Learning theory in the quantum universe

Hsin-Yuan Huang (Robert)

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• A central goal of science is to learn how our universe operates.



Examples of scientific disciplines

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- world could lead to many advances.



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• Because our universe is inherently quantum, the ability to efficiently learn in the quantum





understand how to design better algorithms to learn in the quantum universe.



A cartoon depiction of learning

• To accelerate and automate the development of (quantum) science, it is important to





- Learning is the combination of:

 - 1. receiving information about the universe, 2. processing that information to form models, **3.** storing the models and, subsequently, **4.** using the models to predict in new scenarios.



A cartoon depiction of learning





Overview

Learning with classical machines

What can classical machines learn? Can classical ML perform better than non-ML algorithms?



How to efficiently learn in the quantum universe?



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Learning with classical machines

What can classical machines learn? Can classical ML perform better than non-ML algorithms?





The big question

How can classical machines "see" quantum many-body systems? \bullet



[HKP20] Hsin-Yuan Huang, Richard Kueng, John Preskill. Predicting many properties of a quantum system from very few measurements, Nature Physics, 2020.



The big question

- What do we mean by "seeing" a quantum system?
- Converting the quantum system to a classical form that accurately captures many properties of the quantum system.



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Standard approach

Quantum state tomography:

Learn the density matrix representation of the *n*-qubit state ρ . $(2^n \times 2^n \text{ PSD matrix with trace 1})$

• Sample-optimal protocol (Haah et al.; O'Donnel, Wright):









Classical storage: $\Omega(2^{2n})$



• Classical post-processing: $\Omega(2^{2n})$

Quantum resource: $\Theta(n2^{2n})$ qubits + exponentially long circuits

Deep learning:

Perform simple quantum measurements.

POVM Neural Network Tomography (Carrasquilla et al.):

- ? Sample complexity: Unknown (could be exponential)
- Quantum resource: Simple quantum circuit + measurements 包
- Classical storage: Unknown (depends on sample complexity) ?
- ? Classical post-processing: Unknown (could be very long)

Recent approach

- Train a <u>neural network</u> to represent the quantum state.

What one wants

Find a provably efficient procedure that from very few measurements (not exponential in n). of the quantum state ρ .

Step 1:

Data Acquisition



Unknown Quantum System

- 1. Learns a classical representation of an unknown *n*-qubit state ρ
- 2. Uses the classical representation to predict many properties



What one wants

Find a provably efficient procedure that

1. Learns a classical representation of an unknown *n*-qubit state ρ

from very few measurements (not exponential in n).

2. Uses the classical representation to predict many properties

of the quantum state ρ .



Step 2:

Prediction

Theorem 1 [HKP20]

Let M = # of properties, B = norm bound, $\epsilon = \text{error}$. \exists a procedure that 1. Learns a classical representation of an unknown *n*-qubit state ρ from $T = \mathcal{O}(B \log(M)/\epsilon^2)$ measurements. 2. Given any O_1, \ldots, O_M with $B \ge Tr(O_i^2)$, the procedure can use the

classical representation to predict $\hat{o}_1, \ldots, \hat{o}_M$, where with high prob., $|\hat{o}_i - \operatorname{tr}(O_i \rho)| < \epsilon$, for all $i = 1, \dots, M$.

For example:

Furthermore, we don't need to know O_1, \ldots, O_M in advance.

• $M = 10^6$, B = 1, then naively we need $10^6/\epsilon^2$ measurements. • This theorem shows that we only need $6\log(10)/\epsilon^2$ measurements.

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Repeat the following T times:

Few Repetitions



Data Acquisition Phase

Repeat the following T times:

• Sample a random Clifford circuit U_i to evolve the system.



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Data Acquisition Phase

Measure the system in the computational basis $|b_i\rangle \in \{0,1\}^n$.



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Data Acquisition Phase

Measure the system in the computational basis $|b_i\rangle \in \{0,1\}^n$. • Store the classical shadow: $\hat{\sigma}_i = (2^n + 1)U_i^{\dagger}|b_i\rangle\langle b_i|U_i - I$.



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Classical shadow The Procedure: $\hat{\sigma}_i = (2^n + 1)U_i^{\dagger} |b_i\rangle \langle b_i | U_i - I$ **Prediction Phase**

Given $S_T(\rho) = {\hat{\sigma}_1, ..., \hat{\sigma}_T},$

Compute $X_i = \operatorname{tr}(O\hat{\sigma}_i), \forall i = 1, \dots, T$.

how to predict properties of the quantum state ρ ?





Classical shadow Proof Sketch: $\hat{\sigma}_i = (2^n + 1)U_i^{\dagger} |b_i\rangle \langle b_i | U_i - I$ Moments

- $b_i \in \{0,1\}^n$



Data Acquisition Phase

• 1st moment $\mathbb{E}[\hat{\sigma}_i]$ corresponds to the 4th moment of random Clifford circuit U_i . $\mathbb{E}[\hat{\sigma}_i] = \mathbb{E}_{U_i} \sum_{i=1}^{n} \langle b_i | U_i \rho U_i^{\dagger} | b_i \rangle \left[(2^n + 1) U_i^{\dagger} | b_i \rangle \langle b_i | U_i - I \right]$

• 2nd moment $\mathbb{E}[\hat{\sigma}_i \otimes \hat{\sigma}_i]$ corresponds to the 6th moment of random Clifford circuit U_i .





Classical shadow **Proof Sketch:** $\hat{\sigma}_i = (2^n + 1)U_i^{\dagger} |b_i\rangle \langle b_i | U_i - I$ Weingarten Calculus

- Weingarten calculus fully characterizes moments of random exp(n)-size circuit. The moments of $SU(2^n)$ correspond to the symmetric group.
- Random Clifford circuit matches random exp(n)-size circuit up to the 6th moments.



Data Acquisition Phase





Classical shadow **Proof Sketch:** $\hat{\sigma}_i = (2^n + 1)U_i^{\dagger} |b_i\rangle \langle b_i | U_i - I$ Concentration

 $|b_1\rangle$

- 1st moment $\mathbb{E}[\hat{\sigma}_i] = \rho$. So $\hat{\sigma}_i \approx \rho$ up to random fluctuations.
- 2nd moment $\mathbb{E}[\hat{\sigma}_i \otimes \hat{\sigma}_i]$ is well-controlled despite the $(2^n + 1)$ factor.
- Median-of-means estimator only cares about the first two moments.

Few Repetitions



Data Acquisition Phase



 $|b_2\rangle$



Theorem (Huang et al.; 2020)

[HKP20] Hsin-Yuan Huang, Richard Kueng, John Preskill. Predicting many properties of a quantum system from very few measurements, Nature Physics, 2020.

We can predict any $O_1, ..., O_M$ with $B \ge Tr(O_i^2)$ to ϵ error from $T = \mathcal{O}(B \log(M)/\epsilon^2)$ measurements.



Theorem (Huang et al.; 2020)

Q: Could this be further improved?

Theorem (Huang et al.; 2020)

To predict any $O_1, ..., O_M$ with $B \ge Tr(O_i^2)$ to ϵ error, we need $T = \Omega(B \log(M)/\epsilon^2)$ measurements.

Proved by relating to quantum communication tasks. • This lower bound applies *only* to learning using classical machines.

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We can predict any $O_1, ..., O_M$ with $B \ge ||O_i||_{\text{shadow}}^2$ to ϵ error from $T = \mathcal{O}(B \log(M)/\epsilon^2)$ measurements.

$$\|O\|_{\text{shadow}} = \max_{\sigma \in S_{2^n}} \left(\mathbb{E}_{U \sim \mathcal{U}} \sum_{b \in \{0,1\}^n} \langle b | U \sigma U^{\dagger} | b \rangle \langle b | U \mathcal{M}^{-1}(O) U^{\dagger} | b \rangle^2 \right)^{1/2}$$

Random Clifford Unitary $U \in Cl(2^n)$

$$\mathcal{M}_{n}(\rho) = (\rho + I)/(2^{n} + 1)$$
$$\mathcal{M}_{n}^{-1}(X) = (2^{n} + 1)X - I$$

Random Clifford Circuits



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The corresponding norm:

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 $||O||_{\text{shadow}} \leq \sqrt{\text{tr}(O^2)}$

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Application: Quantum fidelity $|\psi\rangle\langle\psi|$

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Random Local Clifford $U \in Cl(2)^{\otimes n}$

$$\mathcal{M} = \bigotimes_{i=1}^{n} \mathcal{M}_{1}$$
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Random Pauli Measurement



Theorem (Huang et al.; 2020)

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The corresponding norm: $||O||_{\text{shadow}} \le 2^k ||O||_{\infty}$

Observable O acts on k qubits
Classical shadow formalism

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Application: 2-point correlation, local Hamiltonian.

Classical shadow formalism





[HKP20] Hsin-Yuan Huang, Richard Kueng, John Preskill. Predicting many properties of a quantum system from very few measurements, Nature Physics, 2020.







Benchmarking quantum systems

The ability to estimate quantum fidelity and verify entanglement enables efficient benchmarking.

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- Quantum simulation algorithms often need to estimate many properties (e.g., local observables, Hamiltonian).
 - [c] Zhao et al. *Physical Review Letters* (2021). [d] Huggins et al. Nature (2022).



Benchmarking quantum systems

The ability to estimate quantum fidelity and verify Quantum simulation algorithms often need to estimate entanglement enables efficient benchmarking. Many properties (e.g., local observables, Hamiltonian).

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Quantum chemistry simulation

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Universal quantum-to-classical converter

Enable a variety of classical computational techniques (e.g., ML) for addressing quantum problems.

> [e] Huang et al. Nature Communications (2021). [f] Huang et al. *Science* (2022).

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Classical ML for quantum problems

- And can they yield better solutions than non-ML algorithms?

Physical world

[HKT+21] Huang, Kueng, Torlai, Albert, Preskill. Provably efficient machine learning for quantum many-body problems, Science, 2022.

Can classical machines learn to solve challenging problems in quantum physics?

Classical machine

- Given $x \in [-1,1]^m$ that describes an *n*-qubit Hamiltonian H(x), the machine predicts \bullet a classical representation (e.g., classical shadow) of the ground state $\rho(x)$ of H(x).
- Vector x specifies laser intensities, few-body interactions, magnetic fields, etc.

Computational hardness

- This problem is *extremely* hard!
- Consider a smooth class of *n*-qubit 2D Hamiltonians H(x) with spectral gap 1, and the machine only predicts 1-body observable O in ground state $\rho(x)$.
- Furthermore, we only care about average prediction error.

[HKT+21] Huang, Kueng, Torlai, Albert, Preskill. Provably efficient machine learning for quantum many-body problems, Science, 2022.

1D

Computational hardness

- This problem is *extremely* hard!

• Can classical ML algorithms do something useful for this challenging problem?

K Parameters describing a physical Hamiltonian

Service Advantage Advantag a physical Hamiltonian

[HKT+21] Huang, Kueng, Torlai, Albert, Preskill. Provably efficient machine learning for quantum many-body problems, Science, 2022.

Classical representation of the ground state

 $\mathbf{00}$

Proposition 1

Assuming $RP \neq NP$, then no randomized classical algorithm can achieve an average prediction error $\leq 1/4$ within poly(n) time.

Classical algorithm

2D spectral gap 1 **1-body observable**

average prediction error 1/4

Theorem 1: Improved version from [LHT+23]

A classical ML algorithm can achieve an average prediction error $\leq \epsilon$ using log(n) training data and nlog(n) computational time.

Classical algorithm

2D spectral gap 1 **1-body observable** average prediction error 1/4

[LHT+23] Lewis, Huang, Tran, Lehner, Kueng, Preskill. Improved machine learning algorithm for predicting ground state properties, Submitted, 2023. [HKT+22] Huang, Kueng, Torlai, Albert, Preskill. Provably efficient machine learning for quantum many-body problems, Science, 2022.

Classical ML (trained with data)

any constant dimension any constant spectral gap any local observable any average prediction error $\epsilon = \mathcal{O}(1)$

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We proved that a poly-time classical ML algorithm (w/ data) can predict much better than any poly-time classical algorithm.

Classical algorithm 2D spectral gap 1 **1-body observable** average prediction error 1/4

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Classical ML (trained with data)

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The question **(**): Why ML can be more useful than non-ML algorithms?

[HKT+21] Huang, Kueng, Torlai, Albert, Preskill. Provably efficient machine learning for quantum many-body problems, Science, 2022.

The answer 🔂: Generalizing from data can be easier than computing everything

The question O: Why ML can be more useful than non-ML algorithms?

Data contain computational power (e.g., nature operates quantumly)

The answer 😭 Generalizing from data can be easier than computing everything

[HBM+21] Huang, Broughton, et al. Power of data in quantum machine learning. Nature Communications, 2021. [HKT+21] Huang, Kueng, Torlai, Albert, Preskill. Provably efficient machine learning for quantum many-body problems, Science, 2022.

2D random Heisenberg model

[HKT+21] Huang, Kueng, Torlai, Albert, Preskill. Provably efficient machine learning for quantum many-body problems, Science, 2022.

We consider training data size N = 100, T = 500 randomized measurements for constructing classical shadows. The best ML model is chosen from Gaussian kernel method, infinite-width neural networks, and l_2 -Dirichlet kernel.

Classifying quantum phases: Theorem

Theorem 2

learn to classify these phases.

The assumption is believed to hold for gapped quantum systems.

If there exists a nonlinear function of few-body reduced density matrices for classifying the phases, then the classical ML algorithm can efficiently

1D Symmetry protected topological phases

We consider T = 500 randomized measurements to construct classical shadows for each state. The classical **unsupervised** ML model is a kernel PCA using the shadow kernel.

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Classical agent

Storing [HKP21] Huang, Kueng, Preskill. Information-theoretic bounds on quantum advantage in machine learning, Physical Review Letters, 2021. [CCHL21] Chen, Cotler, Huang, Li. Exponential separations in learning with and without quantum memory, FOCS, 2021. [HBC+] Huang, et all. Quantum advantage in learning from experiments, Science, 2022.

Receive, process, and store classical information

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Physical Measurements

> Classical Computation

Processing

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Quantum agent

Receive, process, and store quantum information

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Transduce from quantum sensors

> Quantum Computation

Processing

Classical vs Quantum

- What are the advantage of a quantum agent over a classical agent?

[HKP21] Huang, Kueng, Preskill. Information-theoretic bounds on quantum advantage in machine learning, Physical Review Letters, 2021. [CCHL21] Chen, Cotler, Huang, Li. Exponential separations in learning with and without quantum memory, FOCS, 2021. [HBC+] Huang, et all. Quantum advantage in learning from experiments, Science, 2022.

Could quantum technology fundamentally alter our ability to learn about the physical world?

Learning a state

- Assume the only unknown that we care about is an *n*-qubit state ρ .
- Classical agents can perform any measurement on ρ in each experiment.
- Quantum agents can obtain and store ρ coherently from each experiment.

What can classical agents do?

- We consider a physical source that could generate a single copy of ρ at a time.

We begin with the simpler task of learning about an unknown physical system ρ (density matrix).

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Classical memory storing data from all POVM

Process all classical data

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Predict properties of the physical system ρ

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Quantum memory storing all copies of ρ

Process all quantum data with quantum computation

Predict properties of the physical system ρ

We begin with the simpler task of learning about an unknown physical system ρ (density matrix).



- The classical/quantum agent learns about the unknown *n*-qubit state ρ .

Theorem

Classical agent needs $\Omega(2^n)$ experiments to predict an adversarially chosen P, but quantum agent only needs $\mathcal{O}(n)$ experiments to predict all 4^n observables.

- The classical/quantum agent learns about the unknown *n*-qubit state ρ .

Theorem

Classical agent needs $\Omega(2^n)$ experiments to predict an adversarially chosen P, but quantum agent only needs $\mathcal{O}(n)$ experiments to predict all 4^n observables.

Exponential quantum advantage is present even when the state ρ is a classical distribution over product states (no entanglement!).

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Uncertainty principle significantly hinders the learning ability of classical agents, but surprisingly not the ability of a quantum agent.



Proof Sketch: Tree representation

- Consider the lower bound $\Omega(2^n)$ for classical agents.





• We consider a graphical representation of the memory state of the classical agent.

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- We reduce the prediction task to a many-vs-one distinguishing task.
- Prediction task: Estimate $Tr(P\rho)$ to 1/4 error for all $P \in \{I, X, Y, Z\}^{\otimes n} \setminus \{I^{\otimes n}\}$.
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Information-theoretic bound

- the probability over leaf nodes.



Many-versus-one distinguishing task

• Succeeding in the many-vs-one distinguishing task implies a lower bound on the TV in

• We then upper bound TV with a function of the number of experiments (i.e., samples).





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Uncertainty principle significantly hinders the learning ability of classical agents, but surprisingly not the ability of a quantum agent.

- The classical/quantum agent learns about the unknown *n*-qubit state ρ .
- Subsequently, the agent predicts $Tr(O_i\rho)$ from a known set O_1, \ldots, O_M .

Theorem

Classical agent needs $\tilde{\Omega}(\min(M,2^n))$ experiments to predict the M observables, but quantum agent only needs $O(n \log^2 M)$ experiments to predict the observables.

Quantum agent uses the truly-quantum shadow tomography [Badescu, O'Donnel]: "Online learning" + "Quantum threshold search."

Predicting many incompatible observables

To predict all Pauli observables $\{I, X, Y, Z\}^{\otimes n}$, classical agent needs $\Omega(2^n)$ experiments, quantum agent only needs $\mathcal{O}(n)$ experiments.



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Performing quantum PCA

To estimate property of principal component, classical agent needs exponential time, quantum agent needs polynomial.

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Classifying dynamics with or without time-reversal symmetry gives exponential separations



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Learning physical dynamics

To learn a polynomial-time quantum process, a classical agent requires exponential experiments, a quantum agent only needs polynomial experiments.

Quantum advantage in NISQ

Yes! Rigorous analysis in [HFP22], Experiments in [HBC+].



[HFP22] Huang, Flammia, Preskill. Foundations for learning from noisy quantum experiments, QIP, 2022. [HBC+] Huang, Broughton, Cotler, Chen, Li, Mohseni, Neven, Babbush, Kueng, Preskill, McClean. Quantum advantage in learning from experiments, Science, 2022.

Do these quantum advantages persist in noisy quantum computers?



Utilizing a total of 40 qubits



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Conventional experiments











Conventional experiments



Quantum-enhanced experiments





Apply Clifford rotation randomly (record the rotation)

basis measurement

Computational



Conventional experiments















 ρ









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Conventional experiments



Quantum-enhanced experiments




Conventional experiments



& Quantum-enhanced experiments





Expectation values of all Pauli-Y operators will be zero (if T-symmetry holds)

Comp. Basis Meas.

Conventional experiments



Quantum-enhanced experiments





Rotation to the Bell basis



Quantum-enhanced experiments





satisfies T-symmetry

Comp. Basis Meas.

Repeat multiple times





Predict whether dynamic U satisfies T-symmetry





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Overview



How to efficiently learn in the quantum universe?



Overview



Classical machines \ll Classical learning machines \ll Quantum learning machines (predicting ground states) (uncovering symmetry, ...)

How to efficiently learn in the quantum universe?

Conclusion

- Significant recent progress in understanding how to learn in the quantum universe. But most on lower-level tasks (e.g., predicting properties).
- How to create rigorous ML algorithms for higher-level tasks: designing quantum circuits / protocols / algorithms, discovering new physics?



Long-term ambitions

- Could we develop an algorithmic theory to accelerate/automate (quantum) science and the discovery of new physical phenomena?
- Could we build a quantum machine capable of learning and discovering new facets of our universe beyond humans and classical machines?



Al imagination of itself learning and discovering new facets of our quantum universe (Credit: DALL·E)