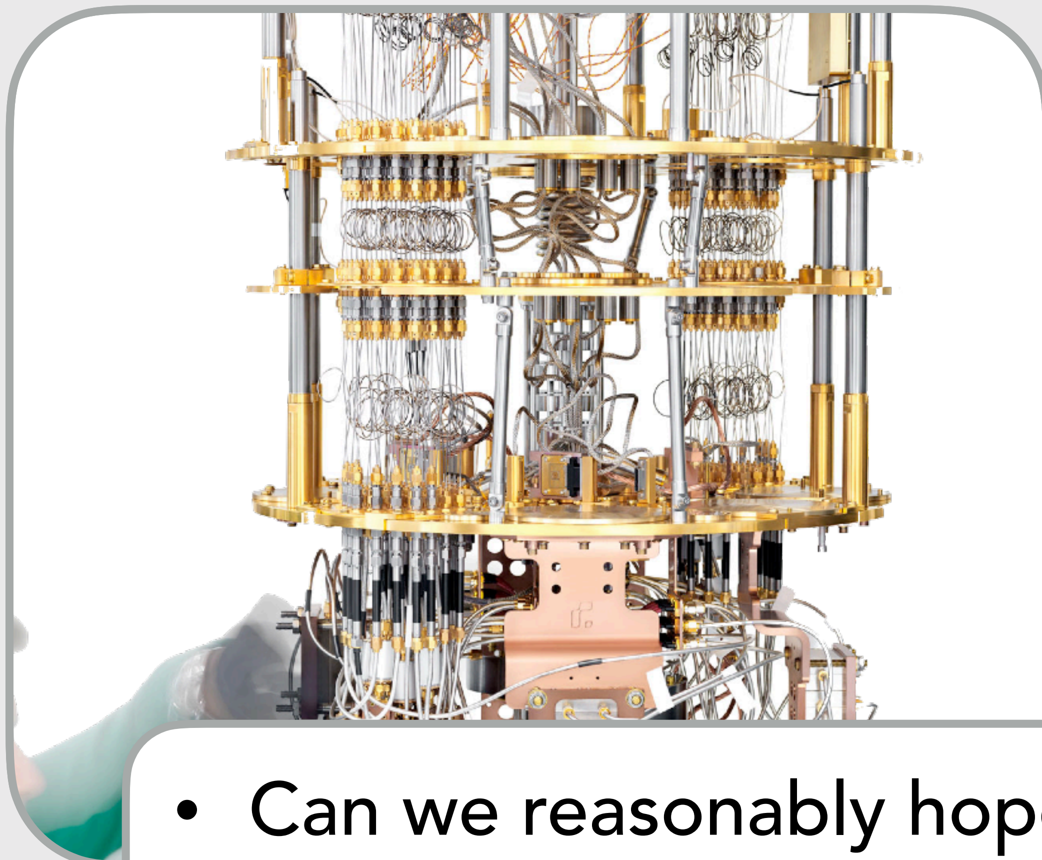
# NEAR-TERM QUANTUM COMPUTERS

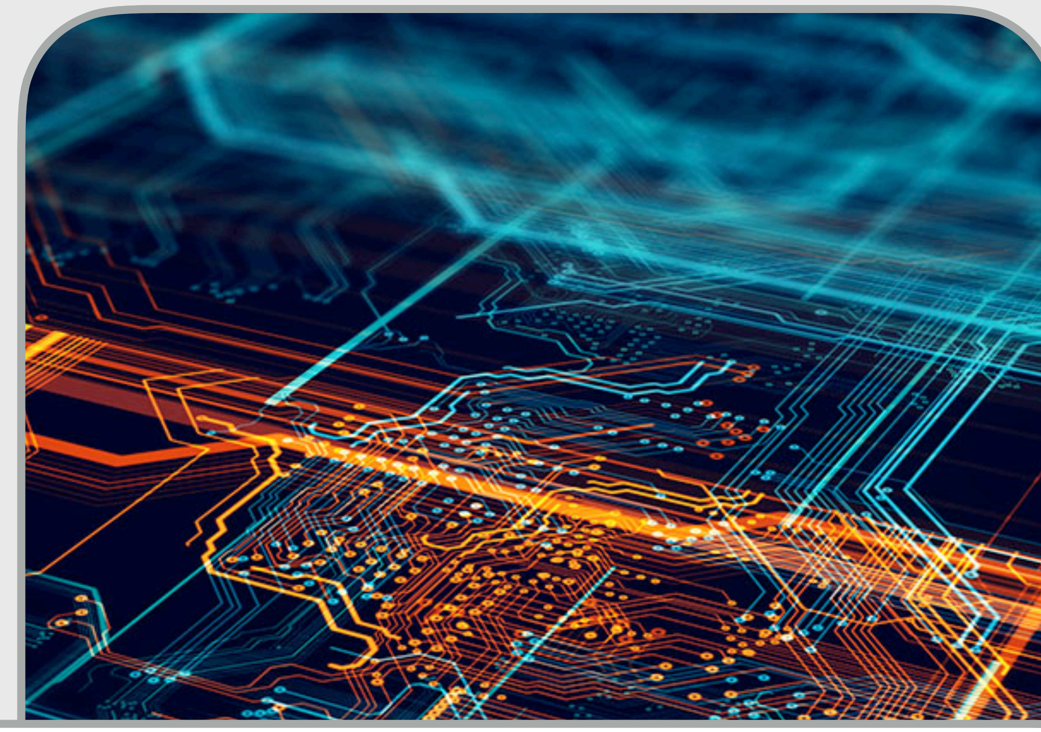
- **Quantum computers** promise advantages in computational tasks



Arute, Arya, ..., Martinis, Nature 574, 505 (2019)
Wang, Qin, Ding, Chen, Chen, You, He, Jiang, Wang, You, Renema, Hoefling, Lu, Pan, Phys Rev Lett 123, 250503 (2019)

- Quantum computers promise advantages in computational tasks





- Can we reasonably hope noisy, realistic quantum devices to provide a speedup over classical computers?

Arute, Arya, ..., Martinis, Nature 574, 505 (2019)
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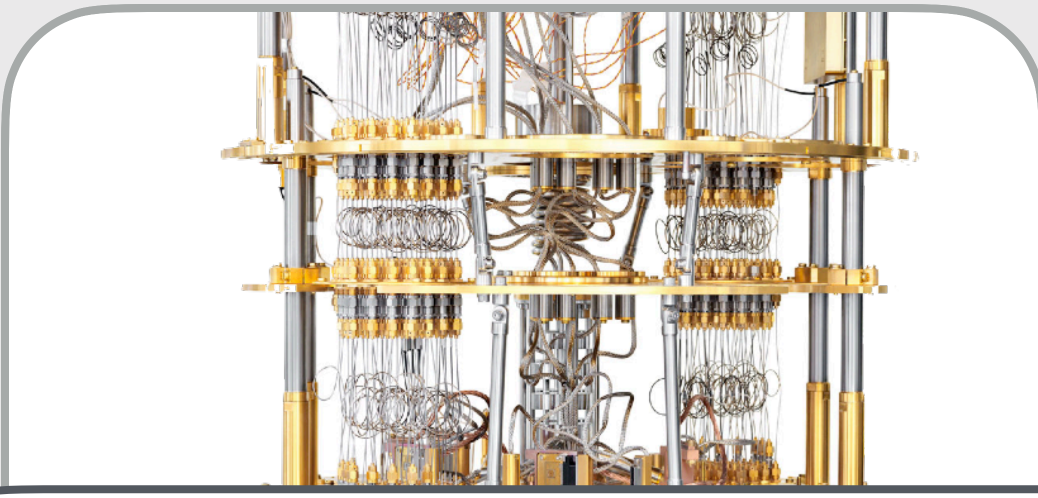
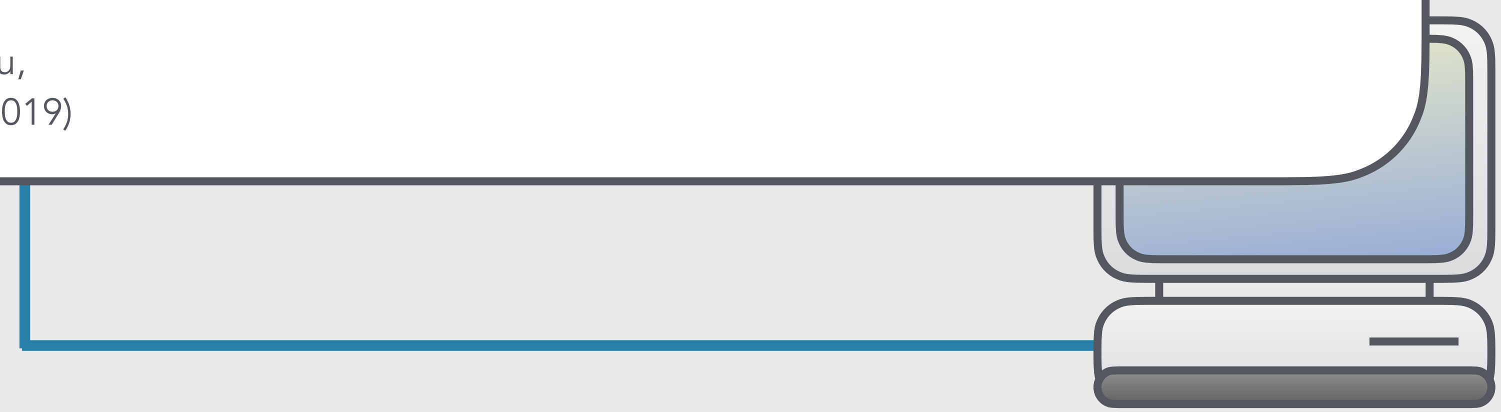
# QUANTUM ADVANTAGES

- **Quantum computers** promise advantages in computational tasks



- **Quantum advantage** for paradigmatic sampling problems
- Sampling up to a constant error in $||.||_{l_1}$ distance is **classically hard**

Arute, Arya, ..., Martinis, Nature 574, 505 (2019)
Wang, Qin, Ding, Chen, Chen, You, He, Jiang, Wang, You, Renema, Hoefling, Lu, Pan, Phys Rev Lett 123, 250503 (2019)



Review: Hangleiter, Eisert, Rev Mod Phys 95, 035001 (2023)

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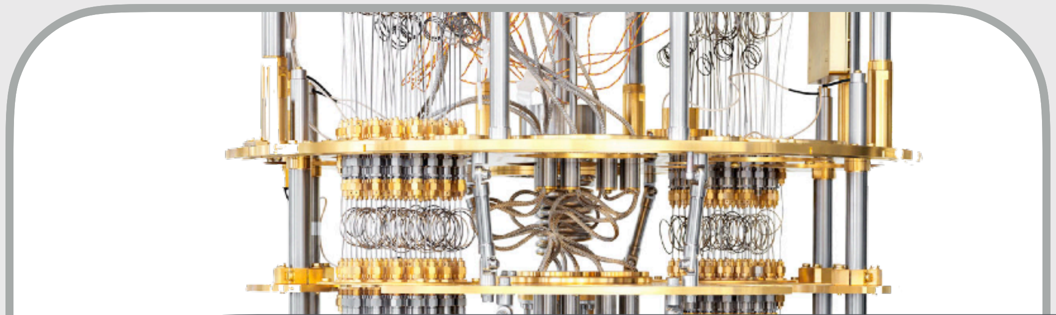
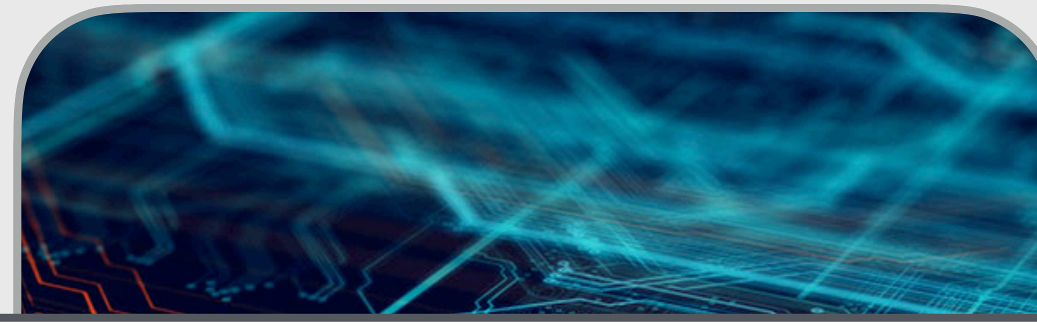
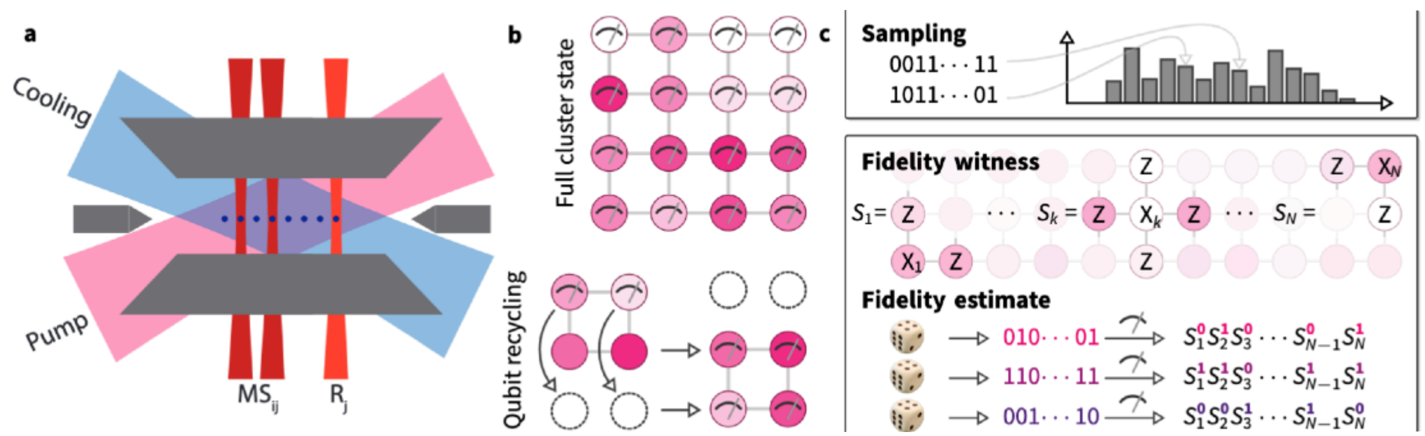




- **Verification** is subtle: XEB benchmarking, no black-box verification

  Hangleiter, Kliesch, Eisert, Gogolin, Phys Rev Lett 122, 210502 (2019)
  Neil, …, Martinis, Science, 360, 195 (2018)
  Boixo, Isakov, Smelyanskiy, Babbush, Ding, Jiang, Bremner, Martinis, Neven, Nature Phys 14, 595 (2018)
  Deshpande, Mehta, Vincent, Quesada, Hinsche, Ioannou, Madsen, Lavoie, Qi, Eisert, Hangleiter, Fefferman, Dhand, Science Advances 8, eabi7894 (2022)

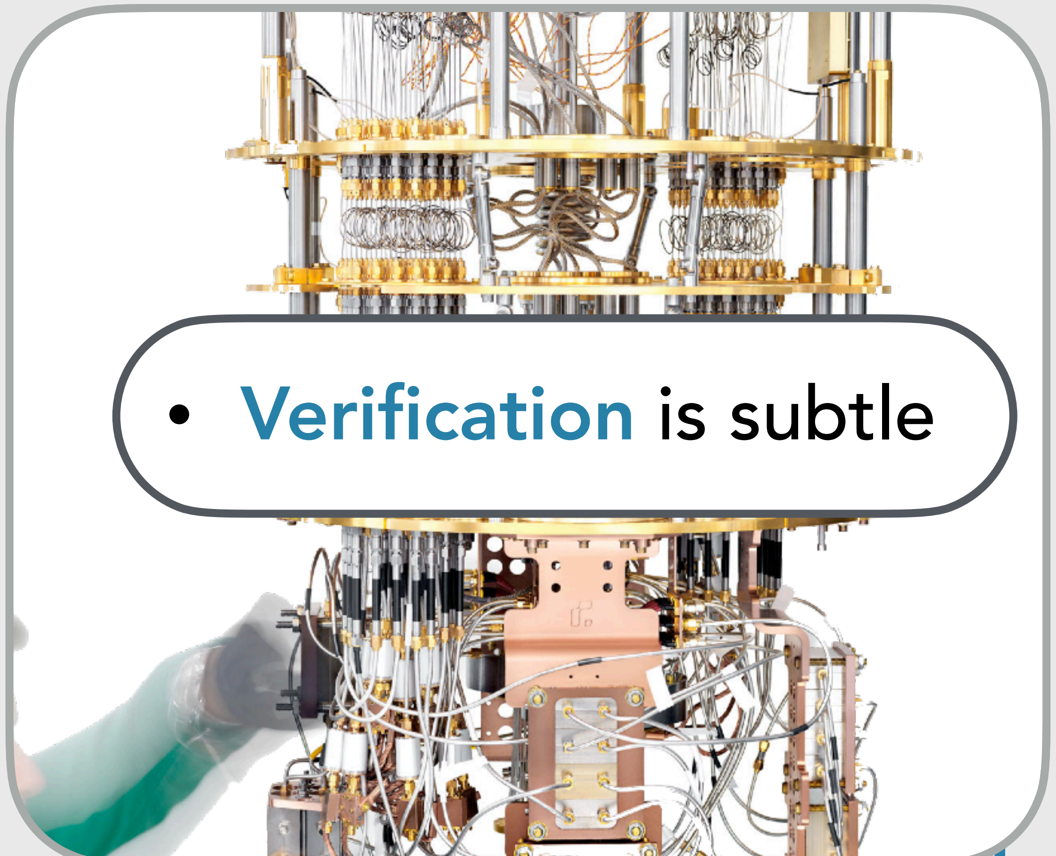
- **Quantum** verification: Trapped ions (with Ringbauer-Blatt-Monz)




Ringbauer, Hinsche, Feldker, Faehrmann, Bermejo-Vega, Edmunds, Stricker, Marciniak, Meth, Pogorelov, Postler, Blatt, Schindler, Eisert, Monz, Hangleiter, arXiv:2307.14424 (2023)

Review: Hangleiter, Eisert, Rev Mod Phys 95, 035001 (2023)

- **Quantum computers** promise advantages in computational tasks



- **Verification** is subtle

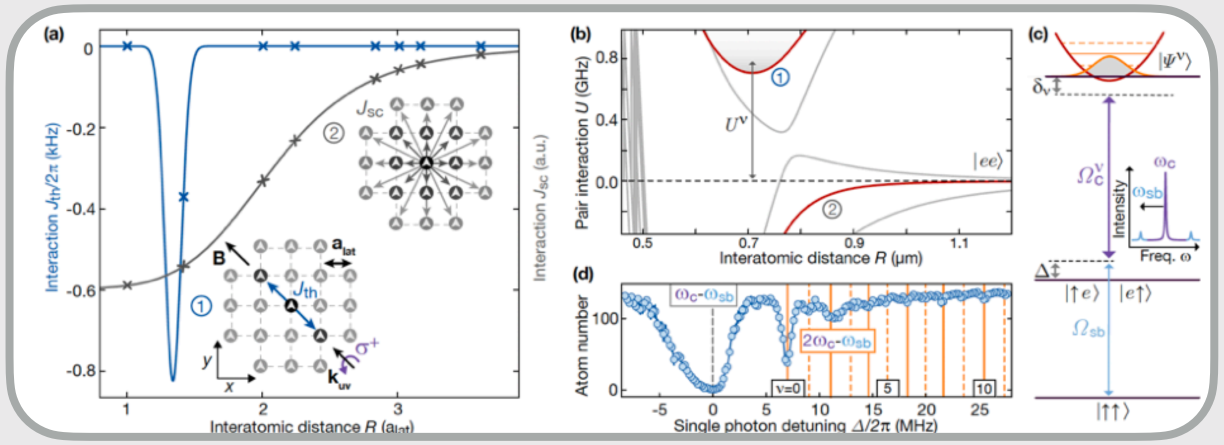
M Caro, "Classical verification of quantum learning"

Review: Hangleiter, Eisert, Rev Mod Phys 95, 035001 (2023)

# QUANTUM ADVANTAGES

- Encouraging - but what **next**?

- **Programmable** quantum simulators




Hollerith, Srakaew, Wei, Rubio-Abadal, Adler, Weckesser, Kruckenhauser, Walther, Bijnen, Rui, Gross, Bloch, Zeiher, Phys Rev Lett 128, 113602 (2022)

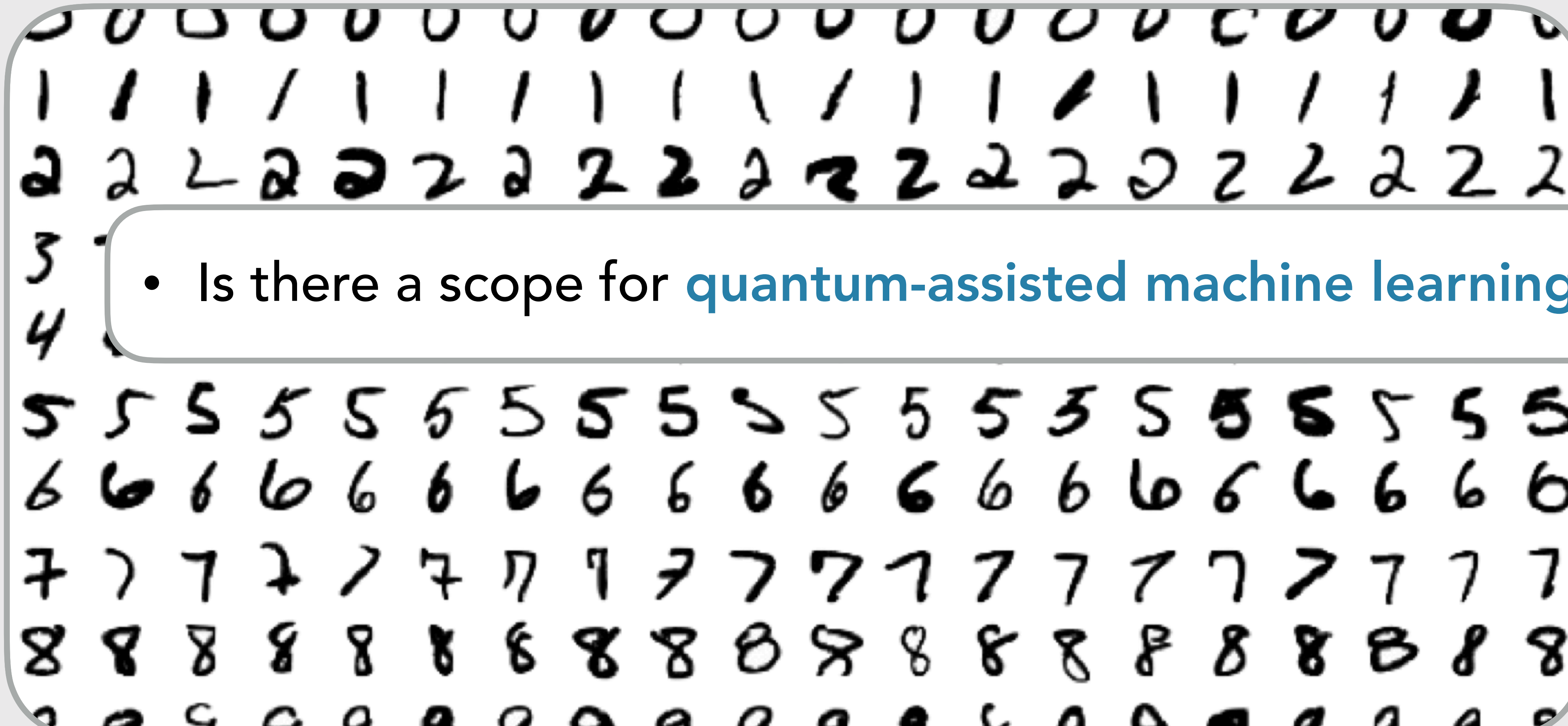
- **Variational** quantum algorithms

McClean, Romero, Babbush, Aspuru-Guzik, New J Phys 18, 023023 (2016)
Farhi, Goldsone, Gutmann, arXiv:1411.4028 (2014)

- Instances of quantum-assisted **machine learning**

# MACHINE LEARNING

- Machine learning has changed the world we live in
  - Chat-GPT, DALL-E 2
  - Machine learning in study of materials and quantum chemistry

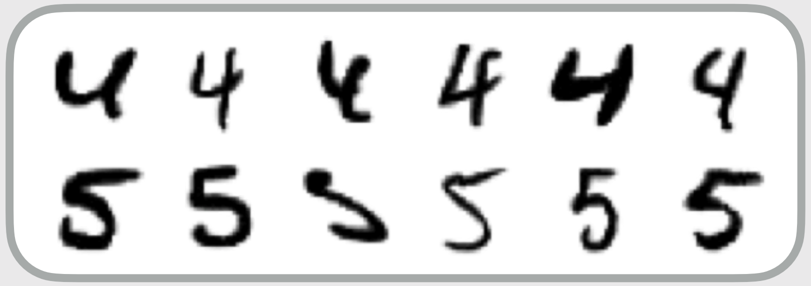


- Is there a scope for quantum-assisted machine learning?

Dunjko, Briegel, Rep Prog Phys 81, 074001 (2018)
Biamonte, Wittek, Pancotti, Rebentrost, Wiebe, Lloyd, Nature 549, 195 (2017)
Arunachalam, de Wolf, arXiv:1701.06806 (2017)

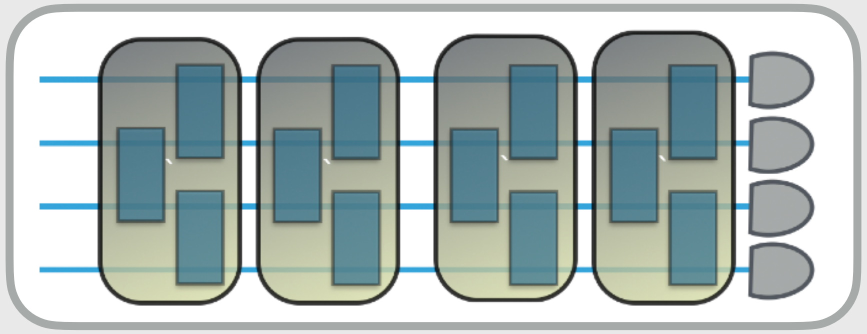
# QUANTUM ADVANTAGES IN LEARNING TASKS?

- We would like to have **rigorous guarantees** for **state-of-the-art learning** algorithms applied on **real-world datasets**



- **Data set**
  - Pictures of cats and dogs
  - Protein configurations
  - Stock market data

- **Algorithm**
  - Stochastic gradient descent
  - Expectation maximization

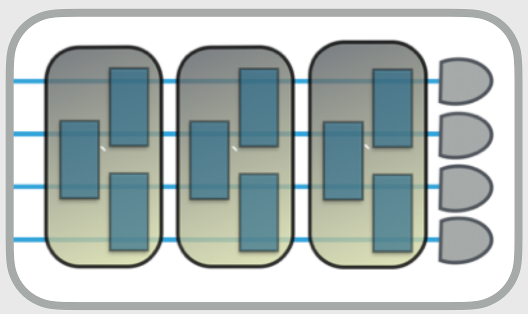


- **Model**
  - Neural networks
  - Parameterised quantum circuits
  - Probabilistic graphical models

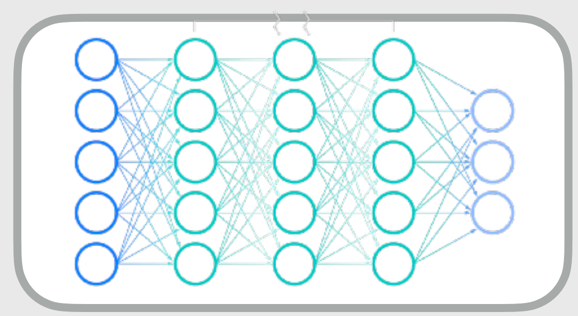
- **Sample complexity**, **computational complexity, generalisation bounds**

# QUANTUM ADVANTAGES IN LEARNING TASKS?

- Even better would be "**classical lower bounds**" and "**quantum upper bounds**"



- **Algorithm**
  - Stochastic gradient descent
- **Model**
  - Parametrized quantum circuits



- **All classical learners**
  - All models
  - All algorithms

- **Quantum**

- **Classical**

- Can we make substantial progress for **fine-tuned data sets**?

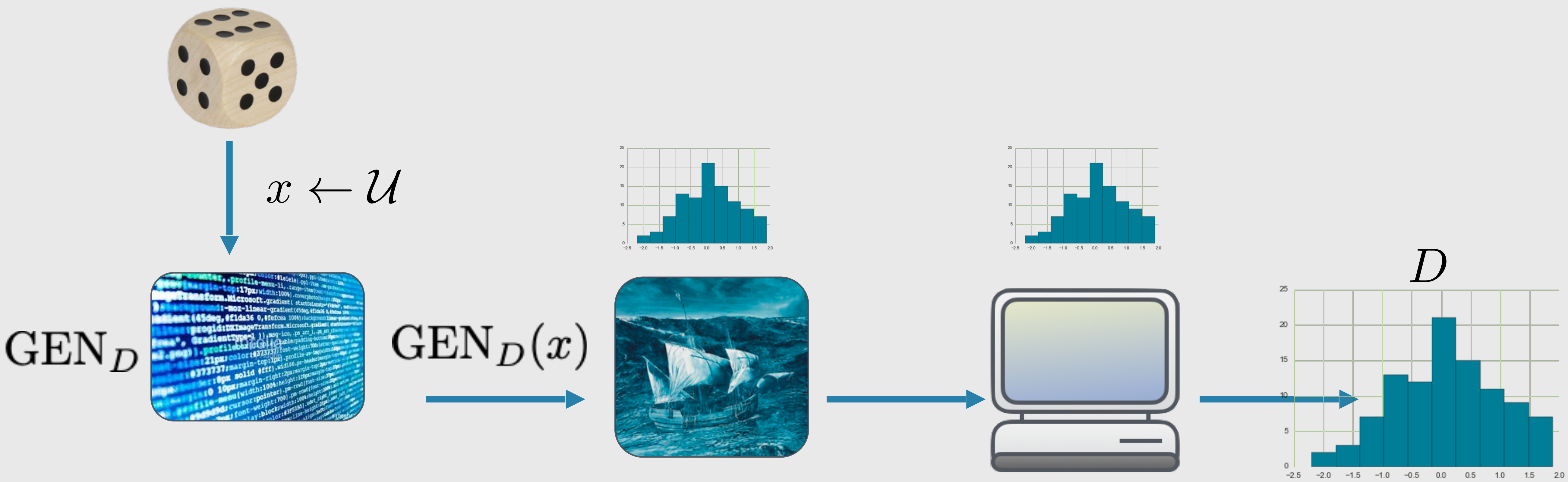
- Quantum

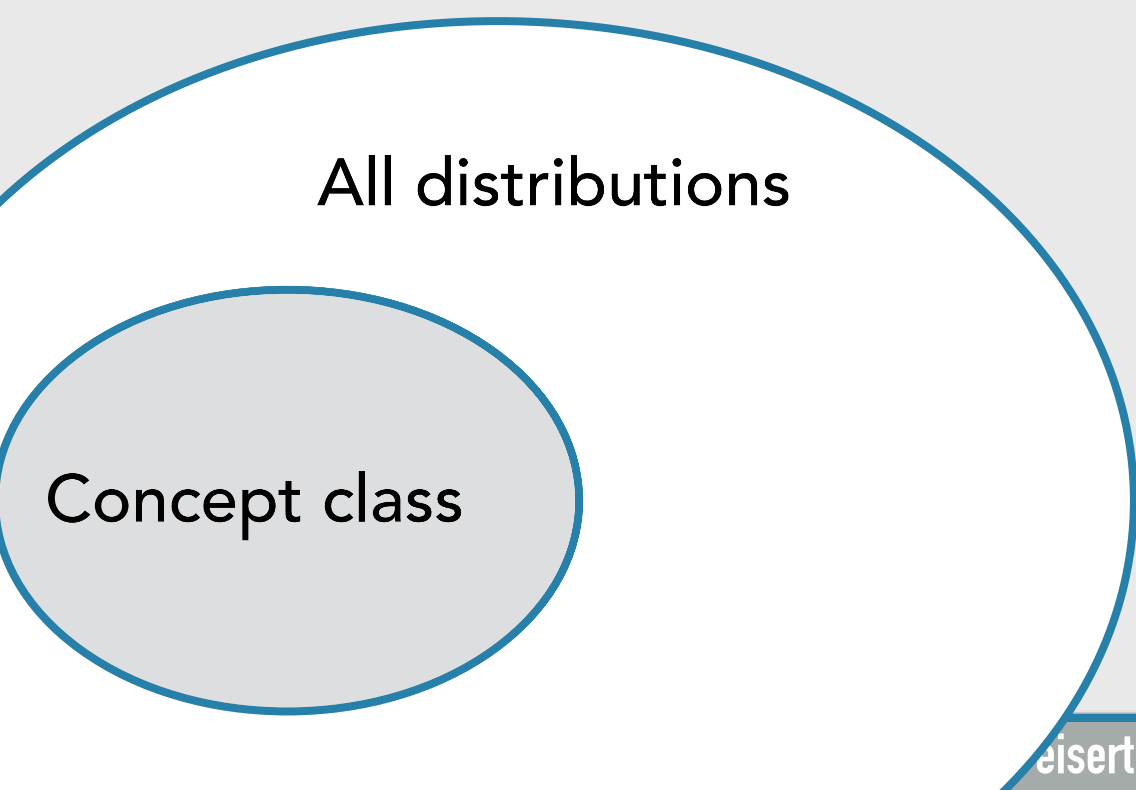
- Classical

# PROVEN QUANTUM ADVANTAGES IN LEARNING TASKS?

Sweke, Seifert, Hangleiter, Eisert, Quantum 5, 417 (2021)
Pirnay, Sweke, Eisert, Seifert, Phys Rev A 107, 042416 (2023)
Liu, Liu, Liu, Ye, Alexeev, Eisert, Liang, arXiv:2303.03428 (2023)

# PROBABILISTIC MODELLING




Pirnay, Sweke, Eisert, Seifert, Phys Rev A 107, 042416 (2023)

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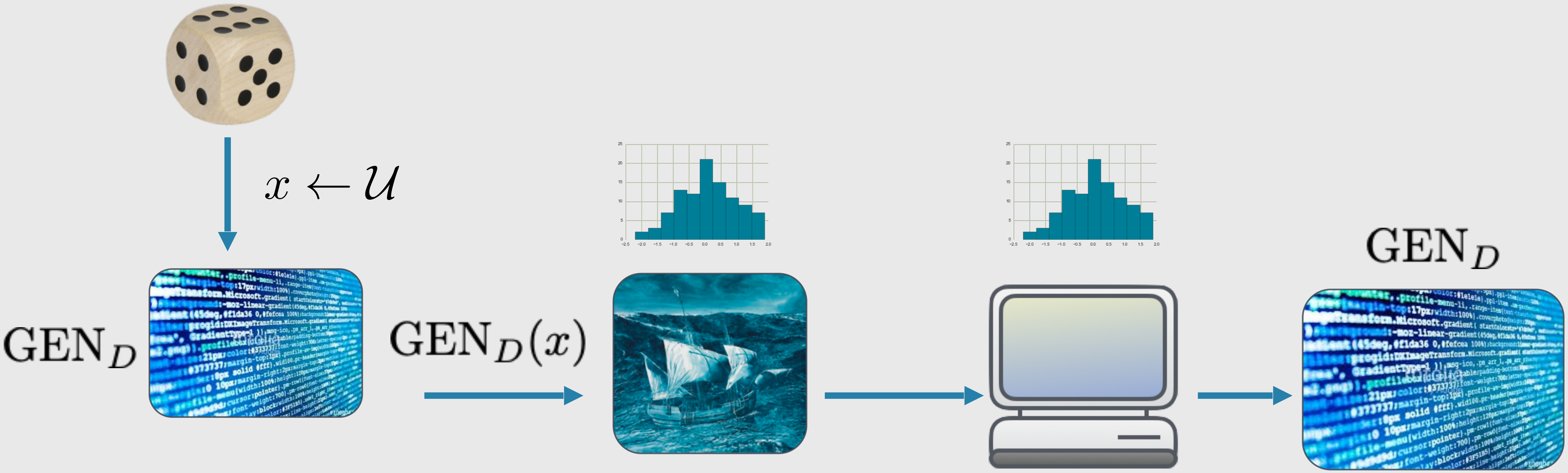
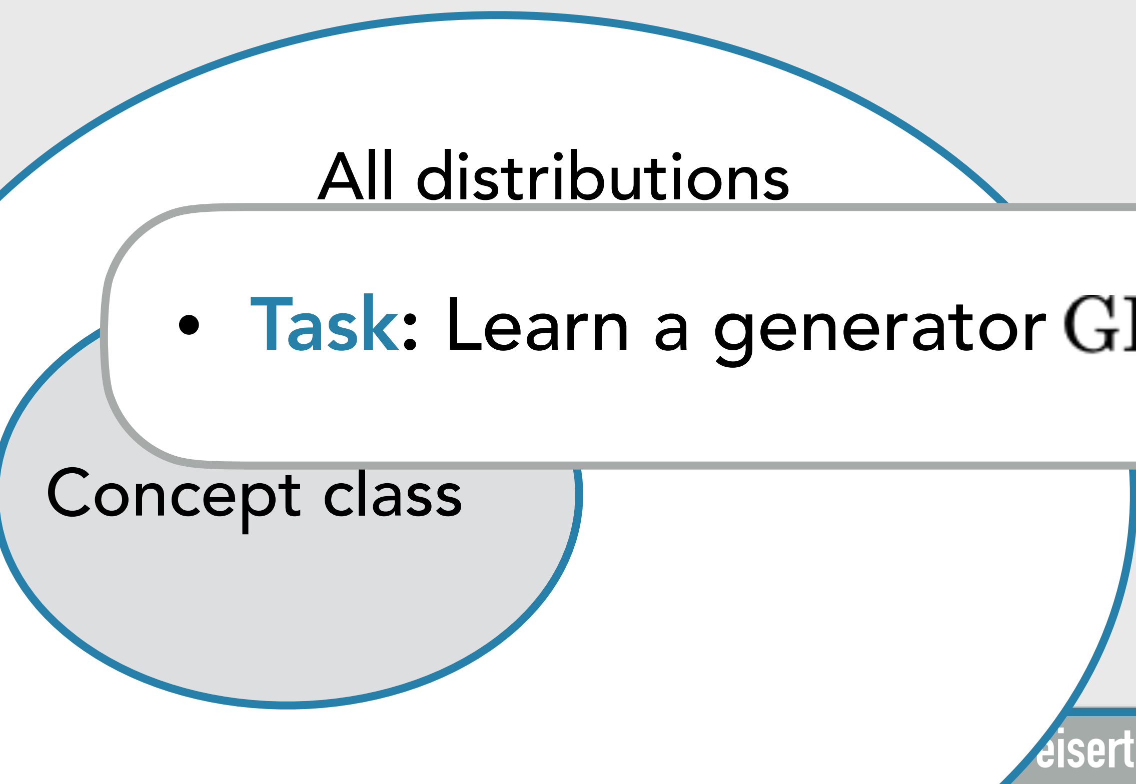


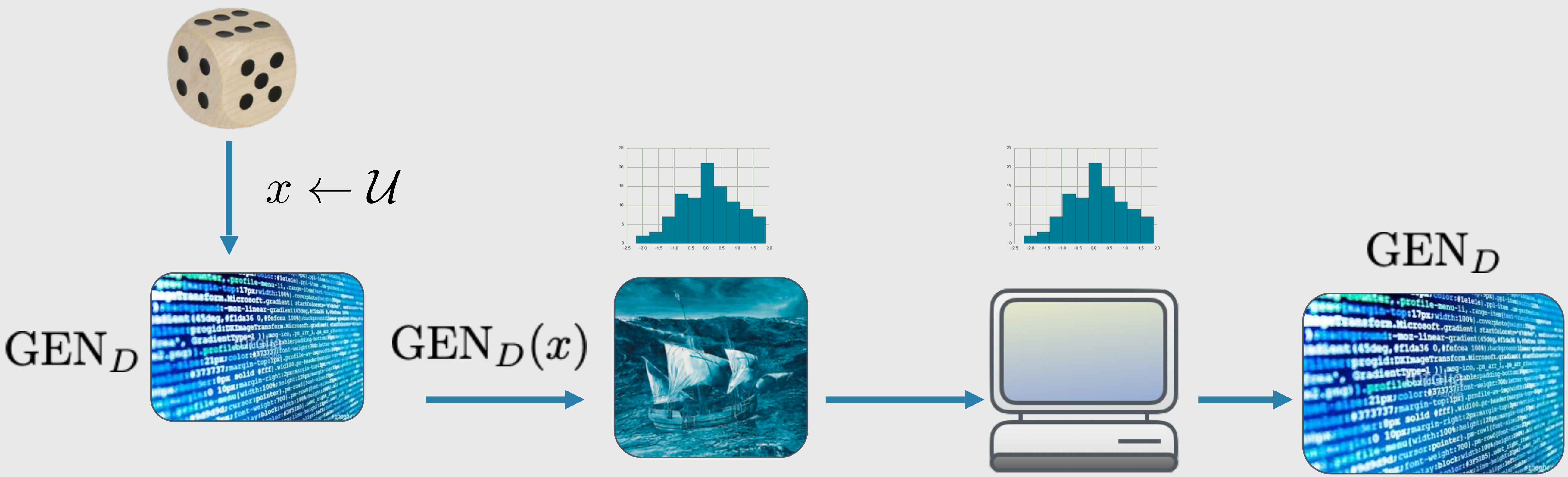

- **Generator modelling**
  Create samples




- **Task**: Learn a generator $\mathrm{GEN}_D$ of a distribution $D$

Sweke, Seifert, Hangleiter, Eisert, Quantum 5, 417 (2021)

# QUANTUM ADVANTAGES IN PROBABILISTIC MODELLING?




- **Can there be a quantum advantage**? Are there distibutions that

- are **efficiently** **quantum** generator learnable?
- are **not efficiently** **classical** generator learnable?

- **Quantum**
- **Classical**

Sweke, Seifert, Hangleiter, Eisert, Quantum 5, 417 (2021)

- "Probably approximately correct" learning of distribution classes
  - A distribution class $\mathcal{C}$ is efficiently PAC learnable w.r.t. distance $d$ if there is an algorithm $\mathcal{A}$ which for every $D \in \mathcal{C}$ and every $\epsilon, \delta > 0$ given access to an oracle $O(D)$, outputs in time $\mathrm{poly}(|D|, 1/\epsilon, 1/\delta)$
  - with probability at least $1 - \delta$ ("probably") a generator $\mathrm{GEN}_{D'}$ of a distribution $D'$ such that

$$d(D, D') < \epsilon$$

("approximately correct")

$\mathrm{GEN}_D$

- Quantum
- Classical

Sweke, Seifert, Hangleiter, Eisert, Quantum 5, 417 (2021)

- **Can there be a quantum advantage**? Are there distibutions that

- are **efficiently quantum** generator learnable?
- are **not efficiently classical** generator learnable?

- For the Kullback Leibler divergence and the natural sample oracle?

# YES! *

* under the decisional Diffie-Hellman assumption for the group family of quadratic residues

- Quantum
- Classical

Sweke, Seifert, Hangleiter, Eisert, Quantum 5, 417 (2021)

# HINT AT THE PROOF

- It is **quantum** easy

- Is it **classical** hard

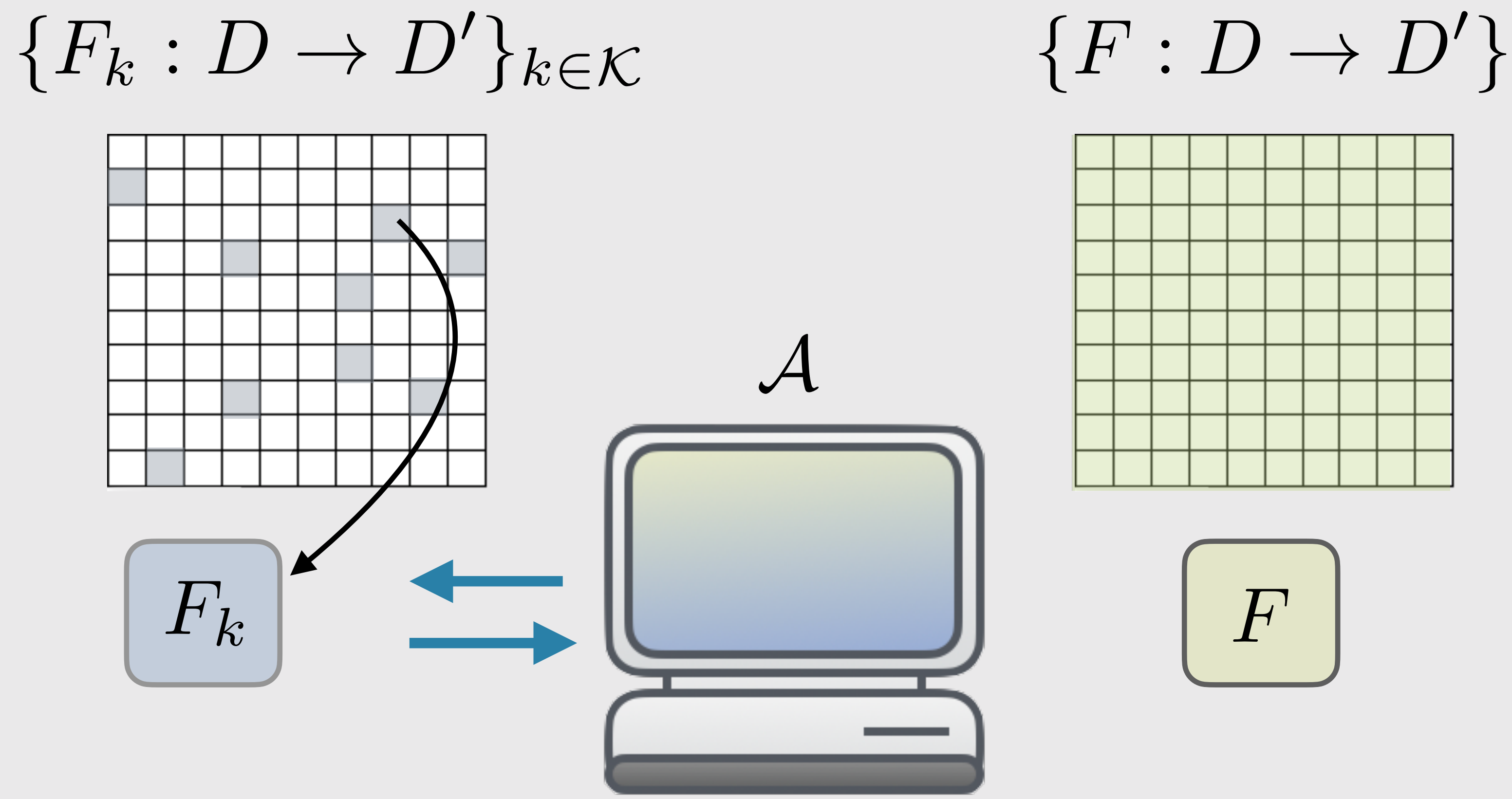
- **Quantum**

- **Classical**

Sweke, Seifert, Hangleiter, Eisert, Quantum 5, 417 (2021)

# HINT AT THE PROOF: CLASSICAL PART

- **Pseudorandom function**
  - A collection of keyed functions $\{F_k : D \to D'\}_{k \in \mathcal{K}}$ that cannot be distinguished from uniformly random functions $\{F : D \to D'\}$ by any polynomial time algorithm $\mathcal{A}$




- Quantum
- Classical

Kearns, Mansour, Sellie, STOC (1994)

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$\{F_k : D \to D'\}_{k\in\mathcal{K}}$ $\{F : D \to D'\}$

- **Theorem**
  - Given a classical secure pseudorandom function $\{F_k\}_k$, the distribution class $\{D_k\}_k$ defined by the "Kearns generator"

$$\mathrm{KGEN}_k(x) = x||F_k(x)$$

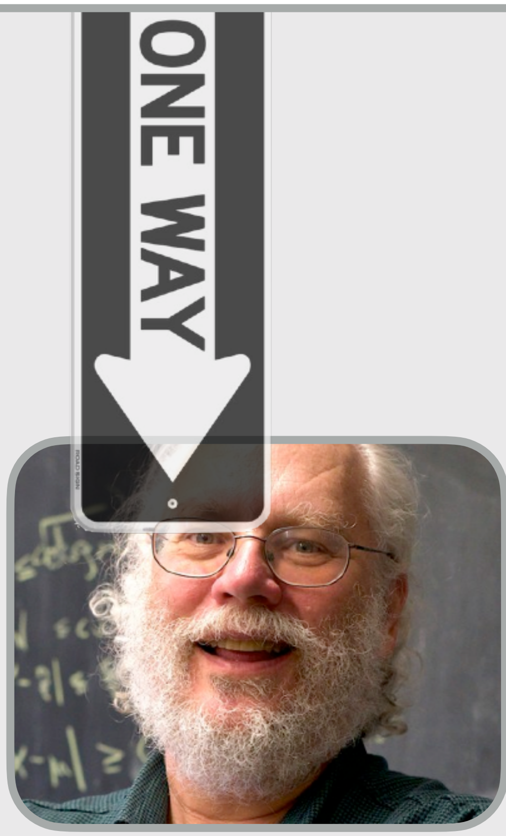
cannot be efficiently classically generator learned

- Quantum
- Classical

Kearns, Mansour, Sellie, STOC (1994)

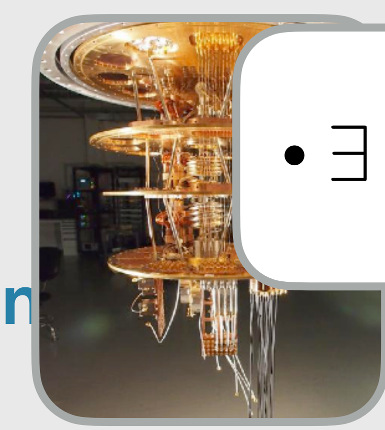
# HINT AT THE PROOF: QUANTUM PART

- ∃ one-way function




- ∃ pseudorandom generator

- ∃ pseudorandom function



- ∃ hard-to-learn distribution class

- Quan

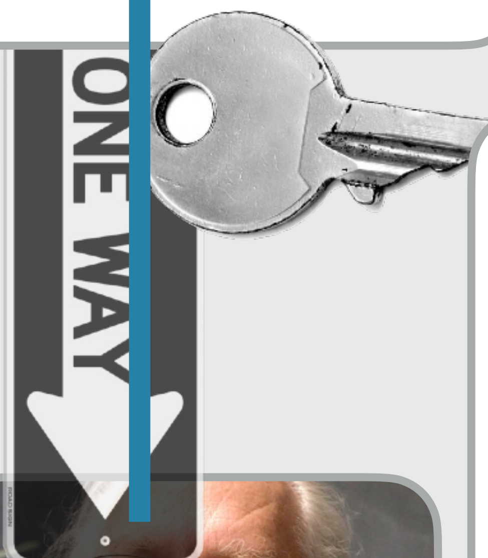
- Classical

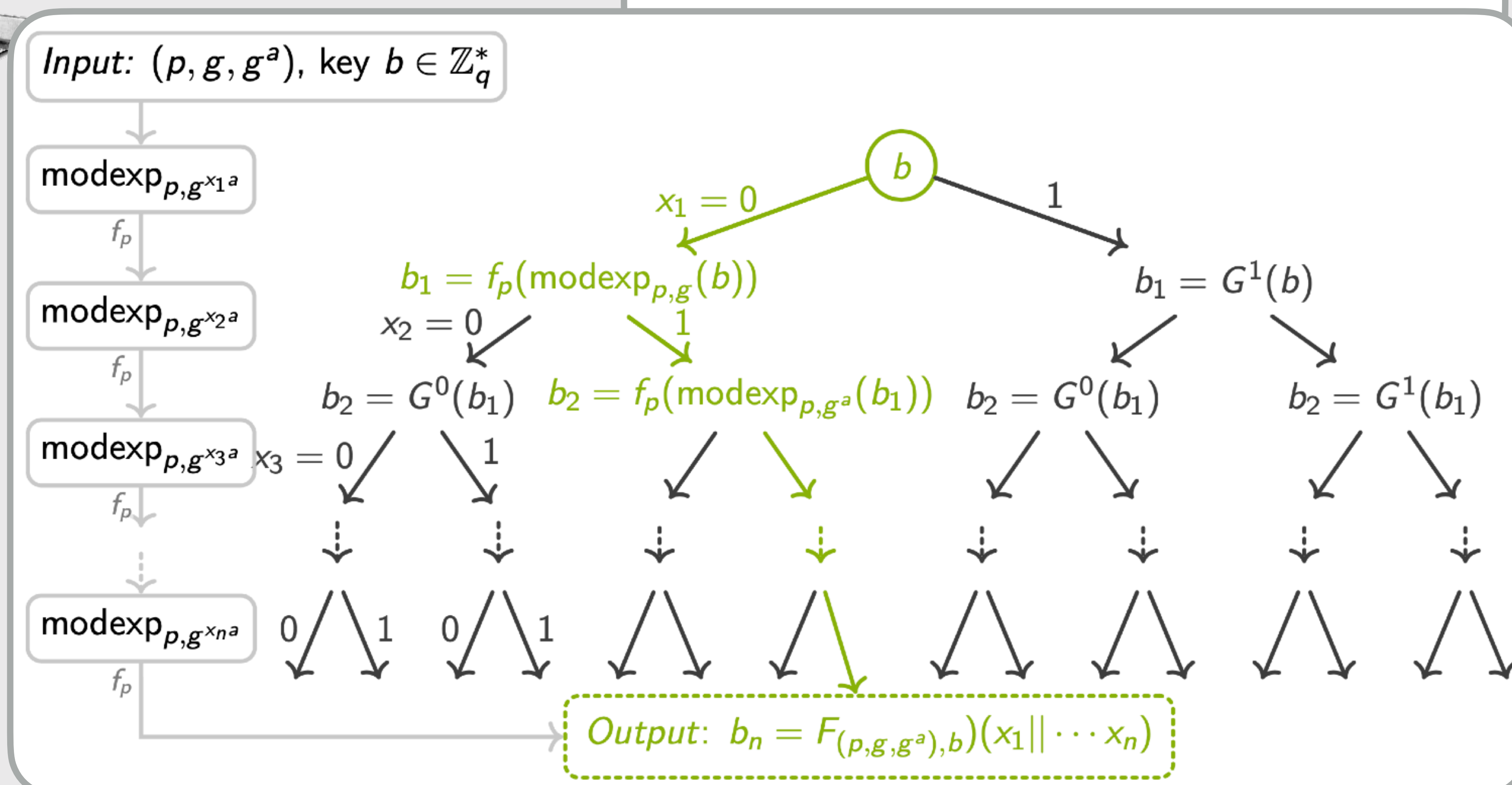
Sweke, Seifert, Hangleiter, Eisert, Quantum 5, 417 (2021)
Shor, SIAM J Comp 26, 1484 (1997)

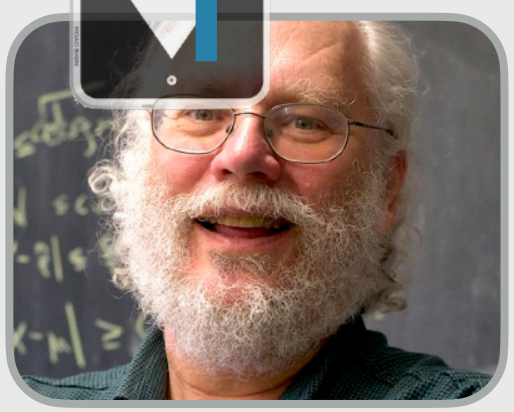
# HINT AT THE PROOF: QUANTUM PART

- ∃ one-way function

- ∃ pseudorandom generator



- Cracking hard-to-learn distribution classes with a QC

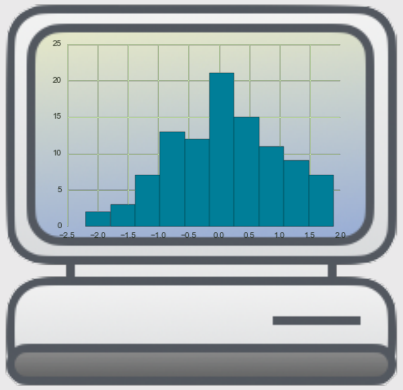
- Quan

- Classical

Sweke, Seifert, Hangleiter, Eisert, Quantum 5, 417 (2021)
Shor, SIAM J Comp 26, 1484 (1997)

# LESSON

- **There is a proven** **quantum generator learning advantage**:
  "Quantum computers learn (exponentially) more efficiently"



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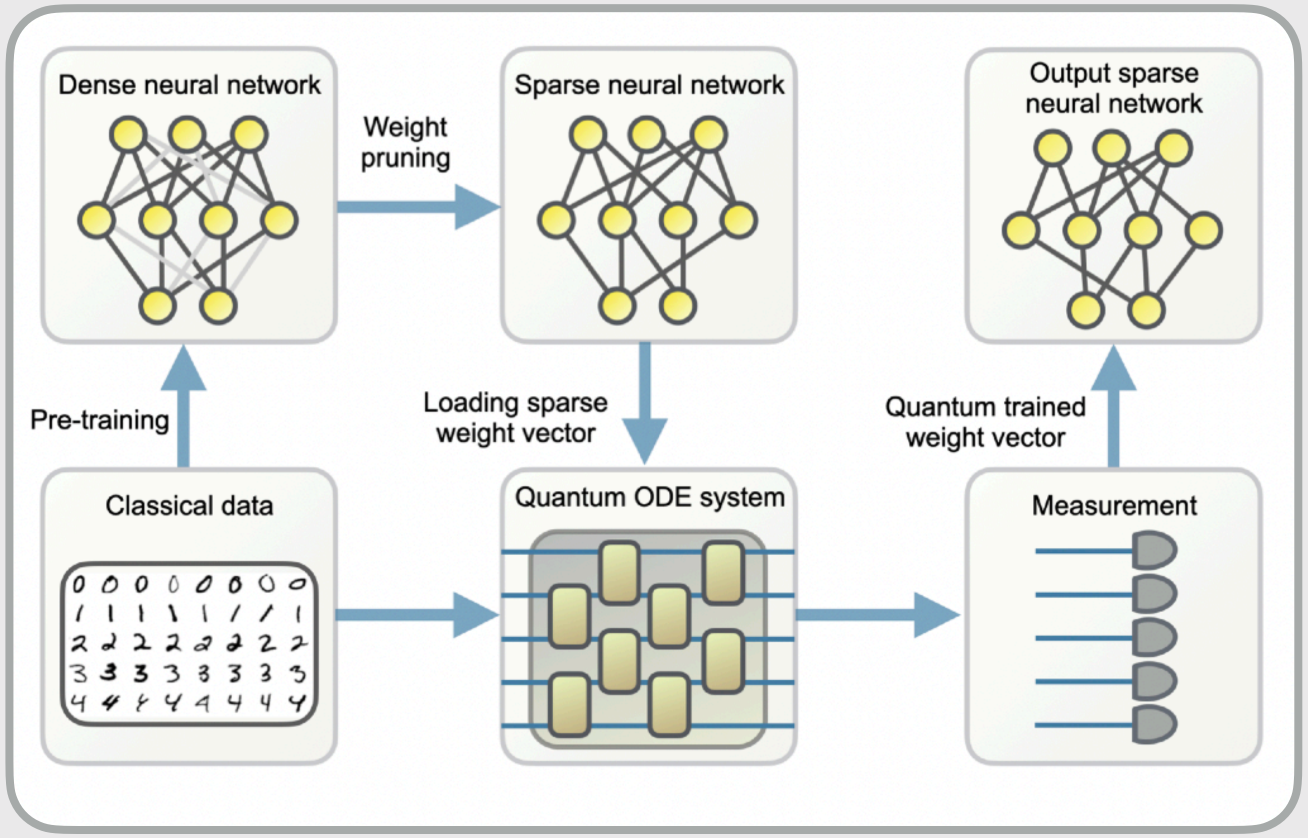
Sweke, Seifert, Hangleiter, Eisert, Quantum 5, 417 (2021)
Pirnay, Sweke, Eisert, Seifert, Phys Rev A 107, 042416 (2023)
Liu, Arunachalam, Temme, Nature Phys 17, 1013 (2021)

# CAN QUANTUM COMPUTERS HELP IN CLASSICAL TRAINING?

- Can **classical networks** be better trained with quantum computers?




Liu, Liu, Liu, Ye, Alexeev, Eisert, Liang, submitted to Nature Communications, arXiv:2303.03428 (2023)

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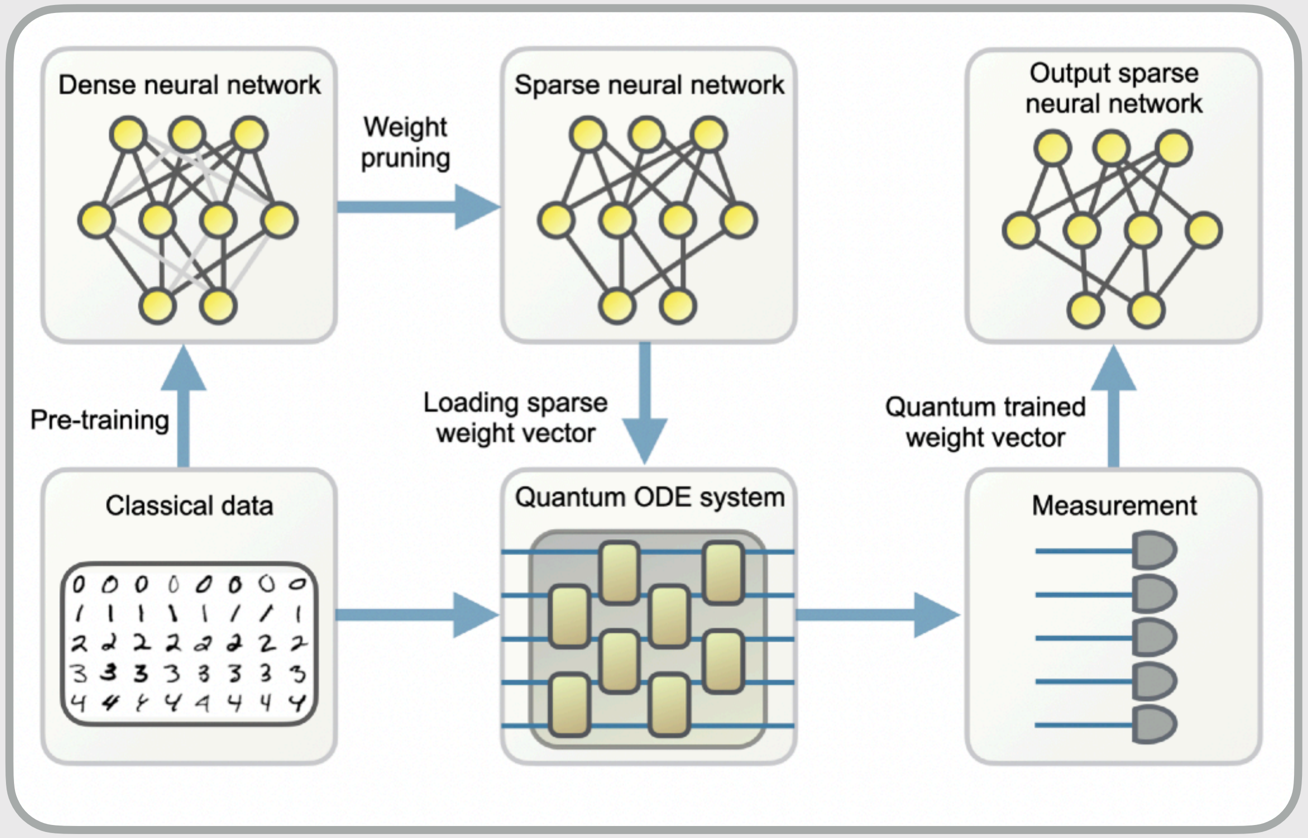


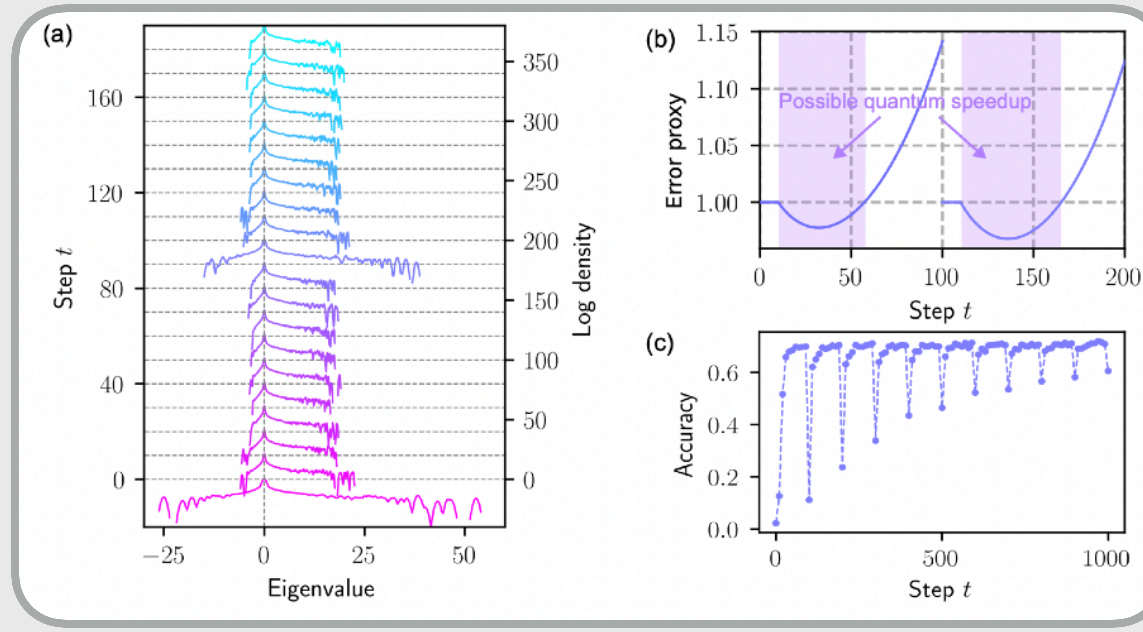


**Theorem 1** (Informal). *For a sparse machine learning model with model size $n$, running $T$ iterations, with the algorithm being fully dissipative with small learning rates (whose formal definition is given in the supplemental material), there is a quantum algorithm that runs in*

$$\mathcal{O}\left(T \times \text{poly}\left(\log n, \frac{1}{\epsilon}\right)\right) \qquad (1)$$

*time with precision $\epsilon > 0$. The sparsity condition also ensures the efficiency of uploading and downloading quantum states towards classical processors.*

- Devise variants of HHL for training sparse (pruned) classical networks

Liu, Liu, Liu, Ye, Alexeev, Eisert, Liang, submitted to Nature Communications, arXiv:2303.03428 (2023)

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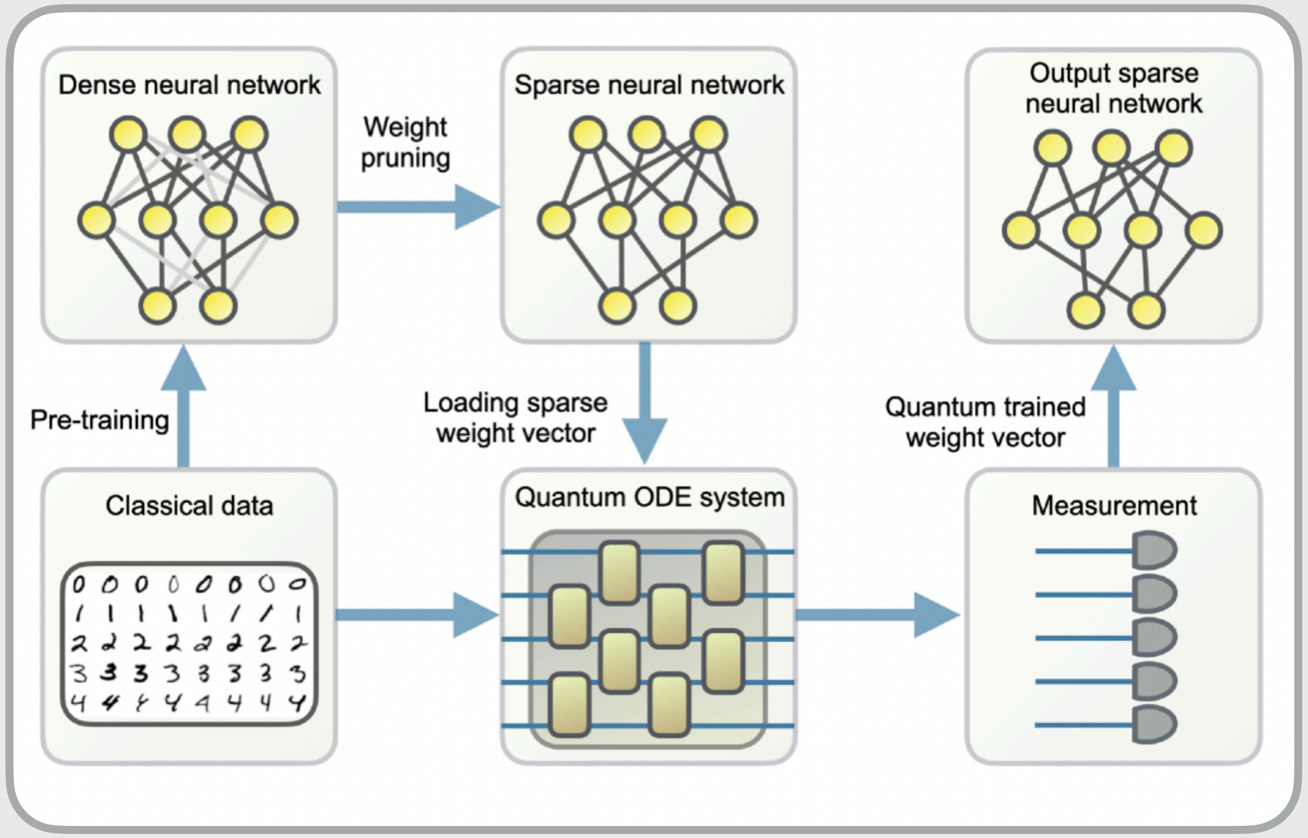


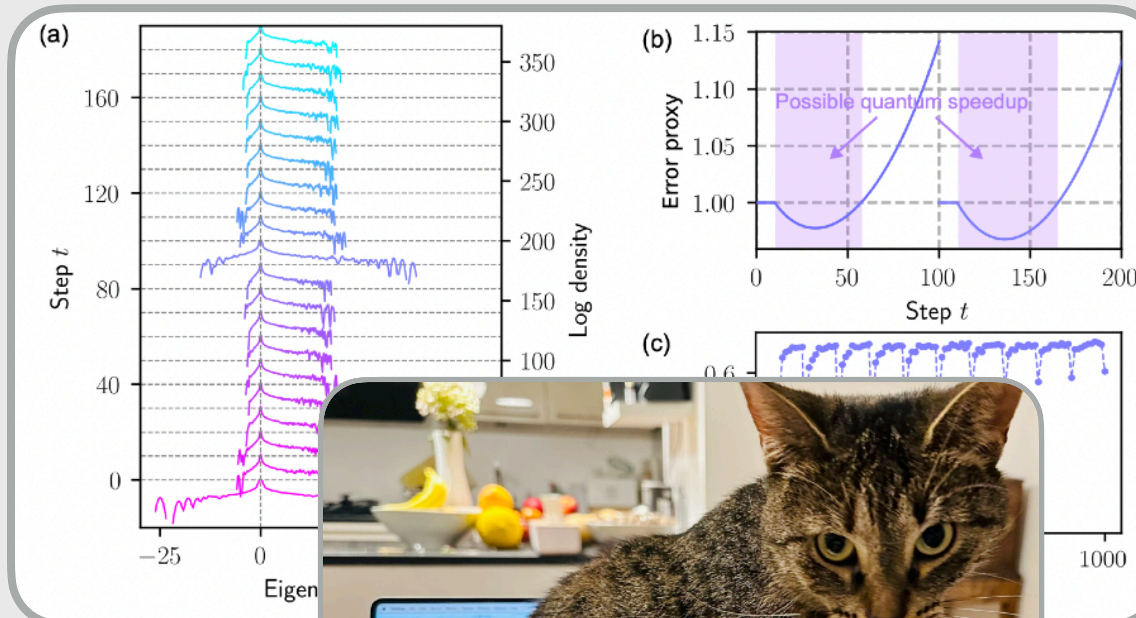


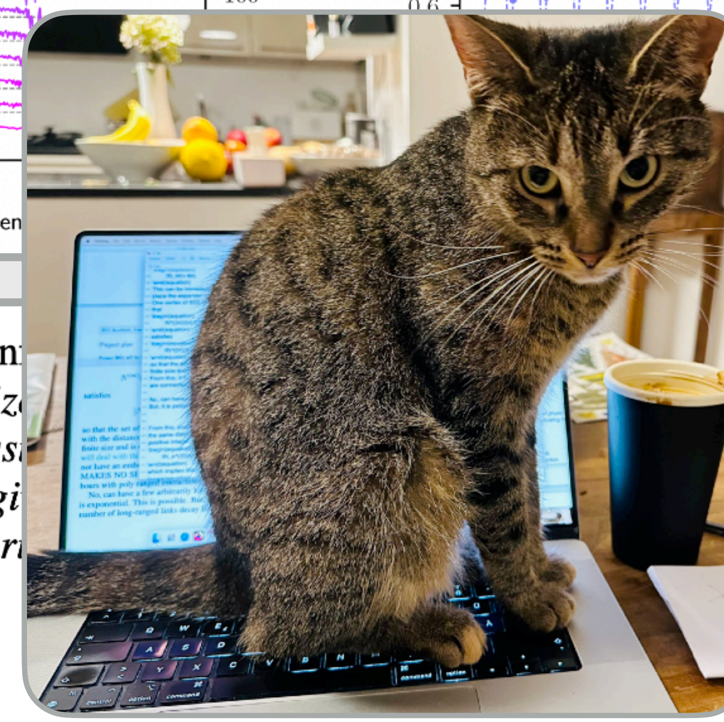

**Theorem 1** (In ... *with model siz ... being fully diss ... definition is gi ... quantum algori* ... *del ... hm ... nal ... s a* ... (1) ... *sures ... tes*

- Devise variants of **HHL** for training sparse (pruned) classical networks

- Quantum algorithms can have superpolynomial speedups over stochastic gradient descent in **training** pruned classical networks

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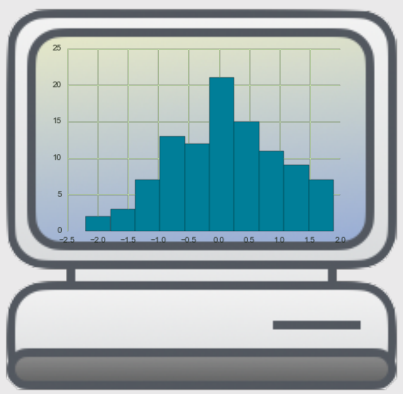

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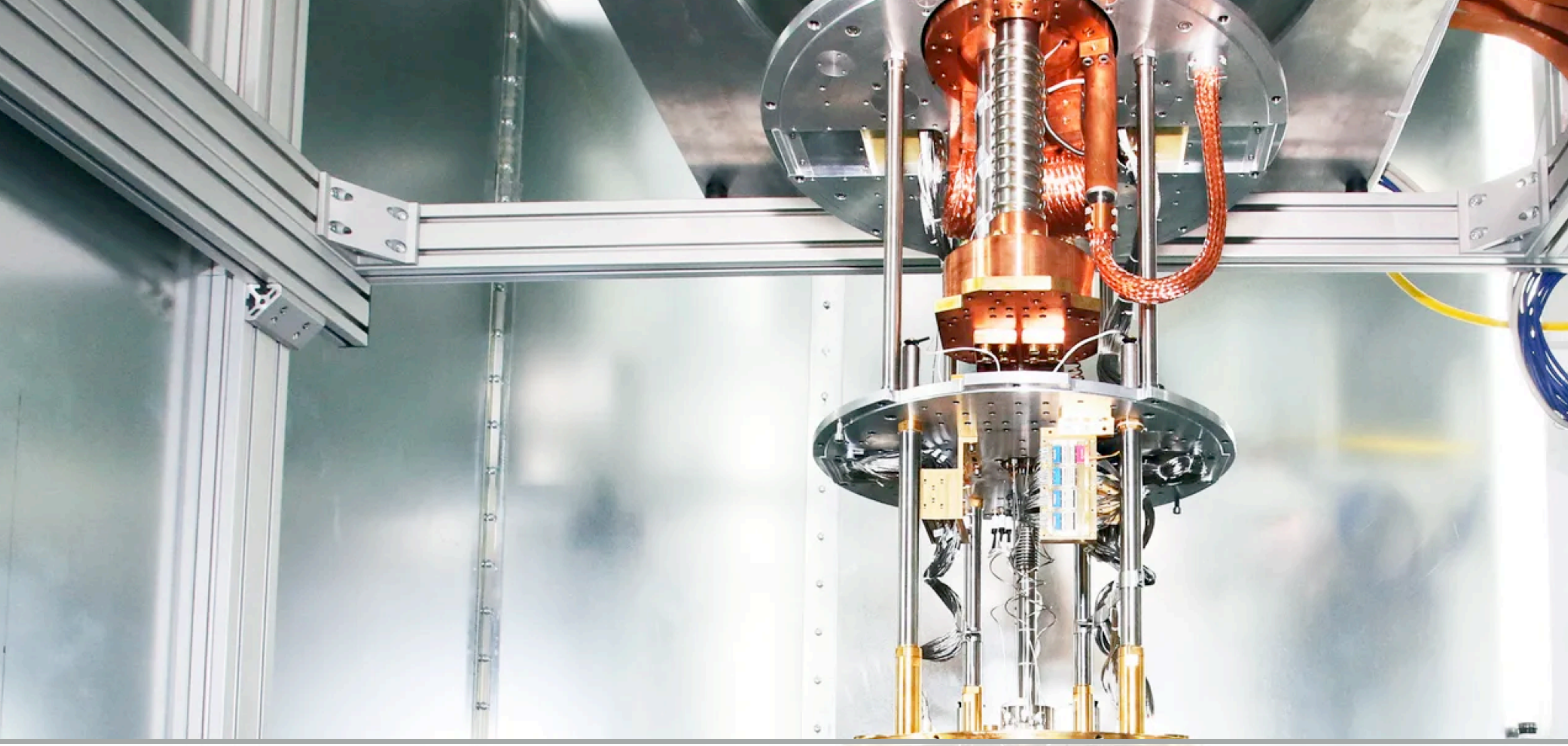
- For unstructured data, but still **fault tolerant quantum computers**

- Quantum algorithms can have superpolynomial speedups over stochastic gradient descent in **training** pruned classical networks

Liu, Liu, Liu, Ye, Alexeev, Eisert, Liang, submitted to Nature Communications, arXiv:2303.03428 (2023)

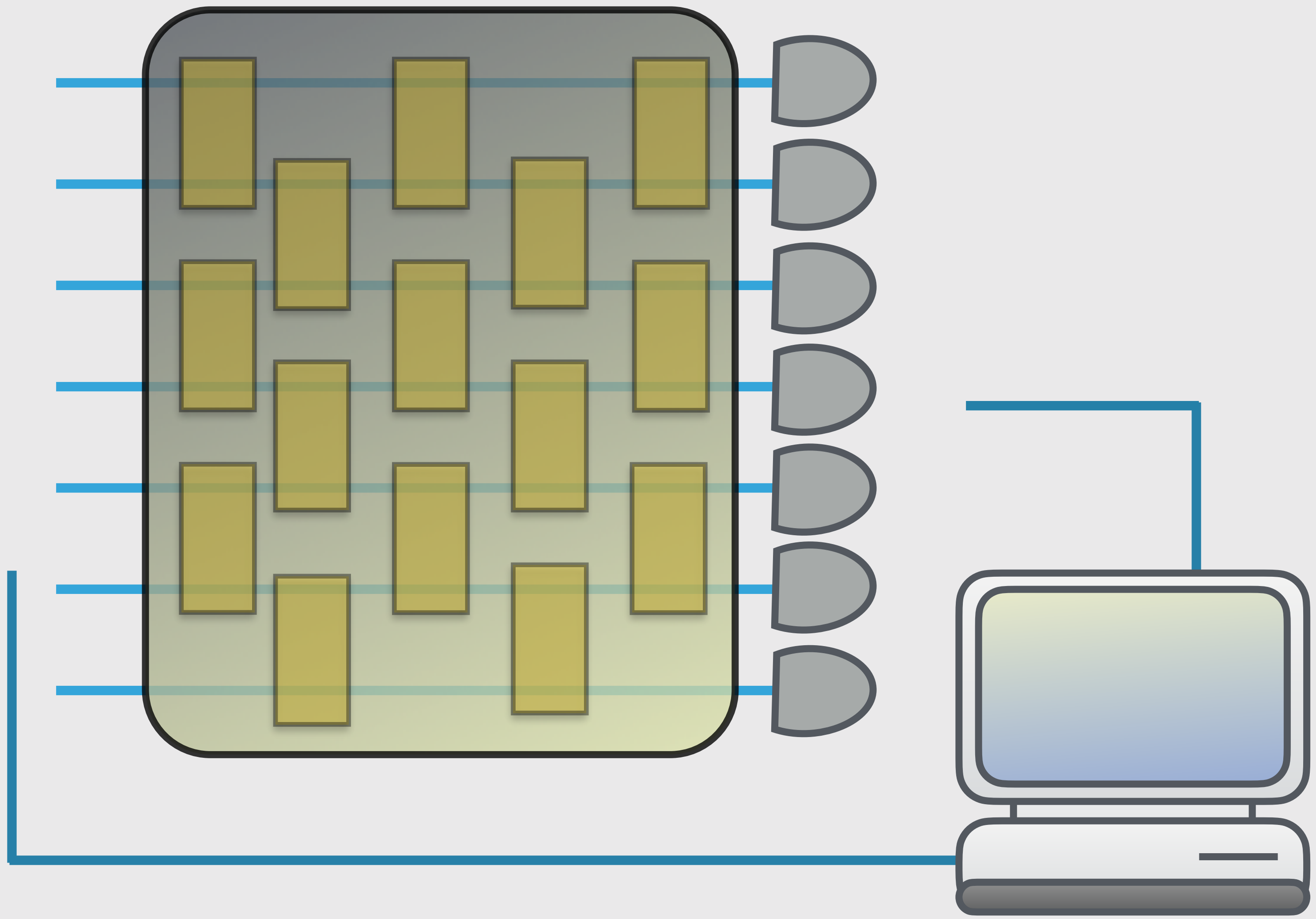


# LEARNING THE OUTPUT DISTRIBUTION OF QUANTUM CIRCUITS

Hinsche, Ioannou, Nietner, Haferkamp, Quek, Hangleiter, Seifert, Eisert, Sweke, Phys Rev Lett 130, 240602 (2023)
Haferkamp, Montealegre-Mora, Heinrich, Eisert, Gross, Roth, Comm Math Phys 397, 995-1041 (2023)
Nietner, Ioannou, Sweke, Kueng, Eisert, Hinsche, Haferkamp, arXiv:2305.05765 (2023)
Hinsche, Ioannou, Nietner, Haferkamp, Quek, Hangleiter, Seifert, Eisert, Sweke, arXiv:2110.05517 (2021)

# TOWARDS NEAR-TERM SEPARATIONS?

- **Question:** Can we find a near-term quantum-classical separation?



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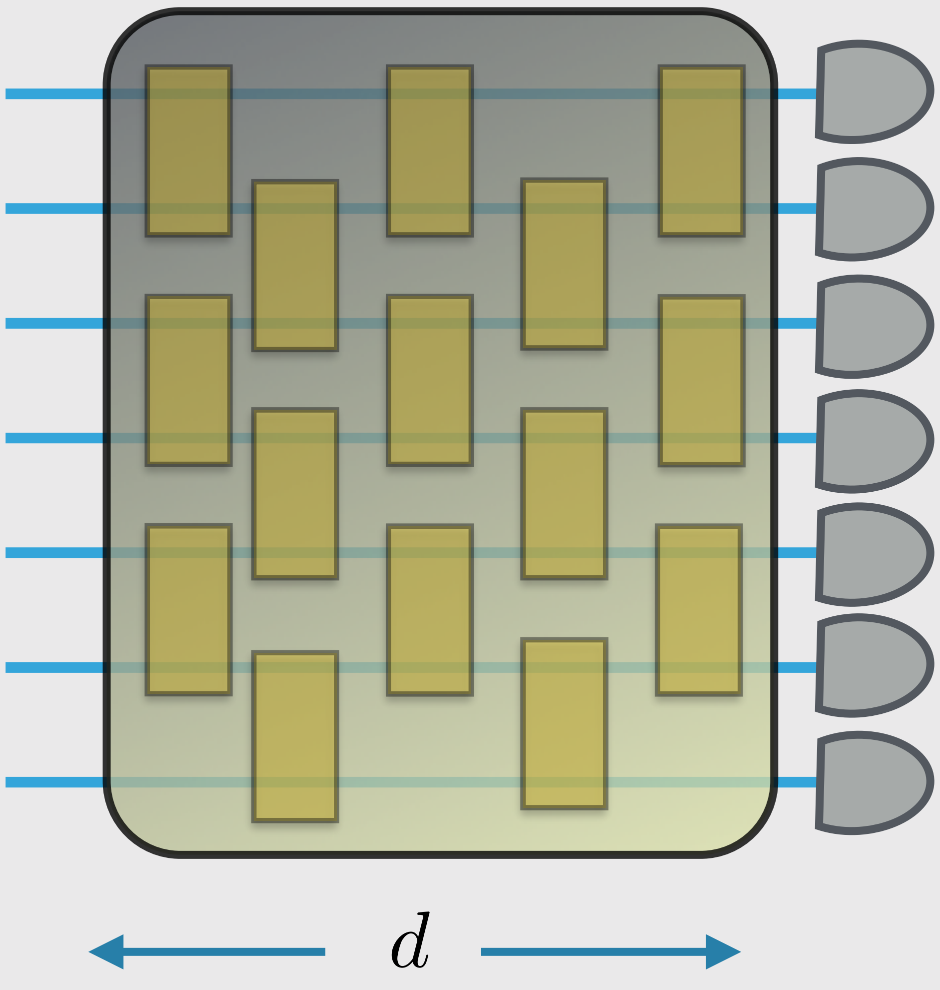
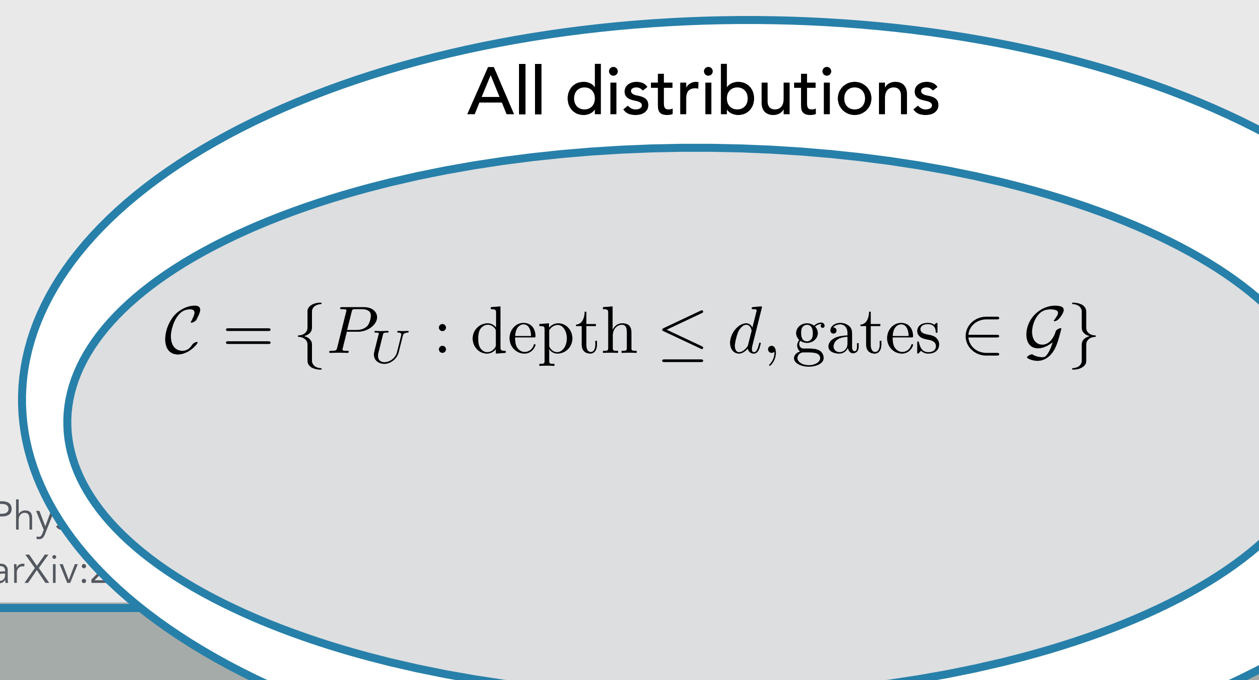
## TOWARDS NEAR-TERM SEPARATIONS?

- Take **output distributions of quantum circuits** themselves




- Hard to classically simulate, but to learn?
  - $d = 1$: product distributions
  - $d \to \infty$: all distributions

- When does **hardness** set in?

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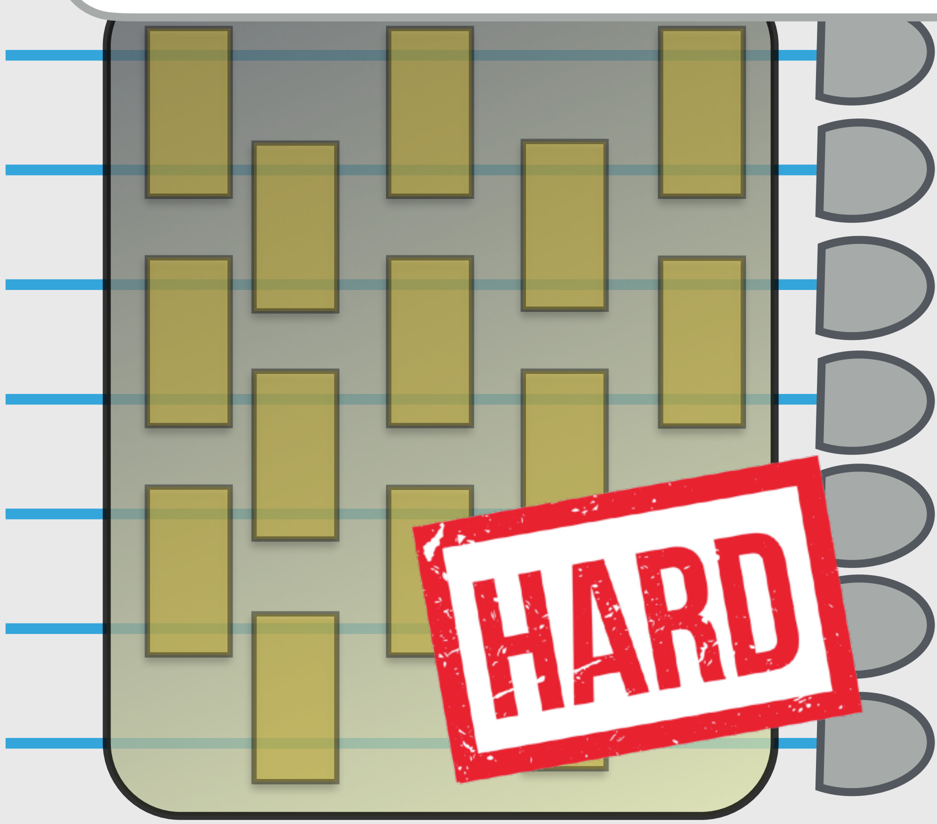
Hinsche, Ioannou, Nietner, Haferkamp, Quek, Hangleiter, Seifert, Eisert, Sweke, arXiv:2

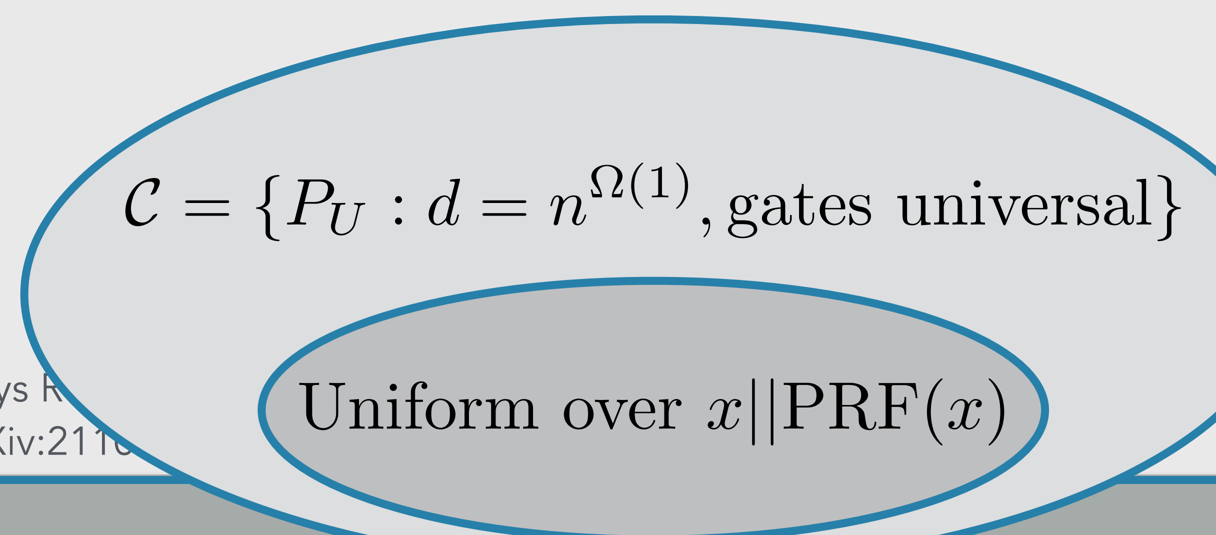
# TOWARDS NEAR-TERM SEPARATIONS?

- **Theorem 2:** If "standard-secure PRFs" exist, then depth $d = n^{\Omega(1)}$ is hard, for generative and density modelling




$d = n^{\Omega(1)}$

- Any (quantum and classical) algorithm!
- This time, PRFs cannot even be broken by quantum algorithms: No advantage

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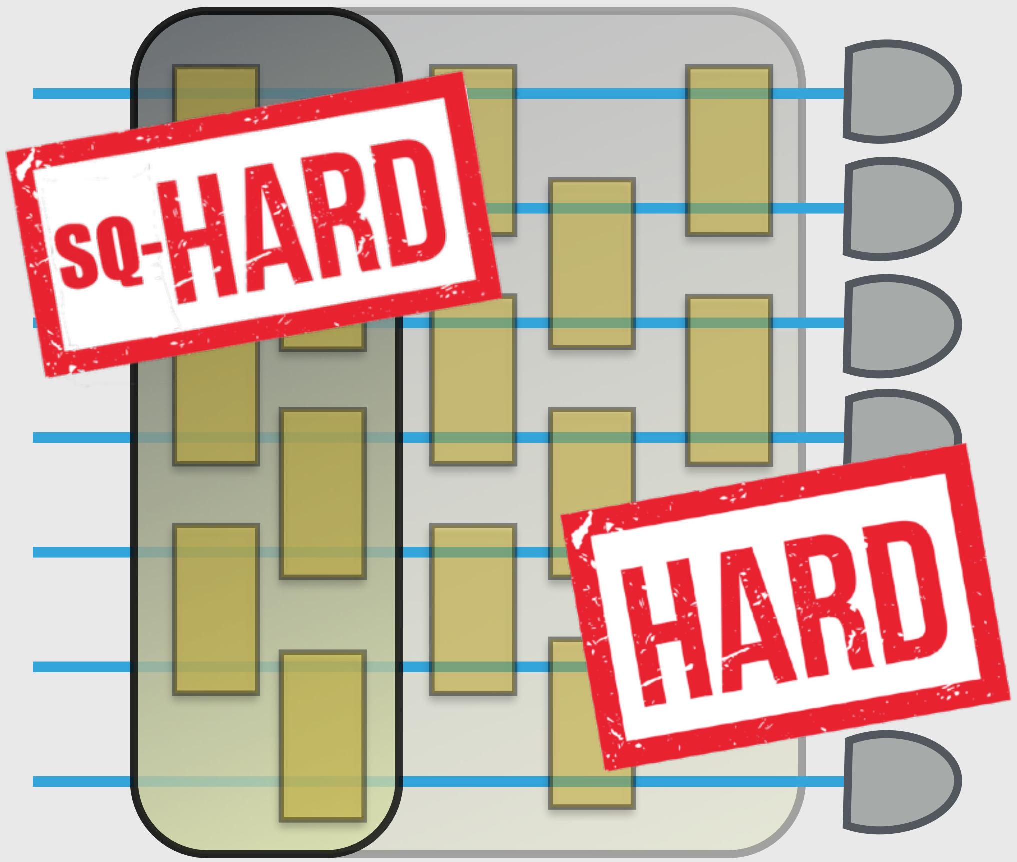
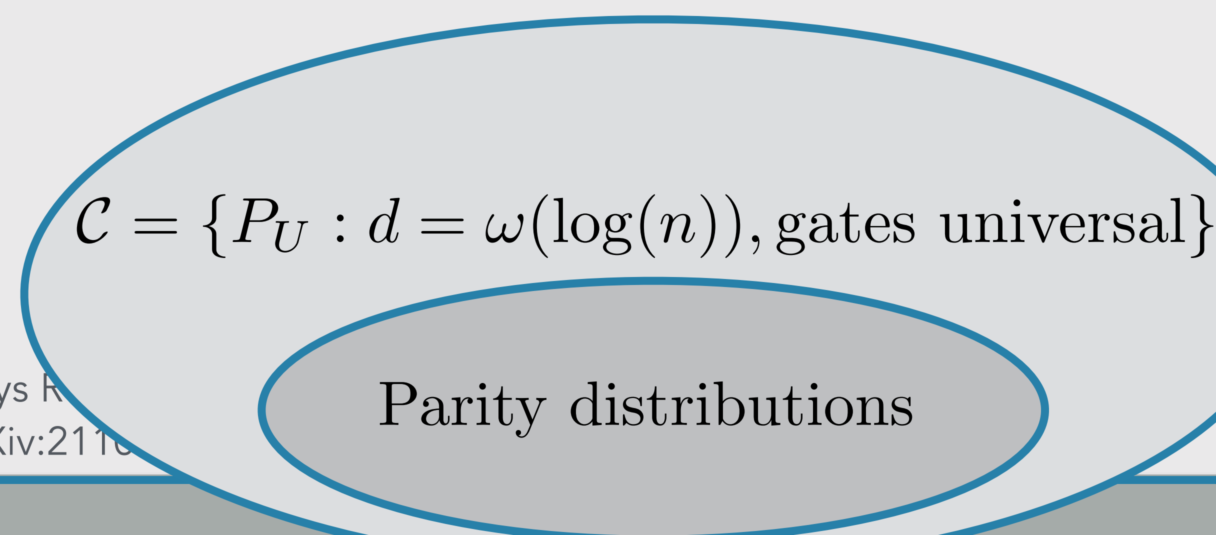
# TOWARDS NEAR-TERM SEPARATIONS?

- **Theorem 3:** Depth $d = \omega(\log(n))$ is **hard**, from **statistical queries**




$d = \omega(\log(n))$ $\quad$ $d = n^{\Omega(1)}$

- Think of "generic" learning algorithms

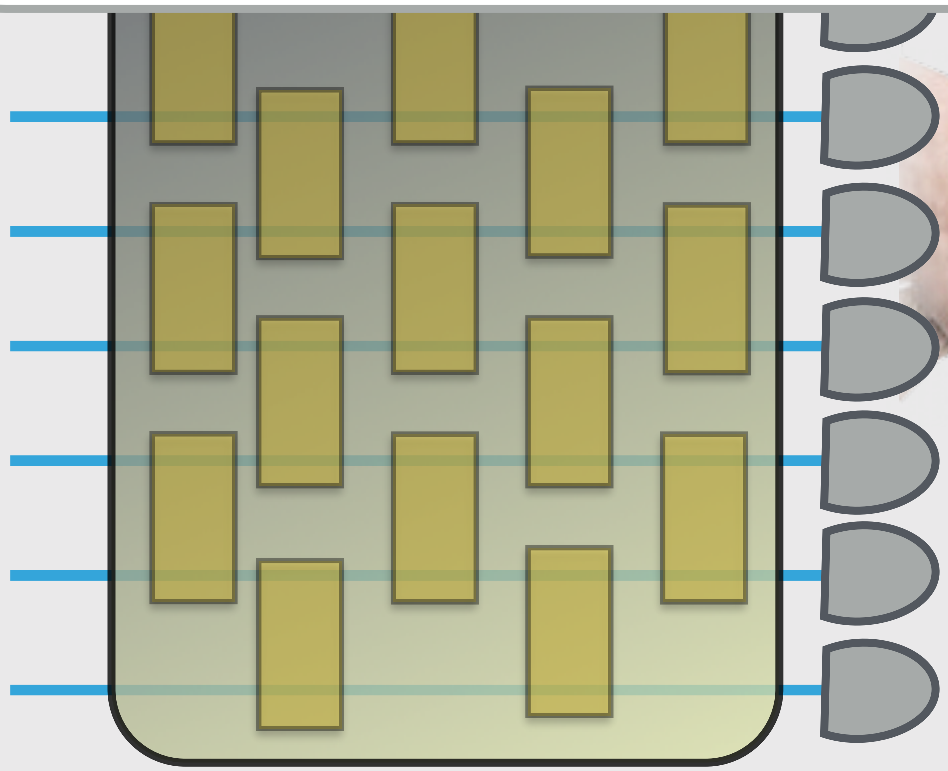


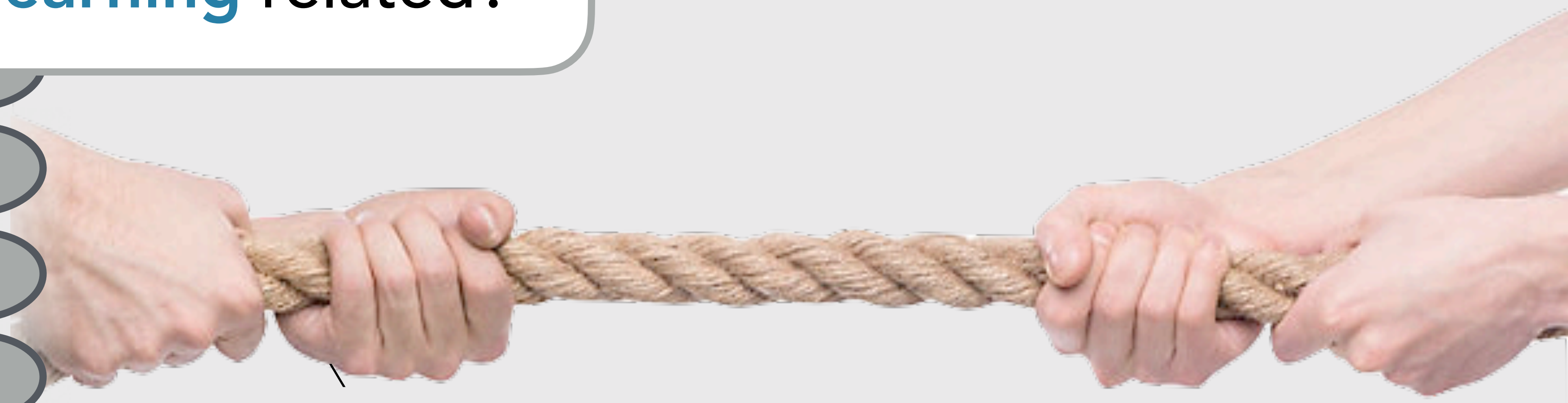


Hinsche, Ioannou, Nietner, Haferkamp, Quek, Hangleiter, Seifert, Eisert, Sweke, Phys R
Hinsche, Ioannou, Nietner, Haferkamp, Quek, Hangleiter, Seifert, Eisert, Sweke, arXiv:211

# CLASSICAL SIMULATION AND LEARNING

- How are **simulation** and **learning** related?

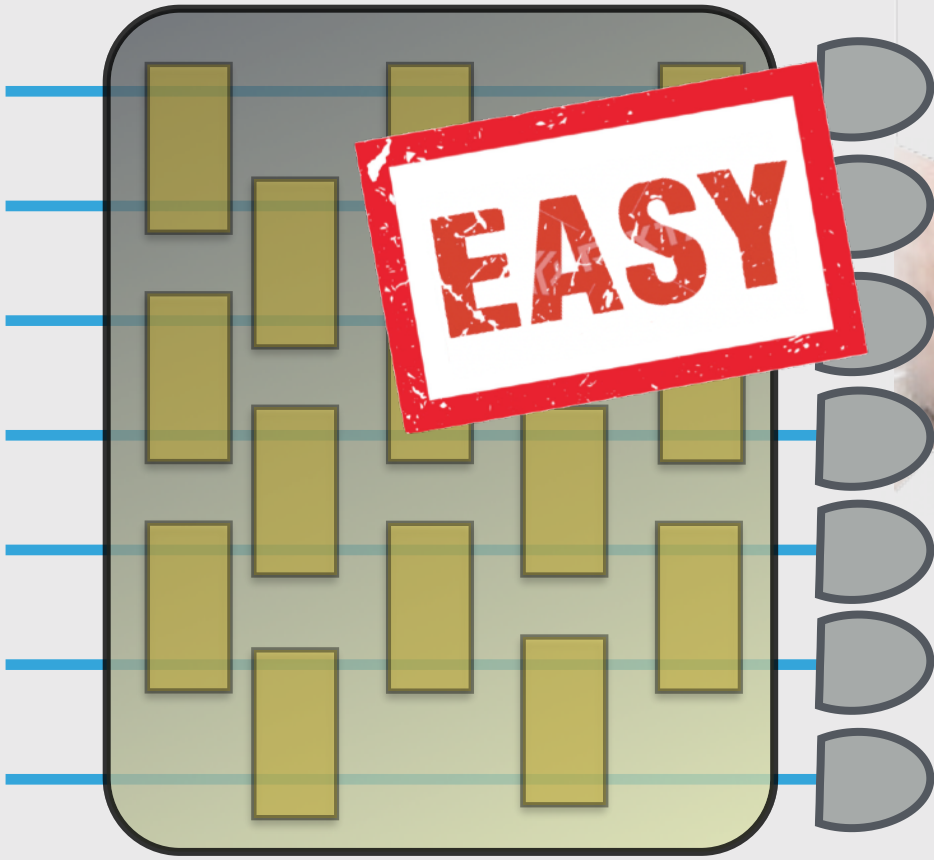





Hinsche, Ioannou, Nietner, Haferkamp, Quek, Hangleiter, Seifert, Eisert, Sweke, Phys Rev Lett 130, 240602 (2023)
Hinsche, Ioannou, Nietner, Haferkamp, Quek, Hangleiter, Seifert, Eisert, Sweke, arXiv:2110.05517 (2021)

# CLASSICAL SIMULATION AND LEARNING

- **Theorem 4:**




Clifford circuits, any depth

Simulation

Learning

- Clifford-circuit output distributions are uniform over affine subspaces of the finite $n$-dimensional vector space $\mathbb{F}_2^n$

Hinsche, Ioannou, Nietner, Haferkamp, Quek, Hangleiter, Seifert, Eisert, Sweke, Phys Rev Lett 130, 240602 (2023)
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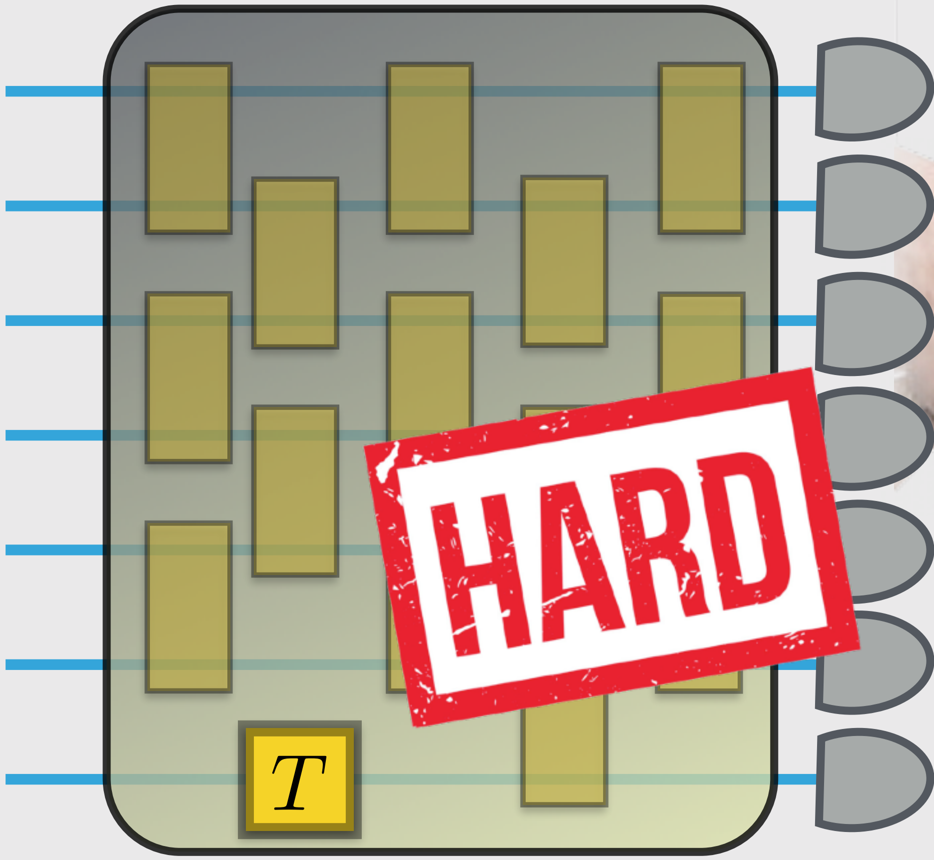
# CLASSICAL SIMULATION AND LEARNING

- **Theorem 5: A single $T$-gate renders learning hard**




Clifford circuits and a single $T$-gate, linear depth

Simulation

Learning

- But circuits with logarithmically many $T$-gates can be efficiently simulated


Hinsche, Ioannou, Nietner, Haferkamp, Quek, Hangleiter, Seifert, Eisert, Sweke, Phys Rev Lett 130, 240602 (2023)
Hinsche, Ioannou, Nietner, Haferkamp, Quek, Hangleiter, Seifert, Eisert, Sweke, arXiv:2110.05517 (2021)

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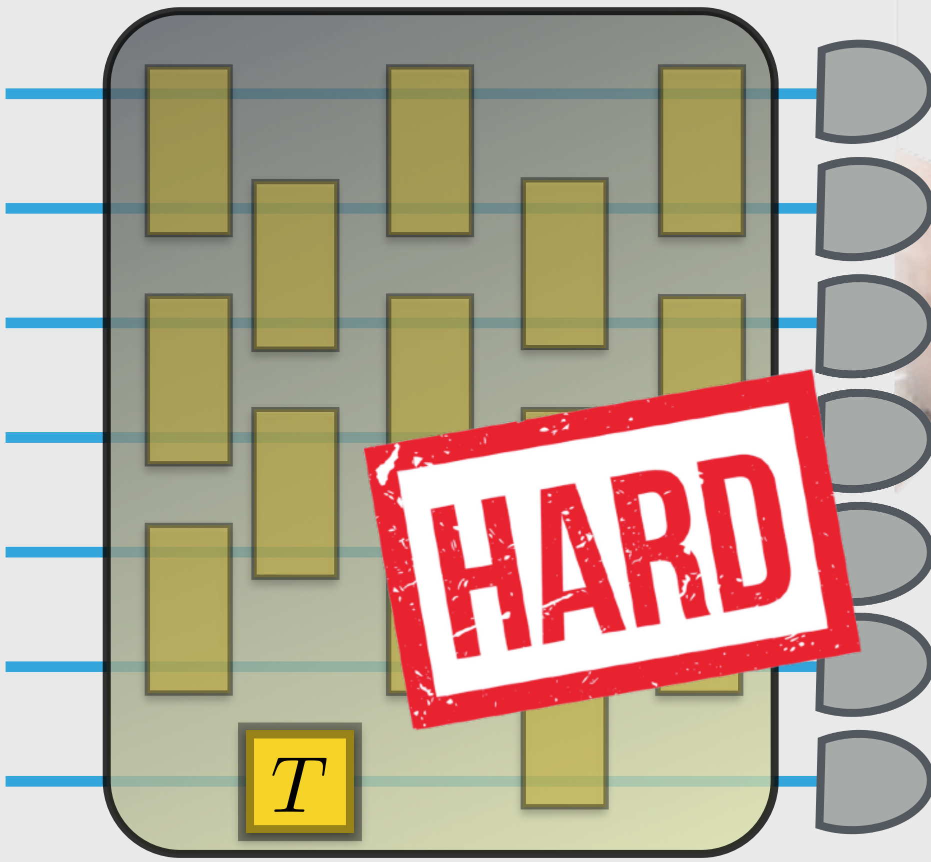


Clifford circuits and a single $T$-gate, linear depth

Simulation

Learning

R Sweke: "Should we use parametrized quantum circuits for machine learning?"

Hinsche, Ioannou, Nietner, Haferkamp, Quek, Hangleiter, Seifert, Eisert, Sweke, Phys Rev Lett 130, 240602 (2023)
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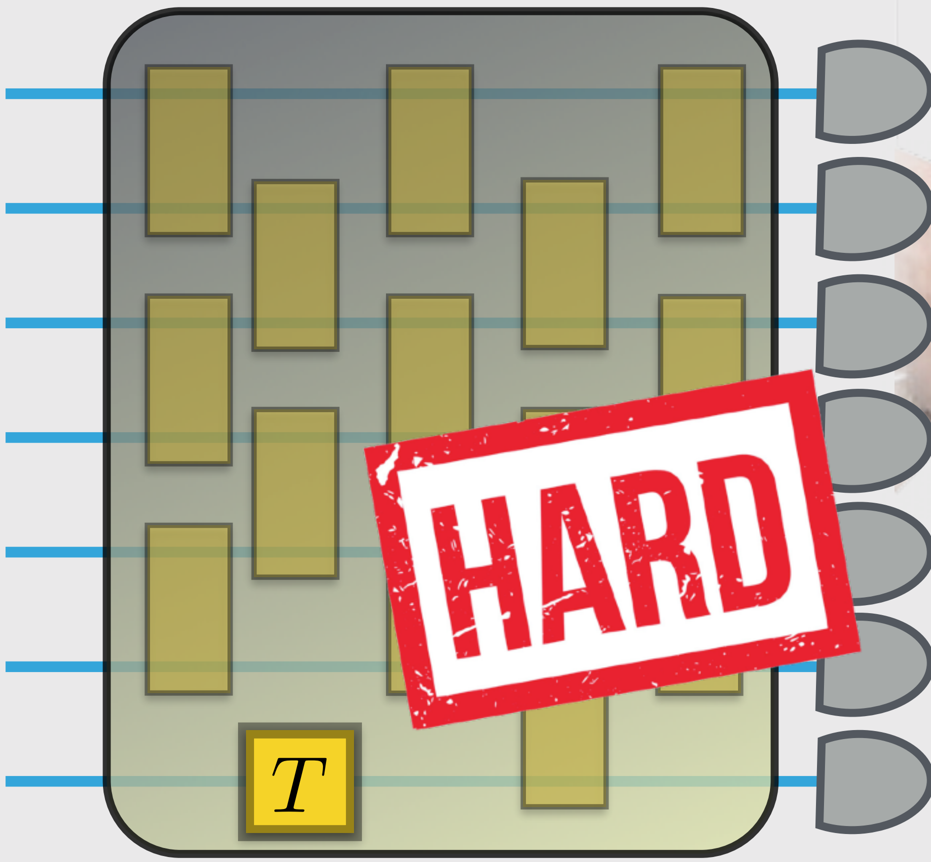
# CLASSICAL SIMULATION AND LE

- **Theorem 5:** A single $T$-gate




Clifford circuits and a single $T$-gate, linear depth

- Random Clifford circuits are unitary 3-designs
- $T$-gates uplift then to (approximate) arbitrary order designs
- **Theorem 6:** A constant (!) number of $T$-gates is sufficient

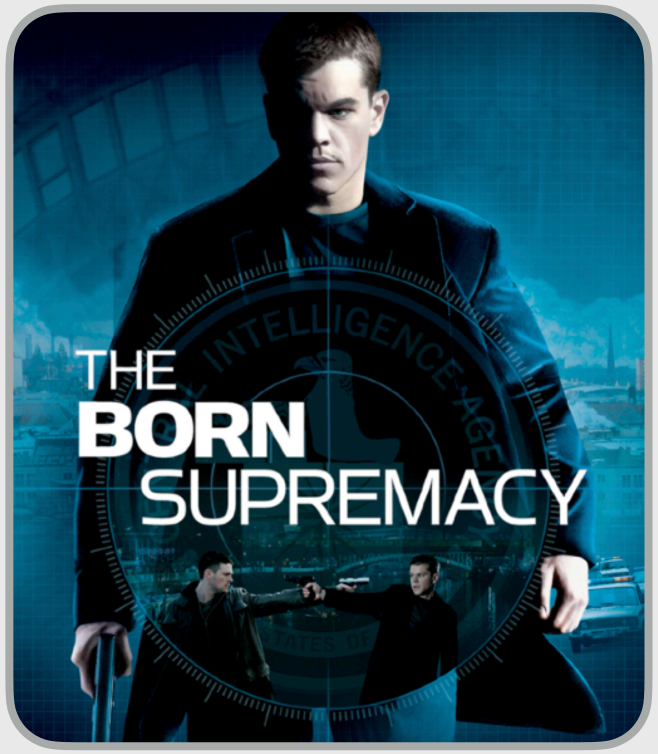
Haferkamp, Montaelegre-Mora, Heinrich, Eisert, Gross, Roth, Commun Math Phys 397, 995-1041 (2023)

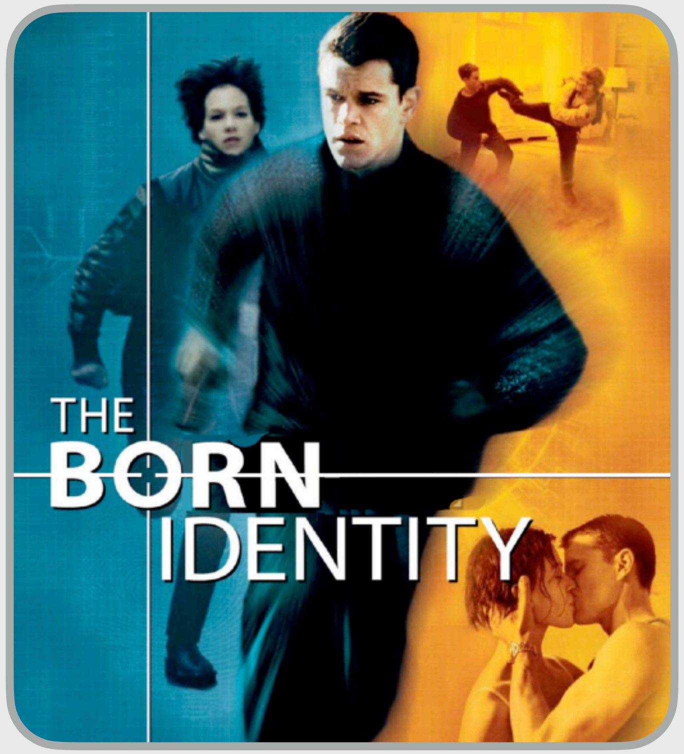
Hinsche, Ioannou, Nietner, Haferkamp, Quek, Hangleiter, Seifert, Eis
Hinsche, Ioannou, Nietner, Haferkamp, Quek, Hangleiter, Seifert, Eisert, Sweke, arXiv:2110.05517 (2021)

# LESSON

- The connection between **simulation** and **learning** is intricate
- **Obstruction** to proving **quantum advantages**

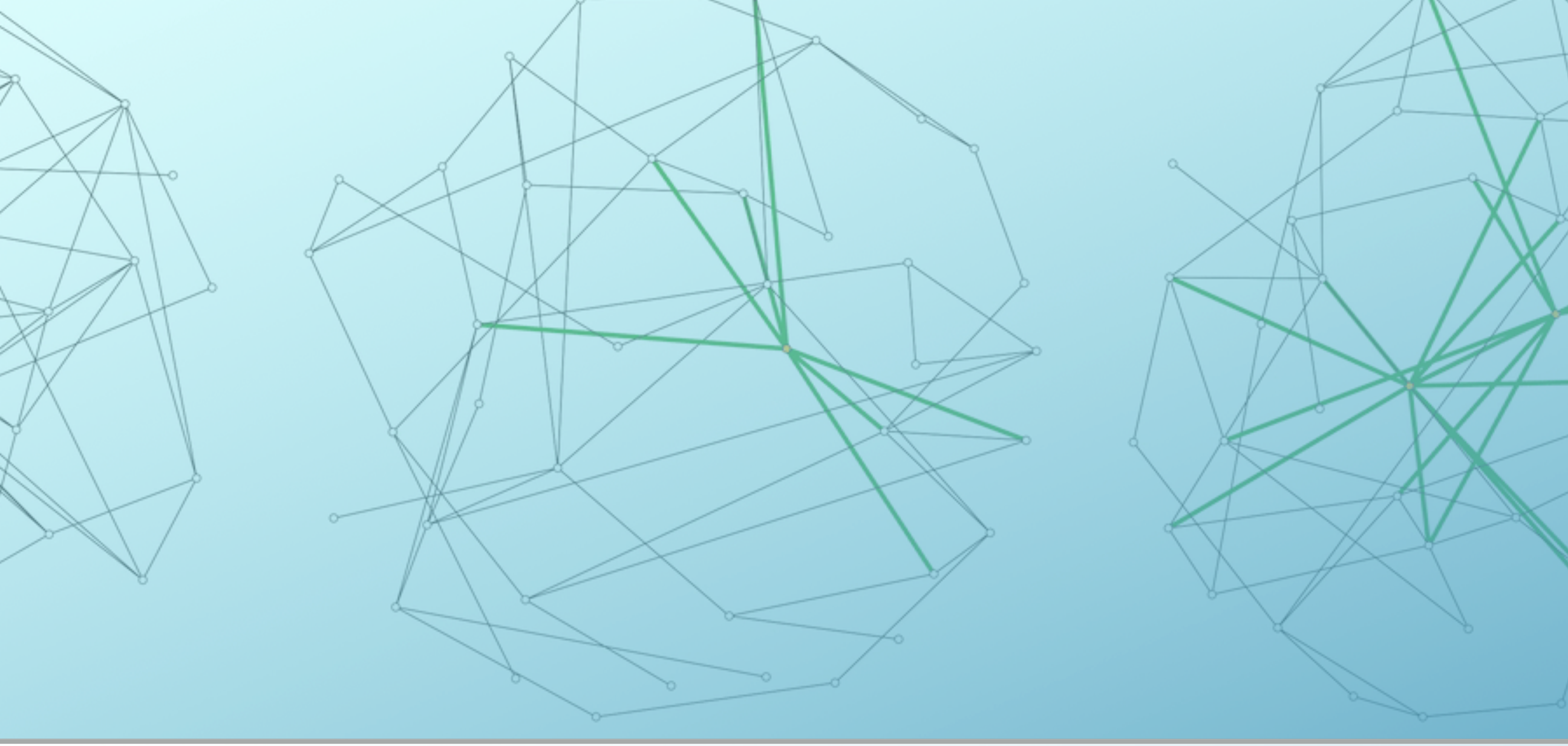
- First rigorous insights into learnability of outputs of **quantum circuits**





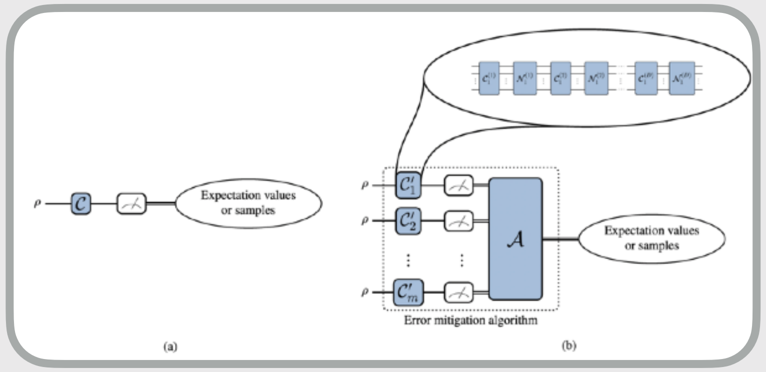
Hinsche, Ioannou, Nietner, Haferkamp, Quek, Hangleiter, Seifert, Eisert, Sweke, Phys Rev Lett 130, 240602 (2023)
Hinsche, Ioannou, Nietner, Haferkamp, Quek, Hangleiter, Seifert, Eisert, Sweke, arXiv:2110.05517 (2021)
Compare Coyle, Mills, Danos, Kashefi, npj Quant Inf 6, 1 (2020)



# CODA ON QUANTUM ADVANTAGES FOR COMBINATORIAL OPTIMIZATION

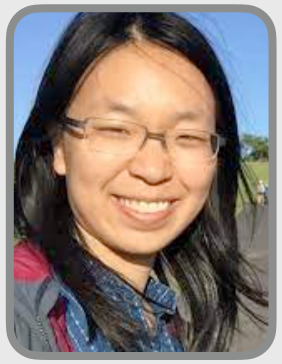
Pirnay, Ulitzsch, Wilde, Eisert, Seifert, arXiv:2212.08678 (2022)

# NEAR TERM QUANTUM COMPUTERS AND LEARNING TASKS



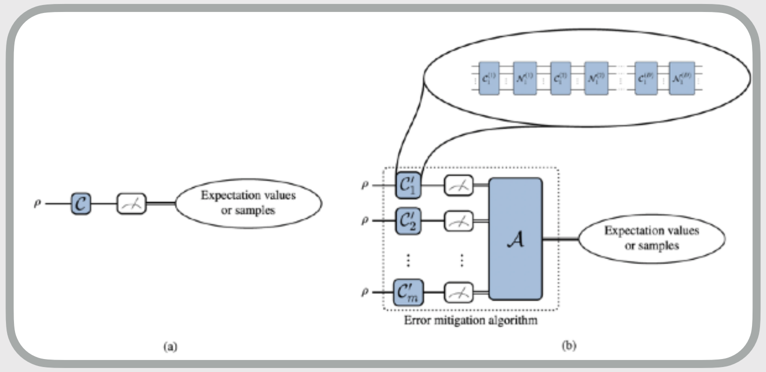

- Strong **limitations** for **quantum error mitigation**

Quek, Franca, Khatri, Meyer, Eisert, arXiv:2210.11505 (2022)



Y Quek: “The signal and the noise: Learning with random quantum ciruits and other agents of noise”

# NEAR TERM QUANTUM C… …RNIN…



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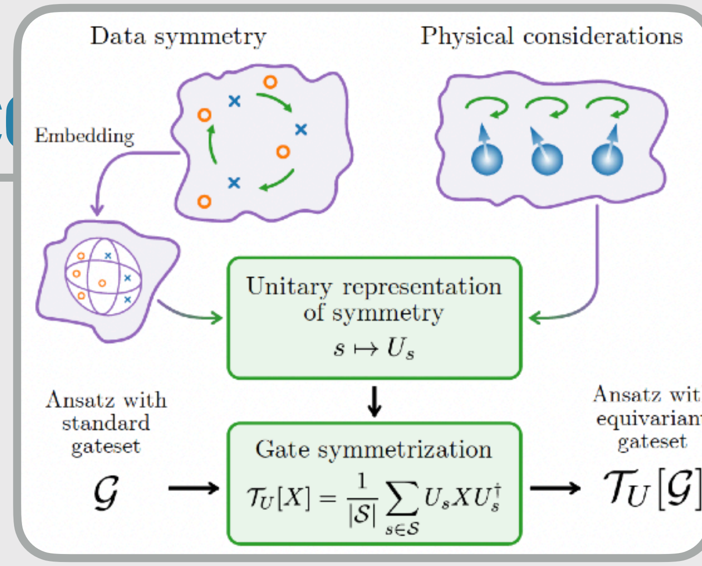
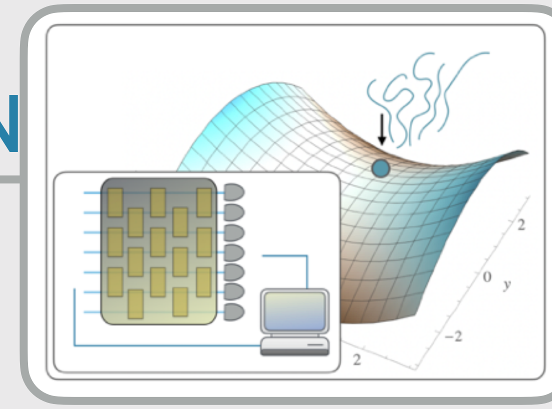



- **Symmetry** helps
  Meyer, Mularski, Gil-Fuster, Mele, Arzani, Wilms, Eisert, PRX Quantum 4, 010328 (2023)



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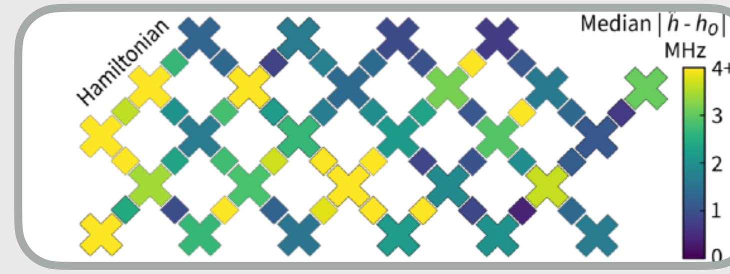
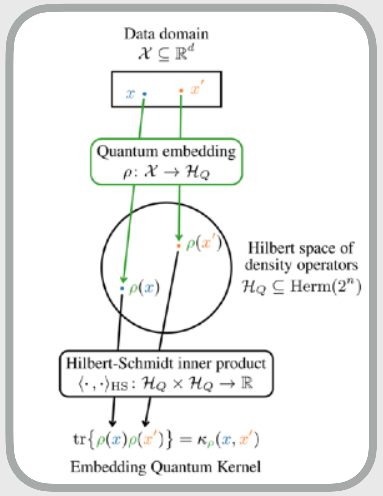
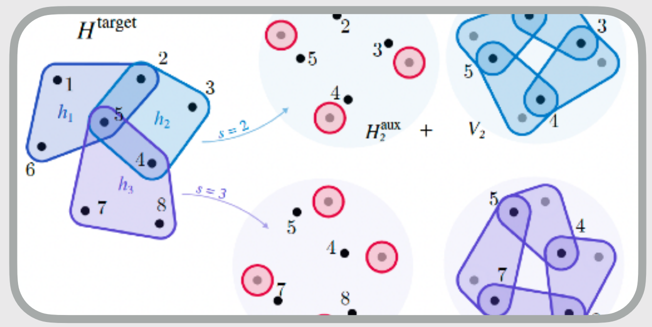
  Liu, Wilde, Mele, Jiang, Eisert, arXiv:2210.06723 (2022)




- **Hamiltonian** learning
  Wilde, Kshetrimayum, Roth, Hangleiter, Sweke, Eisert, arXiv:2209.14328 (2022)
  Hangleiter, Roth, Eisert, Roushan, arXiv:2108.08319 (2021)

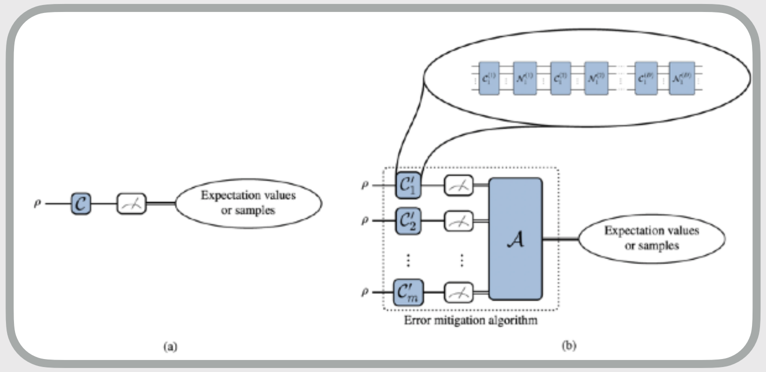



- **Gadgets** against barren plateaus
  Simon, Faehrmann, Khatri, Eisert, arXiv:2210.03099 (2022)

- **Expressivity** of embedding quantum kernels
  Gil-Fuster, Eisert, Dunjko, arXiv:2303.14419 (2023)

# NEAR TERM QUANTUM C RNIN

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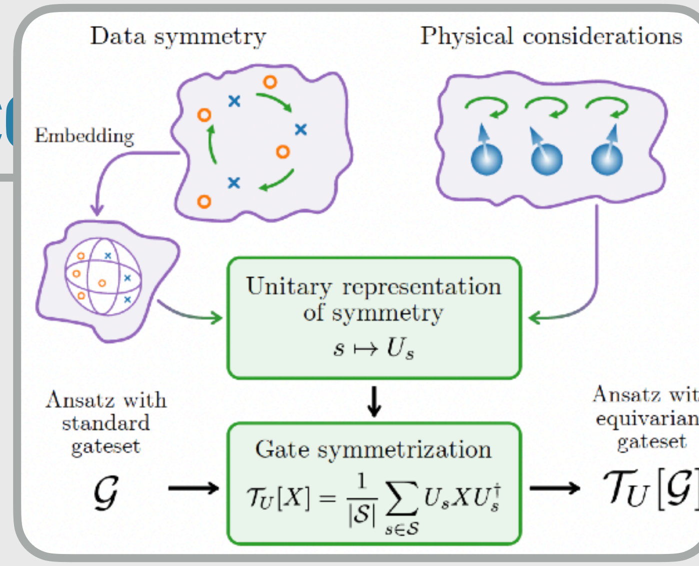


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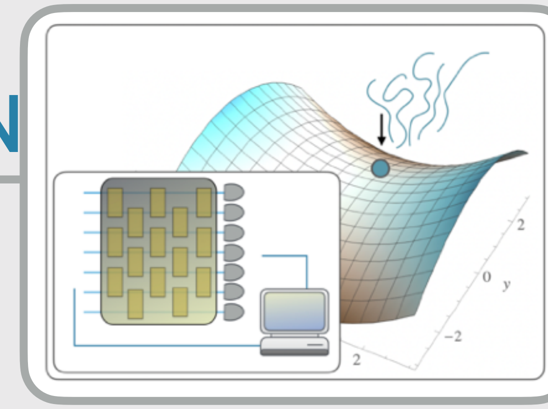

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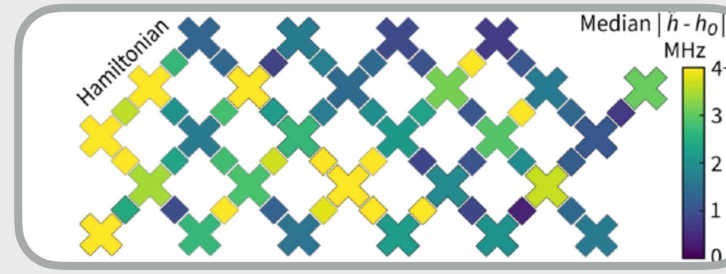


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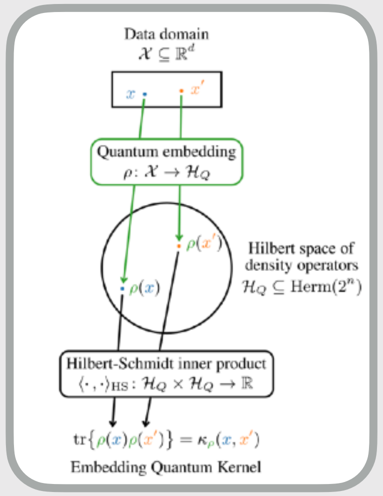


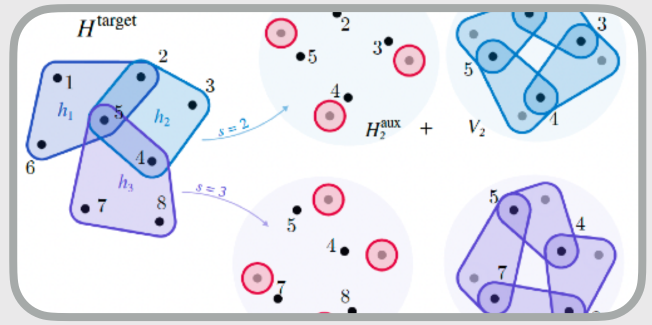


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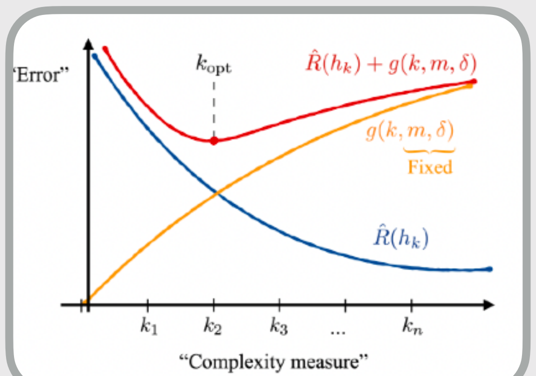
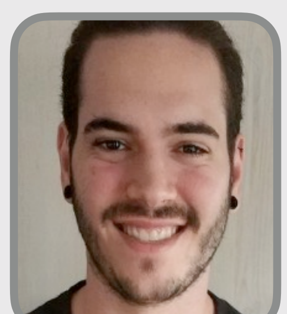


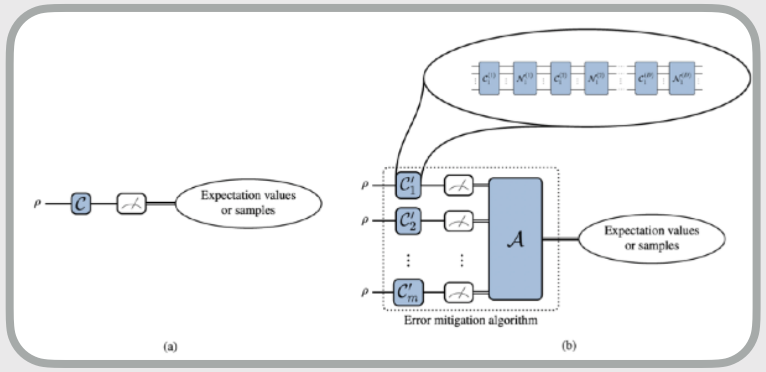

- **Generalization** bounds for parametrized quantum circuits

Caro, Gil-Fuster, Meyer, Eisert, Sweke, Quantum 5, 582 (2021)
Gil-Fuster, Eisert, Bravo-Prieto, arXiv:2306.13461 (2023)



C Bravo Prieto: "Understanding quantum machine learning also requires rethinking generalization"

# NEAR TERM QUANTUM C RNIN

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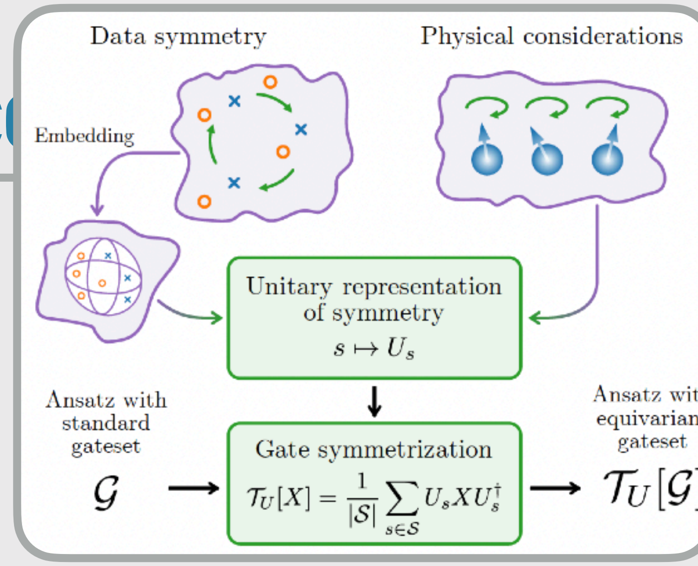


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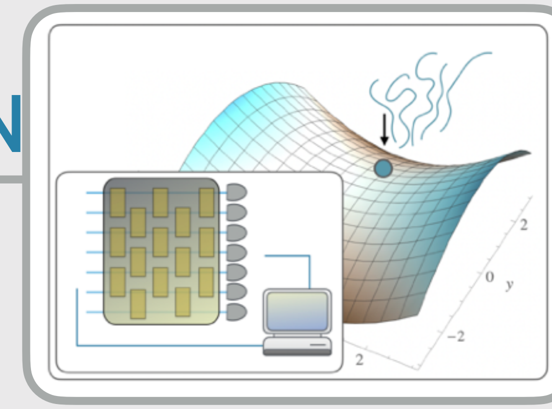

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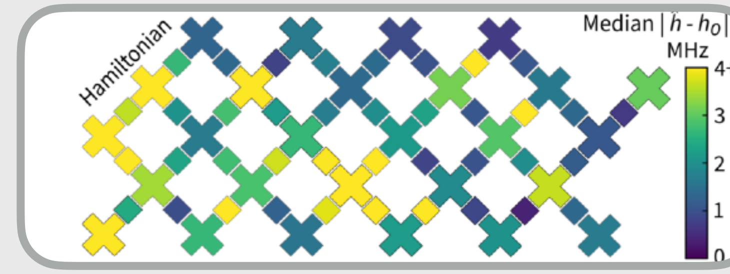


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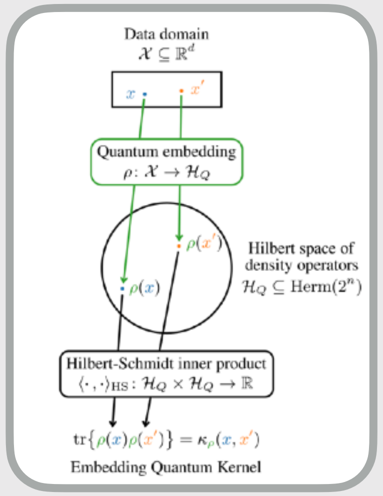


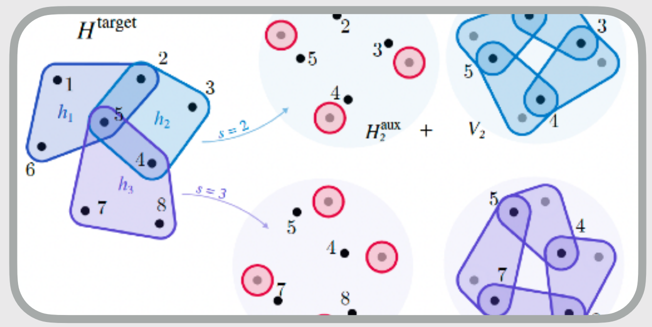


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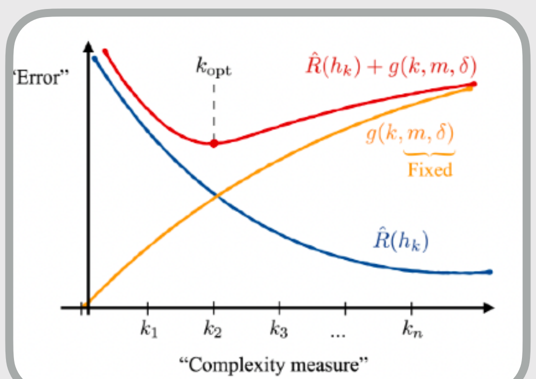


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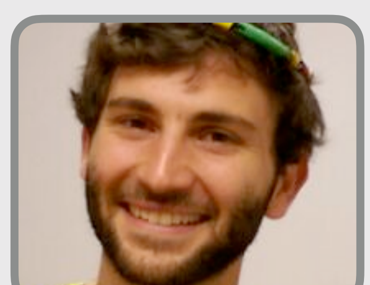
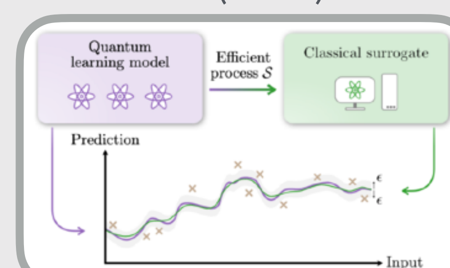




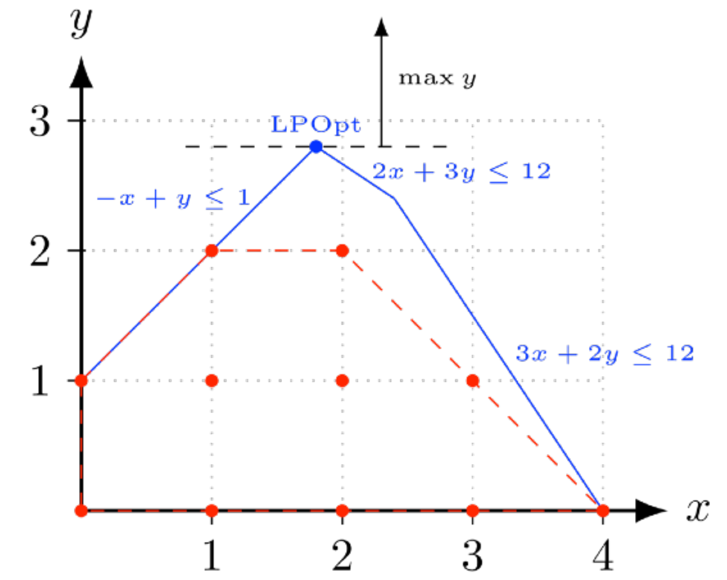

- **Classical surrogates** for quantum learning models

Schreiber, Eisert, Meyer, Phys Rev Lett 131, 100803 (2023)
Sweke, Recio, Jerbi, Gil-Fuster, Fuller, Eisert, Meyer, arXiv:2309.11647 (2023)

- Can we hope to see advantages for **combinatorial optimization**?

- Commonly **NP-hard** in worst case complexity
- E.g., **integer programming**

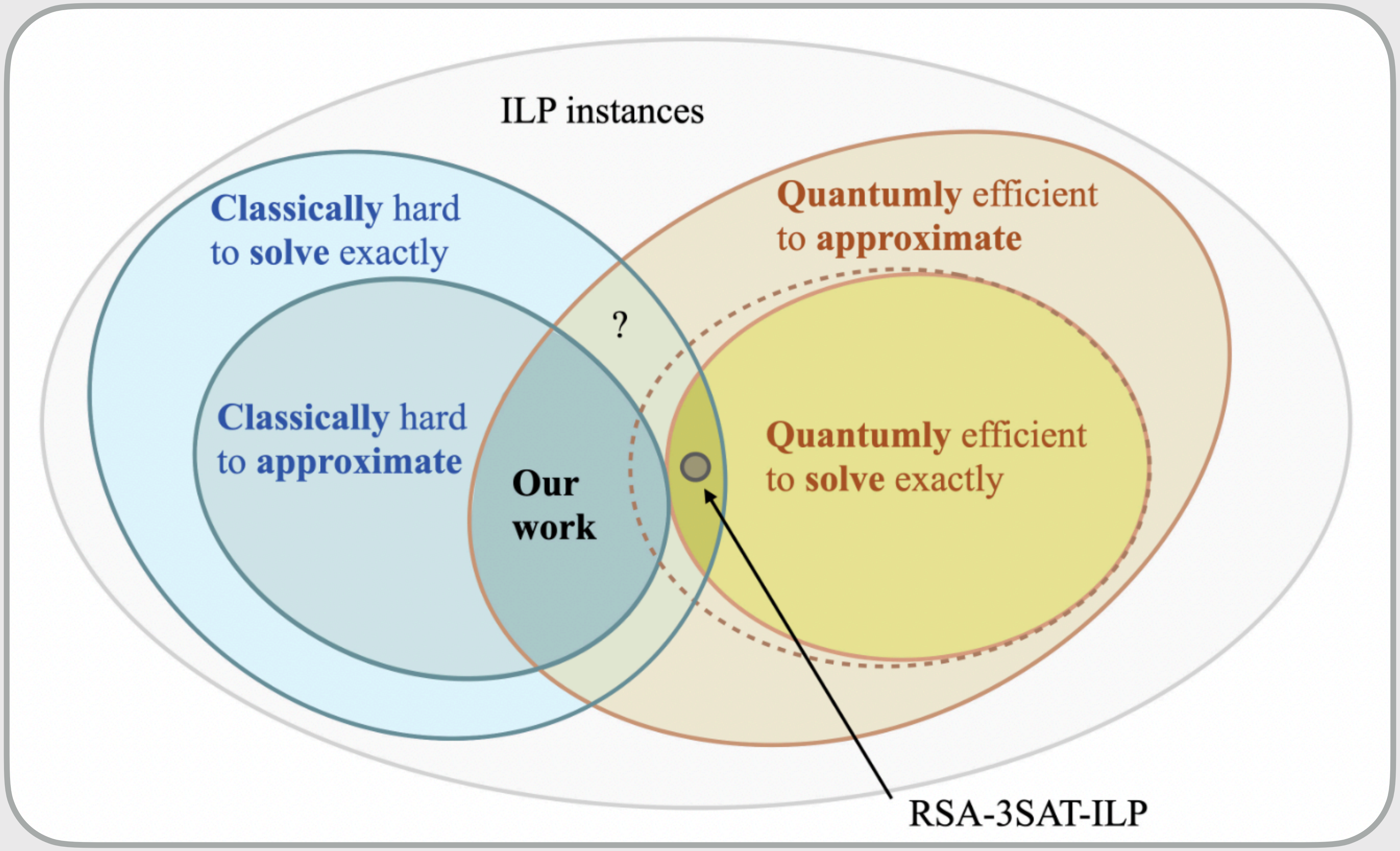
$$
\begin{array}{ll}
\text{maximize} & \mathbf{c}^{\mathrm{T}}\mathbf{x} \\
\text{subject to} & A\mathbf{x} \leq \mathbf{b}, \\
 & \mathbf{x} \geq \mathbf{0}, \\
\text{and} & \mathbf{x} \in \mathbb{Z}^n
\end{array}
$$




- In what sense can there be **quantum advantages**?

Pirnay, Ulitzsch, Wilde, Eisert, Seifert, arXiv:2212.08678 (2022)

- A quantum computer can **approximate hard instances**, originating from the hardness of inverting the RSA function, in polynomial time




Pirnay, Ulitzsch, Wilde, Eisert, Seifert, arXiv:2212.08678 (2022)

# QUANTUM COMPUTING FOR COMBINATORIAL OPTIMIZATION?

- A quantum computer can **approximate hard instances**, originating from the hardness of inverting the RSA function, in polynomial time

- So there is a **super-polynomial speed-up!**

- Can be translated to variational **Hamiltonian problems**?
- **Constructive:** Find classically "hard instances"
- Connection to **QAOA?**

Pirnay, Ulitzsch, Wilde, Eisert, Seifert, arXiv:2212.08678 (2022)

# OUTLOOK

## OUTLOOK

- Can we hope noisy, realistic quantum devices to provide a speedup over classical computers?


- Yes, find super-polynomial advantages in PAC-learning!
- A little noise can be a good thing!
- We have found super-polynomial advantages in optimization!


- Well, but how much structure do we need?
- A single-T gate renders learning hard
- But error mitigation already fails for log-log deep circuits
- Let us see …

## OUTLOOK

- Can we hope noisy, realistic quantum devices to provide a **speedup over classical computers**?



Definitely Maybe

- Yes, ... advan...
- A little noise can be a **good thing**!
- We have found **super-polynomial advantages in optimization**!

- ...nders ... learning hard
- But **error mitigation already fails** for log-log deep circuits
- Let us see ...

# THANKS FOR YOUR ATTENTION

## WE ARE HIRING