Optimized signal fitting for Quantum phase estimation on a early fault-tolerant quantum computer

Zhiyan Ding

UC Berkeley

CQCWS1: Quantum Algorithms for Scientific Computation

Joint work with Yulong Dong, Lin Lin, Yu Tong

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Method

Problem

Optimized signal fitting for Quantum phase estimation on a early fault-tolerant quantum computer

Device (constraint)

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Outlines:

- Introduction

main problem, input, complexity

- Method: QCELS

informal result, algorithm, informal proof

- Other applications

multiple eigenvalue, depolarizing noise

- Conclusion

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Main problem:

Given a Hamiltonian $H \in \mathbb{C}^{d\times d}$, we estimate the smallest eigenvalue λ_0

$d \gg 1$

Ground state energy

Assume: $\lambda_0 \in [-\pi, \pi]$

Corresponding eigenvector: $\left|\phi_0\right\rangle \in \mathbb{C}^d$, $\left\| \left|\phi_0\right\rangle \right\| = 1$

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Input:

1. Initial state: $\left|\psi_0\right\rangle \in \mathbb{C}^d, \left\| \left|\psi_0\right\rangle \right\| = 1$

$$0 < p_0 = \left|\left\langle \psi_0 \middle| \phi_0 \right\rangle\right|^2 < 1$$ Initial overlap

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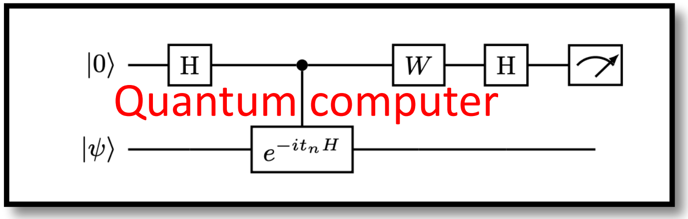
Input:

1. Initial state: $\left|\psi_0\right\rangle \in \mathbb{C}^d, \ \left\| \left|\psi_0\right\rangle \right\| = 1$ $\quad p_0 = \left|\left\langle \psi_0 \middle| \phi_0 \right\rangle\right|^2$

2. Quantum oracle: Given any $t \in \mathbb{R}$

$t \in \mathbb{R}$ →



→ $Z_t = \pm 1 \pm 1i$

$$\mathbb{E}(Z_t) = \left\langle \psi_0 \right| \exp(-itH) \left| \psi_0 \right\rangle = \sum_{k=0}^{d-1} p_k \exp(-it\lambda_k)$$

Signal in t

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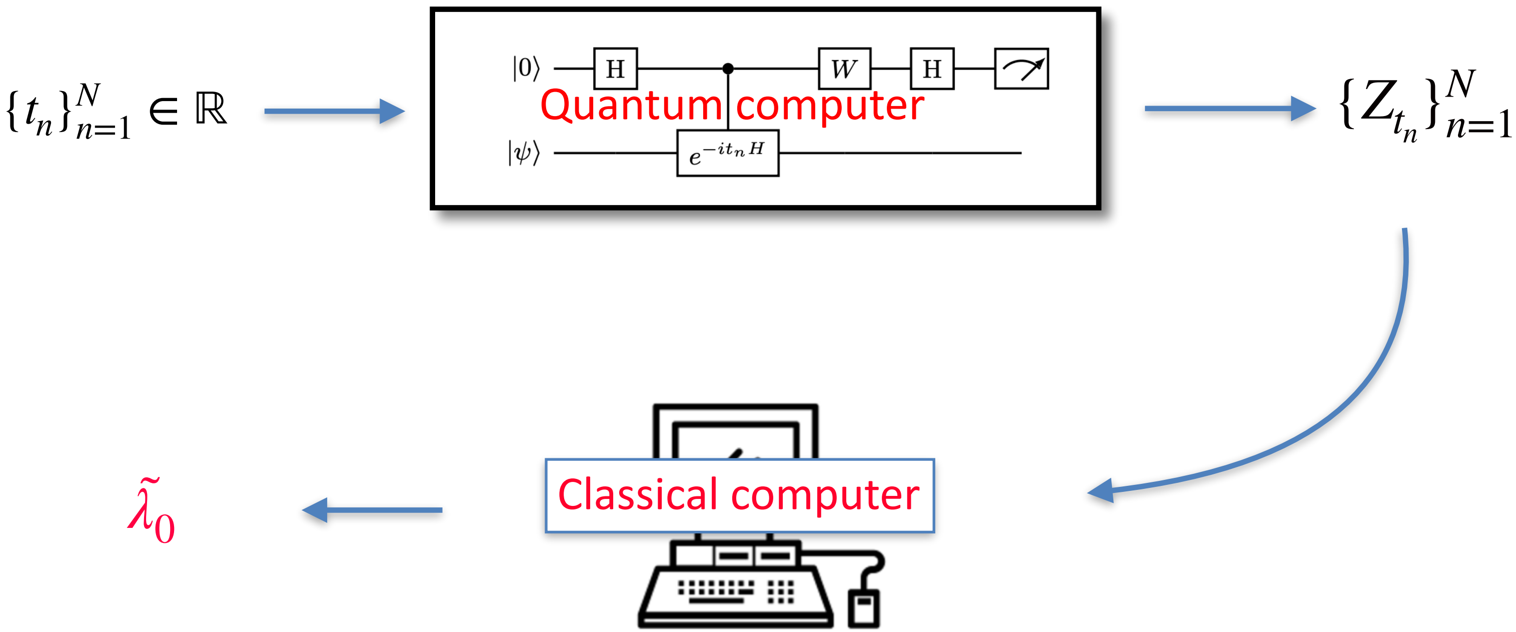
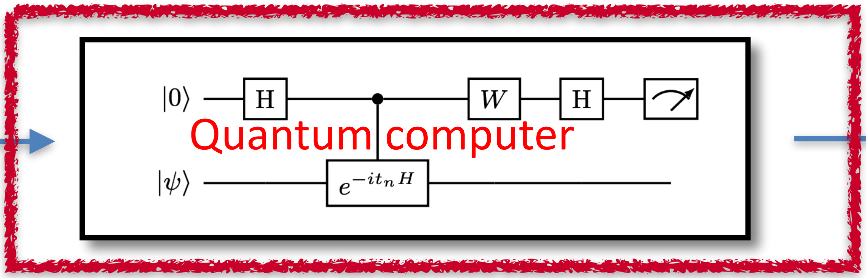
General Algorithm:



Complexity:

$\{t_n\}_{n=1}^{N} \in \mathbb{R}$ →  → $\{Z_{t_n}\}_{n=1}^{N}$

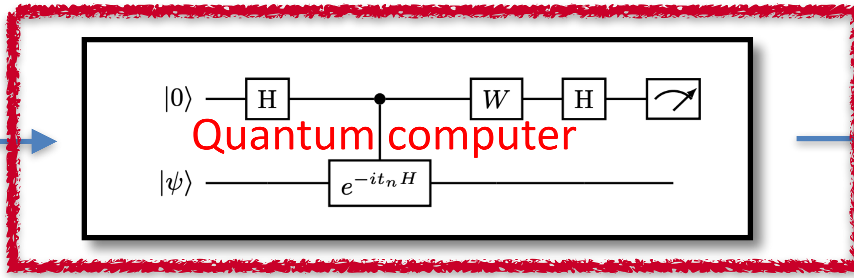
- Maximal running time: $T_{\max} = \max_n \{ \ t_n \ \}_{n=1}^{N}$

  How large our quantum computer needs to be?

- Total running time: $T_{total} = \sum_n \ t_n$

  How long we need to run?

Complexity:

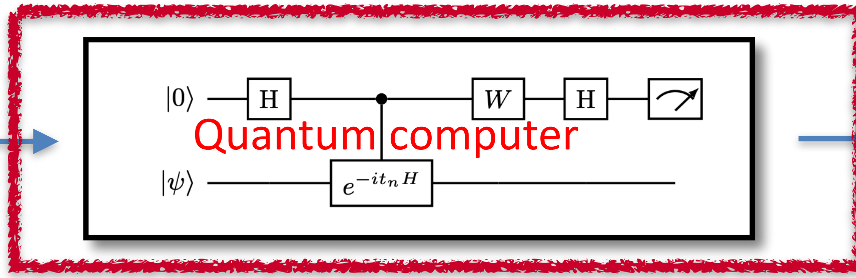
$\{t_n\}_{n=1}^{N} \in \mathbb{R}$ →  → $\{Z_{t_n}\}_{n=1}^{N}$

- Maximal running time: $T_{\max} = \max_n \{ \; t_n \; \}_{n=1}^{N}$
- Total running time: $T_{total} = \sum_n \; t_n$

How large our quantum computer needs to be?

Early fault-tolerant quantum computer

Complexity:

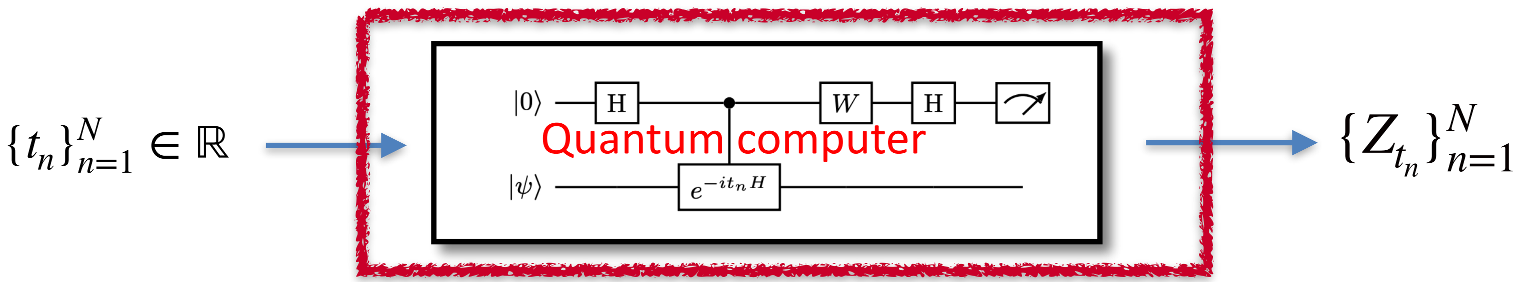
$\{t_n\}_{n=1}^{N} \in \mathbb{R}$ →  → $\{Z_{t_n}\}_{n=1}^{N}$

Classical result: To ensure $\tilde{\lambda}_0 - \lambda_0 \quad \leq \epsilon$

- When $p_0 = 1$, $T_{\max} = \mathcal{O}(1)$, $T_{total} = \mathcal{O}(1/\epsilon^2)$ — Hadamard test
- Any p_0, $T_{\max} > \pi/\epsilon$, $T_{total} = \mathcal{O}(1/\epsilon)$ — QPE, QET-U,..

Heisenberg-limited scaling

Complexity:



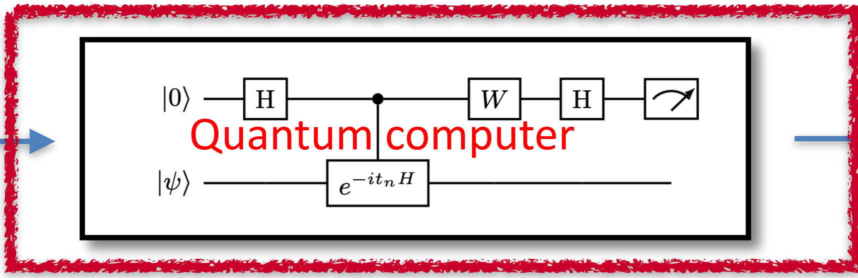
Classical result: To ensure $\tilde{\lambda}_0 - \lambda_0 \leq \epsilon$

- When $p_0 = 1$, $T_{\max} = \mathcal{O}(1)$, $T_{total} = \mathcal{O}(1/\epsilon^2)$ Hadamard test
- Any p_0, $T_{\max} > \pi/\epsilon$, $T_{total} = \mathcal{O}(1/\epsilon)$ QPE, QET-U,..

Reduce

Complexity:

$\{t_n\}_{n=1}^{N} \in \mathbb{R}$



$\{Z_{t_n}\}_{n=1}^{N}$

Classical result: To ensure $\tilde{\lambda}_0 - \lambda_0 \leq \epsilon$

- When $p_0 = 1$, $T_{\max} = \mathcal{O}(1)$, $T_{total} = \mathcal{O}(1/\epsilon^2)$ Hadamard test

  $p_0 > 0.5, T_{\max} \ll \pi/\epsilon, T_{total} = \mathcal{O}(1/\epsilon)$ Our method

- Any p_0, $T_{\max} > \pi/\epsilon, T_{total} = \mathcal{O}(1/\epsilon)$ QPE, QET-U,..

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Informal theorem:

[Ding, Lin, 2023] Assume $p_0 > 0.5$. There exists an algorithm(QCELS) such that, with high probability, it outputs

$$\left|\tilde{\lambda}_0 - \lambda_0\right| \leq \epsilon,$$

with

$$T_{\max} = \mathcal{O}\left(\frac{1-p_0}{\epsilon}\right), \quad T_{total} = \mathcal{O}\left(\frac{1}{\epsilon}\right)$$

- When $p_0 \approx 1, \quad T_{\max} \ll \frac{\pi}{\epsilon}$

[1]. Zhiyan Ding, Lin Lin, Even shorter quantum circuit for phase estimation on early fault-tolerant quantum computers with applications to ground-state energy estimation, PRX Quantum, 2023. ($\sqrt{1-p_0}/\epsilon$)

[2]. Zhiyan Ding, Lin Lin, Simultaneous estimation of multiple eigenvalues with short-depth quantum circuit on early fault-tolerant quantum computers, to appear, Quantum, 2023.

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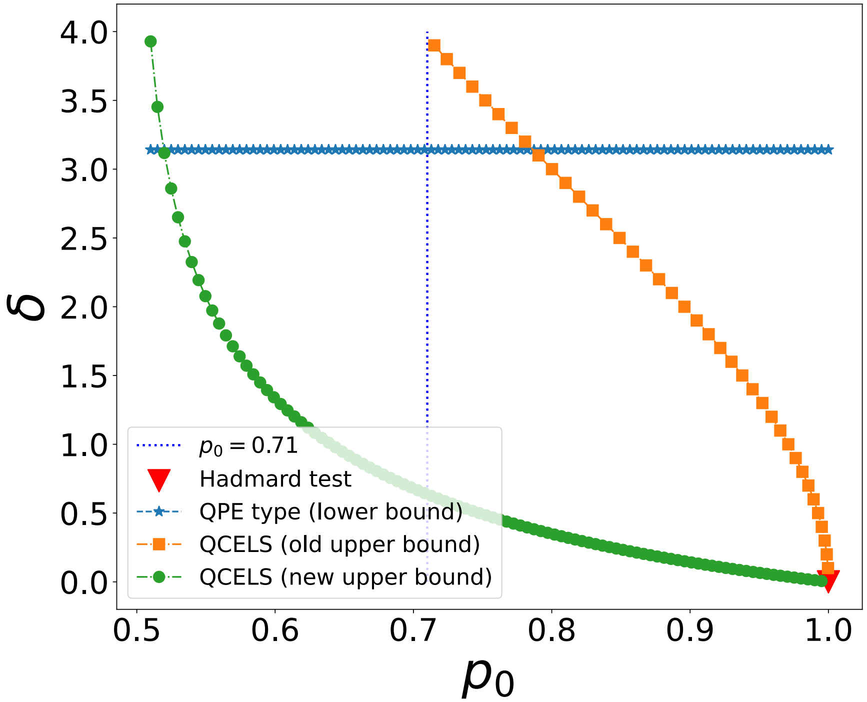
Informal theorem: $T_{\max} = \mathcal{O}\left(\frac{1-p_0}{\epsilon}\right)$



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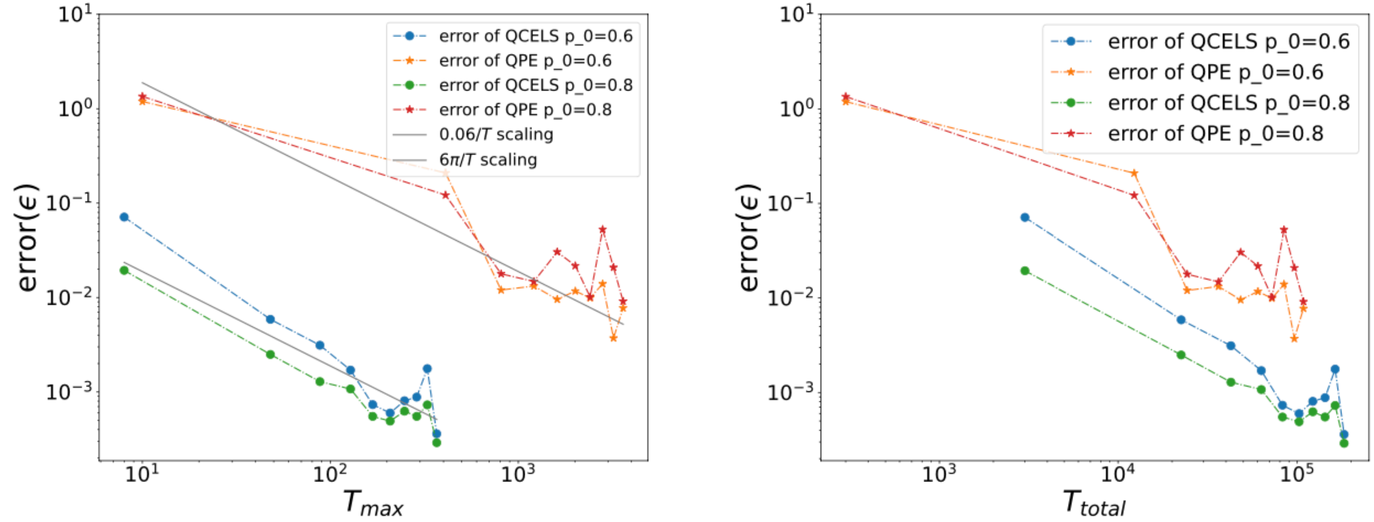
Numerical evidence:

Transverse field Ising model (IFIM):



- Two order of magnitude reduction of maximal running time

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Rough algorithm:

Step 1: Draw N independent samples $\{t_n\}_{n=1}^{N}$ from a truncated Gaussian with density:

$$a(t) = \frac{1}{Z_{T,\gamma}} \exp\left(-\frac{t^2}{2T^2}\right) \mathbf{I}_{\,t \leq \gamma T}$$

Step 2: For each $\{t_n\}_{n=1}^{N}$, using quantum computer to generate one sample Z_{t_n} such that

$$\mathbb{E}(Z_t) = \left\langle \psi_0 \right| \exp(-iHt) \left| \psi_0 \right\rangle = \sum_{k=0}^{d-1} p_k \exp(-i\lambda_k t)$$

Step 3: Solve optimization problem and $\tilde{\lambda}_0 = \theta^*$

$$(r^*, \theta^*) = \min_{r,\theta} \frac{1}{N} \sum_{n=1}^{N} \left| Z_{t_n} - r \exp(-i\theta t_n) \right|^2$$ (QCELS)

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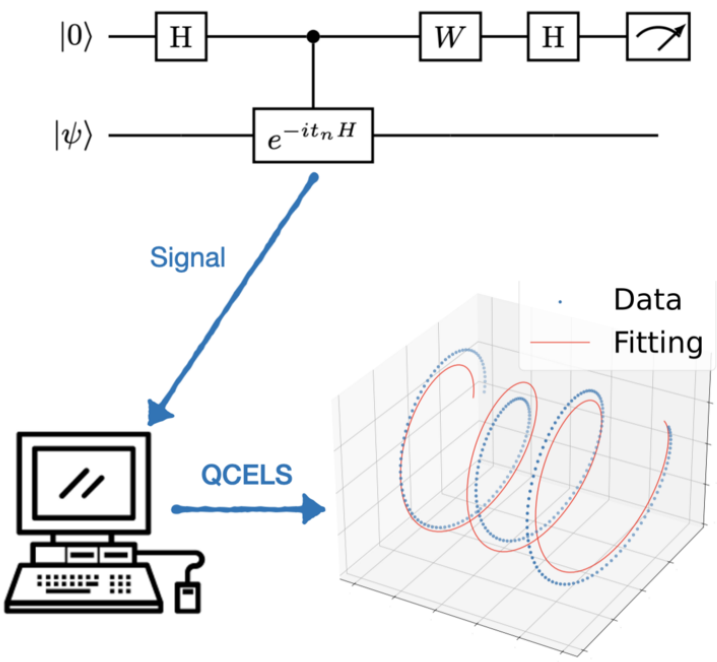
Rough algorithm:

$$(r^*, \theta^*) = \min_{r,\theta} \frac{1}{N} \sum_{n=1}^{N} \left| Z_{t_n} - r \exp(-i\theta t_n) \right|^2 \quad \text{(QCELS)}$$



Fitting:

$$r \exp(-i\theta t) \approx Z_t \approx \sum_{k=0}^{d-1} p_k \exp(-i\lambda_k t)$$

When $p_0 > 0.5$, $\theta^* \approx \lambda_0$.

Informal proof:

$$(r^*, \theta^*) = \min_{r,\theta} \frac{1}{N} \sum_{n=1}^{N} \left| Z_{t_n} - r \exp(-i\theta t_n) \right|^2 \quad \text{(QCELS)}$$

How?

$$p_0 > 0.5, T_{\max} = \mathcal{O}((1 - p_0)/\epsilon), T_{total} = \mathcal{O}(1/\epsilon)$$

Note: $T_{\max} = \max_n \; t_n \; \leq \gamma T$

$$t_n \sim a(t) = \frac{1}{Z_{T,\gamma}} \exp\left(-\frac{t^2}{2T^2}\right) \mathbf{I}_{t \leq \gamma T}$$

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Informal proof:

$$(r^*, \theta^*) = \min_{r,\theta} \frac{1}{N} \sum_{n=1}^{N} \left| Z_{t_n} - r \exp(-i\theta t_n) \right|^2 \quad \text{(QCELS)}$$

$N \gg 1$ Recall: $t_n \sim a(t)$

$$(r^*, \theta^*) = \min_{r,\theta} \int_{-\gamma T}^{\gamma T} a(t) \left| \mathbb{E}(Z_t) - r \exp(-i\theta t) \right|^2 \mathrm{d}t$$

$$= \min_{r,\theta} \int_{-\gamma T}^{\gamma T} a(t) \left| \mathbb{E}(Z_t) \exp(i\theta t) - r \right|^2 \mathrm{d}t$$

Explicitly solve r

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Informal proof:

$$(r^*, \theta^*) = \min_{r,\theta} \frac{1}{N} \sum_{n=1}^{N} \left| Z_{t_n} - r \exp(-i\theta t_n) \right|^2 \quad \text{(QCELS)}$$

$$(r^*, \theta^*) = \min_{r,\theta} \int_{-\gamma T}^{\gamma T} a(t) \left| Z_t - r \exp(-i\theta t) \right|^2 \mathrm{d}t$$

$$\mathbb{E}(Z_t) = \sum_{k=0}^{d-1} p_k \exp(-i\lambda_k t)$$

$$\theta^* = \max_{\theta} \left| \sum_{k=0}^{d-1} p_k \left(\int_{-\gamma T}^{\gamma T} a(t) \exp(i(\theta - \lambda_k)t) \mathrm{d}t \right) \right|^2$$

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$$\theta^* = \max_{\theta} \left| \sum_{k=0}^{d-1} p_k \left(\int_{-\gamma T}^{\gamma T} a(t) \exp(i(\theta - \lambda_k)t) \mathrm{d}t \right) \right|^2 = \max_{\theta} \left| \sum_{k=0}^{d-1} p_k F(\theta - \lambda_k) \right|^2$$

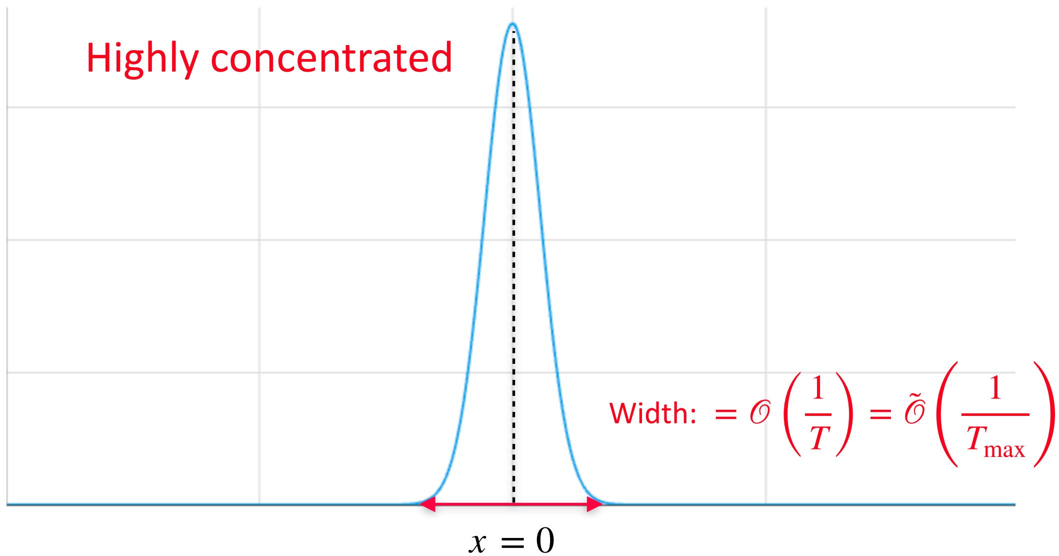
IFT: $F(x) = \int_{-\gamma T}^{\gamma T} a(t) \exp(ixt) dt$

$$\theta^* = \max_{\theta} \left| \sum_{k=0}^{d-1} p_k \left(\int_{-\gamma T}^{\gamma T} a(t) \exp(i(\theta - \lambda_k)t) \mathrm{d}t \right) \right|^2 = \max_{\theta} \sum_{k=0}^{d-1} p_k F(\theta - \lambda_k)$$

$$a(t) = \frac{1}{Z_{T,\gamma}} \exp\left(-\frac{t^2}{2T^2} \right) \mathbf{I}_{t \leq \gamma T}$$

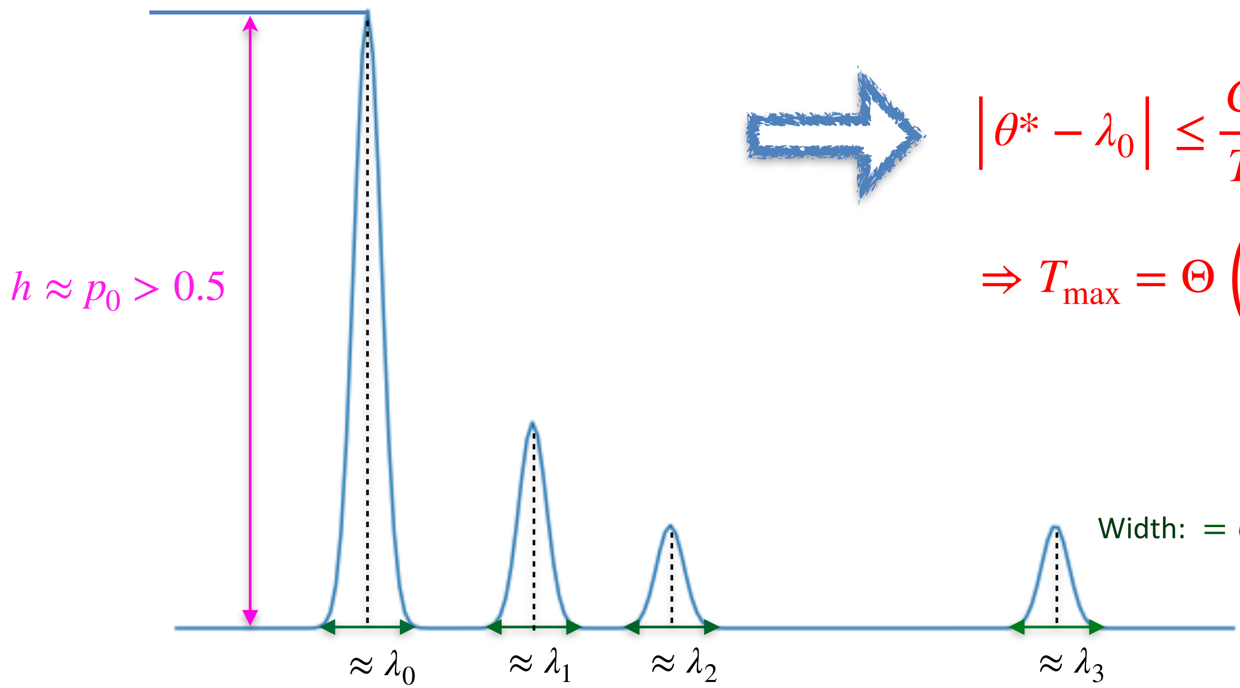
$\gamma = \tilde{\mathcal{O}}(1)$

$$F(x) \approx \exp\left(-\frac{T^2 x^2}{2} \right) \in \mathbb{R}_{>0}$$



$$\theta^* \approx \max_{\theta} \sum_{k=0}^{d-1} p_k \exp\left(-\frac{T^2(\theta-\lambda_k)^2}{2}\right)$$

Step 1:



$$\Rightarrow \quad \left|\theta^* - \lambda_0\right| \le \frac{C}{T} \approx \frac{C}{T_{\max}}$$

$$\Rightarrow T_{\max} = \Theta\left(\frac{C}{\epsilon}\right)$$

Width: $= \mathcal{O}\left(\frac{1}{T}\right) = \tilde{\mathcal{O}}\left(\frac{1}{T_{\max}}\right)$

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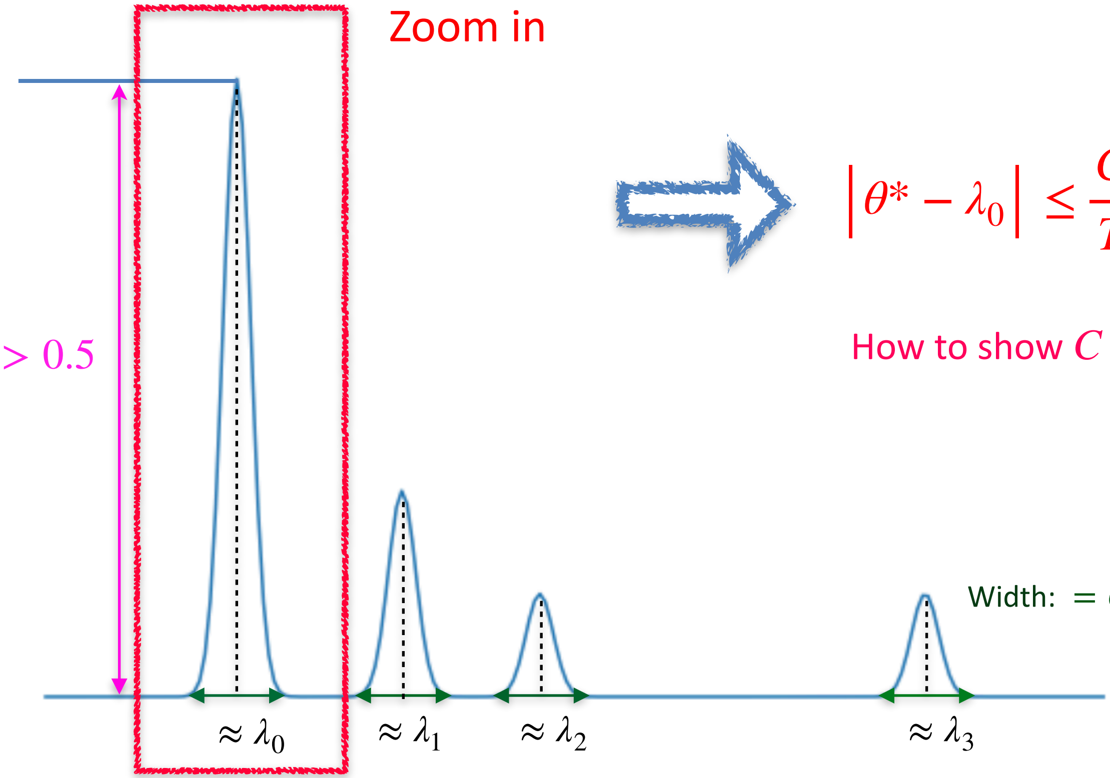
$$\theta^* \approx \max_{\theta} \sum_{k=0}^{d-1} p_k \exp\left(-\frac{T^2(\theta - \lambda_k)^2}{2}\right)$$

Step 1:



$\Rightarrow$ $$\left|\theta^* - \lambda_0\right| \leq \frac{C}{T} \approx \frac{C}{T_{\max}}$$

How to show $C \ll 1$?

Width: $= \mathcal{O}\left(\frac{1}{T}\right) = \tilde{\mathcal{O}}\left(\frac{1}{T_{\max}}\right)$

$$\theta^* \approx \max_{\theta} \sum_{k=0}^{d-1} p_k \exp\left(-\frac{T^2(\theta - \lambda_k)^2}{2}\right)$$

Step 2:

Notice:

$$\sum_{k=0}^{d-1} p_k \exp\left(-\frac{T^2(\theta - \lambda_k)^2}{2}\right) = p_0 \exp\left(-\frac{T^2(\theta - \lambda_0)^2}{2}\right) + Error(\theta)$$

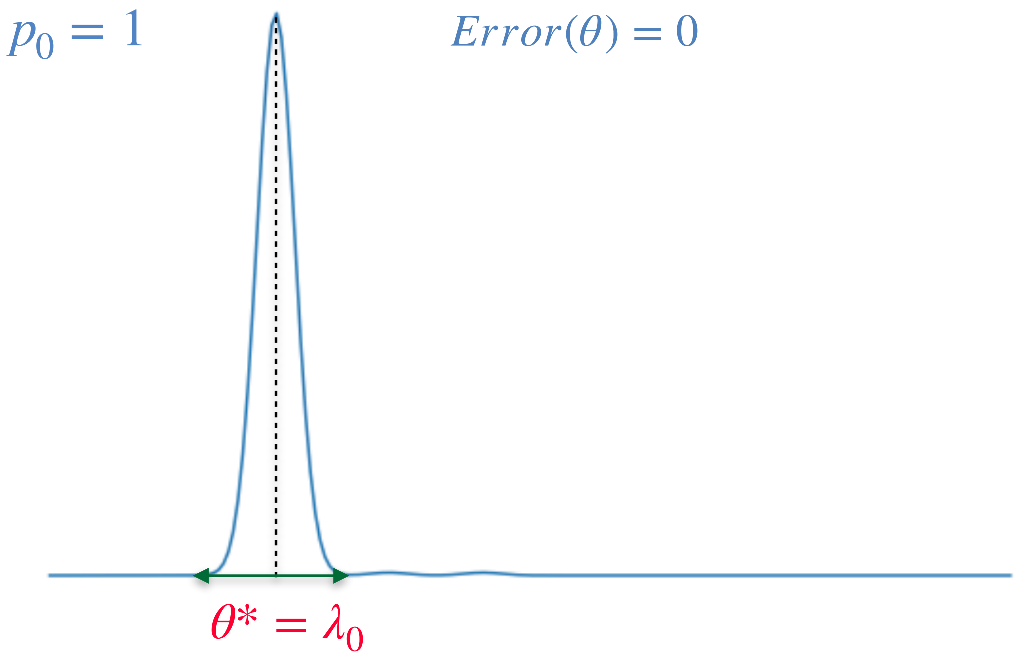
where

$$\left|Error(\theta)\right| \leq 1 - p_0$$

Focus on the region: $\theta \approx \lambda_0$

$$\theta^* \approx \max_{\theta} p_0 \exp\left(-\frac{T^2(\theta - \lambda_0)^2}{2}\right) + Error(\theta)$$

Step 2:



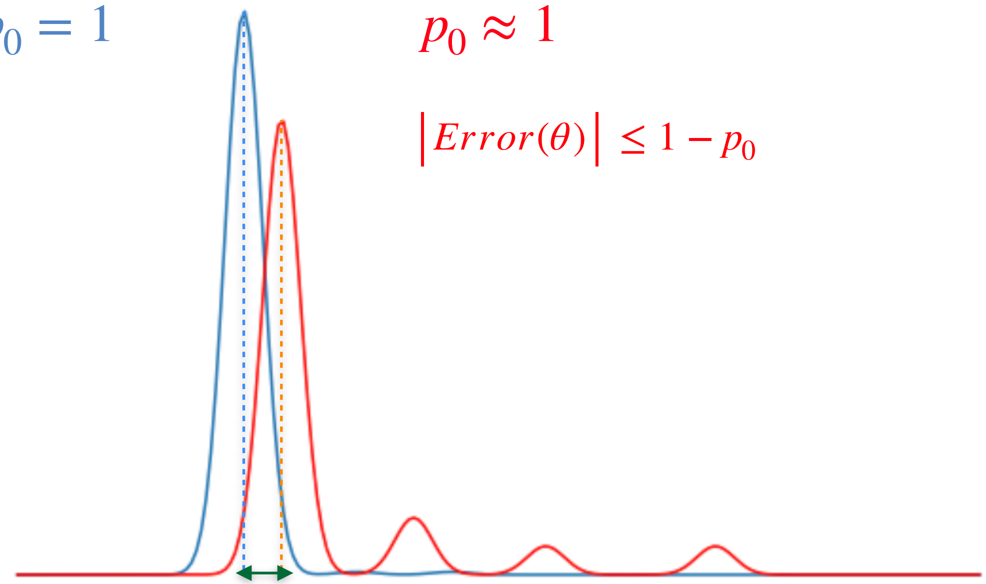
$$\theta^* \approx \max_{\theta} p_0 \exp\left(-\frac{T^2(\theta - \lambda_0)^2}{2}\right) + Error(\theta)$$

Step 2:

$p_0 = 1$

$p_0 \approx 1$

$\left|Error(\theta)\right| \leq 1 - p_0$



$$\left|\theta^* - \lambda_0\right| = \mathcal{O}\left(\frac{1-p_0}{T}\right) \quad \Rightarrow T_{\max} = \Theta\left(\frac{1-p_0}{\epsilon}\right)$$

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Informal theorem:

[Ding, Lin, 2023] Assume $p_0 > 0.5$. There exists an algorithm(QCELS) such that, with high probability, it outputs

$$\left|\tilde{\lambda}_0 - \lambda_0\right| \leq \epsilon,$$

with

$$T_{\max} = \mathcal{O}\left(\frac{1-p_0}{\epsilon}\right), \quad T_{total} = \mathcal{O}\left(\frac{1}{\epsilon}\right)$$

Uniformly bound the effect of random error (in Z_t) for all θ

$$(r^*, \theta^*) = \min_{r,\theta} \frac{1}{N} \sum_{n=1}^{N} \left| Z_{t_n} - r \exp(-i\theta t_n) \right|^2 \quad \text{(QCELS)}$$

Informal theorem:

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$$T_{\max} = \mathcal{O}\left(\frac{1-p_0}{\epsilon}\right), \quad T_{total} = \mathcal{O}\left(\frac{1}{\epsilon}\right)$$

- To achieve $\mathcal{O}(1/\epsilon)$ scaling, (theoretically) multi-level QCELS is needed:

One T → An increasing sequence: $T_n = 2^n T_0$

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Informal theorem:

[Ding, Lin, 2023] Assume $p_0 > 0.5$. There exists an algorithm(QCELS) such that, with high probability, it outputs

$$\left|\tilde{\lambda}_0 - \lambda_0\right| \leq \epsilon,$$

with

$$T_{\max} = \mathcal{O}\left(\frac{1-p_0}{\epsilon}\right), \quad T_{total} = \mathcal{O}\left(\frac{1}{\epsilon}\right)$$

- To achieve $\mathcal{O}(1/\epsilon)$ scaling, (theoretically) multi-level QCELS is needed:
- Or $(r^*, \theta^*) = \min_{r,\theta\in[-\pi,\pi]_\epsilon} \frac{1}{N}\sum_{n=1}^{N} \left|Z_{t_n} - r\exp(-i\theta t_n)\right|^2$ (Discrete QCELS)

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Multiple eigenvalue estimation:

Consider K dominant eigenvalues $\{(\lambda_{n_k}, p_{n_k})\}_{k=1}^{K}$ such that

$$p_{n_k} > \sum_{i \neq n_{1 \leq k \leq K}} p_i,$$

we want estimate $\{\lambda_{n_k}\}_{k=1}^{K}$.

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Multiple eigenvalue estimation:

Consider K dominant eigenvalues $\{(\lambda_{n_k}, p_{n_k})\}_{k=1}^{K}$ such that

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we want estimate $\{\lambda_{n_k}\}_{k=1}^{K}$.

Multi-QCELS:

$$(r_k^*, \theta_k^*) = \min_{r,\theta} \frac{1}{N} \sum_{n=1}^{N} \left| Z_{t_n} - \sum_{k=1}^{K} r_k \exp(-i\theta_k t_n) \right|^2$$

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Multiple eigenvalue estimation:

[Ding, Lin, 2023] Define $\Delta = \min_k \lambda_{n_{k+1}} - \lambda_{n_k}$, $p_D = \sum_k p_{n_k}$. There exists an algorithm(Multi-QCELS) such that, with high probability, it outputs

$$\left|\tilde{\lambda}_k - \lambda_{n_k}\right| \le \epsilon,$$

with $T_{\max} = \Omega(\Delta^{-1})$

$$T_{\max} = \mathcal{O}\left(\frac{1 - p_D}{(\min_k p_{n_k})\epsilon}\right), \quad T_{total} = \mathcal{O}\left(\frac{1}{\epsilon}\right)$$

Ensure λ_{n_k} are separated by different Gaussian filters

[2]. Zhiyan Ding, Lin Lin, Simultaneous estimation of multiple eigenvalues with short-depth quantum circuit on early fault-tolerant quantum computers, to appear, Quantum, 2023.

Multiple eigenvalue estimation:

[Ding, Lin, 2023] Define $\Delta = \min_k \lambda_{n_{k+1}} - \lambda_{n_k}$, $p_D = \sum_k p_{n_k}$. There exists an algorithm(Multi-QCELS) such that, with high probability, it outputs

$$\left|\tilde{\lambda}_k - \lambda_{n_k}\right| \leq \epsilon,$$

with $T_{\max} = \Omega(\Delta^{-1})$

Improve

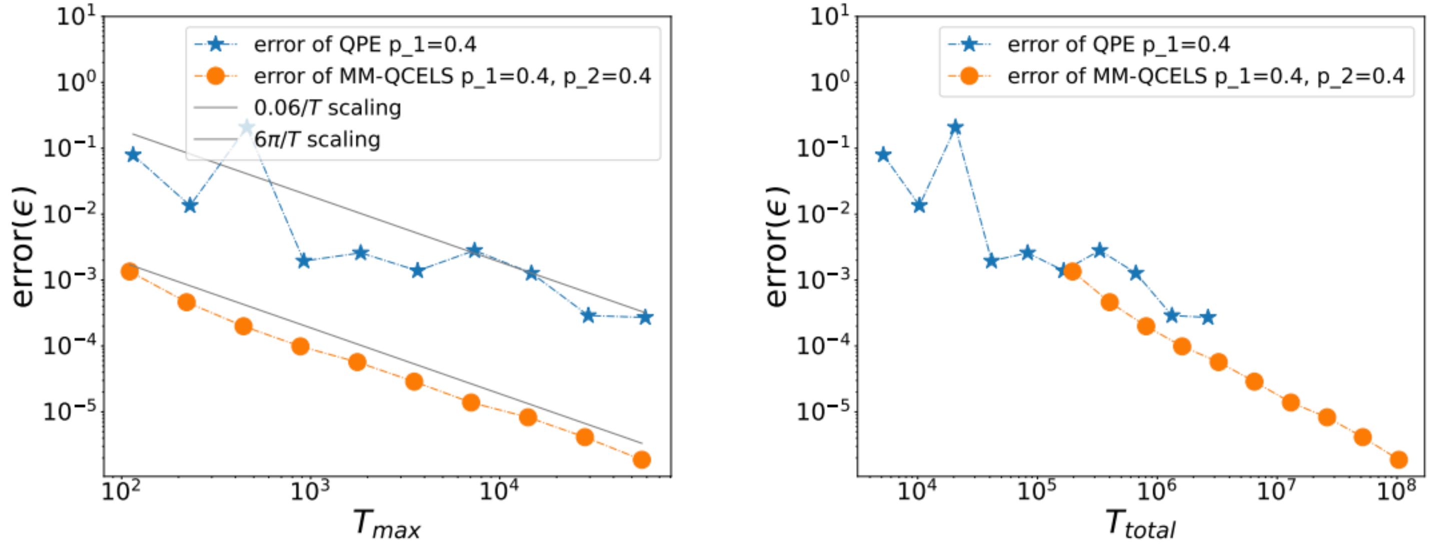
$$T_{\max} = \mathcal{O}\left(\frac{1 - p_D}{(\min_k p_{n_k})\epsilon}\right), \quad T_{total} = \mathcal{O}\left(\frac{1}{\epsilon}\right)$$

If $\lambda_{n_{k+1}} - \lambda_{n_k} \ll \epsilon$, we approximately see $\lambda_{n_{k+1}}, \lambda_{n_k}$ as one eigenvalue and solve

$$(r^*, \theta^*) = \min_{r,\theta} \frac{1}{N} \sum_{n=1}^{N} \left| Z_{t_n} - \sum_{k=1}^{K-1} r_k \exp(-i\theta_k t_n) \right|^2$$

Numerical evidence:

Transverse field Ising model (IFIM): p_0, p_1 large

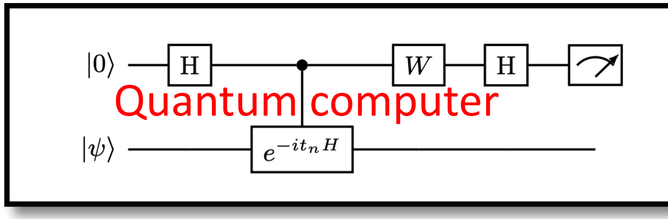


- Two order of magnitude reduction of maximal running time

Depolarizing noise:

Noisy quantum oracle:

$t \in \mathbb{R}$ →



→ $Z_t = \pm 1 \pm 1i$

$$\mathbb{E}(Z_t) = \exp(-\alpha t)\left\langle \psi_0 \right| \exp(-itH) \left| \psi_0 \right\rangle = \exp(-\alpha t) \sum_{k=0}^{d-1} p_k \exp(-it\lambda_k)$$

Depolarizing noise:

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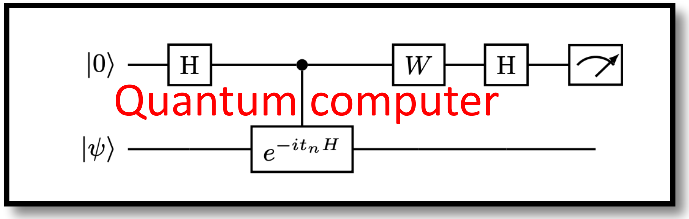
$t \in \mathbb{R}$ → 

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Difficulty: Assume we know $\beta \approx \alpha$, we recover the signal using $Z_{\beta,t} = \exp(\beta t) Z_t$

$$\mathbb{E}(Z_{\beta,t}) \approx \sum p_k \exp(-it\lambda_k), \qquad \mathrm{Var}(Z_{\beta,t}) = \Omega(\exp(\beta t))$$

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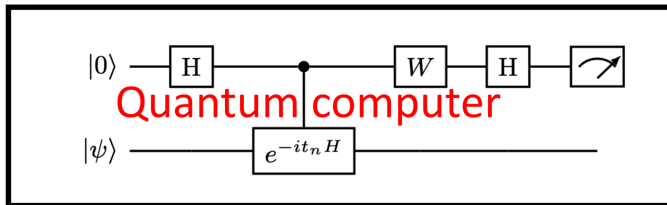
Depolarizing noise:

Noisy quantum oracle:

$t \in \mathbb{R}$



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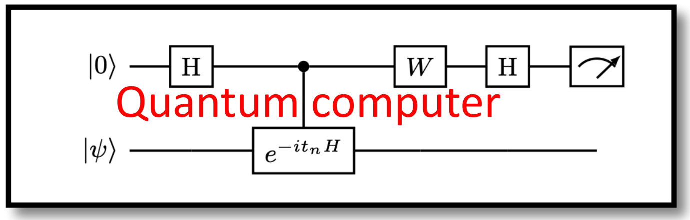
Difficulty: Assume we know $\beta \approx \alpha$, we recover the signal using $Z_{\beta,t} = \exp(\beta t) Z_t$

$\mathrm{Var}(Z_{\beta,t}) = \Omega(\exp(\beta t))$ → $N = \Omega(\exp(2\beta t))$ Number of repetitions

Depolarizing noise:

Noisy quantum oracle:

$t \in \mathbb{R}$ →  → $Z_t = \pm 1 \pm 1i$

$$\mathbb{E}(Z_t) = \exp(-\alpha t)\left\langle \psi_0 \middle| \exp(-itH) \middle| \psi_0 \right\rangle = \exp(-\alpha t)\sum_{k=0}^{d-1} p_k \exp(-it\lambda_k)$$

Difficulty: Assume we know $\beta \approx \alpha$, we recover the signal using $Z_{\beta,t} = \exp(\beta t) Z_t$

$\mathrm{Var}(Z_{\beta,t}) = \Omega(\exp(\beta t))$ → $N = \Omega(\exp(2\beta t))$ Number of repetitions

→ $T_{total} = \Omega(\exp(\alpha/\epsilon))$ if $T_{\max} = \Theta(1/\epsilon)$

Previous method is expensive

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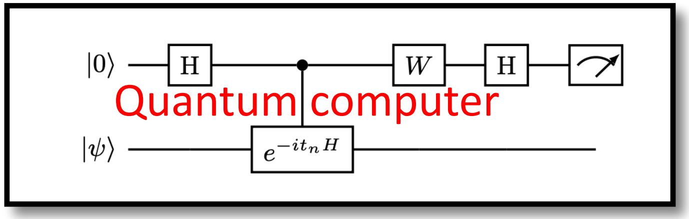
Depolarizing noise:

Noisy quantum oracle:

$t \in \mathbb{R}$ →  → $Z_t = \pm 1 \pm 1i$

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Need to find an algorithm such that $T_{\max} = \mathcal{O}(\log(1/\epsilon))$

Depolarizing noise:

[Ding, Dong, Tong, Lin, 2023] Define $\Delta = \lambda_1 - \lambda_0$ and assume $p_0 > 0.5$. Given $\beta - \alpha = \mathcal{O}(\epsilon)$, there exists an algorithm(Noisy-QCELS) such that, with high probability, it outputs

$$\left|\tilde{\lambda}_0 - \lambda_0\right| \le \epsilon,$$

with

$$T_{\max} = \mathcal{O}\left(\frac{1}{\Delta}\log\left(\frac{1}{\epsilon}\right)\right), \quad T_{total} = \mathcal{O}\left(\left(\frac{1}{\epsilon}\right)^{\frac{\alpha}{\Delta}+2}\right)$$

No Heisenberg-limited scaling

Some lower bound result: $T_{\max} = \Omega(1/\epsilon)$

[3]. Zhiyan Ding, Yulong Dong, Lin Lin, Yu Tong, Robust ground-state energy estimation in the presence of global deplorazing noise, arXiv/2307.11257

Depolarizing noise:

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Noisy-QCELS:

$$(r^*, \theta^*) = \min_{r,\theta} \frac{1}{N}\sum_{n=1}^{N}\left|\exp(\beta t_n) Z_{t_n} - r\exp(-i\theta t_n)\right|^2$$

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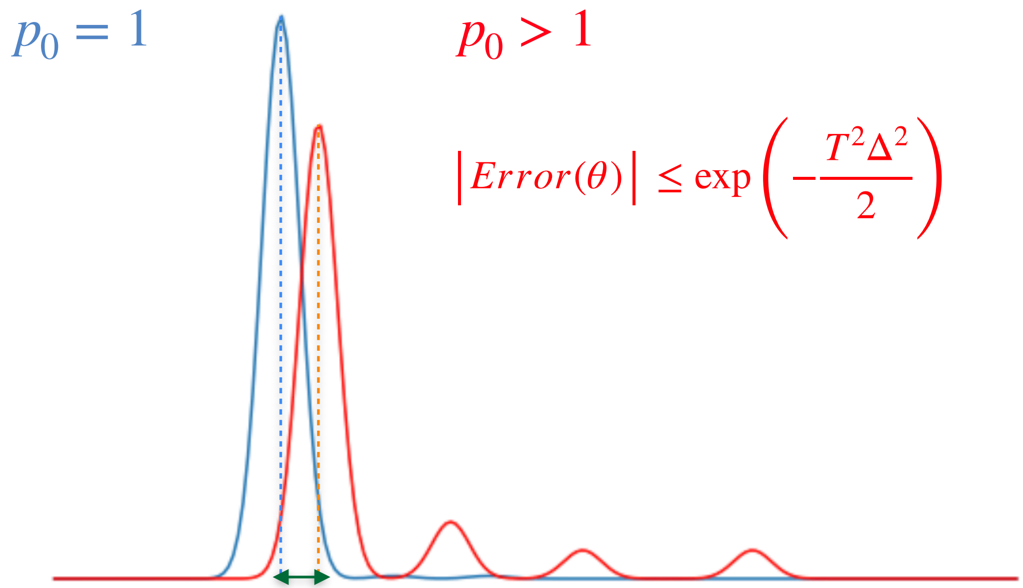
Why even smaller $T_{\max}$?:

Recall: $\theta^* \approx \max_{\theta} p_0 \exp\left(-\frac{T^2(\theta-\lambda_0)^2}{2}\right) + Error(\theta)$



$$\left|\theta^* - \lambda_0\right| = \mathcal{O}\left(\frac{\exp\left(-T^2\Delta^2/2\right)}{T}\right) \Rightarrow T_{\max} = \mathcal{O}\left(\frac{1}{\Delta}\log\left(\frac{1}{\epsilon}\right)\right)$$

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Conclusion:

Quantum phase estimation → Signal processing problem

Optimized signal fitting (QCELS)

- Ground state energy estimation
- Multiple eigenvalue estimation
- Depolarizing noise

Conclusion:

Quantum phase estimation → Signal processing problem

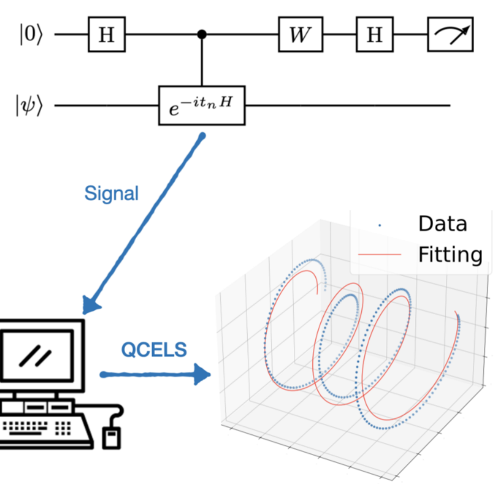
- Other signal processing method: RMPE, ESPRIT, SPF

[1]. G. Wang, D. Stilck-Fran¸ca, R. Zhang, S. Zhu, and P. D. Johnson. Quantum algorithm for ground state energy estimation using circuit depth with exponentially improved dependence on precision. arXiv/2209.06811, 2022.
[2]. Hongkang Ni, Haoya Li, Lexing Ying, On low-depth algorithms for quantum phase estimation. arxiv/2302.02454, 2023.
[3]. Haoya Li, Hongkang Ni, Lexing Ying, On adaptive low-depth quantum algorithms for robust multiple-phase estimation. arxiv/2303.08099, 2023.

Conclusion:

Quantum phase estimation → Signal processing problem

Optimized signal fitting (QCELS)



[1]. Zhiyan Ding, Lin Lin, Even shorter quantum circuit for phase estimation on early fault-tolerant quantum computers with applications to ground-state energy estimation, PRX Quantum, 2023.

[2]. Zhiyan Ding, Lin Lin, Simultaneous estimation of multiple eigenvalues with short-depth quantum circuit on early fault-tolerant quantum computers, to appear, Quantum, 2023.

[3]. Zhiyan Ding, Yulong Dong, Lin Lin, Yu Tong, Robust ground-state energy estimation in the presence of global depolarizing noise, arxiv/2307.11257, 2023.

Questions?

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