CAM-HOMME: A locally conservative mimetic spectral finite element method for the CCSM

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IPAM – Numerical Hierarchies for Climate Modeling,

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U.S. Department of Energy

Office of Science

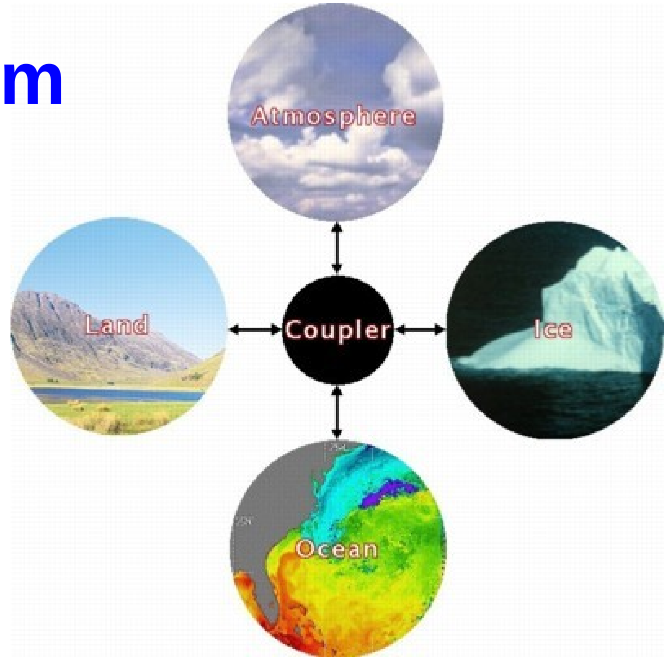
CCPP
CLIMATE CHANGE PREDICTION PROGRAM

SciDAC
Scientific Discovery through Advanced Computing

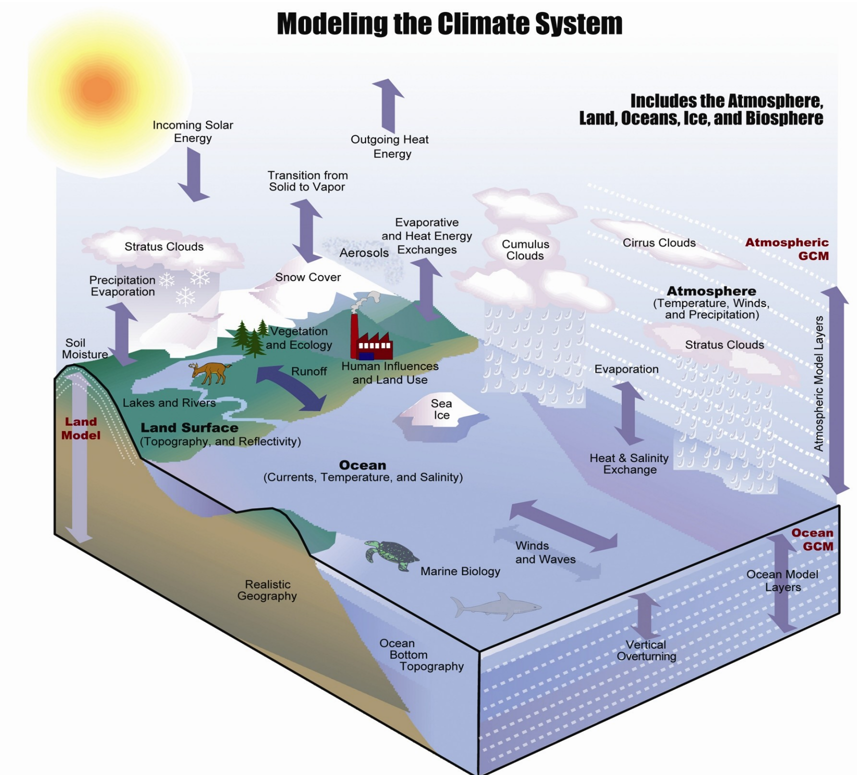
Outline

- HOMME: spectral finite element dynamical core
  - Motivation: improve scalability of the CCSM
- Spectral elements are *compatible/mimetic*
  - Leads to local conservation (mass, PV, Energy) and monitone transport
- Aqua planet:
  - Spectra & energy budgets
  - Equivalent resolution tests
- Real planet model intercomparison.

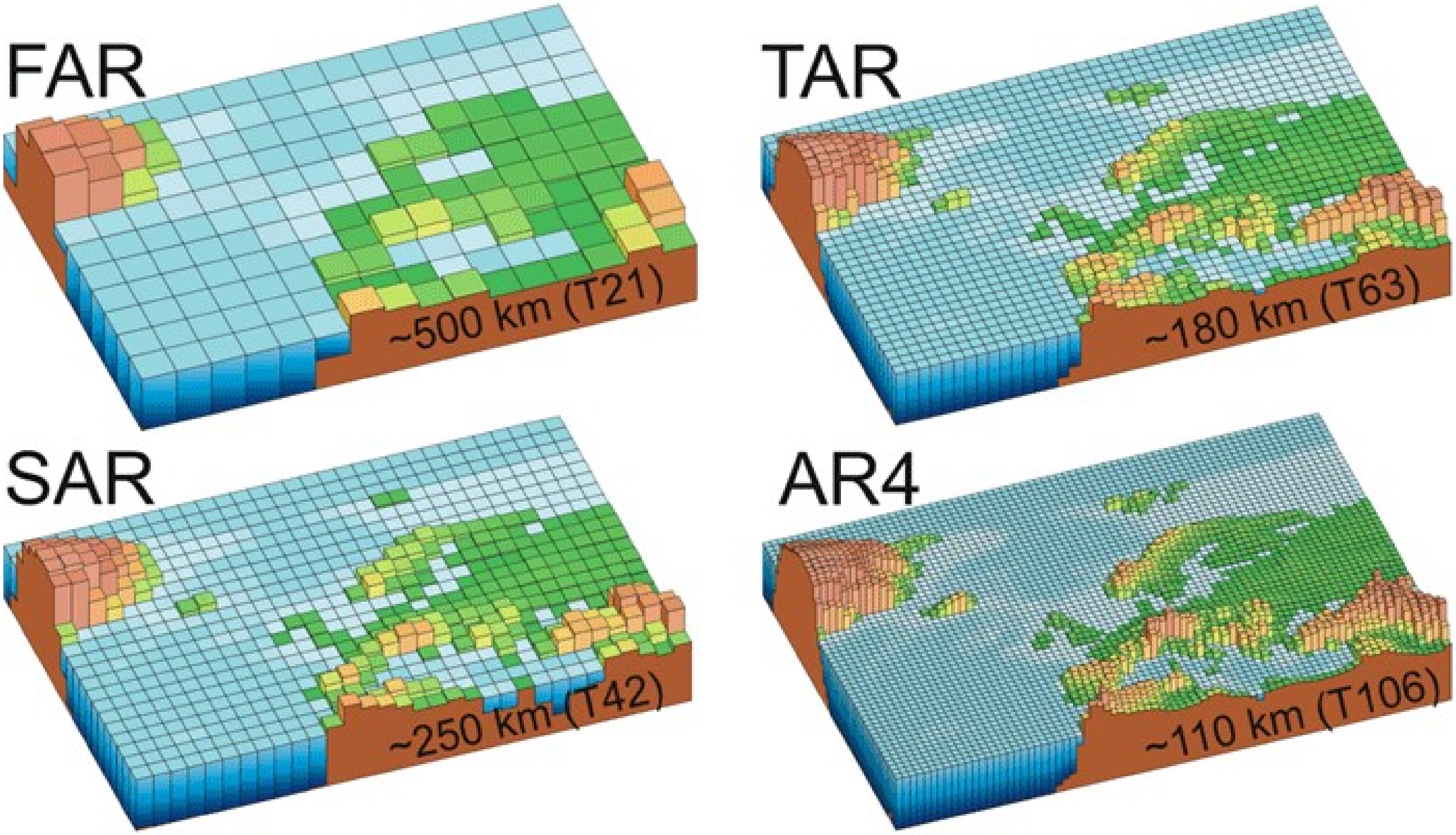
The Community Climate System Model (CCSM)



- IPCC-class model:
  - Seasonal and interannual variability in the climate
  - Explore the history of Earth's climate
  - Estimate future of environment for policy formulation
  - Contribute to assessments
- Developed by NCAR, National Labs and Universities.
- Fully documented, supported and freely distributed
- Runs on multiple platforms and resolutions
- CCSM4 (Released April 1, 2010): Higher resolution and increasing complexity

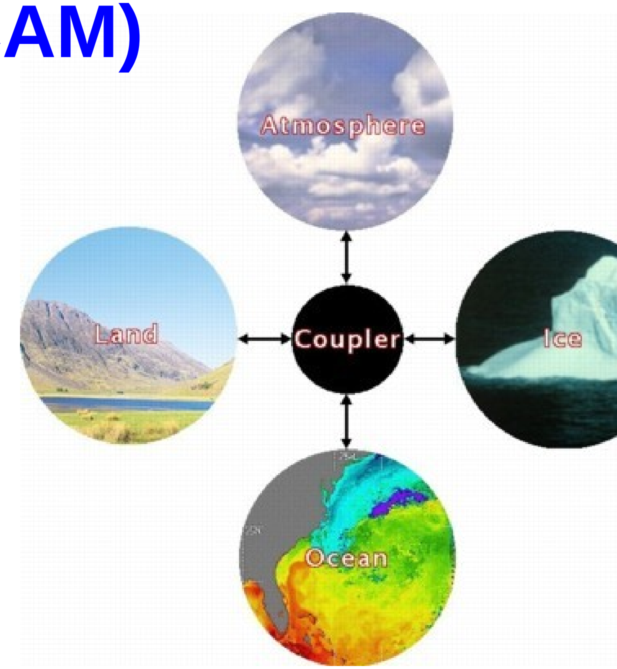


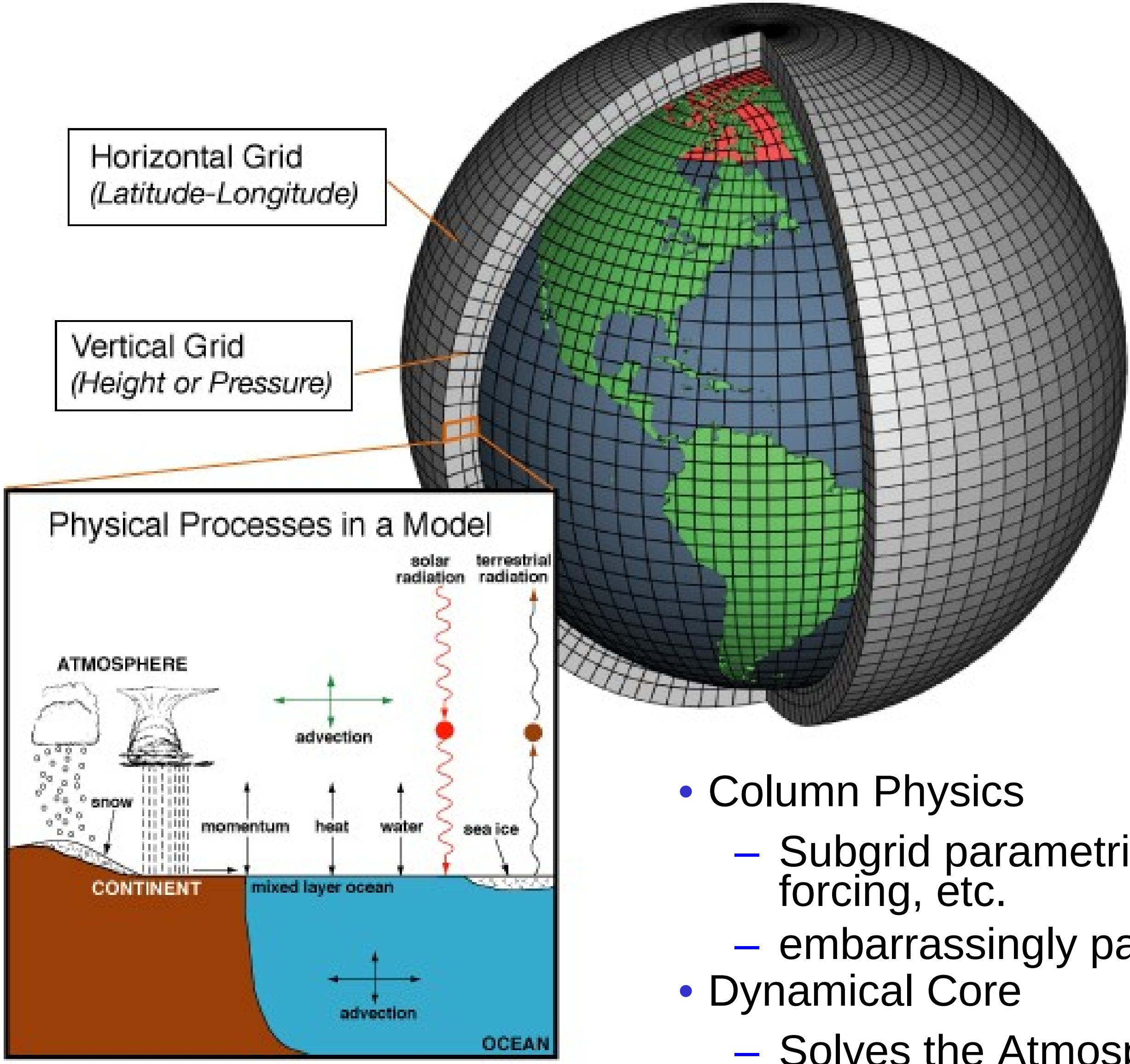
Horizontal Grid Resolution



Source: IPPC 4th Assessment Report

CCSM Atmosphere Component (CAM)





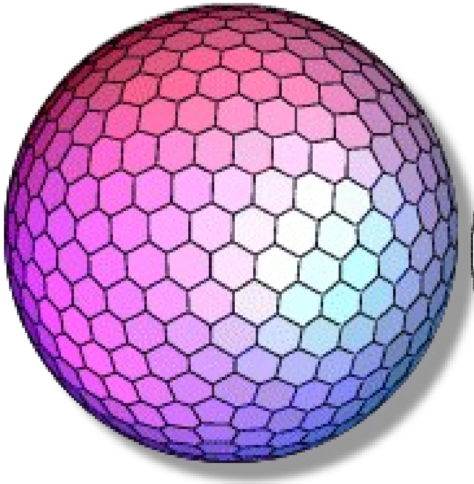
- Column Physics
  - Subgrid parametrizations: precipitation, radiative forcing, etc.
  - embarrassingly parallel with 2D domain decomp
- Dynamical Core
  - Solves the Atmospheric Primitive Equations
  - Scalability bottleneck!

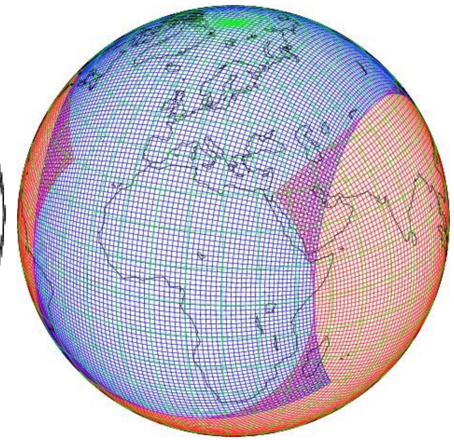
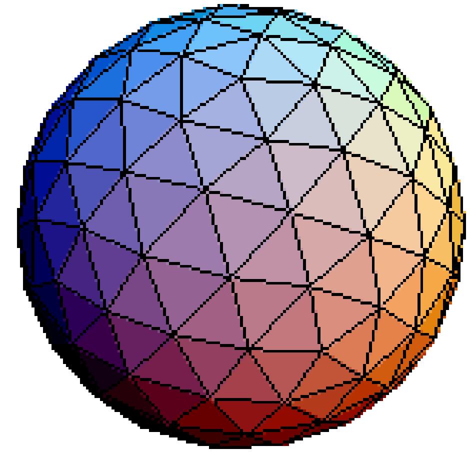
Source: http://celebrating200years.noaa.gov/breakthroughs/climate_model/welcome.html

The Dynamical Core Scalability Bottleneck

- Most dynamical cores in operational models use latitude-longitude grids:
  - Well proven. Many good solutions to the “pole problem”: Spherical harmonics, polar filtering, implicit methods
  - But these approaches degrade parallel scalability

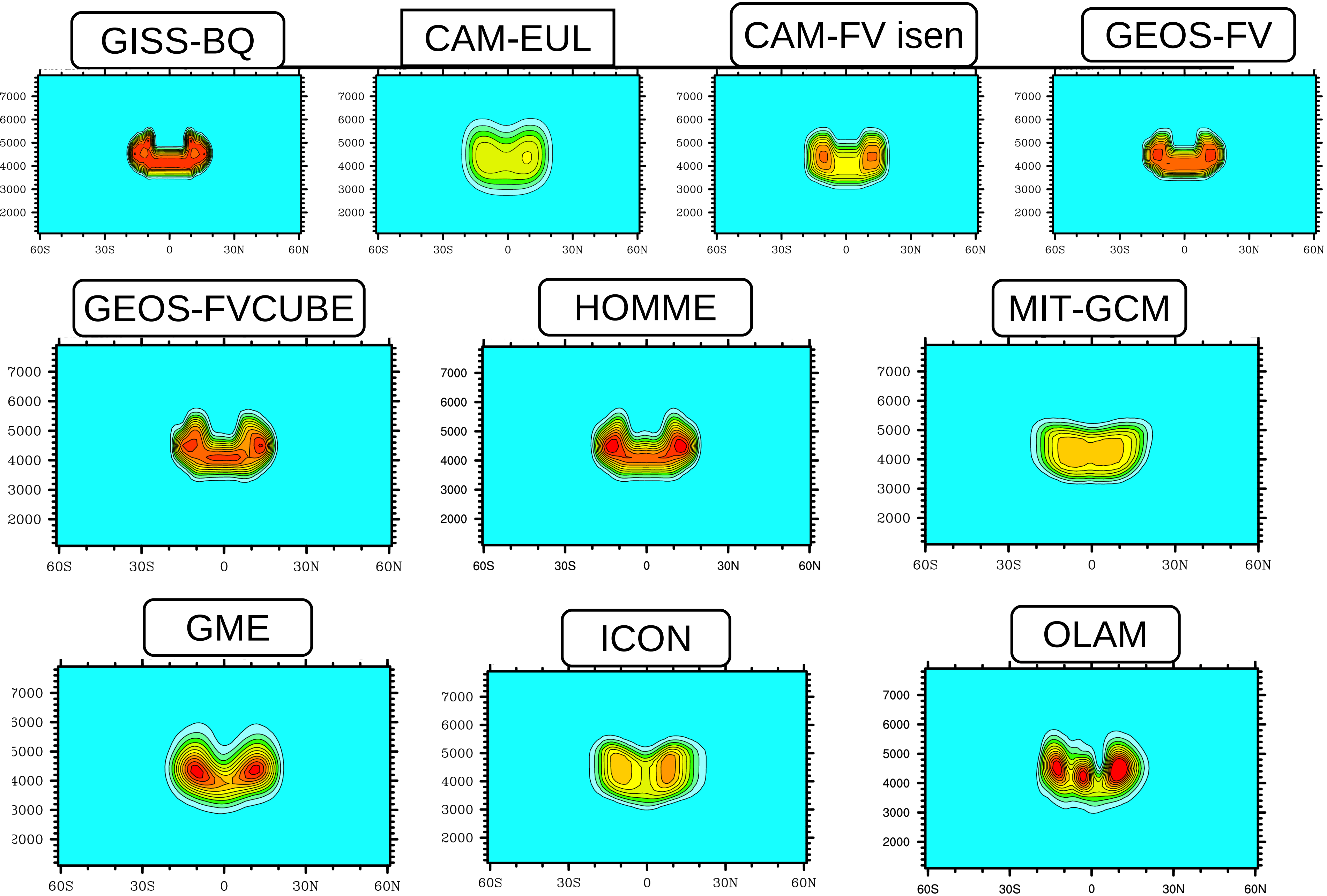
The Dynamical Core Scalability Bottleneck



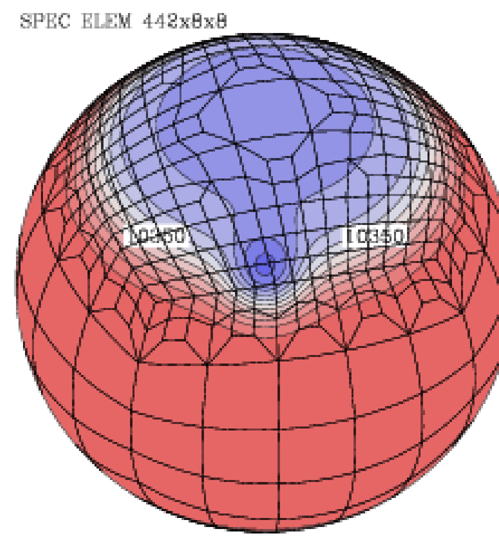
- Petascale dynamical core:
  - Quasi-uniform grids avoid the pole problem
  - Can use full 2D domain decomposition in horizontal directions
  - Each column in the vertical/radial direction kept on processor
  - Equations can be solved explicitly, only nearest neighbor communication
  - Numerical methods that perform as well as lat/lon based methods remains a challenge.

Advection Results from the NCAR 2008 ASP Coloquium Dynamical Core Experiment



HOMME *optional* dynamical core included in CCSM4

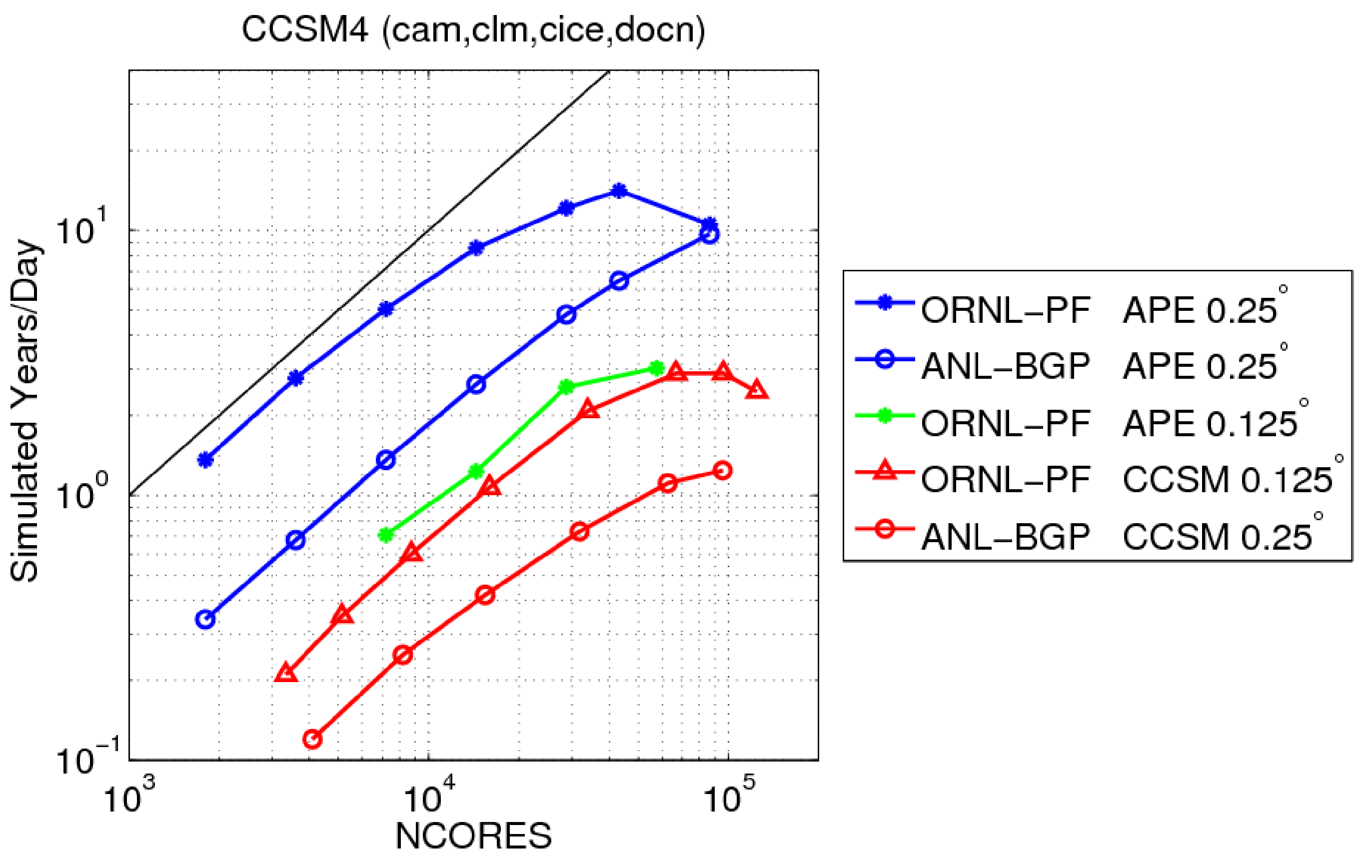
- HOMME: NCAR's High-Order Method Modeling Environment.
- Dynamics (modeled after CAM-Eulerian)
  - Hydrostatic finite-element based dynamical core
  - 4'th order accurate finite element method (quadrilateral elements): Locally conserves mass, PV(2d) and moist energy
  - S&B MWR 1981 vertical coordinate
  - Enstrophy/KE dissipation: CAM-Eulerian type hyper-viscosity
- Transport (modeled after CAM-FV)
  - 3'rd order accurate, locally conservative, quasi-monotone finite element advection for horizontal
  - Vertically-Lagrangian (Lin 2004) w/UKMO conservative monotone reconstruction (Zerroukat et. al, QJRMS 2005)
- All properties preserved on arbitrary unstructured quadrilateral grids (CCSM configurations use cubed-sphere grid)



Two *time-slice* configurations included in CCSM4

- 1 Degree (~100km)
  - Atmosphere: cubed-sphere grid, equatorial grid spacing 1°
  - Land: 2° lat-lon
  - Data ocean + cice (prescribed ice extent) gx1v6
  - Physics/Tracer/Dynamics timesteps: 1800 / 360 / 90s
  - AMWG Diagnostics: `http://users.nccs.gov/~taylorm`
- 1/8 Degree (~12.5km)
  - Atmosphere: cubed-sphere grid, equatorial grid spacing 1/8°
  - Land: 1/4° lat-lon
  - Data ocean + cice (prescribed ice extent) gx1v6
  - Physics/Tracer/Dynamics timesteps: 900 / 45 / 11.25s
  - For testing scalability out to O(400,000) cores

CCSM/HOMME Scalability



- BGP (4 cores/node): Excellent scalability down to 1 element per processor (86,200 processors at 0.25 degree resolution). 1/8 degree CCSM running at > 1 SYPD.
- JaguarPF (12 cores/node): 2-3x faster per core than BGP, scaling not as good. 1/8 degree CCSM running at 3 SYPD.

Compatible Finite Elements

The CG Finite Element Method

- Tile the sphere with quadrilateral elements
- H_0^d = piecewise polynomials (degree d) within each element.
- $H_1^d = C_0 \cap H_0^d$ (continuous piecewise polynomials)
- Quadrature based inner product <f,g >
- Solve the Galerkin formulation in H_1^d
- Differential operators more-or-less determined by choice of H_0^d

P: Projection operator from H_0^d to H_1^d
Orthogonal w.r.t. the quadrature based inner product.

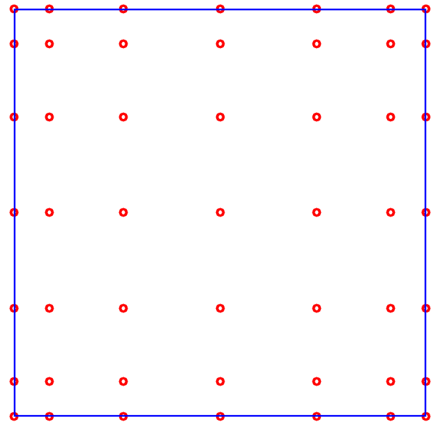
Discrete Inner Product: Gauss-Lobatto (d+1)x(d+1) Quadrature

$$f, g \in H_0^d$$

$$\int f\, g \approx \sum_{\text{elements}} \sum_i w_i\, J(x_i, y_i) f(x_i, y_i)\, g(x_i, y_i) = \langle f\, g \rangle$$

w_i = Gauss-Lobatto weights

J = Jacobian of mapping from spherical element to reference element



Finite Elements
Example: Tracer Advection

Advection Equation:

$$\frac{\partial h}{\partial t} = -\nabla \cdot (h\boldsymbol{u})$$

Forward Euler:
(or convex combinations like SSP RK)

$$h^{t+1} = h^t - \Delta t \, \nabla \cdot (h\boldsymbol{u})$$

Weak Form:
Solve system of scalar equations for

$$\langle \phi h^{t+1} \rangle = \langle \phi h^t \rangle - \langle \phi \nabla_h \cdot (h\boldsymbol{u}) \rangle$$

$$h^{t+1} \in H_1^d \qquad \forall \phi \in H_1^d$$

Finite Elements: Tracer Advection

Advection:

$$h^{t+1} = h^t - \Delta t \, P \, \nabla \cdot (h \boldsymbol{u})$$

FE Galerkin solution:

$$h^* = h^{t-1} - \Delta t \, \nabla_h \cdot (h \boldsymbol{u}) \qquad h^* \in H_0^d$$

$$h^{t+1} = P h^* \qquad h^{t+1} \in H_1^d$$

- Application of the FE projection operator P requires inverting the FE mass matrix.
- Spectral FE: choice of GLL quadrature based inner product and nodal basis functions gives a diagonal mass matrix. (Maday & Patera 1987)
- All inter-element communication is embed in P. FE provides a clean decoupling of computation & communication.

Compatible Numerical Methods

Discrete operators and discrete integral satisfy continuum properties:

$$\int p \nabla \cdot \boldsymbol{v} + \int \boldsymbol{v} \cdot \nabla p = 0$$

$$\int \boldsymbol{v} \cdot \nabla \times \boldsymbol{u} - \int \boldsymbol{u} \cdot \nabla \times \boldsymbol{v} = 0$$

$$\nabla \times \nabla p = 0$$

$$\nabla \cdot \nabla \times \boldsymbol{u} = 0$$

- Integration by parts insures conservation
- Curl Grad = 0 preserves Lagrangian nature of vorticity evolution
- Many schemes have this property on orthogonal Cartesian grids
- The SEM is one of the few methods to retain these properties on arbitrary grids in general curvilinear coordinates.

•Margolin, Shashkov, Int. J. Numer. Meth. Fluids, 2007

•Arnold, Bochev, Lehoucq, Nicolaided, Shashkov, *Compatible Spatial Discretizations (The IMA Volumes in Mathematics and its Applications),* Springer 2006

Compatible Numerical Methods

Discrete operators and discrete integral satisfy similar properties:

$$\langle p\,\nabla_h \cdot \boldsymbol{v} \rangle + \langle \boldsymbol{v} \cdot \nabla_h p \rangle = 0$$

$$\langle \boldsymbol{v} \cdot \nabla_h \times \boldsymbol{u} \rangle - \langle \boldsymbol{u} \cdot \nabla_h \times \boldsymbol{v} \rangle = 0$$

$$P\left(\nabla_h \times P\left(\nabla_h p\right)\right) = 0$$

$$P\left(\nabla_h \cdot P\left(\nabla_h \times \boldsymbol{u}\right)\right) = 0$$

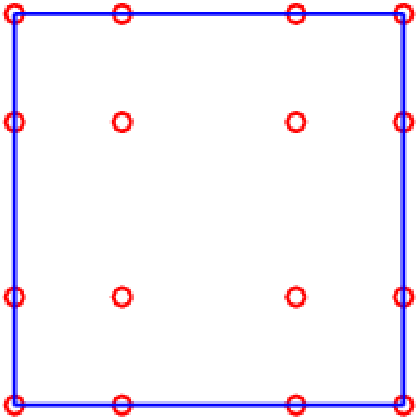
- Integration by parts formulas hold for arbitrary conforming grids in curvilinear coordinates.
- Un-projected annihilator properties are trivial for FE
- Projected annihilator property is more difficult, but holds for spectral elements (Taylor & Fournier, JCP 2010)

Compatibility: Local Version

Continuum Identity for any area Ω:

$$\int_{\Omega} p \nabla \cdot \boldsymbol{v} + \int_{\Omega} \boldsymbol{v} \cdot \nabla p = \oint_{\partial \Omega} p \boldsymbol{v} \cdot \hat{\boldsymbol{n}}$$

Discrete Identity holds for Ω = a single element:



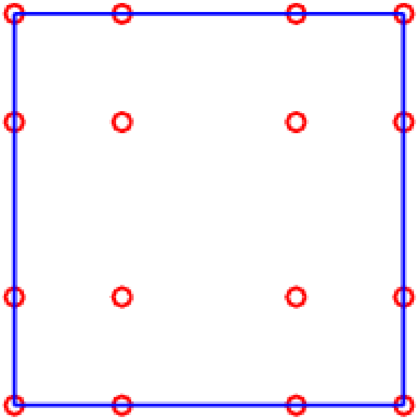
$$\sum_{\Omega} \boldsymbol{v} \cdot \nabla_h p + \sum_{\Omega} p \nabla_h \cdot \boldsymbol{v} = \sum_{\partial \Omega} p \boldsymbol{v} \cdot \hat{\boldsymbol{n}}$$

- Discrete boundary integral is the natural Gauss-Lobatto quadrature approximation along the element boundary
- *p* and ***v*** are continuous, thus element edge “flux” is equal and opposite to flux computed by adjacent elements
- For curvilinear mappings, requires only that the mapping is continuous across element edges

Local Conservation: Mass

- Relatively new result for finite elements:
  - Hughes et al. JCP 2000: *The Continuous Galerken Method is Locally Conservative*
  - Work presented here extends this result to inexact integration formulation with curvilinear elements (spectral elements).
- Local conservation often obtained by replacing divergence operator by control volume flux. In SEM, we use the numerical divergence operator directly and rely on the discrete form of the identity, for any element Ω :

$$\int_{\Omega} \nabla \cdot p\boldsymbol{v} = \oint_{\partial\Omega} p\boldsymbol{v} \cdot \hat{\boldsymbol{n}}$$



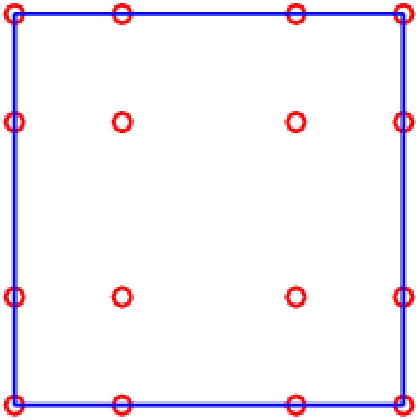
- Identity holds for any curvilinear conforming mesh
- Since p & v are continuous in a CG formulation, element edge flux is always continuous

Local Conservation: Energy

- Local conservation of energy is obtained by using the vector invariant formulation of the equations, combined with the SEM discrete form of the integration by parts identity: for any element Ω :

$$\int_{\Omega} p \nabla \cdot \boldsymbol{v} + \int_{\Omega} \boldsymbol{v} \cdot \nabla p = \oint_{\partial \Omega} p \boldsymbol{v} \cdot \hat{\boldsymbol{n}}$$

- Energy conservation obtained by exact, term-by-term balance of all adiabatic terms in the KE, IE and PE budgets.
- Identity holds on any curvilinear conforming mesh
- Energy conservation is semi-discrete: exact with exact time integration.



PV conservation (2D shallow water)

Shallow water equations, vector invariant form:

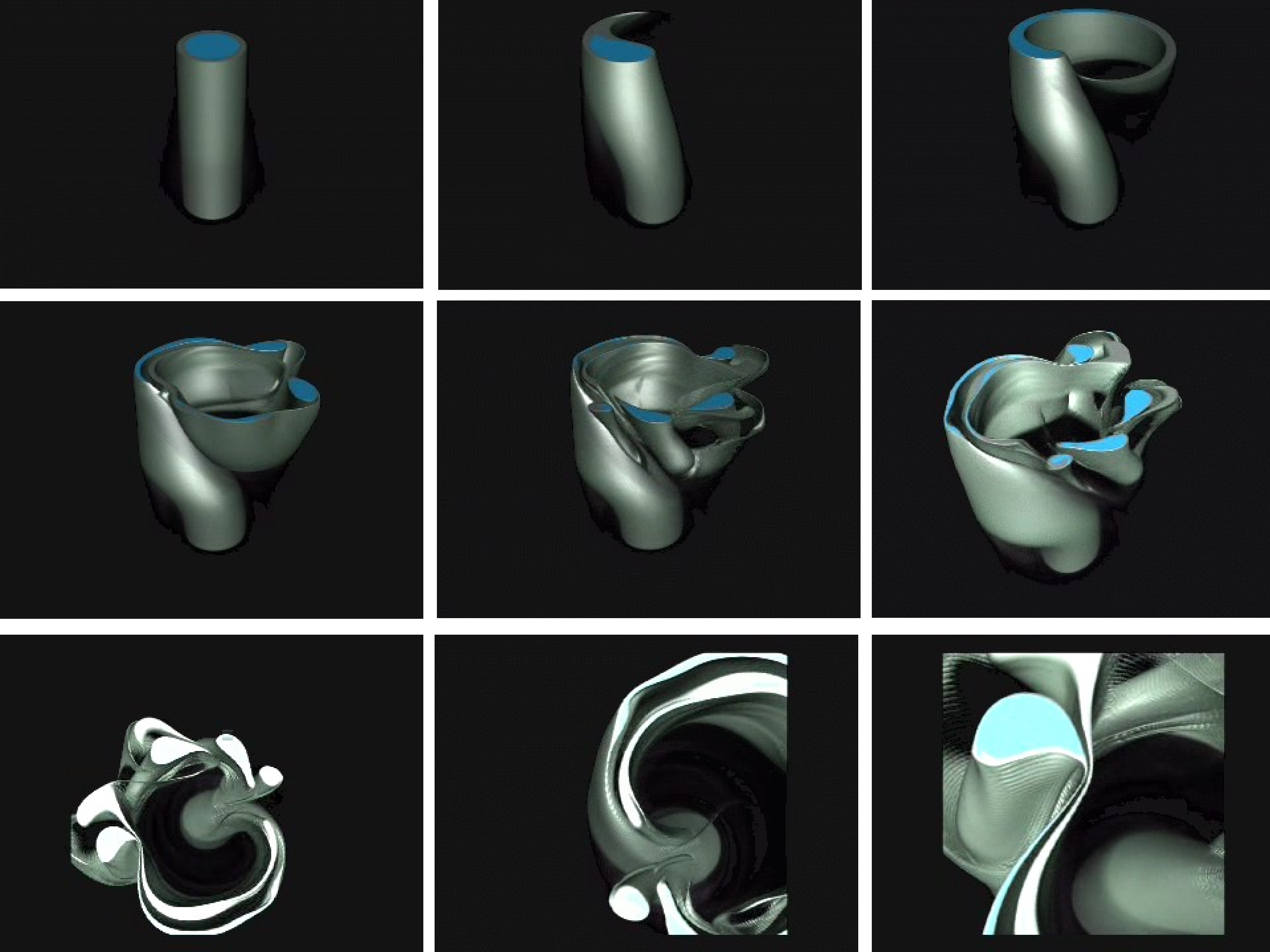
$$\frac{\partial \boldsymbol{u}}{\partial t}+(\omega+f)\hat{\boldsymbol{k}}\times\boldsymbol{u}+\nabla\left(\frac{1}{2}\boldsymbol{u}^{2}+gh\right)=0$$

Using that P CURL P GRAD = 0, we can show that for a SEM solution u above, the projected vorticity,

$$\omega=\nabla\times\boldsymbol{u} \qquad \hat{\omega}=P\,\omega \qquad q=\frac{\hat{\omega}+f}{h}$$

Solves the SEM discrete equation for potential vorticity, *q*, in conservation form (and thus conserves potential vorticity):

$$\frac{\partial}{\partial t}(h\,q)+\nabla\cdot h\,q\,\boldsymbol{u}=0$$



Monotone or Sign Preserving Finite Element Advection

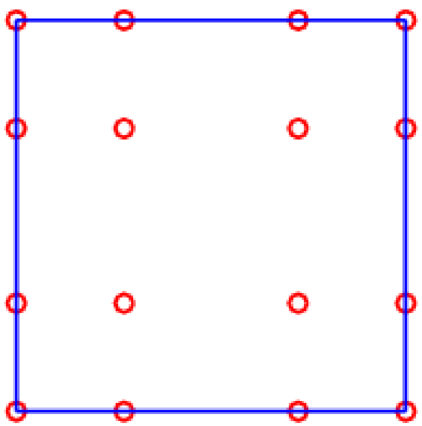
Finite Element Tracer Advection

Discrete Equation: $$h^{t+1} = h^t - \Delta t \, P \, \nabla \cdot (h \boldsymbol{u})$$

Equivalent to: $$h^* = h^{t-1} - \Delta t \, \nabla_h \cdot (h \boldsymbol{u}) \qquad h^* \in H_0^d$$

$$h^{t+1} = P h^* \qquad h^{t+1} \in H_1^d$$

- Within each element, h* can have oscillations and new extrema, but in any compatible finite element method, the element averages of h* are monotone - no new local min or max. (Taylor, St.Cyr, Fourner, ICCS 2009).



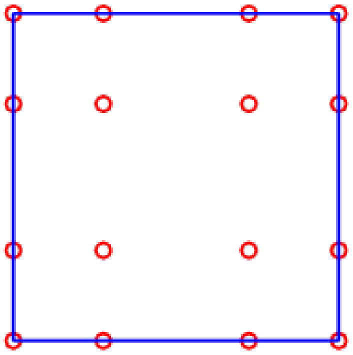
Finite Element Tracer Advection

$$h^{*} = h^{t-1} - \Delta t \, \nabla_h (h \boldsymbol{u})$$

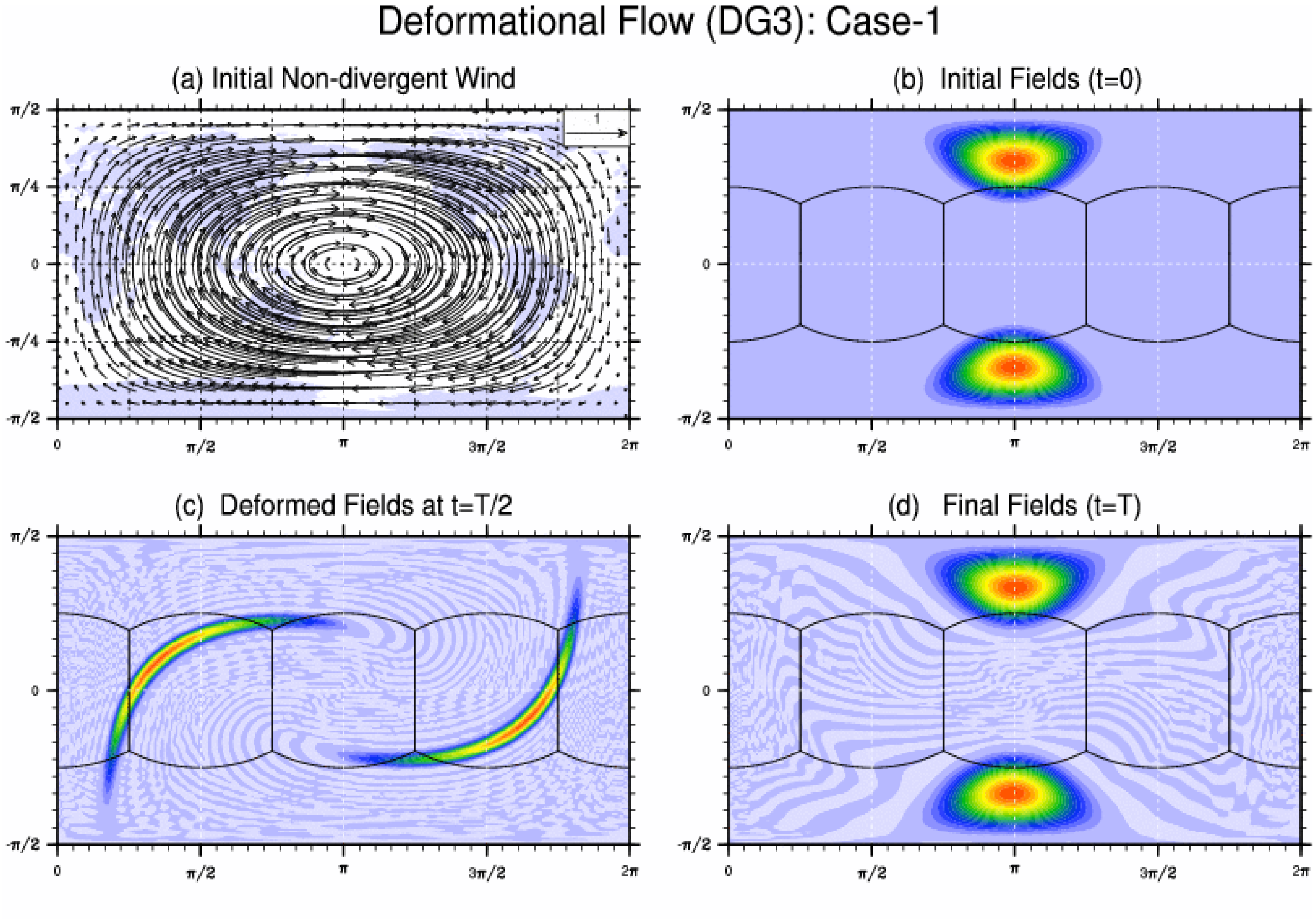
$$h^{**} = \mathrm{limiter}(h^{*})$$

$$h^{t+1} = P h^{**}$$

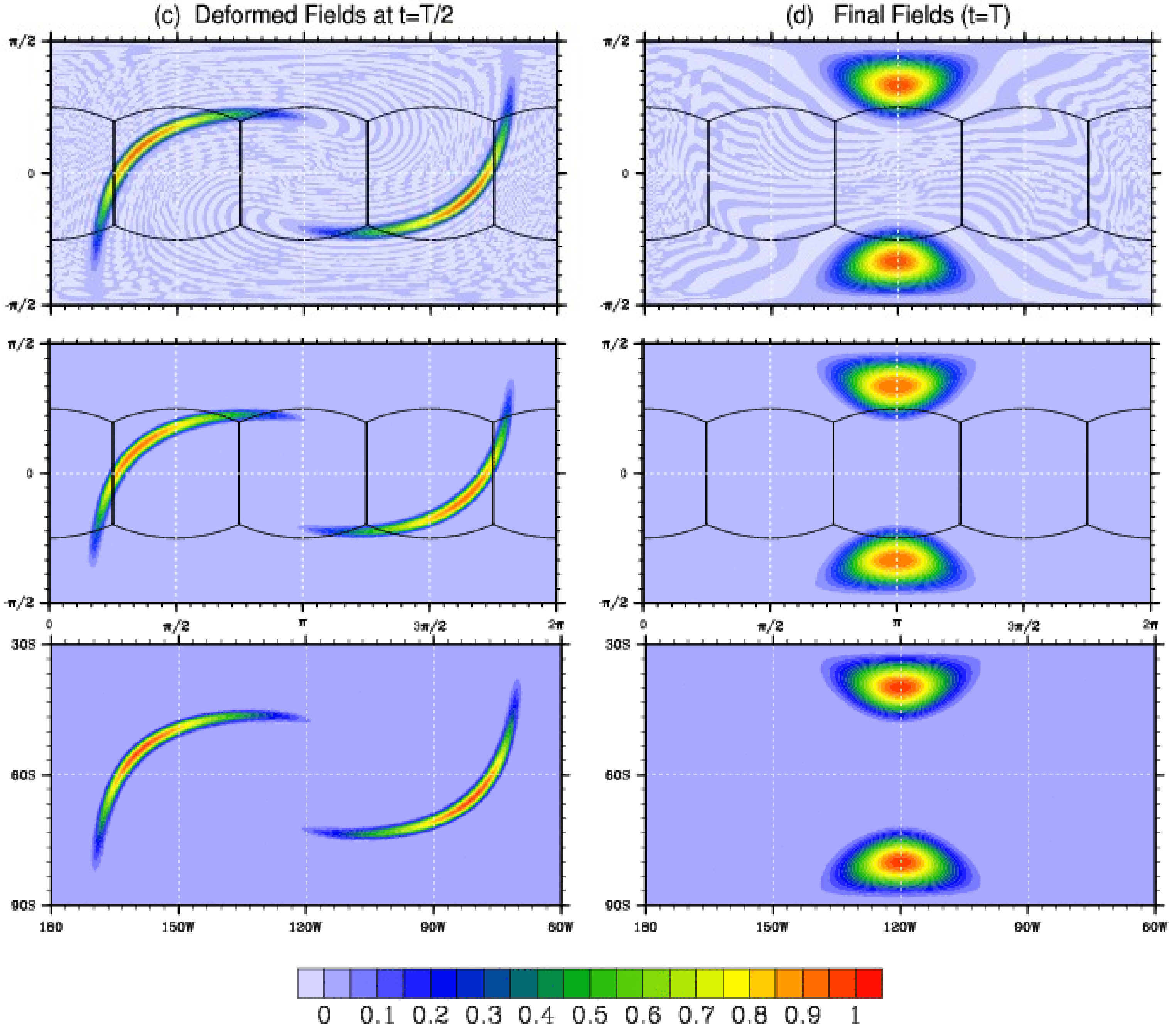
- Element average monotonicity property means that h* can be replaced by any mean-preserving reconstruction (monotone, sign-preserving, others).
- Reconstruction requires no inter-element communication.
- Projection must be monotone preserving – true for spectral FE, but not for general FE



Deformational Flow Test Case for the sphere
(Nair, Lauritzen, under review, JCP)



Nair-Lauritzen Deformational Flow Test Case for the sphere



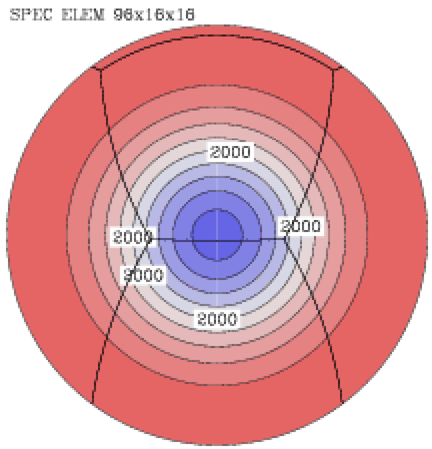
Errors

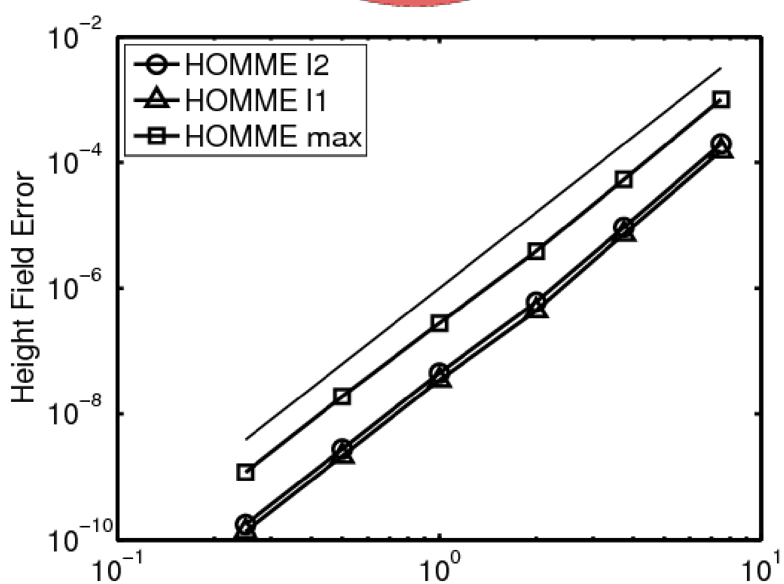
	L1	L2	Max Norm	Max	Min
DG	0.045	0.029	0.031	-0.001	-0.018
CSLAM	0.025	0.019	0.029	-0.002	-0.019
HOMME	0.030	0.020	0.025	-0.004	0.000

Idealized Test Cases

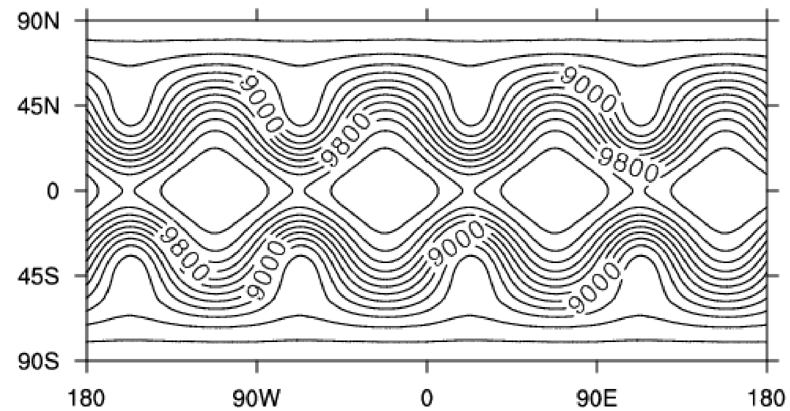
Shallow Water Equations on the Sphere

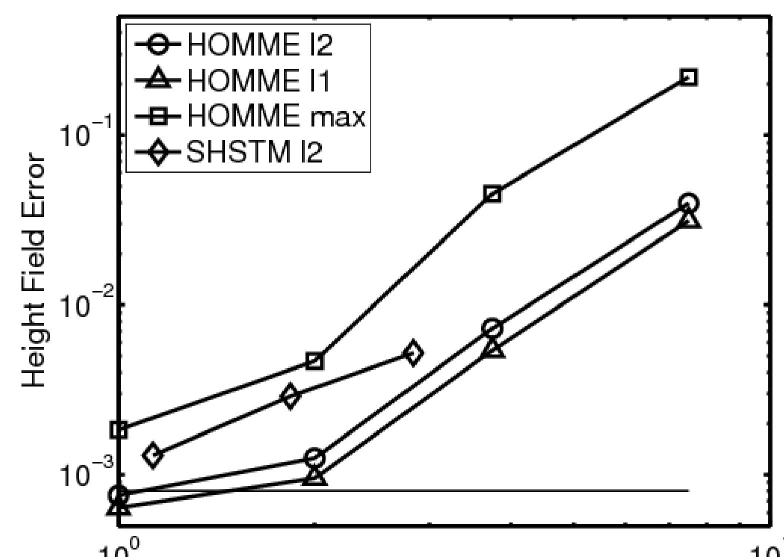
Test 2





Test 5

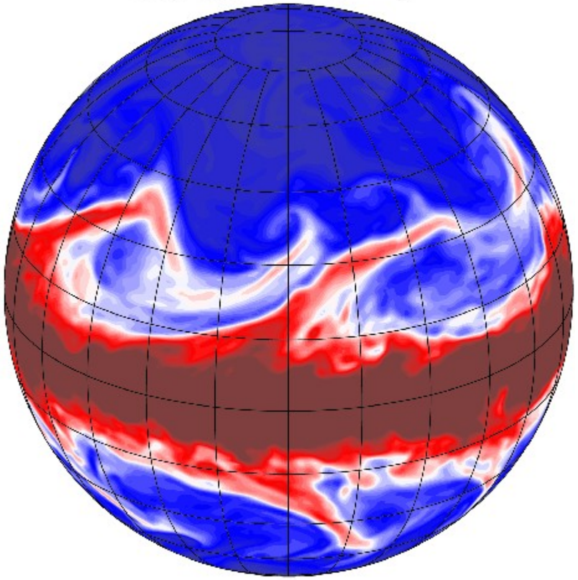




Test case 2 (analytical solution): perfect 4'th order convergence
Test case 5 (reference solution): convergence rate of 2.6 until error is reduced to the level of uncertainty in the reference solution.

Aqua Planet

- Full Atmospheric physics and dynamics, on a planet with no land and a fixed sea surface temperature
- Follow Williamson equivalent resolution methodology (Williamson, Tellus 2008)
  - No convergence under mesh refinement, as expected due to the nature of many of the subgrid physics parametrizations
  - Strong signal with resolution
  - However, agreement between dynamical cores establishes equivalent resolutions



Moist Atmospheric Primitive Equations

Assuming terrain following pressure coordinate $\sigma = p/p_s$:

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{\zeta} + f)\hat{\boldsymbol{k}} \times \boldsymbol{u} + \nabla\left(\frac{1}{2}\boldsymbol{u}^2 + \Phi\right) + \dot{\sigma}\frac{\partial \boldsymbol{u}}{\partial \sigma} + \frac{RT_v}{p}\nabla p_s = 0$$

$$\frac{\partial T}{\partial t} + \boldsymbol{u}\cdot\nabla T + \dot{\sigma}\frac{\partial T}{\partial \sigma} - \frac{RT_v}{c_p^*}\frac{\omega}{p} = 0$$

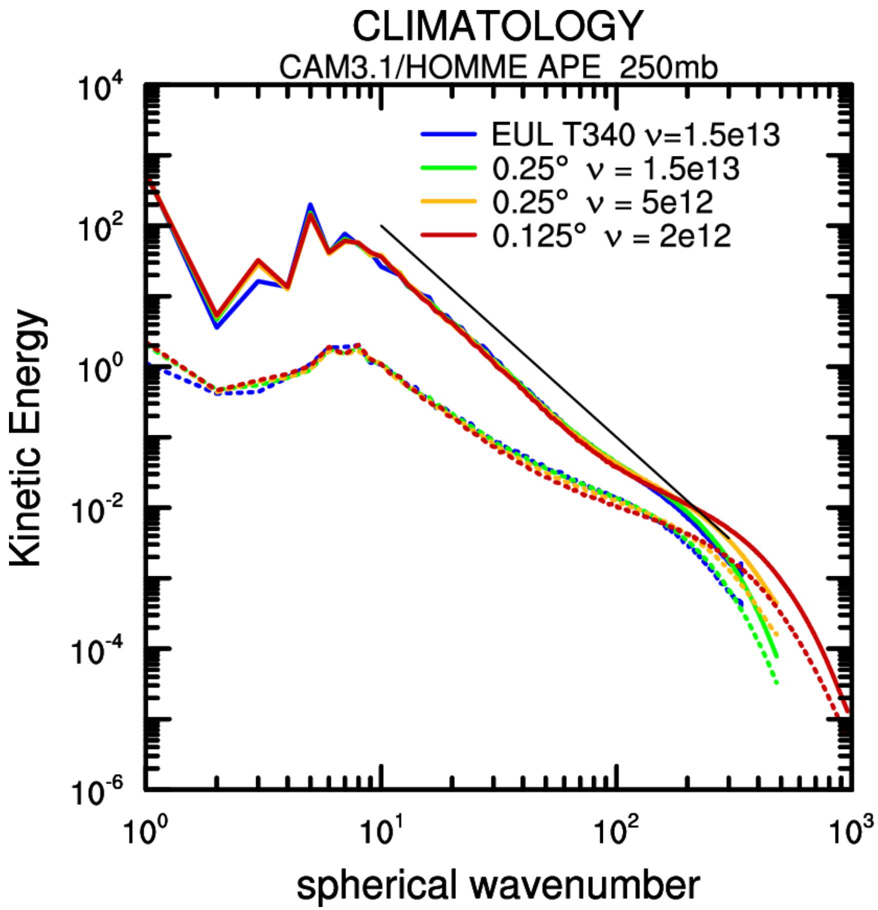
$$\frac{\partial p_s}{\partial t} + \nabla\cdot(p_s \boldsymbol{u}) + \frac{\partial}{\partial \sigma}(p_s \dot{\sigma}) = 0$$ (combining p_s and sigma-dot equations)

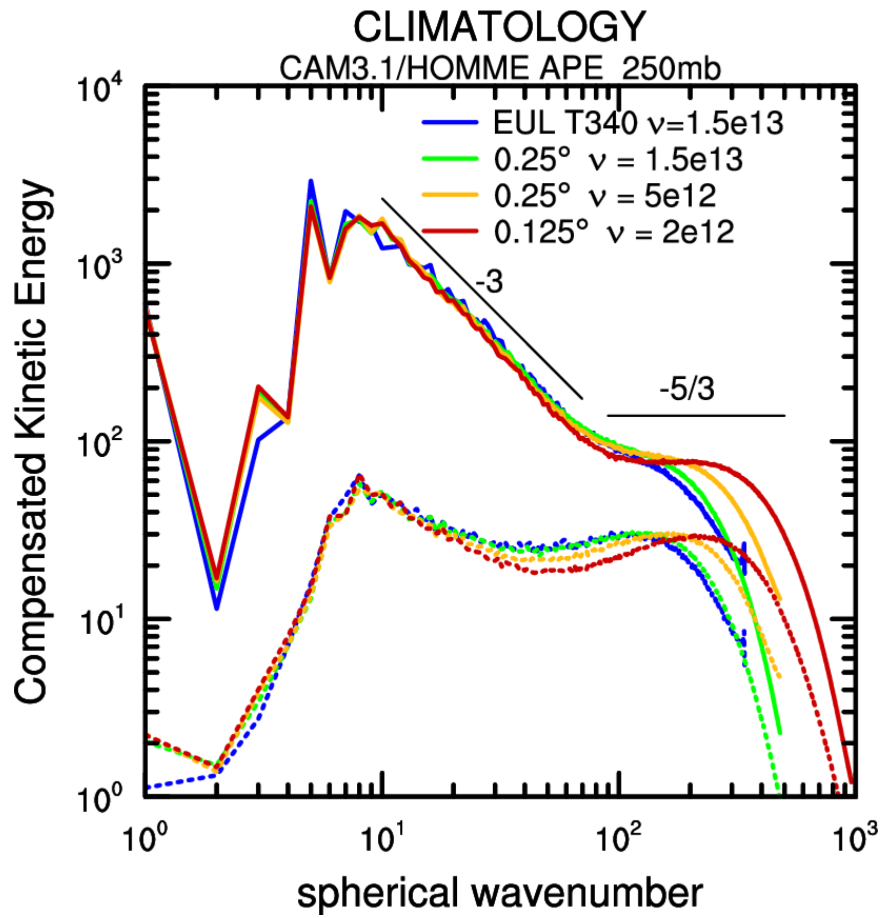
$$\frac{\partial p_s q}{\partial t} + \nabla\cdot(p_s q \boldsymbol{u}) + \frac{\partial}{\partial \sigma}(p_s q \dot{\sigma}) = 0$$

$\boldsymbol{u}$ = velocity field
p_s = surface pressure
T = temperature

$\boldsymbol{\zeta}$ = vorticity
ω = pressure vertical velocity
σ = sigma vertical coordinate
Φ = geopotential

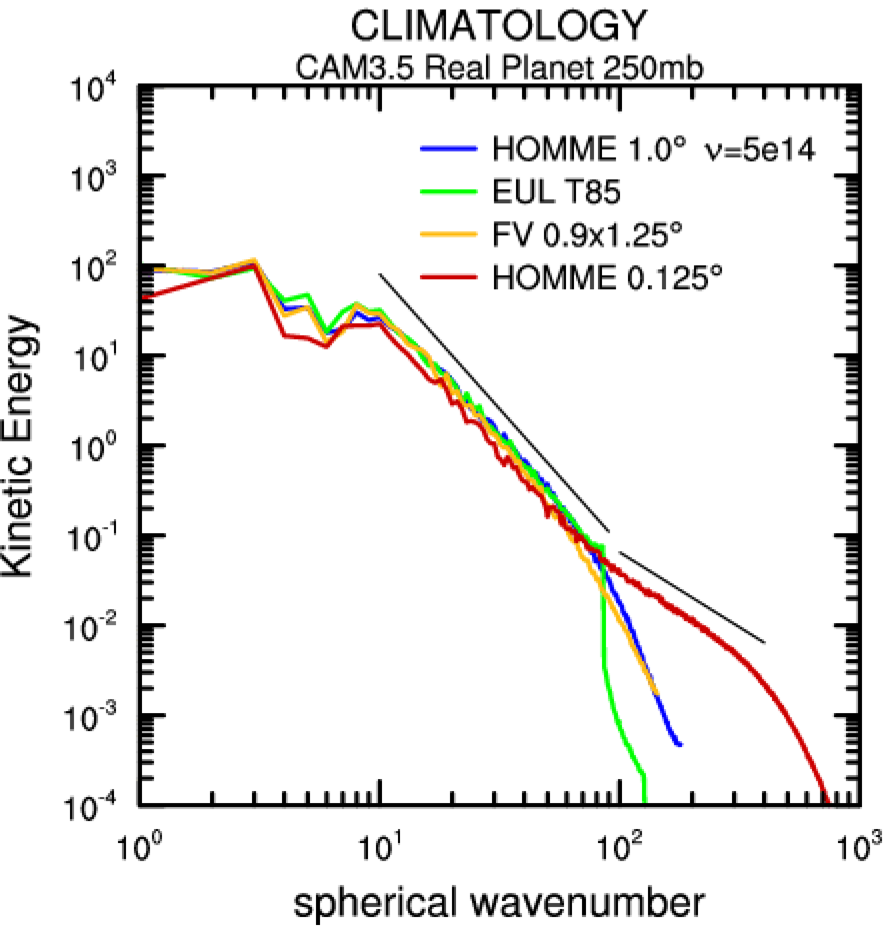
Aqua Planet - CAM 3.4 Physics

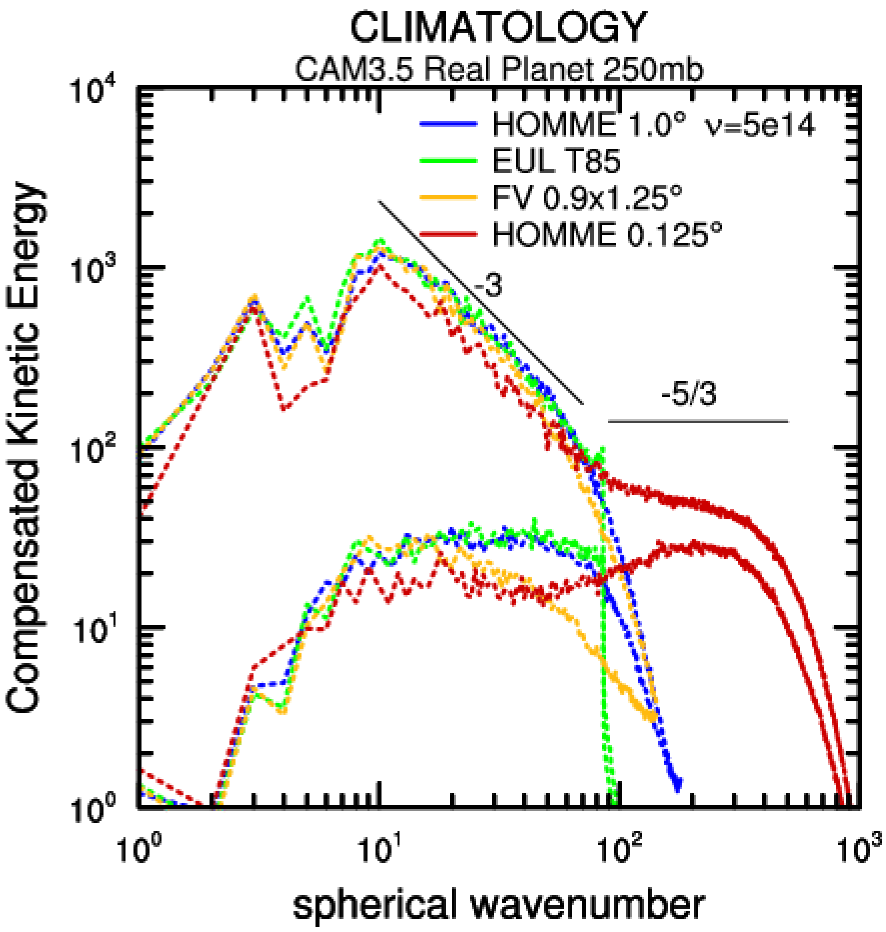




Total (solid lines) and compressible (dotted lines) components

Real Planet: KE spectra





Rotational (top) and compressible (bottom) components

Moist Energy Conservation

$$E = \frac{1}{2}\iint p_s \boldsymbol{u}\cdot\boldsymbol{u} + \iint c_p^* p_s T + \int p_s \Phi_s$$

- Total Energy conservation: adiabatic case: (no forcing, dissipation, limiters, Robert filter)
  - dE/dt = 1e-4 W/m^2
  - Decreases to machine precision as: $O(\Delta t^2)$
  - Must include moist contributions to c_p to show conservation
  - $c_p^* = c_p + (c_{pv} - c_p)q$
  - Using dry energy formula, dE/dt = 0.5 W/m^2.

Moist Energy Conservation

$$E = \frac{1}{2}\iint p_s \boldsymbol{u}\cdot\boldsymbol{u} + \iint c_p^* p_s T + \int p_s \Phi_s$$

- Full Model: (0.5 degree resolution) W/m^2

	Robert Filter, q limiters	Hyperviscosity	Physics	Internal Transfer
dKE/dt		-0.6	-2.5	3.2
dIE/dt		0.6	2.3 +/- 6	-3.2
dE/dt	-0.013	0.0		0.0

Note: At 2 degree resolution, models are criticized for dissipating too much KE in order to achieve sufficient enstrophy dissipation. Hyperviscosity (CAM-Eulerian, CAM-HOMME) or ILES (CAM-FV) or semi-lagrange (Bowler QJRMS 2009) are 1-2 W/m^2.

CAM-HOMME at ½ degree (above) 0.6 W/m^2
CAM-HOMME at 1/8 degree: 0.3 W/m^2

Aqua Planet Global Mean Quantities

Resolution	Physics dt	Viscosity	PRECC	PRECL	CLDTOT	TMQ
EUL T42	5m	1.0E+16	1.71	1.11	0.64	20.21
HOMME 1.9	5m	1.0E+16	1.76	1.14	0.66	20.09
EUL T85	5m	1.0E+15	1.59	1.38	0.60	19.63
HOMME 1.0	5.5m	1.0E+15	1.59	1.43	0.61	19.67
EUL T170	5m	1.5E+14	1.44	1.62	0.55	19.13
HOMME 0.5	5m	1.5E+14	1.48	1.62	0.55	19.36
T340	5m	1.5E+13	1.36	1.75	0.50	18.75

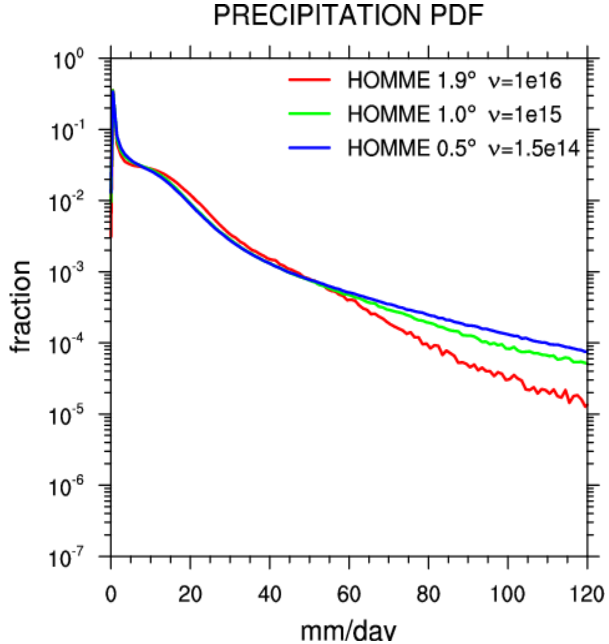
Compared to the size of the resolution signal, there is a remarkable agreement between CAM/HOMME and CAM/Eulerian

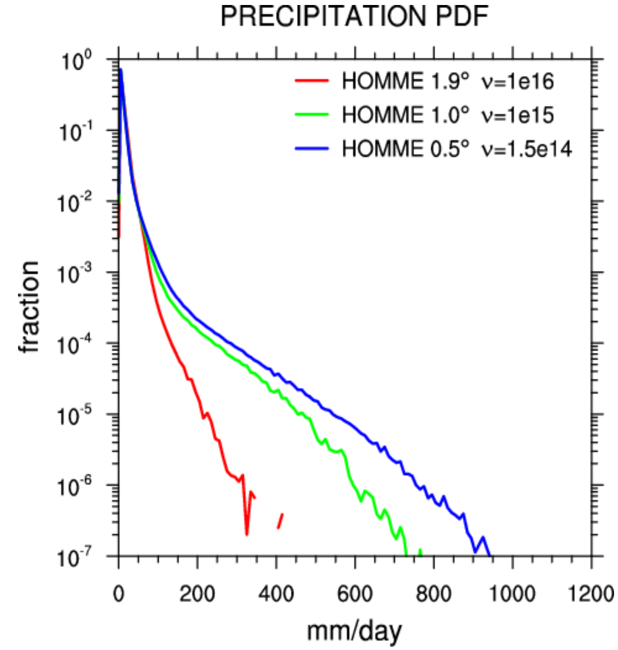
Precipitation PDFs

1mm bin-size

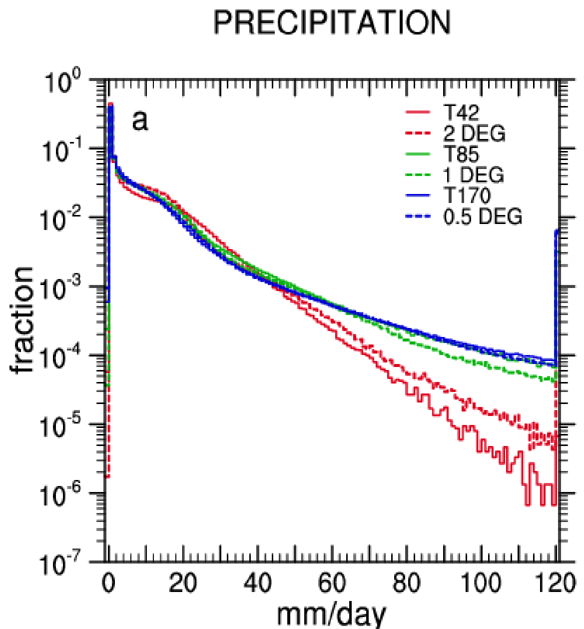
10mm bin-size

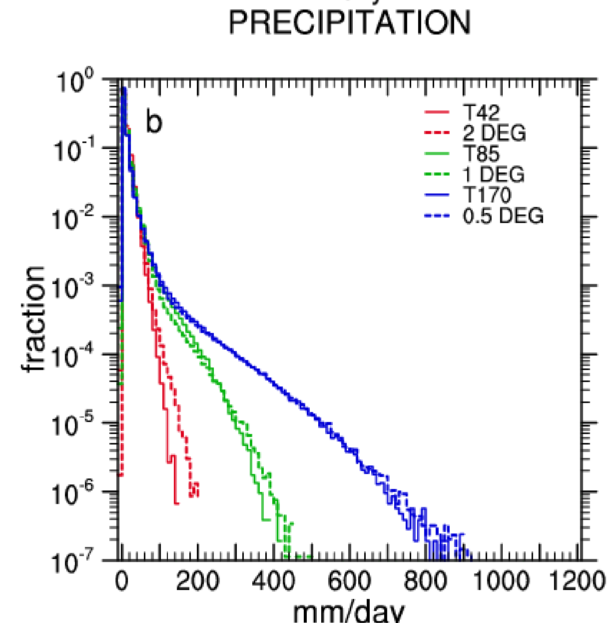
HOMME





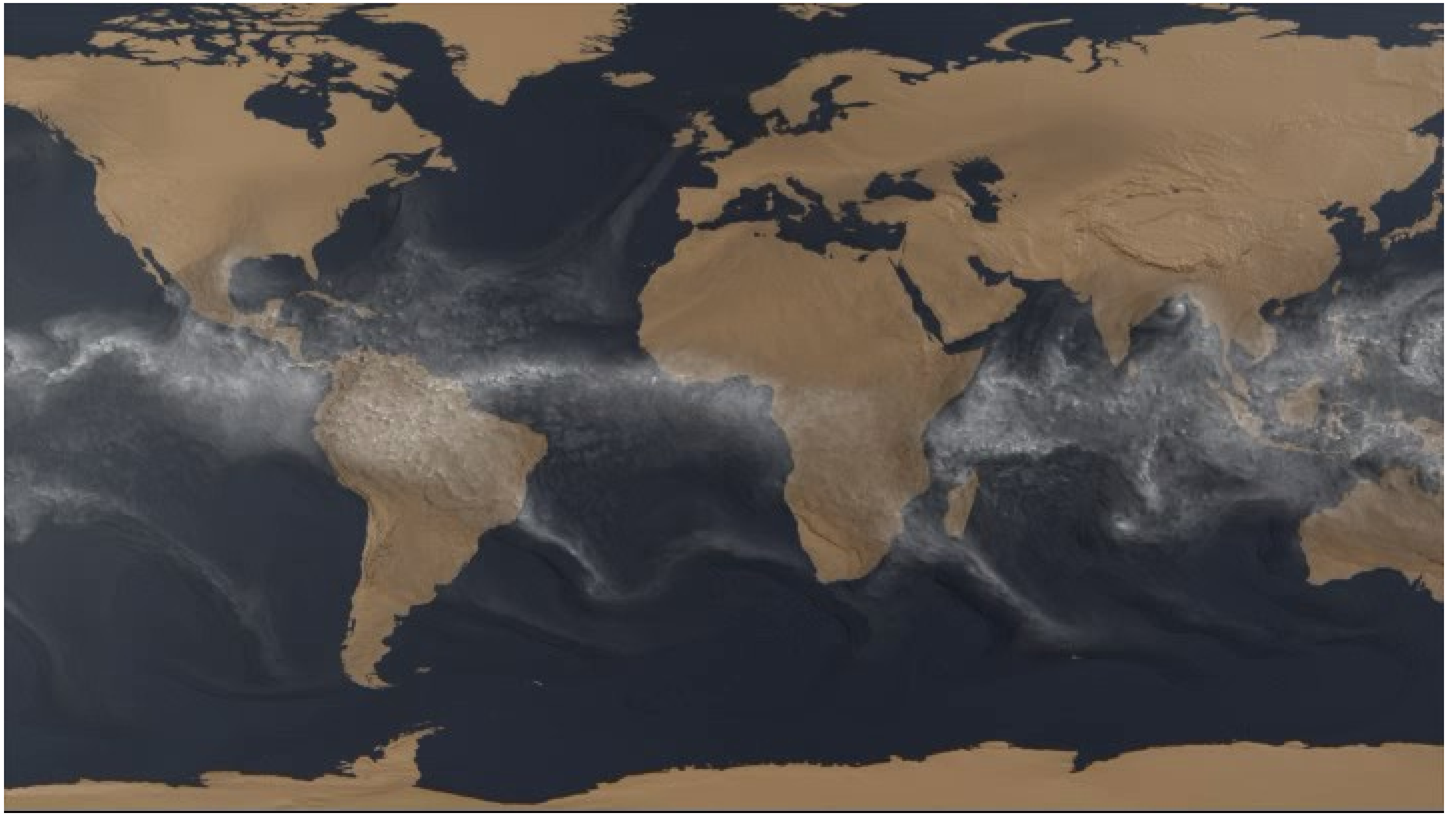
FV & Eulerian





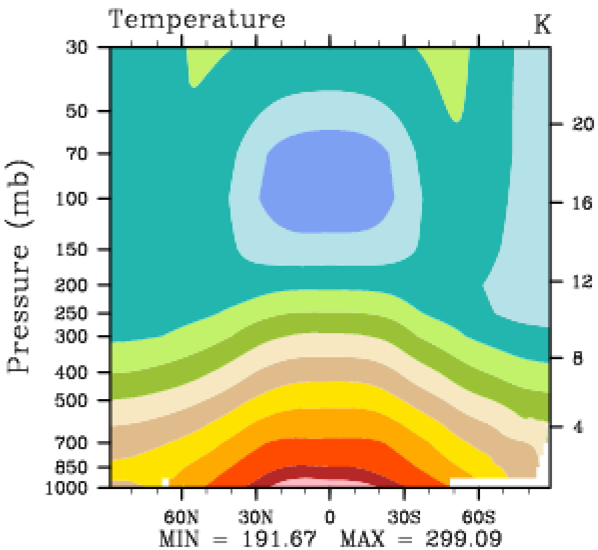
Real Planet Simulations
Comparisons with Observations

1/8 degree simulations on JaguarPF

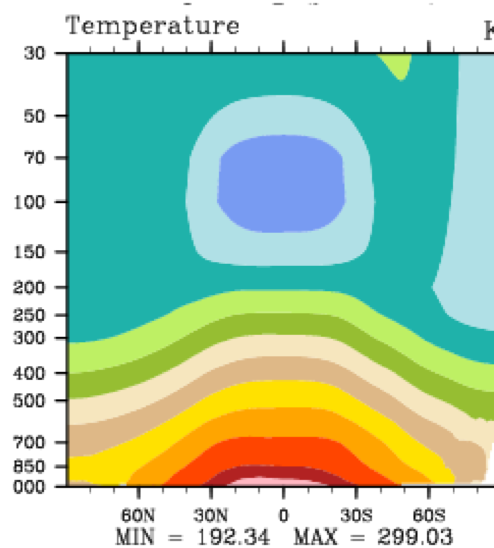


Real Planet: Zonal Mean Temperature

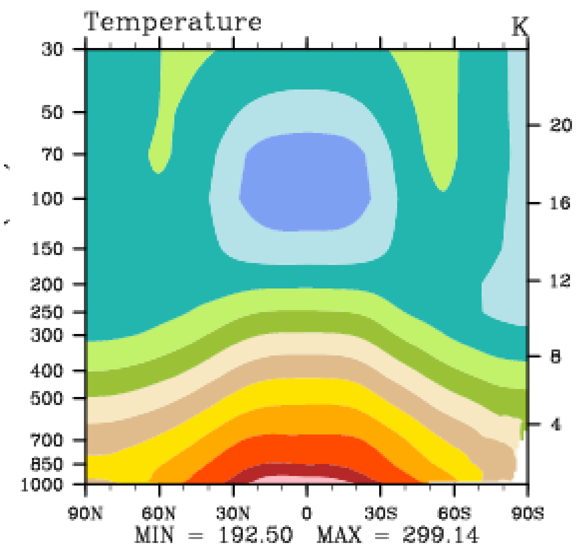
CCSM-EUL
T85

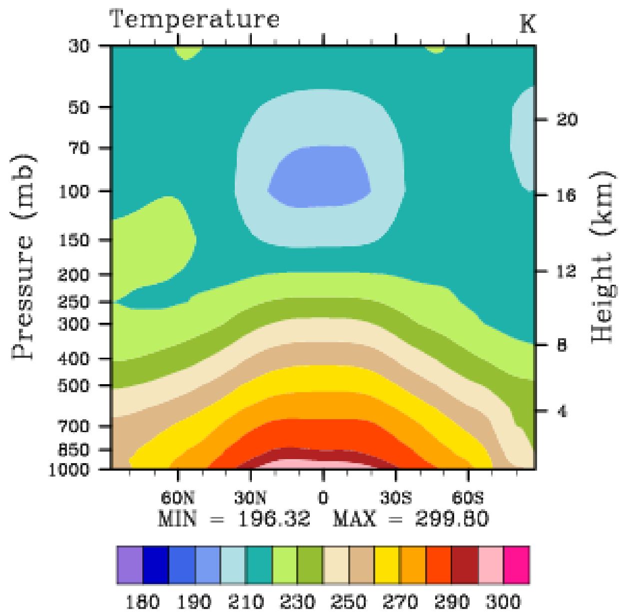


CCSM-HOMME
1 Degree



CAM-FV
0.9x1.25

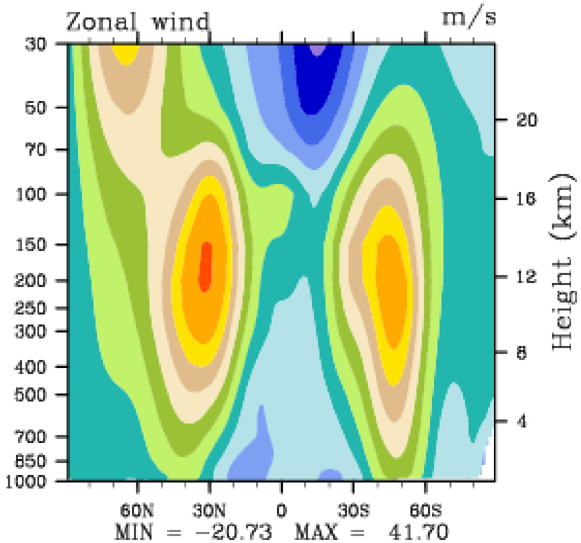




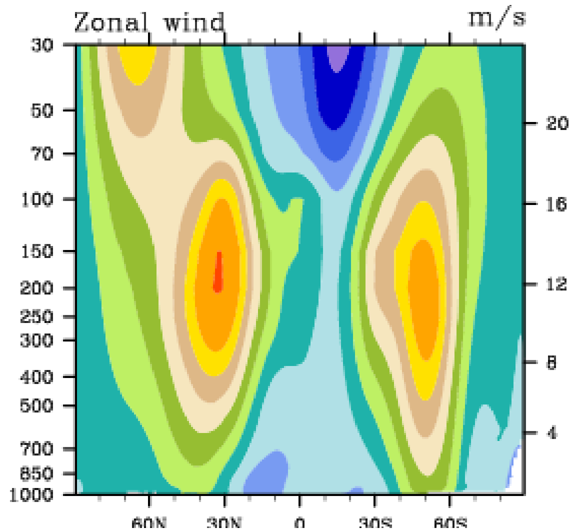
Observations

Zonal Mean Zonal Wind (DFJ)

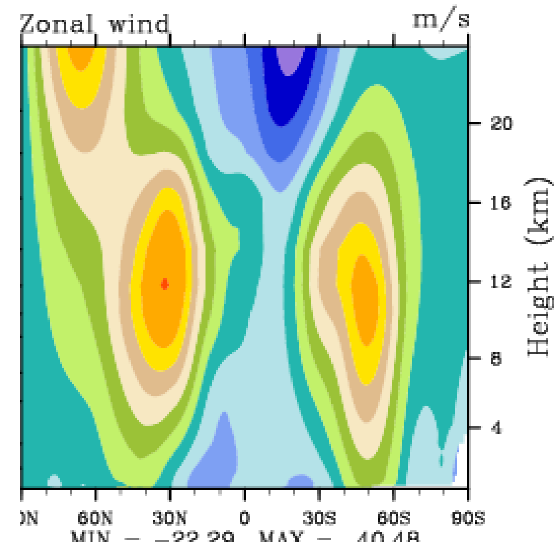
CCSM-EUL
T85



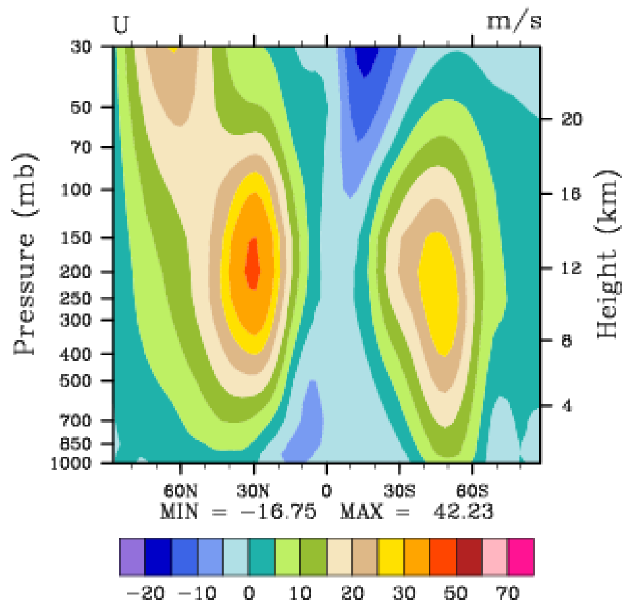
CCSM-HOMME
1 Degree



CAM-FV
0.9x1.25



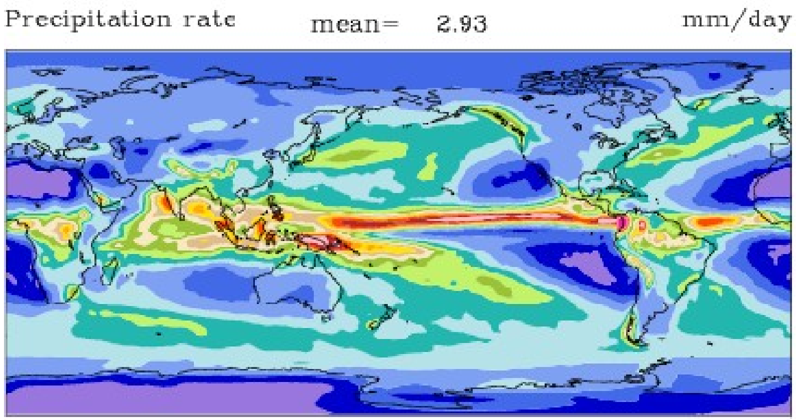
NCEP



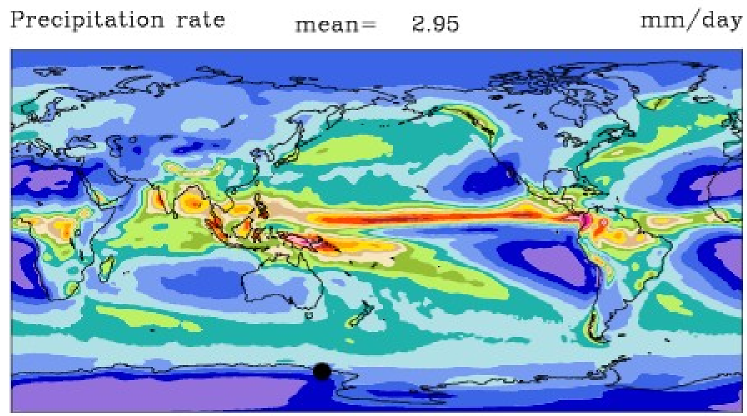
Observations

Precipitation Rate

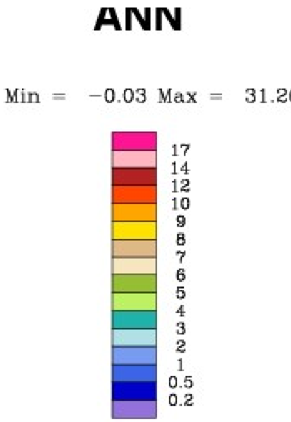
CCSM-EUL T85



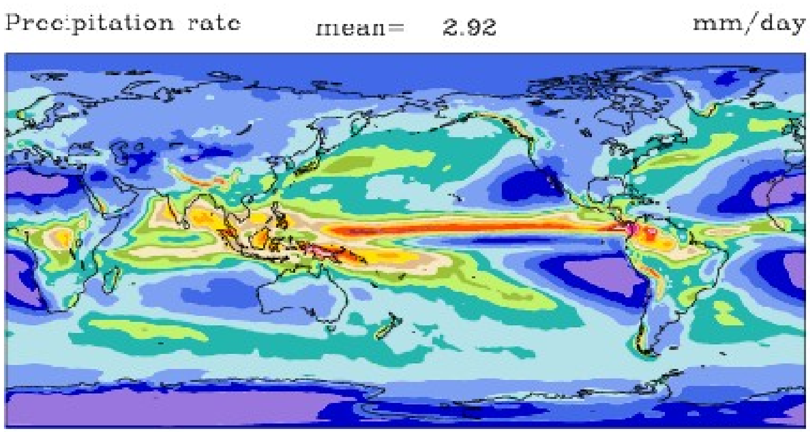
CCSM-HOMME 1 Degree



amipt85b (yrs 2-6)

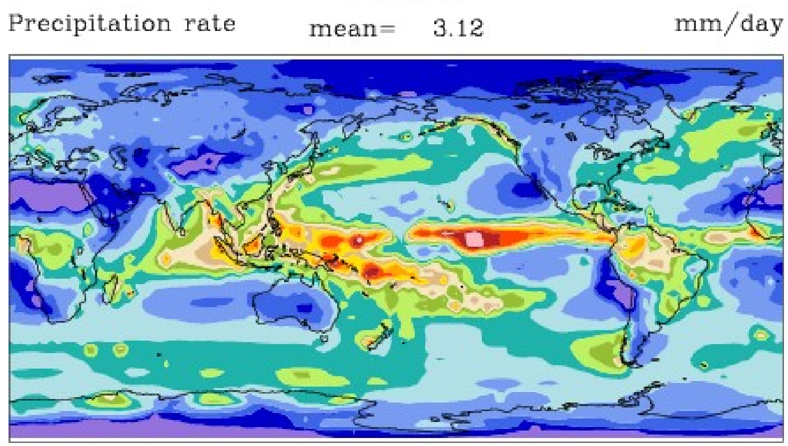


CAM-FV 0.9x1.25



OBSERVATIONS

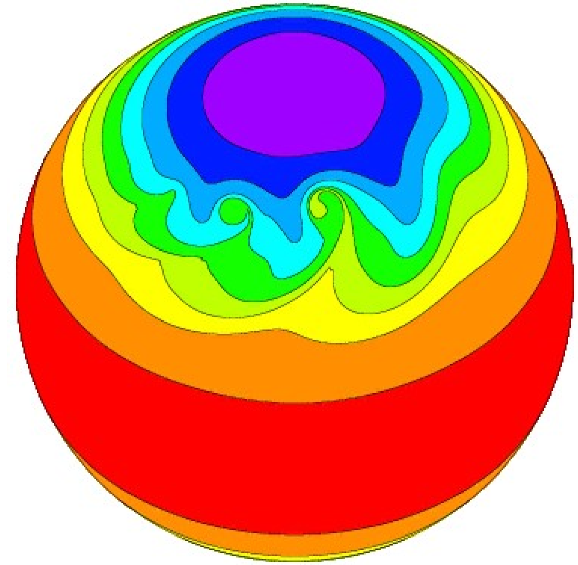
LEGATES



Conclusions

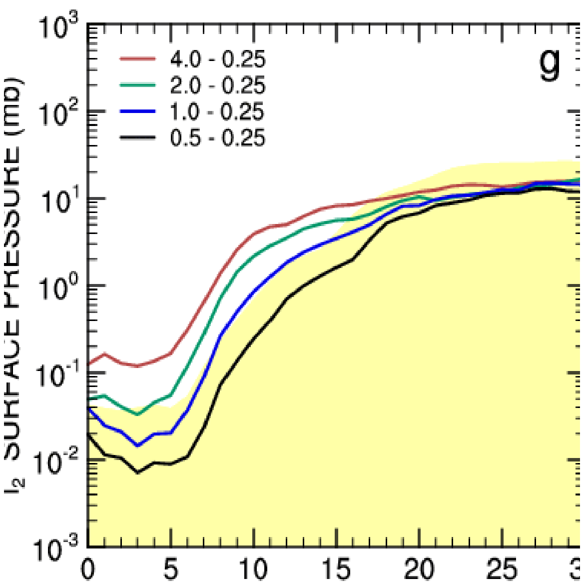
- Compatible version of SEM:
  - SEM: efficient, scalable explicit method which obtains high-order accuracy on unstructured grids.
  - Compatible formulation: locally conservative and preserves Lagrangian properties of vorticity
- Allows the CCSM to scale to O(100,000) processors.

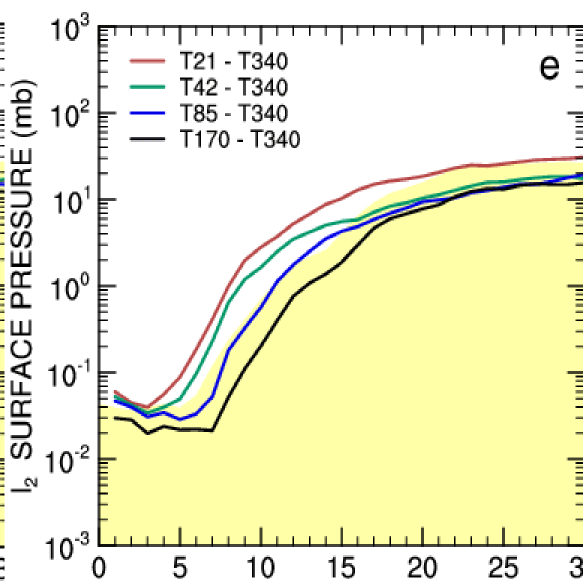
Test 2: Baroclinic Instability Test

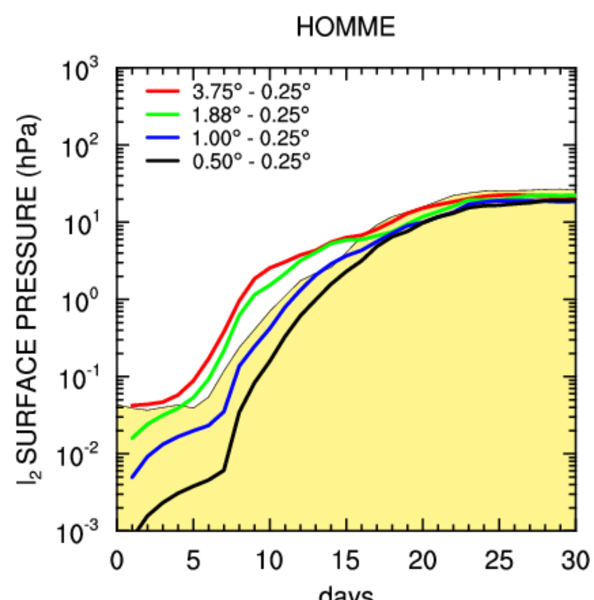


Jablonowski and Williamson, *A Baroclinic Instability Test Case for Atmospheric Model Dynamical Cores*, Q.J.R. Meteorol. Soc. (2006)

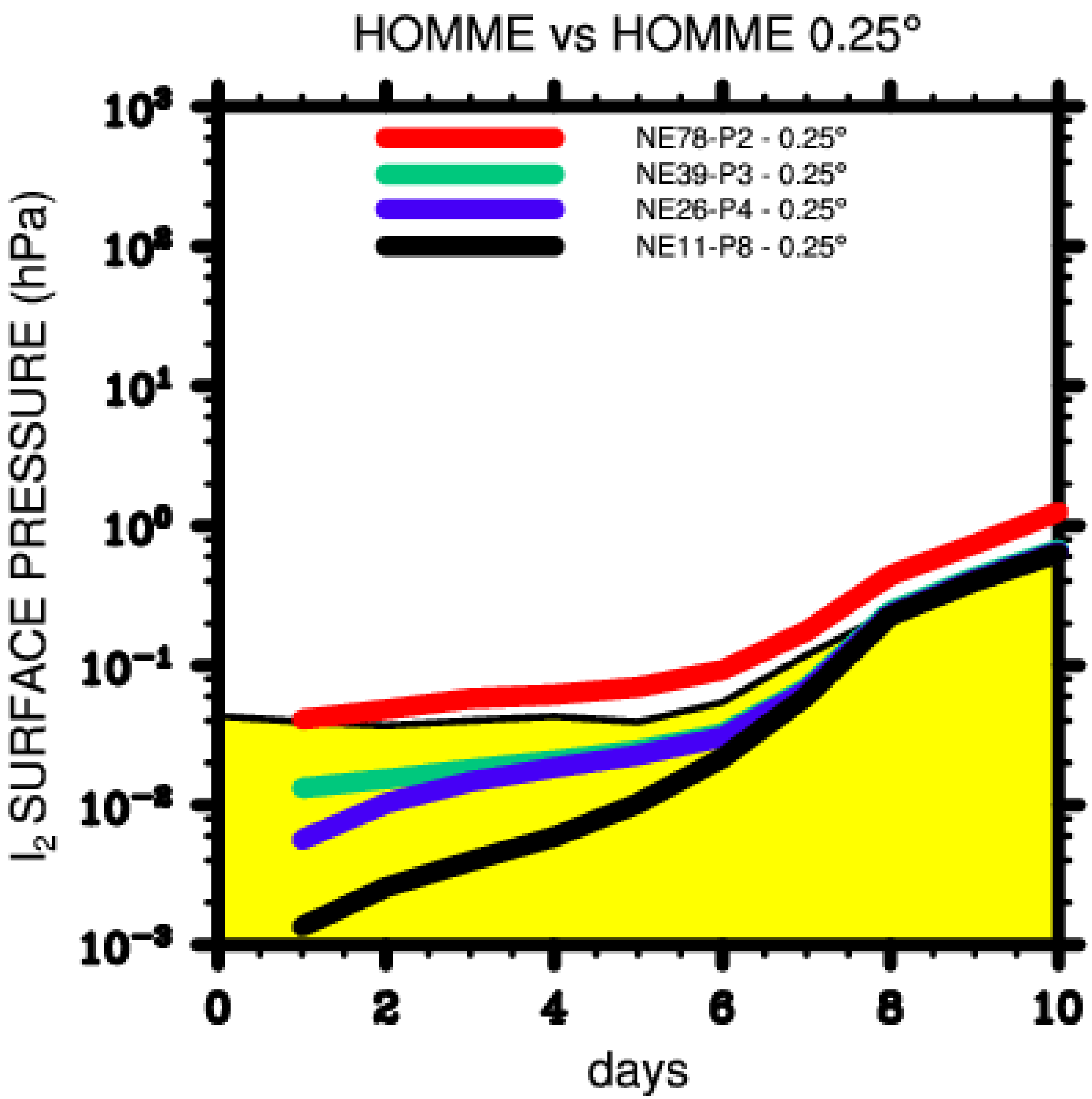
- Dynamical core only: no atmospheric physics
- L2 error in surface pressure as a function of time shown below
- Converges under mesh refinement to reference solution (uncertainty in reference solution is yellow shaded region)

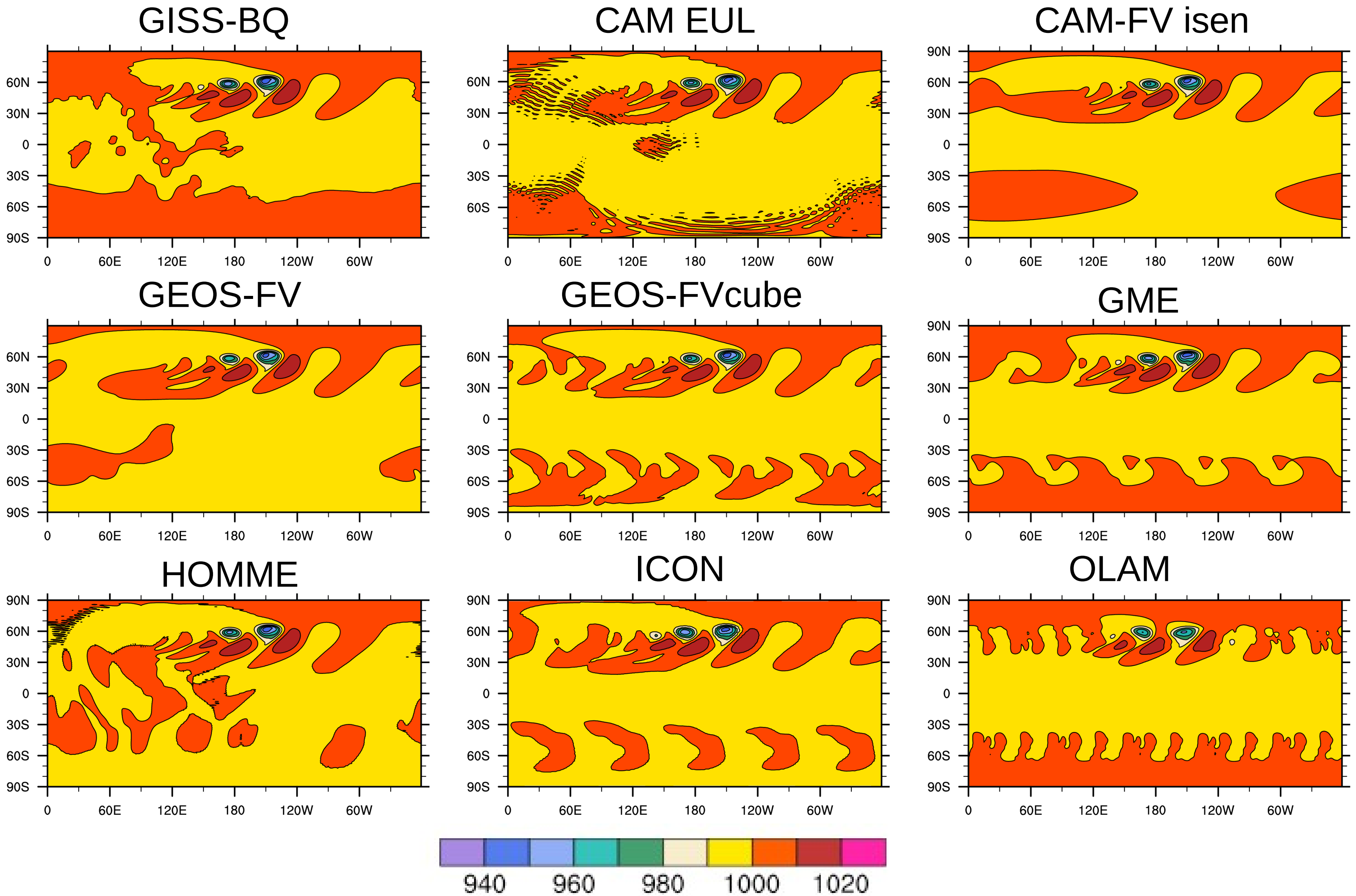






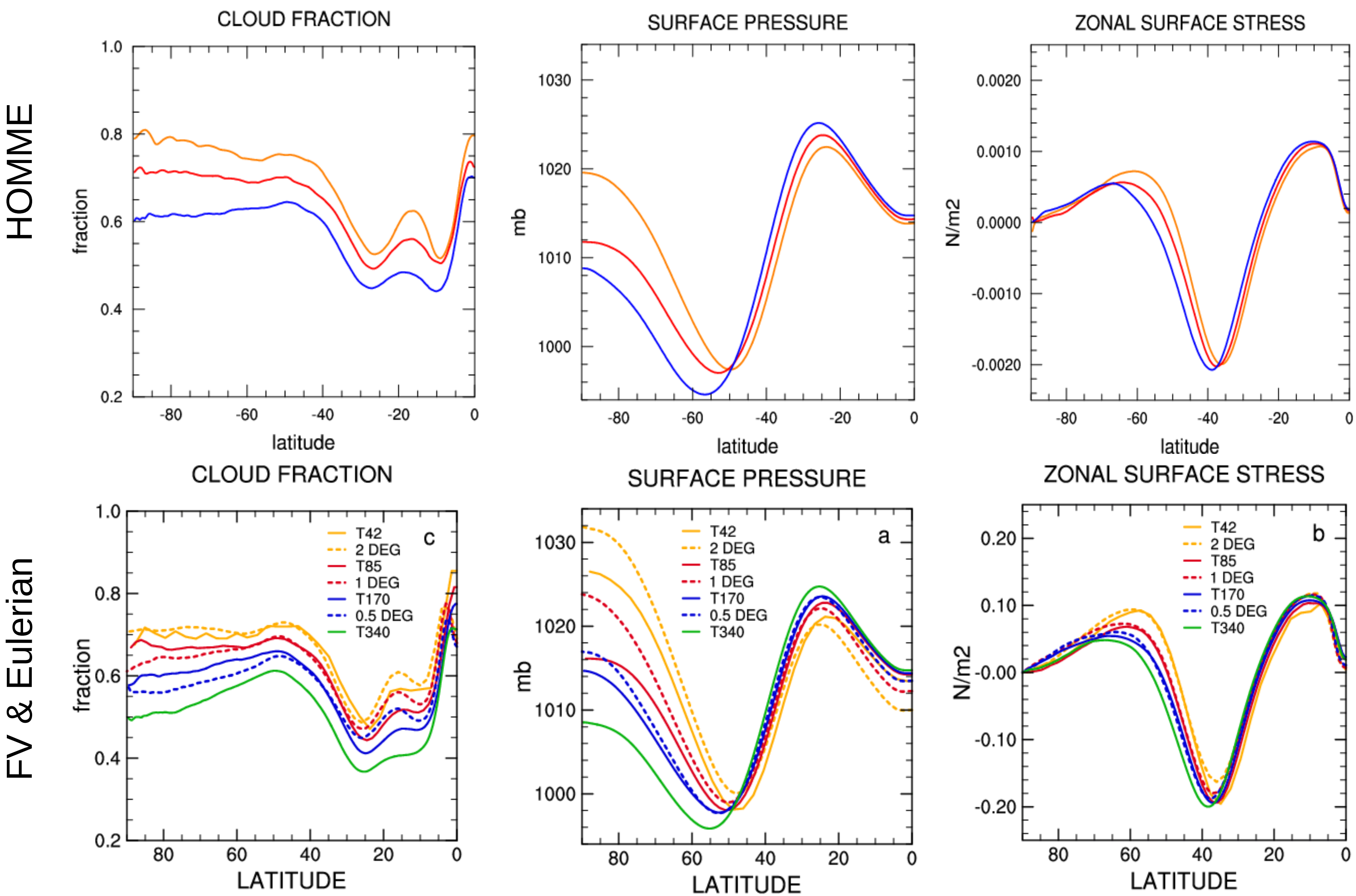
Order of Accuracy Comparison





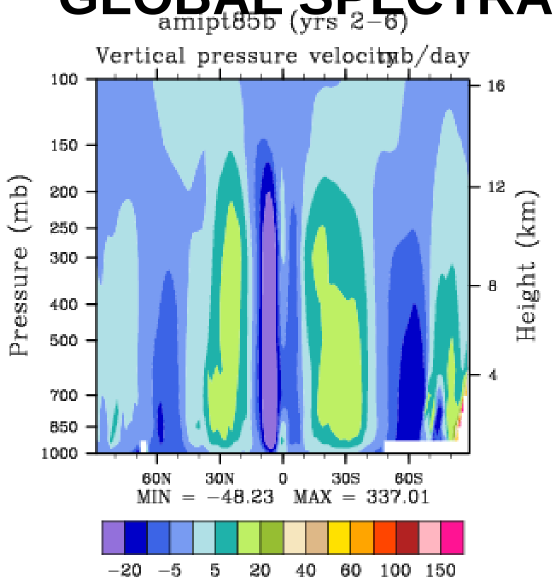
Test 2: Baroclinic instability. Surface pressure at day 9. The tests starts with balanced initial conditions that are overlaid by a Gaussian hill perturbation. The perturbation grows into a baroclinic wave. Some models show cubed-sphere or icosahedral grid imprinting in the Southern Hemisphere. High order methods show spectral ringing in the 1000mb contour.

Aqua Planet Experiment: Zonal Data
Comparison with FV & Eulerian Dycore

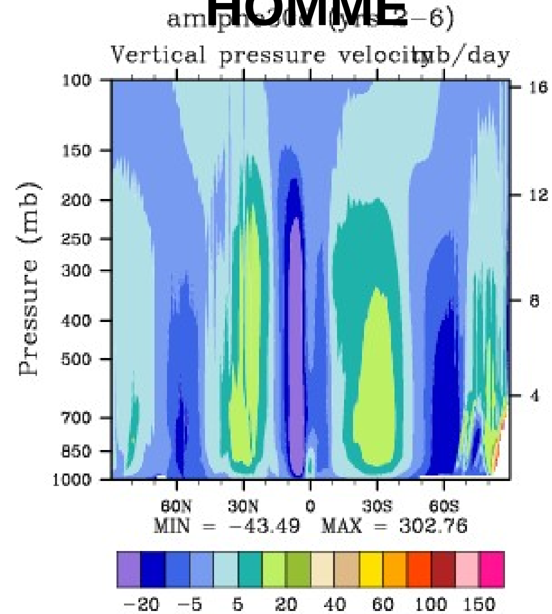


Zonal Mean Pressure Vertical Velocity

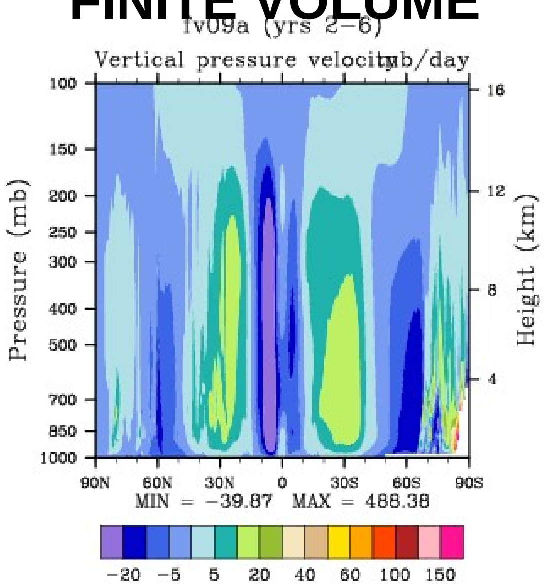
GLOBAL SPECTRAL



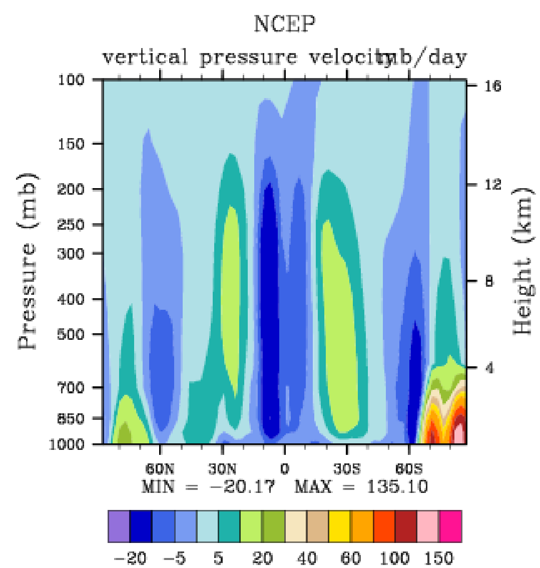
HOMME



FINITE VOLUME

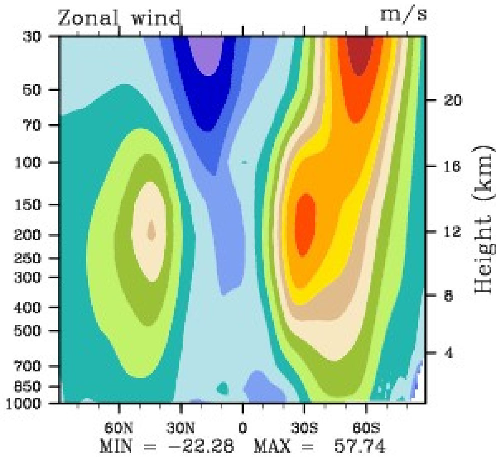


OBSERVATIONS

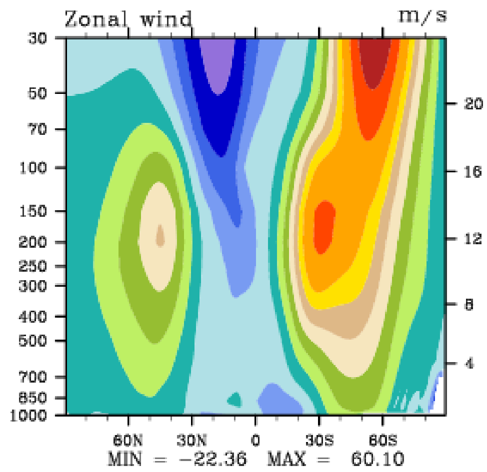


Real Planet: Zonal Means (DFJ, JJA)

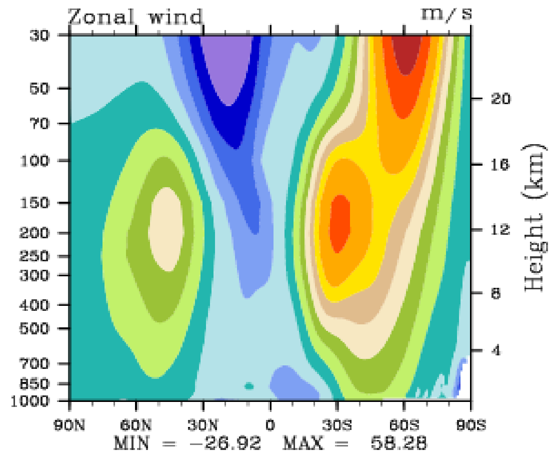
CCSM-EUL
T85

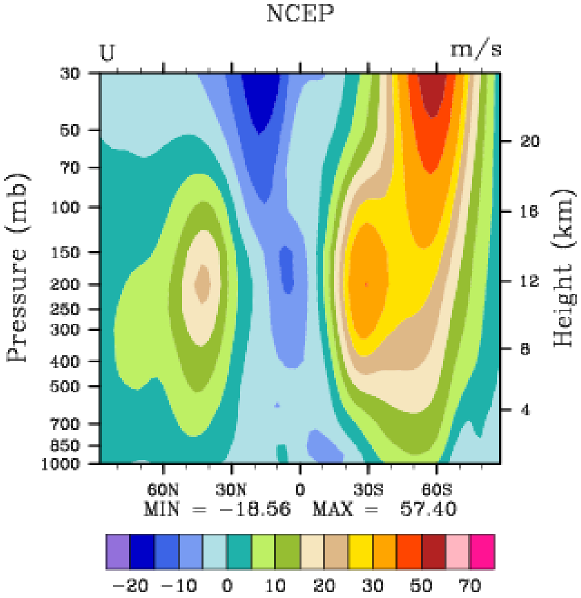


CCSM-HOMME
1 Degree



CAM-FV
0.9x1.25





Observations