Energy and Enstrophy Cascades in Weather and Climate Models

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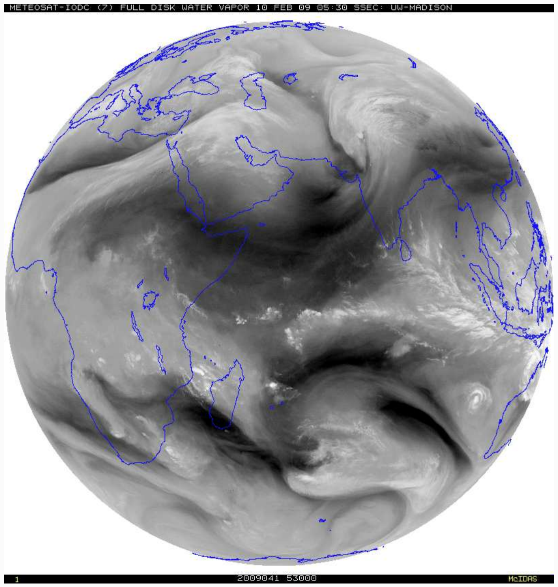
IPAM Workshop on Numerical Hierarchies for Climate Modeling,

April 2010

- Energy and (potential) enstrophy are conserved by the adiabatic, frictionless governing equations...

- ...but nonlinearity leads to systematic transfers between scales



Meteosat water vapour image

- How well do numerical models handle those transfers, especially near the truncation limit?

Outline

- Potential enstrophy and energy cascades
- The need to remove potential enstrophy without dissipating too much energy
- BVE as a model problem

    Effect of unresolved scales on enstrophy and energy spectra

    Effect of some numerical schemes on enstrophy and energy spectra

    Parameterization of energy backscatter

Turbulence theory

For BVE or QG turbulence,

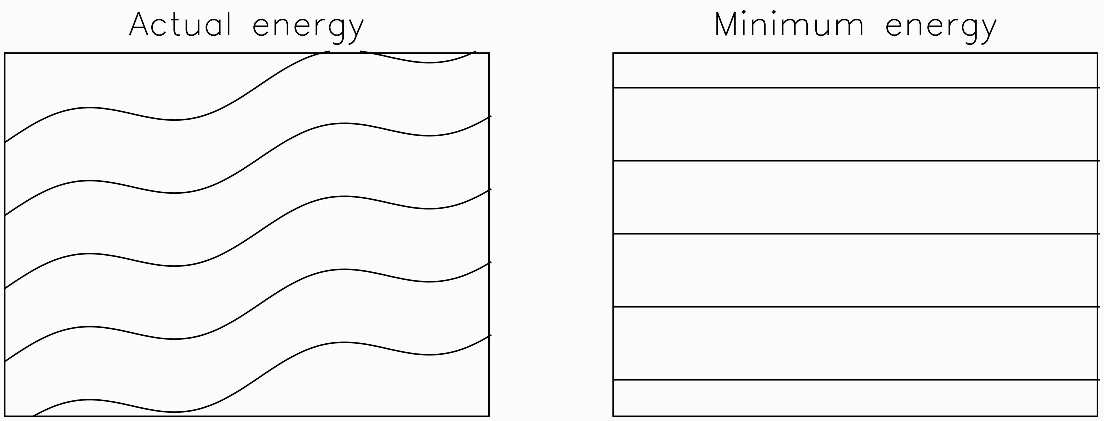
energy is transferred mainly upscale,

(potential) enstrophy is transferred mainly downscale.

(Initial value problem or forced dissipative statistically steady state.)

Energy

Total energy ($\sim 3 \times 10^9 \mathrm{Jm}^{-2}$) is made up of available and unavailable contributions.



	Unavailable PE	Available PE	KE
Ratio:	2000	4	1

Unavailable and available energy are separately conserved.

Unavailable energy is a function of the mass per unit θ - almost robust.

Can conserve mass in each model isentropic layer by using θ as a vertical coordinate.

There is some evidence that about 5-10% of available energy cascades downscale in the free atmosphere. (Mechanism?)

The rest goes upscale before being dissipated mainly by the boundary layer.

Numerical representation of energy and potential enstrophy transfers

Typically either

(a) use conservative numerics supplemented by some scale-selective dissipation such as $\kappa\nabla^{2n}$

or

(b) use inherently dissipative numerics such as semi-Lagrangian or non-oscillatory finite volume (ILES).

Note the multiple roles of scale-selective dissipation.

Implicit Large Eddy Simulation (ILES)

Finite resolution => need to represent effects of unresolved scales: SG model.

At the same time, all numerical methods have truncation errors.

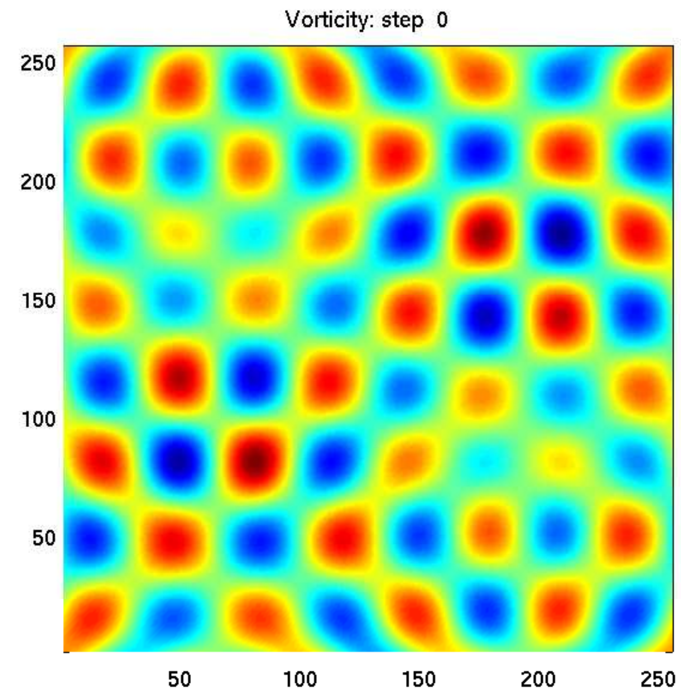
Can truncation errors play the role of a SG model?

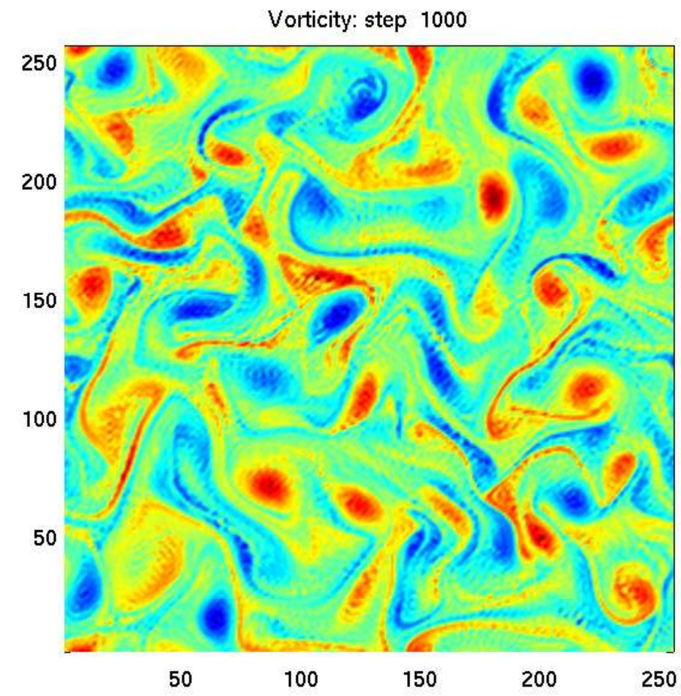
Some success claimed for 3D turbulence. (Except when upscale effects are important, e.g. near a wall.)

What about (layerwise) 2D turbulence?

Upscale energy transfers, but steeper spectrum so stronger slaving of small scales to large.

What if we don't remove resolved enstrophy?





Evidence that models dissipate too much energy

Frederiksen et al. (JAS 1996): Effect on spectra of various dissipation schemes.

Williamson (WGNE 2003): Numerical and scale-selective dissipation comparable to boundary layer.

Shutts (QJRMS 2005): Reviews various pieces of evidence.

Bowler et al. (QJRMS 2009): Estimate of dissipation due to semi-Lagrangian interpolation.

If we remove enstrophy at horizontal wavenumber k at a rate $\dot{Z}$ then we necessarily remove KE at a rate $\dot{E} = \dot{Z}/k^2 \geq \dot{Z}/k^2_{\max}$.

At current climate resolutions this is too large.

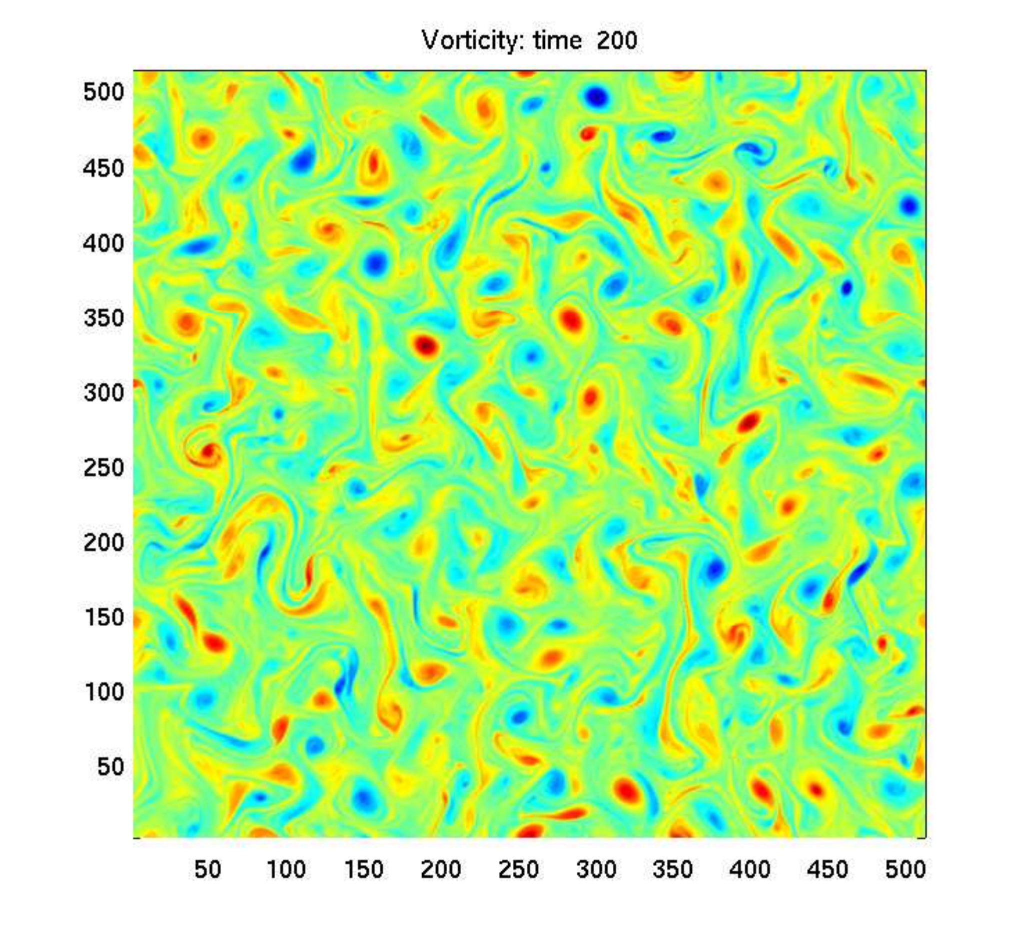
What does ILES or any explicit SG model need to capture?

Barotropic vorticity equation as model problem:

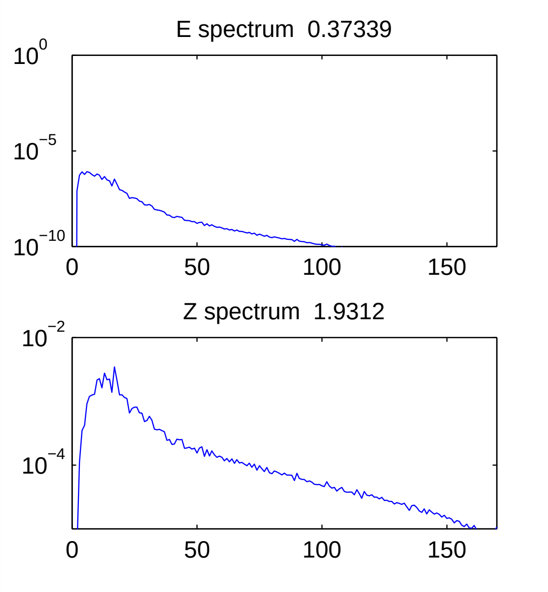
$$\frac{D\zeta}{Dt} = 0; \qquad \nabla^2\psi = \zeta; \qquad \mathbf{v} = \nabla^{\perp}\psi$$

Statistically steady turbulence

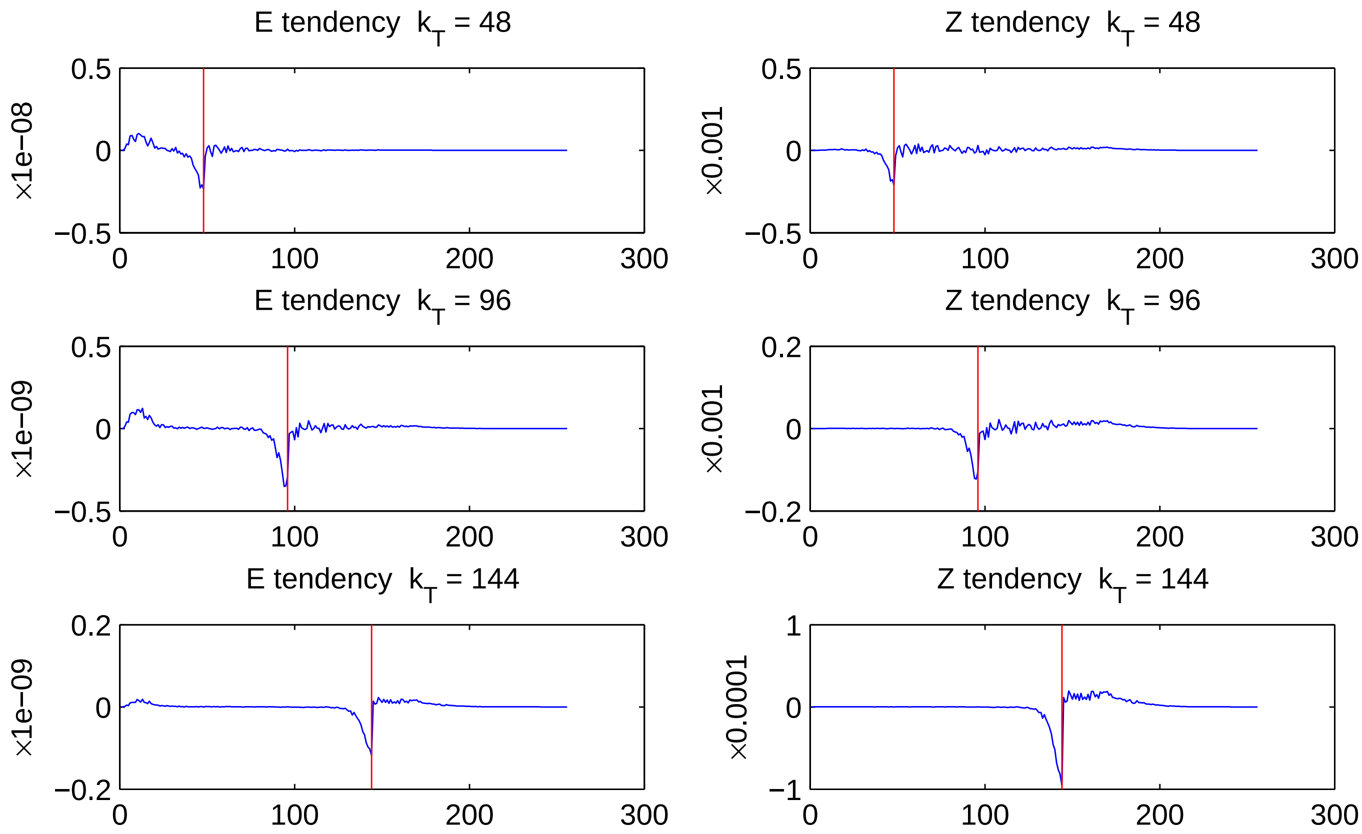
$t = 200$



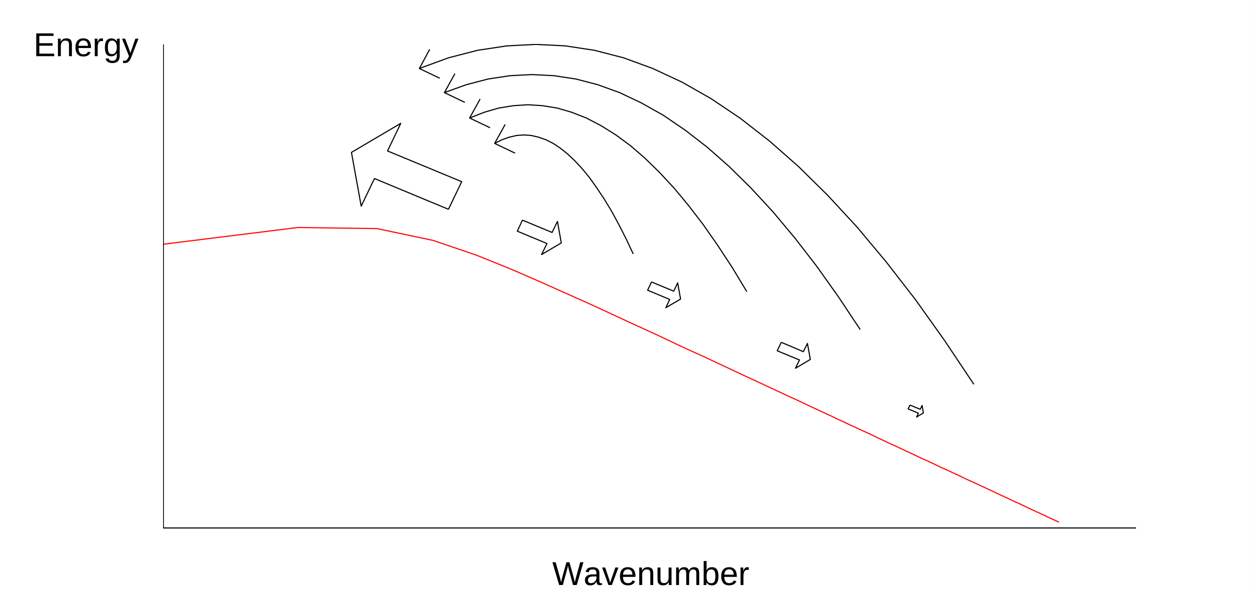
Forcing at $n = 16$; scale-independent dissipation; and ∇^8 small-scale dissipation.



Spectral interactions associated with truncated scales

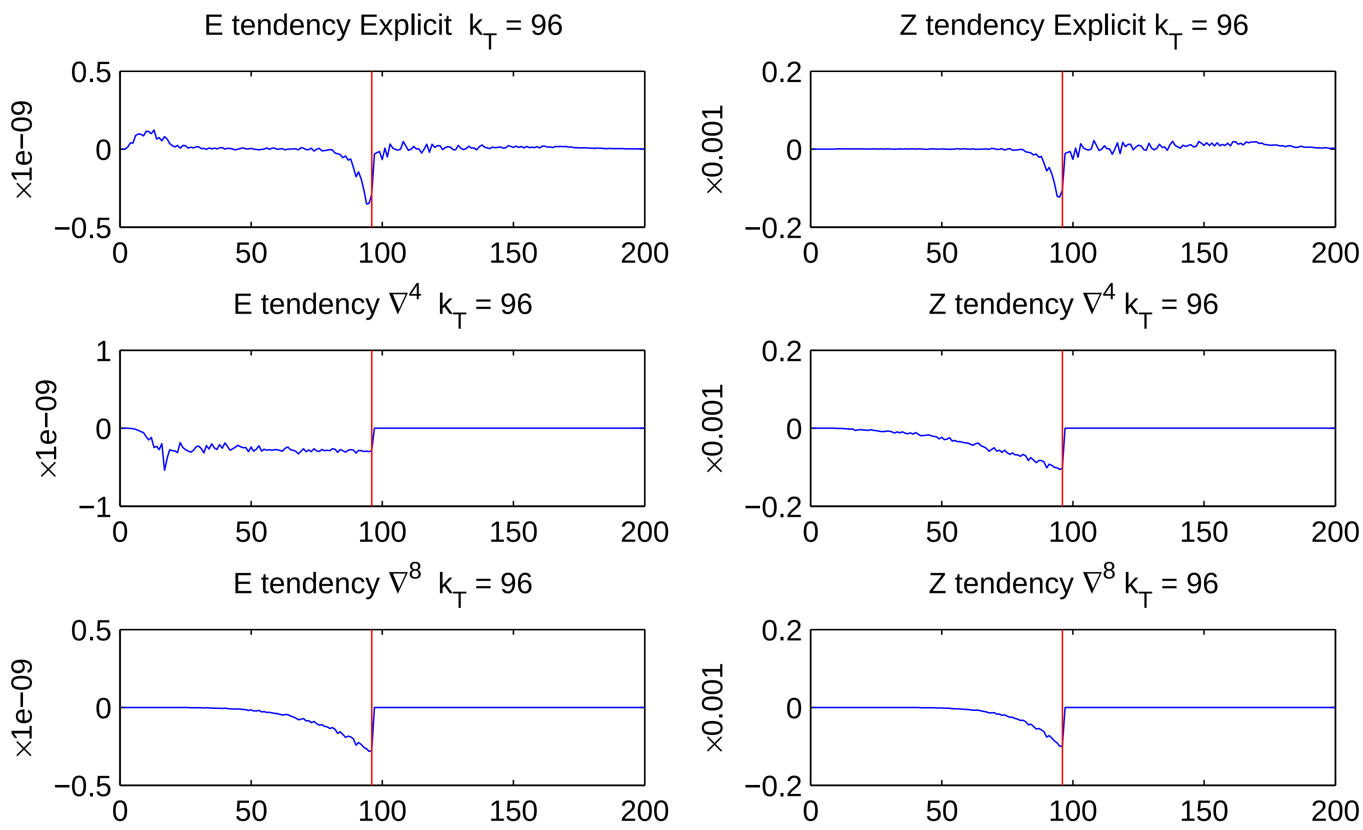


Schematic of energy transfers



Cascade local in k; backscatter nonlocal.

Spectral interactions as represented by ∇^4 and ∇^8



Anticipated Potential Vorticity Method

Sadourny and Basdevant (1985).

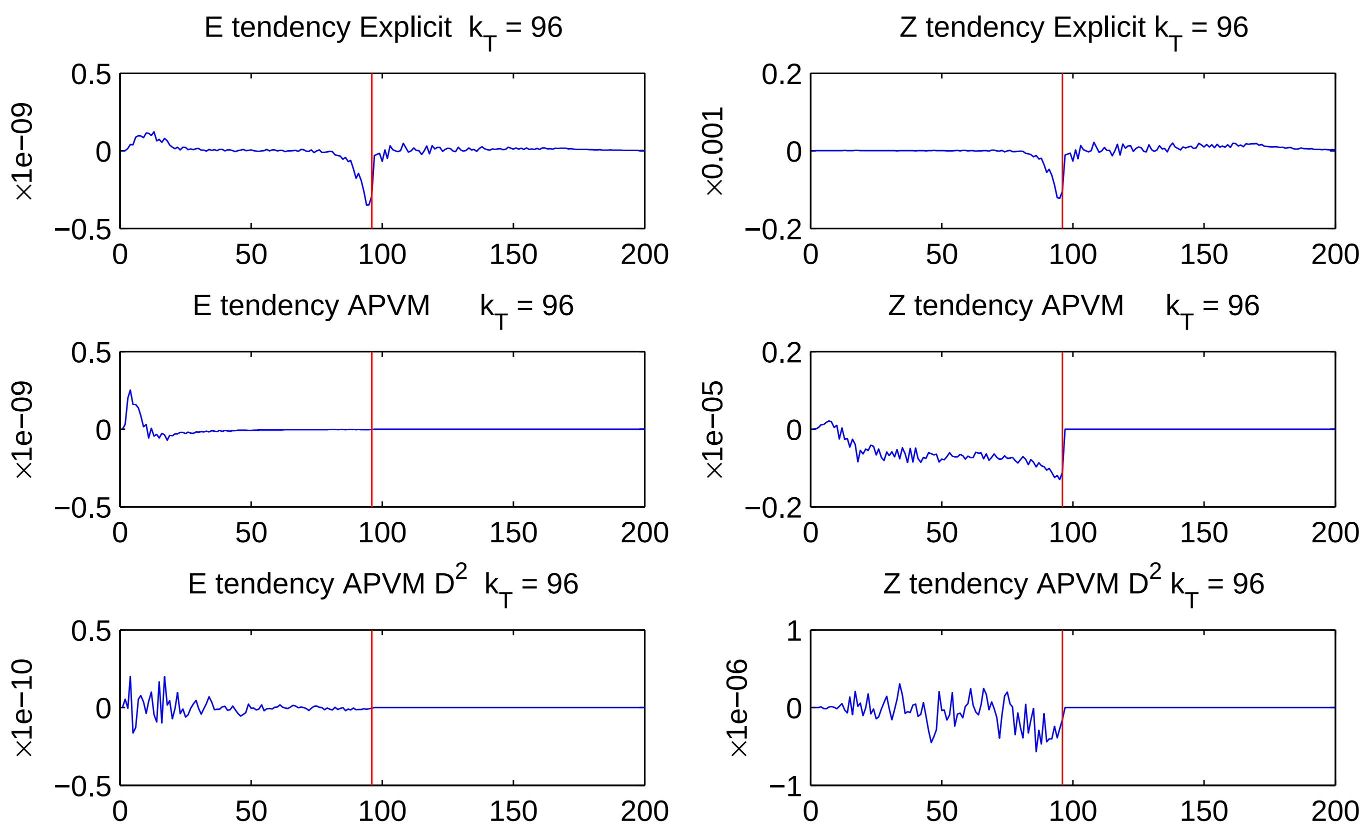
$$\frac{\partial \mathbf{v}}{\partial t} + (\zeta - D)\hat{\mathbf{k}} \times \mathbf{v} + \nabla\left(p + \frac{\mathbf{v}^2}{2}\right) = 0$$

$$\frac{\partial \zeta}{\partial t} + \nabla.(\mathbf{v}\zeta) = \nabla.(\mathbf{v}D)$$

Choose $D = \theta\mathcal{L}(\mathbf{v}.\nabla\zeta)$. Here $\mathcal{L} \equiv 1$ or $\mathcal{L} \equiv -\nabla^2$

$$\dot{Z} = -\theta \int (\mathbf{v}.\nabla\zeta)^2 \, dA \quad \text{or} \quad \dot{Z} = -\theta \int \left(\nabla(\mathbf{v}.\nabla\zeta)\right)^2 \, dA$$

Spectral interactions as represented by APVM



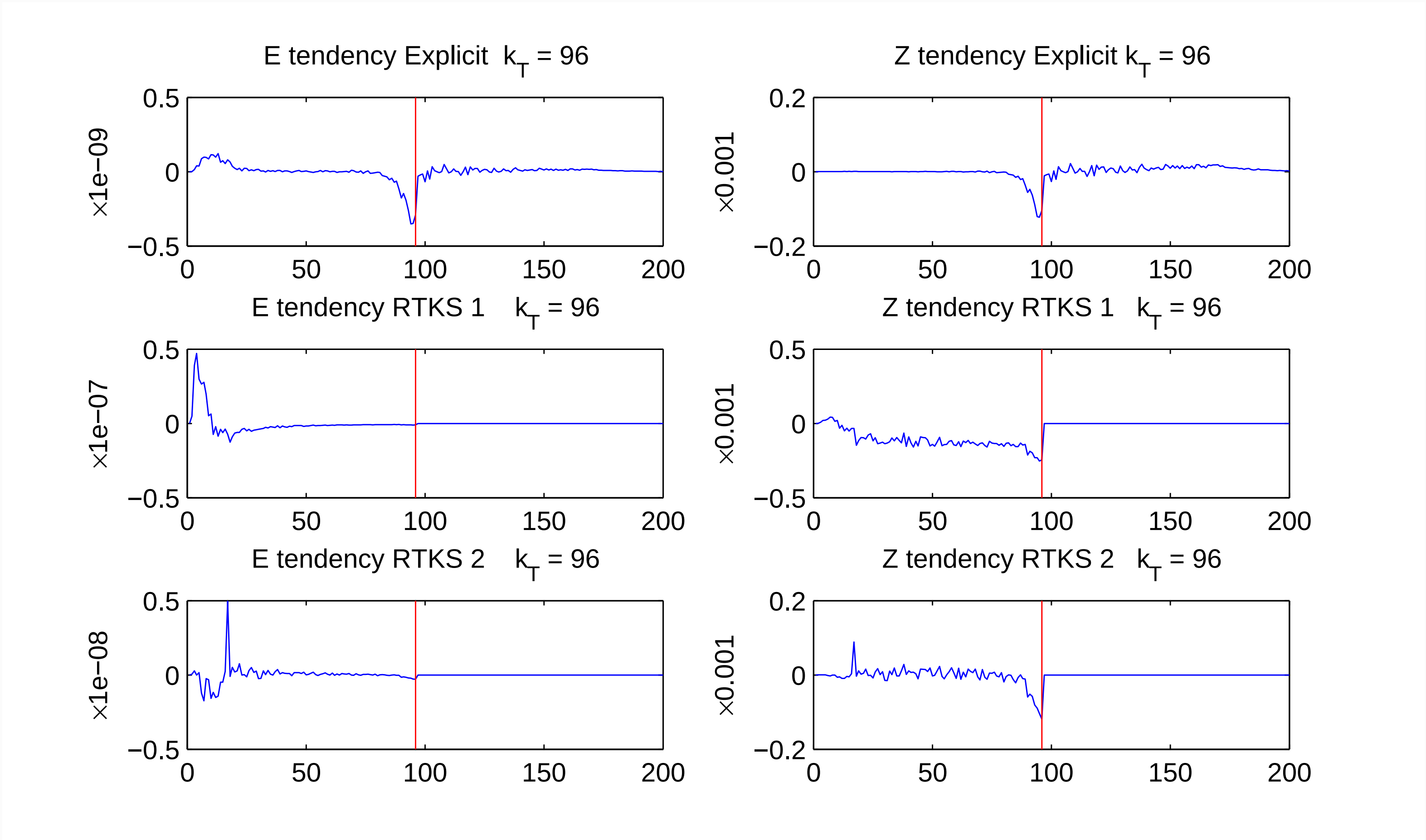
Ringler et al. scheme

RTKS (J. Comput. Phys. 2010 in press) developed a family of energy conserving C-grid schemes for the SWEs on arbitrary polygonal grid cells.

$$\frac{\partial \mathbf{v}}{\partial t} + \zeta \hat{\mathbf{k}} \times \mathbf{v} + \nabla \left(\Phi + \frac{\mathbf{v}^2}{2} \right) = \mathbf{0}$$

Can arrange for potential enstrophy to be dissipated.

Spectral interactions as represented by RTKS scheme



UTOPIA advection of ζ

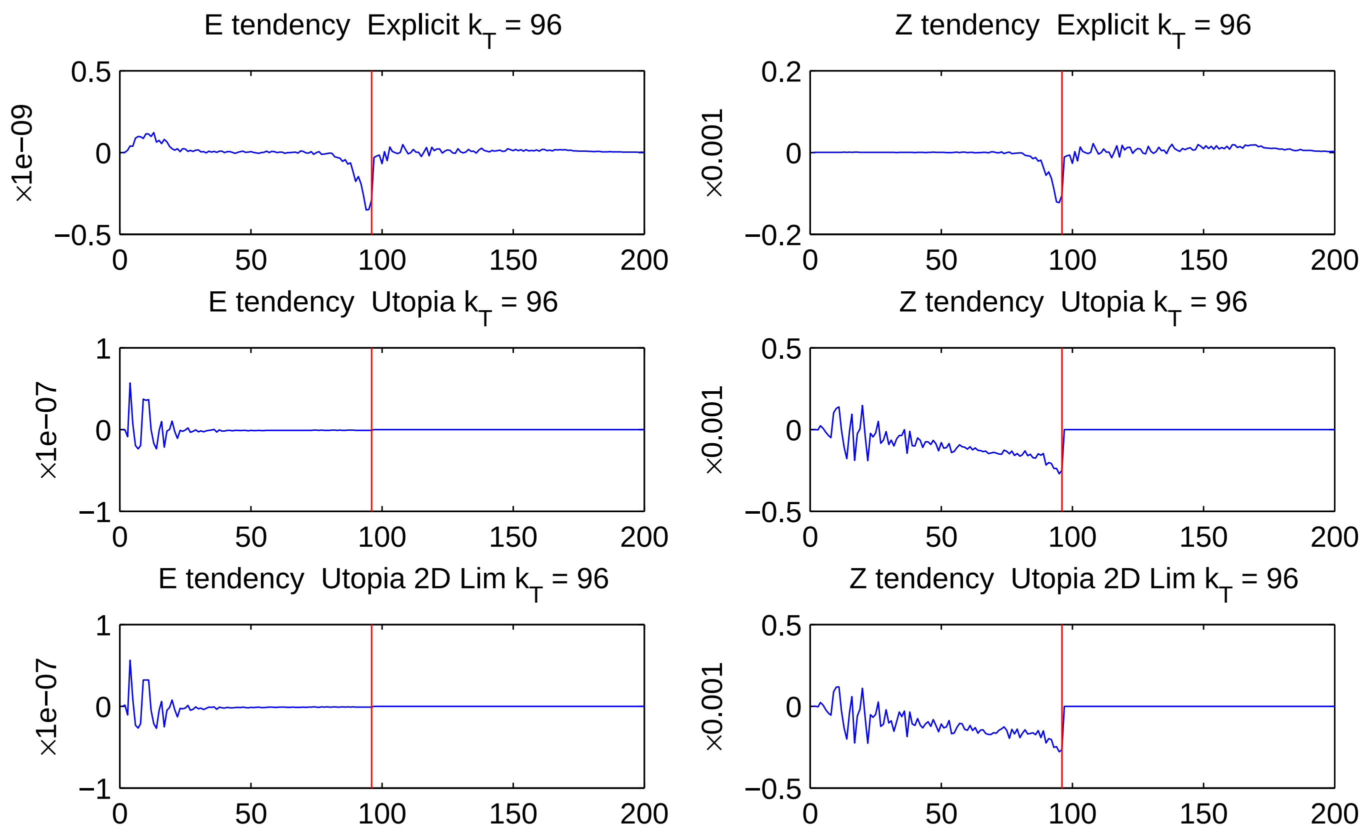
Quasi-third-order upwind scheme.

Inherently dissipative, but more scale-selective than first-order upwind.

Should be comparable to semi-Lagrangian with cubic interpolation.

Can include a flux limiter to prevent over/under-shoots.

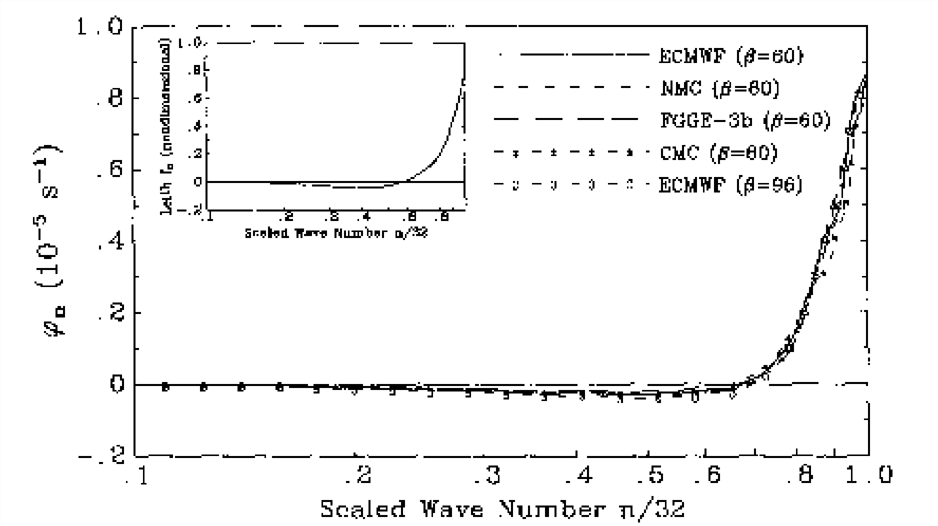
Spectral interactions as represented by UTOPIA scheme



Can we represent the energy backscatter to large scales?

Scale-dependent dissipation/anti-dissipation

Koshyk and Boer (1995)



$$I_n = I_n^{\mathrm{R}} + I_n^{\mathrm{U}}; \qquad I_n^{\mathrm{U}} = -2\phi_n E_n$$

A simple backscatter scheme for BVE

Let $\zeta^* = \mathrm{UTOPIA}(\zeta^n)$

and let $\delta E = E(\zeta^n) - E(\zeta^*)$

Choose a vorticity pattern $\delta\zeta$ and let $\zeta^{n+1} = \zeta^* + \alpha\delta\zeta$.

$$\alpha = -\frac{\delta E}{\int \psi \delta\zeta \, dA}$$

gives energy conservation (to an excellent approximation).

Which vorticity pattern $\delta\zeta$ to use?

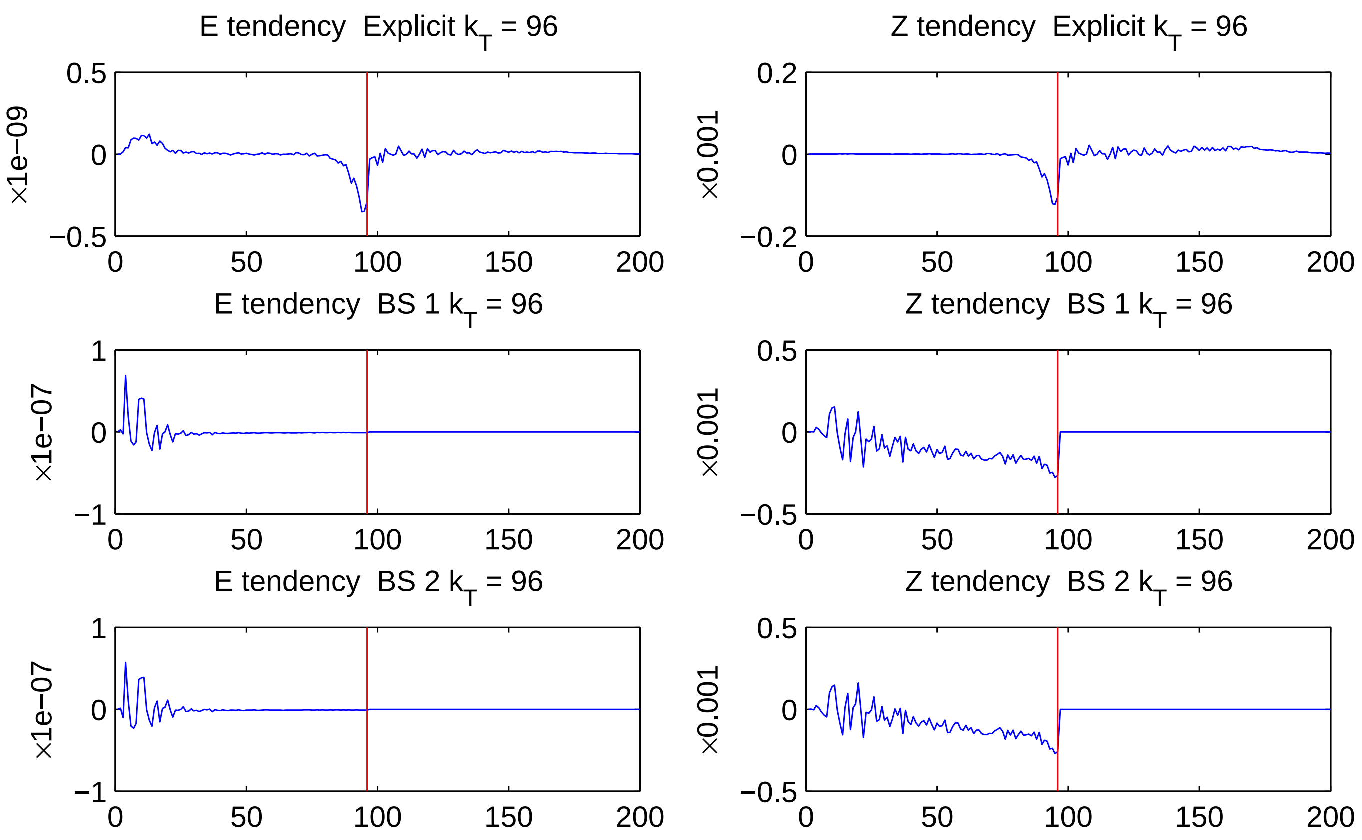
E.g.

$\delta\zeta_1 = \overline{\zeta}^{4\Delta x}$ (large scales)

$\delta\zeta_2 = \zeta - \overline{\zeta}^{4\Delta x}$ (small scales)

$\delta\zeta_2$ was found to work better in numerical tests, giving better energy statistics and also a small but measurable improvement in l_2 errors.

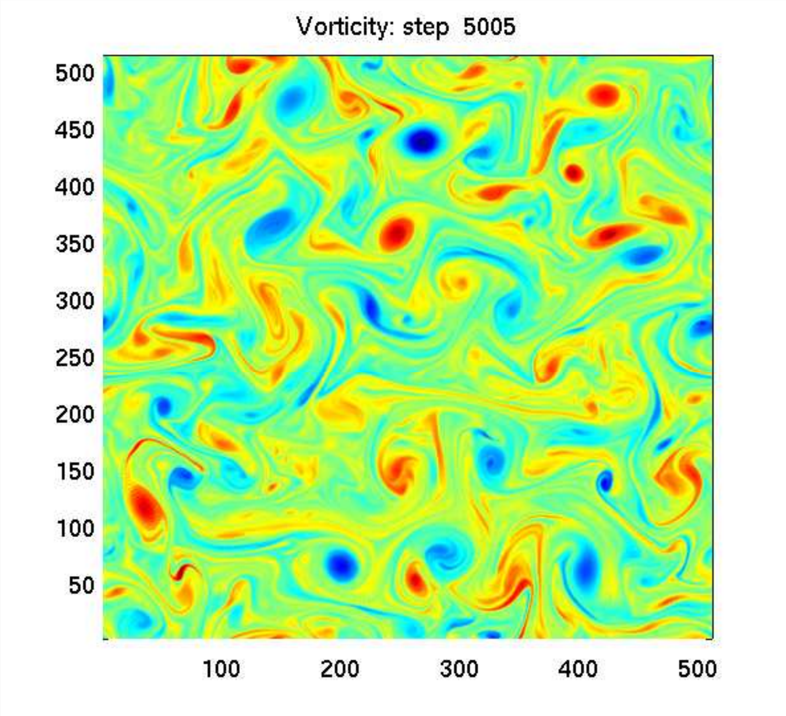
Spectral interactions as represented by UTOPIA + backscatter scheme

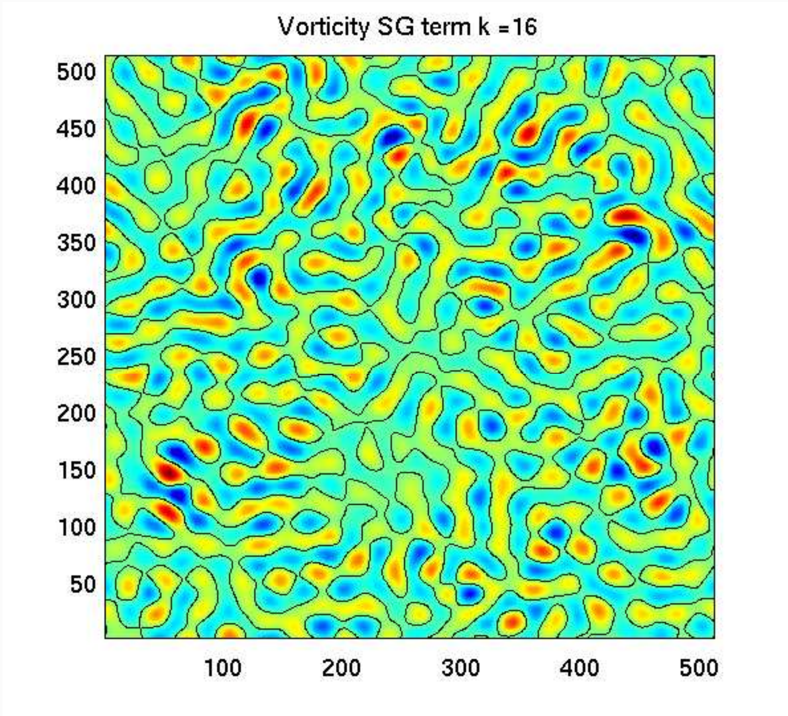


Possible improvements

- Use scale similarity to derive $\delta\zeta$
- Use spectral dissipation characteristics of basic scheme to derive $\delta\zeta$

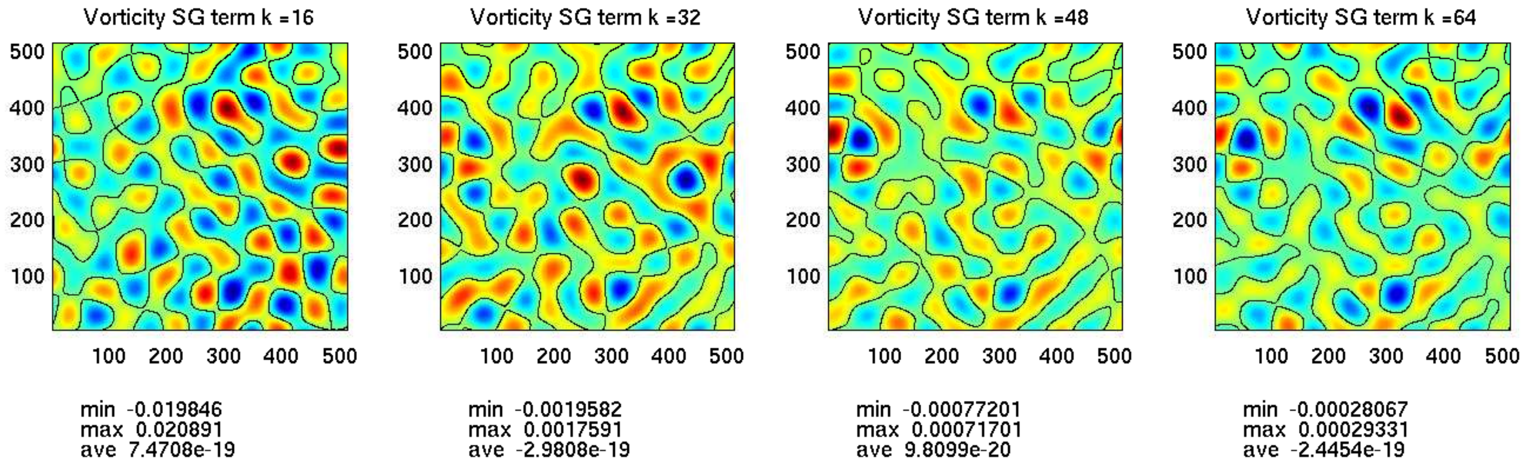
Subgrid forcing of vorticity





$$\partial_t \overline{\zeta} + \partial_j \left(\overline{v_j} \overline{\zeta} \right) = \mathrm{SG} = \partial_j \left(\overline{v_j} \overline{\zeta} - \overline{v_j \zeta} \right)$$

Scale similarity of backscatter?



Conclusions

• For the BVE, explicit calculation of the effects of unresolved scales shows enstrophy removal near the truncation limit and energy input at the most energetic scales. Very robust.

• Both ILES schemes and simple explicit dissipation schemes can remove enstrophy at small scales (but are typically not scale-selective enough

• Neither ILES schemes nor standard SG models capture the energy backscatter.

• A simple backscatter model can improve energy statistics and l_2 errors.

• There is room to improve the backscatter model.