Observations on Equation-Based Rate Control*

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The Control that We Study

Data send rate set as:

\[ X_n = f(\tilde{p}_n) \]
\[ X(t) = X_n, \; T_n < t \leq T_{n+1} \]

We call it basic control.

- \( \{T_n\} \) a point process on \( \mathbb{R} \); \( S_n \overset{\text{def}}{=} T_{n+1} - T_n \)
- \( \tilde{p} \), long-run loss event ratio
  \[ \tilde{p} = \lim_{t \to \infty} \frac{\sum_{k>0} 1_{[0,t)}(T_k)}{\int_0^t X(s)\,ds} \]
- \( \hat{p}_n \), estimator of \( \tilde{p} \) at \( T_n \)
- \( f : [0,1] \to \mathbb{R}^+ \), non-increasing
- \( f \) is typically TCP loss-throughput function; it is also function of some statistics of round-trip time; not considered here
The Control that We Study (cont’d)

Comprehensive control – the basic control, plus: if at $t$, the amount of data sent since the most recent loss event, $\theta(t)$, would increase the value of the estimator $\hat{\theta}(t)$, then use it as a sample

Sample-Paths:

![Basic Control Diagram]

![Comprehensive Control Diagram]
What is the Problem?

P: Is it true

\[ \mathbb{E}[X(0)] \leq \mathbb{E}[X_{tcp}(0)] \]

Here \( \mathbb{E}[X_{tcp}(0)] \) is TCP throughput under the same operating conditions. If yes, we say the control is TCP-friendly.

We subdivide P into three subproblems:

P1:

\[ \mathbb{E}[X(0)] \leq f(\bar{p}) \]

If yes, we say the control is conservative.

P2: Does it hold

\[ \bar{p} \geq \bar{p}_{tcp} \]

P3: Does TCP satisfy its equation,

\[ \mathbb{E}[X_{tcp}(0)] = f(\bar{p}_{tcp}) \]
Why is the Problem of Interest?

- Several equation-based rate controls proposed with various definitions of \( \{T_n\}, \hat{p}_n \)

  **Ex.** See:
  
  http://www.psc.edu/networking/tcp_friendly.html
  
  http://www.icir.org/tfrc

- Common objective:
  
  - smooth send rate dynamics, but still responsive to sustained congestion

  - TCP-friendly: the long-run throughput less than or equal to TCP throughput under the same operating conditions
Two Additional Assumptions

(A1) \( \{T_n\} \) is point process of loss events

(A2) \( \hat{p}_n = \frac{1}{\hat{\theta}_n}, \hat{\theta}_n = \sum_{l=1}^{L} w_l \theta_{n-l} \)

- \((w_l)_{l=1}^{L}\) positive-valued, \(\sum_{l=1}^{L} w_l = 1\)

- \(\theta_n\) is amount of data sent in \([T_n, T_{n+1})\); we call it as in TFRC: loss-event interval

Ob. \(1/\hat{p}_n\) is unbiased estimator of \(1/\bar{p}\)
Some Functions $f$

**SQRT:**

$$f(p) = \frac{1}{c_1 R \sqrt{p}}$$

**PFTK-standard:**

$$f(p) = \frac{1}{c_1 R \sqrt{p} + Q \min[1, c_2 \sqrt{p}] \left(p + 32p^3\right)}$$

**PFTK-simplified:**

$$f(p) = \frac{1}{c_1 R \sqrt{p} + Q c_2 \left(p^{3/2} + 32p^{7/2}\right)}$$

- $R$: expected round-trip time
- $Q$: expected retransmit timeout
- $c_1, c_2$: positive-valued constants
We Will See

1. Throughput Representation

2. What Makes the Basic Control Conservative or Not?
   - For the basic control: two sets of conditions for either conservative or non-conservative control
   - Suggestions of the analytical results, validation by numerical and ns-2 experiments

3. We expect $\bar{p} \geq \bar{p}_{tcp}$.

4. It may be $\mathbb{E}[X(0)] > \mathbb{E}[X_{tcp}(0)]$, even though $\mathbb{E}[X(0)] \leq f(\bar{p})$, and $\bar{p} \geq \bar{p}_{tcp}$.

5. Conclusion
Throughput Representation

Palm inversion formula:

\[ \mathbb{E}[X(0)] = \lambda \mathbb{E}_T[\int_0^{S_0} X(s)ds] \]

- \( \lambda = 1/\mathbb{E}_T[S_0] \)
- \( \mathbb{E}_T \) is expectation w.r.t. Palm probability (given a point at 0; \( T_0 = 0 \))

We suppose stability, i.e. \( \{X_n\} \) and \( \{S_n\} \) are stationary ergodic
Throughput Representation (cont’d)

Basic control:

\[ \mathbb{E}[X(t)] = \frac{\mathbb{E}[\theta_n]}{\mathbb{E}\left[\frac{\theta_n}{f(1/\theta_n)}\right]} \]

Comprehensive control (PFTK-simplified):

\[ \mathbb{E}[X(t)] \leq \frac{\mathbb{E}[\theta_n]}{\mathbb{E}\left[\frac{\theta_n}{f(1/\theta_n)}\right] - \mathbb{E}[V_n1_{\theta_{n+1}>\theta_n}]} \]

\[ V_n = \frac{1}{w_i} \left[ -2c_1R(\bar{\theta}_{n+1}^{\frac{1}{2}} - \bar{\theta}_{n}^{\frac{1}{2}}) + 2c_2Q(\bar{\theta}_{n+1}^{\frac{3}{2}} - \bar{\theta}_{n}^{\frac{3}{2}}) - \\
\quad + 2c_3Q(\bar{\theta}_{n+1}^{-\frac{5}{2}} - \bar{\theta}_{n}^{-\frac{5}{2}}) + (\bar{\theta}_{n+1} - \bar{\theta}_n)\frac{1}{f(1/\theta_n)} \right] \]

**Ob.** From the joint law of \( \theta_n, \ldots, \theta_{n-L} \) one may compute the throughput
First Set of Sufficient Conditions for Conservativeness

Assume

(F1) \( \frac{1}{f(1/x)} \) is convex with \( x \)

(C1) \( \text{Cov}[\theta_n, \hat{\theta}_n] \leq 0 \)

Then, the basic control is conservative.

Moreover,

\[
\mathbb{E}[X(t)] \leq f(\bar{p}) \frac{1}{1 + \frac{f'(\bar{p})\bar{p}^3}{f(\bar{p})}\text{Cov}[\theta_n, \hat{\theta}_n]}
\]

Ob. \( \text{Cov}[\theta_n, \hat{\theta}_n] = \sum_{i=1}^{L} w_i \text{Cov}[\theta_n, \theta_{n-i}] \)

Ex. (C1) is indeed true for \( \{\theta_n\} \) renewal process (i.i.d.)
For SQRT and PFTK-simplified, \( \frac{1}{f(1/x)} \) is Convex
For PFTK-standard, $\frac{1}{f(1/x)}$ is Almost Convex

- Note: $g(x) \overset{def}{=} \frac{1}{f(1/x)}$, $g^*$ is convex conjugate of $g$
First Suggestion

Assume $\theta_n$ and $\hat{\theta}_n$ are negatively or lightly correlated. Consider the function $f$ in the region where $\hat{\theta}_n$ takes values.

1. The more convex $\frac{1}{f(1/x)}$, the more conservative the control is.

2. The more variable $\hat{\theta}_n$, the more conservative the control is.
First Suggestion: Numerical Example

\{\theta_n\} i.i.d. with generalized exponential density

\[ f_{\theta_n}(x) = \lambda \exp(-\lambda(x - x_0)), \ x \geq x_0, \ \lambda, x_0 \geq 0 \]

- \( \mathbb{E}[\theta_n] = x_0 + \lambda \)

- Coeff. of variation: \( \frac{\lambda}{\sqrt{\lambda + x_0}} \)

- Skewness: 2

- Kurtosis: 6
**First Suggestion: Numerical Example**

**SQRT**

**PFTK-simplified**

**Ob.** PFTK-simplified: the larger \( \bar{p} \), the more convex \( \frac{1}{f(1/x)} \), the more conservative the control.
First Suggestion: Numerical Example
(PFTK-simplified, fixed \( \bar{p} \))

\[ \bar{p} = 0.01 \]

\[ \bar{p} = 0.1 \]

\textbf{Ob.} The more variable \( \bar{\theta}_n \), the more conservative the control.
First Suggestion: Numerical Example  
(Comprehensive Control)

**SQRT**

**PFTK-simplified**

Ob. Qualitatively the same as for the basic control, but somewhat less pronounced.
First Suggestion: ns-2 Experiment (TFRC, PFTK-standard)

Setting: single RED link shared by TFRC and TCP-Sack1 connections (link capacity 15 Mb/s, round-trip time about 50 ms)
First Suggestion: ns-2 Experiment (TFRC, SQRT)
Second Set of Sufficient Conditions for Conservativeness

Assume

(F2) $f(1/x)$ is concave with $x$

(C2) $\text{Cov}[X_n, S_n] \leq 0$

Then, the basic control is conservative.

Conversely, if

(F2') $f(1/x)$ is convex with $x$

(C2') $\text{Cov}[X_n, S_n] \geq 0$

(V) $\hat{\theta}_n$ has non-zero variance

Then, the basic control is non-conservative.
When $f(1/x)$ is concave or convex?

- **SQRT**: $f(1/x)$ concave
- **PFTK**: $f(1/x)$ concave for light loss, but convex for heavy loss
Importance of Feller’s Paradox Type of Effects

Feller’s Paradox: for a point process, average interval between two points seen by a random observer is larger than as seen at the interval boundaries.

Recall (C2): $\text{Cov}[X_0, S_0] \leq 0$

Thus, by Palm inversion formula:

$$\mathbb{E}[X(0)] = \mathbb{E}_T[X(0)] + \frac{\text{Cov}[X_0, S_0]}{E_T[S_0]}$$

it follows (C2) $\Rightarrow \mathbb{E}[X(0)] \leq \mathbb{E}_T[X(0)]$

Interpretation:

1. a random observer would more likely pick larger interval $S_n$

2. (C2) implies, on average, she would see smaller rate than as seen at $\{T_n\}$
When (C2) is True?

**Ob.** $\mathbb{E}[S_n | X_n = x]$ non-increasing with $x \Rightarrow \text{Cov}[X_n, S_n] \leq 0$ (C2)

**Ex.** If $\theta_n$ is independent of data send rate $X_n$, then

$$\mathbb{E}[S_n | X_n = x] = \frac{1}{px}$$

Thus, (C2) holds.
Second Suggestion

- Assume $S_n$ and $X_n$ are negatively or non correlated.
  
  If $f(1/x)$ is concave in the region where $\hat{\theta}_n$ takes its values, the control tends to be conservative.

- Conversely, assume $S_n$ and $X_n$ are positively or non correlated.
  
  If $f(1/x)$ is convex in the region where $\hat{\theta}_n$ takes its values, the control is non conservative.

In both cases, more variable $\hat{\theta}_n$ is, stronger the effect.
Second Suggestion: ns-2 Experiment

\[ L = 4 \]

Setting: a rate controlled source with fixed packet send rate (each 20 ms) through a single loss link (fixed drop probability independent of the packet size)
Second Suggestion: ns-2 Experiment

$L = 8$

**Ob.** Qualitatively the same as for $L = 4$, but the effects less pronounced
Third Suggestion (Loss Event Ratios Seen by Sources)

The loss event ratios for TCP, our adaptive equation based rate controlled source (A), and a non-adaptive source (P) (Poisson) should be in the relation

\[(*) \quad \bar{p}_{tcp} \leq \bar{p}_A \leq \bar{p}_P\]

The more responsive source A is, the closer $\bar{p}_A$ should be to $\bar{p}_{tcp}$.

\textbf{Ob.} If $\mathbb{E}[X(0)] \leq f(\bar{p}_A)$, and $(*)$, then

$$\mathbb{E}[X(0)] \leq f(\bar{p}_{tcp})$$

(Conservativeness implies TCP-friendliness, for a TCP source that attains throughput $\geq f(\bar{p}_{tcp})$, with equality if $f$ is accurate loss-throughput function of the given TCP)
Third Suggestion: ns-2 Experiment
\( f \) may be Inaccurate Loss-throughput Function

**Ob.** TCP-Sack1 does not verify PFTK loss-throughput function \( f \)
**f** may be Inaccurate Loss-throughput Function

**Ob.** Control may be not friendly to the given TCP, even though being conservative and friendly to the function \( f \)

**Ob.** This is NOT problem of the control, but merely due to inaccuracy of \( f \)
Conclusion

Two causes of $\mathbb{E}[X(t)] \neq f(\bar{p})$:

- time versus event averages

- convexity properties of $f(1/x), 1/f(1/x)$

Important to separate:

- conservativeness

- this control loss event ratio versus TCP loss event ratio

- obedience of TCP to given function $f$
Pointers


Available at: http://lcawww.epfl.ch
Appendix

Comparison of (Non-)Conservativeness Conditions

(F1) \( \frac{1}{f(1/x)} \) convex \hspace{1cm} (F2) \( f(1/x) \) concave

(F1') \( \frac{1}{f(1/x)} \) concave \hspace{1cm} (F2') \( f(1/x) \) convex

(F2) \( \Rightarrow \) (F1)
Comparison of (Non-)Conservativeness Conditions (cont’d)

\((C1)\) \(\text{Cov}[\theta_n, \tilde{\theta}_n] \leq 0\) \hspace{1cm} \((C2)\) \(\text{Cov}[X_n, S_n] \leq 0\)

\((C1')\) \(\text{Cov}[\theta_n, \tilde{\theta}_n] > 0\) \hspace{1cm} \((C2')\) \(\text{Cov}[X_n, S_n] > 0\)

\((C2) \Leftrightarrow \)

\(\text{Cov}[\theta_n, \frac{1}{f(1/\tilde{\theta}_n)}] \geq \mathbb{E}[\theta_n] \left( \frac{1}{\mathbb{E}[f(1/\tilde{\theta}_n)]} - \mathbb{E}[\frac{1}{f(1/\tilde{\theta}_n)}] \right)\)

\((C2') \Leftrightarrow \)

\(\text{Cov}[\theta_n, \frac{1}{f(1/\tilde{\theta}_n)}] < \mathbb{E}[\theta_n] \left( \frac{1}{\mathbb{E}[f(1/\tilde{\theta}_n)]} - \mathbb{E}[\frac{1}{f(1/\tilde{\theta}_n)}] \right)\)

**Ob.** The RHS is negative.

Assume \(g(x) = \frac{1}{f(1/x)}\) non-increasing convex.

\((C2) \Leftrightarrow \text{Cov}[\theta_n, \tilde{\theta}_n] \leq \frac{\mathbb{E}[\theta_n]}{g'(\mathbb{E}[\theta_n])} \left( \frac{1}{\mathbb{E}[f(1/\tilde{\theta}_n)]} - \frac{1}{f(1/\mathbb{E}[\theta_n])} \right)\)

\((C2') \Rightarrow \text{Cov}[\theta_n, \tilde{\theta}_n] > \frac{\mathbb{E}[\theta_n]}{g'(\mathbb{E}[\theta_n])} \left( \frac{1}{\mathbb{E}[f(1/\tilde{\theta}_n)]} - \frac{1}{f(1/\mathbb{E}[\theta_n])} \right)\)

**Ob.** For \(f(1/x)\) convex with \(x\), the RHS is positive. In this case, if \((C2')\) holds, then necessarily \((C1')\) holds.
Example of Non-Conservative Control

Assume:

- $\{\theta_n, Z_n\}$ is semi-Markov process

- $[p_{ij}]$ transition matrix of DTMC $\{Z_n\}$

- $P(Z_{n+1} = j, \theta_n = m|Z_n = i) = p_{ij}g_i(m)$

Consider:

- $\{Z_n\}$ two-state DTMC with state space $\{g, b\}$

- periodic losses while in a given state; $P(\theta_n = n_g|Z_n = g) = 1$ and $P(\theta_n = n_b|Z_n = b) = 1$
Example of Non-Conservative Control
(cont’d)

Slow MC limit, $p_{gb}, p_{bg} \to 0$, $\frac{p_{gb}}{p_{bg}} = u$,

$$
\mathbb{E}[X(0)] \to \frac{p_{bg} n_g + p_{gb} n_b}{p_{bg} f(1/n_g) + p_{gb} f(1/n_b)}
$$

Maximum attained for $\frac{p_{bg}}{p_{gb}} = \sqrt{\frac{n_b}{n_g}}$; maximum normalized throughput $(\mathbb{E}[X(0)]/f(\bar{p}))$,

$$
x^* = \frac{1}{2} \sqrt{\frac{n_g}{n_b} + \frac{1}{\sqrt{\frac{n_g}{n_b}}}}
$$
Example of Non-Conservative Control
(cont’d)

$\mathbb{E}[X(0)]/f(\bar{p})$ vs. $p_{gb}$ and $p_{bg}$ (Basic Control)
Example of Non-Conservative Control (cont’d)

$E[X(0)]/f(\tilde{p})$ vs. $p_{gb}$ and $p_{bg}$ (Comprehensive Control)
Example of Non-Conservative Control (cont’d)

Maximum $\mathbb{E}[X(0)]/f(\bar{p})$ for slow MC limit ($x^*$)