Quantum Walks

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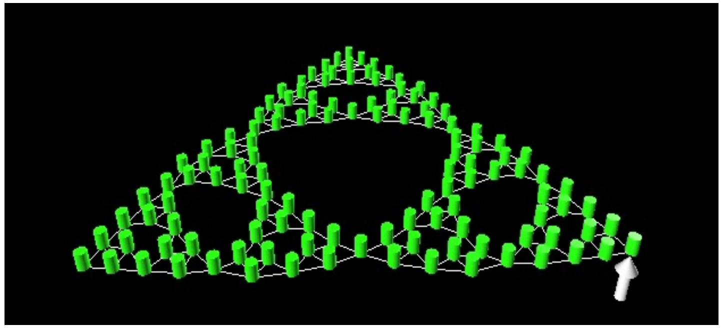
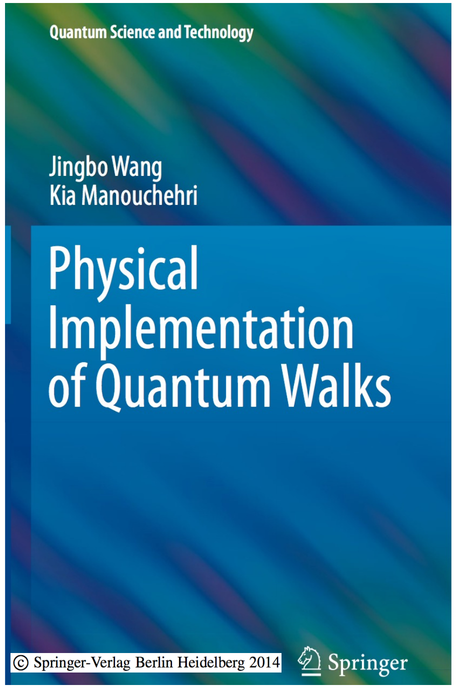
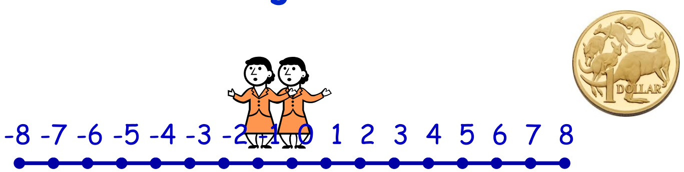
University of Western Australia, Perth



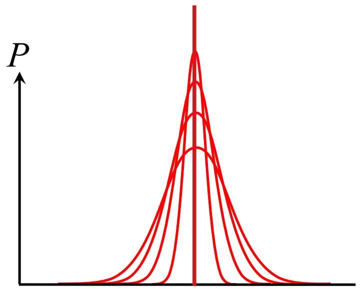


Walking on a line

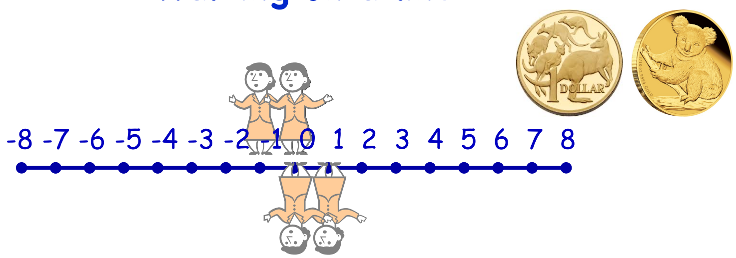


Classical Random Walk:

1. Start at origin
2. Toss a coin
3. Move left or right accordingly
4. Repeat



Walking on a line



Quantum Walk:

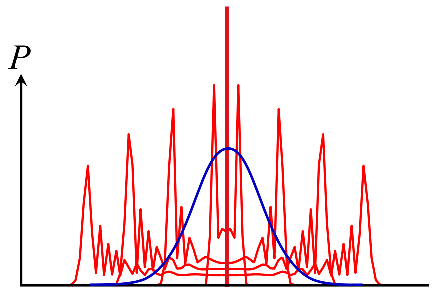
1. Start at origin
2. Toss a quantum coin, e.g.

$$I\otimes C = \frac{1}{\sqrt{2}}\left(|i,\uparrow\rangle + |i,\downarrow\rangle\right)\langle i,\uparrow| + \frac{1}{\sqrt{2}}\left(|i,\uparrow\rangle - |i,\downarrow\rangle\right)\langle i,\downarrow|$$

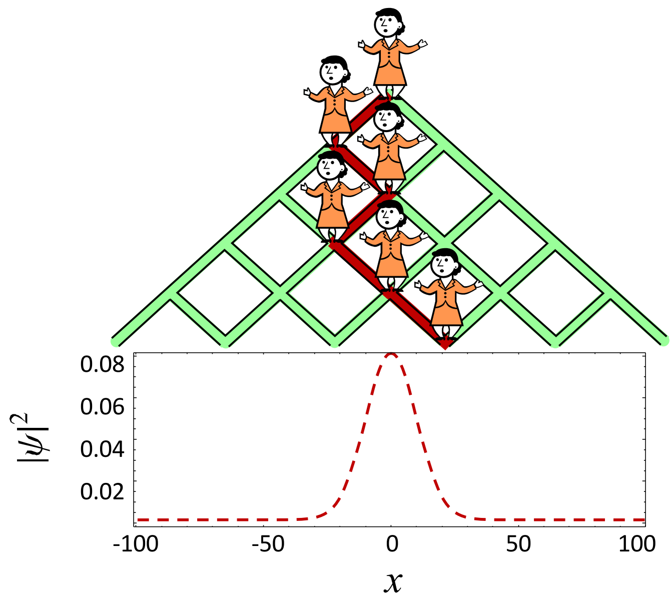
3. Move left and right simultaneously according to coin state

$$S = |i+1,\uparrow\rangle\langle i,\uparrow| + |i-1,\downarrow\rangle\langle i,\downarrow|$$

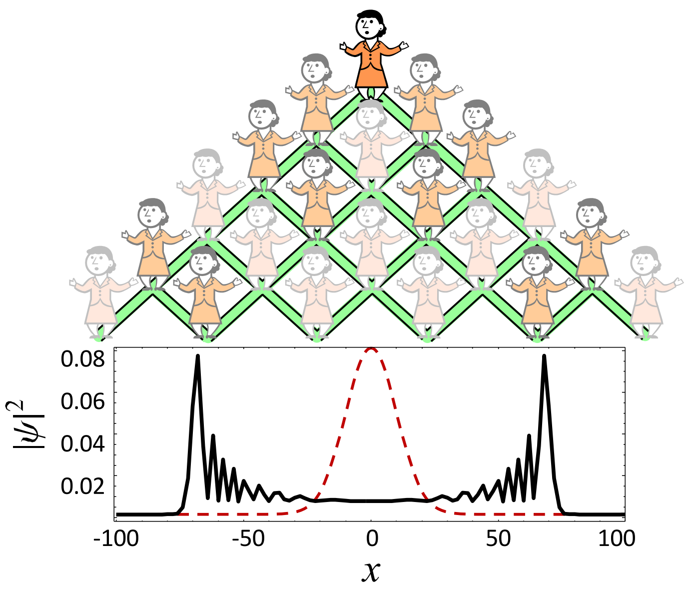
4. Repeat



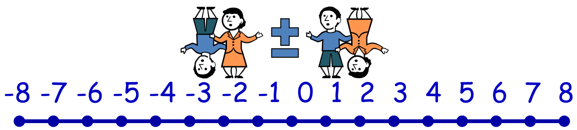
Artist Impression – *classical random walk*



Artist Impression – *quantum random walk*



Walking on a line



Entangled Quantum Walk:

1. Start at origin
2. Apply quantum coins

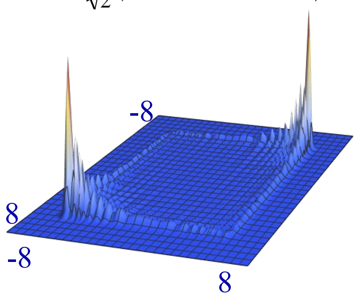
$C_{12} = (I \otimes C_1) \otimes (I \otimes C_2)$

3. Move left and right simultaneously according to coin state

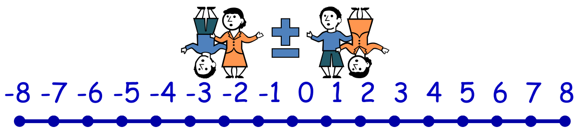
$$\mathbf{S} = |i+1,\uparrow; j+1,\uparrow\rangle\langle i,\uparrow; j,\uparrow| + |i+1,\uparrow; j-1,\downarrow\rangle\langle i,\uparrow; j,\downarrow| + |i-1,\downarrow; j+1,\uparrow\rangle\langle i,\downarrow; j,\uparrow| + |i-1,\downarrow; j-1,\downarrow\rangle\langle i,\downarrow; j,\downarrow|$$

4. Repeat

$$|\psi_0^-\rangle = \frac{1}{\sqrt{2}}\left(|0,\uparrow;0,\downarrow\rangle - |0,\downarrow;0,\uparrow\rangle\right)$$



Walking on a line



Entangled Quantum Walk:

1. Start at origin
2. Apply quantum coins

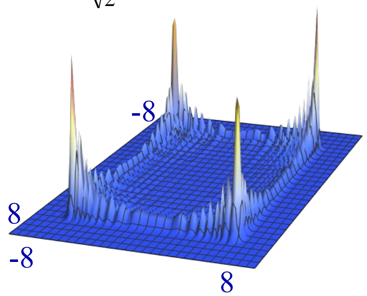
$C_{12} = (I \otimes C_1) \otimes (I \otimes C_2)$

3. Move left and right simultaneously according to coin state

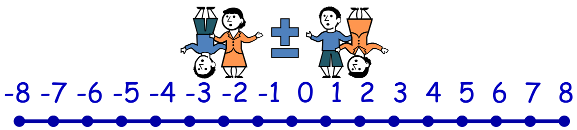
$$\mathbf{S} = |i+1,\uparrow; j+1,\uparrow\rangle\langle i,\uparrow; j,\uparrow| + |i+1,\uparrow; j-1,\downarrow\rangle\langle i,\uparrow; j,\downarrow| + |i-1,\downarrow; j+1,\uparrow\rangle\langle i,\downarrow; j,\uparrow| + |i-1,\downarrow; j-1,\downarrow\rangle\langle i,\downarrow; j,\downarrow|$$

4. Repeat

$$|\psi_0^+\rangle = \frac{1}{\sqrt{2}}\left(|0,\uparrow;0,\downarrow\rangle + |0,\downarrow;0,\uparrow\rangle\right)$$



Walking on a line



Entangled Quantum Walk:

$$|\psi_0^{\varphi}\rangle = \frac{1}{\sqrt{2}}\left(|0,\uparrow;0,\downarrow\rangle + e^{i\varphi}|0,\downarrow;0,\uparrow\rangle\right)$$

(with interaction)

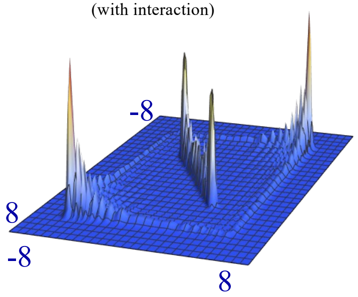
1. Start at origin
2. Apply quantum coins

$$C_{12} = (I \otimes C_1) \otimes (I \otimes C_2)$$

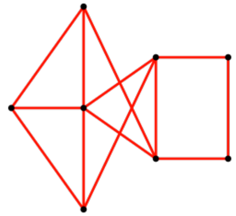
3. Move left and right simultaneously according to coin state

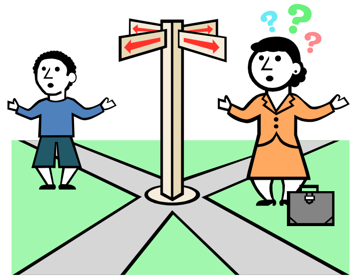
$$\begin{aligned} S =\ & |i+1,\uparrow;j+1,\uparrow\rangle\langle i,\uparrow;j,\uparrow| + |i+1,\uparrow;j-1,\downarrow\rangle\langle i,\uparrow;j,\downarrow| \\ & + |i-1,\downarrow;j+1,\uparrow\rangle\langle i,\downarrow;j,\uparrow| + |i-1,\downarrow;j-1,\downarrow\rangle\langle i,\downarrow;j,\downarrow| \end{aligned}$$

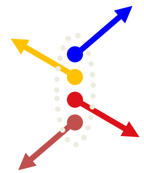
4. Repeat



Walking on a graph





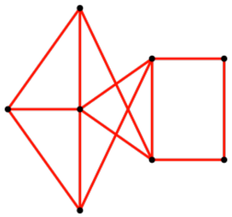


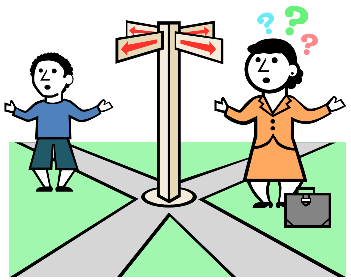
Classical : coin flip and move (stochastic)

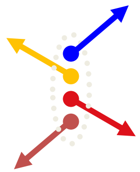
$$\begin{pmatrix} p'_1 \\ \cdots \\ p'_n \end{pmatrix} = \begin{pmatrix} q_{11} & \cdots & q_{1n} \\ \cdots & \cdots & \cdots \\ q_{n1} & \cdots & q_{nn} \end{pmatrix} \begin{pmatrix} p_1 \\ \cdots \\ p_n \end{pmatrix}$$

probability

Walking on a graph







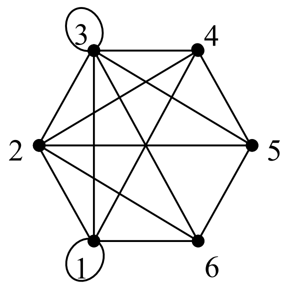
Quantum : apply coin operator and evolve (unitary)

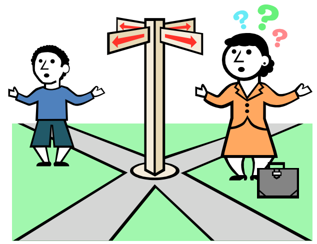
$$\begin{pmatrix} \psi'_1 \\ \cdots \\ \psi'_n \end{pmatrix} = \begin{pmatrix} u_{11} & \cdots & u_{1n} \\ \cdots & \cdots & \cdots \\ u_{n1} & \cdots & u_{nn} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \cdots \\ \psi_n \end{pmatrix}$$

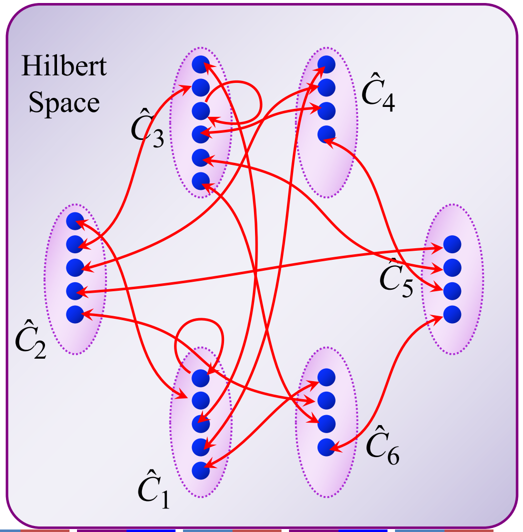
amplitude

Walking on a graph

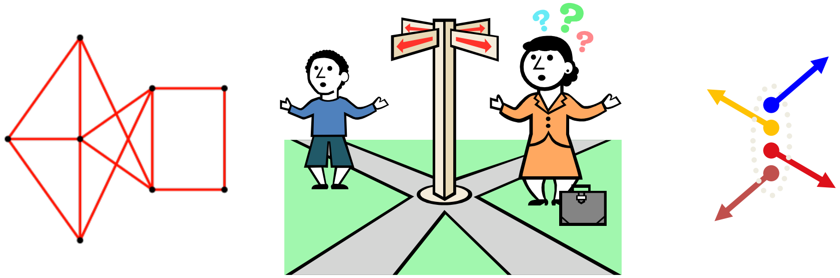
$$|\psi_f\rangle = \ldots \hat{T}\hat{C}\hat{T}\hat{C}\hat{T}\hat{C}\hat{T}\hat{C}\hat{T}\hat{C}|\psi_0\rangle$$







Walking on a graph



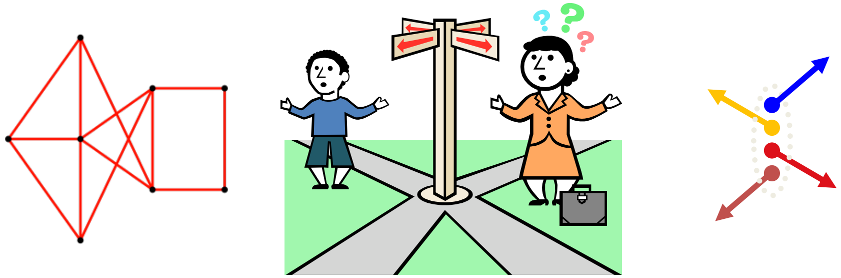
Classical : stochastic Markov chains

$$P(t) = e^{-Ht} P(0) \qquad \text{probability}$$

Quantum : unitary evolution

$$\psi(t) = e^{-i\hat{H}t} \psi(0) \qquad \text{amplitude}$$

Walking on a graph



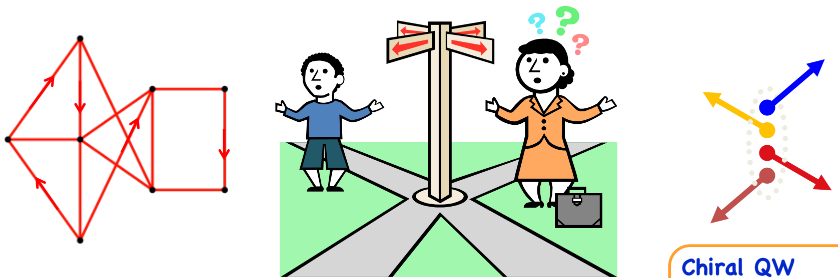
Discrete-time quantum walk

$$\psi_{n+1} = \hat{C}\hat{T}\psi_n = \hat{U}\psi_n \qquad (\hat{U} : \textit{unitary})$$

Continuous-time quantum walk

$$\psi(t) = e^{-i\hat{H}t} \psi(0) \qquad (\hat{H} : \textit{Hermitian})$$

Walking on a directed/weighted graph



Chiral QW
Szegedy QW
PT-symmetric QW
Stochastic QW

Discrete-time quantum walk

$$\psi_{n+1} = \hat{C}\hat{T}\psi_n = \hat{A}\psi_n \qquad (\hat{A} : \textit{non-unitary})$$

Continuous-time quantum walk

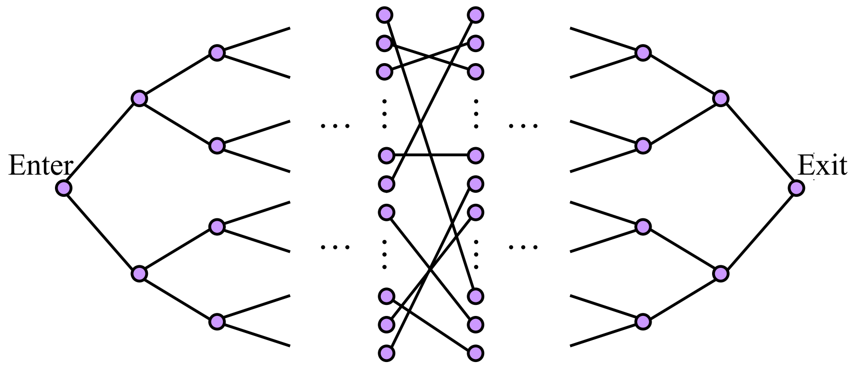
$$\psi(t) = e^{-i\hat{B}t} \psi(0) \qquad (\hat{B} : \textit{non-Hermitian})$$

Classical random walk applications

Examples: DNA synopsis, animal foraging strategies, diffusion and mobility in materials, exchange rate forecast, stock market analysis, solving differential equations, quantum monte carlo, optimization, clustering and classification, graph connectivity, fractal theory, structure analysis of facebook, Google, MSN and Yahoo search engines, etc.

Quantum walk applications ?

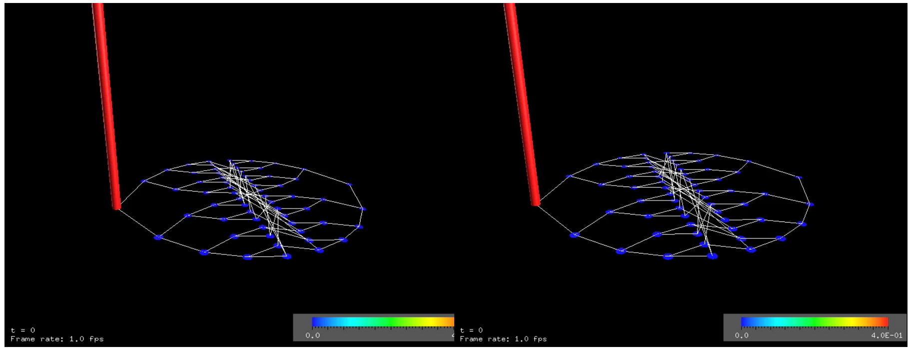
Quantum walk provides exponential speedup traversing a glued tree



Childs, Cleve, Deotto, Farhi, Gutmann, Spielman
35th ACM Theory of Computing, 59 (2003)

Quantum walk provides exponential speedup traversing a glued tree

Classical random walk | Quantum walk



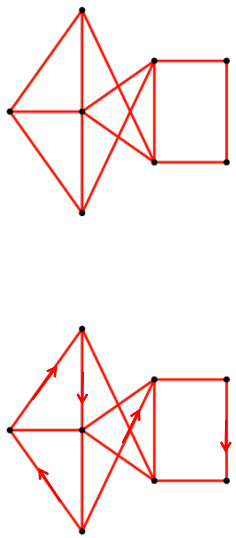
Childs et al 35th ACM Theory of Computing, 59 (2003)

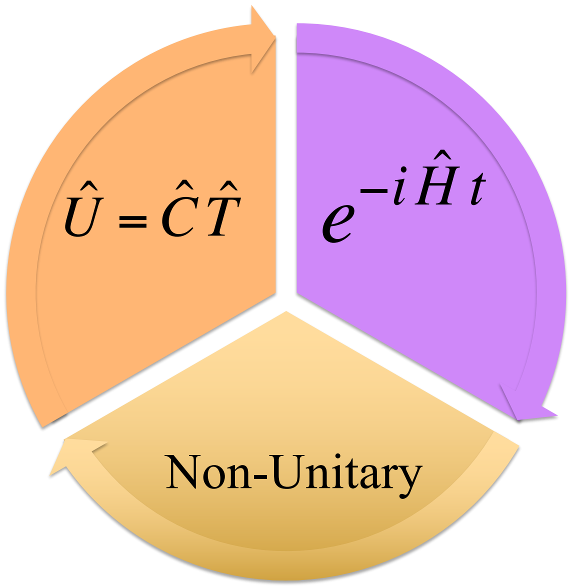
Quantum Algorithms for the NISQ Era

(https://quisa.tech/publications/)

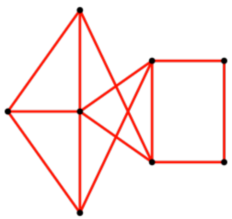
- Combinatorial optimization via highly efficient quantum walks
- Quantum optimisation of financial portfolios
- Quantum optimisation of capacitated vehicle routing
- Quantum algorithm for network analysis and centrality ranking
- Quantum walk based algorithms for graph similarity, isomorphism, and other graph-theoretic quantum algorithms
- Quantum algorithm for video visual tracking
- Quantum informatics: protein sequence engineering
- Quantum data compression by principal component analysis
- Gibbs partition function using quantum Clifford sampling
- Quantum predictive algorithms on phase transition and criticality
- …

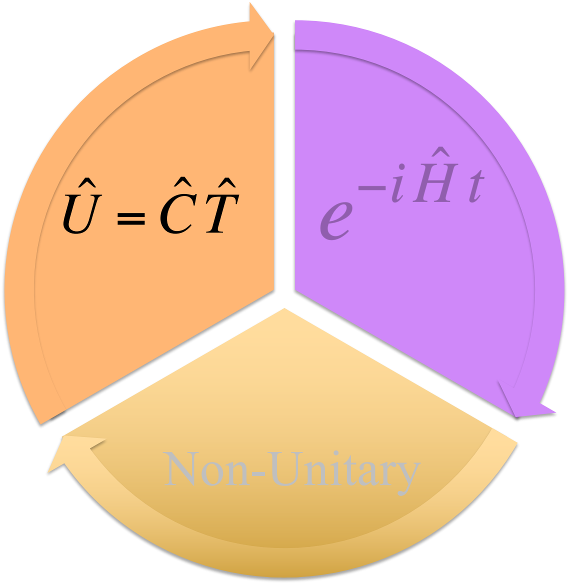
Efficient quantum Circuit Implementation





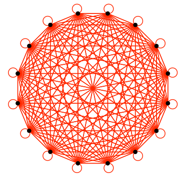
Efficient quantum Circuit Implementation



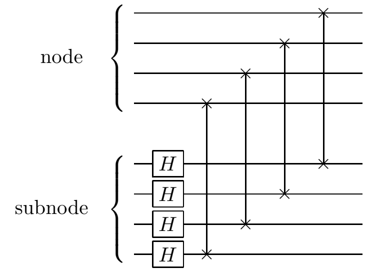


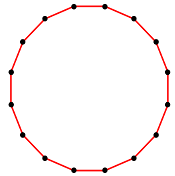
$\hat{C}\hat{T}$

1. Graphs with certain symmetry

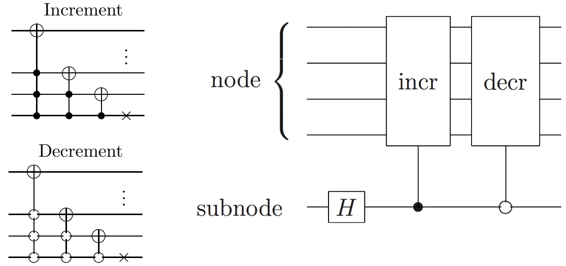


Complete 16-graph



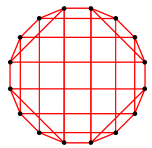


16-cycle

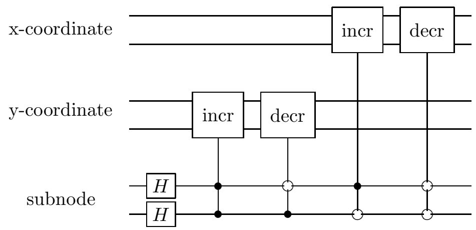


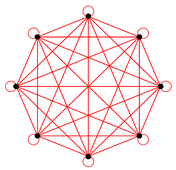
$\hat{C}\hat{T}$

1. Graphs with certain symmetry

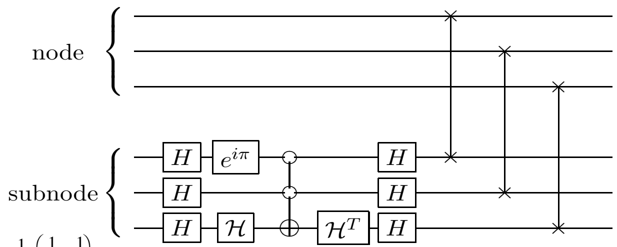


2D hypercycle





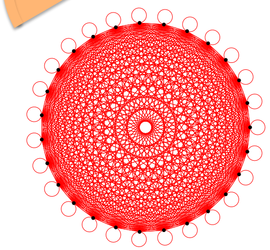
Complete 8-graph with Grover Coin



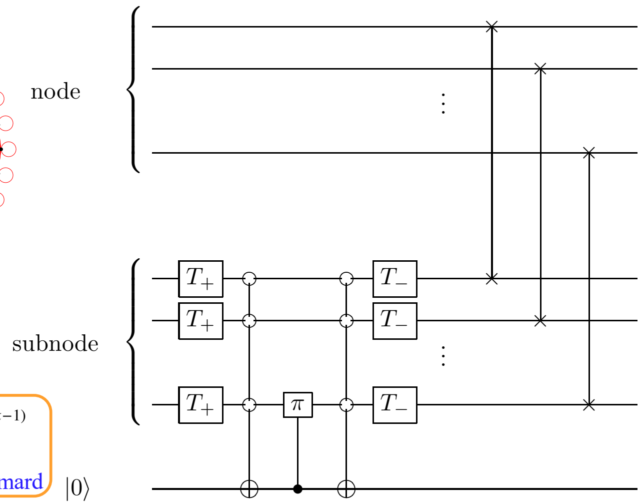
$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$; $\mathcal{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

$\hat{C}\hat{T}$

1. Graphs with certain symmetry



Complete 3^n-graph

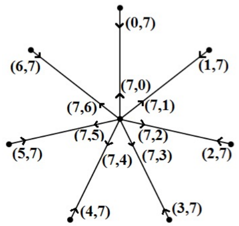


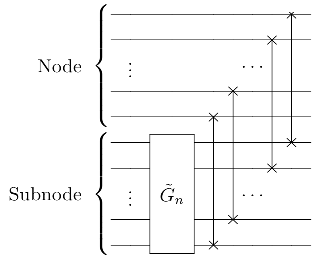
$$(T_\pm)_{j,k} = \frac{1}{\sqrt{3}} e^{\pm i\frac{2\pi}{3}(j-1)(k-1)}$$

Qutrit equivalent of Hadamard

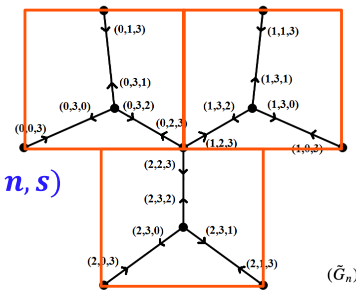
1. Graphs with certain symmetry

star graph

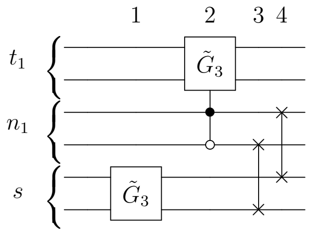




3CT-2

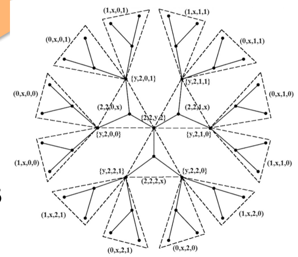


(t, n, s)

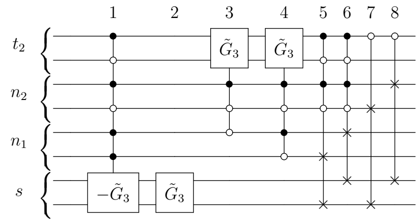


$$(\tilde{G}_n)_{i,j} = \begin{cases} \frac{2}{n} - \delta_{i,j}, & i \leqslant n, j \leqslant n, \\ \delta_{i,j}, & \text{otherwise,} \end{cases}$$

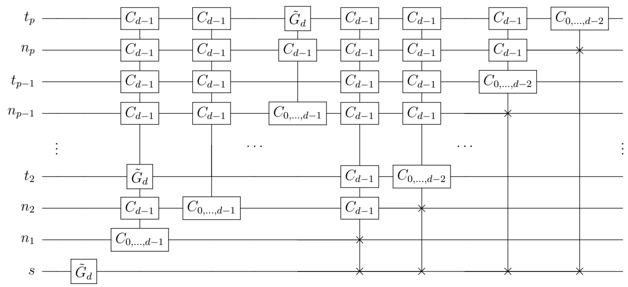
1. Graphs with certain symmetry



3CT-3

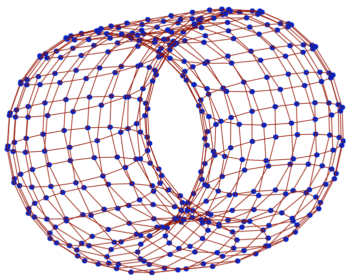


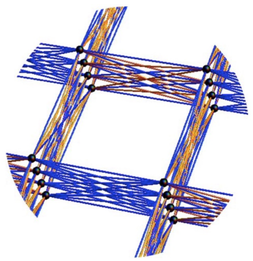
dCT-n

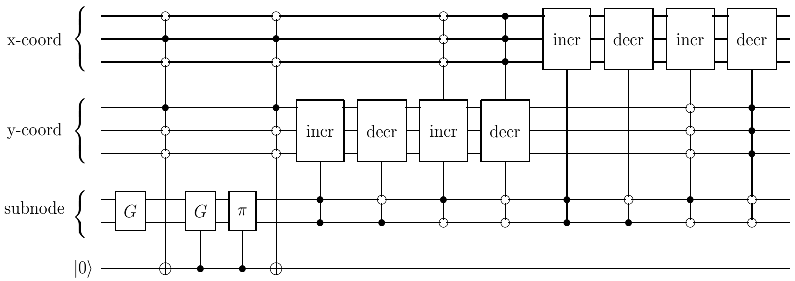


1. Graphs with certain symmetry

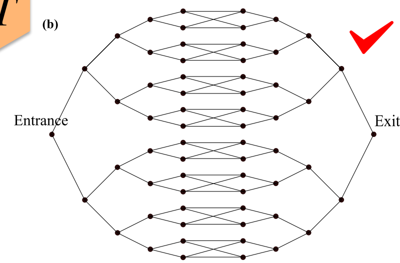
"Twisted" toroidal lattice

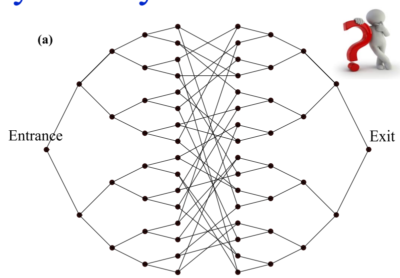


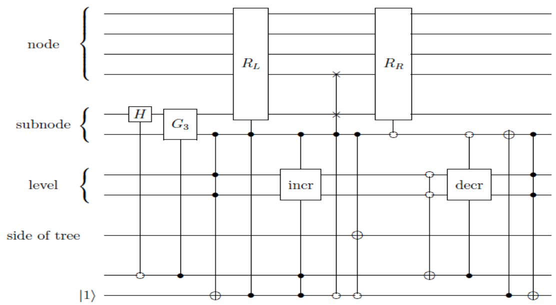




1. Graphs with certain symmetry







Glued-trees

sparse

2. Sparse graphs

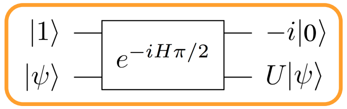
$$H = \begin{pmatrix} 0 & U \\ U^\dagger & 0 \end{pmatrix}; \qquad \begin{array}{l} U: unitary \\ H: Hermitian;\ H^2 = I;\ \|H\| = 1 \end{array}$$

$$e^{-iH\theta} = \cos(\theta) I - i \sin(\theta) H$$

$$e^{-iH\pi/2} = -iH = -i \begin{pmatrix} 0 & U \\ U^+ & 0 \end{pmatrix}$$

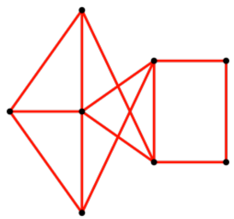
$$\therefore\ e^{-iH\pi/2} \begin{pmatrix} 0 \\ \psi \end{pmatrix} = -i \begin{pmatrix} U\psi \\ 0 \end{pmatrix}$$

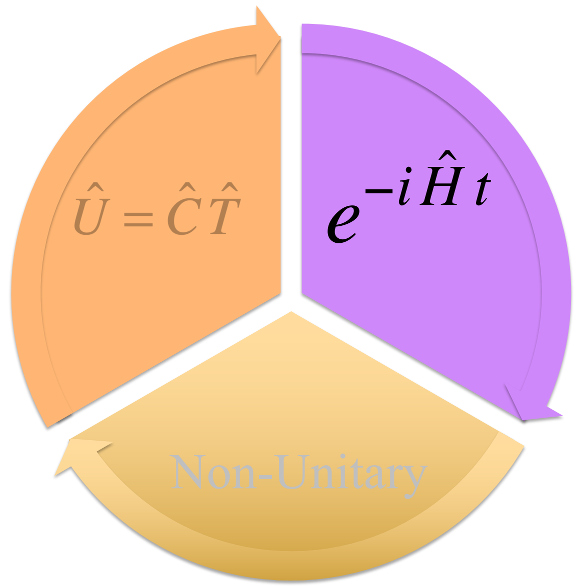
$$\therefore\ e^{-iH\pi/2} |1\rangle |\psi\rangle = -i |0\rangle U |\psi\rangle$$



Jordan and Wocjan PRA 80, 062301 (2009)

Quantum Circuit Implementation





1. Sparse Hamiltonian with efficiently computable entries

- Lloyd (1996)
- Aharonov and Ta-Shma (2003)
- Christandl, Datta, Ekert, Landahl, (2004)
- Berry, Ahokas, Cleve, Sanders (2007)
- Childs (2008)
- Wiebe, Berry, Høyer, Sanders (2010)
- Childs (2010)
- Poulin, Qarry, Somma, Verstraete (2011)
- Childs and Wiebe (2012)
- Berry and Childs (2012)
- Berry, Childs, Kothari (2015)
- Berry and Novo (2016)
- Low and Chuang (2017, 2019)
- Chen, Dalzell, Berta, Brandão, Tropp (2023)

2. Efficiently diagonalizable dense graph

Spectral theorem $\quad e^{-iHt} = Q^\dagger e^{-i\Lambda t} Q$

Λ: diagonal matrix of eigenvalues of H

Q: matrix of column eigenvectors of H

$$C = \begin{pmatrix} c_1 & c_2 & \cdots & c_n \\ c_n & c_1 & \cdots & c_{n-1} \\ \vdots & \vdots & \ddots & \vdots \\ c_2 & c_3 & \cdots & c_1 \end{pmatrix}$$

Circulant

Fourier

$$Q \equiv F_{jk} = \omega^{jk} / \sqrt{n}, \quad \omega = e^{2\pi i/n}$$

$$\Lambda = diag(\sqrt{n}\, F\{c_1, c_2 \cdots c_n\})$$

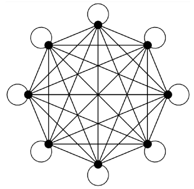
DFT: O(n log n)
QFT: O($\log^2$ n)

Nature Communication 7:11511 (2016)

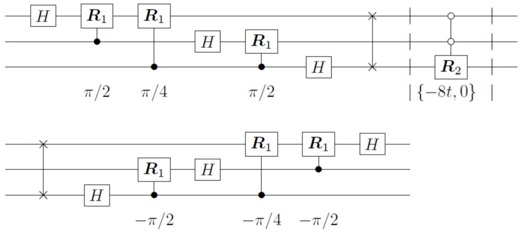
$e^{-i\hat{H}t}$

Circulant graphs $\quad e^{-iH_c t} = F^\dagger e^{-i\Lambda t} F$

(1) $O(\text{poly}\log(n))$ distinct eigenvalues

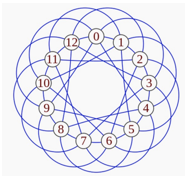


Complete K_m



$\Lambda = diag(\{m,0,0,0,0,0,0,0\})$

\# gates: $2\log^2(n)+1$



Paley Graph (SRG)

$\Lambda = diag(\{a,b,b,b,b,b,b,c,c,c,c,c,c\})$

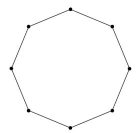
(all SRGs have at most three distinct eigenvalues)

Nature Communication 7:11511 (2016)

$e^{-i\hat{H}t}$

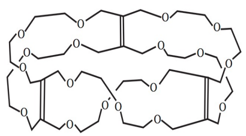
Circulant graphs $\quad e^{-iH_c t} = F^\dagger e^{-i\Lambda t} F$

(2) eigenvalues can be computed efficiently



Cycles

$$\Lambda = diag\left(\left\{2\cos\left(\frac{2k\pi}{n}\right) \middle|\ k=0,1,\cdots,n-1\right\}\right)$$



Möbius

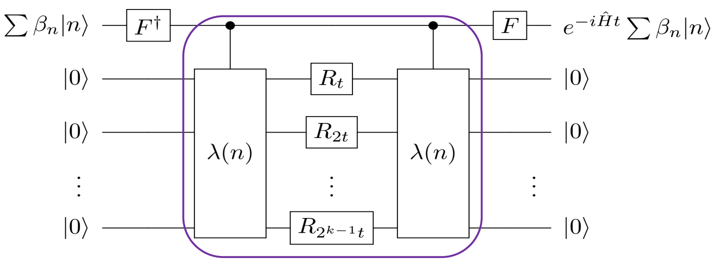
$$\Lambda = diag\left(\left\{4\sin^2\left(\frac{k\pi}{n}\right)+\left(1-(-1)^k\right) \middle|\ k=0,1,\cdots,n-1\right\}\right)$$

Nature Communication 7:11511 (2016)

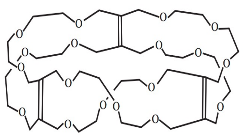
$e^{-i\hat{H}t}$

Circulant graphs $\quad e^{-iH_c t} = F^\dagger e^{-i\Lambda t} F$

(2) eigenvalues can be computed efficiently



Childs PhD thesis



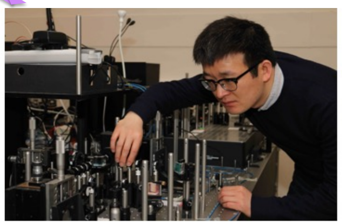
Möbius

$$\Lambda = diag\left(\left\{4\sin^2\left(\frac{k\pi}{n}\right)+\left(1-(-1)^k\right) \middle|\ k=0,1,\cdots,n-1\right\}\right)$$

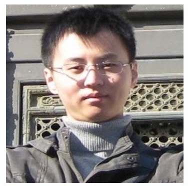
Nature Communication 7:11511 (2016)

$e^{-i\hat{H}t}$

Circulant graphs $\quad e^{-iH_c t} = F^\dagger e^{-i\Lambda t} F$



Xiaogang Qiang



Xiaoqi Zhou

Kanin Aungskunsiri

University of BRISTOL

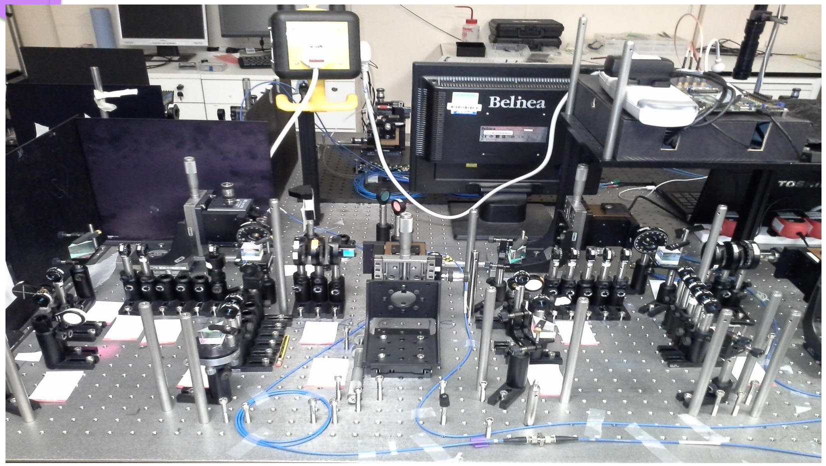
Ashley Montanaro

Jonathan Matthews

Jeremy O'Brien

$e^{-i\hat{H}t}$

♪ Circulant graphs $\quad e^{-iH_c t} = F^{\dagger} e^{-i\Lambda t} F$



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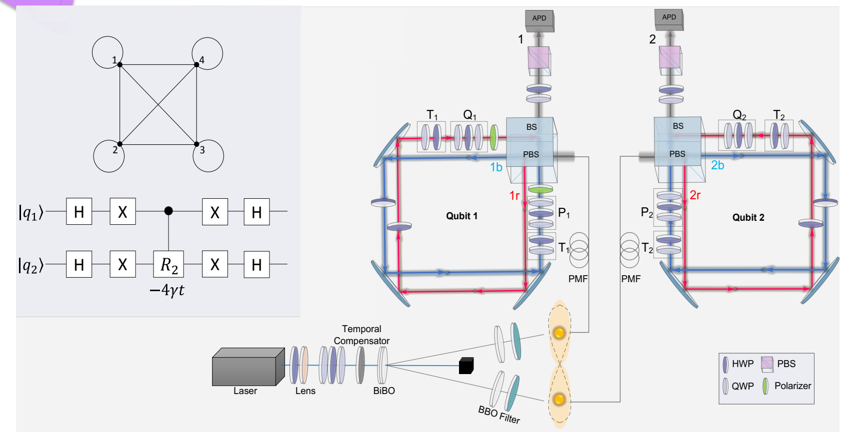
$e^{-i\hat{H}t}$

♪ Circulant graphs $\quad e^{-iH_c t} = F^{\dagger} e^{-i\Lambda t} F$



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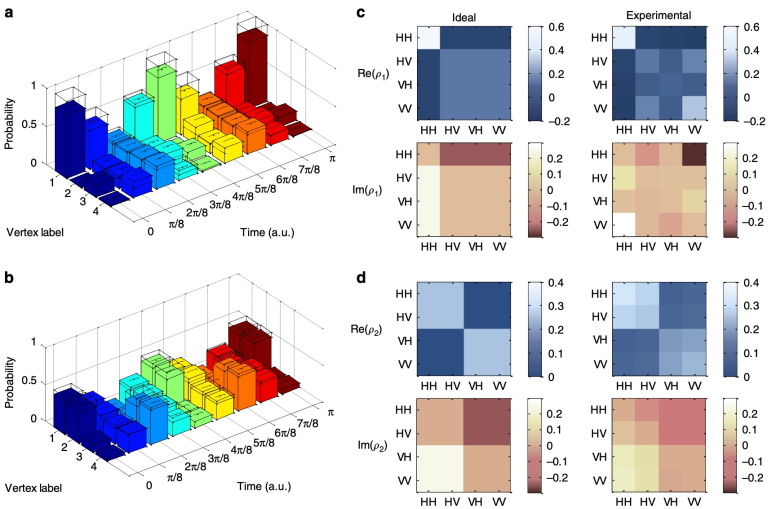

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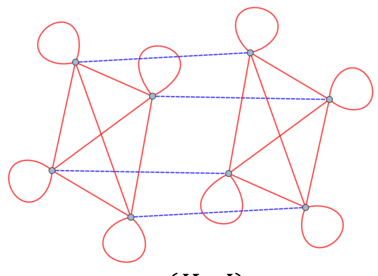
$e^{-i\hat{H}t}$

3. Composite graphs

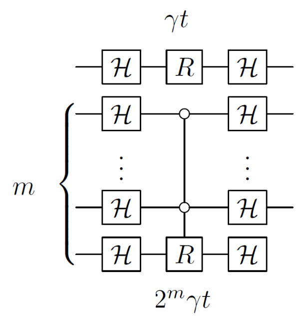
(1) COMMUTING GRAPHS

$$e^{-i\gamma(A+B)t} \equiv e^{-i\gamma At} e^{-i\gamma Bt}$$



(K_4, I)

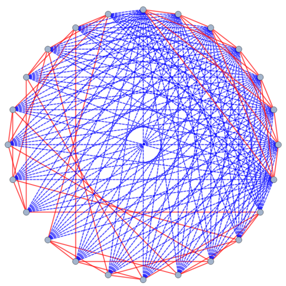
identity interconnection between two complete graph

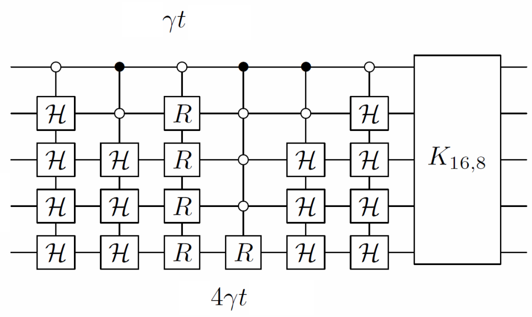


2. Composite graphs

(1) COMMUTING GRAPHS

$$e^{-i\gamma(A+B)t} \equiv e^{-i\gamma At} e^{-i\gamma Bt}$$





$(Q_4, K_{16,8})$

complete interconnection between two disjoint degree-regular graphs

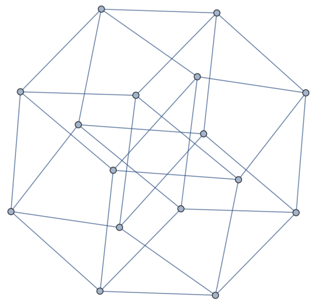
JPhysA 50, 055303 (2017)

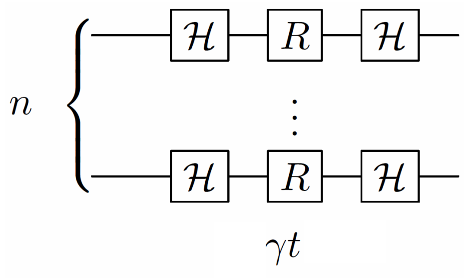
2. Composite graphs

(2) CARTESIAN PRODUCT OF GRAPHS

$$H_1 \oplus H_2 = H_1 \otimes I_{n_2} + I_{n_1} \otimes H_2$$

$$e^{-i(H_1 \oplus H_2)t} \equiv e^{-iH_1 t} \otimes e^{-iH_2 t}$$





hypercube graph $Q_n = K_2^{\oplus n}$

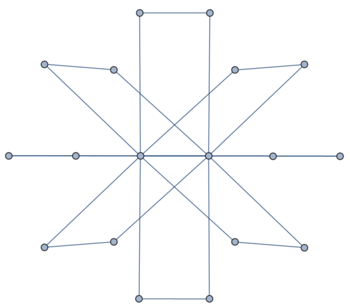
JPhysA 50, 055303 (2017)

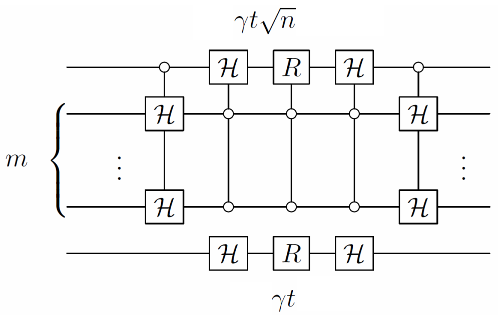
2. Composite graphs

(2) CARTESIAN PRODUCT OF GRAPHS

$$H_1 \oplus H_2 = H_1 \otimes I_{n_2} + I_{n_1} \otimes H_2$$

$$e^{-i(H_1 \oplus H_2)t} \equiv e^{-iH_1 t} \otimes e^{-iH_2 t}$$



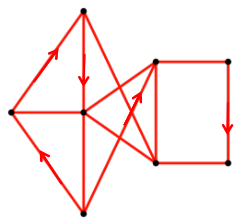


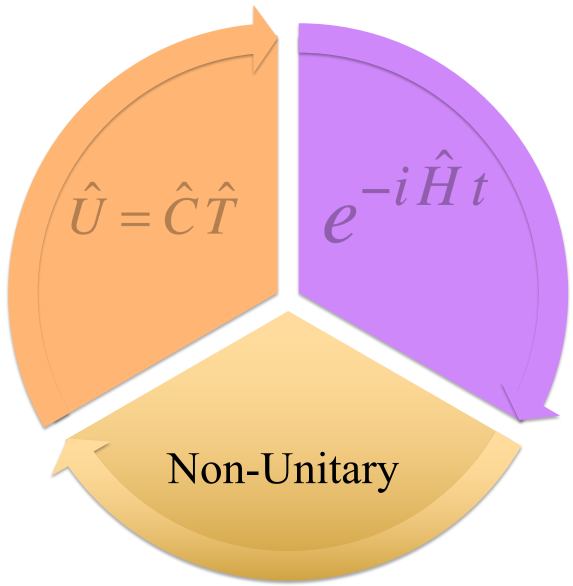
Cartesian product of star and path graphs

$S_{N+1} \oplus K_2$

JPhysA 50, 055303 (2017)

Quantum Circuit Implementation





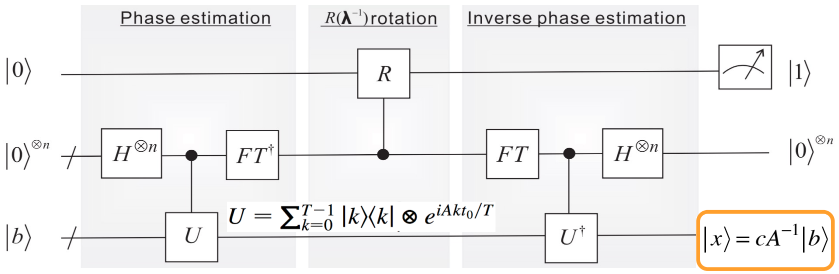
Can we implement Non-Unitary operations efficiently?

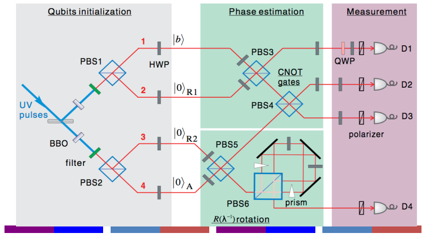
Non-Unitary

1. Arbitrary sparse matrix and inverse

Harrow, Hassidim, Lloyd (2009)

Cai, Weedbrook, Su, Chen, Gu, Zhu, Li, Liu, Lu, Pan (2013)





Non-Unitary

2. Dense Circulant, Toeplize & Hankel

Every Matrix is a Product of Toeplitz Matrices

Ke Ye · Lek-Heng Lim (2015)

FOUNDATIONS OF COMPUTATIONAL MATHEMATICS

The Journal of the Society for the Foundations of Computational Mathematics

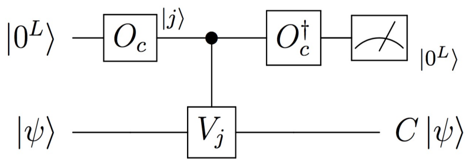
The choice of Toeplitz factors is natural for two reasons. Firstly, Toeplitz matrices are ubiquitous and are one of the most well-studied and understood classes of structured matrices. They arise in pure mathematics: algebra [6], algebraic geometry [46], analysis [31], combinatorics [35], differential geometry [40], graph theory [26], integral equations [5], operator algebra [23], partial differential equations [50], probability [45], representation theory [25], topology [42], as well as in applied mathematics: approximation theory [54], compressive sensing [32], numerical integral equations [39], numerical integration [51], numerical partial differential equations [52], image processing [19], optimal control [44], quantum mechanics [24], queueing networks [7], signal processing [53], statistics [22], time series analysis [18], and among other areas.

Non-Unitary

2. Dense Circulant, Toeplize & Hankel

Circulant Matrices

$$C = \begin{pmatrix} c_0 & c_1 & \cdots & c_{n-1} \\ c_{n-1} & c_0 & \cdots & c_{n-2} \\ \vdots & \vdots & \ddots & \vdots \\ c_1 & c_2 & \cdots & c_0 \end{pmatrix} = c_0 \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} + c_1 \begin{pmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & \vdots \\ \vdots & \vdots & \ddots & 1 \\ 1 & 0 & \cdots & 0 \end{pmatrix} + \cdots = \sum_{j=0}^{n-1} c_j V_j$$



$O(\log^2 n / \| C|\psi\rangle \|)$

where $V_j = \sum_{k=0}^{n-1} |(k-j) \mod n\rangle \langle k|$

$O_c |0^L\rangle = \sum_{j=0}^{n-1} \sqrt{c_j}\, |j\rangle$

JPhysA 49, 275301 (2016)

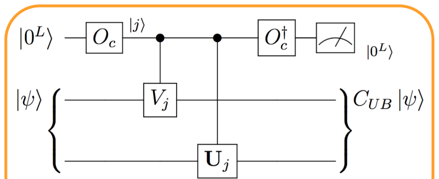
R. Soc. Open Sci. 4, 160906 (2017)

Non-Unitary

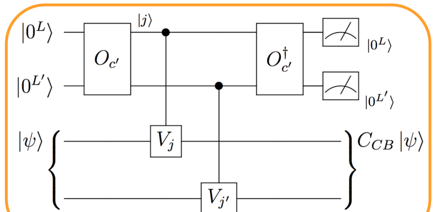
2. Dense Circulant, Toeplize & Hankel

Block Circulant Matrices

$$C_{BC} = \begin{pmatrix} \mathbf{C}_0 & \mathbf{C}_1 & \cdots & \mathbf{C}_{n-1} \\ \mathbf{C}_{n-1} & \mathbf{C}_0 & \cdots & \mathbf{C}_{n-2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}_1 & \mathbf{C}_2 & \cdots & \mathbf{C}_0 \end{pmatrix}$$



(a) unitary blocks



(b) circulant blocks

JPhysA 49, 275301 (2016)

R. Soc. Open Sci. 4, 160906 (2017)

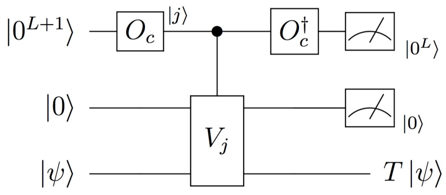
Non-Unitary

2. Dense Circulant, Toeplize & Hankel

Toeplize & Hankel

$$T = \begin{pmatrix} t_0 & t_{-1} & t_{-2} & \cdots & t_{-(n-1)} \\ t_1 & t_0 & t_{-1} & \cdots & t_{-(n-2)} \\ t_2 & t_1 & t_0 & \cdots & t_{-(n-3)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ t_{n-1} & t_{n-2} & t_{n-3} & \cdots & t_0 \end{pmatrix} \quad B_T = \begin{pmatrix} 0 & t_{n-1} & \cdots & t_2 & t_1 \\ t_{-(n-1)} & 0 & \cdots & t_3 & t_2 \\ t_{-(n-2)} & t_{-(n-1)} & \cdots & t_4 & t_3 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ t_{-1} & t_{-2} & \cdots & t_{-(n-1)} & 0 \end{pmatrix}$$

$$C_T = \begin{pmatrix} T & B_T \\ B_T & T \end{pmatrix} \begin{pmatrix} \psi \\ 0 \end{pmatrix} = \begin{pmatrix} T\psi \\ B_T\psi \end{pmatrix}$$



JPhysA 49, 275301 (2016)
R. Soc. Open Sci. 4, 160906 (2017)

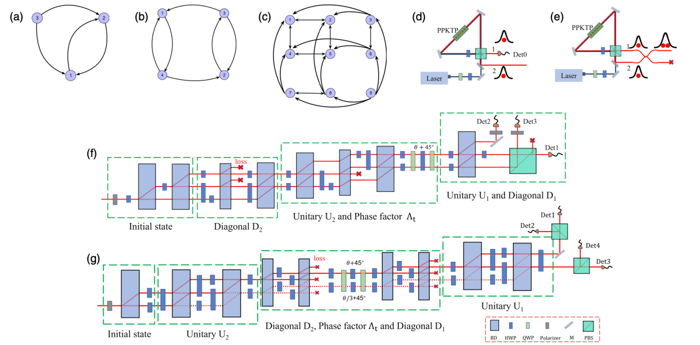
Non-Unitary

3. PT-symmetric Quantum Walk

PHYSICAL REVIEW LETTERS **125,** 240501 (2020)

Experimental Parity-Time Symmetric Quantum Walks for Centrality Ranking on Directed Graphs

Tong Wu (吴通),[1] J. A. Izaac,[2] Zi-Xi Li (黎子溪),[1] Kai Wang (王凯),[1] Zhao-Zhong Chen (陈召忠),[1] Shining Zhu (祝世宁),[1] J. B. Wang,[2] and Xiao-Song Ma (马小松)[1,*]



PRL 125, 240501 (2020)

Non-Unitary

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PHYSICAL REVIEW LETTERS **125,** 240501 (2020)

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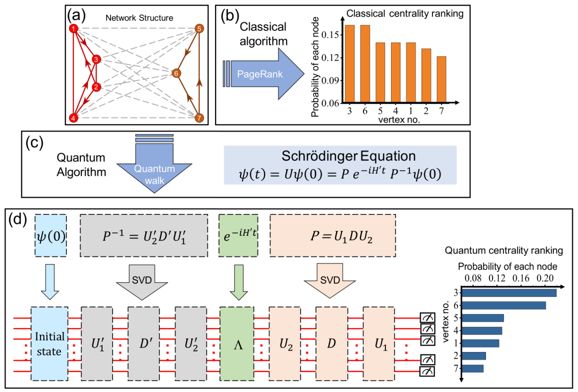


PRL 125, 240501 (2020)

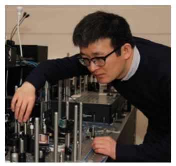
Non-Unitary

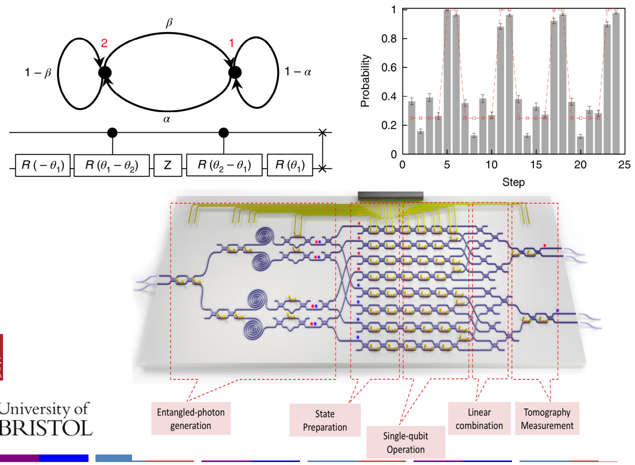
4. Szgedy quantum walk

NATURE PHOTONICS | VOL 12 | SEPTEMBER 2018 | 534–539 |

Large-scale silicon quantum photonics implementing arbitrary two-qubit processing

Xiaogang Qiang[1,2,3], Xiaoqi Zhou[4*], Jianwei Wang[1,5], Callum M. Wilkes[1], Thomas Loke[6], Sean O'Gara[1], Laurent Kling[1], Graham D. Marshall[1], Raffaele Santagati[1], Timothy C. Ralph[7], Jingbo B. Wang[6], Jeremy L. O'Brien[1], Mark G. Thompson[1] and Jonathan C. F. Matthews[1*]





University of BRISTOL

Non-Unitary

5. Unitary dilation

$$U_A(t)=\begin{pmatrix}\frac{G(t)}{\Lambda(t)} & \sqrt{1-\frac{G(t)}{\Lambda(t)}\frac{G^\dagger(t)}{\Lambda(t)}}\\ \sqrt{1-\frac{G^\dagger(t)}{\Lambda(t)}\frac{G(t)}{\Lambda(t)}} & -\frac{G^\dagger(t)}{\Lambda(t)}\end{pmatrix}$$

1524 Vol. 7, No. 11 / November 2020 / Optica Research Article

optica

Experimental realization of continuous-time quantum walks on directed graphs and their application in PageRank

Kunkun Wang,[1] Yuhao Shi,[2] Lei Xiao,[1] Jingbo Wang,[3] Yogesh N. Joglekar,[4,5] and Peng Xue[1,*]

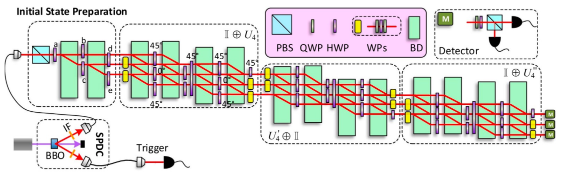


Fig. 4. Experimental setup. Polarization-degenerated photon pairs are generated by type I spontaneous parametric downconversion (SPDC) in a 0.5 mm thick nonlinear β-barium-borate (BBO) crystal pumped by a 400.8 nm continuous wave diode laser with 90 mW of power. The single photon is generated by triggering of the other photon. Interference filters are used to restrict the photon bandwidth to 3 nm. With a polarizing beam splitter (PBS), wave plates (WPs), and two beam displacers (BDs), we can prepare an arbitrary six-dimensional qudit state. CTQW on directed graphs is simulated via $U_A(t)$, which can be realized via linear optics.

Optica 7, 1524 (2020)

Non-Unitary

5. Unitary dilation

$$U_A(t)=\begin{pmatrix}\frac{G(t)}{\Lambda(t)} & \sqrt{1-\frac{G(t)}{\Lambda(t)}\frac{G^\dagger(t)}{\Lambda(t)}}\\ \sqrt{1-\frac{G^\dagger(t)}{\Lambda(t)}\frac{G(t)}{\Lambda(t)}} & -\frac{G^\dagger(t)}{\Lambda(t)}\end{pmatrix}$$

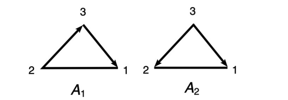


Fig. 1. Distinct directed graphs with three nodes. Both graphs A_1 and A_2 have two directed edges. Graph A_1 is a reversible directed graph, and A_2 is irreversible.

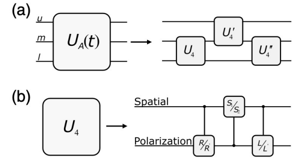
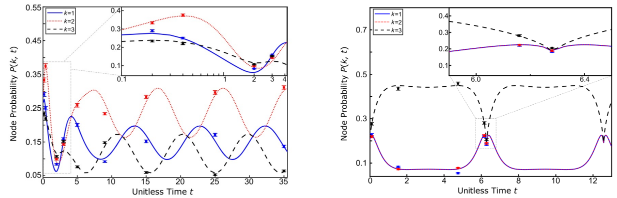


Fig. 2. (a) The circuit diagram for the 6×6 unitary transformation $U_A(t)$. (b) The circuit diagram for the 4×4 unitary transformation U_4.

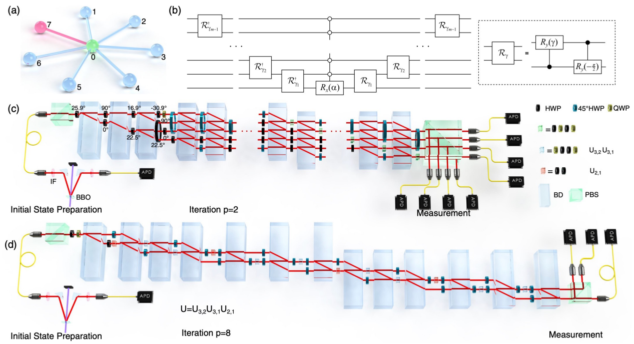


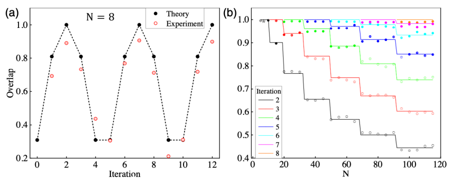
Optica 7, 1524 (2020)

PHYSICAL REVIEW LETTERS **128**, 050501 (2022)

Deterministic Search on Star Graphs via Quantum Walks

Dengke Qu,[1,2,*] Samuel Marsh,[3,*] Kunkun Wang,[1] Lei Xiao,[1] Jingbo Wang,[3,†] and Peng Xue[1,‡]



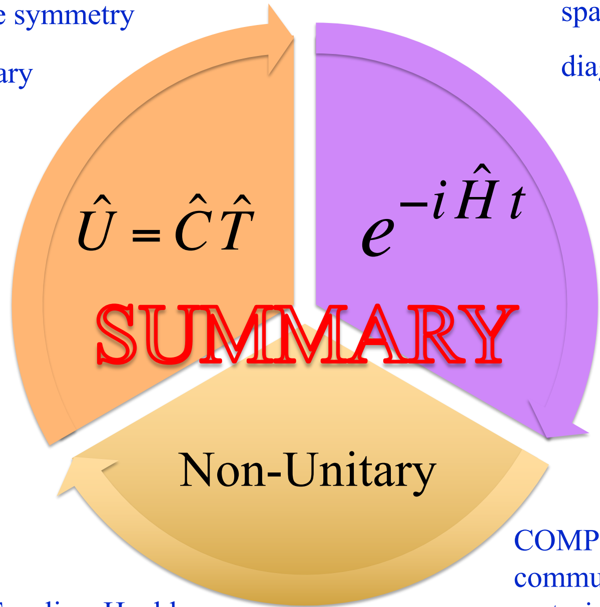


high degree symmetry

sparse unitary

sparse

diagonalizable



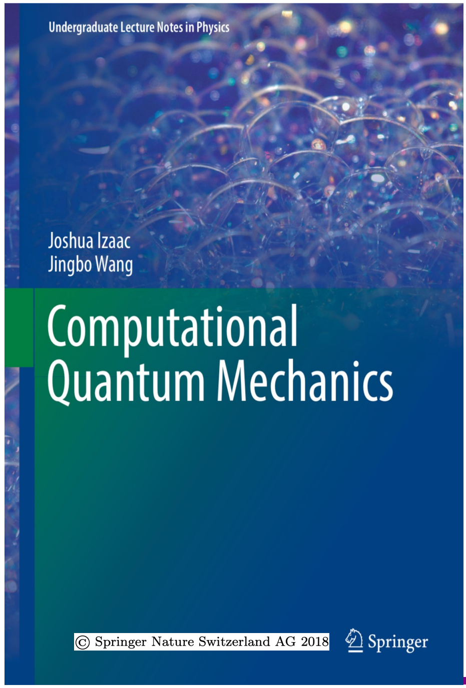
sparse

circulant, Toeplize, Hankle

COMPOSITE

commuting graphs

cartesian product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Questions?