Exploring a Multi-Resolution Approach within the Shallow-Water System

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Climate, Ocean and Sea-Ice Modeling Project
http://public.lanl.gov/ringler/ringler.html

Acknowledgments:

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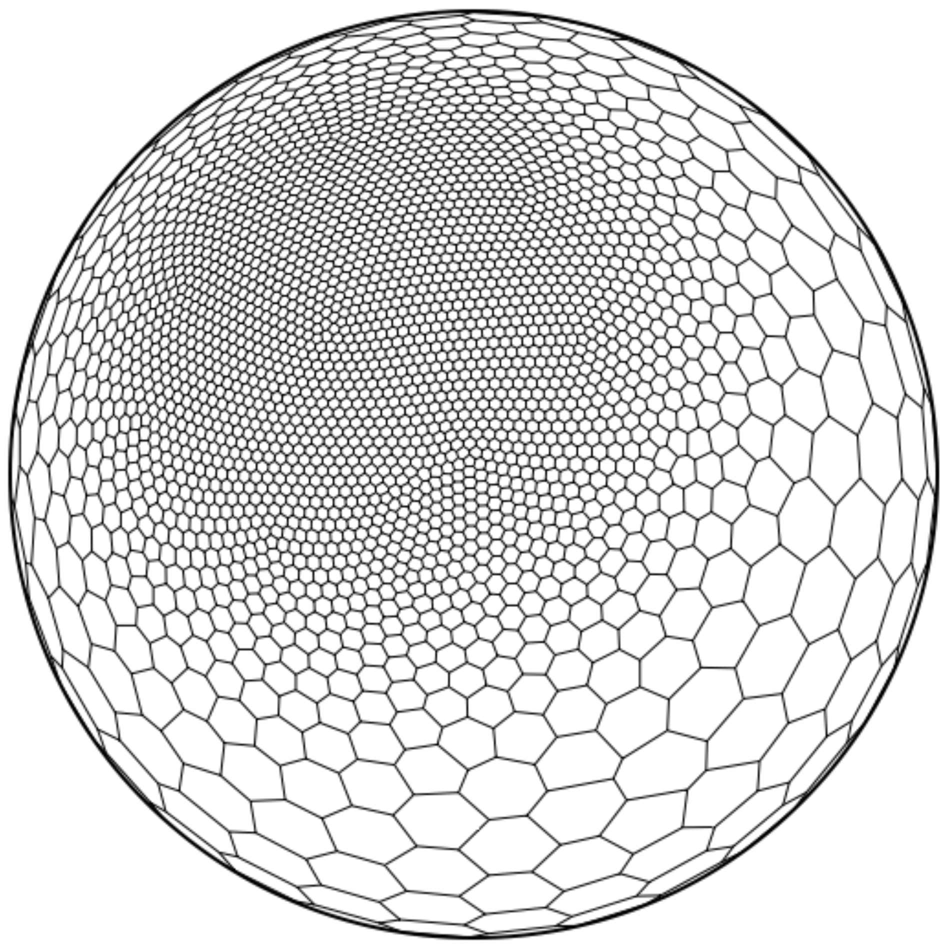
Research Topics:

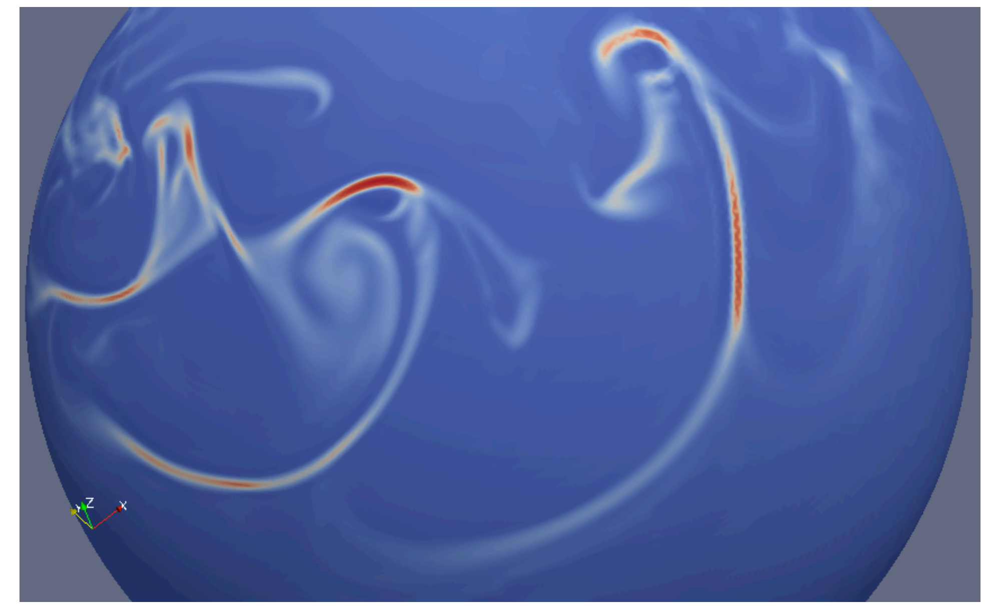
Mesh Generation
Numerical Methods
Analysis of Large-Scale Dynamics
Implementation on hybrid CPU/GPU architectures

Outline

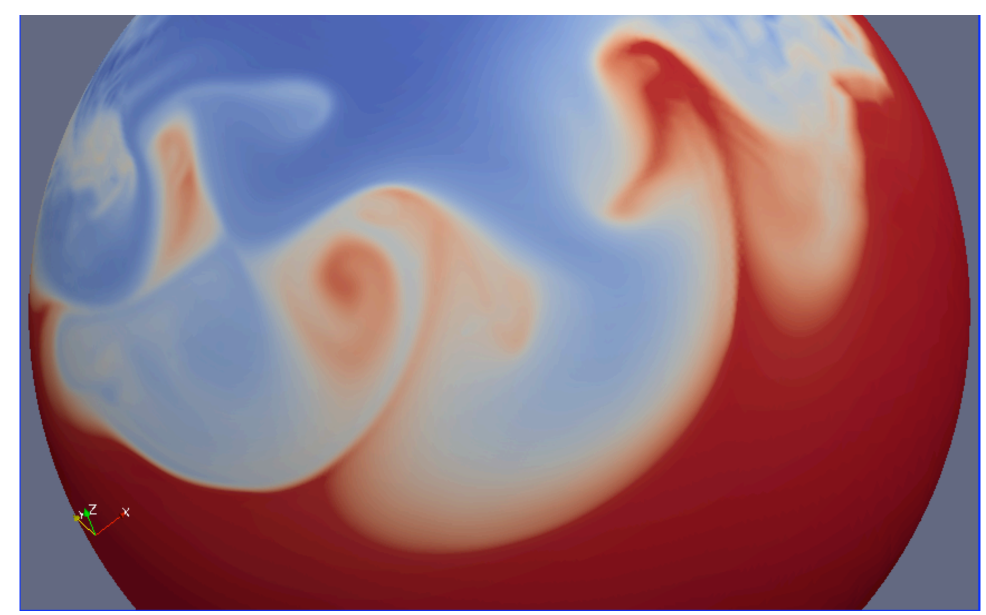
1. Some motivation for a multi-resolution approach.
2. Mesh generation for multi-resolution approaches.
3. A numerical method for variable resolution meshes.
4. An exploration of results.
5. Where to from here?

The system I will discuss is working in the setting of the primitive equations ....





relative vorticity



potential temperature

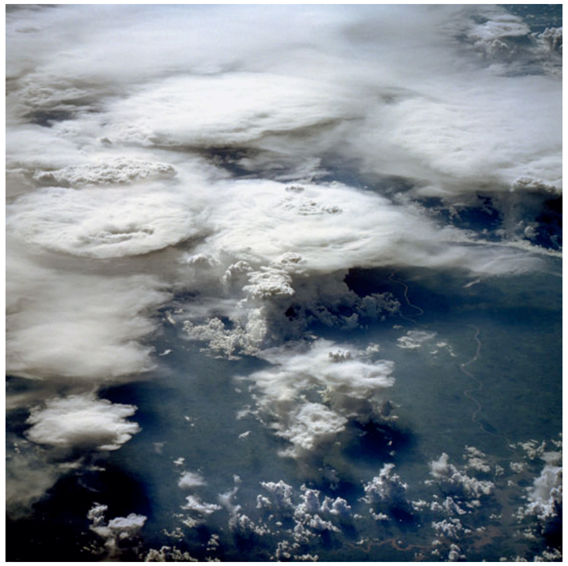
This is the Jablonowski and Williams baroclinic eddy test case on a mesh with 40 km grid spacing in high resolution zone and 320 km grid spacing in low resolution zone.

Section 1:

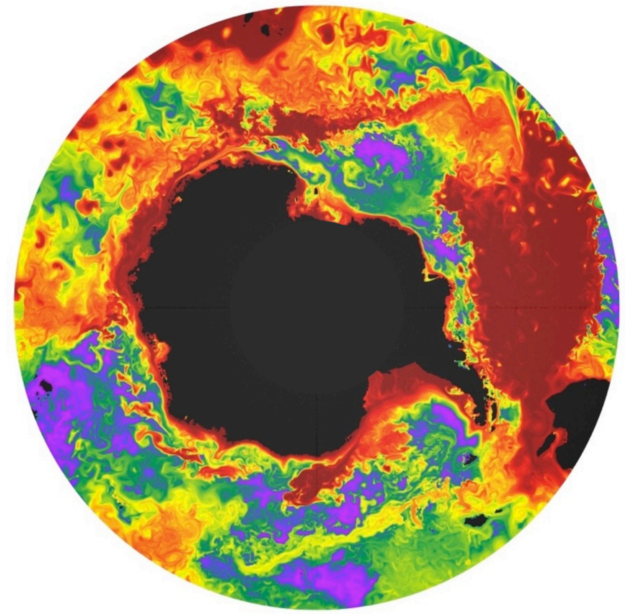
Motivation

Examples of processes that are currently unresolved.

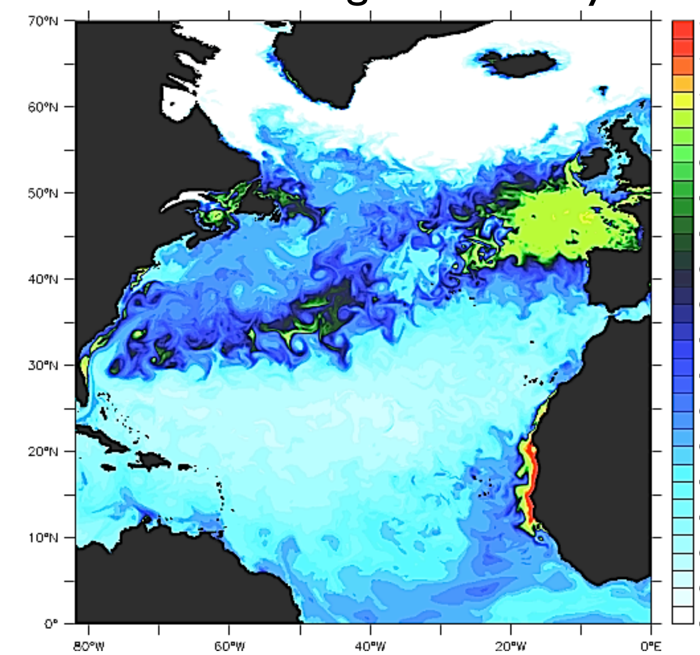
Cloud Processes



Ocean/Atmosphere Interaction



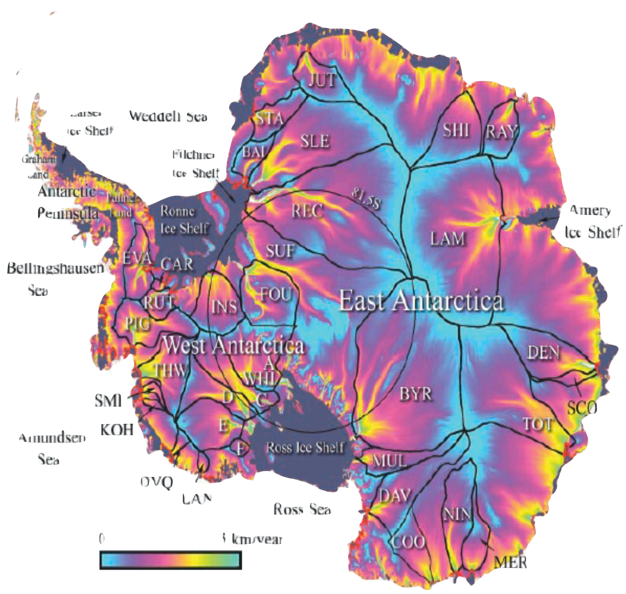
Ocean Biogeochemistry



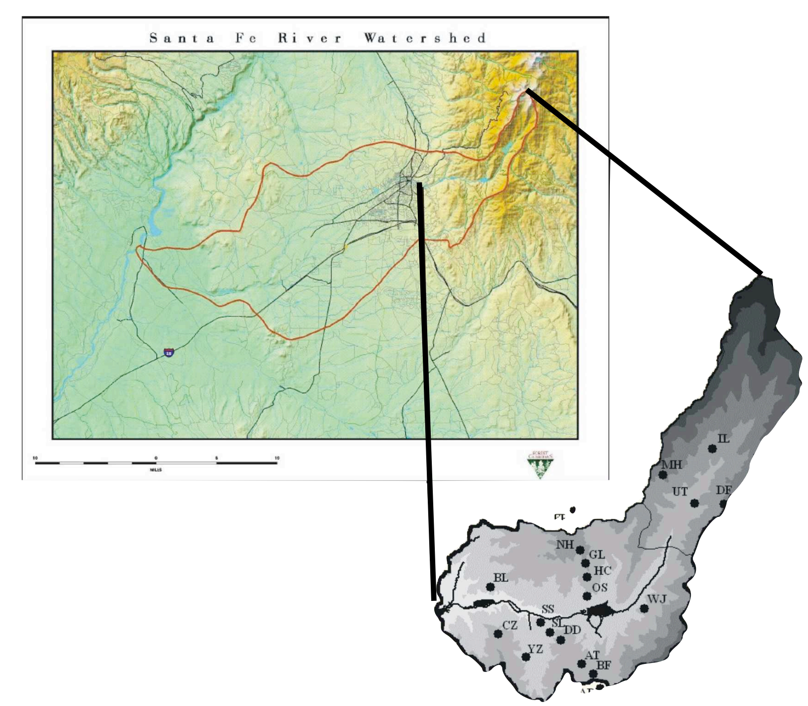
Each of these examples demonstrate scale-sensitive processes that might impact the climate system in a fundamental and important way.

The length scale of these processes is O(km).

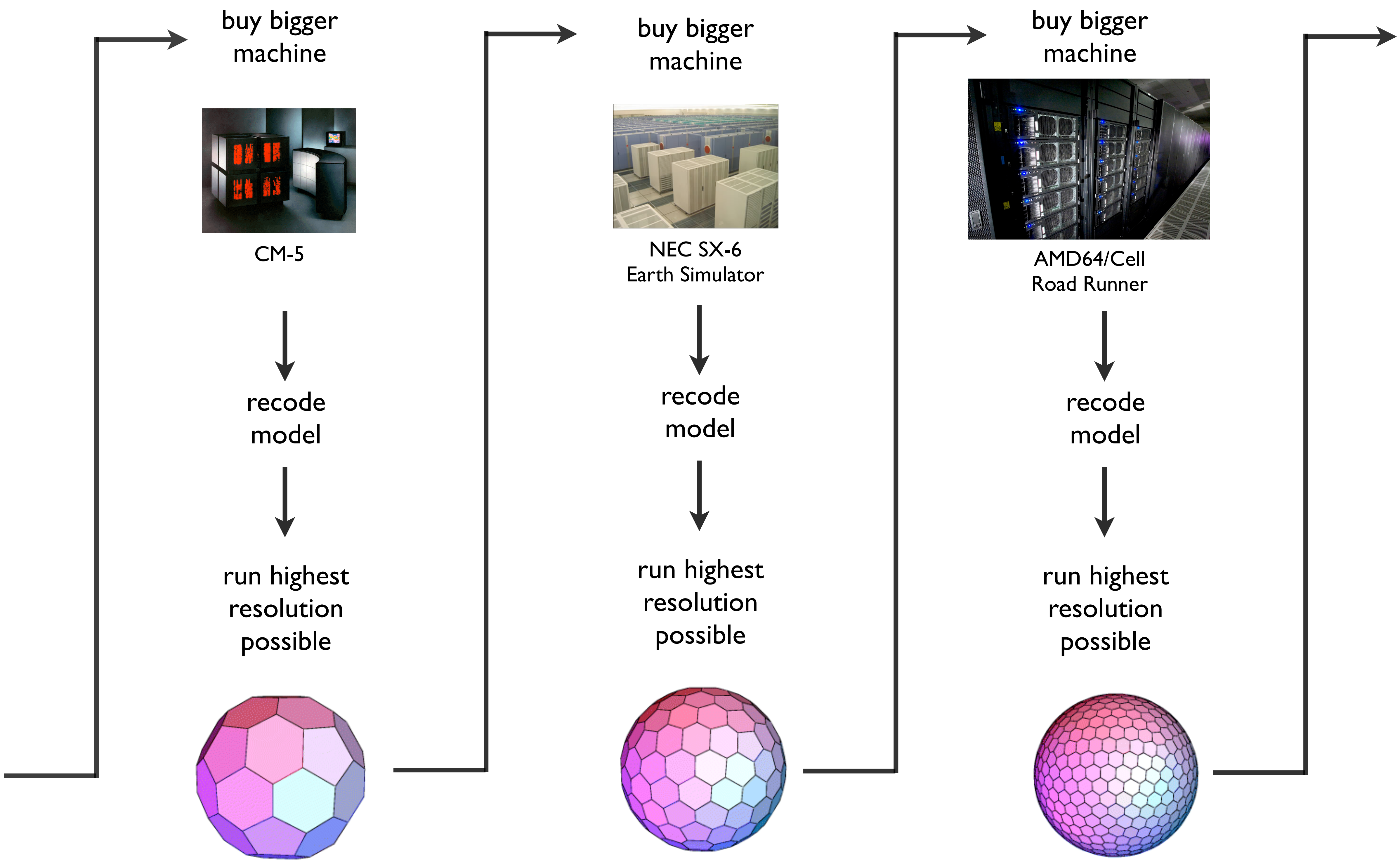
Ocean/Ice Shelf Interaction



Hydrology in Complex Terrain



Modus Operandi of Climate System Modeling



There is absolutely nothing wrong with this approach. Clearly it works.
The fact that it works should not preclude other complementary approaches.

How do we get there?

A quick back-of-the-envelope analysis should be very concerning to mid-career scientists:
IPCC resolution ~ 100 km
Target resolution ~ 1 km
Ratio of where we are to where we want to be ~ 2^7
Increase in computational resource required ~ 8^7 ~ 2e6
Time to reach target assuming a doubling in resources every 18 months ~ 22 years

Looking at the massive effort required to increase the resolution climate models, 22 years is not unrealistic. Maybe it is 15 years, or maybe it it 25 years. <u>Either way, it is a long time.</u>

The climate modeling community would benefit from an <u>additional</u> approach that allows <u>some</u> of these processes to be resolved at <u>some</u> locations.

We could then consider the notion of approaching the target from additional directions, for example by expanding the areas over which these fine-scale processes are simulated.

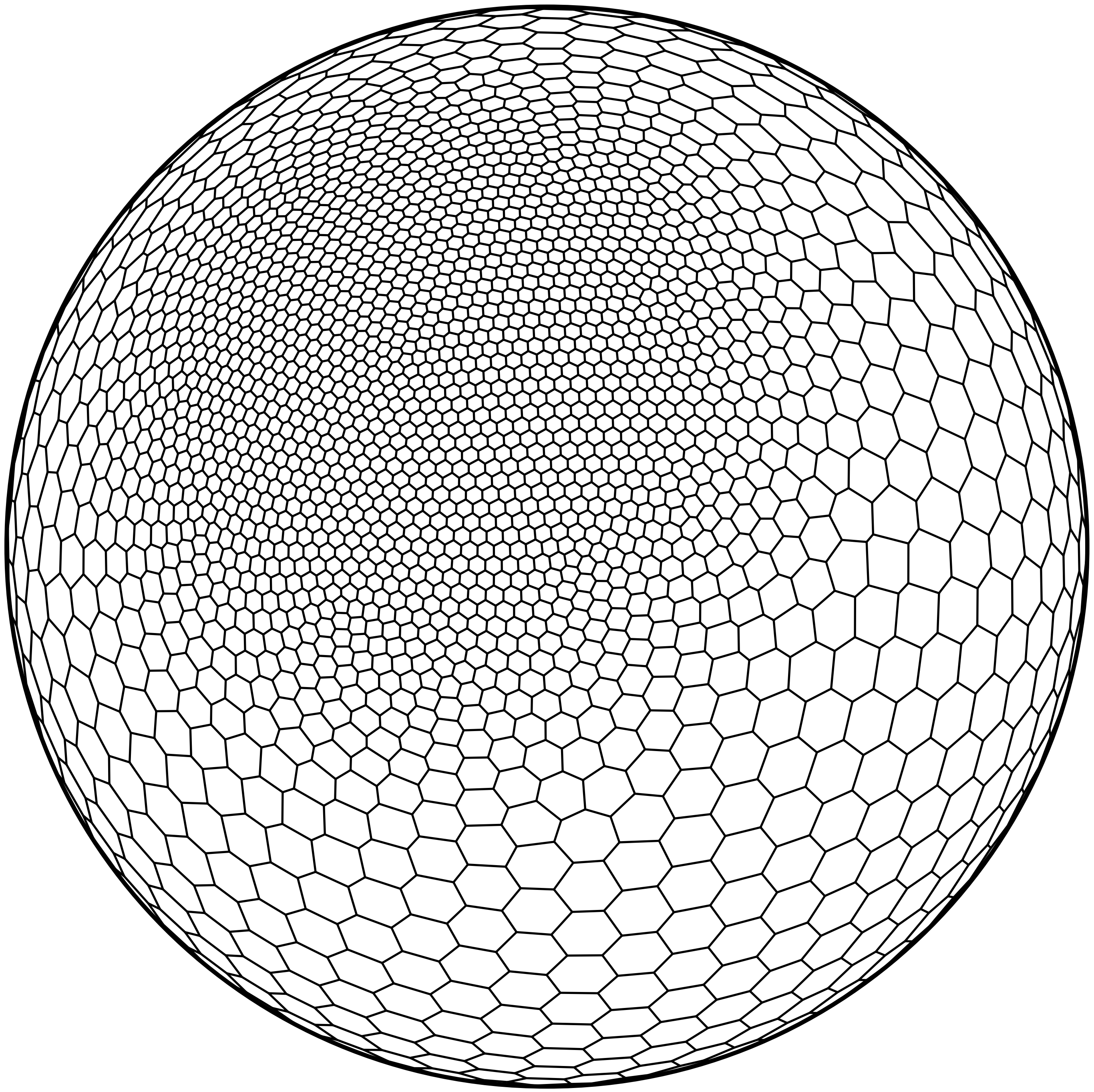
A multi-resolution approach based on Spherical Centroidal Voronoi Tessellations

Underlying principles of this approach:

1. This is a global modeling framework

2. The approach allows us to "paint" the sphere with regions enhanced resolution.

3. The resulting mesh is conforming (i.e. no hanging nodes) and is a Voronoi diagram.



Summary of Section 1

1. Having access to a multi-resolution climate system model would make for a powerful research tool.

2. A multi-resolution climate system model might even lead to a better simulation of the observed climate system.

Section 2:

An approach of multi-resolution mesh generation

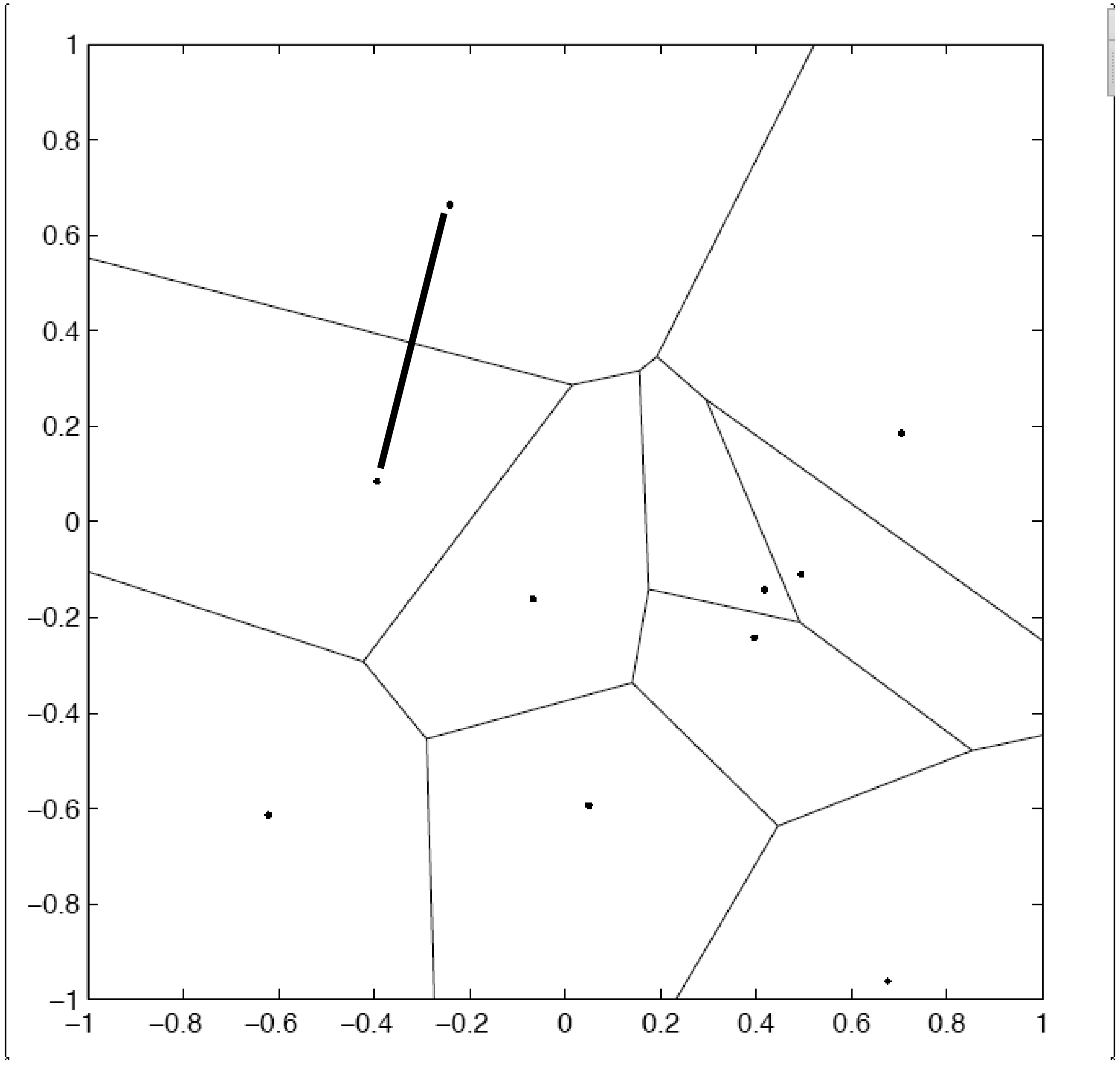
Definition of a Voronoi Tessellations

Given a region, S, and a set of generators, z_i ...

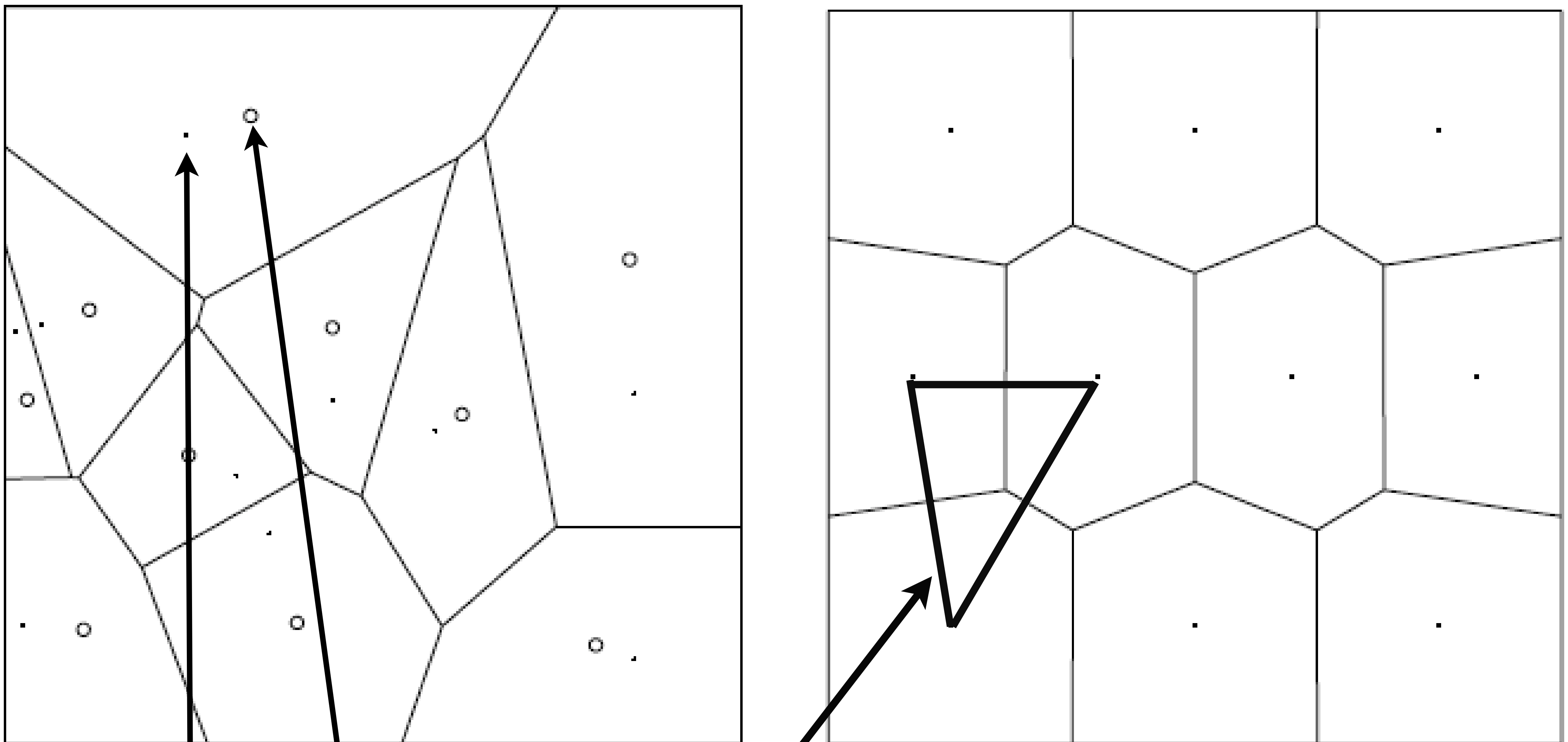
The Voronoi region, V_i, for each z_i is the set of all points closer to z_i than z_j for j not equal to i.

We are guaranteed that each edge (aka face) is shared by exactly two generators.

We are guaranteed that the line connecting generators is orthogonal to the shared edge and is bisected by that edge.



Definition of a Centroidal Voronoi Tessellations



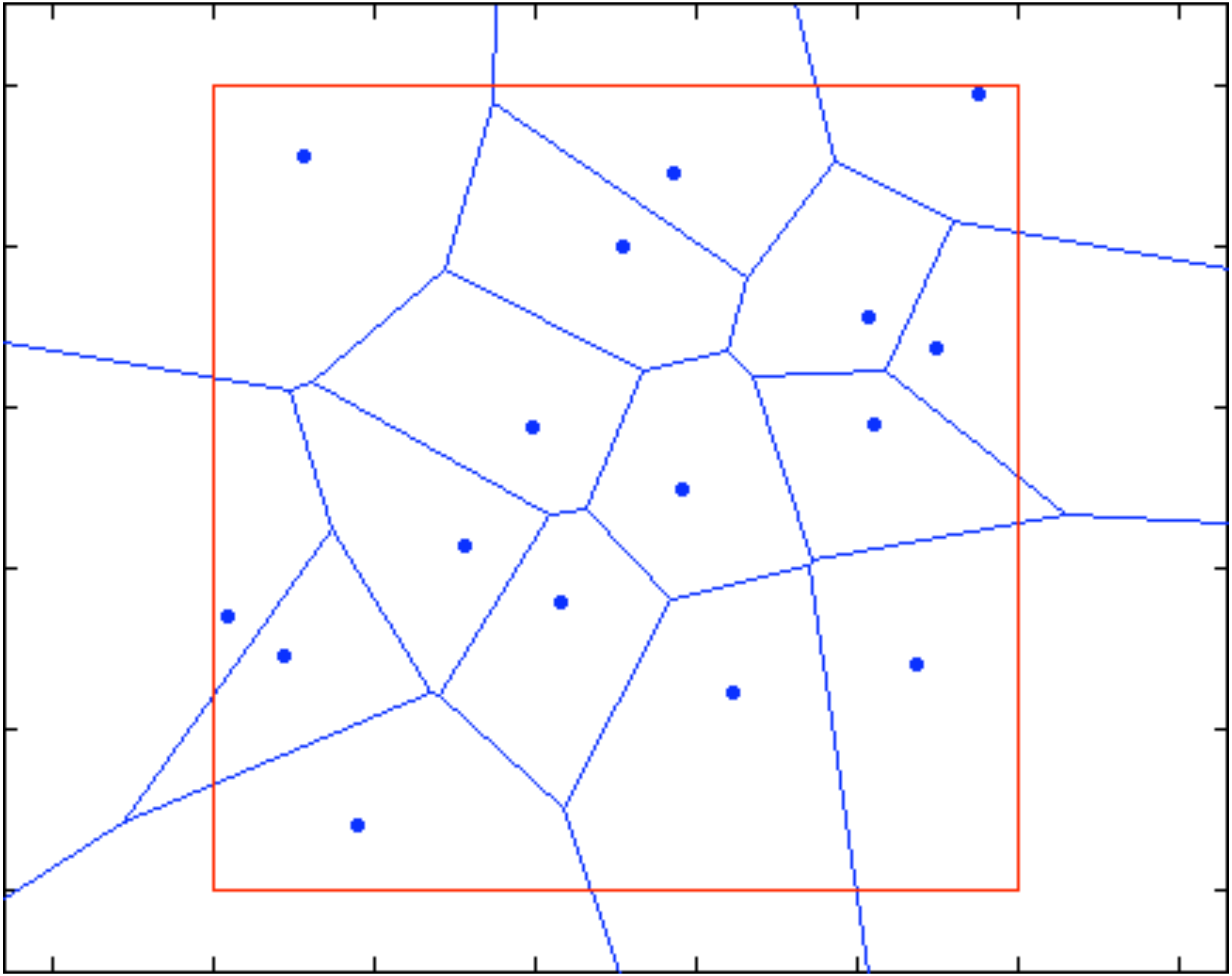
Dual tessellation

z_i

z_i^* = center of mass wrt a **user-defined** density function

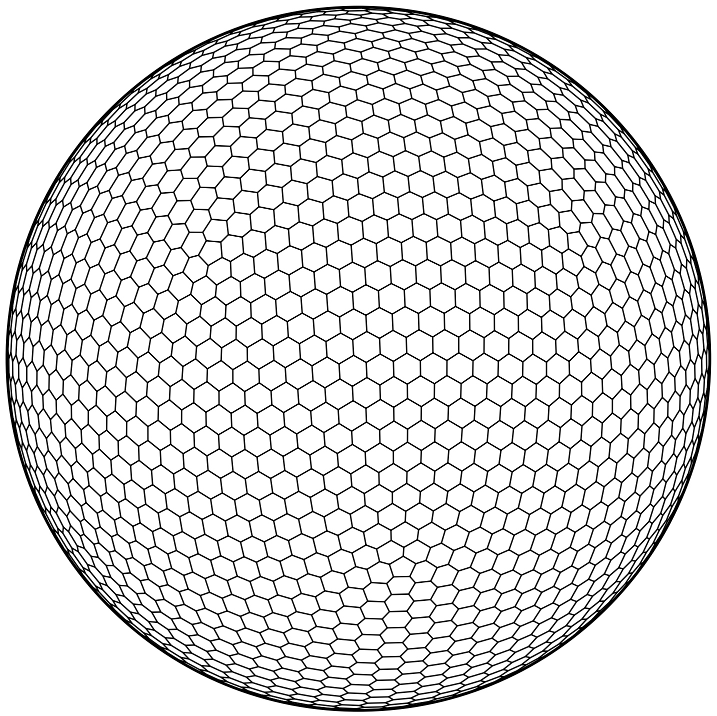
$$z^* = \frac{\int_V w\rho(w)\,dw}{\int_V \rho(w)\,dw}$$

Centroidal Voronoi Diagrams:
Voronoi diagram where generator is also center of mass.

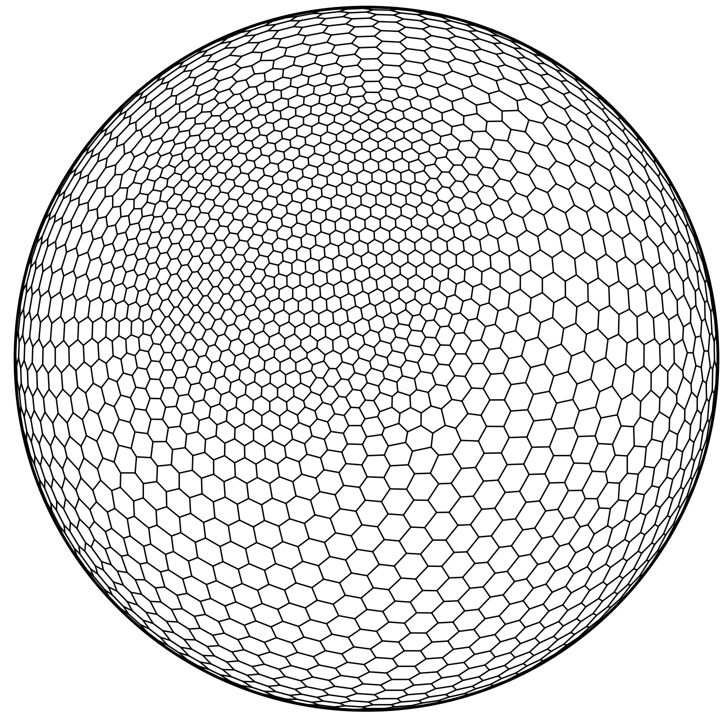


A peek to where we are heading ....

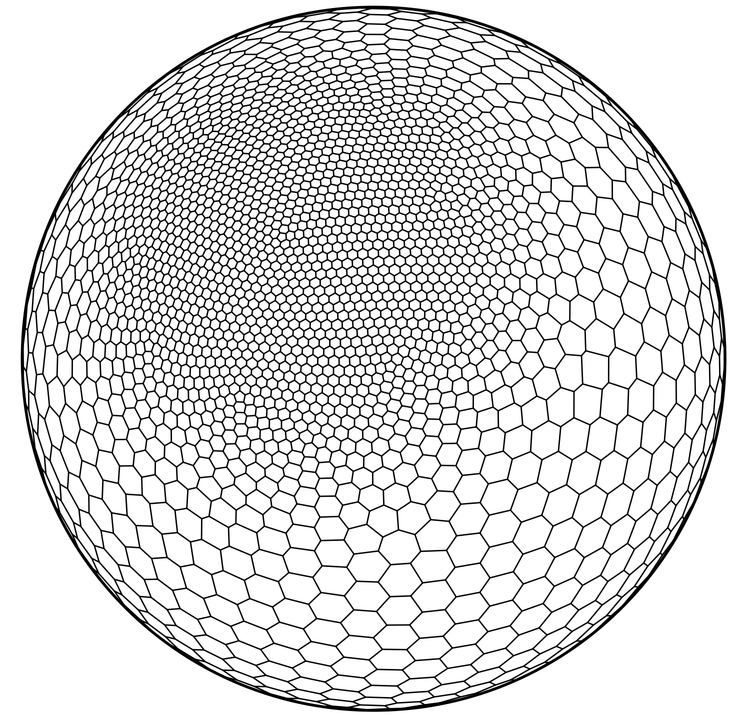
these are Spherical Centroidal Voronoi Tessellations (SCVTs)



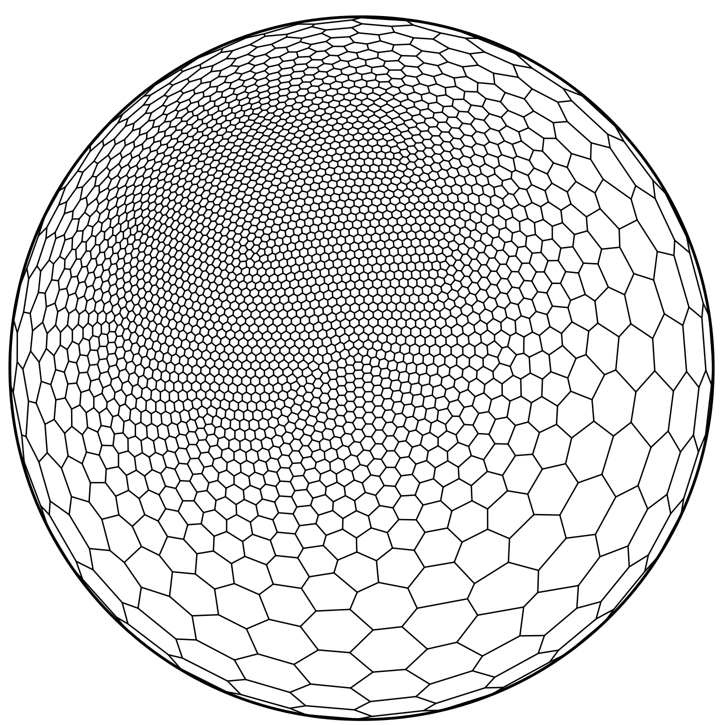
x1



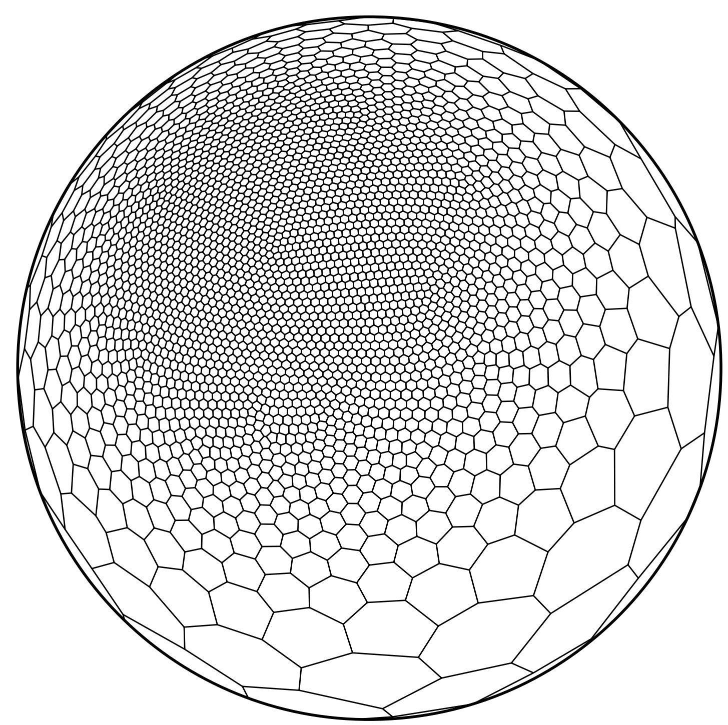
x2



x4



x8



x16

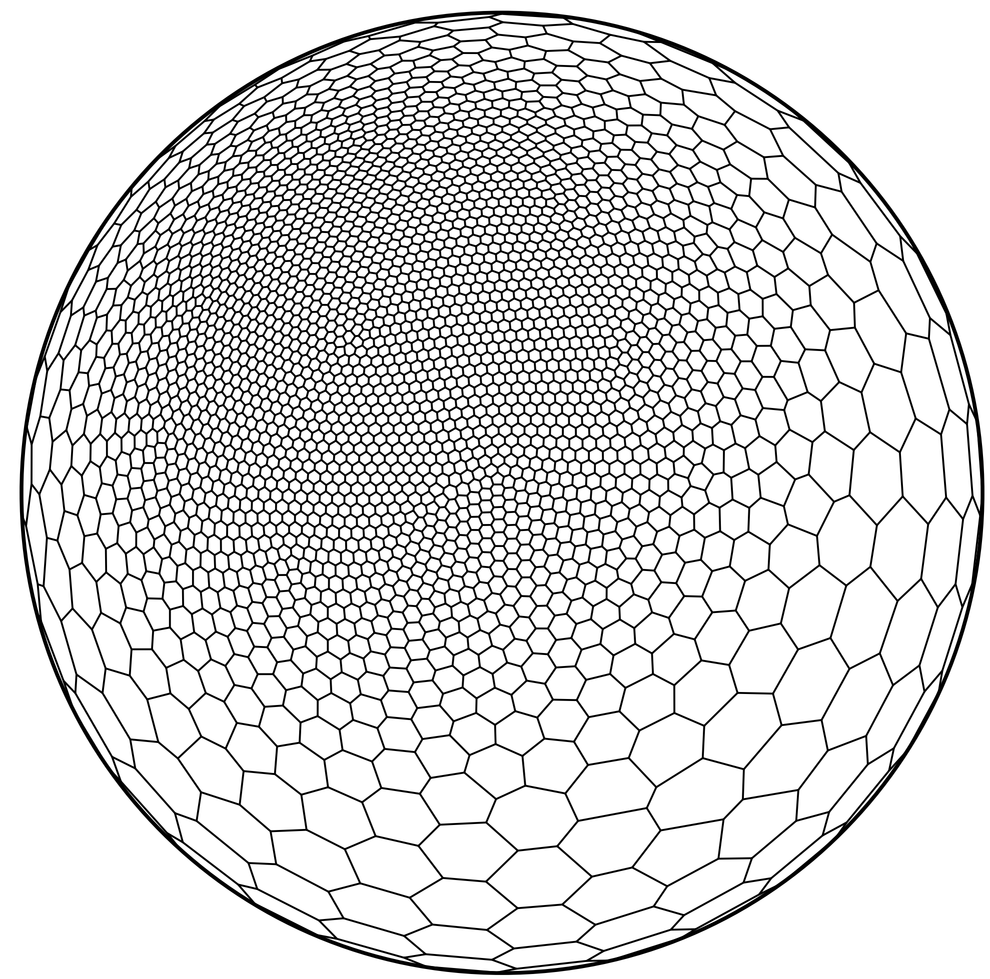
As far as mesh generation goes, SCVTs have a strong mathematical foundation

$$\frac{h(\mathbf{x_i})}{h(\mathbf{x_j})} \approx \left(\frac{\rho(\mathbf{x_j})}{\rho(\mathbf{x_i})}\right)^{d+2}$$

d : dimension of space to tessellate

$h(\mathbf{x})$: nominal grid spacing

$\rho(\mathbf{x})$: user-defined density function



Defining a family of density functions ....

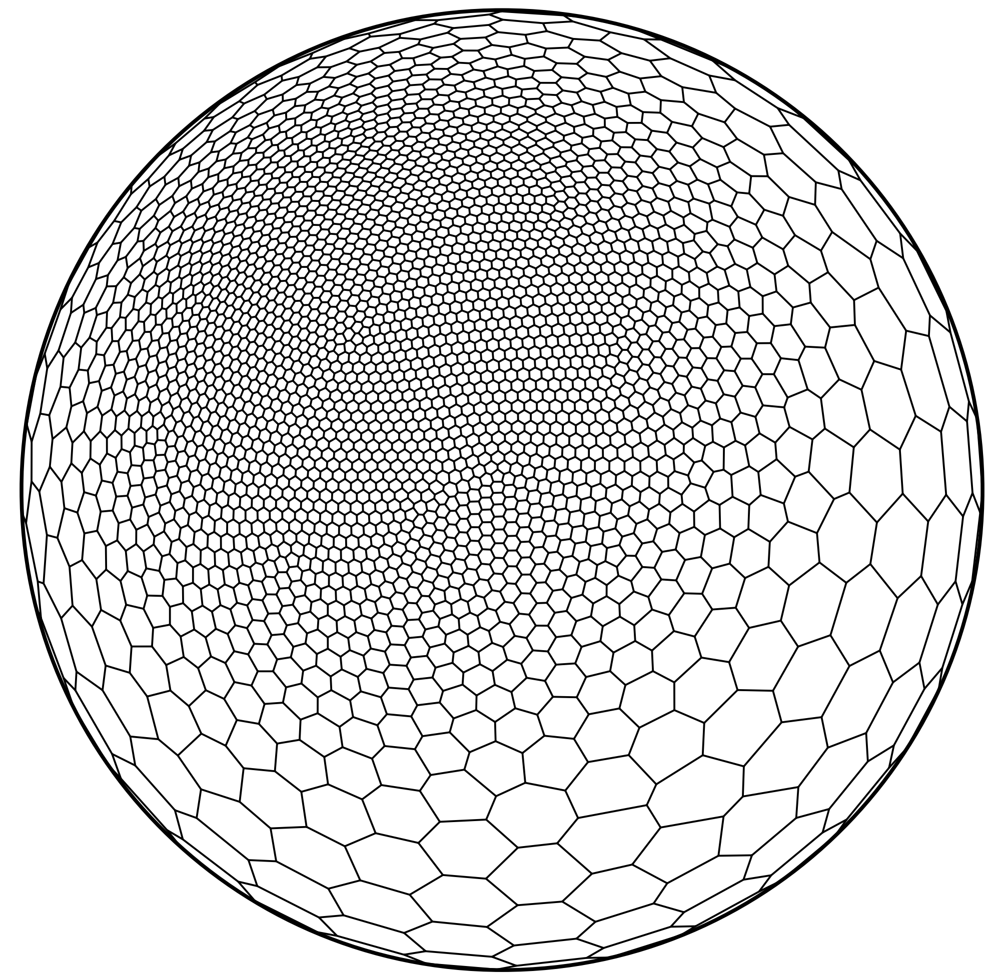
$$\rho(\mathbf{x}_i) = \frac{1}{2\beta}\left(\tanh\left(\frac{size - r}{width}\right) + 1\right) + ratio$$

$size$: size of high-res region

$width$: width of transition zone

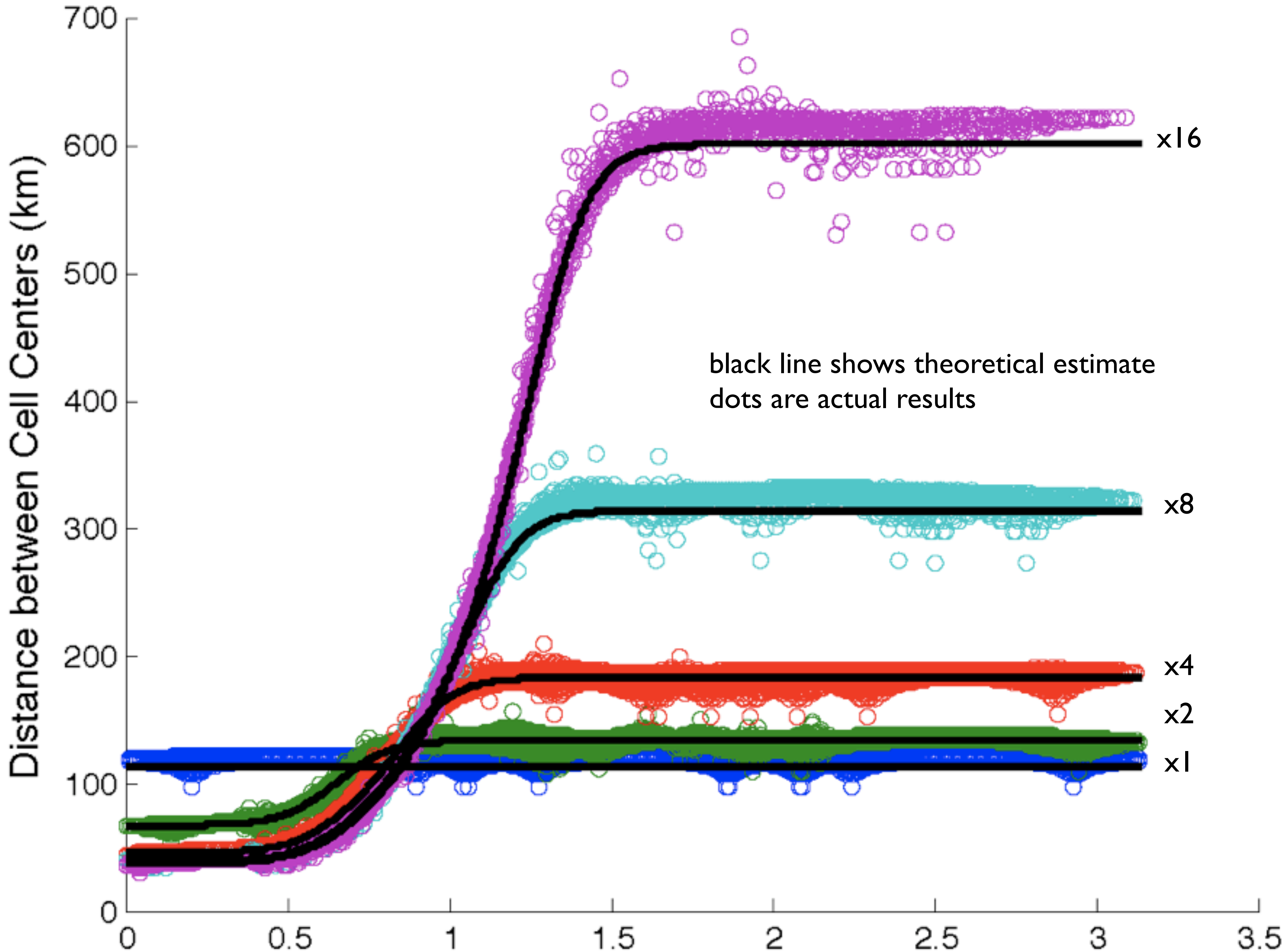
$$ratio = \left[\frac{h(\mathbf{x}_{low})}{h(\mathbf{x}_{high})}\right]^{\frac{1}{4}}$$

r : distance from center of high-res region



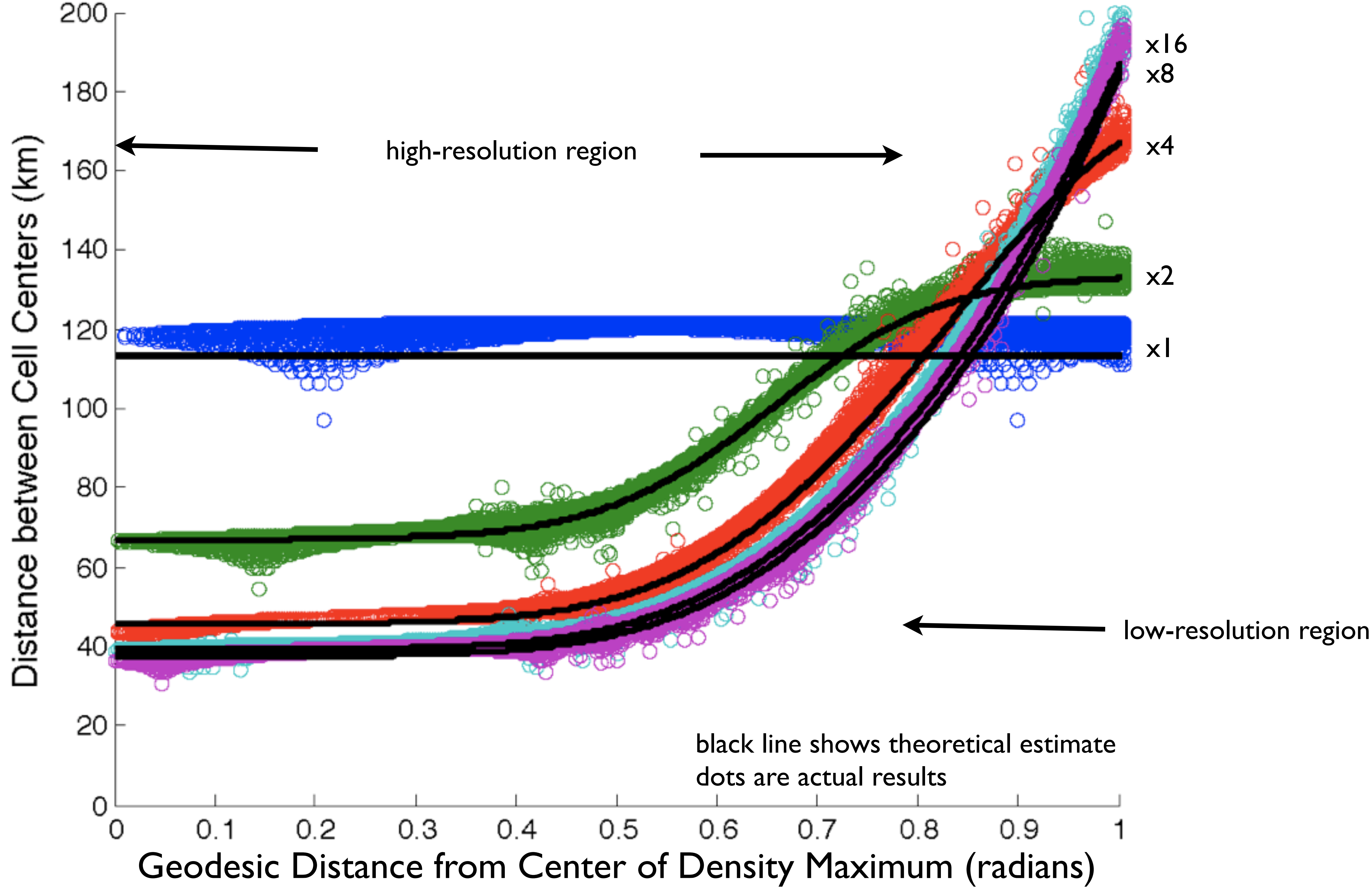
The density function is a simple three parameter system from which only the impact of the ratio parameter will be explored.

Some grid metrics with 40962 nodes ....

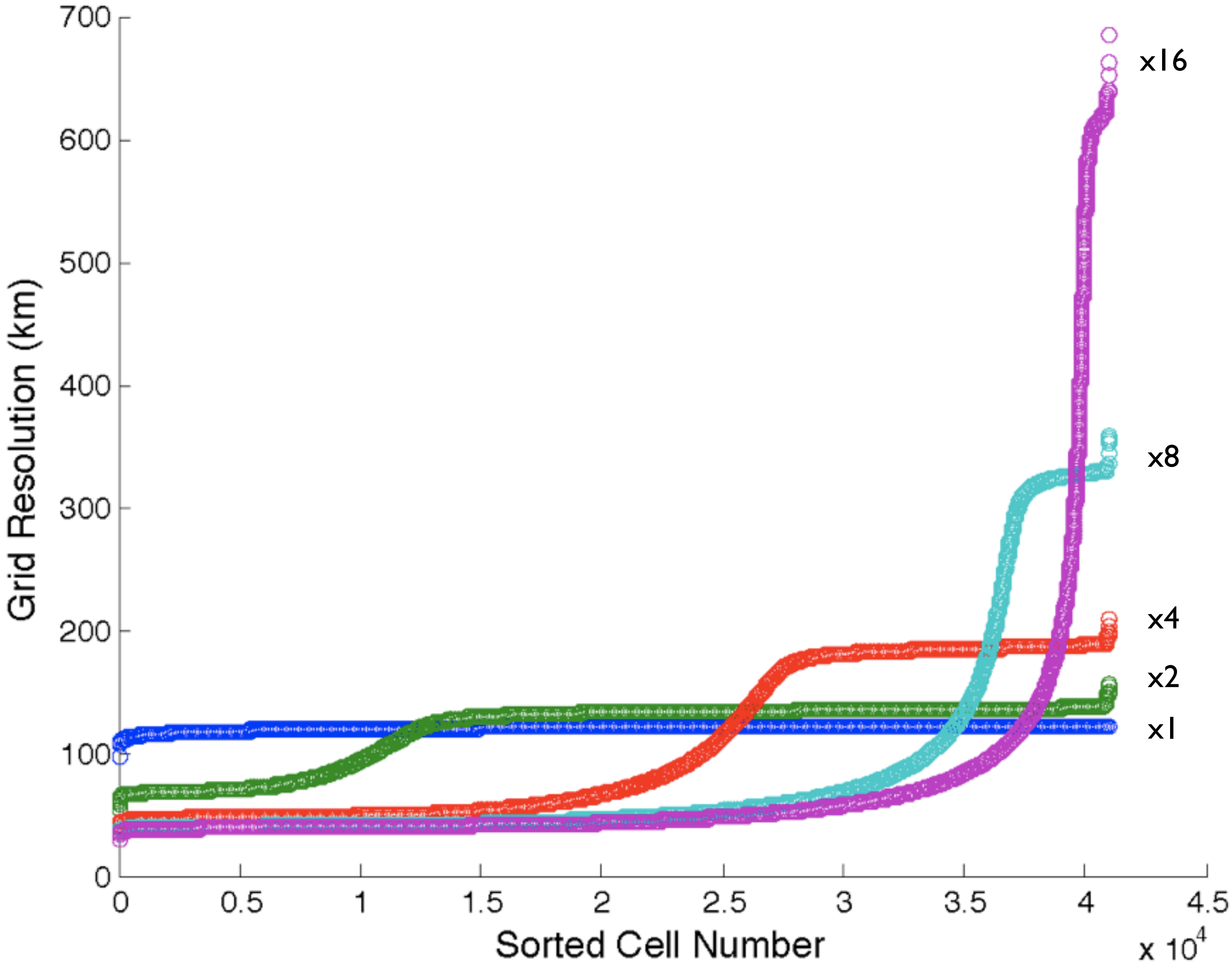


Geodesic Distance from Center of Density Maximum (radians)

Some grid metrics with 40962 nodes ....



Some grid metrics with 40962 nodes ....

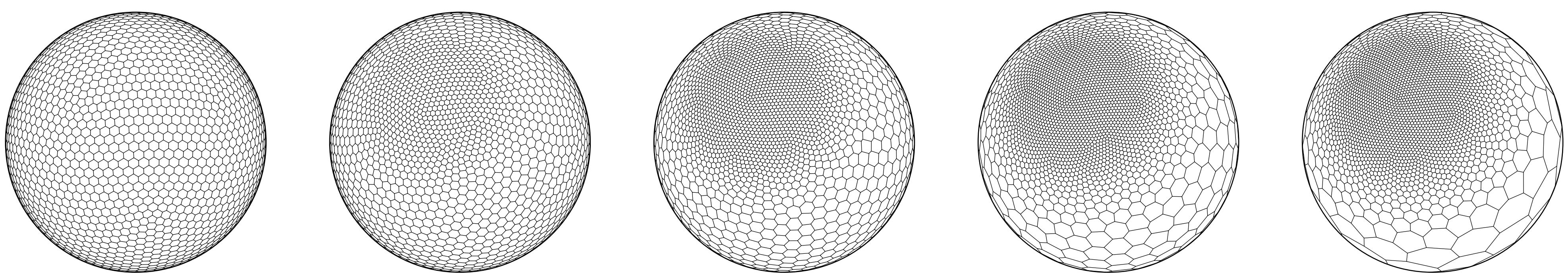


Summary of Section 2

1. We have precise control over positioning of grid cells.

2. These meshes are simple to generate.

3. The mesh are of high quality.



Section 3:

A numerical method applicable to variable-resolution meshes

Equation Set ....

PDE:

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{u}) = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + q(h\mathbf{u}^{\perp}) = -g\nabla\left(h + h_s\right) - \nabla K$$

definition:

$$\eta = \nabla \times \mathbf{u} + f$$

$$\mathbf{u}^{\perp} = \mathbf{k} \times \mathbf{u}$$

$$q = \frac{\eta}{h}$$

Relationship between nonlinear Coriolis force and potential vorticity flux

nonlinear Coriolis force

$$\mathbf{k}\cdot\nabla\times\ \left[\frac{\partial \mathbf{u}}{\partial t} + q(h\mathbf{u}^{\perp}) = -g\nabla\left(h+h_s\right) - \nabla K\right]$$

$$\frac{\partial \eta}{\partial t} + \mathbf{k}\cdot\nabla\times\ \left[\eta\mathbf{u}^{\perp}\right] = 0$$

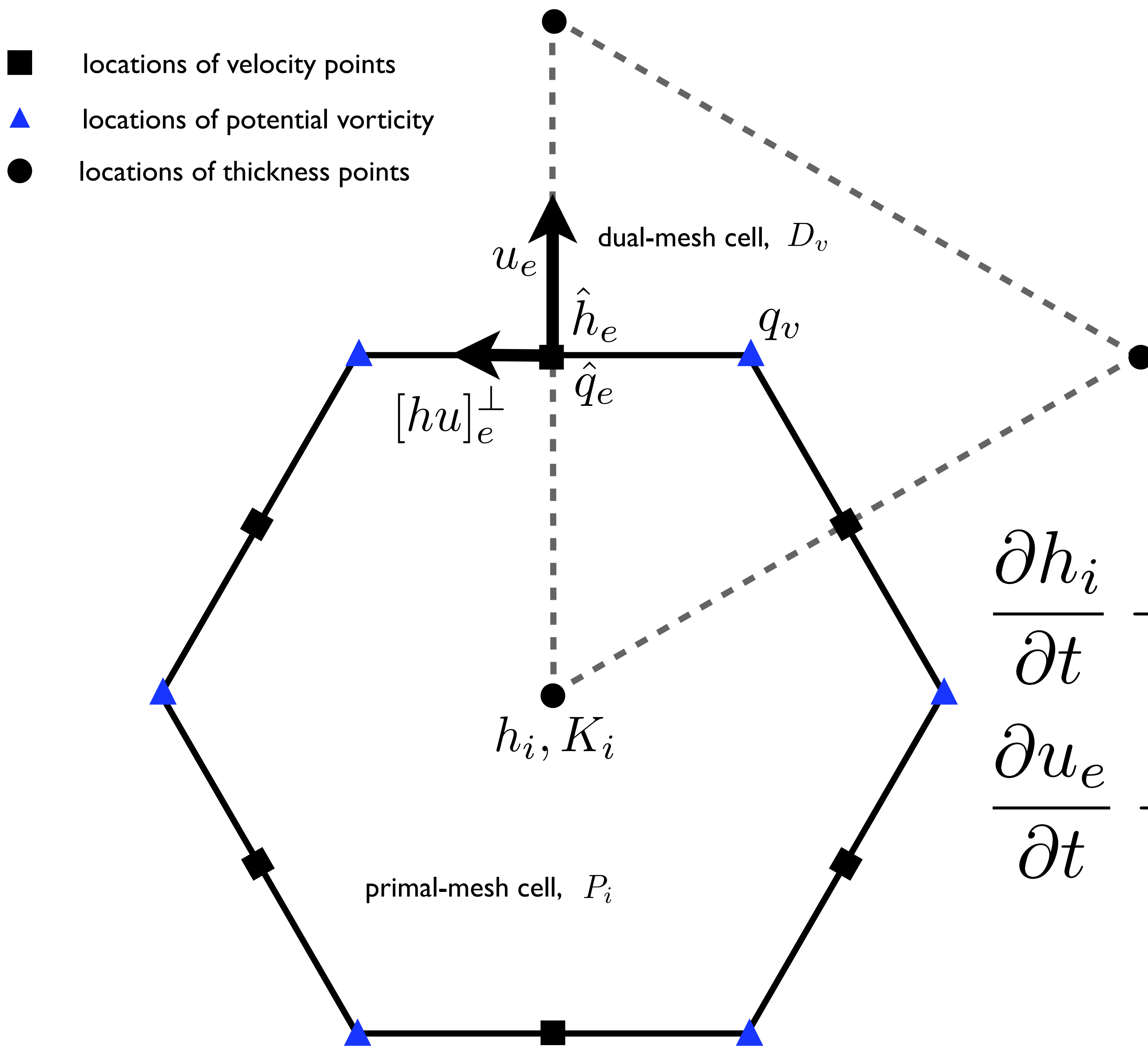
$$\frac{\partial \eta}{\partial t} + \nabla\cdot\left[\eta\mathbf{u}\right] = 0$$

potential vorticity flux

$$\frac{\partial (hq)}{\partial t} + \nabla\cdot\left[hq\mathbf{u}\right] = 0$$

The nonlinear Coriolis force IS the PV flux in the direction perpendicular to the velocity.

Defining the discrete system

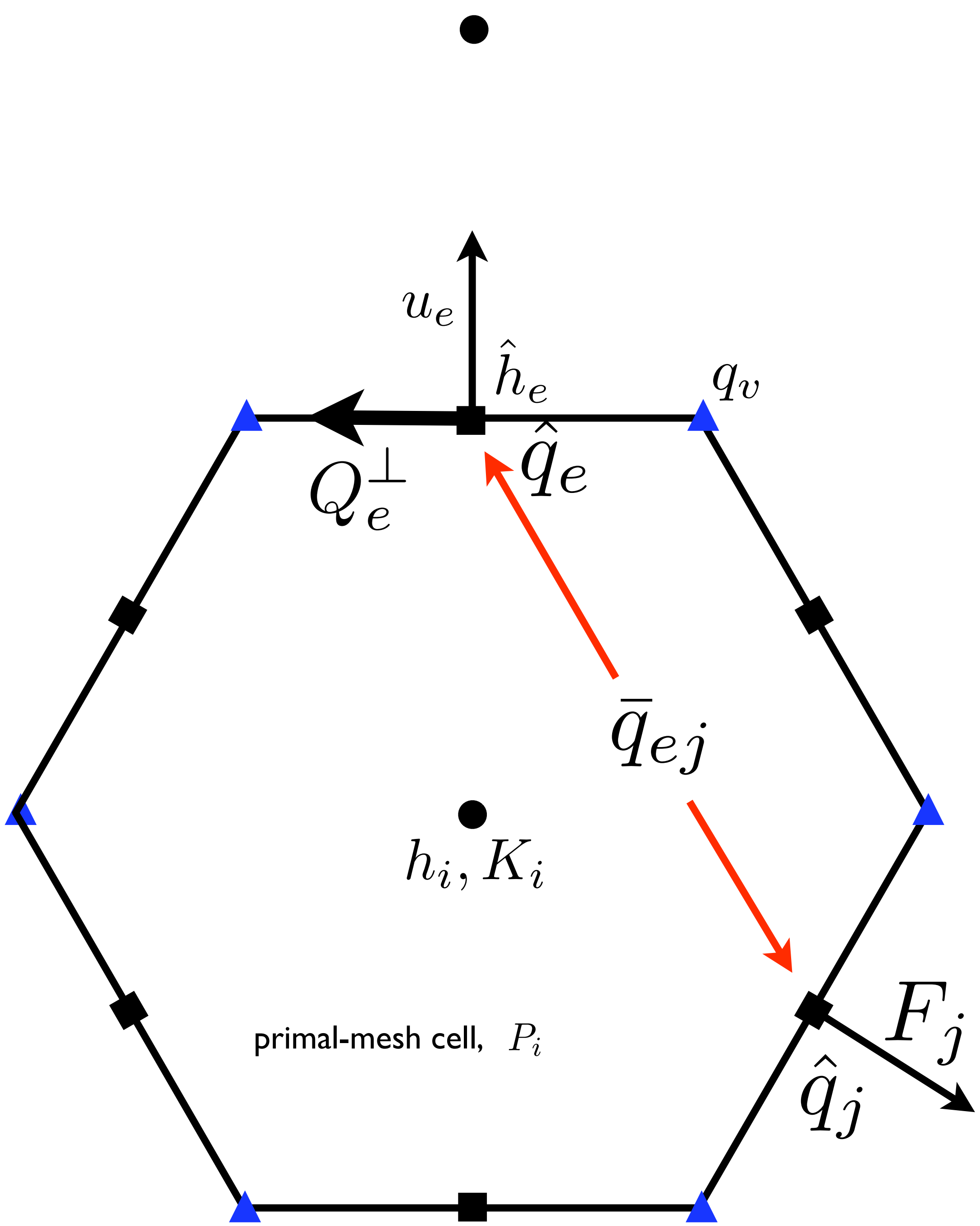


$$\frac{\partial h_i}{\partial t} + \left[\nabla \cdot \left(\widehat{h}_e u_e\right)\right]_i = 0$$

$$\frac{\partial u_e}{\partial t} + \widehat{q}_e \, [hu]_e^\perp = \left[\nabla \left(g h_i + K_i\right)\right]_e$$

Reconstructing the nonlinear Coriolis force

(recall that the nonlinear Coriolis force is the the PV-flux perpendicular to the velocity)



$$\frac{\partial u_e}{\partial t} + \widehat{q}_e \left[hu\right]_e^\perp = \left[\nabla\left(gh_i + K_i\right)\right]_e$$

$$\frac{\partial u_e}{\partial t} + Q_e^\perp = \left[\nabla\left(gh_i + K_i\right)\right]_e$$

$$d_e\, Q_e^\perp = \sum_j w_e^j\, l_j\, F_j\, \bar{q}_{ej}$$

$F_j = \hat{h}_j u_j$ thickness flux

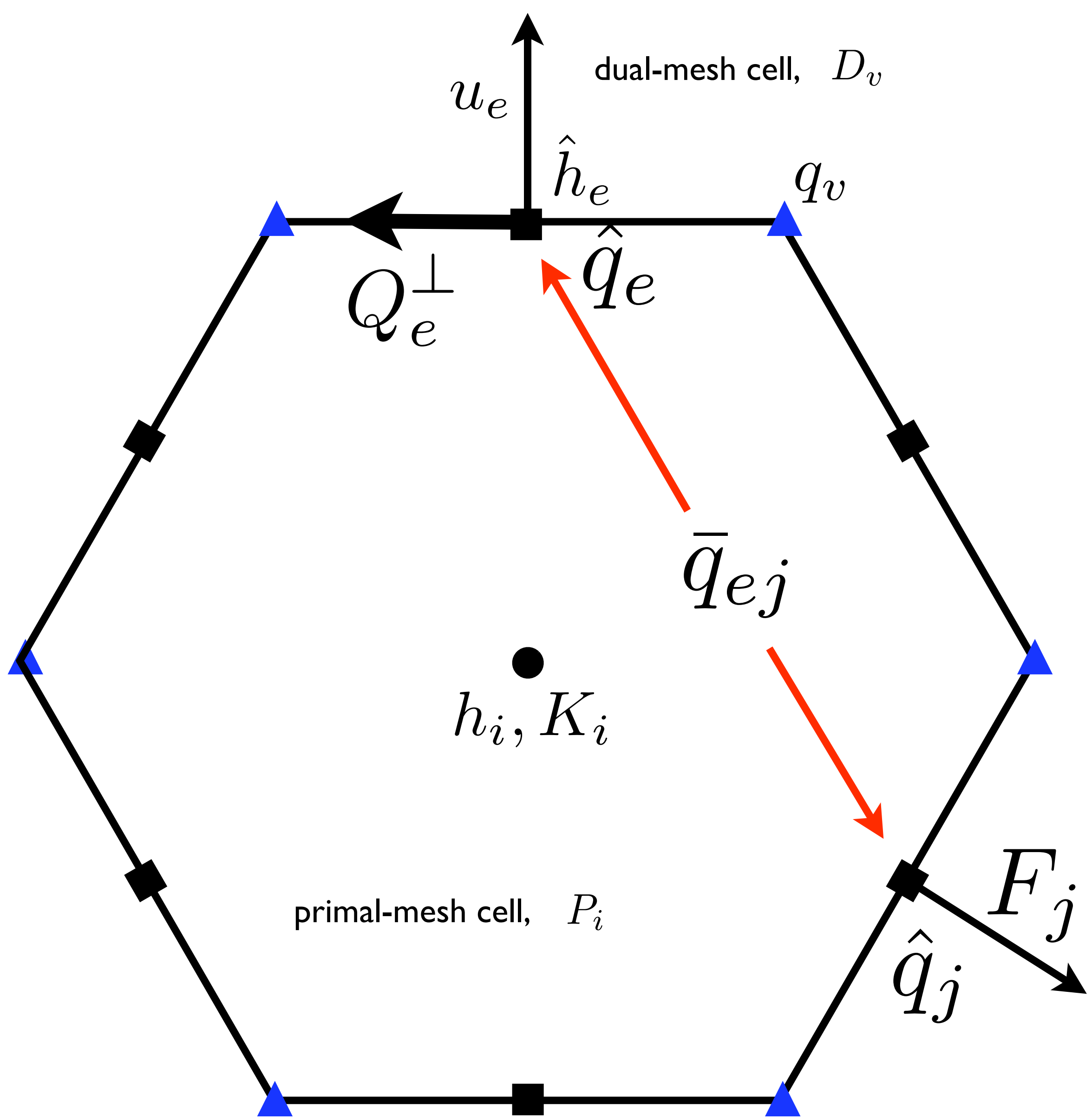
$w_e^j = -w_j^e$ weights are equal and opposite

$\bar{q}_{ej} = \bar{q}_{je}$ PV is symmetric

The nonlinear Coriolis force will be energetically neutral for any $\bar{q}_{ej}$.
This is an extension to what Sadourny (1975) showed for regular meshes.

Reconstructing the nonlinear Coriolis force

(recall that the nonlinear Coriolis force is the the PV-flux perpendicular to the velocity)



$$\frac{\partial u_e}{\partial t} + \widehat{q}_e \, [hu]_e^{\perp} = [\nabla \left(g h_i + K_i \right)]_e$$

$$\frac{\partial u_e}{\partial t} + Q_e^{\perp} = [\nabla \left(g h_i + K_i \right)]_e$$

The curl of the above eq, leads to the below eq.

$$\frac{\partial}{\partial t}\left(h_v q_v\right) + \frac{1}{A_v} \sum_{e \in G(v)} Q_e^{\perp} \, dc_e = 0$$

For a uniform PV field, the above eq reduces (identically) to the vertex thickness eq.

$$\frac{\partial}{\partial t}\left(h_v\right) + \frac{1}{A_v} \sum_{e \in G(v)} F_e^{\perp} \, dc_e = 0$$

The evolution of the discrete velocity field is compatible with the evolution of a valid, discrete PV equation. The compatibility holds to round-off.

Summary of Section 3

1. We have developed a finite-volume method that is applicable to any mesh composed of conforming, convex polygons that are locally-orthogonal.

2. Even though we are integrating the momentum equation, we have as much control over PV as when solving for PV directly.

3. The system conserves total energy to within time truncation error.

Section 4:

An exploration of results

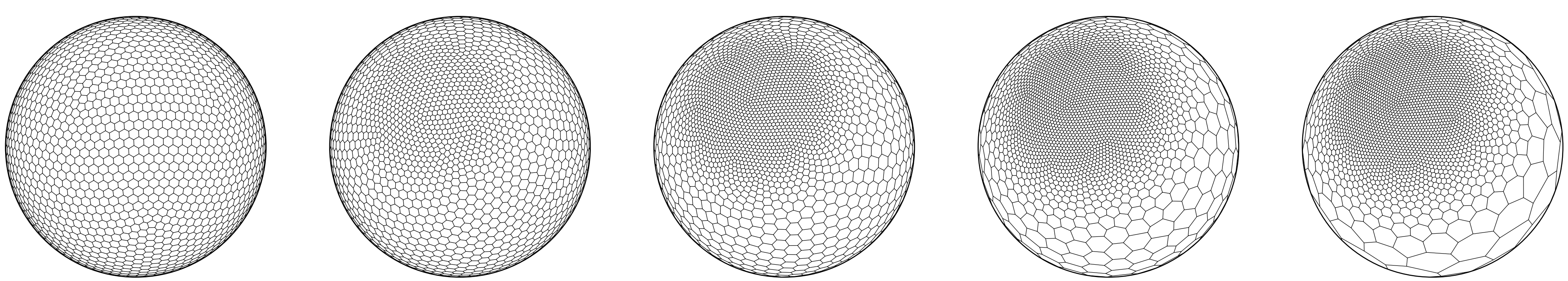
Experimental design

1. Every simulation is conducted with the same executable with the exact same input parameters.

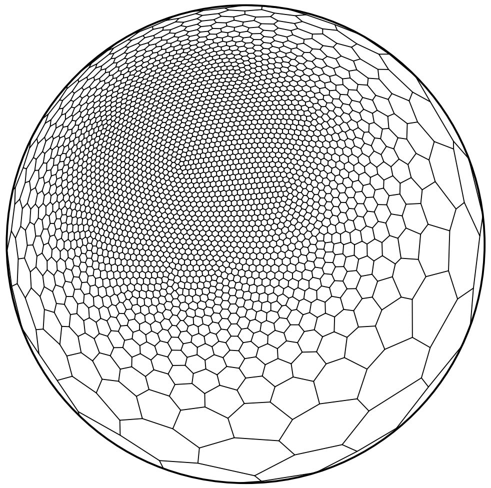
2. There is no explicit dissipation. All numerics (with one important exception!) are ~2nd-order centered in space. The time-stepping is 4th-order Runge Kutta.

3. We use the anticipated potential vorticity method to dissipate potential enstrophy while conserving energy.

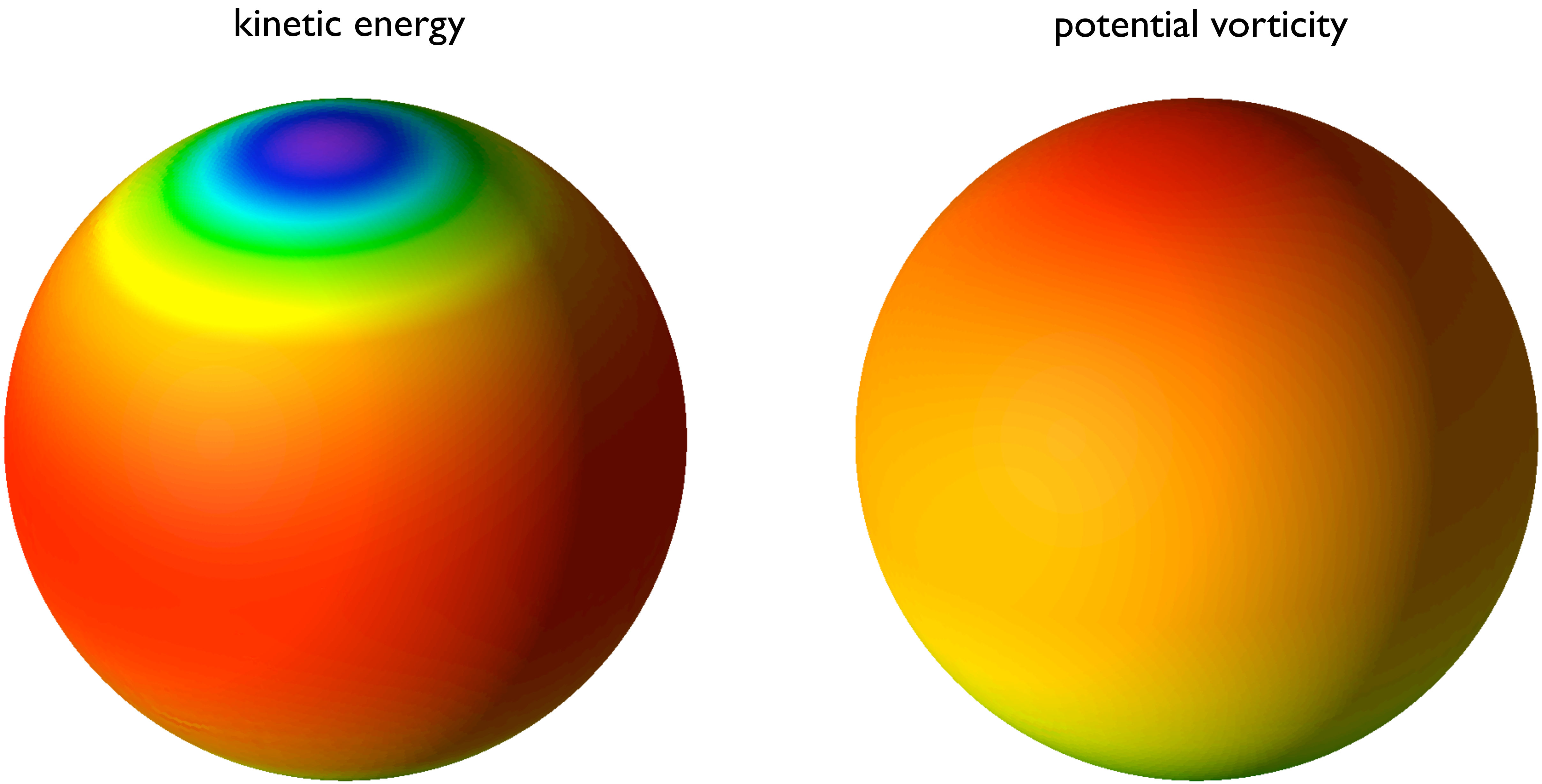
4. We will use shallow-water test cases #2 and #5 as the basis for evaluation.



Let's start with a sanity check ....



x16, 163842 nodes, shallow-water test case #2, day 50
(20 km / 320 km resolution)



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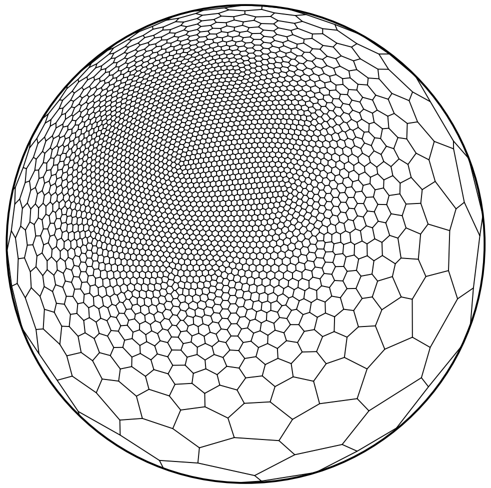
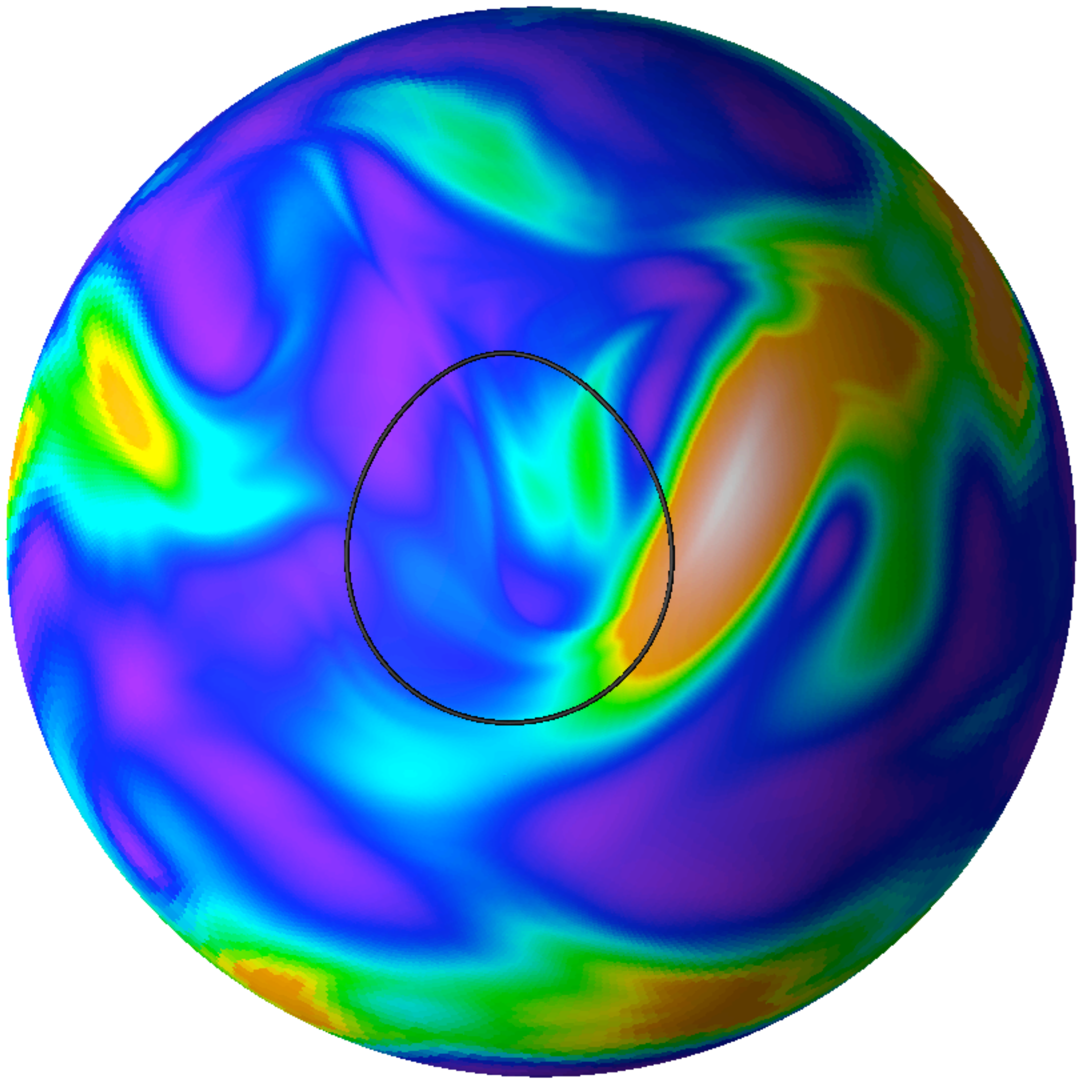


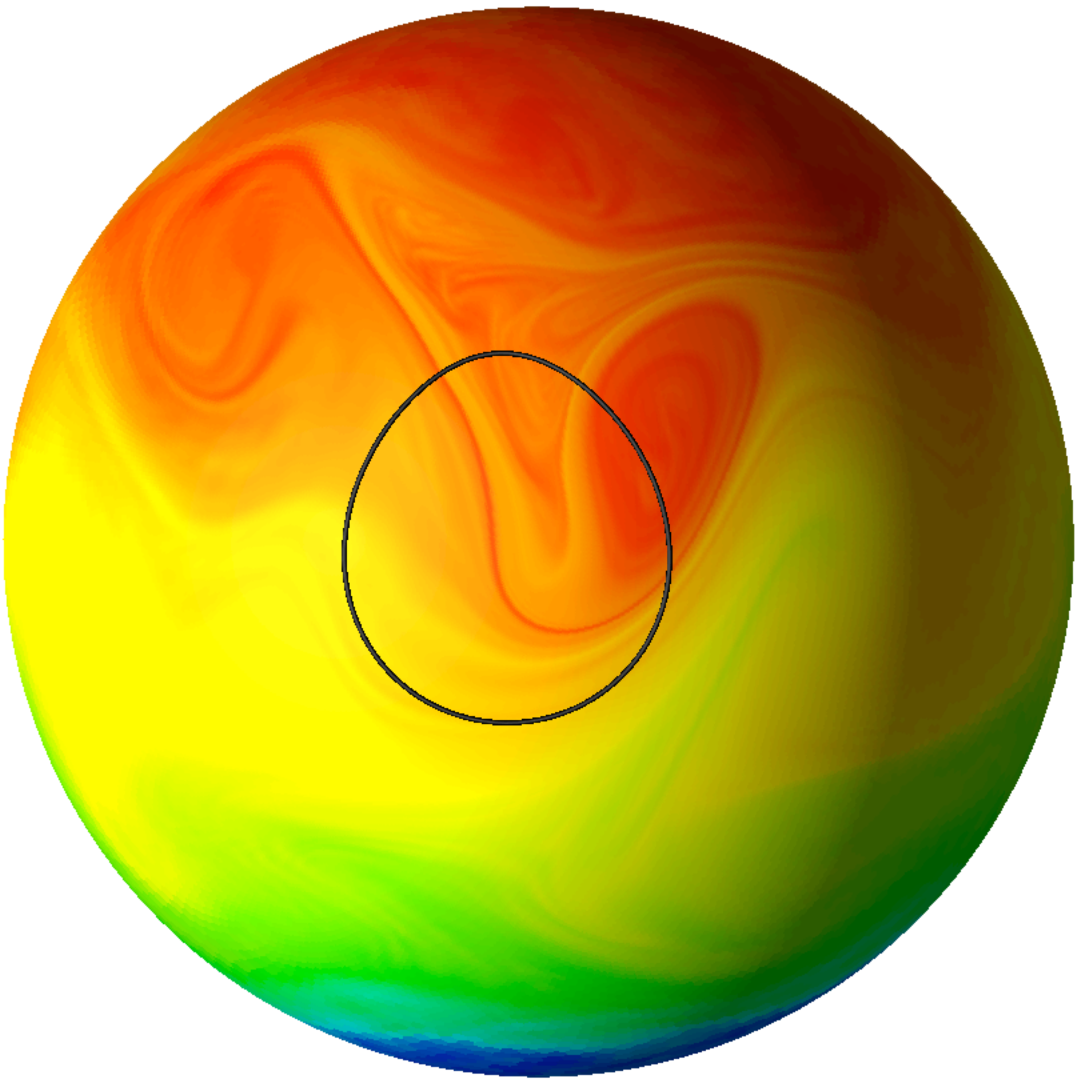
x16, 163842 nodes, shallow-water test case #2, day 50

(20 km / 320 km resolution)

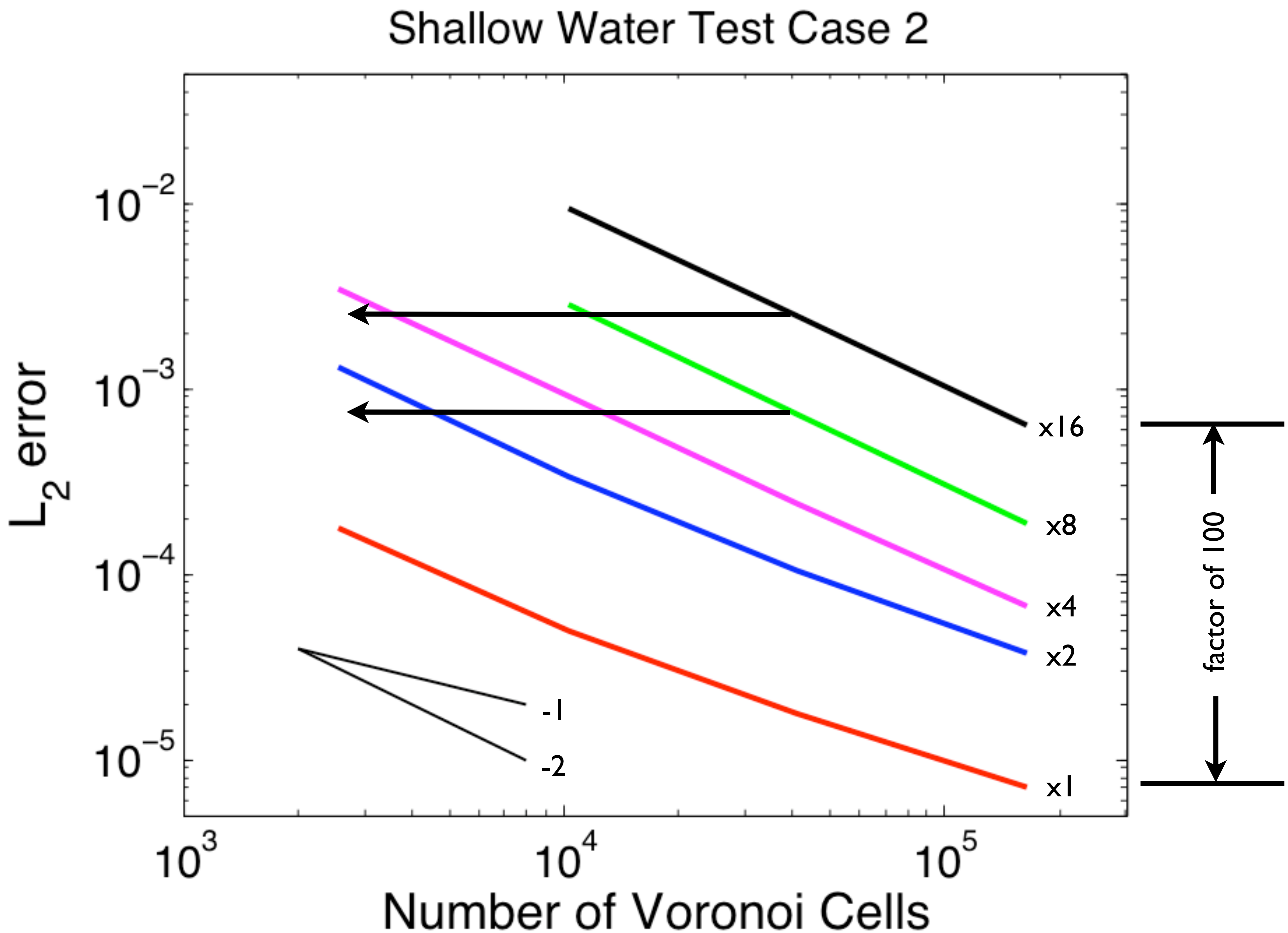
kinetic energy



potential vorticity

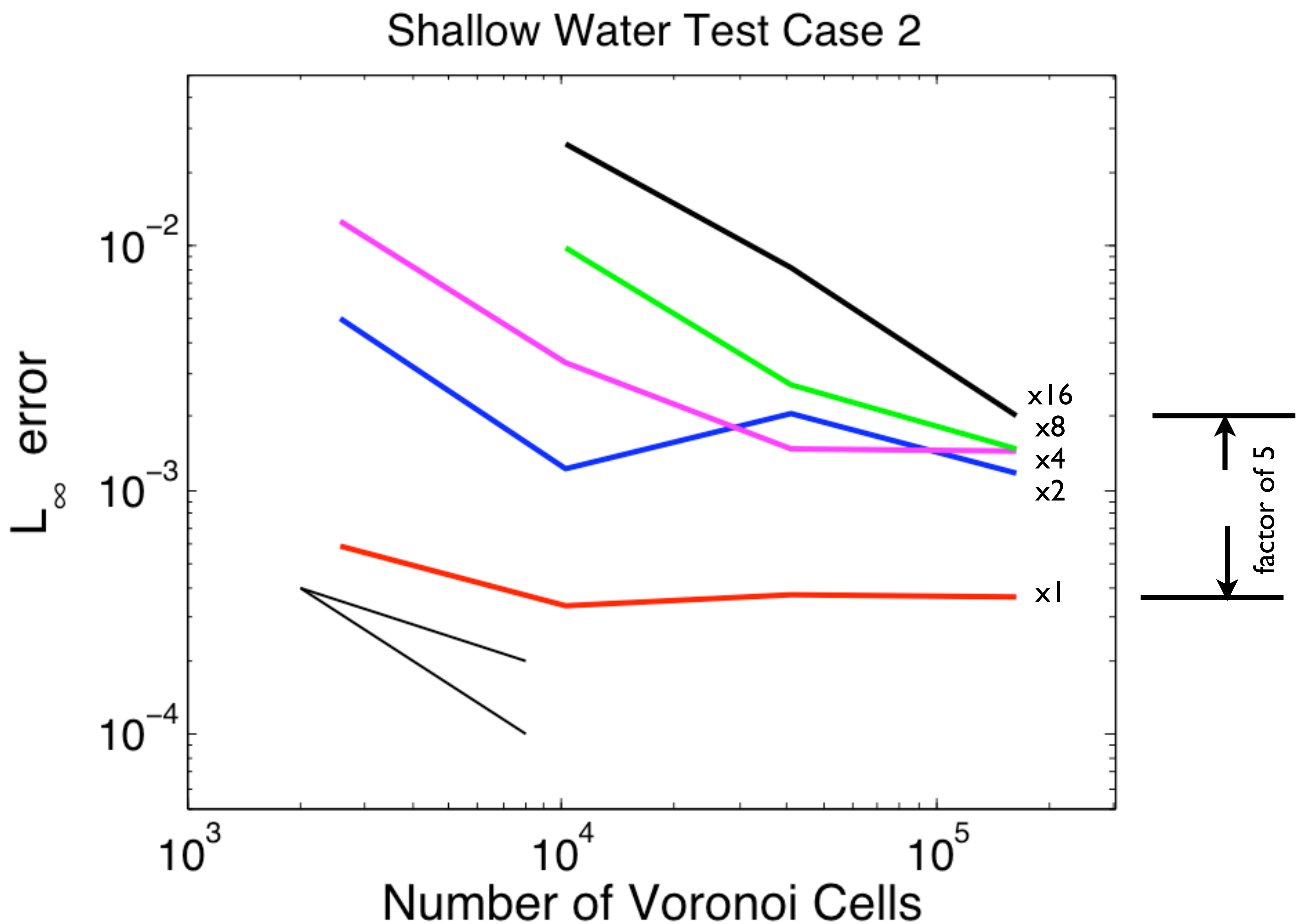


Now something a bit more quantitative ...
L2 error of SWTC#2

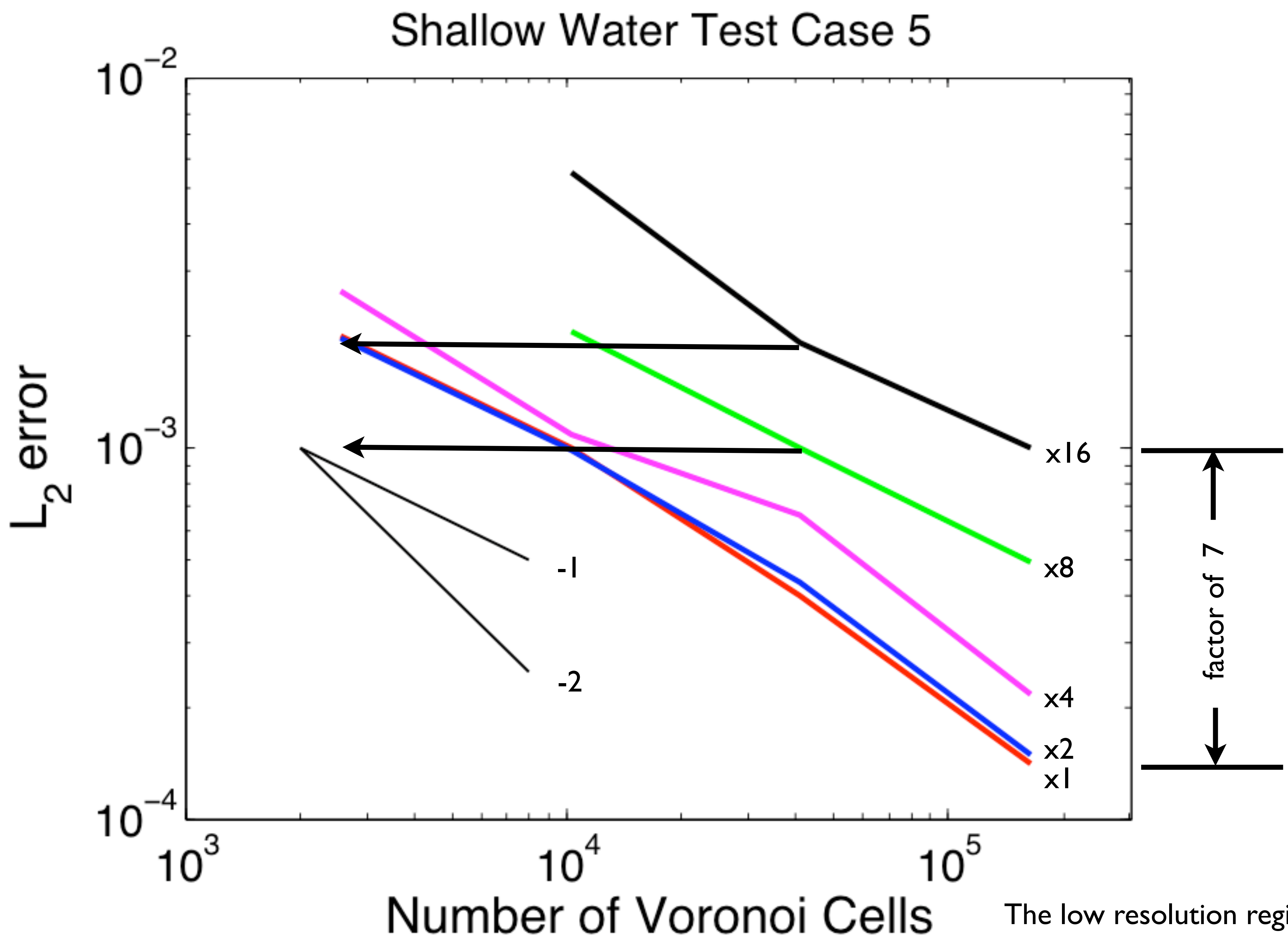


The low resolution region of the
x8 40962 mesh is about the same
as the resolution of the x1 2562 mesh.

Linf error of SWTC#2

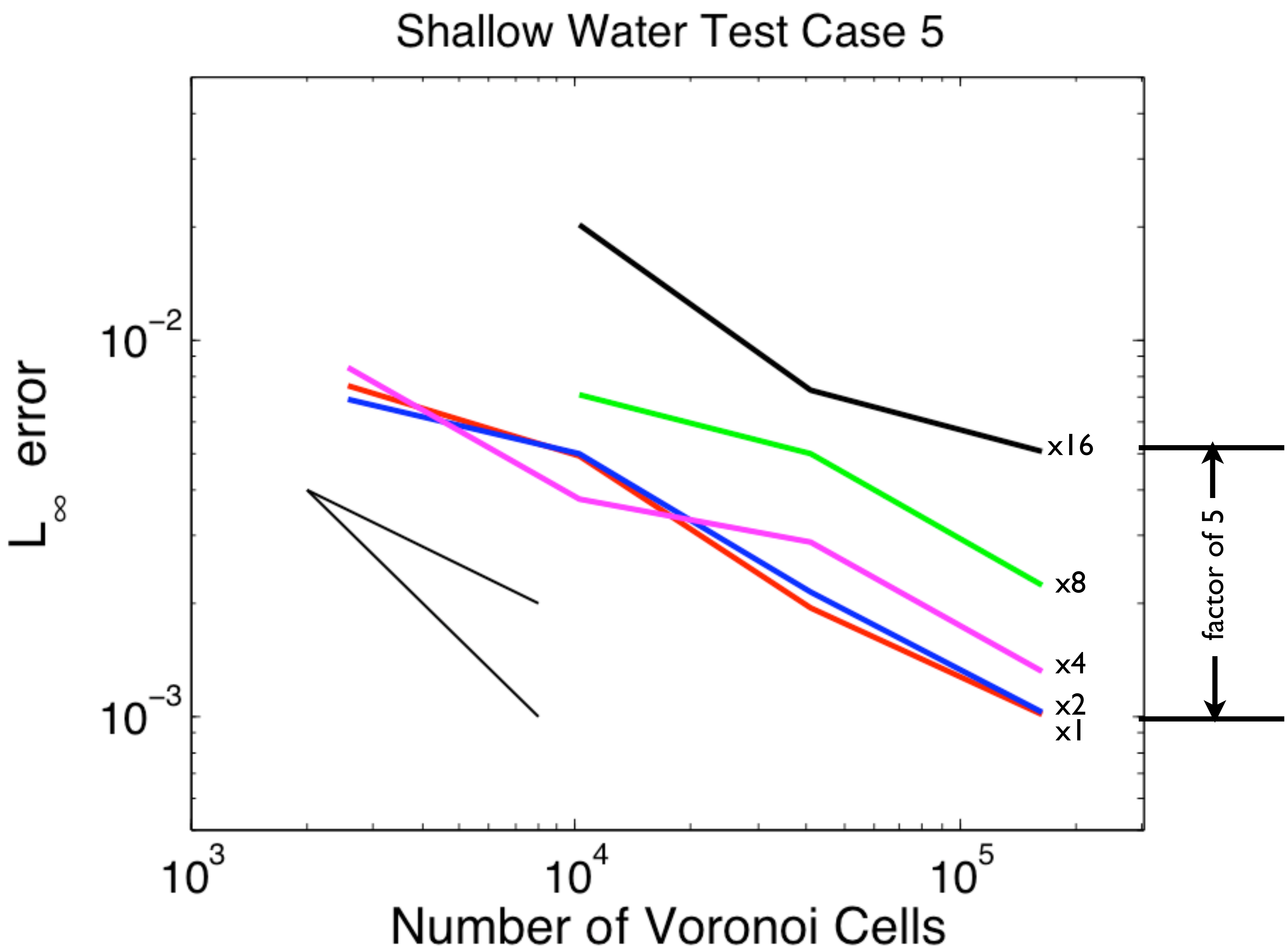


L2 error of SWTC#5



The low resolution region of the x8 40962 mesh is about the same as the resolution of the x1 2562 mesh.

Linf error of SWTC#5

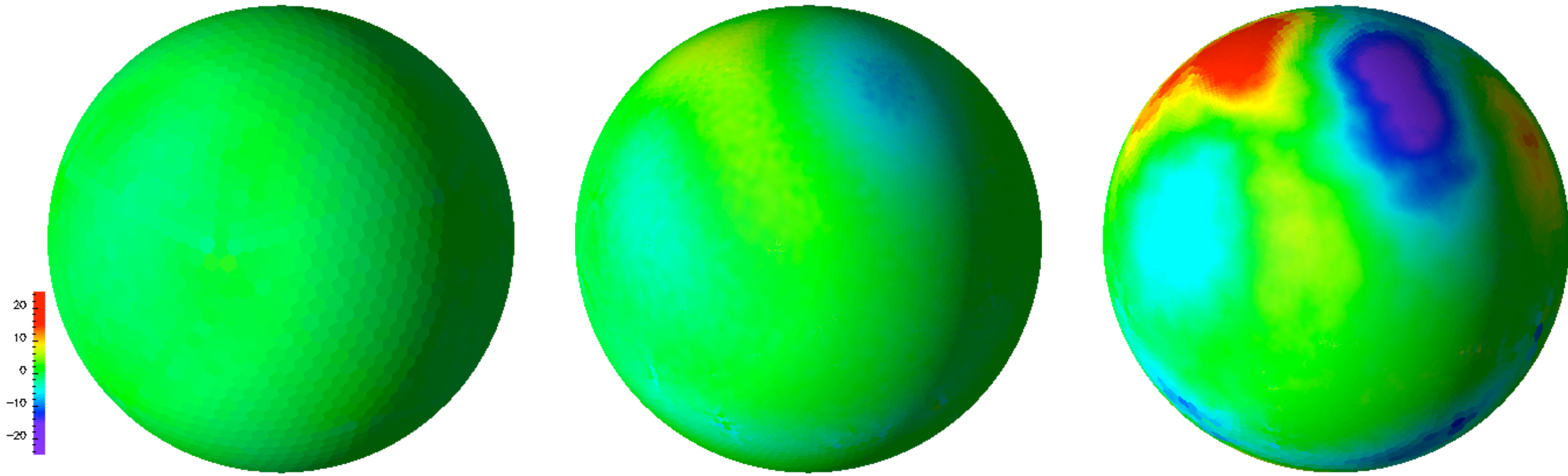


A look at the error distribution in TC2 at day 12 ....

height error x1, 2562

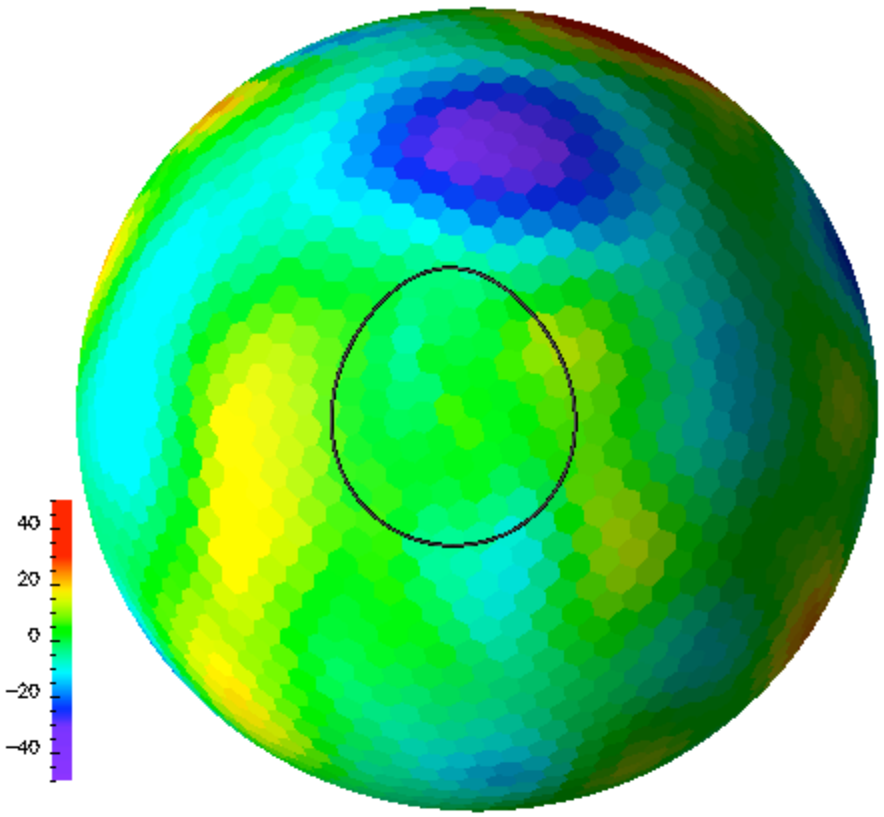
height error x8, 40962

height error x16, 40962

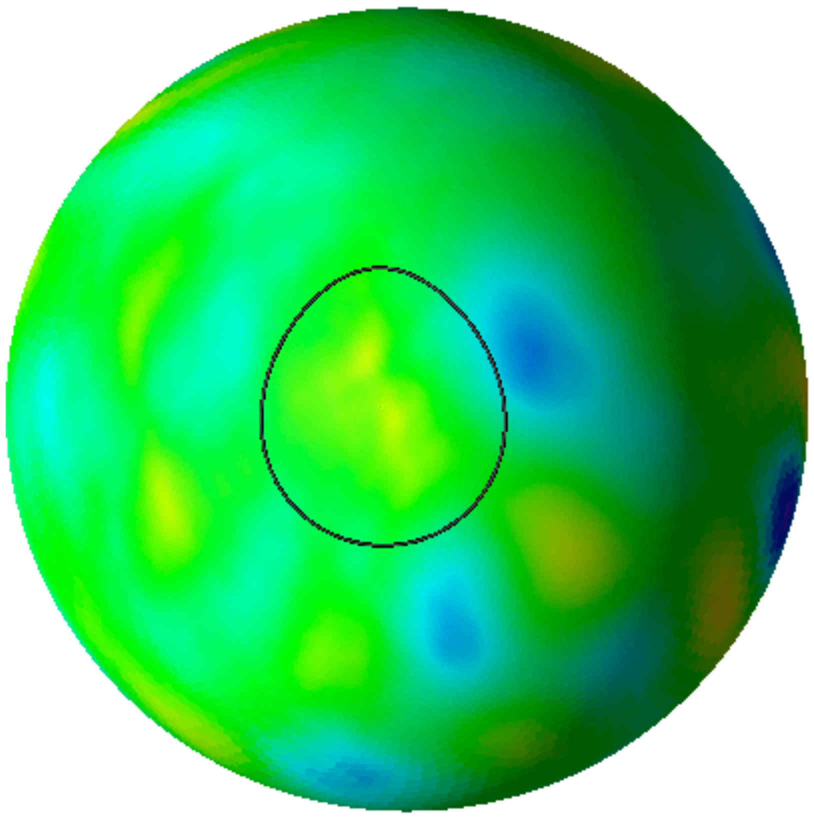


A look at the error distribution in TC5 at day 15 ....

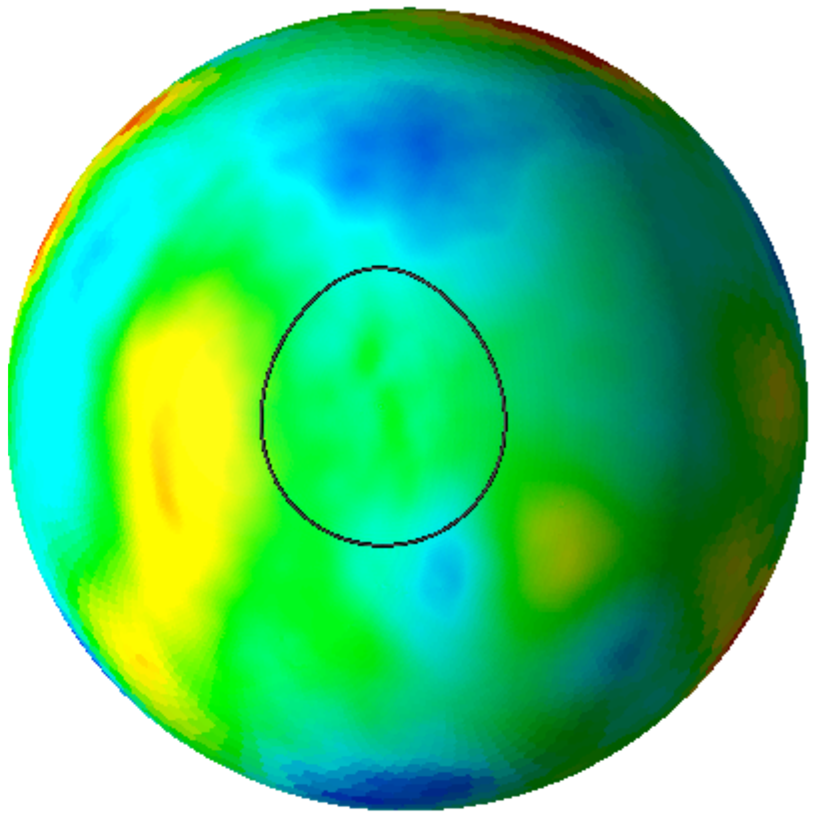
height error
x1, 2562



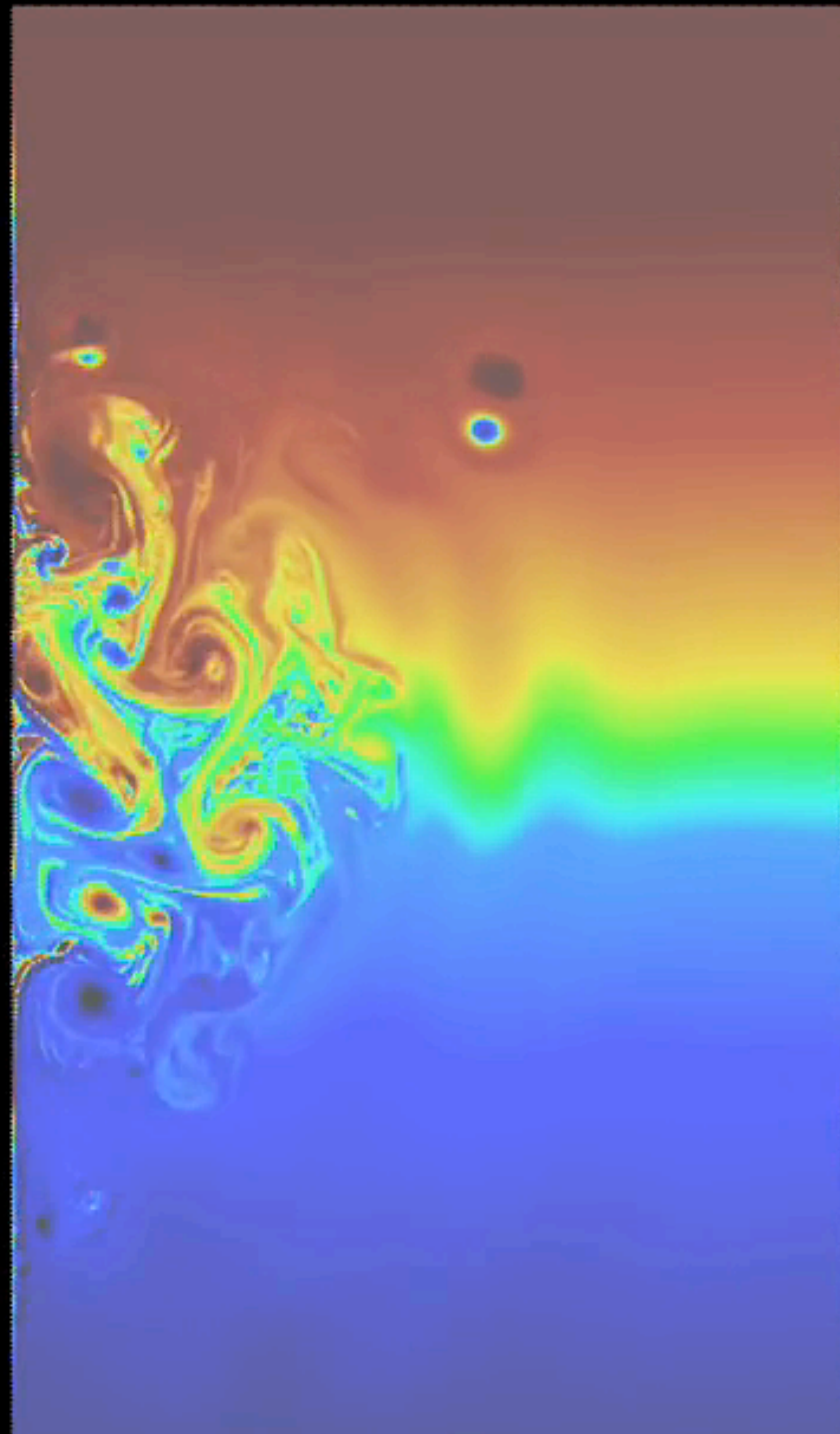
height error
x8, 40962



height error
x16, 40962



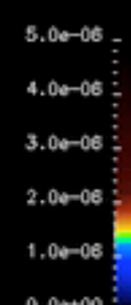
Ocean Double-Gyre Problem



Potential Vorticity: one frame per day

5 km mesh
5000 km x 2500 km domain
Forcing consistent with Greatbatch and Nadiga (2000).
No bottom drag
Laplacian mixing on velocity at 1.0 m^2/s
no-slip boundary conditions

NOTE: The purpose here is to confirm that the numerical scheme can handle strong eddy activity without the need for stabilization through ad hoc dissipation. The scheme is stable for a very wide range of dissipations, even those that clearly lead to an underdamped system.



http://public.lanl.gov/ringler/movies/2009/eddies_18.mov

Summary of Section 4

1. The numerical method is robust, even with meshes that vary in grid resolution by a factor of 16.

2. SWTC#2 provides an opportunity to measure the cost of local mesh refinement in terms of accuracy.

3. SWTC#5 allows us to determine if any of this cost can recouped when features amenable to local mesh refinement are present.

Section 5:

Where to from here?

First, a summary of what (I hope) I discussed ....

1. The climate modeling community would greatly benefit from a model capable of placing resolution in one (or more) areas of interest.

2. We have developed a simple approach to building spherical centroidal Voronoi tessellations with user-specified mesh-density.

3. We have developed a traditional finite-volume method capable of conserving energy and PV on these variable-resolution meshes.

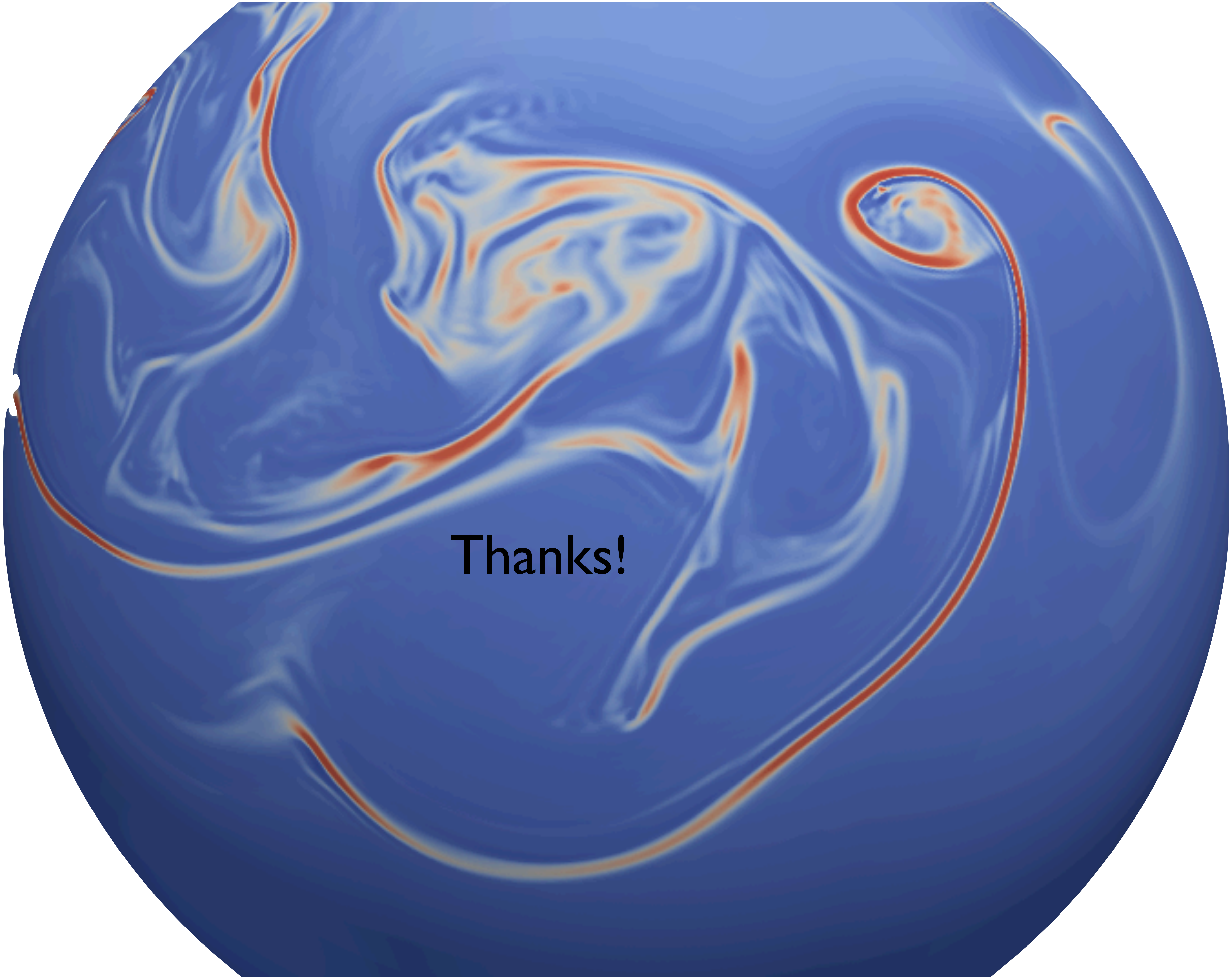
4. We have paired the numerical method and the SCVTs to produce robust simulations of the shallow-water system, even on meshes that vary in resolution by 16X.

A look ahead ....

1. A full physics AGCM and OGCM based on this approach will be functioning by the end of the calendar year.

2. We have extended the Prather method-of-moments scheme to higher-order accuracy on arbitrary convex polygons. We expect that this scheme will prove useful in helping to reduce the errors in the mesh transition zones.

3. We expect that this modeling approach will allow us to explore scientific questions that would be otherwise prohibitively expensive in terms of computation.



Thanks!