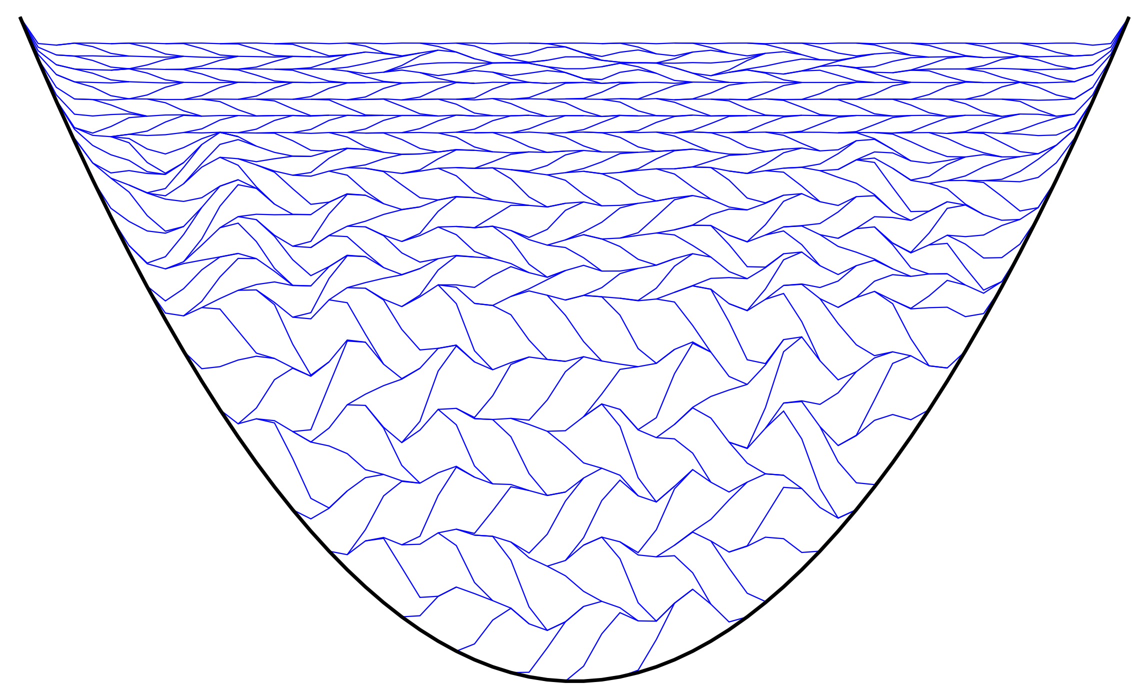
# Lagrangian Modeling of Oceans and Atmospheres

*Patrick Haertel, Yale University*



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# Outline

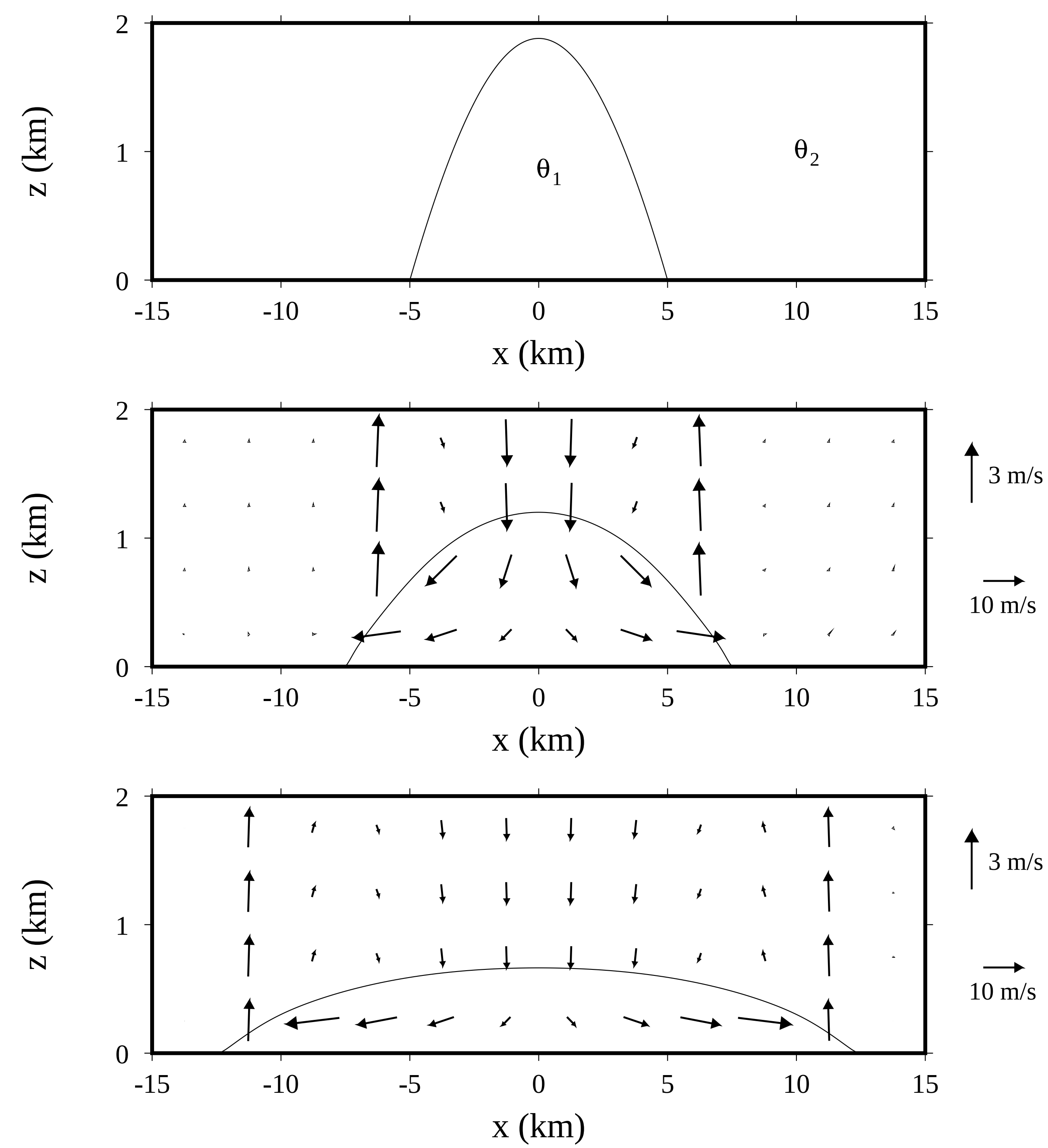
1. Introduction
2. Lagrangian Method
3. Idealized Tests
4. Lake and Ocean Applications
5. Atmospheric Convective Systems
6 Summary and Conclusions

# Introduction

## Where it all started . . .

# Simulations of Thunderstorm Outflows

*Haertel et al. 2001*

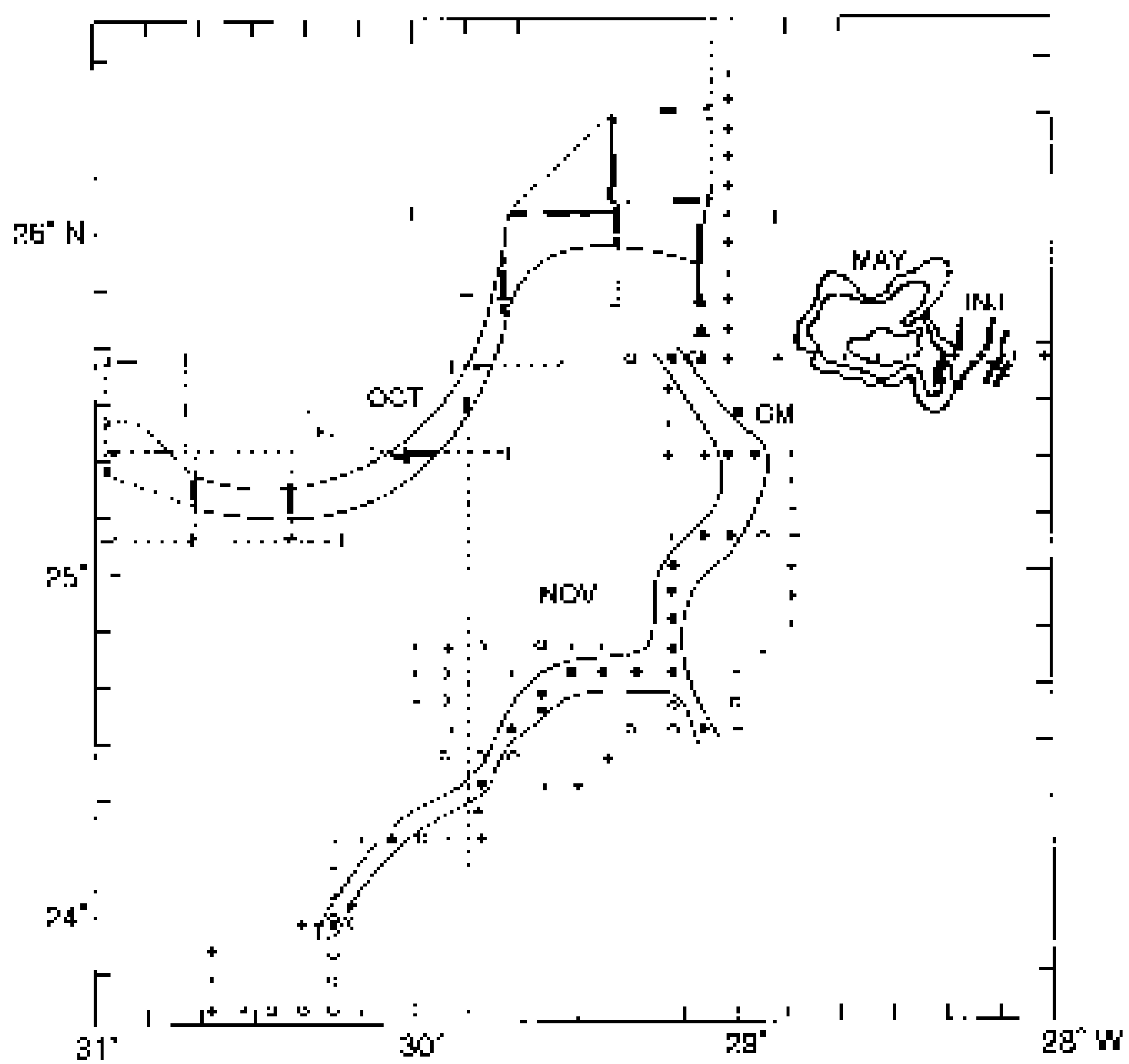


## Why I first applied it to oceans . . .

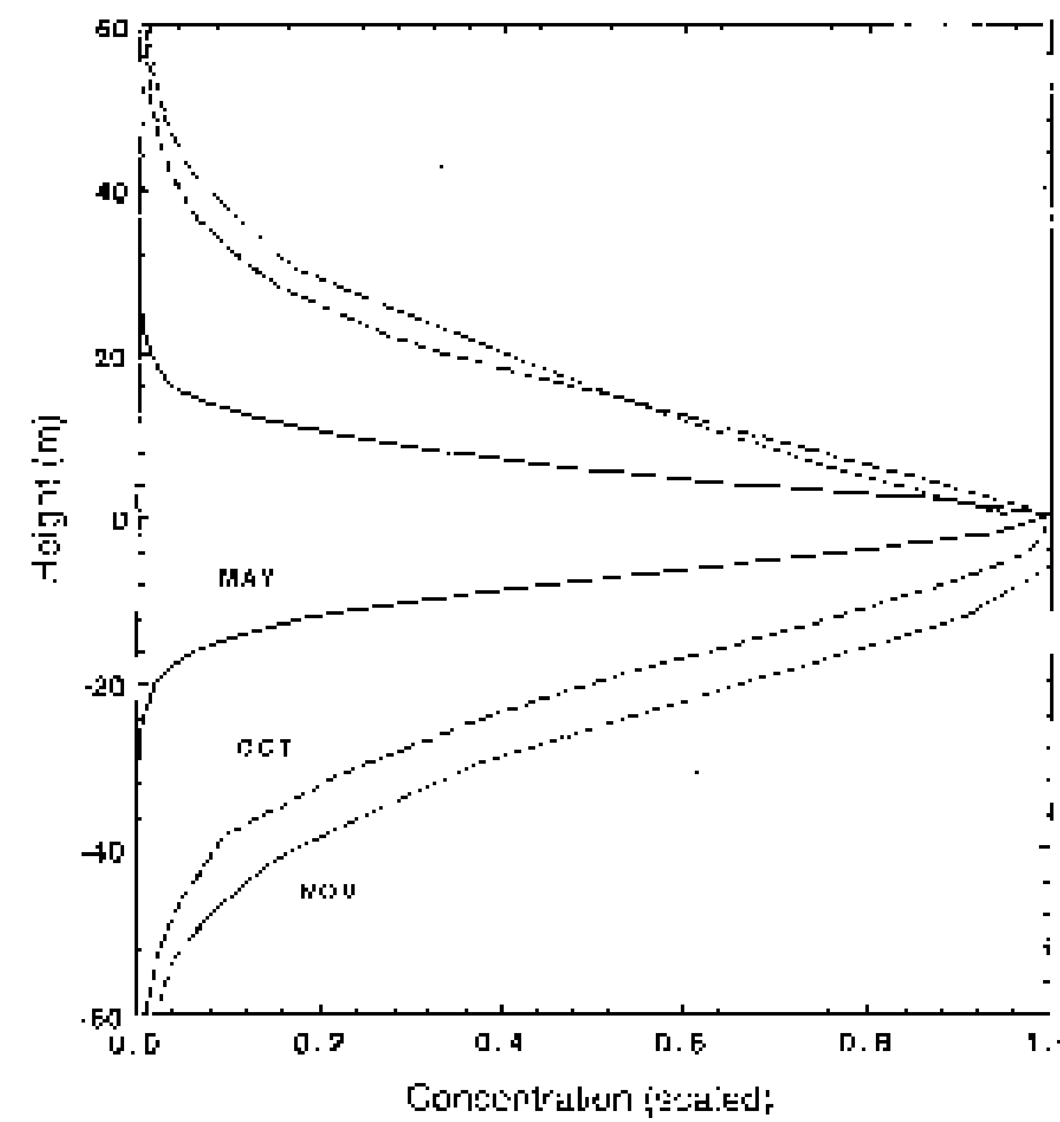
# Tracer Release Experiment

*Ledwell et al. 1993*

Horizontal Diffusion (~3 $m^2 s^{-1}$)




Vertical Diffusion (~0.11 $cm^2 s^{-1}$)



## Advantages of the Lagrangian Method

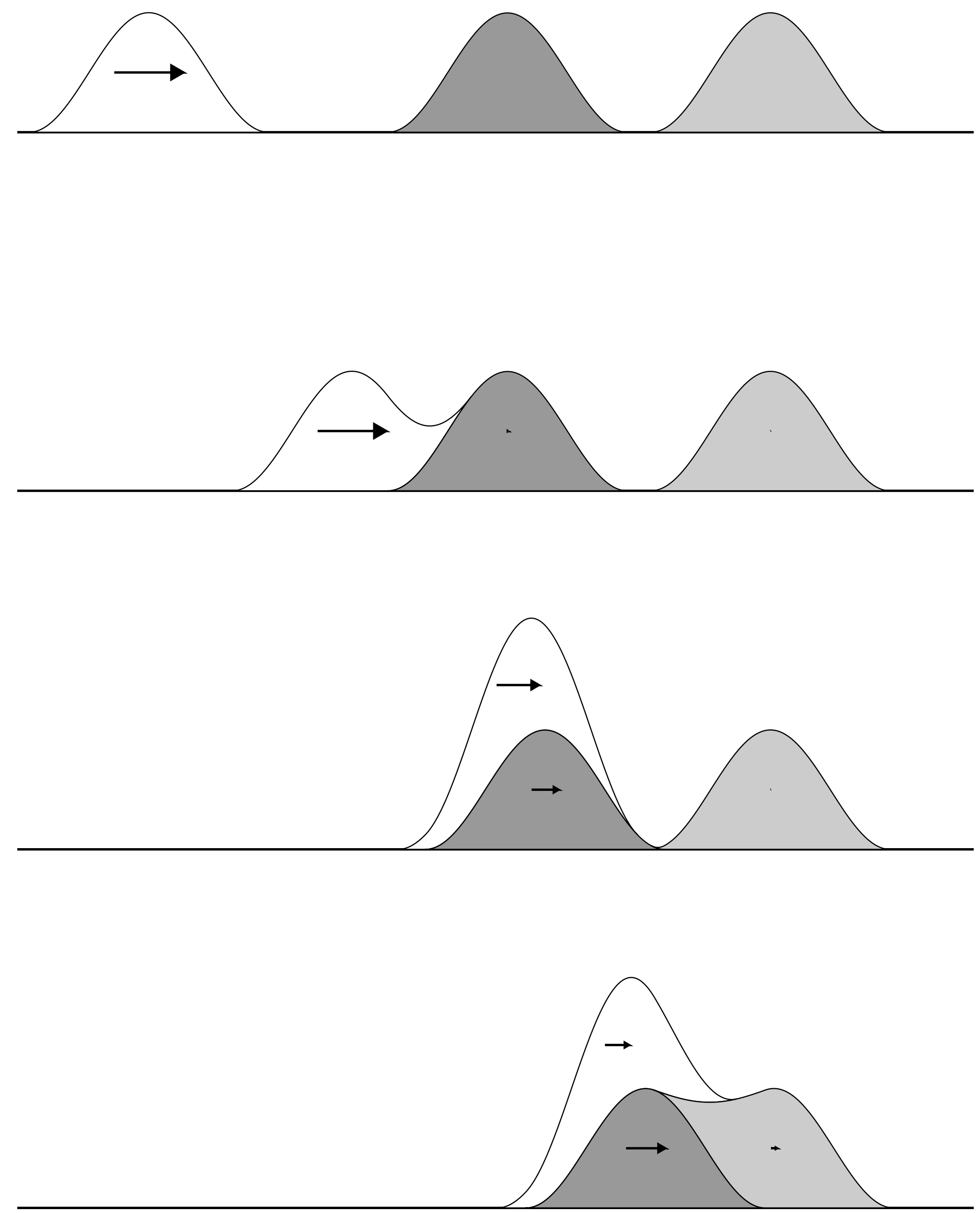
1. Control over mixing
2. Trajectories for every parcel
3. Realistic moist convective systems

# Lagrangian Method

## Properties of Parcels

1. time independent horizontal mass distribution
2. surfaces conform
3. uniform density
4. hydrostatic pressure
5. dense parcels lie beneath not so dense parcels

# Three Parcel System



## Equations of Motion

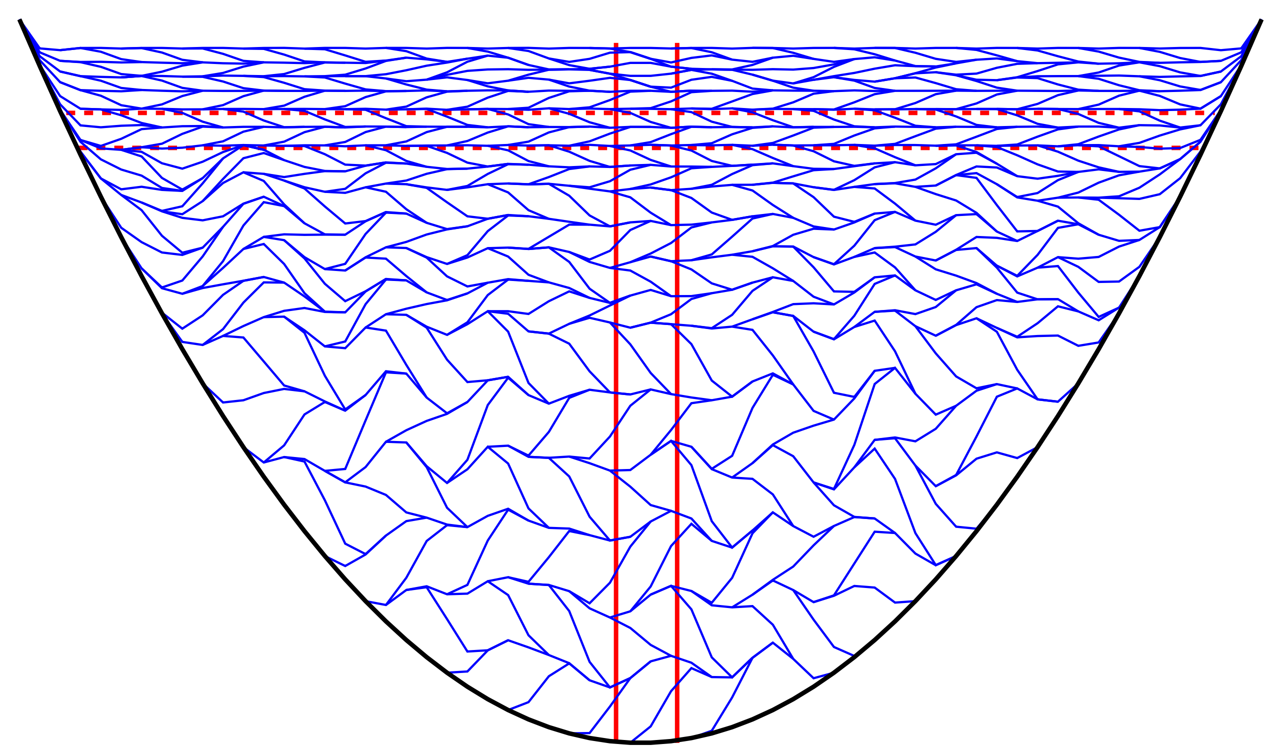
$$\frac{d\mathbf{x}_i}{dt} = \mathbf{v}_i$$

$$\frac{d\mathbf{v}_i}{dt} + f\ \mathbf{k} \times \mathbf{v}_i = \frac{\mathbf{F}_{p_i}}{M_i} + \mathbf{D}_{\mathbf{v}_i}$$

# Pressure Force

$$\mathbf{F}_{p_i} = \int_{S_i} p \, \mathbf{n} \, dA$$

# Mixing Columns and Rows

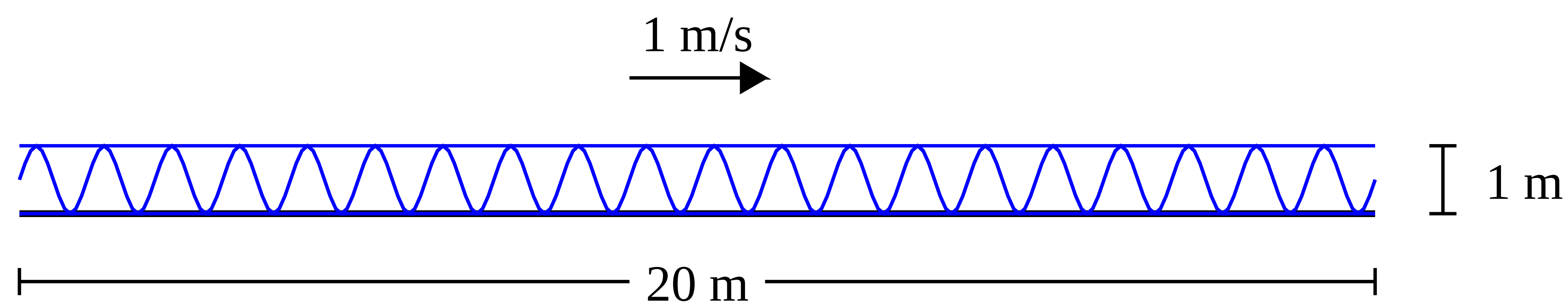


## Computational Efficiency

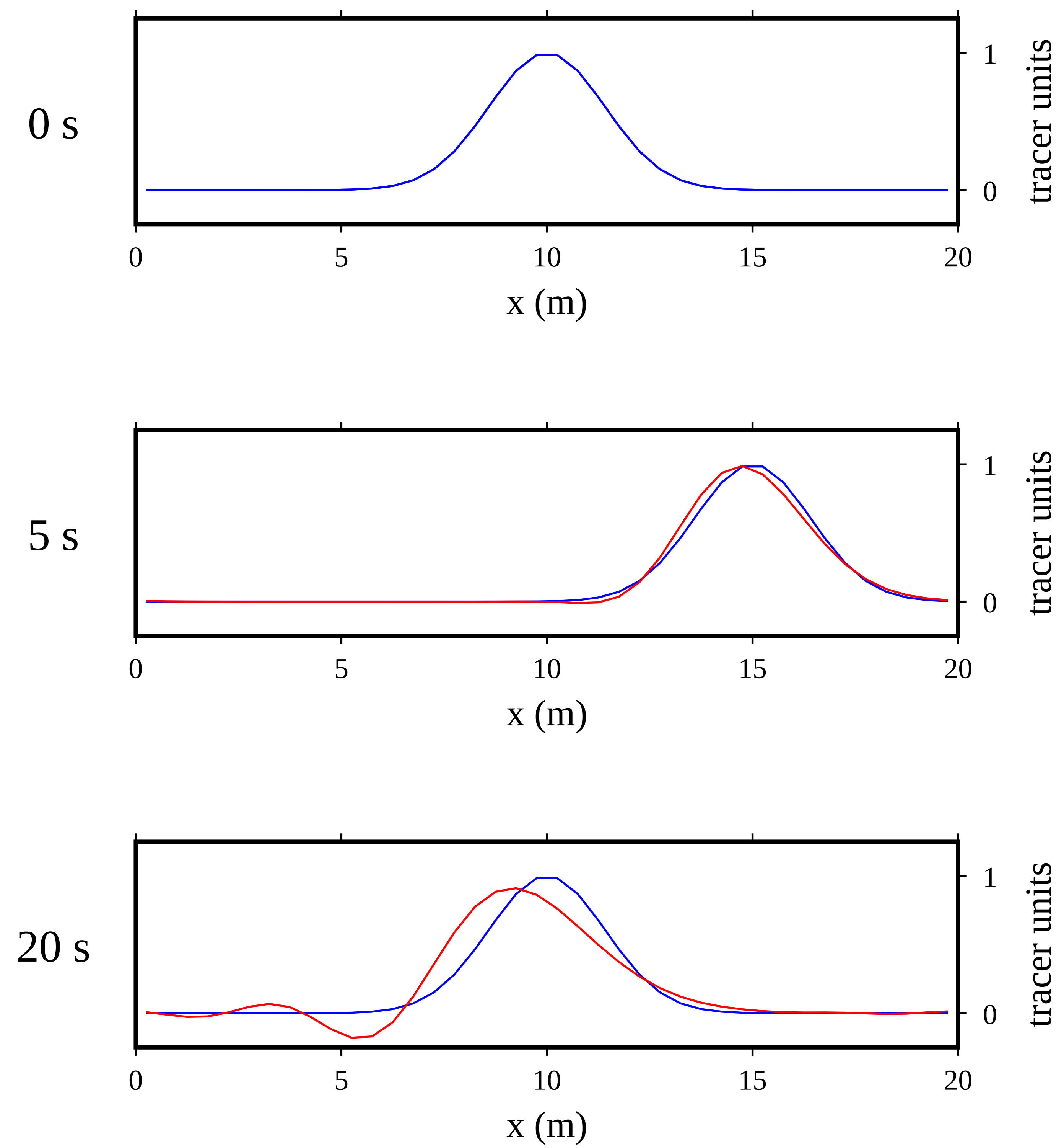
- O(n) where n is number of parcels
- Competitive with Eulerian models when optimized (Haertel et al 2004)

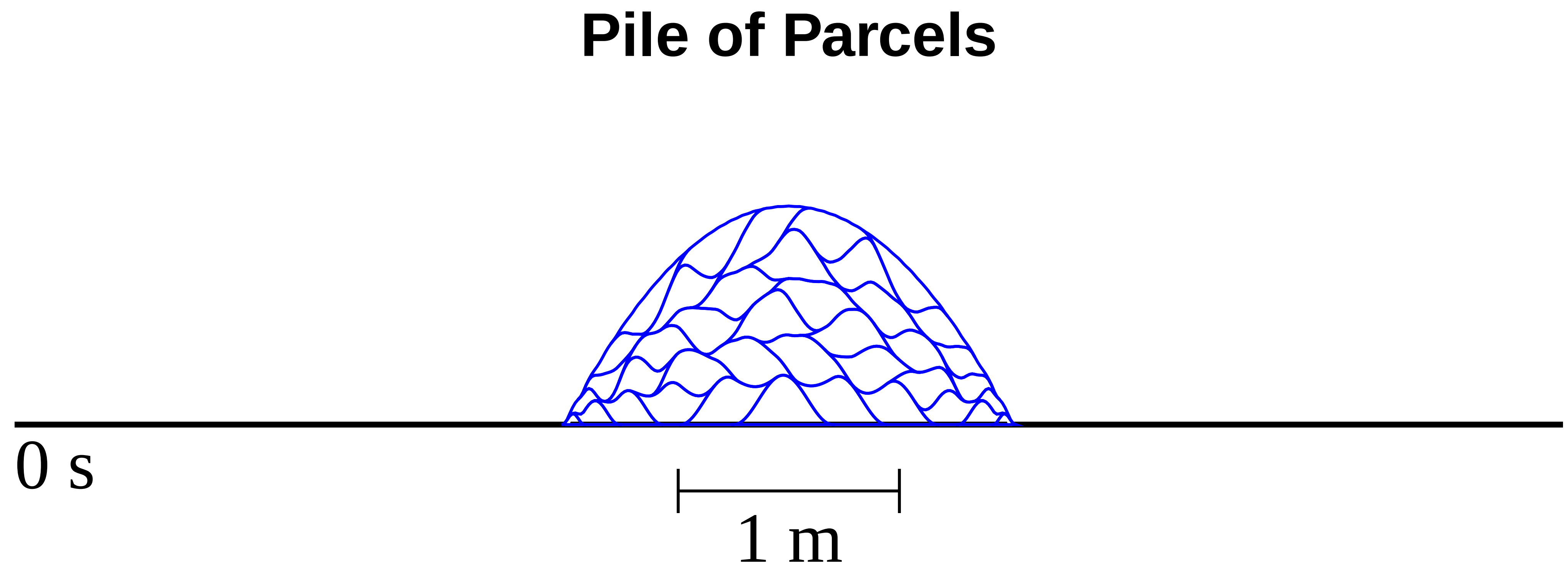
# Idealized Tests

# Advection



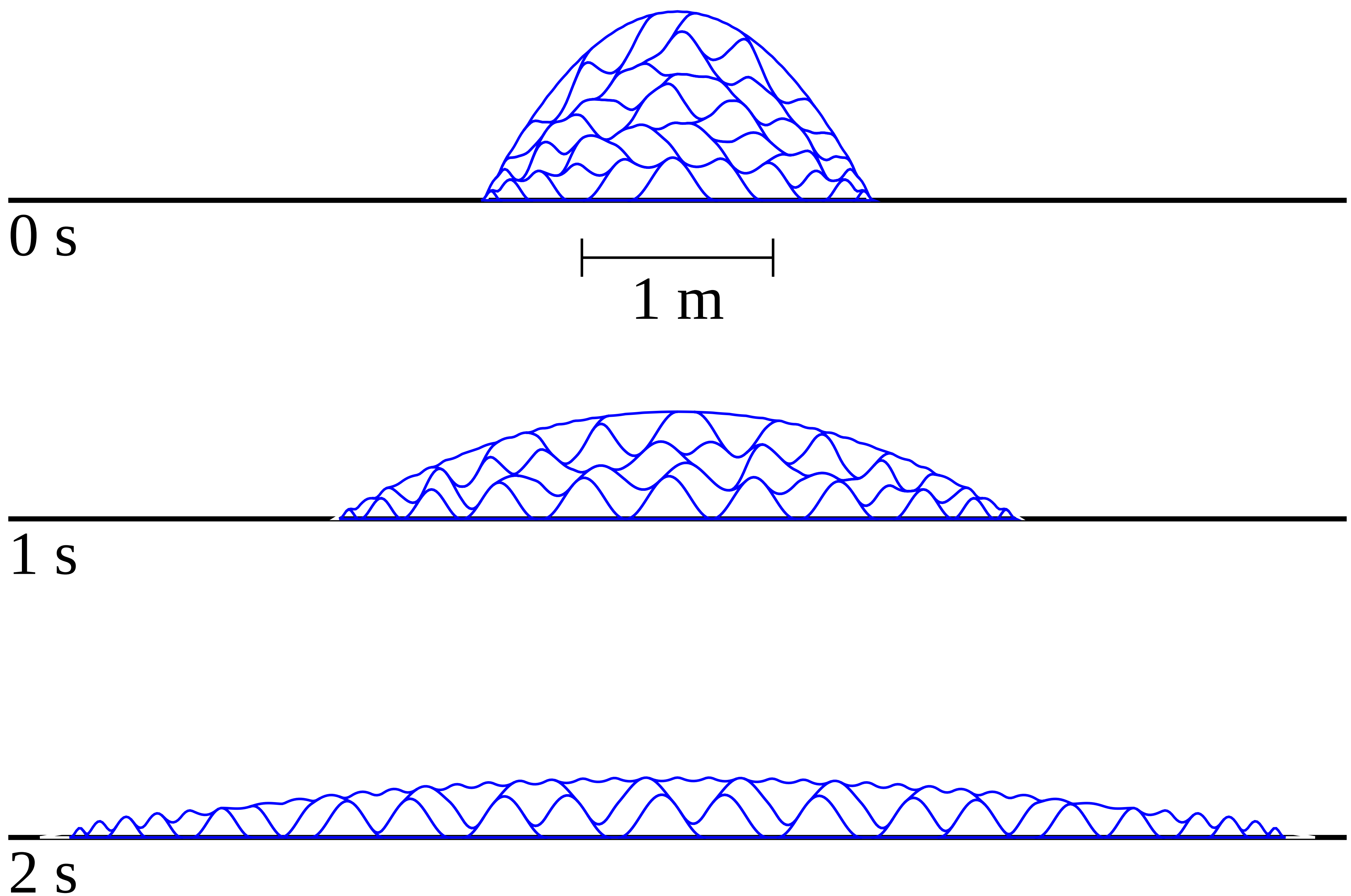
# Tracer Distribution

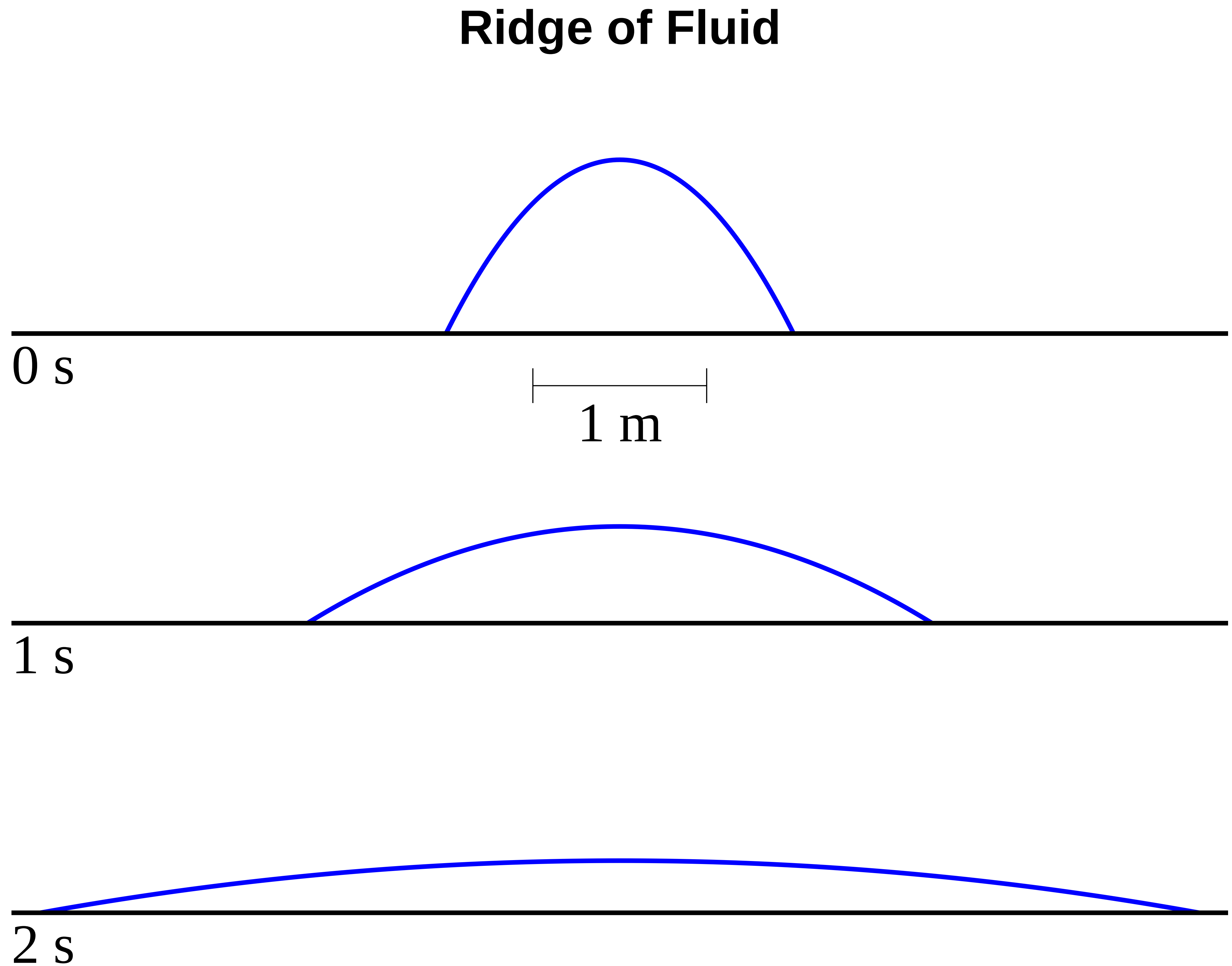




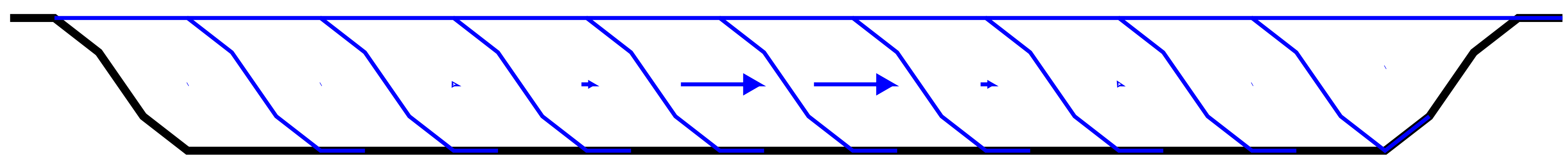
Pile of Parcels
0 s
1 m

## Pile of Parcels

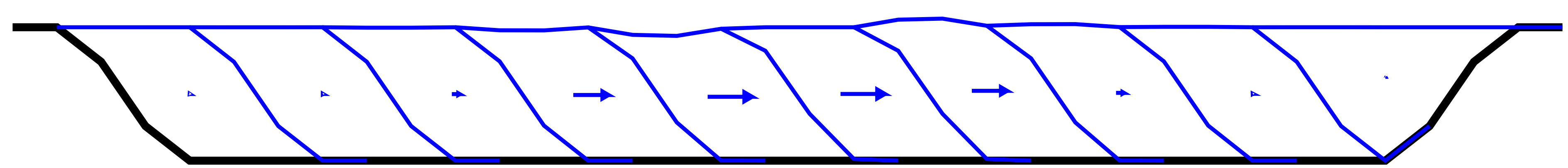



Ridge of Fluid
0 s
1 m
1 s
2 s

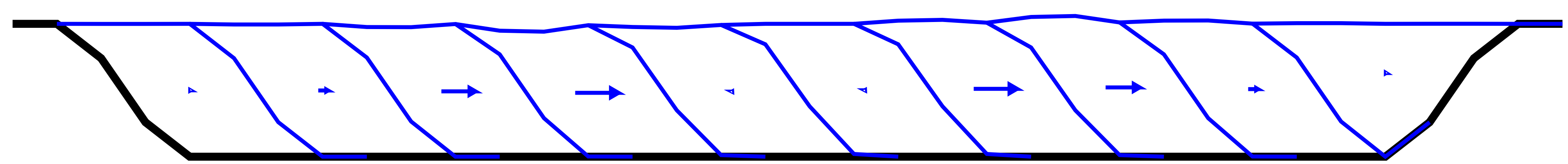
# External Gravity Waves



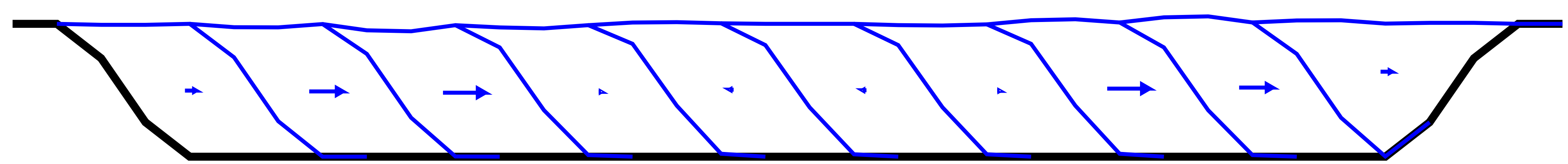
# External Gravity Waves



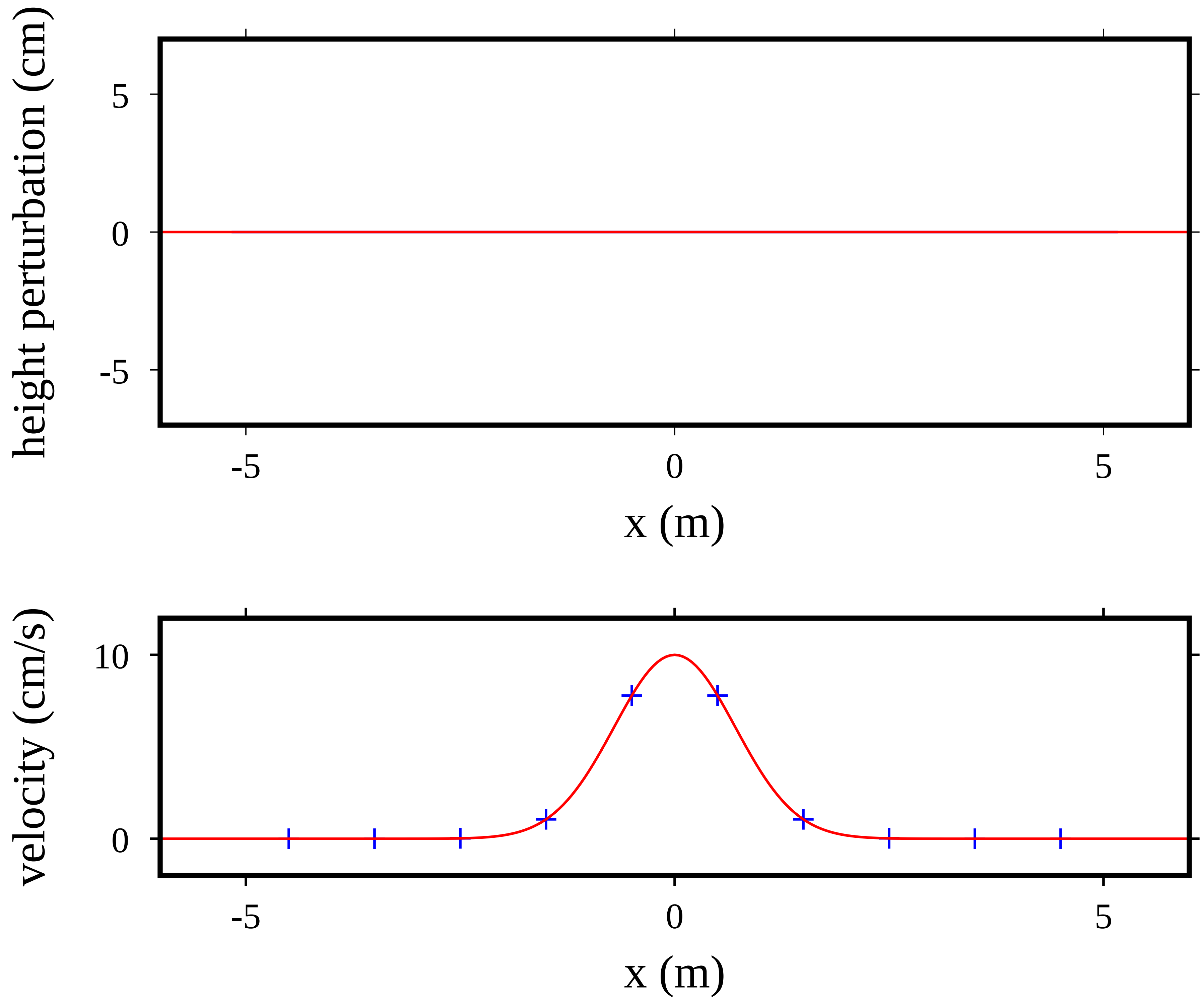
# External Gravity Waves

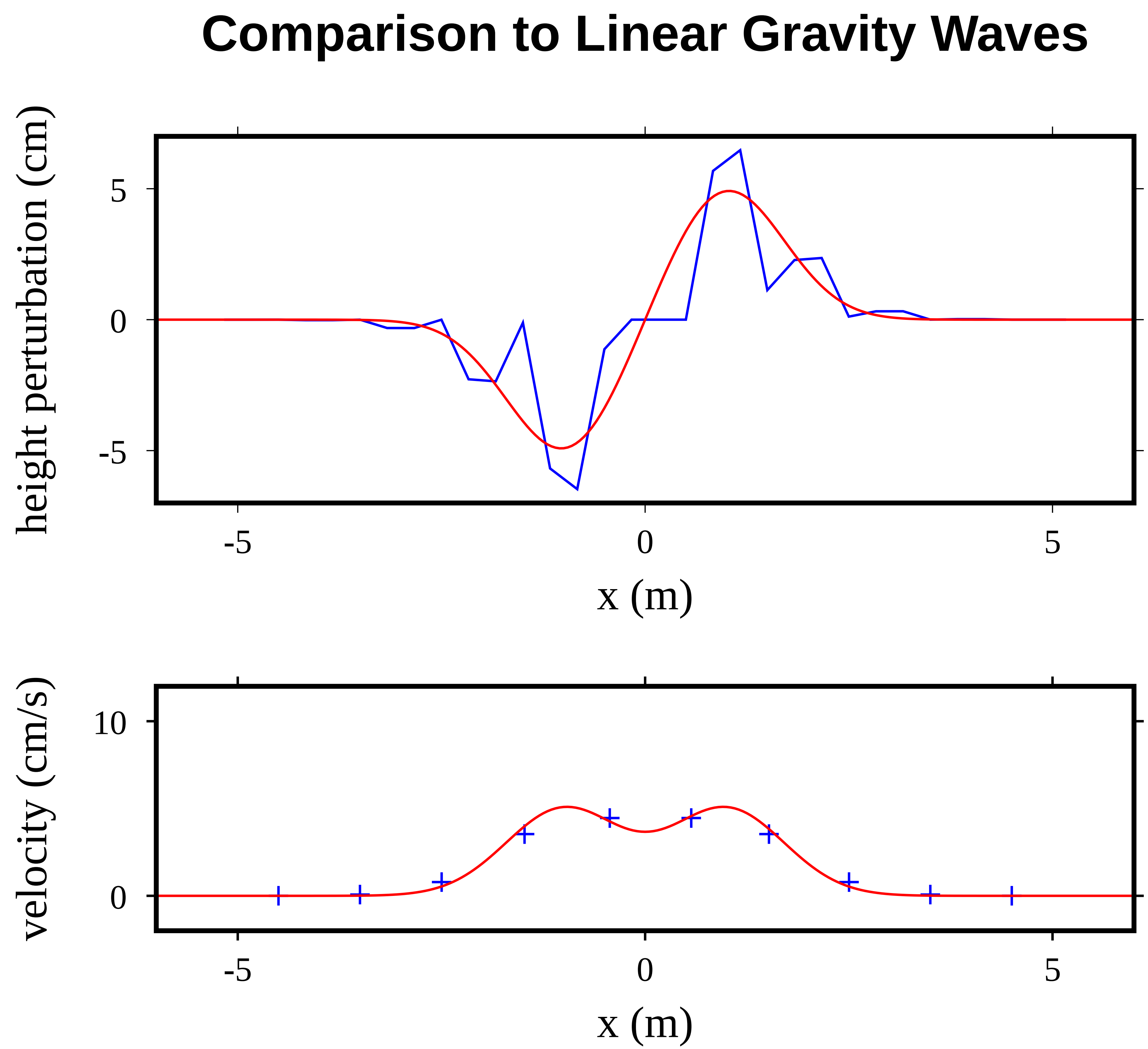


# External Gravity Waves



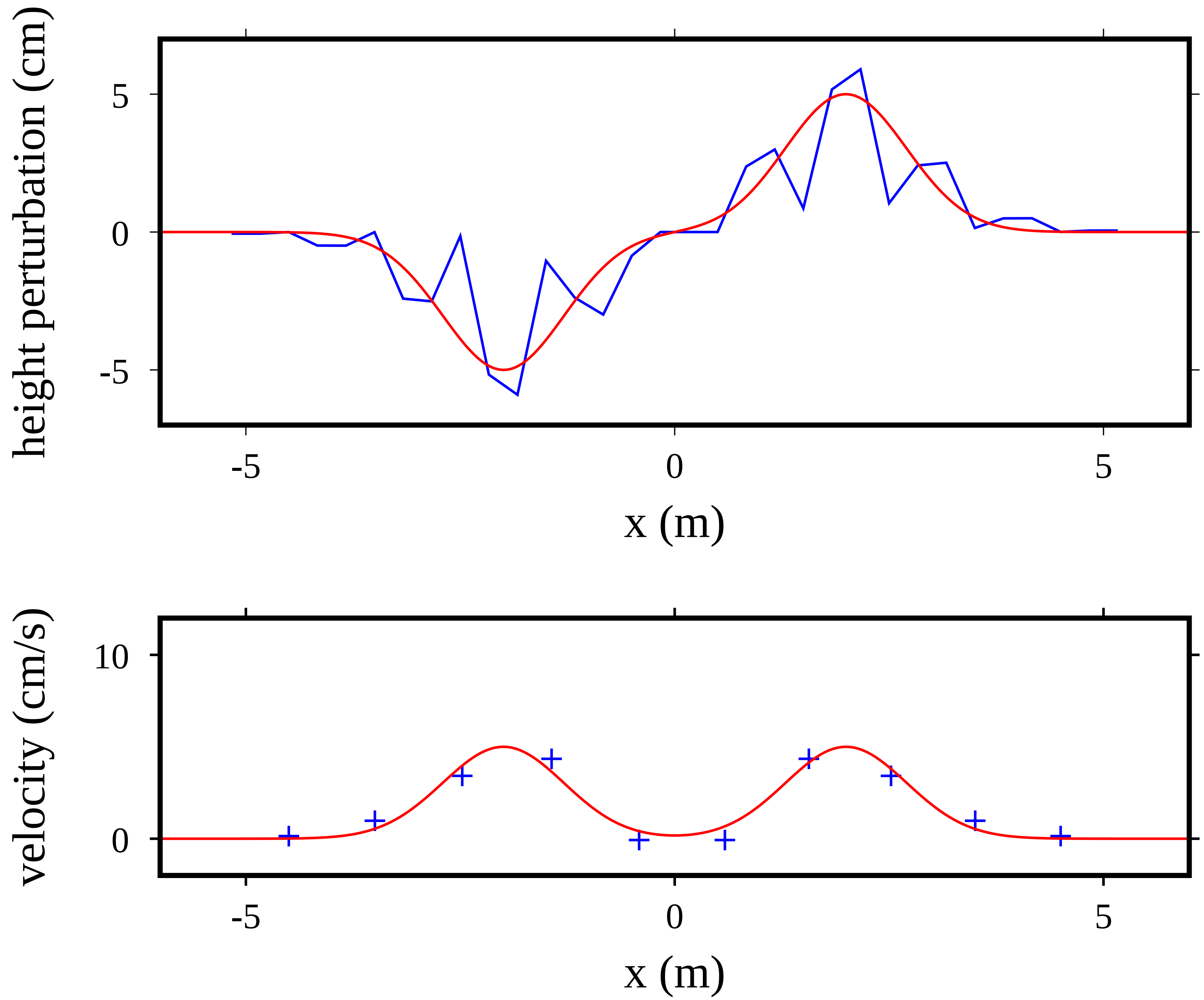
## Comparison to Linear Gravity Waves

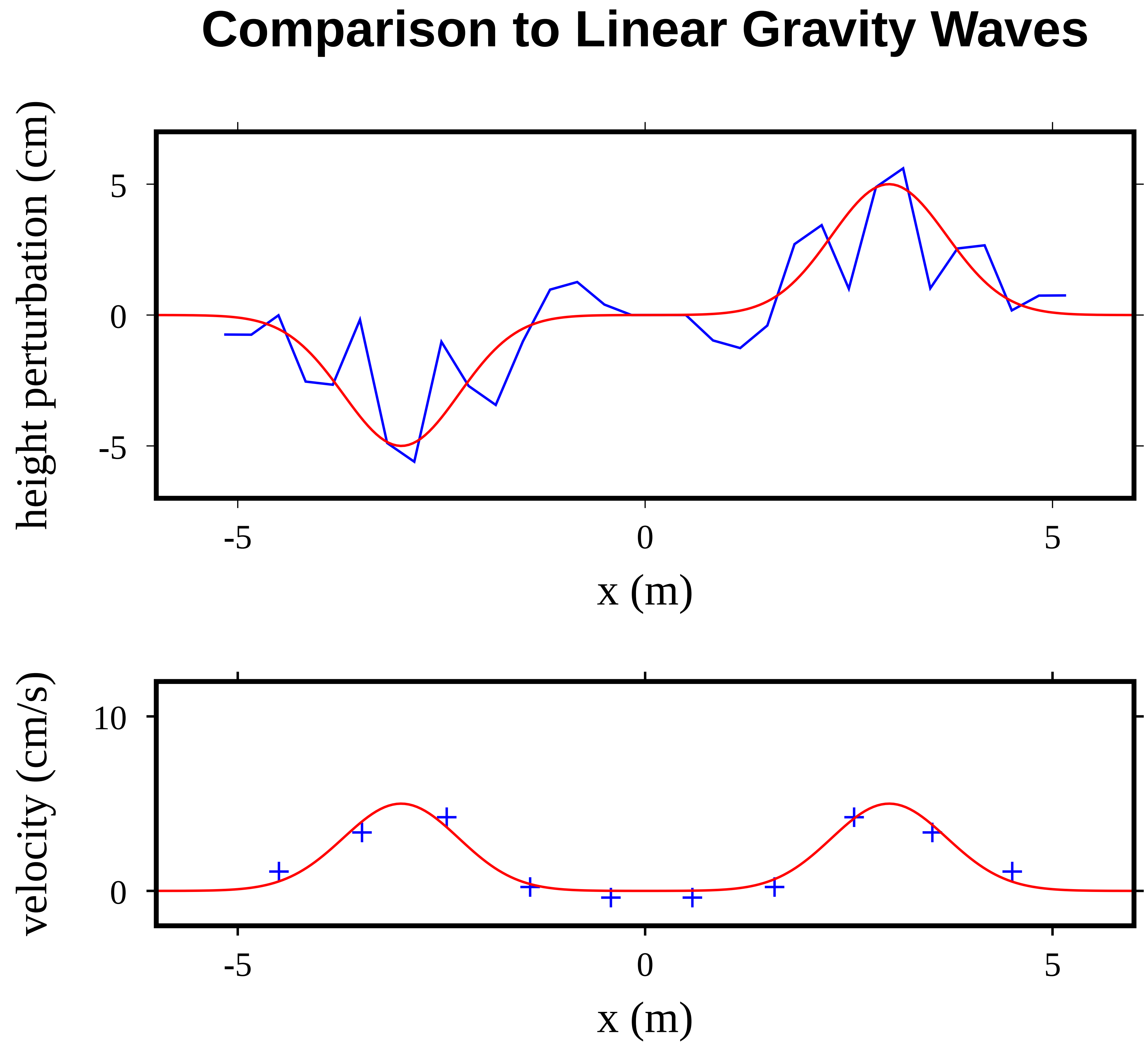


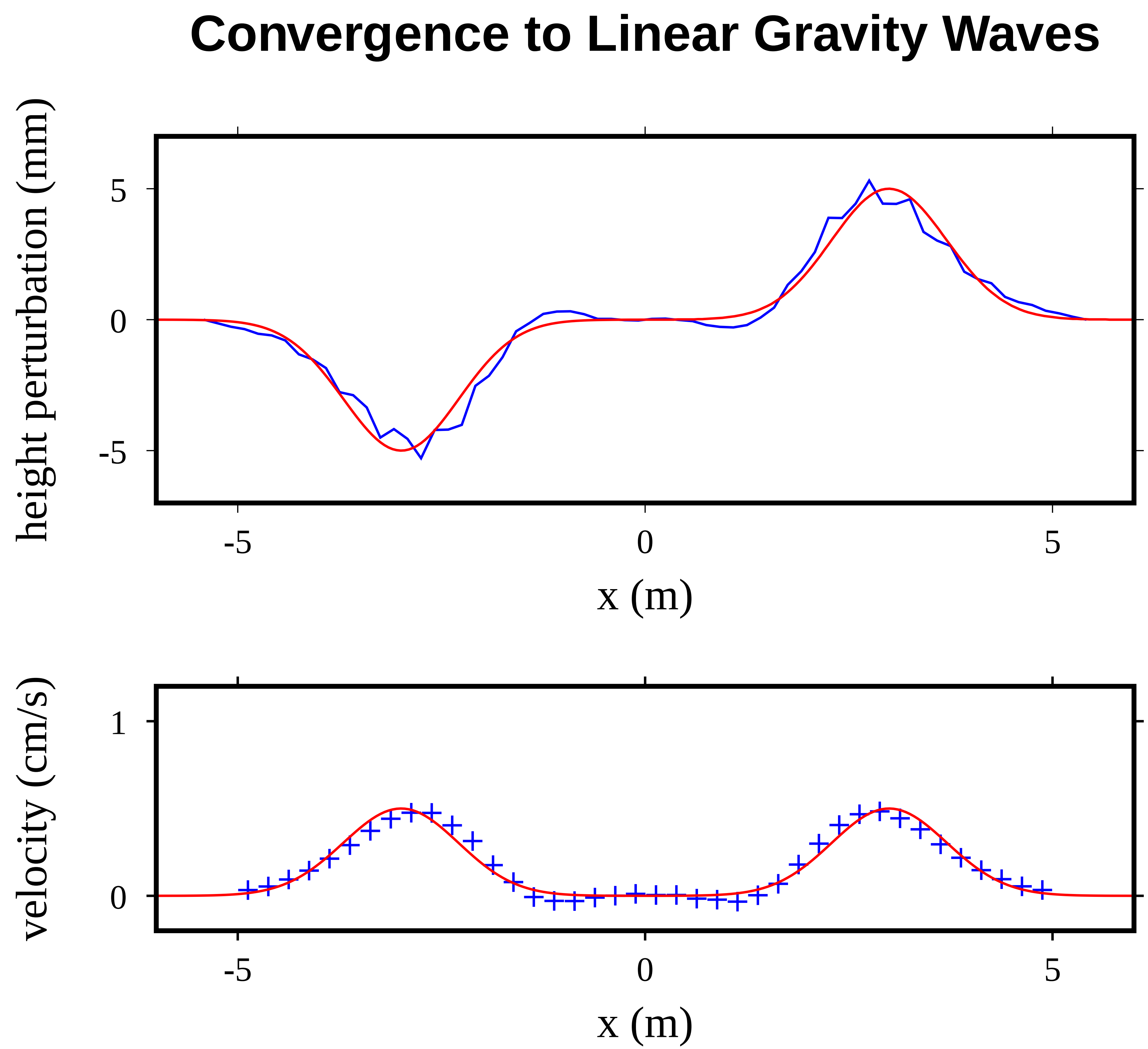


Comparison to Linear Gravity Waves
height perturbation (cm)
5
0
-5
-5
0
5
x (m)
velocity (cm/s)
10
0
-5
0
5
x (m)

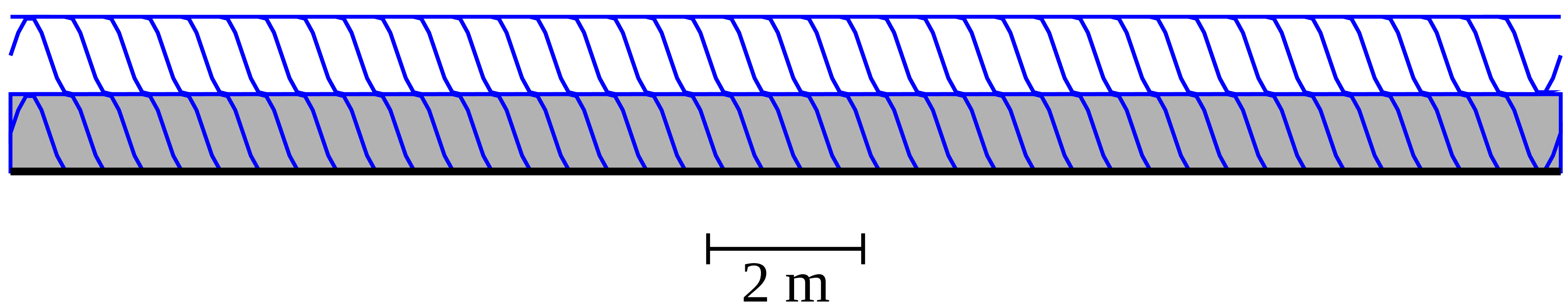
# Comparison to Linear Gravity Waves

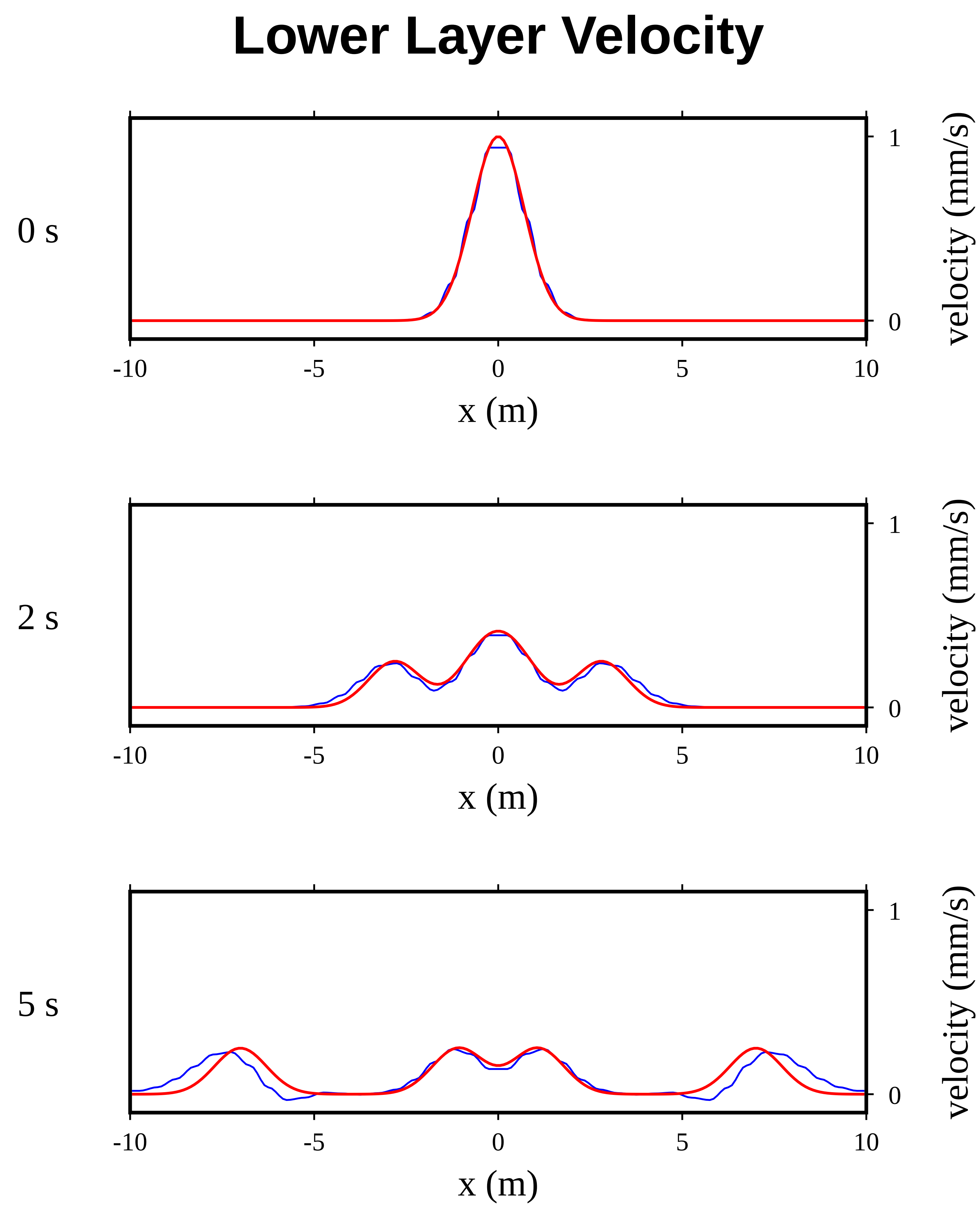



Comparison to Linear Gravity Waves
height perturbation (cm)
5
0
-5
-5
0
5
x (m)
velocity (cm/s)
10
0
-5
0
5
x (m)

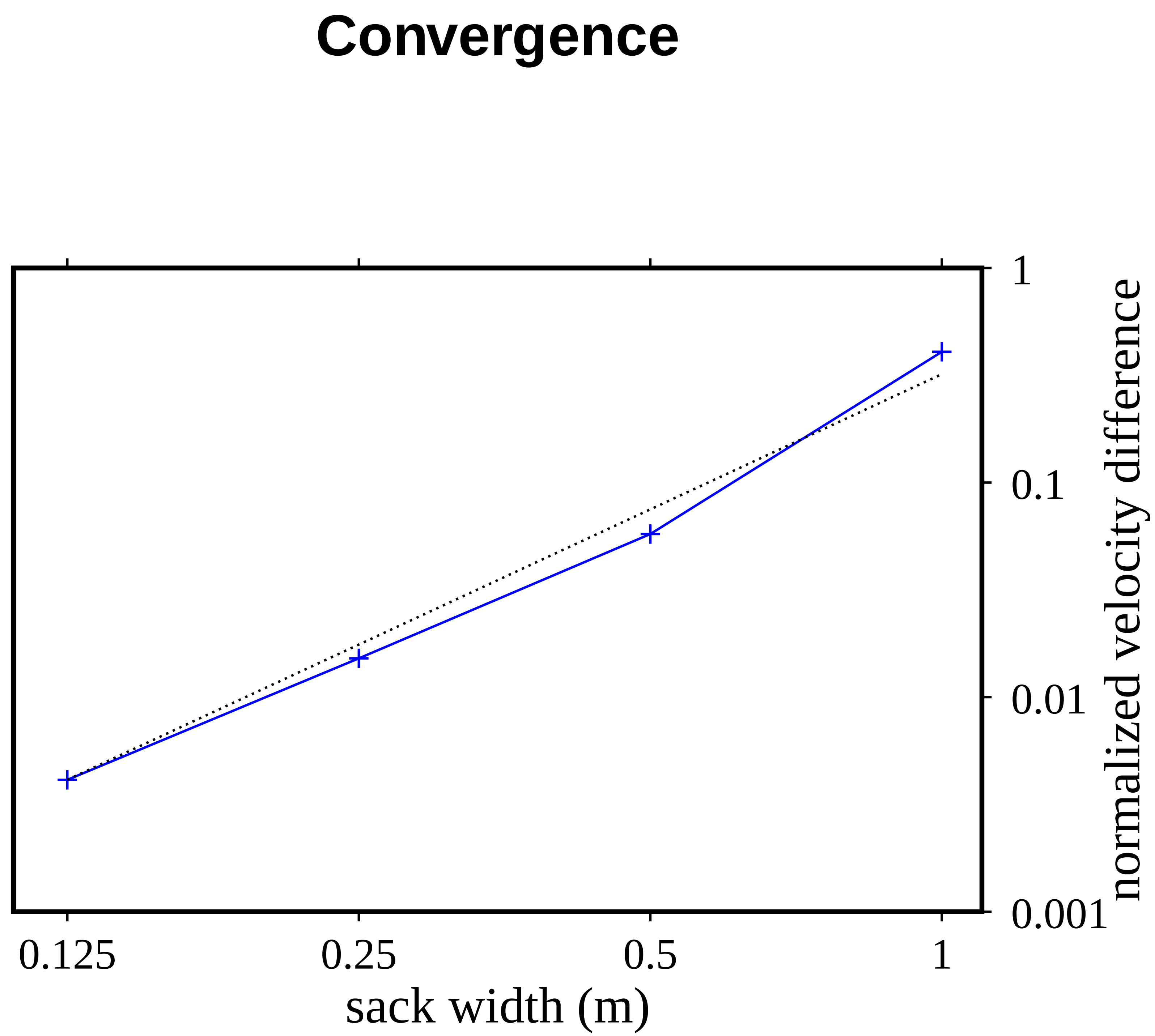

Convergence to Linear Gravity Waves
height perturbation (mm)
5
0
-5
-5
0
5
x (m)
velocity (cm/s)
1
0
-5
0
5
x (m)

# Internal Gravity Waves





Lower Layer Velocity
0 s
2 s
5 s
velocity (mm/s)
x (m)
-10
-5
0
5
10
1
0


Convergence
1
0.1
0.01
0.001
normalized velocity difference
0.125
0.25
0.5
1
sack width (m)

# Lake and Ocean Applications

# Lake Upwelling

# Example from Lake Michigan

*Beletsky et al. 1997*

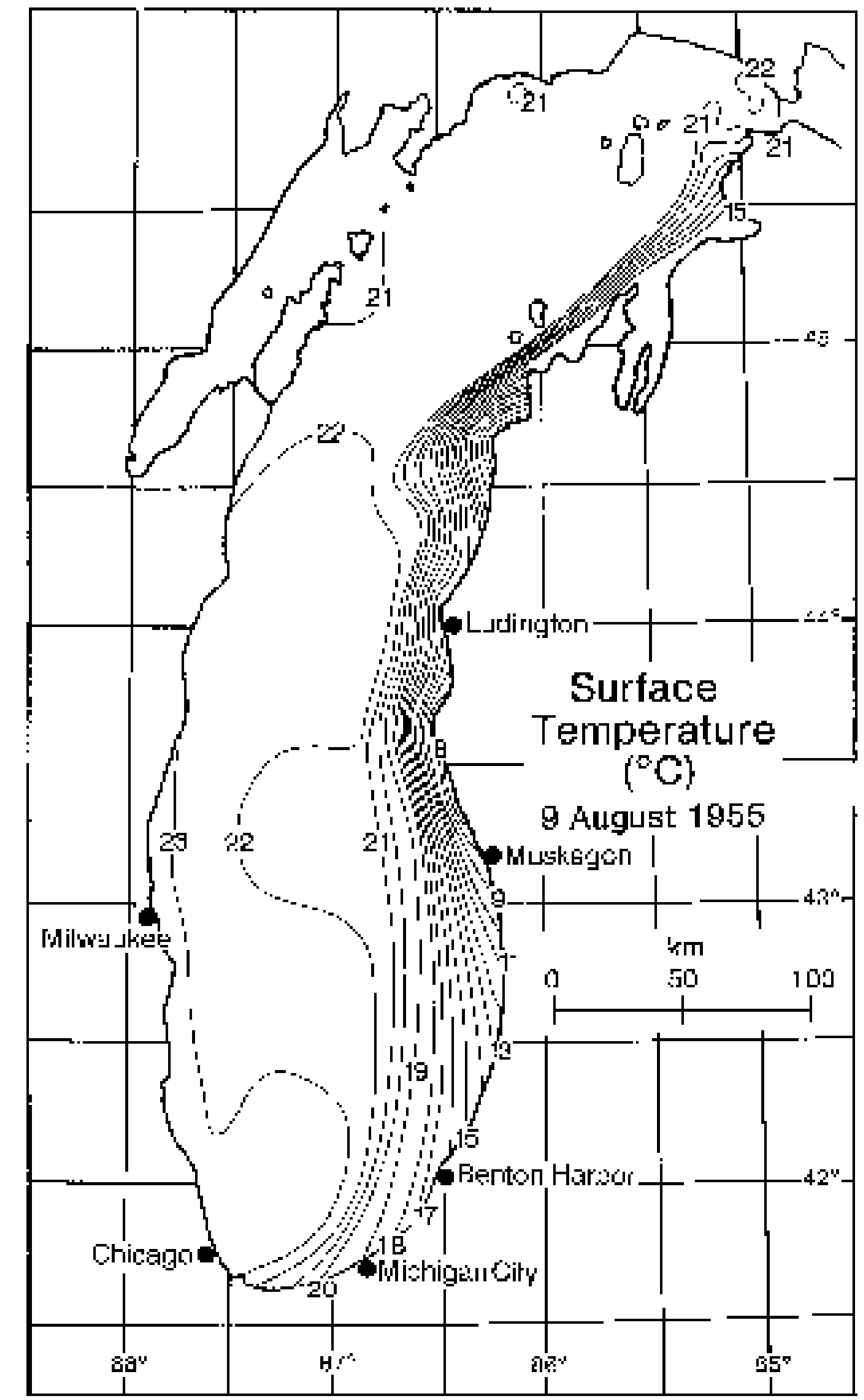
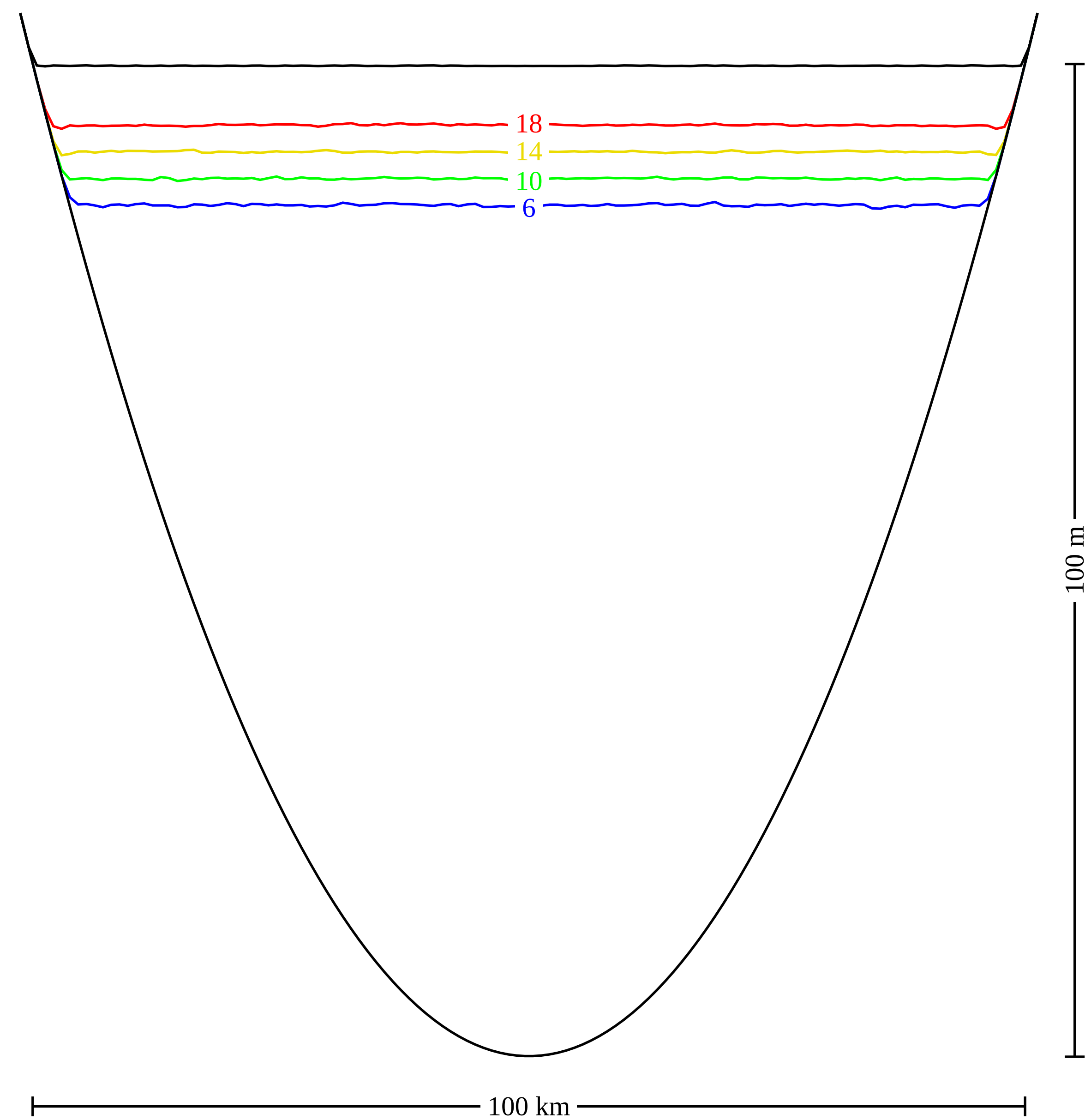



FIG. 14. Surface temperature in Lake Michigan, 9 August 1955, redrawn from Ayers (1958).

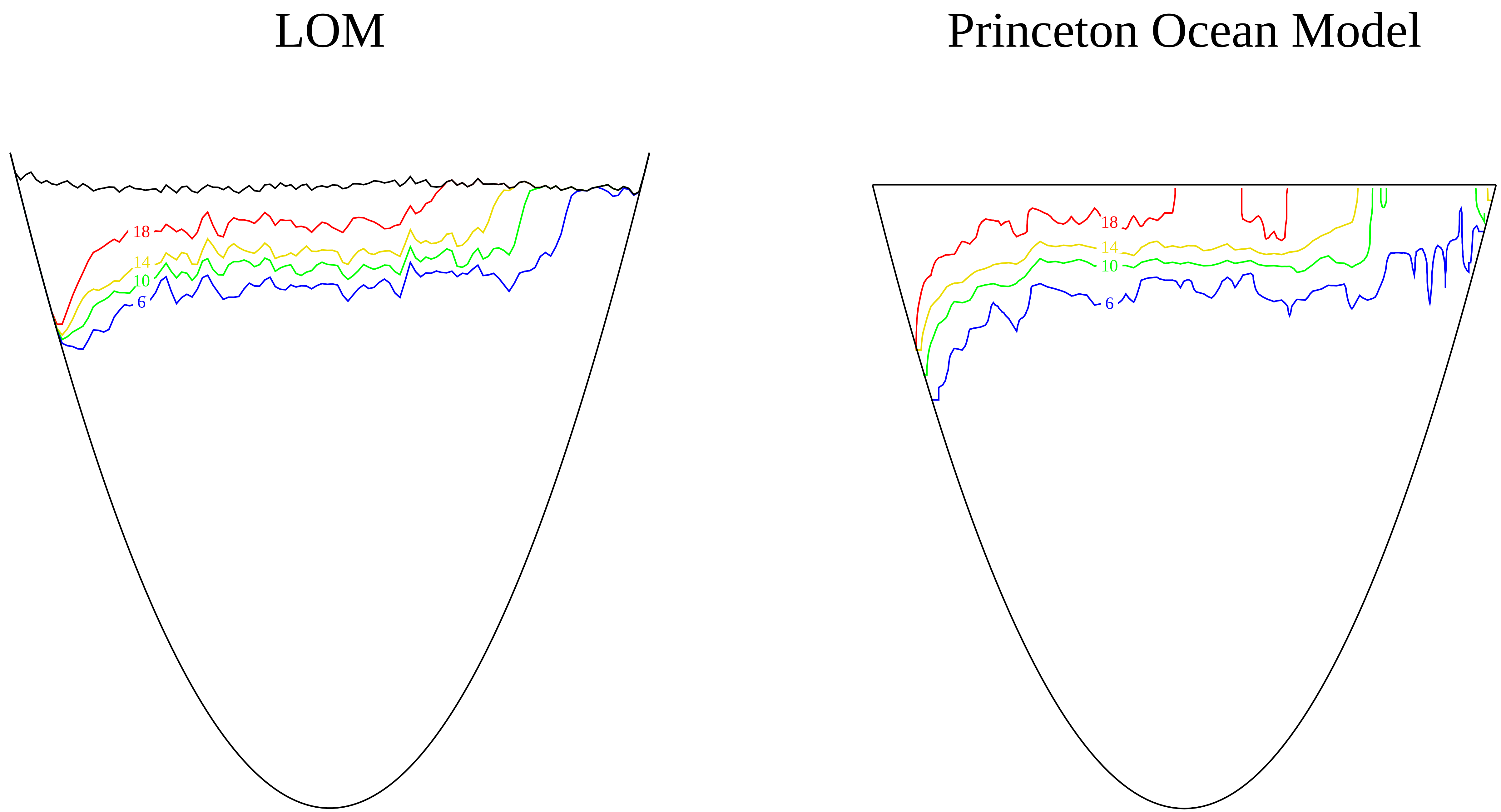
# Initial Condition



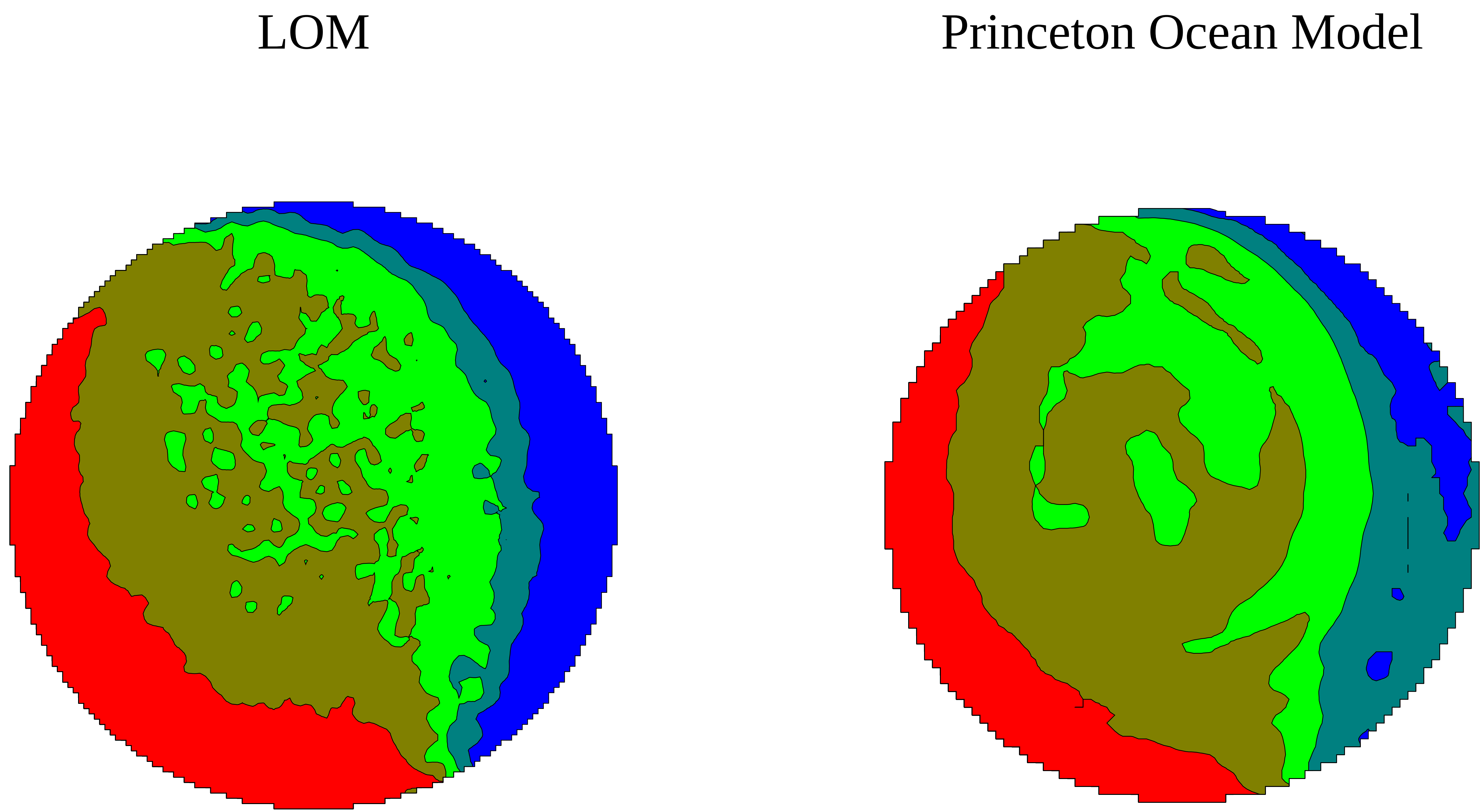
## Forcing

The lake is exposed to northerly winds for 29 hours. The winds ramp up over 18 hours, maintain their maximum strength for 6 hours ($\tau = 0.3\ N\ m^{-2}$, and decay over 5 hours.

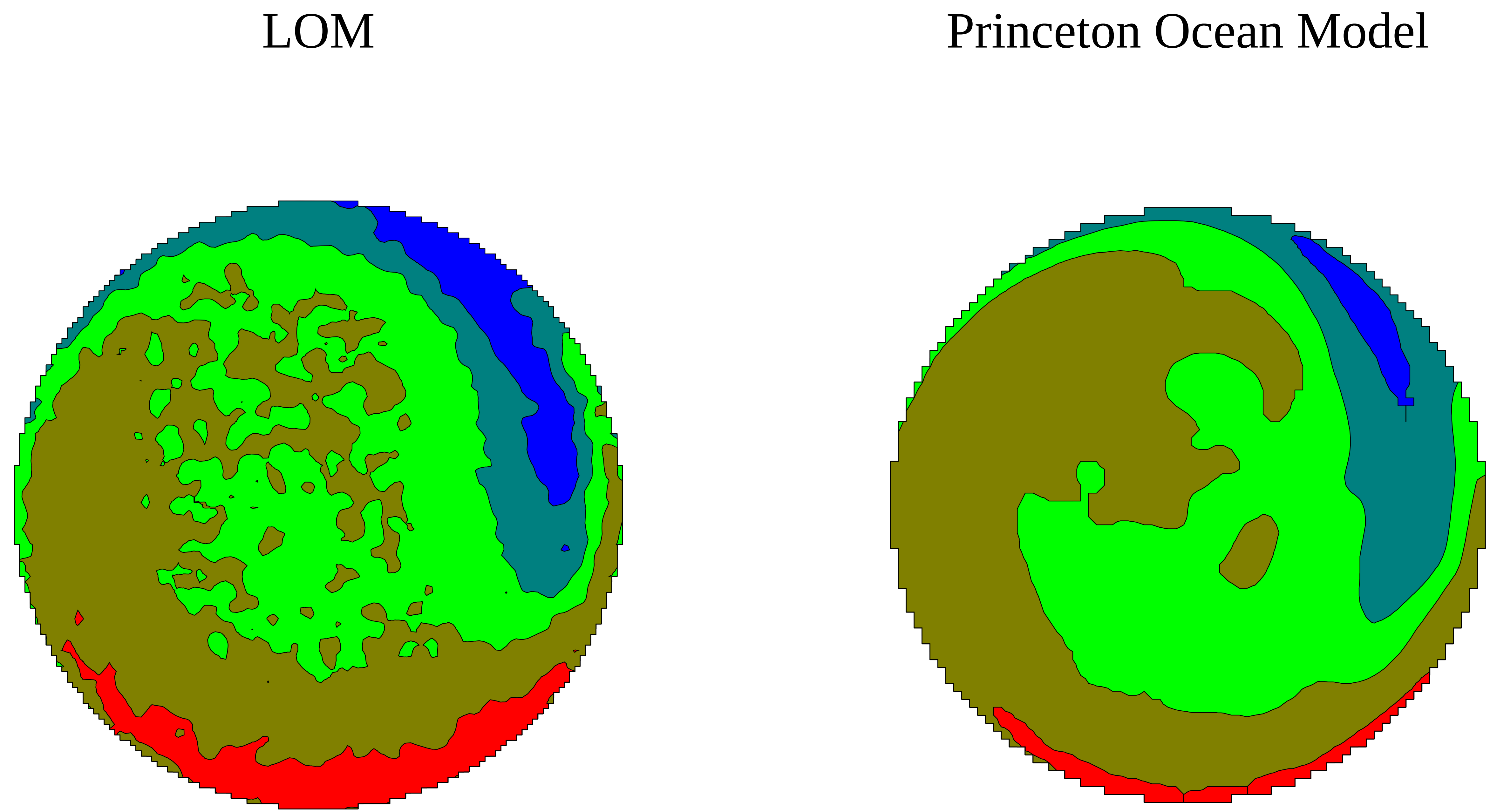
## Vertical cross sections (W-E) of Temperature at 29 *h*



## **Horizontal cross sections (10 *m*) of Temperature at 29 *h***



# Horizontal cross sections (10 $m$) of Temperature at 120 $h$



# Equatorial Oceans

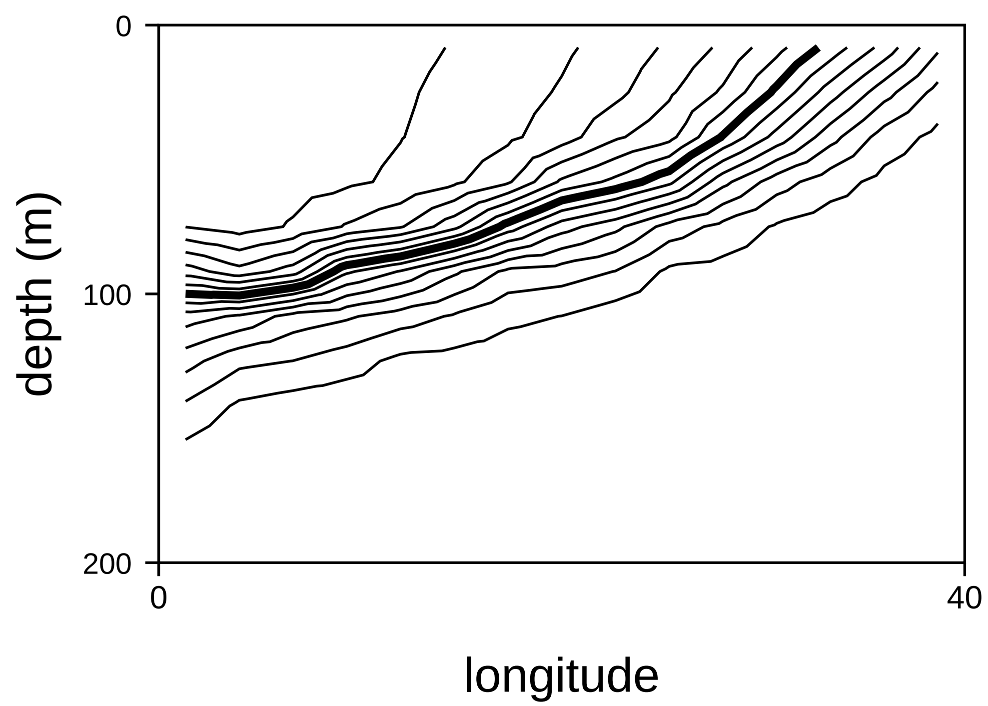
## Model Set-Up

- follows Fedorov et al. (2004)
- $40^o$ by $32^o$ straddling the Equator
- constant westward wind stress: 0.05 $N\ m^{-2}$
- restoring temp.: 25 C from 10 S to 10 N, drops 10 C at 16 S and 16 N

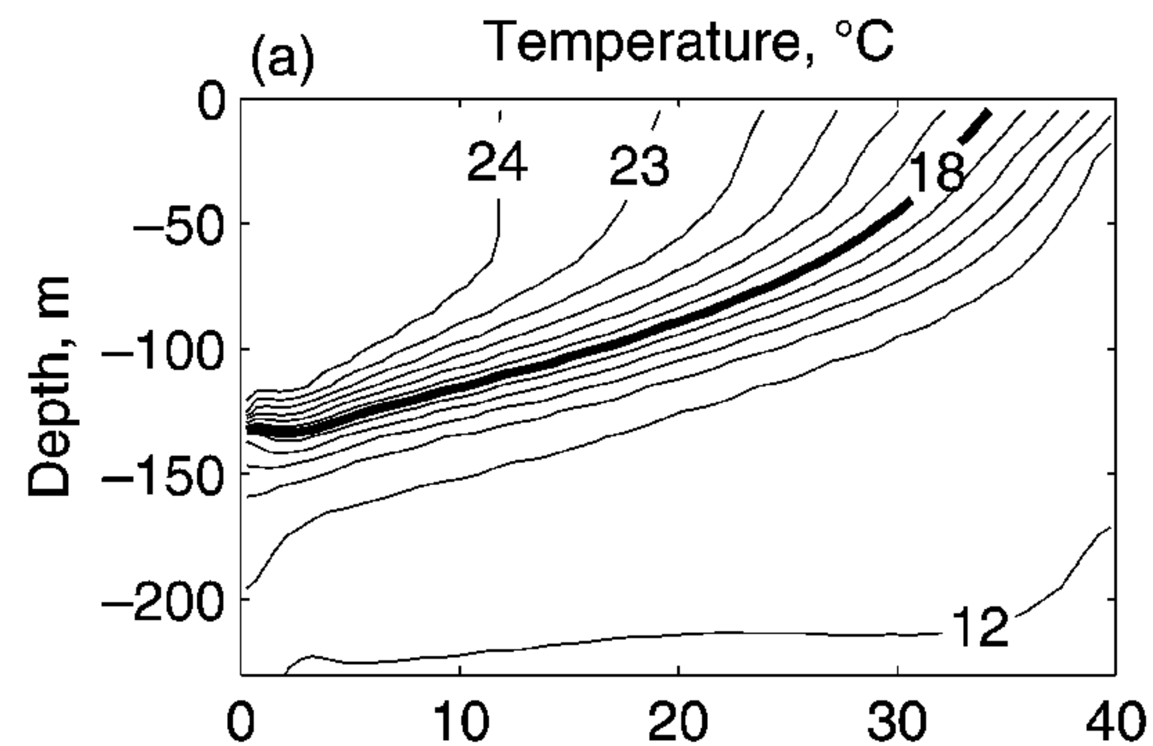
# Quasi-steady solutions

# Temperature Along the Equator

LOM

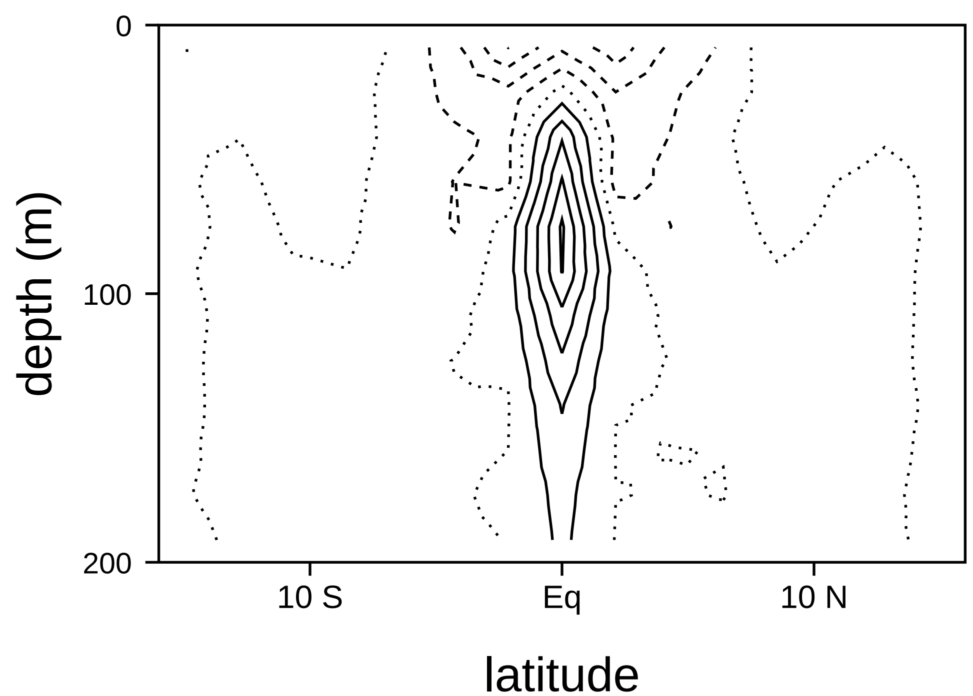



MOM4

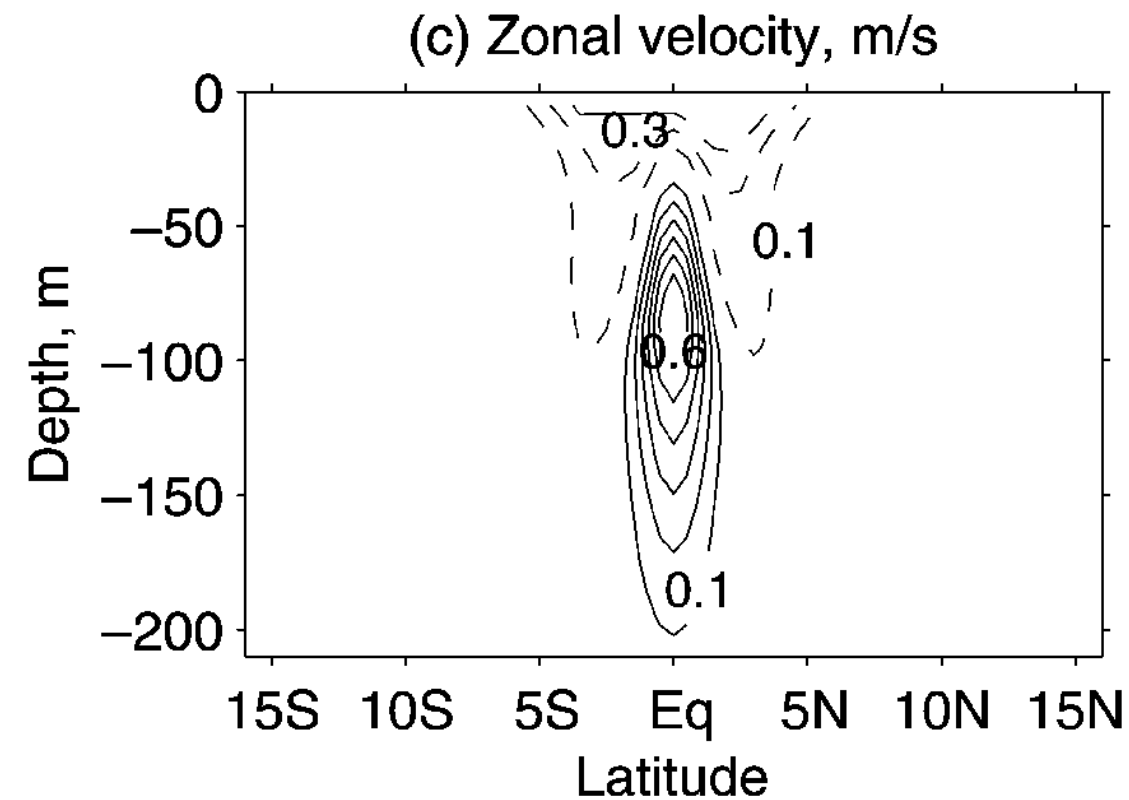


# Zonal Velocity Along 20 E

LOM



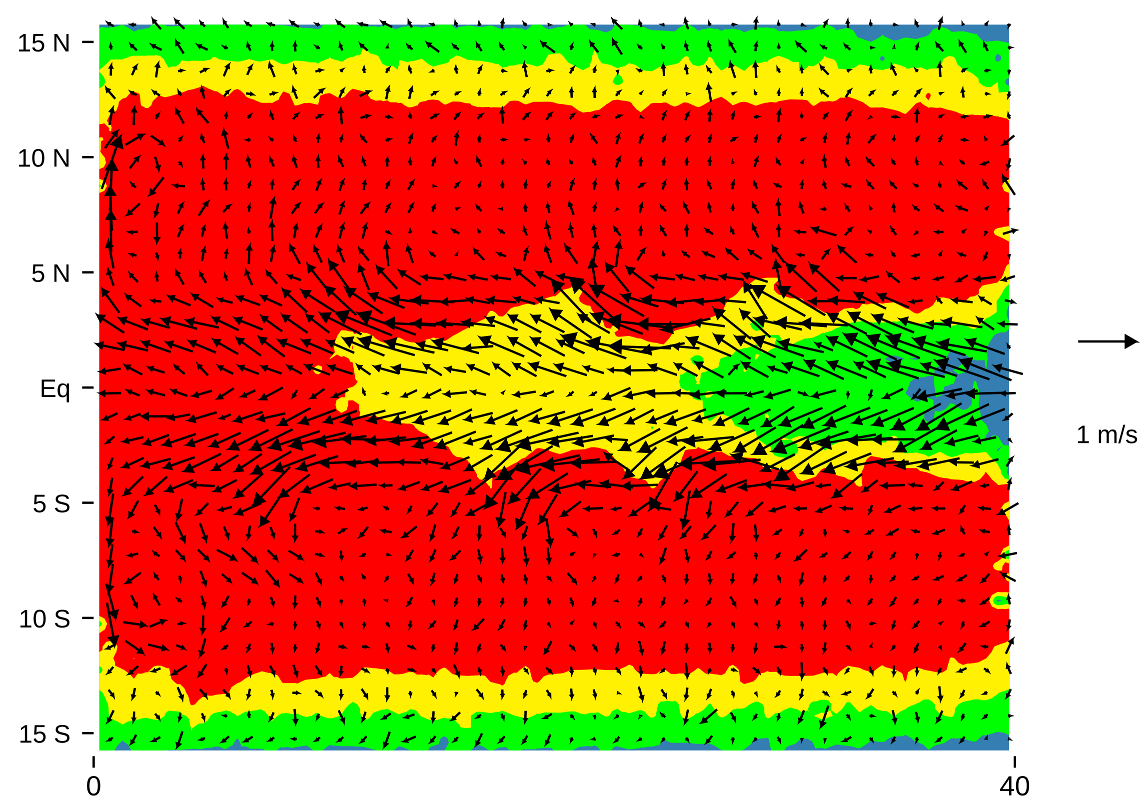

MOM4



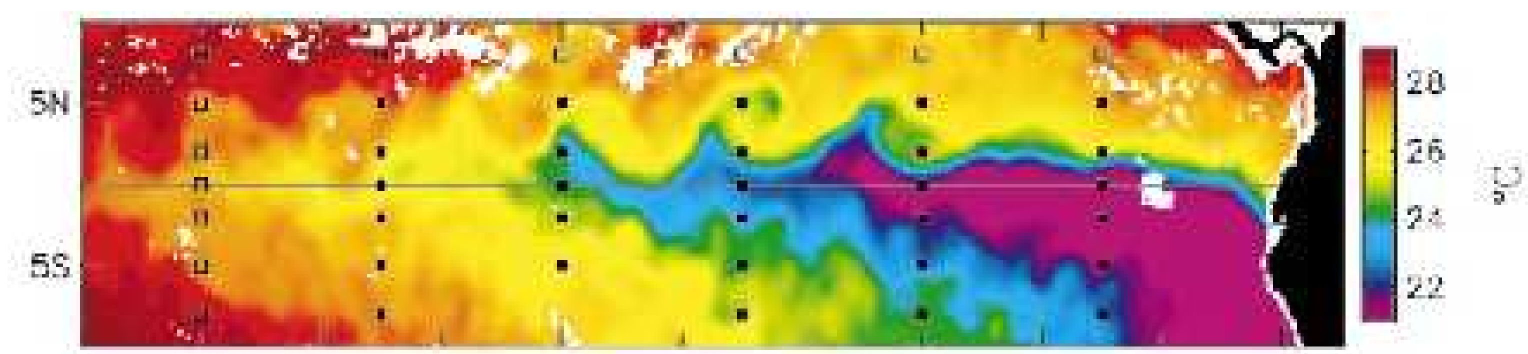
# Tropical Instability Waves

*Surface Temperature*

LOM



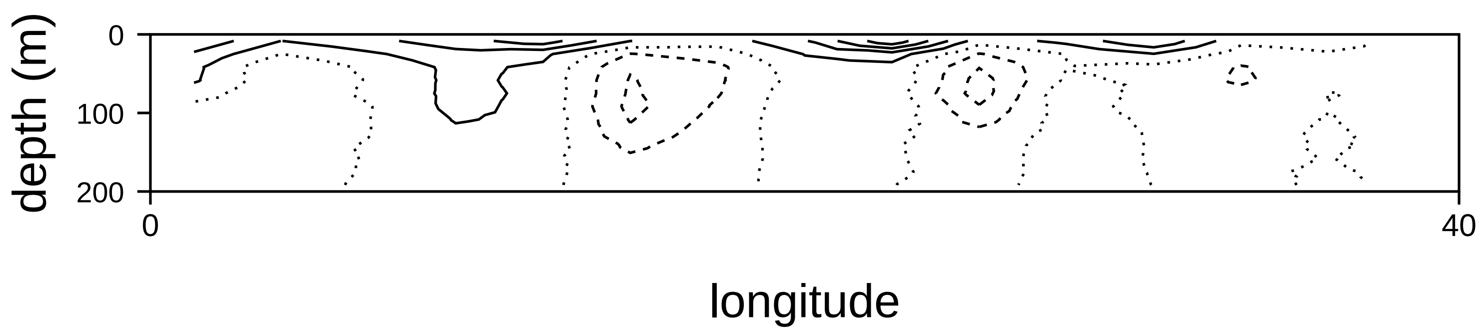

Observed



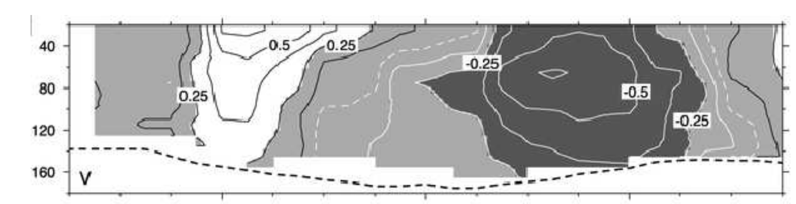
# Tropical Instability Waves

*Meridional Velocity Along 5 N*

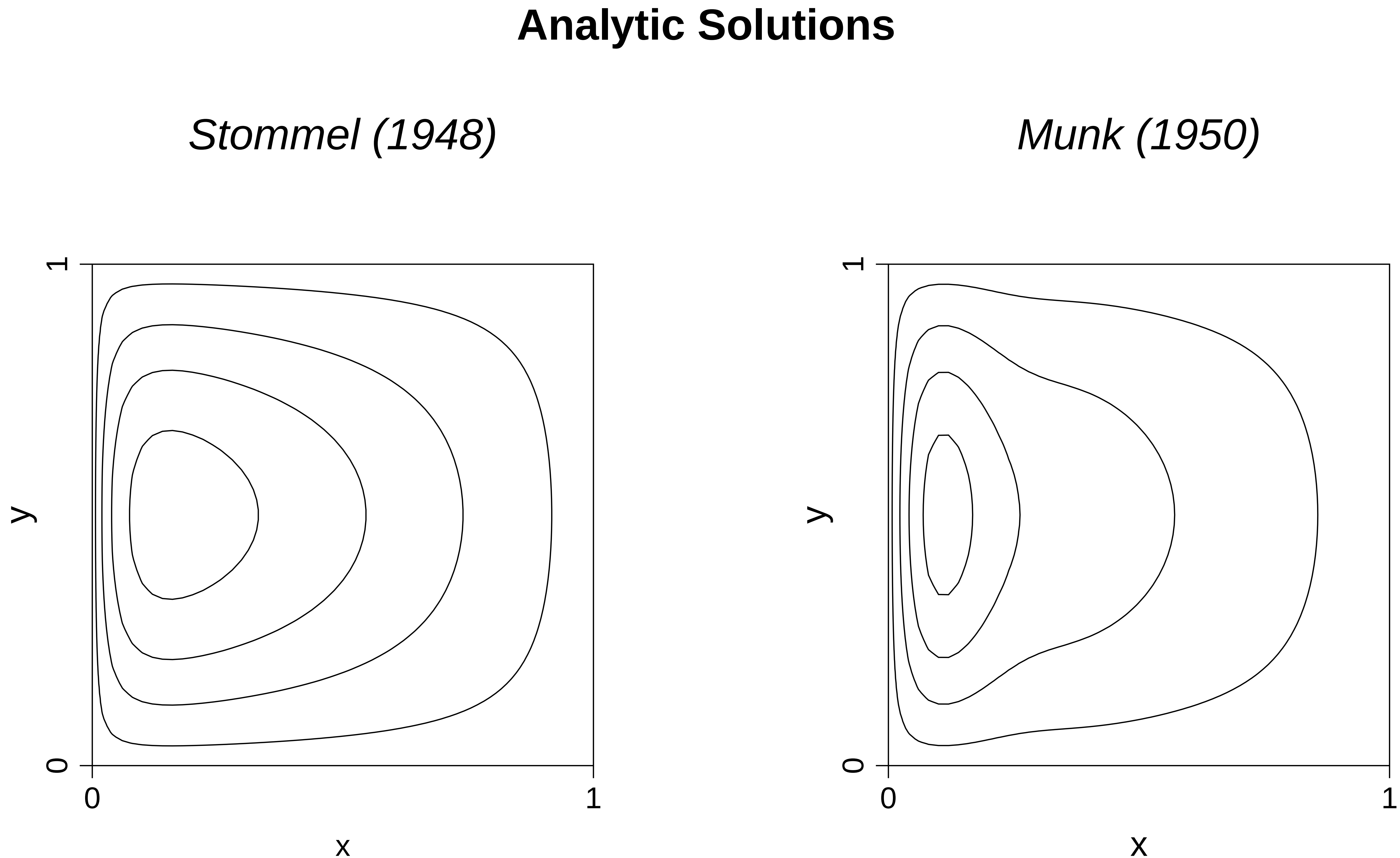
LOM



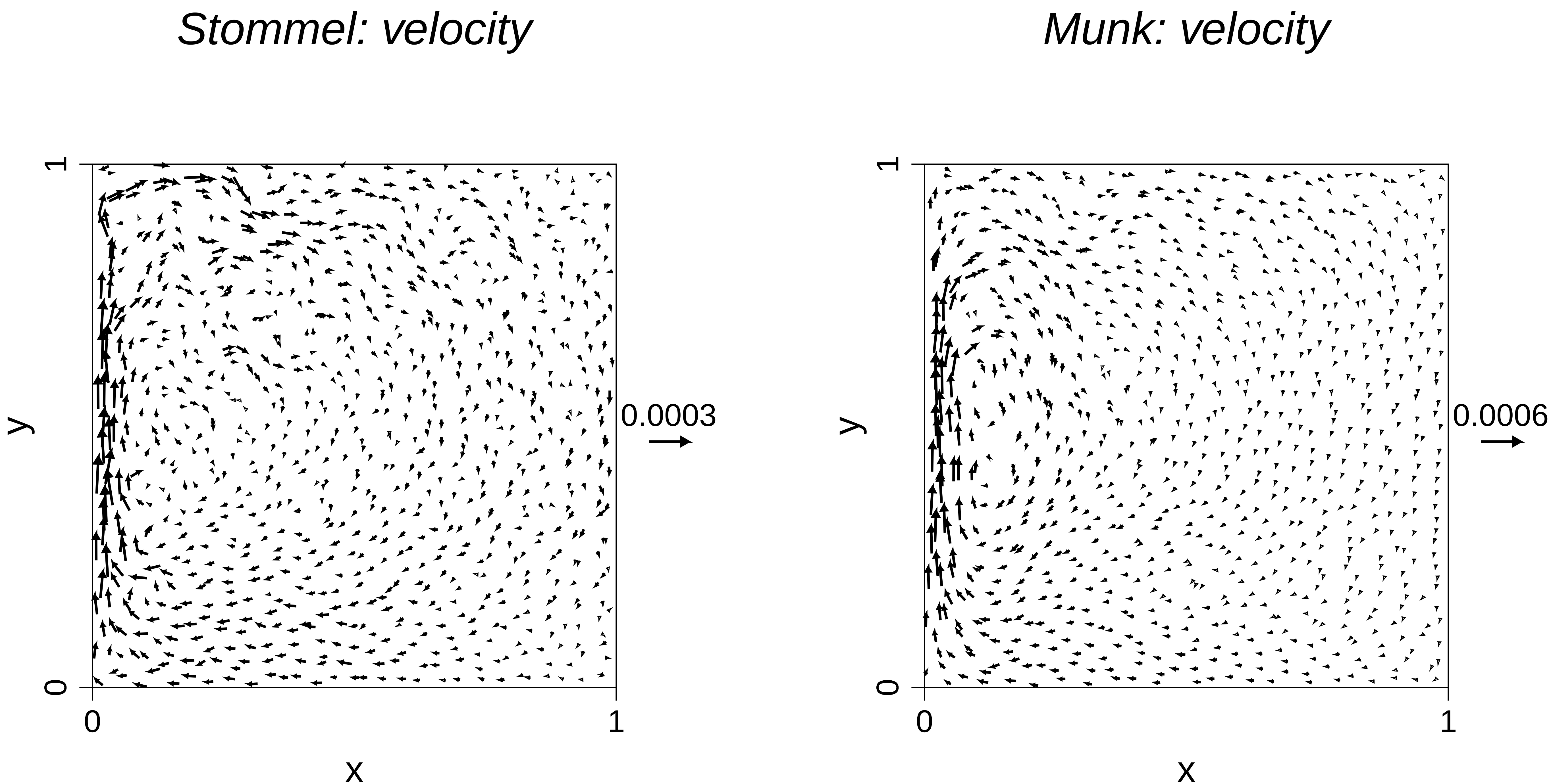

Observed



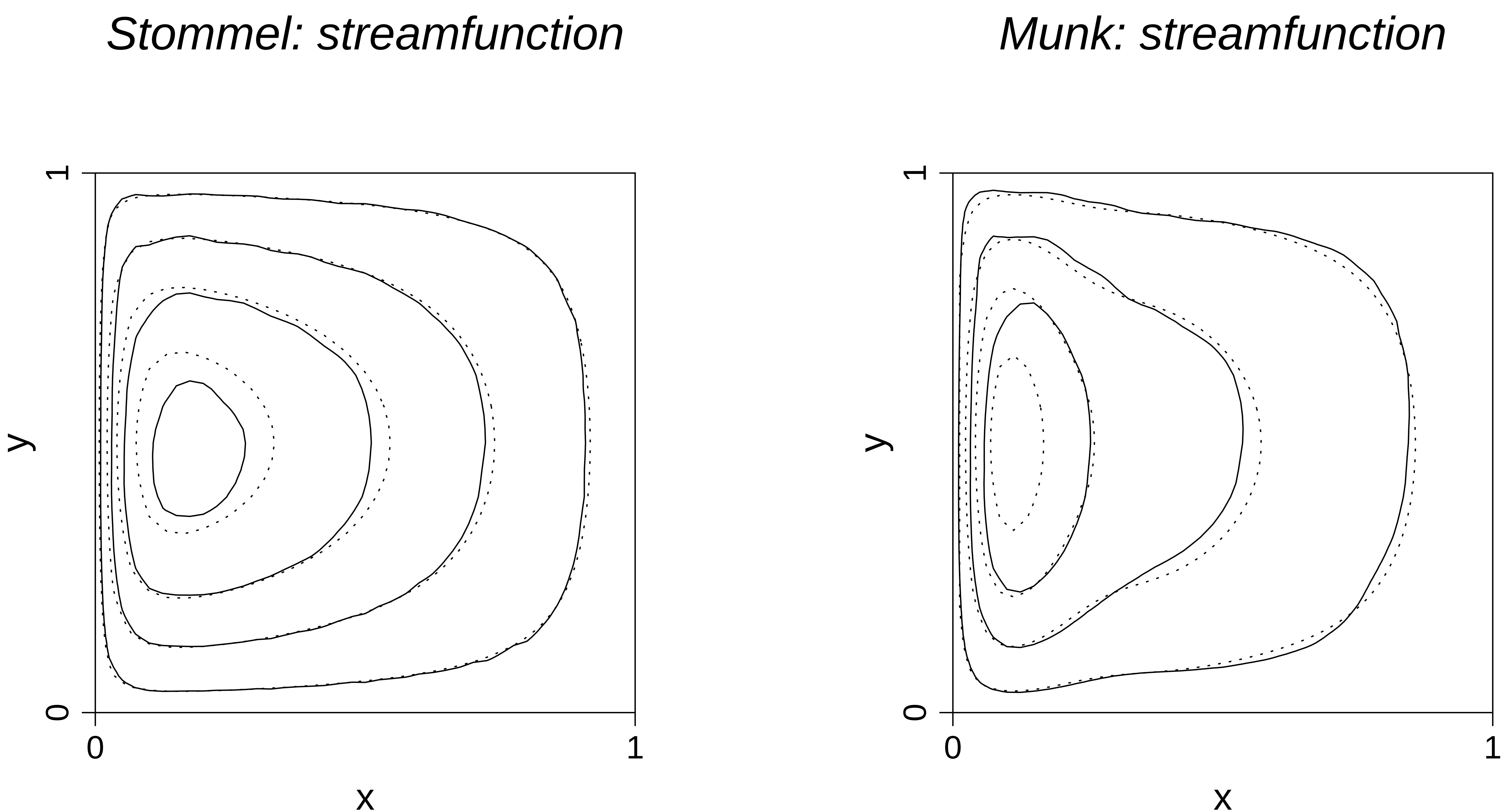
# Western Boundary Currents in Homogeneous Oceans


Analytic Solutions
Stommel (1948)
Munk (1950)
1
y
0
0
1
x
1
y
0
0
1
x

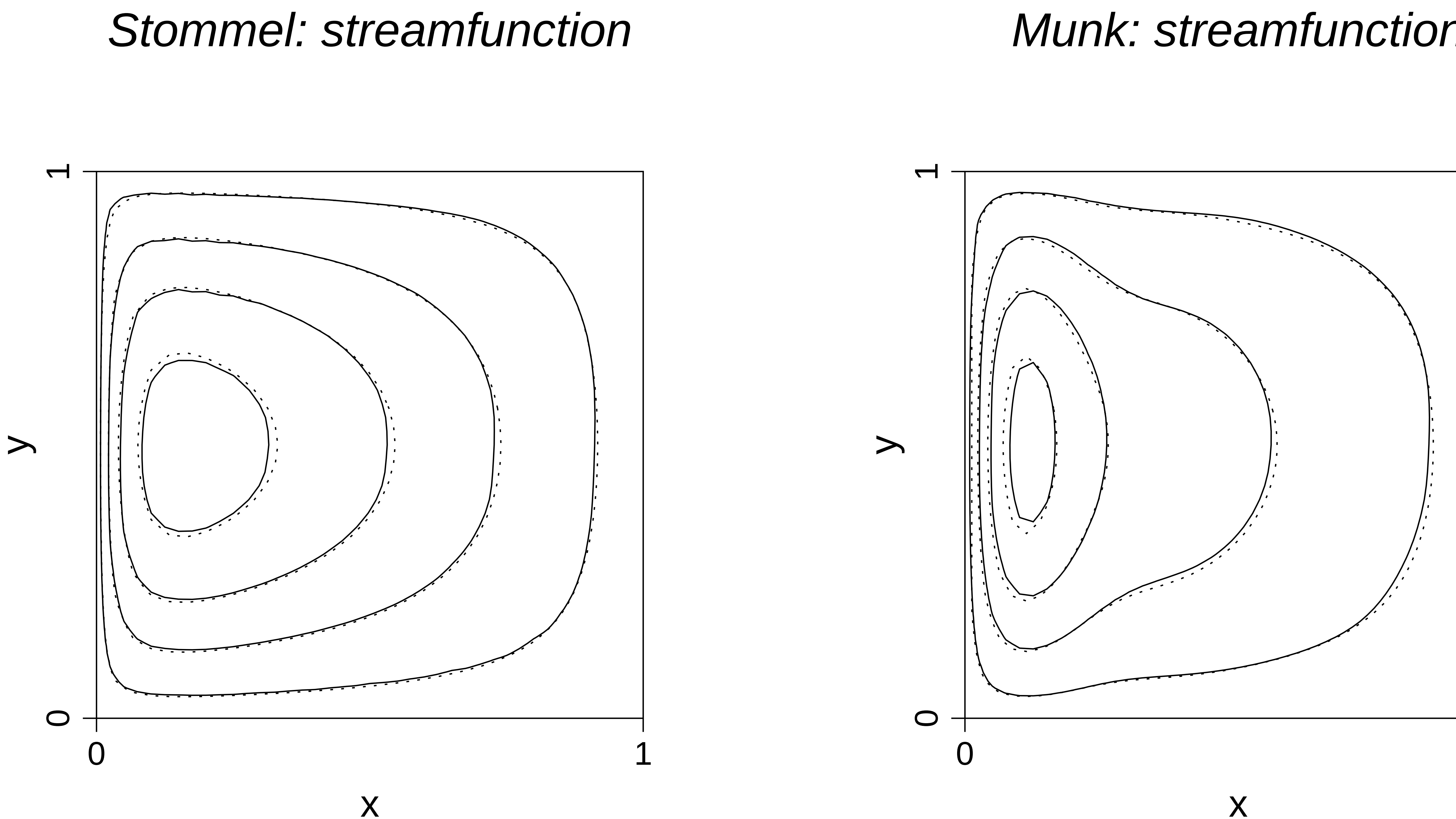
# Low-Resolution LOM Simulations



# Low-Resolution LOM Simulations



# Medium-Resolution LOM Simulations

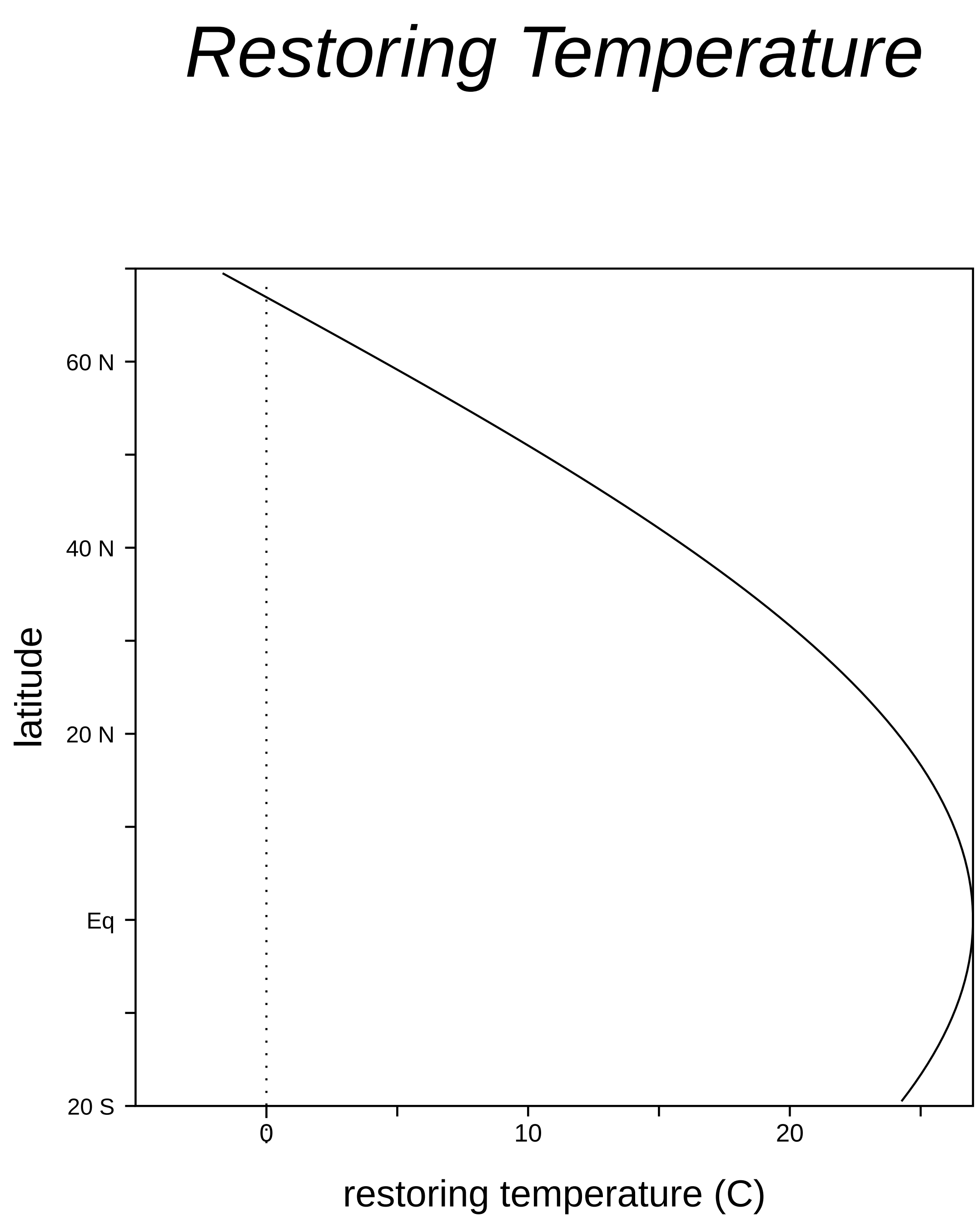


# Idealized Model of the North Atlantic Ocean

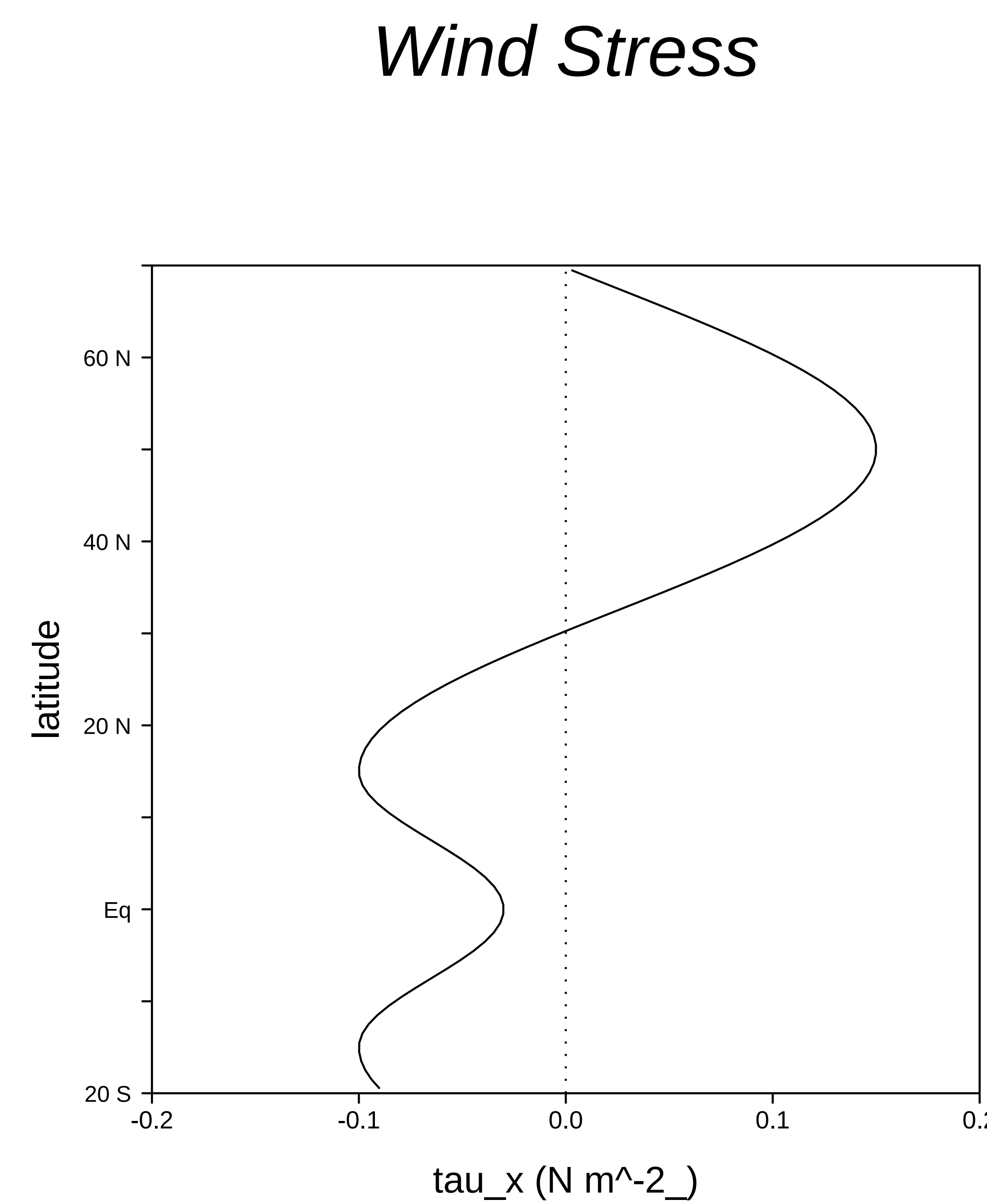
## Model Set-Up

- Basin: $0 – 60^o$ W, $20^o S$ - $70^o N$, 4500 $m$ deep
- analytic wind and surface temperature forcings
- compare $3^o$ LOM and $1^o$ MITgcm runs

# Forcing

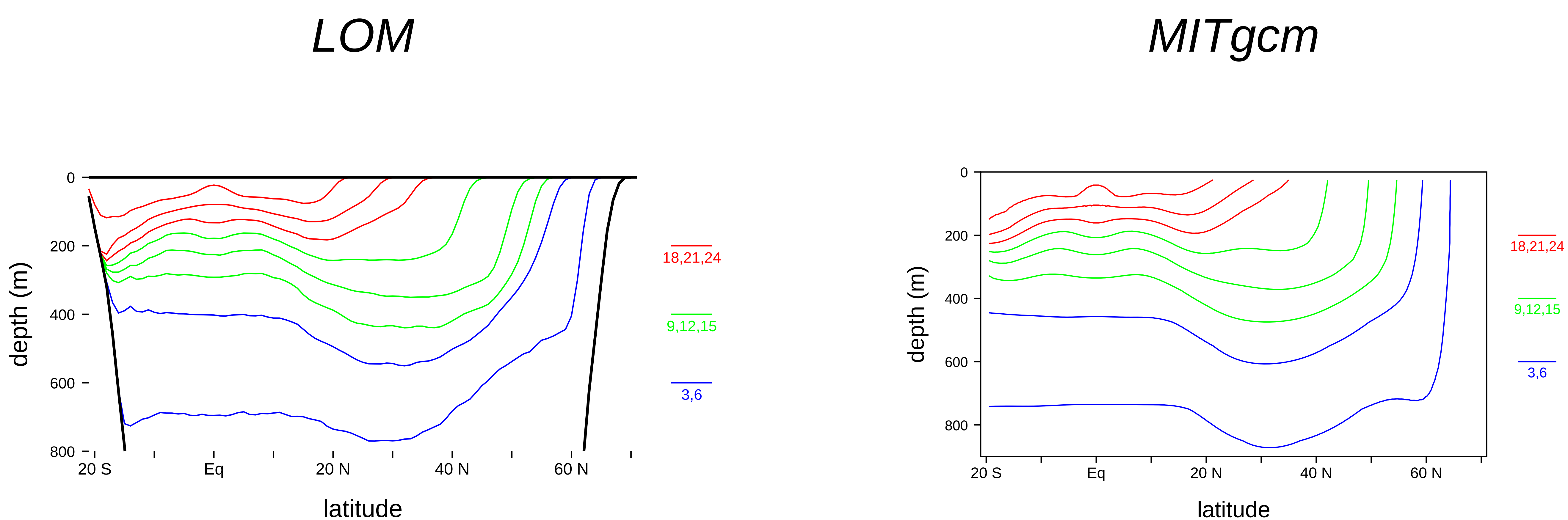


Restoring Temperature
latitude
60 N
40 N
20 N
Eq
20 S
0
10
20
restoring temperature (C)

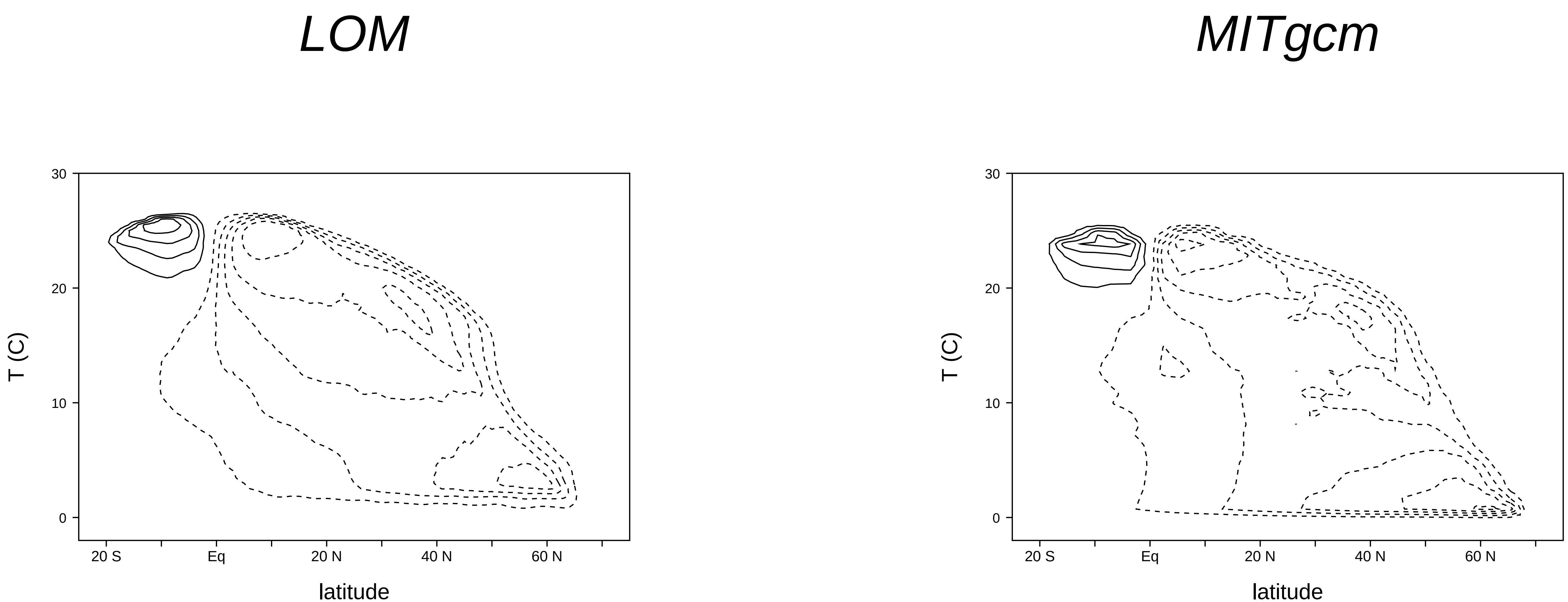



Wind Stress
latitude
60 N
40 N
20 N
Eq
20 S
-0.2
-0.1
0.0
0.1
0.2
tau_x (N m^-2_)

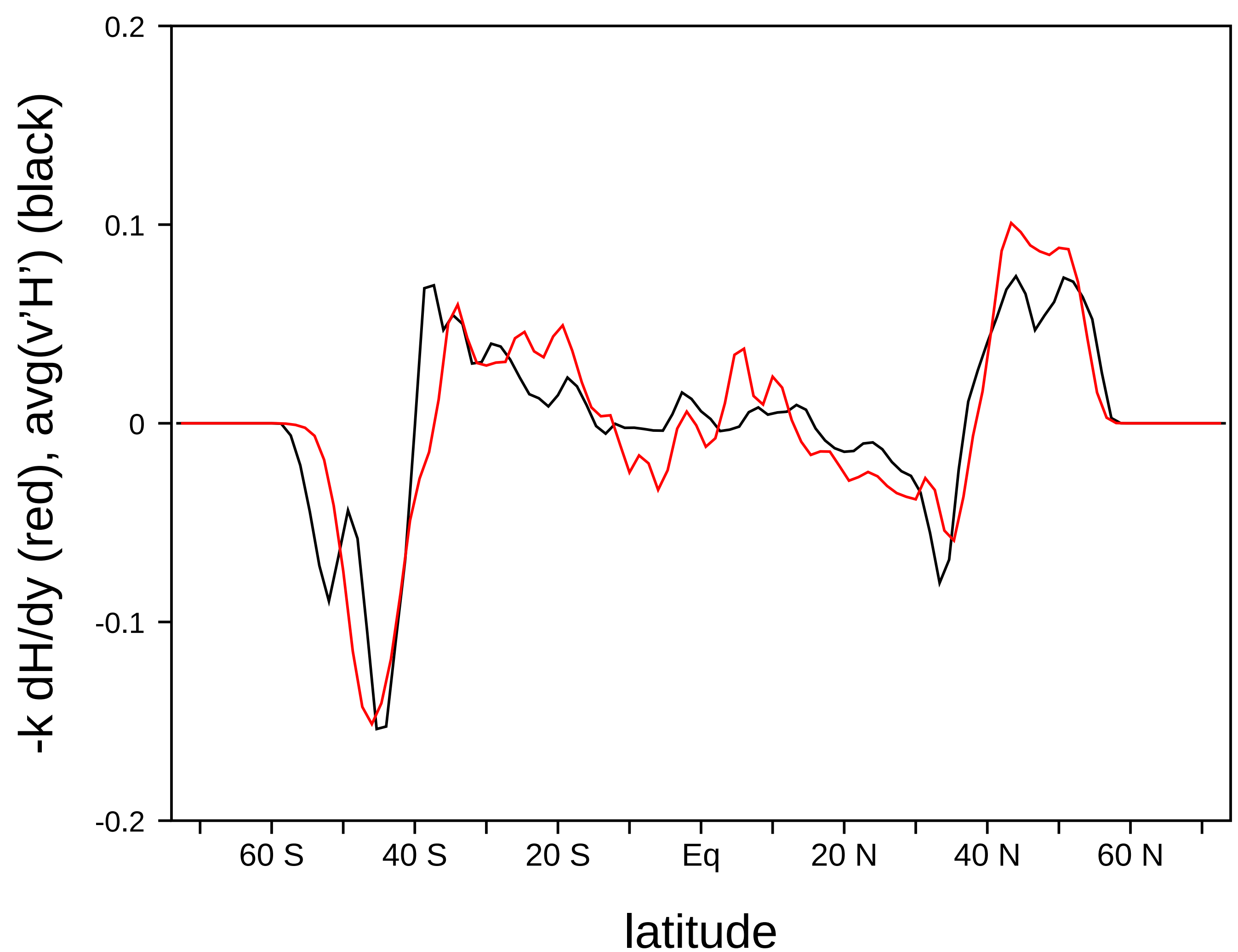
# Thermocline Structure



# Meridional Overturning



# Bolus Tranport of 9-15 C Layer Thickness vs Gent-McWilliams Flux



# Lagrangian Modeling of Atmospheric Convective Systems

## Motivations

1. Convectively coupled equatorial waves are poorly represented in climate models (e.g., D. Williamson talk, Straub and Haertel 2010, J. Climate)

2. Another potential bridge for the gap between GCMs and CRMs (A. Arakawa talk)

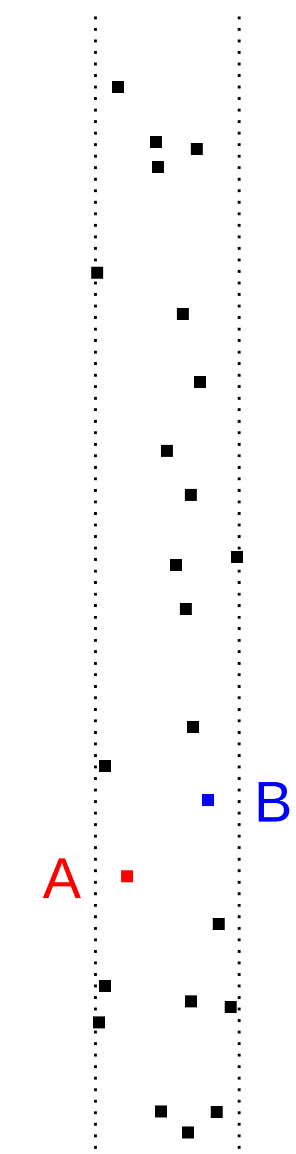
3. Simulating the Madden Julian Oscillation remains a challenge

# Lagrangian Atmospheric Model

- Same general principle as lagrangian ocean model
- Parcels have constant potential temperature instead of constant density
- Slightly different pressure force equation
- Same treatment of viscosity and diffusion
- Simply changed/added ˜300 lines code in ocean model

# Lagrangian Overturning as a Convective Parameterization

Consider two parcels A and B centered in the same column of a lagrangian model. Suppose A lies beneath B. If exchanging the vertical positions of A and B leads to $\theta(A) > \theta(B)$ , then do so!



## Potential Advantages of Lagrangian Overturning

1. Directly models physical transport associated with convection

2. Handles both dry and moist convection

3. Ascending and descending parcels in a given column can have different properties (e.g., up moist / down dry ).

4. The existence and/or depth of convection responds to local perturbations in thermodynamics profiles

5. Descending parcels have a long memory

6. Parcel trajectories are provided at no additional computational cost

7. Few tunable parameters (so far . . . .)

# Single Column Experiments

## Components of Single Column Model

1. Surface Fluxes: Restore T, q of lowest parcel to SST, qsat(SST)

2. Radiation: ˜1 K/day tropospheric cooling, restore stratospheric temperature

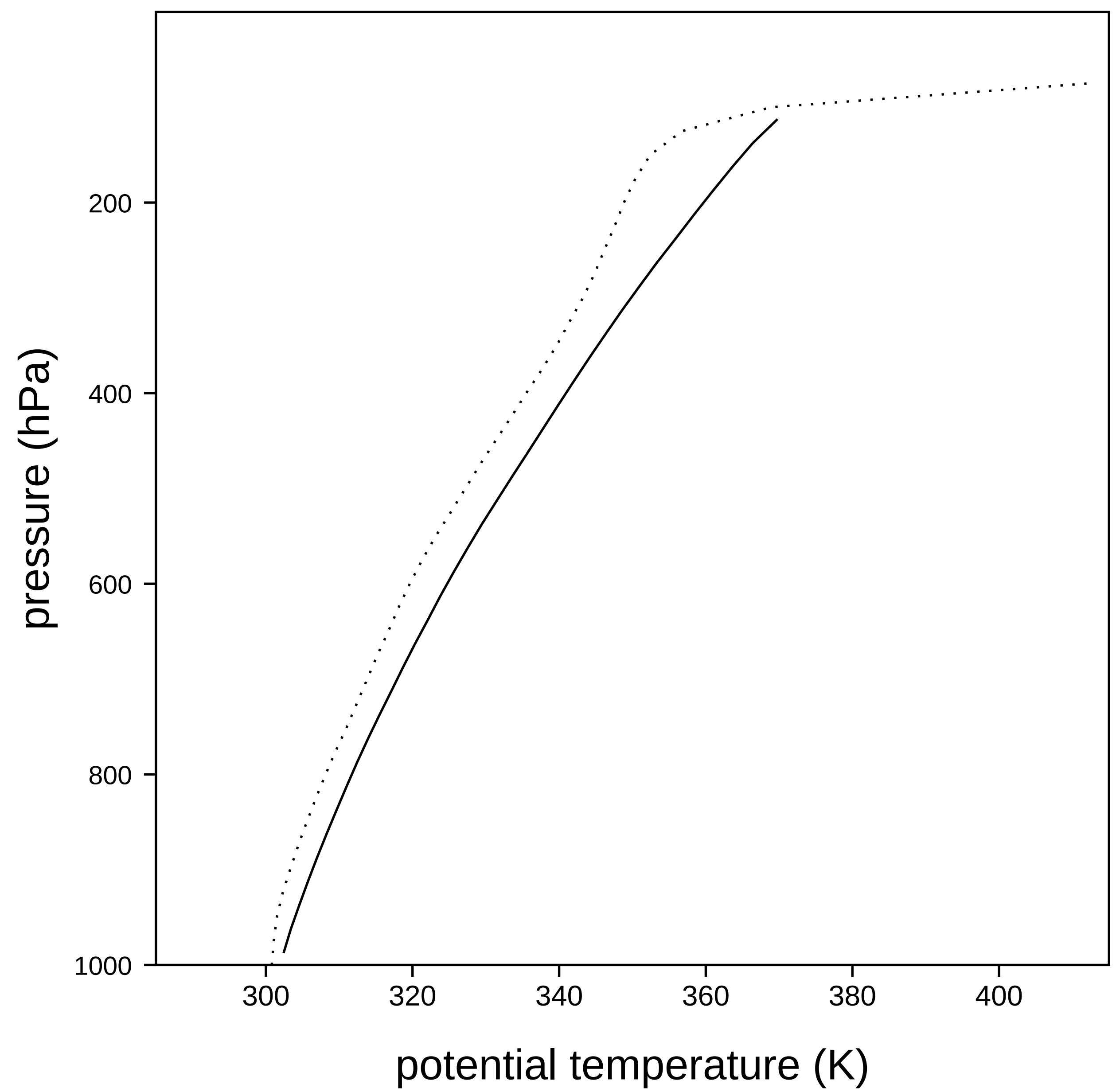
3. Evaporation: Fixed percentage of liquid water evaporates each time step

4. Rain: Condensed water falls to next parcel down each time step

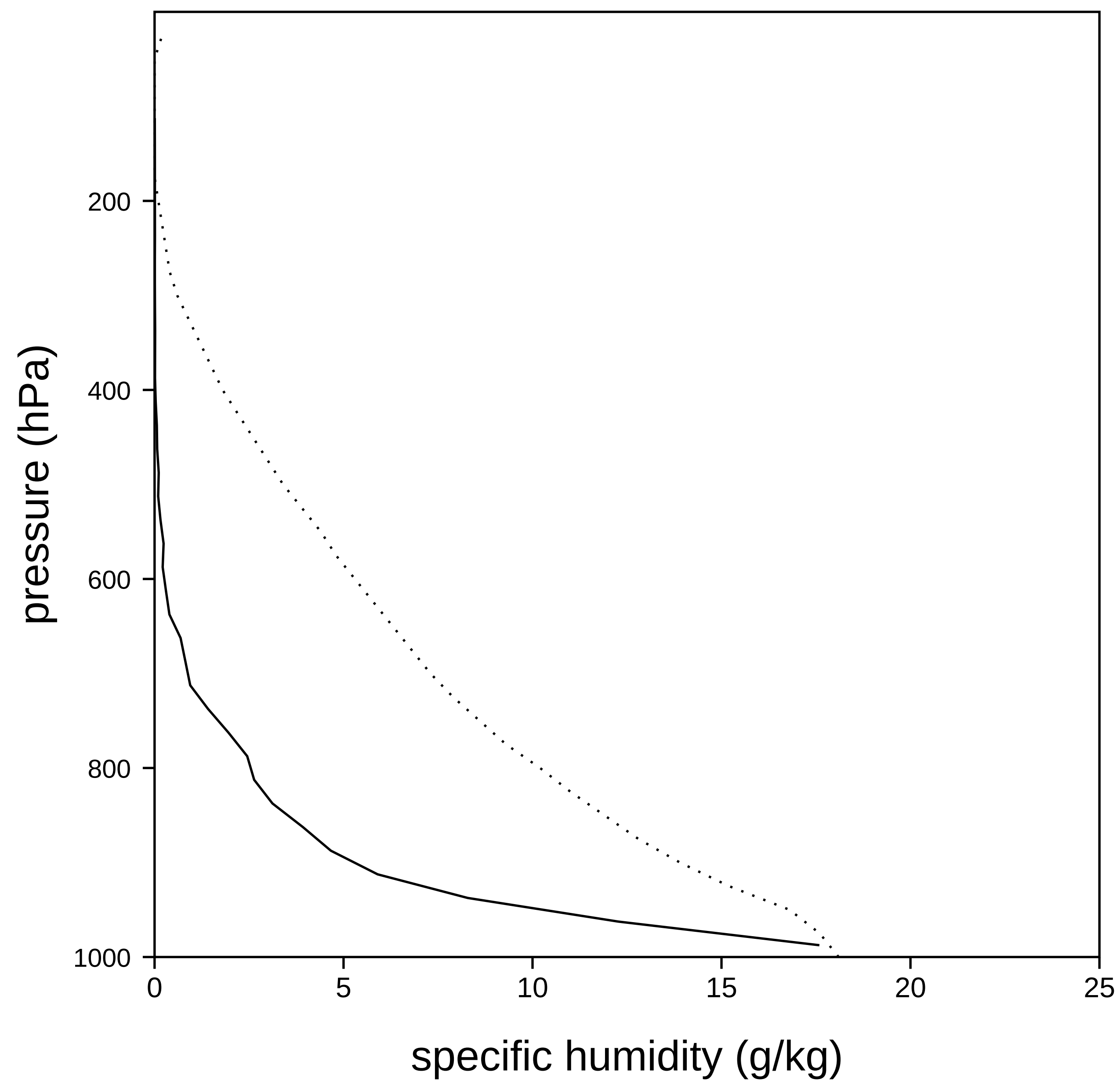
5. Convection: Lagrangian Overturning

# Experiment 1: No mixing, no evaporation of rain

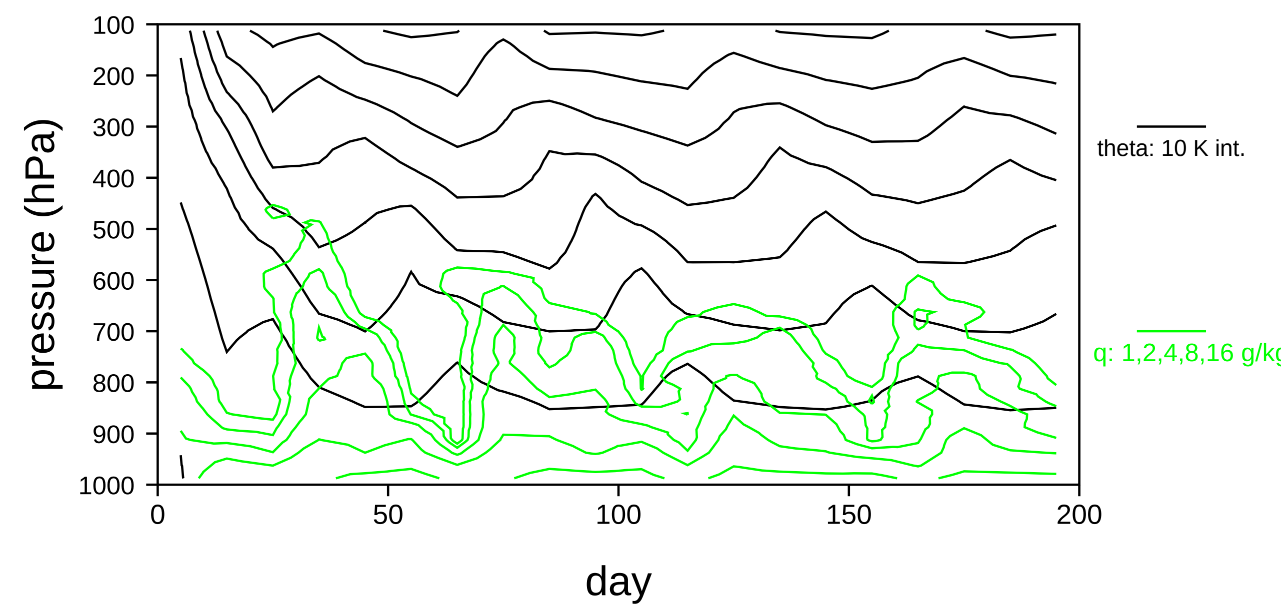
# Average Potential Temperature (LO solid, COARE IFA dashed)



# Average Specific Humidity (LO solid, COARE IFA dashed)

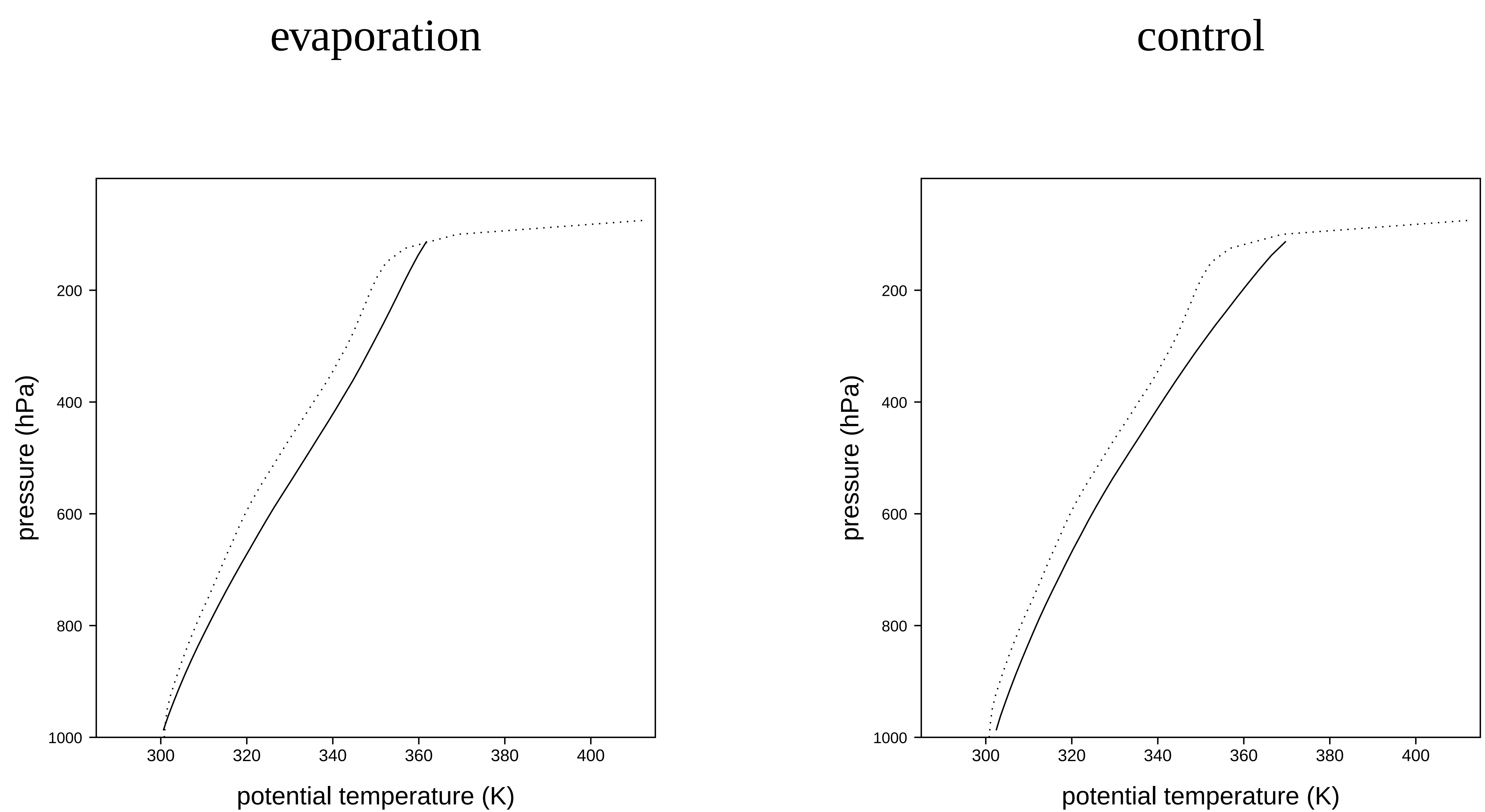


# Time Pressure Series of Potential Temperature, Spec. Humidiy

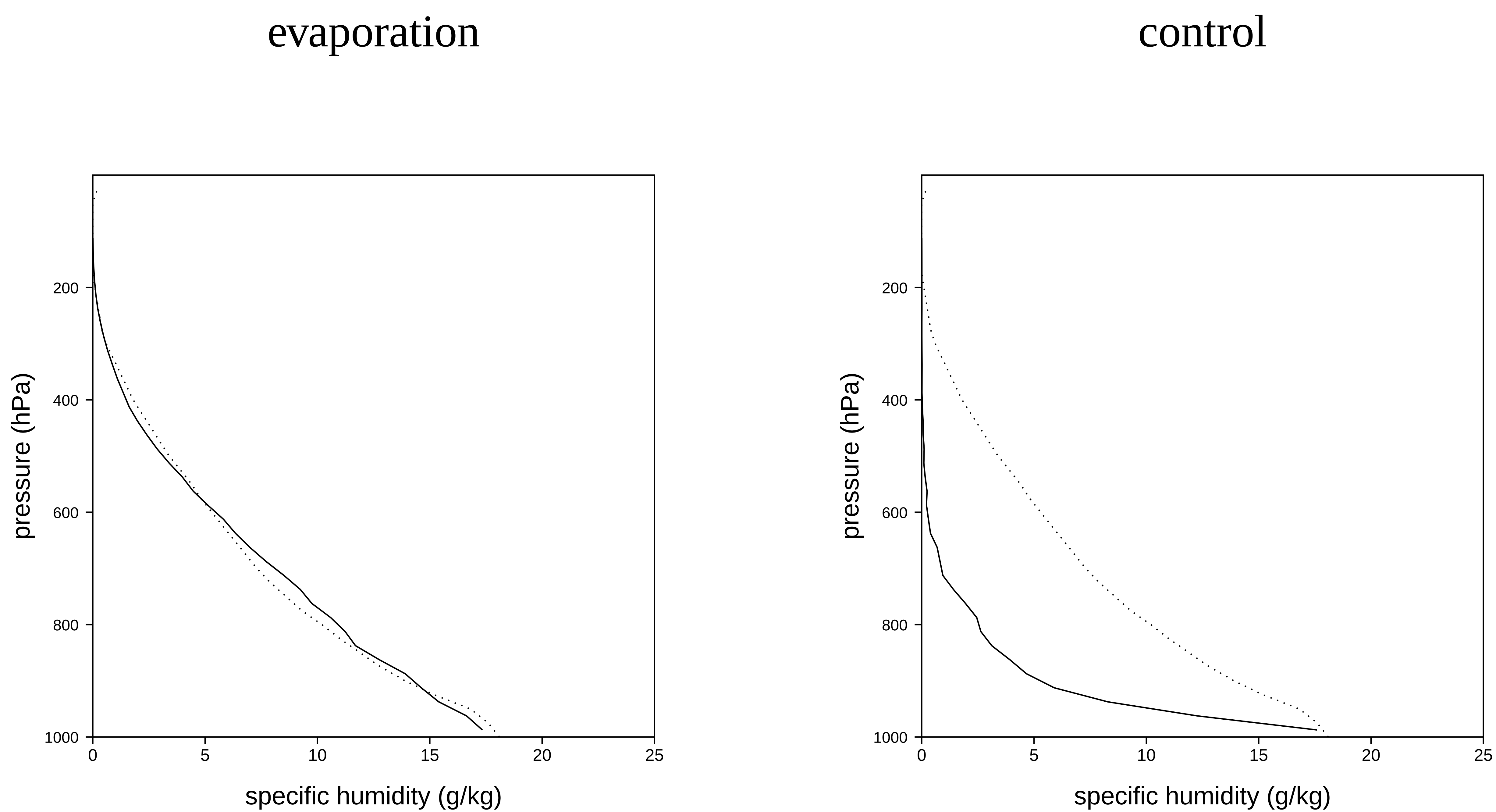


# Experiment 2: Evaporation of rain, no mixing

## Average Pot. Temperature (LO solid, COARE IFA dashed)

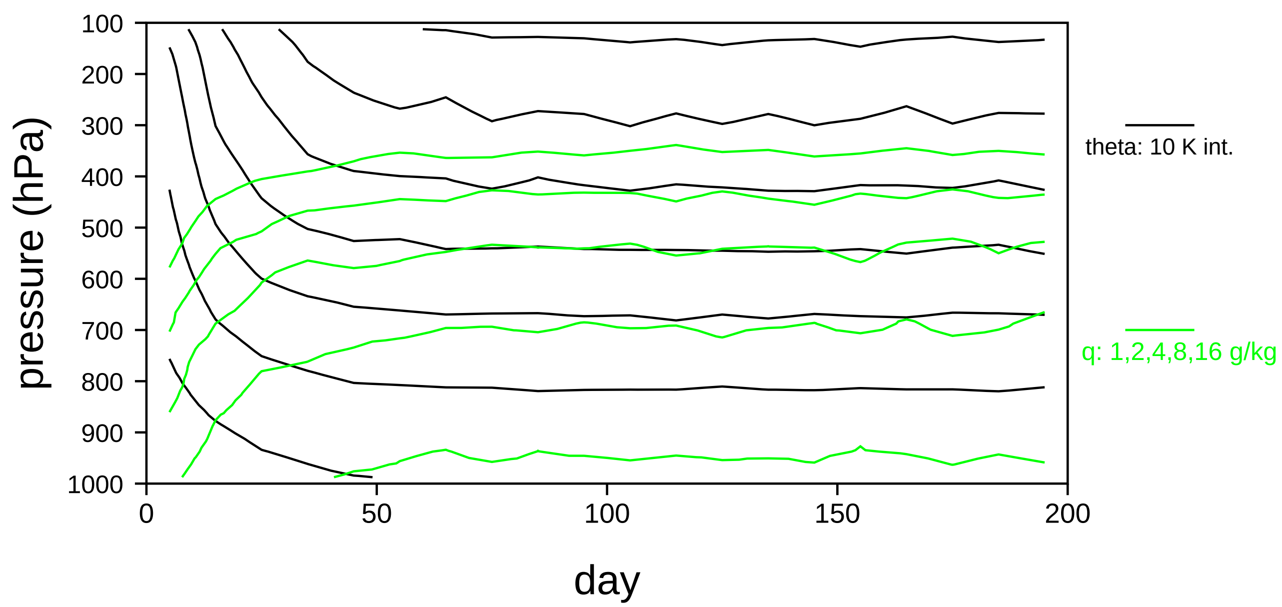


# Average Specific Humidity (LO solid, COARE IFA dashed)

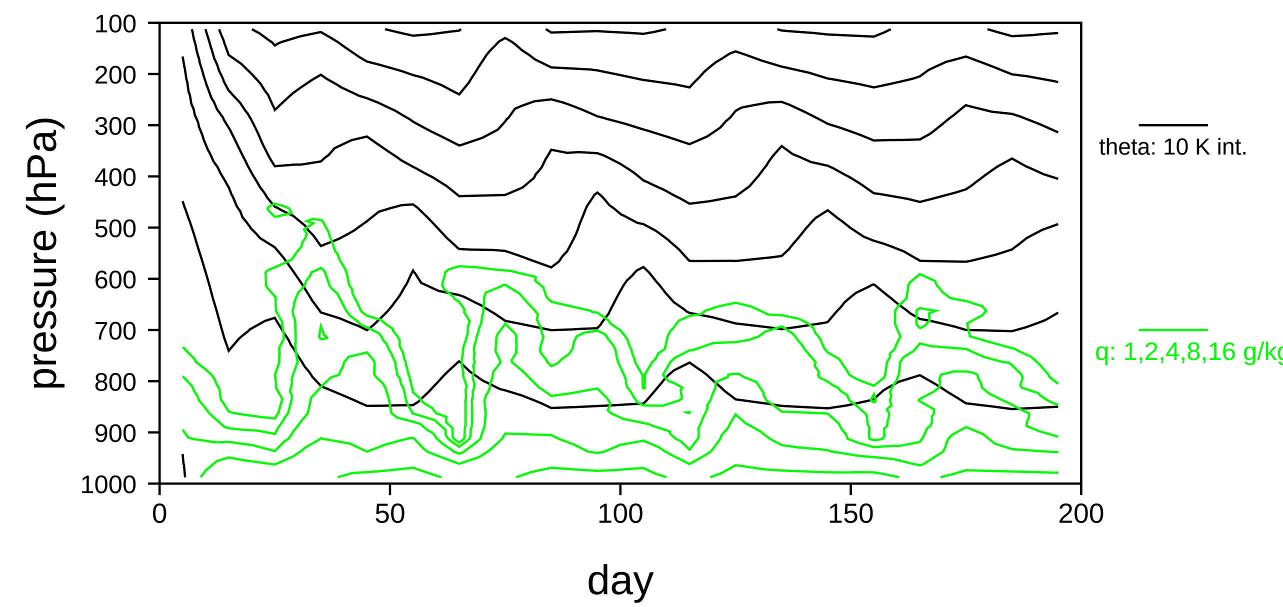


# Time Pressure Series of Potential Temperature, Specific Humidity

evaporation




control



## Conclusions from Single Column Experiments

1. Including evaporation of rain is critical for generating realistic moisture profiles

2. When the mid-troposphere is warm, LO produces relatively shallow convection that moistens the lower troposphere.

3. Interesting oscillation that couples descending temperature anomalies and convective morphology

**Simulating Convectively Coupled Kelvin Waves**

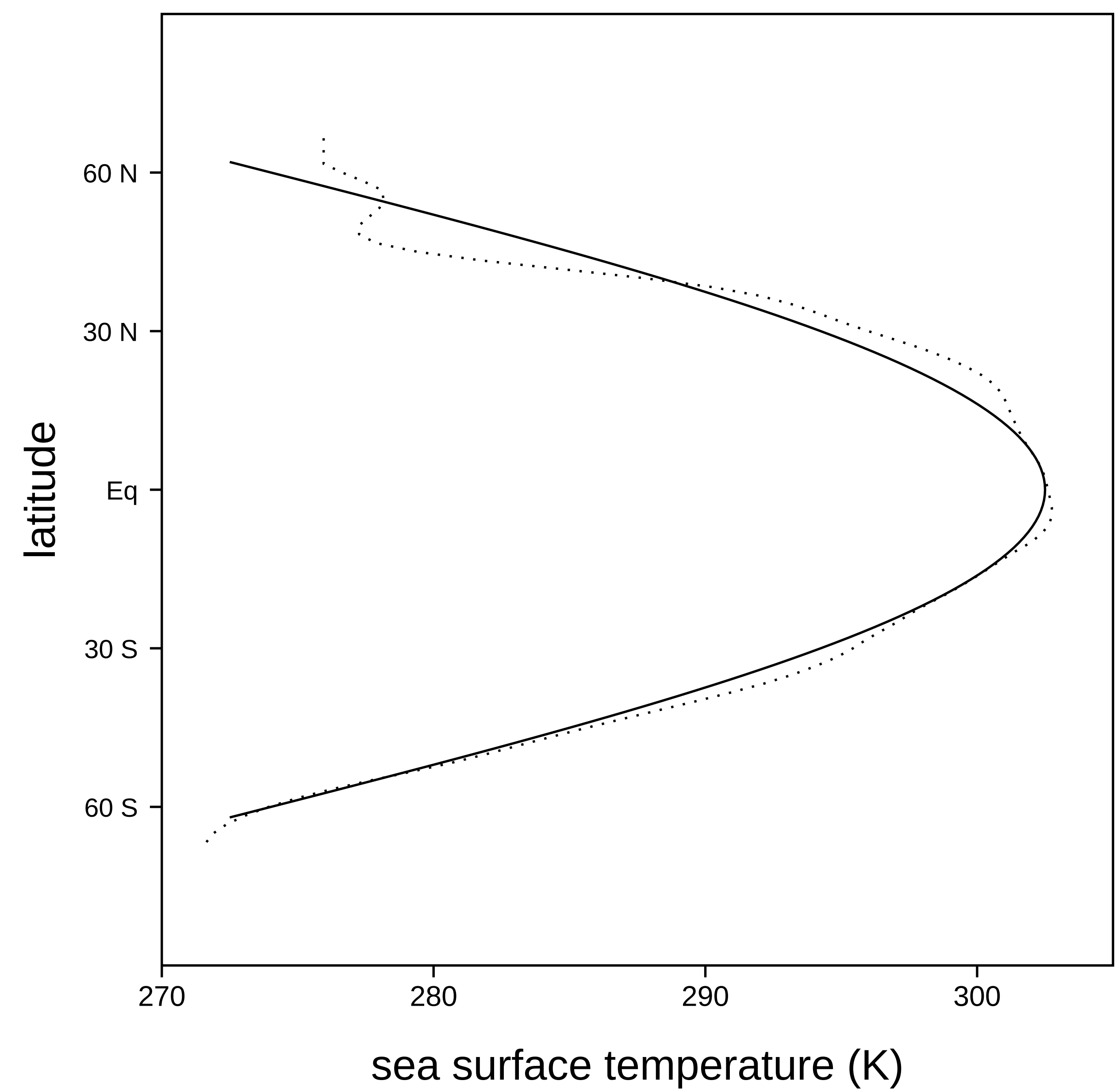
*(Haertel and Straub 2010, QJRMS)*

## Model Characteristics

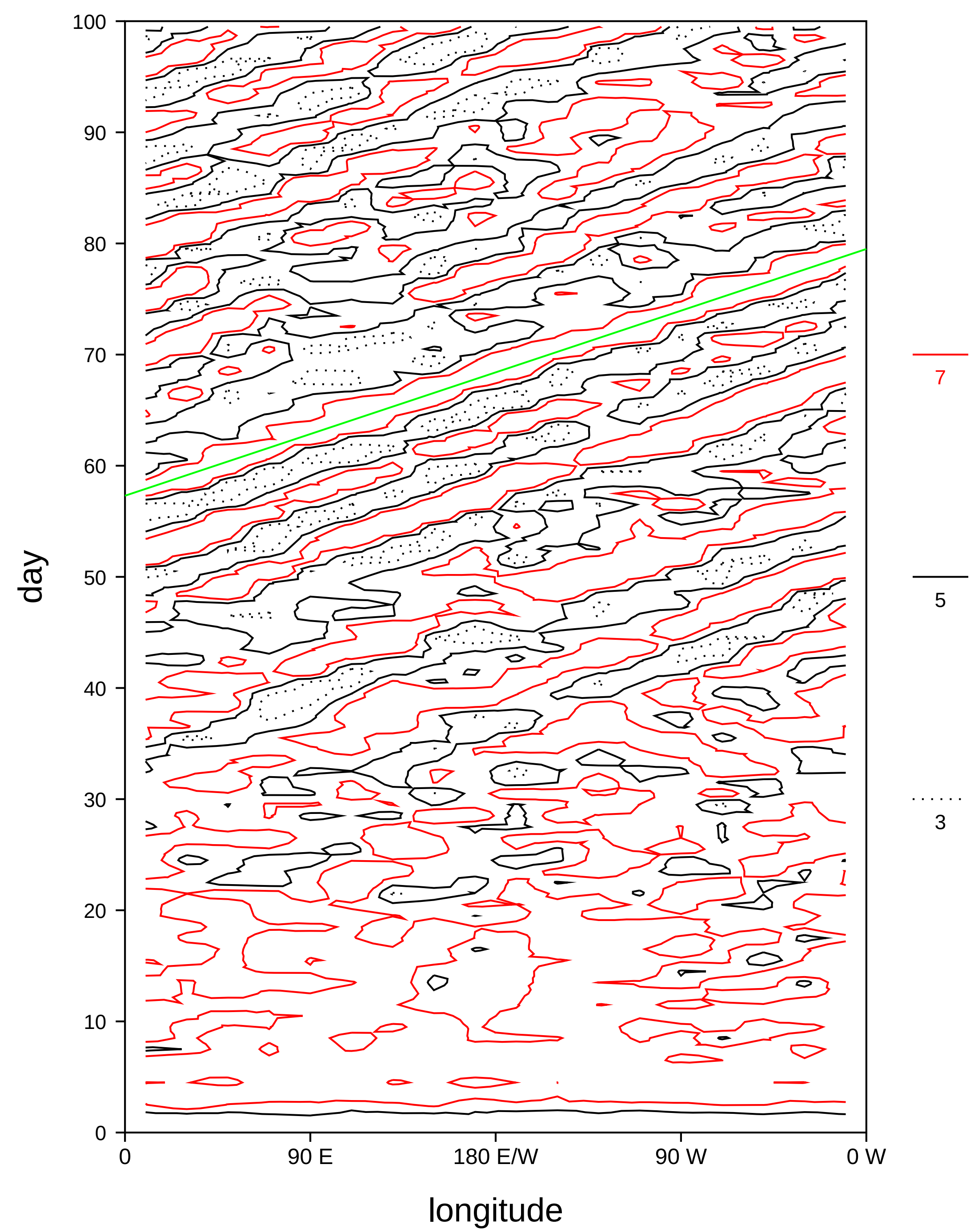
- Tropics of an Aquaplanet
- Zonally symmetric SST
- Lagrangian dynamical core
- Meridional boundaries near 30 N/S

# Sea Surface Temperature

*LO model solid, Levitus 150-160 E dashed*



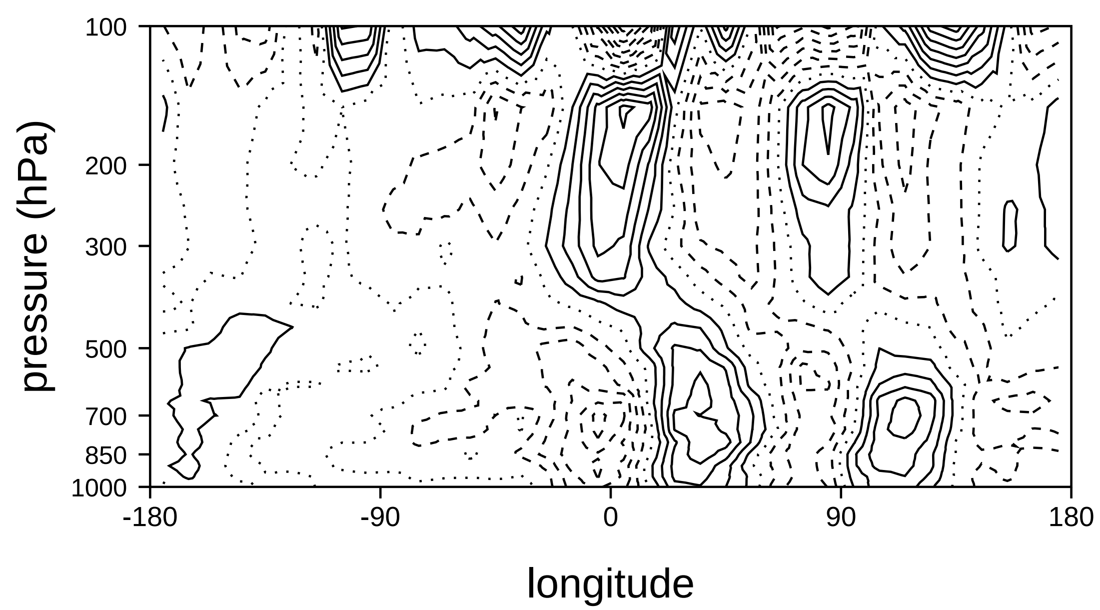
# Time Longitude Series of Rainfall (mm/day, 15 S - 15 N)



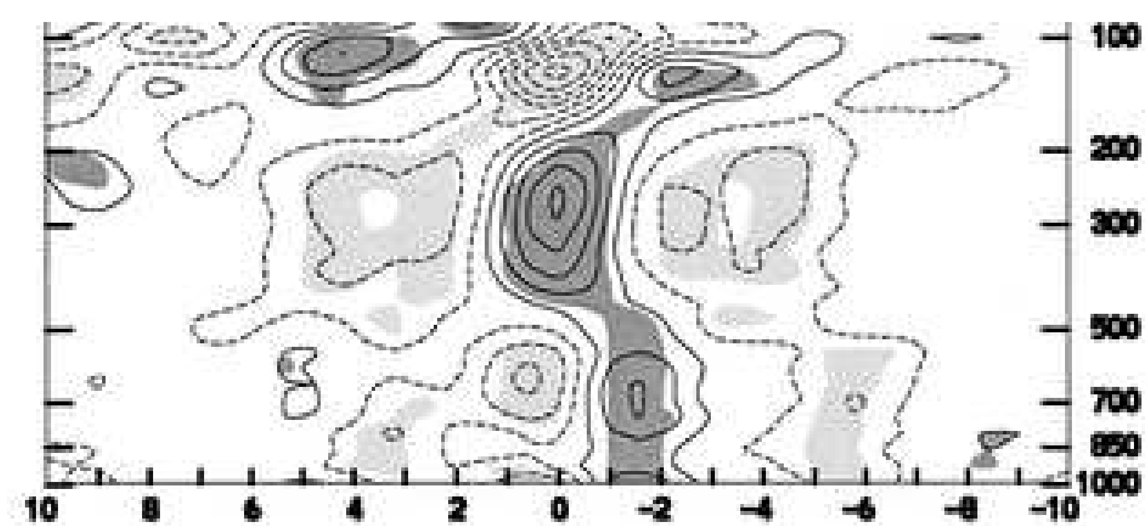
## LO generated Kelvin Wave vs. Straub and Kiladis (2003) Composite

# Temperature

LO (0.5 K)

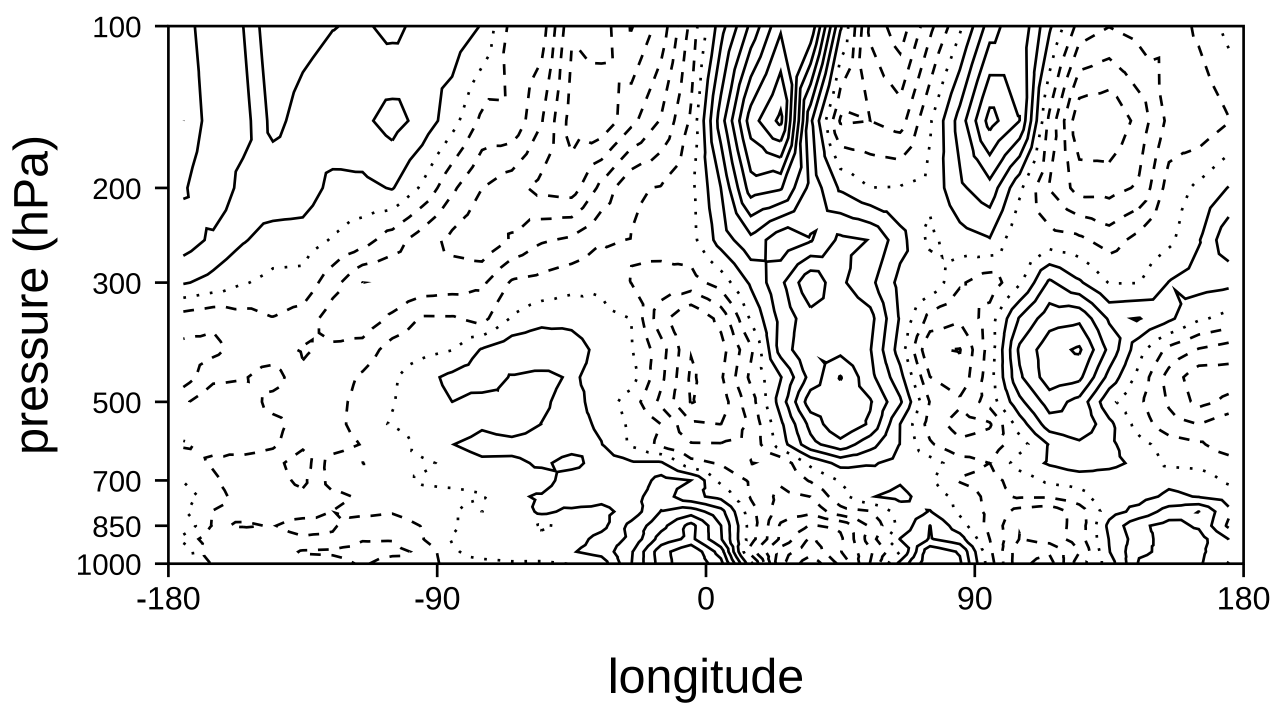



Observed (0.1 K)

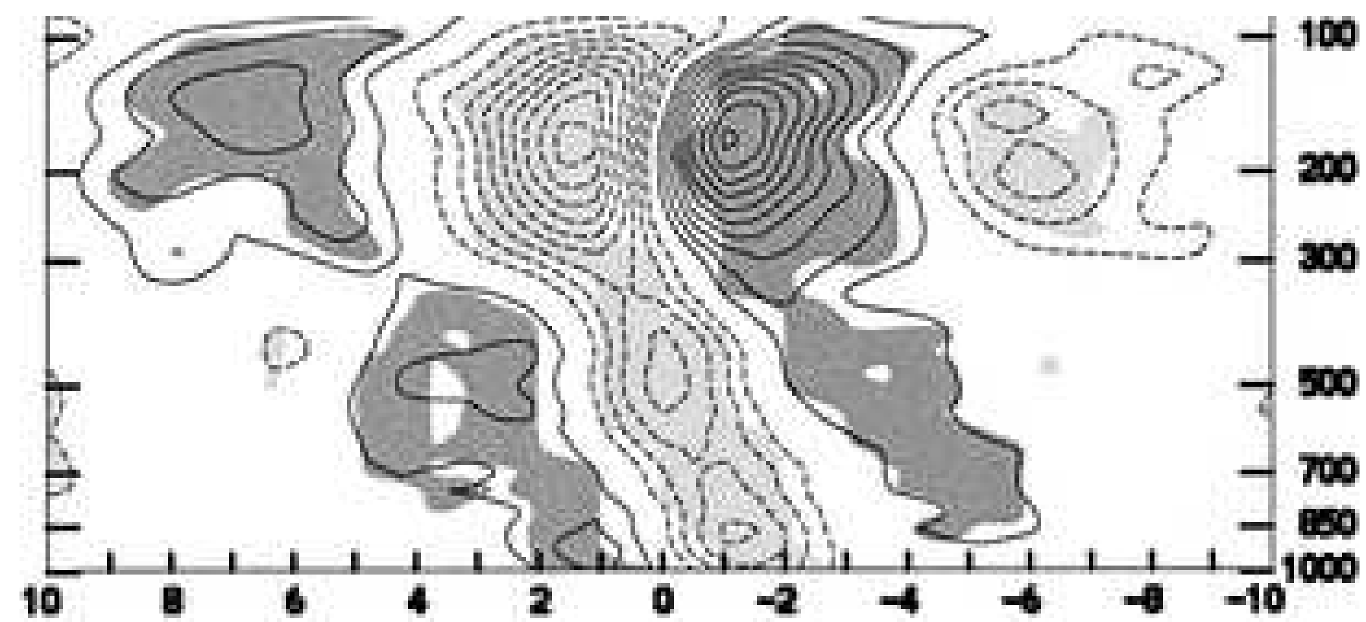


# Zonal Wind

LO (1 m/s)

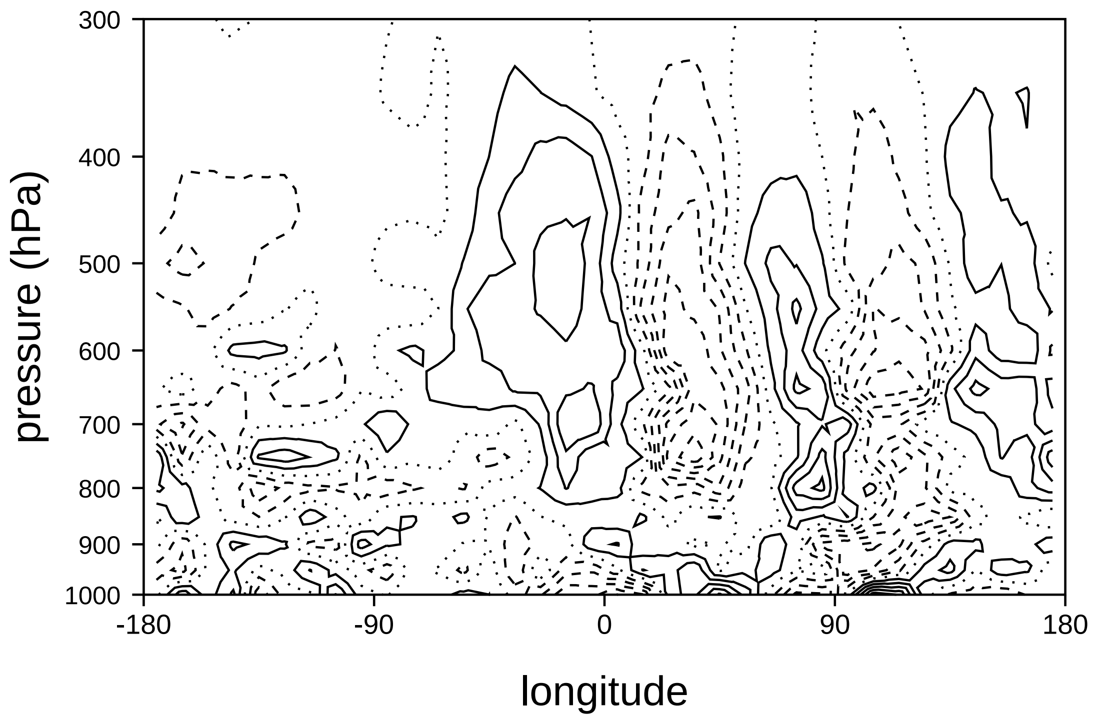



Observed (0.5 m/s)

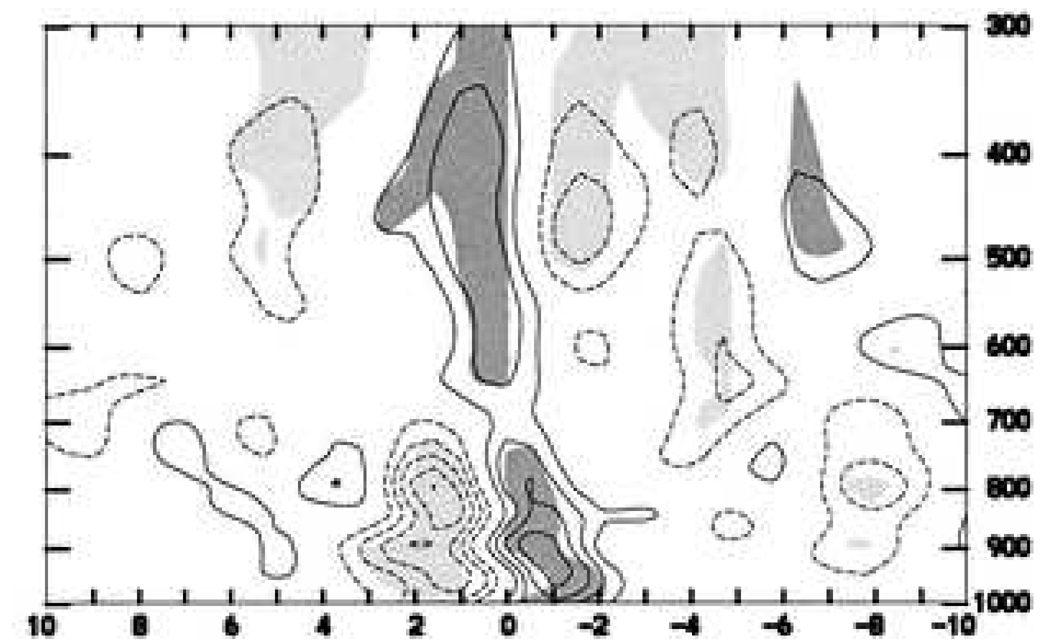


# Specific Humidity

LO (0.2 g/kg)

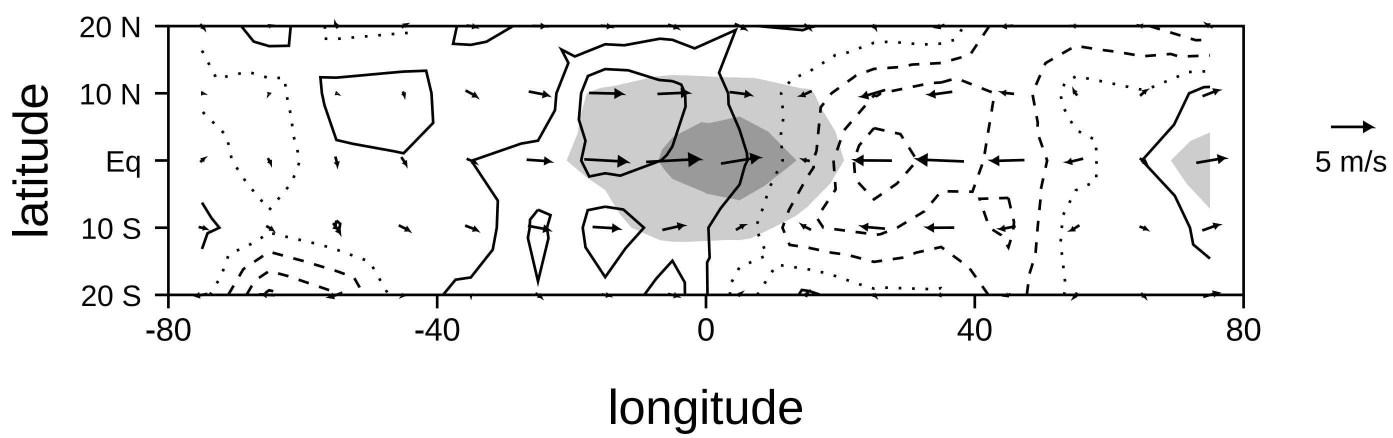



Observed (0.1 g/kg)

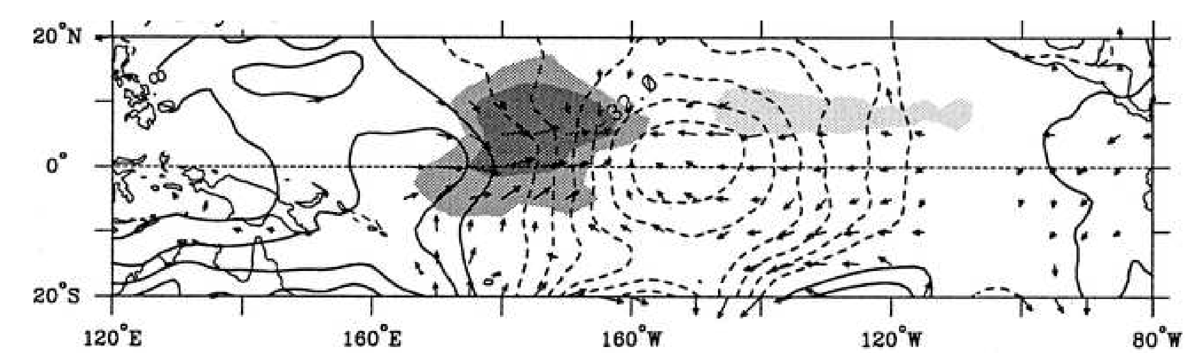


## Surface pressure (height) and velocity perturbations

LO (1 hPa)




Observed (8 m)



## Results of Sensitivity Tests

1. Wave number depends on radiative cooling (lower tropospheric subsidence time scale)

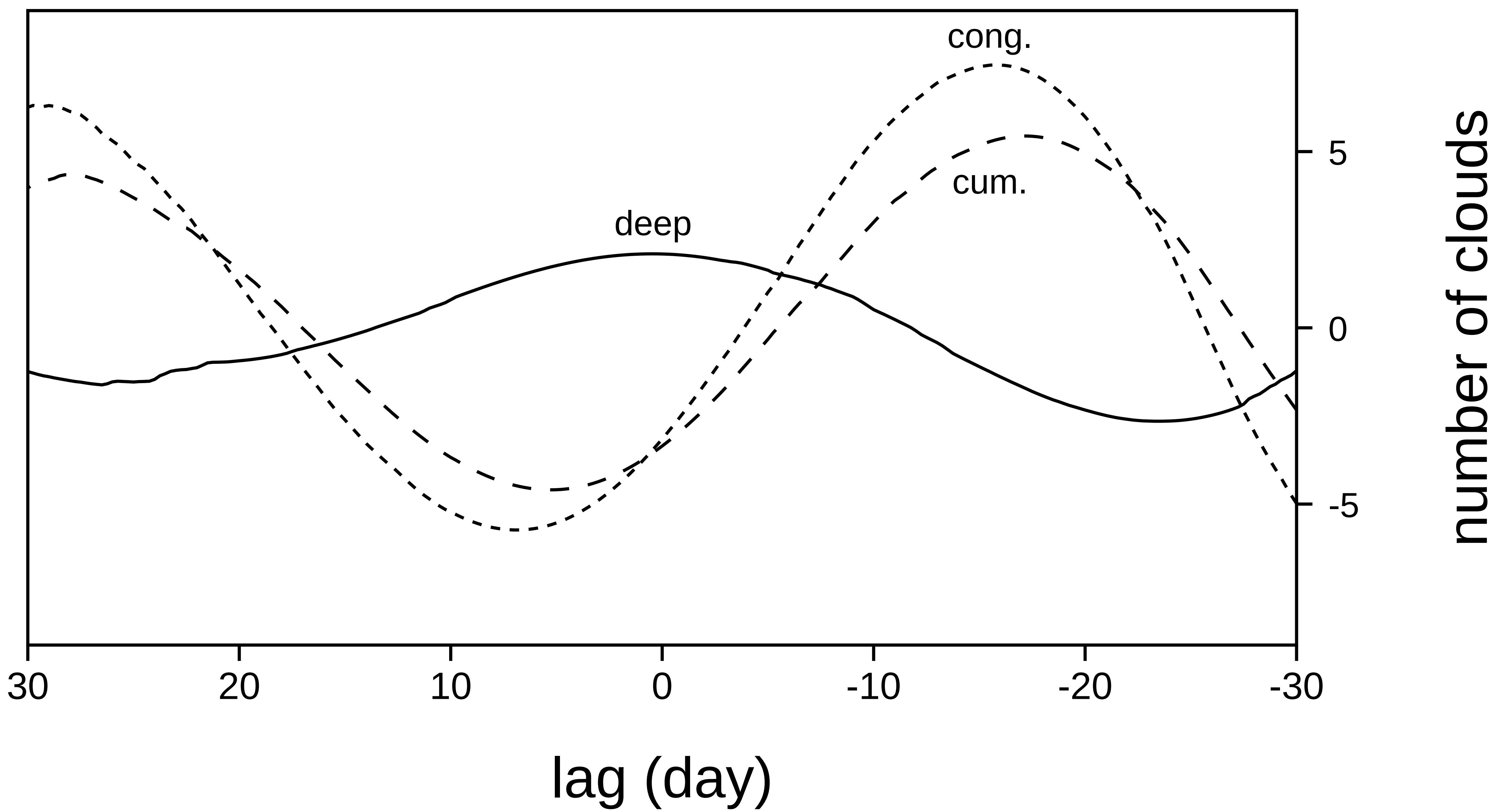
2. Realistic tilted heating structures

3. Evaporative cooling is essential for long-lived waves

# Madden Julian Oscillation

# MJO Convective Morphology from COARE

*Haertel et al. (2008)*



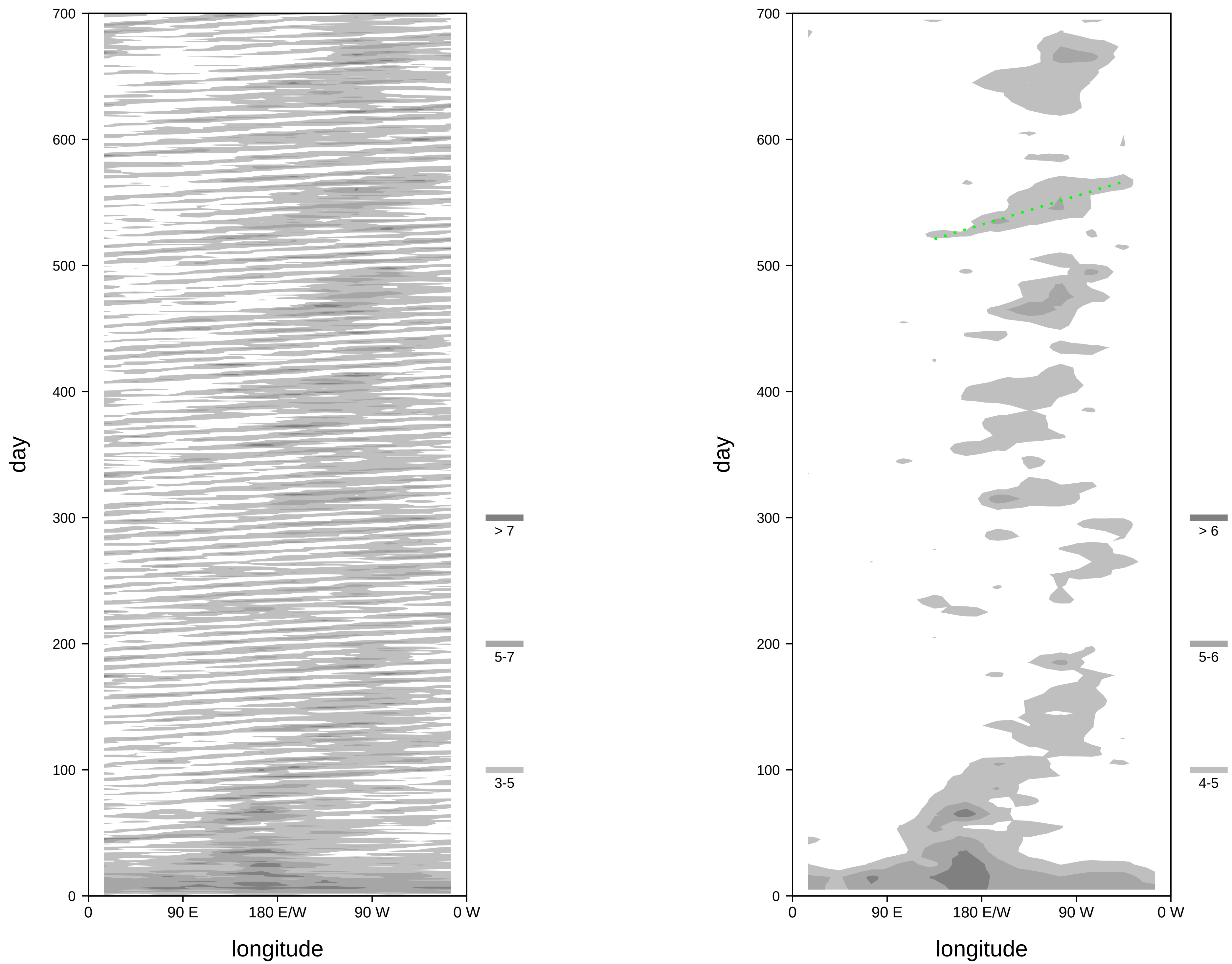
## Model Modifications for Simulating MJO

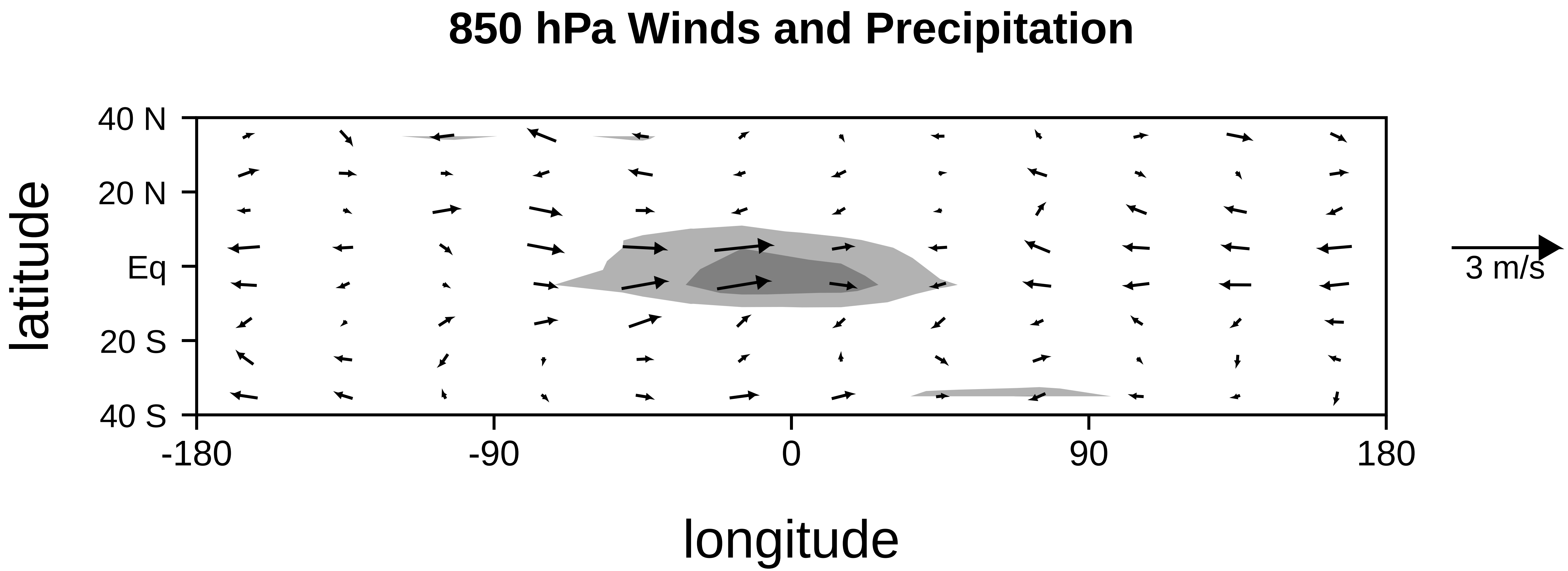
1. Moved Meridional Boundaries Poleward

2. More realistic stratospheric stratification

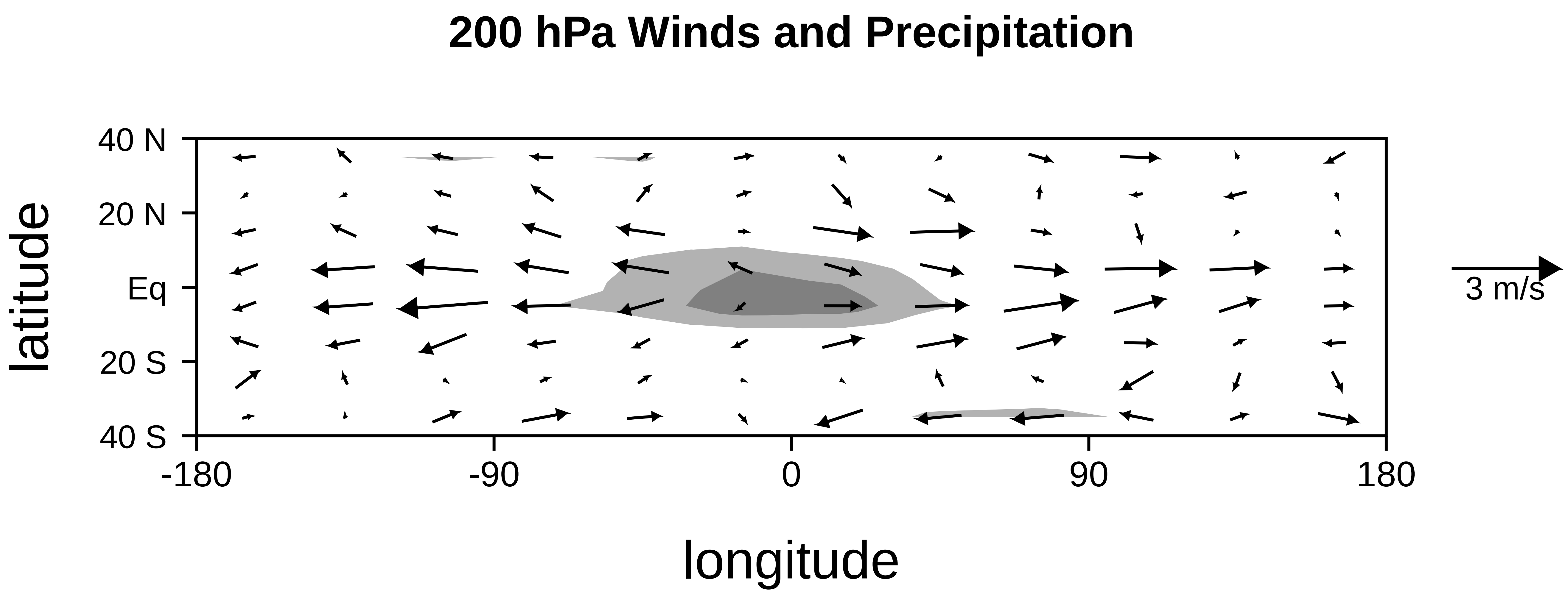
3. Inclusion of a warm pool

# Raw and Low-Pass Time-Series of Precipitation (15 S - 15 N)



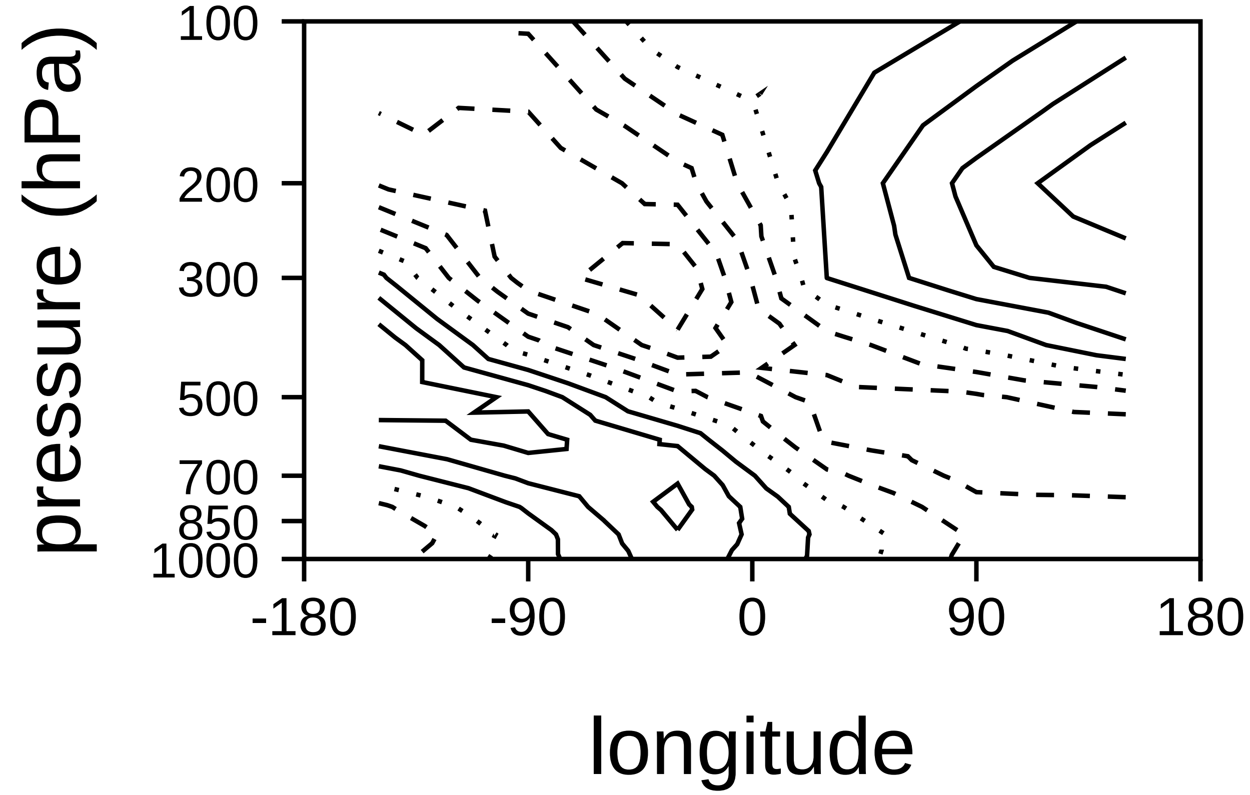


850 hPa Winds and Precipitation
latitude
40 N
20 N
Eq
20 S
40 S
-180
-90
0
90
180
longitude
3 m/s


200 hPa Winds and Precipitation
40 N
20 N
Eq
20 S
40 S
-180
-90
0
90
180
latitude
longitude
3 m/s

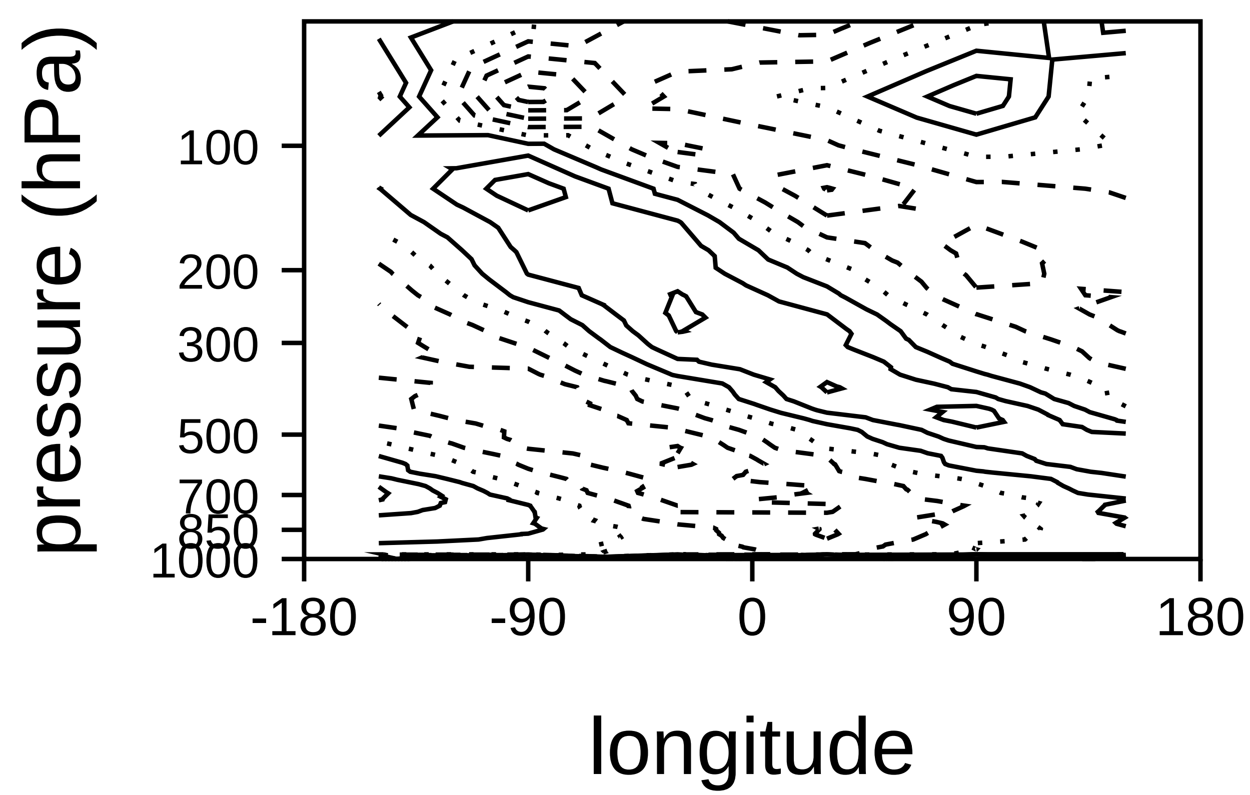
## Zonal Wind

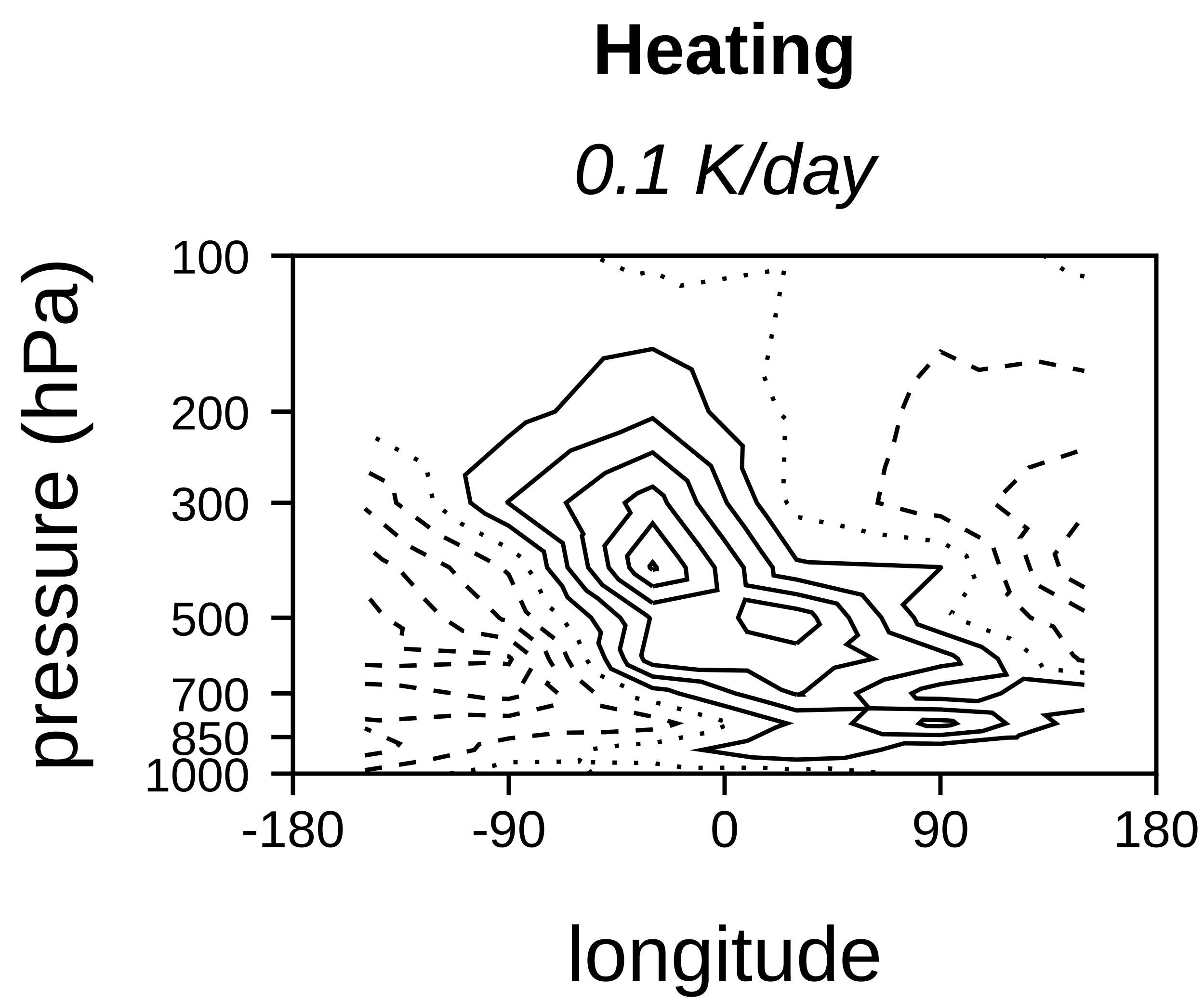
*0.5 m/s*

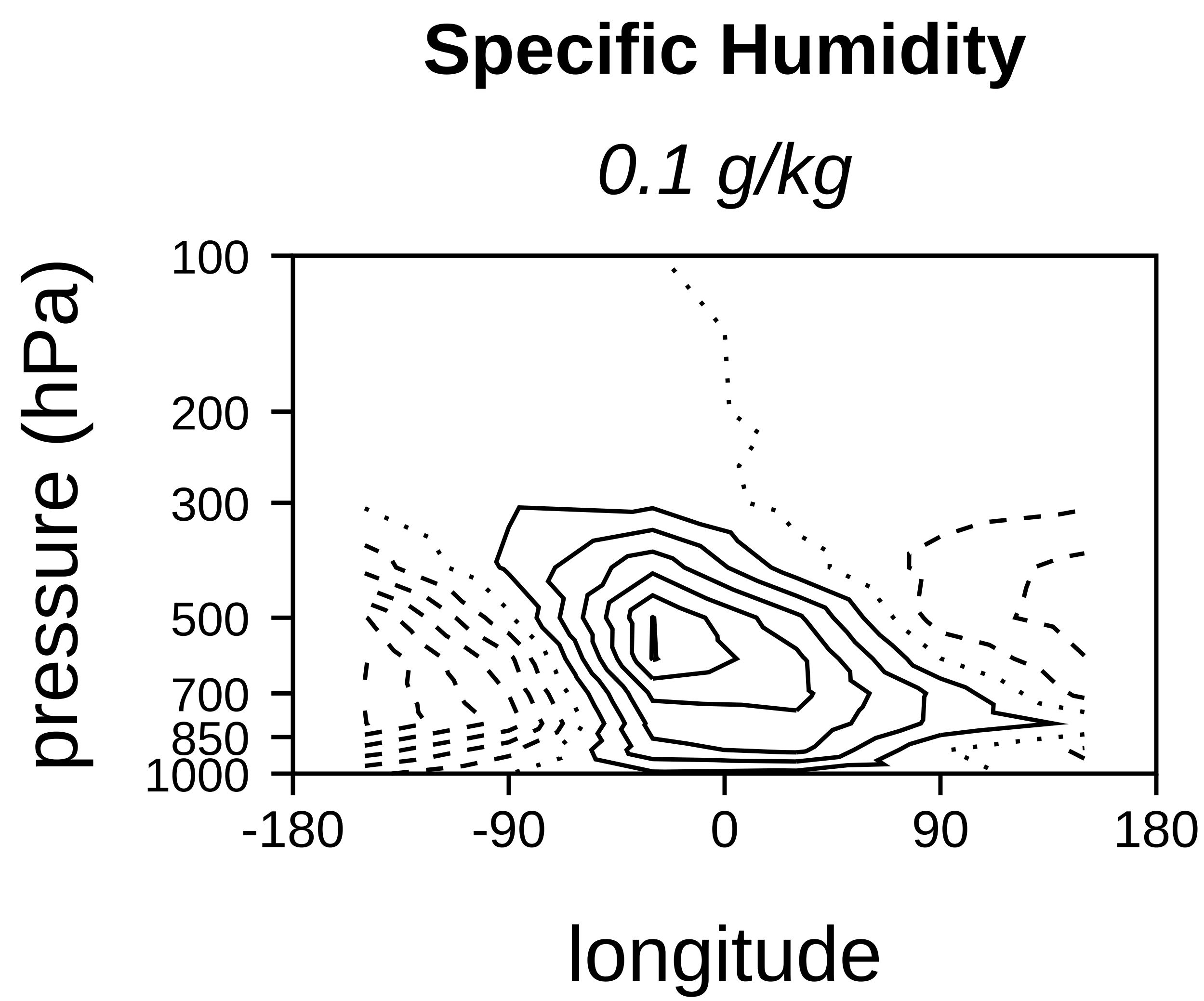


**Temperature**

*0.2 K*




Heating
0.1 K/day
pressure (hPa)
100
200
300
500
700
850
1000
-180
-90
0
90
180
longitude


Specific Humidity
0.1 g/kg
pressure (hPa)
100
200
300
500
700
850
1000
-180
-90
0
90
180
longitude

## Summary

- I have developed a numerical model for simulating the motions of fluid parcels
- Idealized tests suggest that piles of parcels can behave like fluids
- I have successfully simulated lake upwelling, equatorial undercurrents, tropical instability waves, ocean thermocline structure, mid-latitude gyres and meridional overturning
- Initial tests with an atmospheric version of the model produce robust convectively-coupled Kelvin waves, and MJOs for some configurations

## Advantages of the Lagrangian Method

1. Control over mixing
2. Trajectories for all parcels
3. Realistic convective systems

## Want to Help?

1. Figure out dynamics of simulated MJO
2. Optimize atmospheric model
3. New time differencing
4. Global geometry
5. More microphysics
6. Radiation
7. More realistic surface fluxes

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## Pressure Force

$$\mathbf{F}_{p_i} = \int_{D_h} g \nabla H_i \left[ \sum_{j=i+1}^{k} \left( \rho_j - \rho_i \right) H_j + \gamma \rho_i \left( b + \sum_{j=1}^{k} H_j \right) \right] dA$$