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PQM, GENERAL COORDINATE OCEAN MODELS AND THE SPURIOUS MIXING PROBLEM

White & Adcroft, JCP 2008
White, Adcroft & Hallberg, JCP 2009

Background (concerning ocean models)

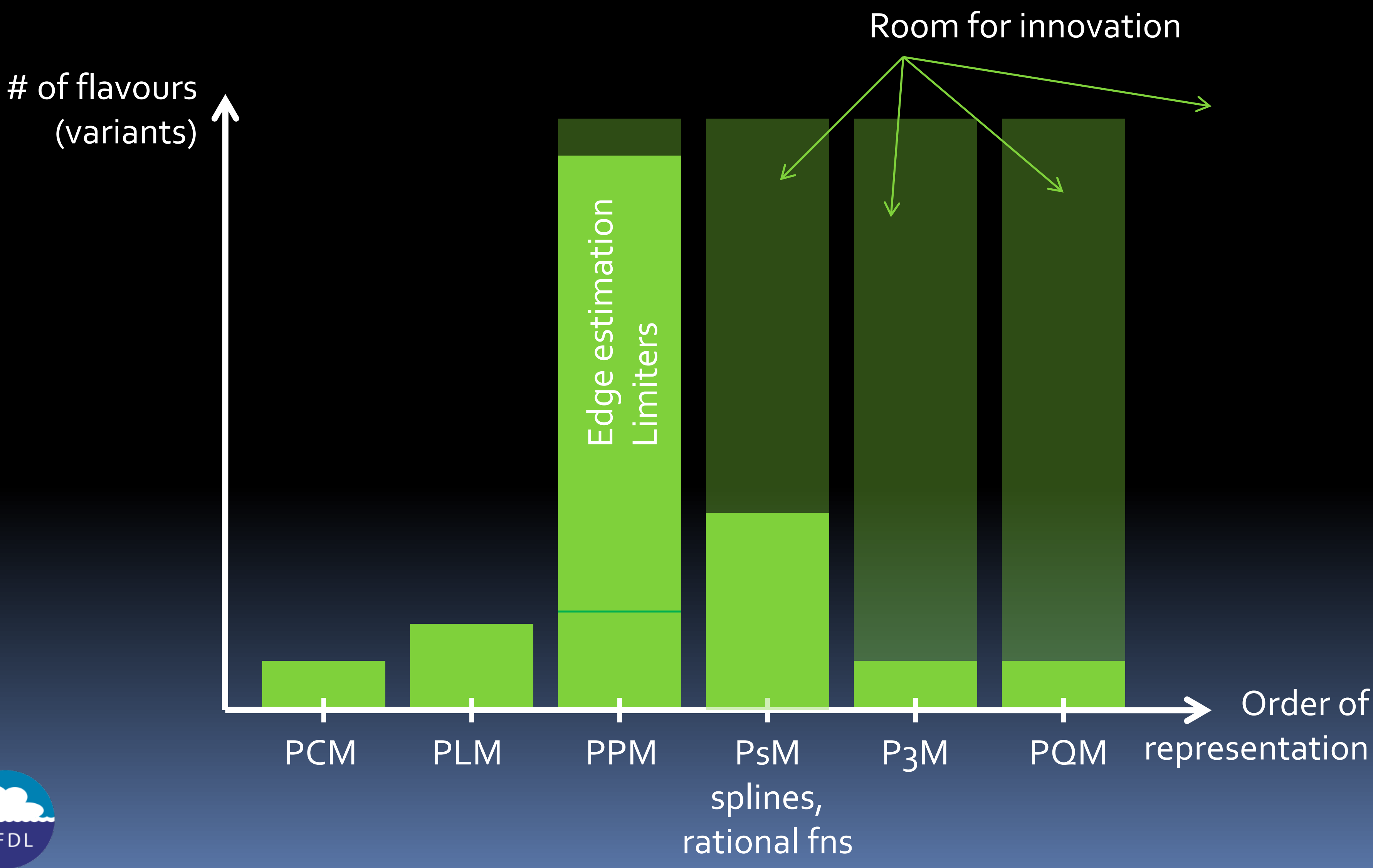
- The coordinate debate: "z" v's isopycnal
  - Representation of processes
    - e.g. Overflows (convective mixing in z, Winton et al., 1998)
    - e.g. Un-stratified column (lack of resolution in isopycnal models)
  - Spurious diapycnal mixing
    - Model inter-comparisons often compare apples and oranges DYNAMO; Legg et al., 2006 (GCE-CPT)
    - Need one model to evaluate coordinate issue
    - Believed to be more imperative at higher resolution (eddy permitting) Griffies et al, 2000
- Hybrid coordinates
  - Starr, 1945; Konor & Arakawa, 1997; Lin 2004
  - HyCOM (Bleck, 2002)
  - Goal: best of both worlds
  - Reality: only part way there
    - Accurate remapping clearly important

In this talk

- Develop better methods for regridding/remapping
  - PQM is a natural follow on from PCM, PLM and PPM
- Assessing methods involves measuring spurious mixing
  - Hard to measure/quantify this spurious mixing
    Griffies et al, 2000; Maqueda & Holloway, 2006; Rennau & Burchard, 2009
  - Will hybrid or eddying models be adiabatic enough?
- Context of these developments:
  - GOLD
    - G = Generalized/GFDL/Great
    - O = Ocean
    - L = Layer/Level/Langrangian
    - D = Dynamics
  - Derived from “classic” isopycnal code (HIM; Hallberg, 2000)
  - Now a robust layered model using FV concepts
    Adcroft et al., 2008; Hallberg & Adcroft, 2009
  - This work adds regridding/remapping for general coordinates
  - Two ESM/ocean models submitted to IPCC: **CM2M** & **CM2G**

GFDL

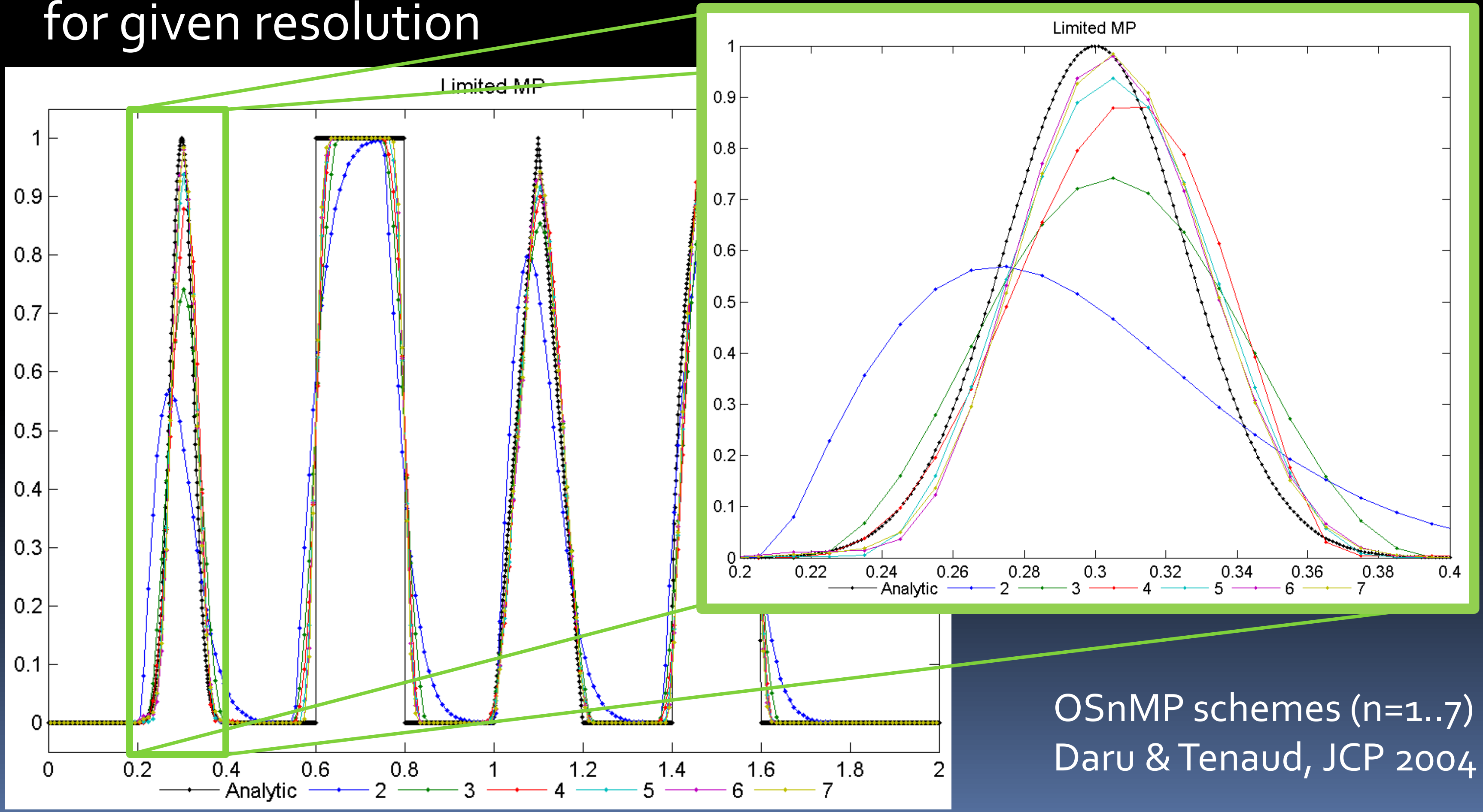
Where to innovate



Why higher order

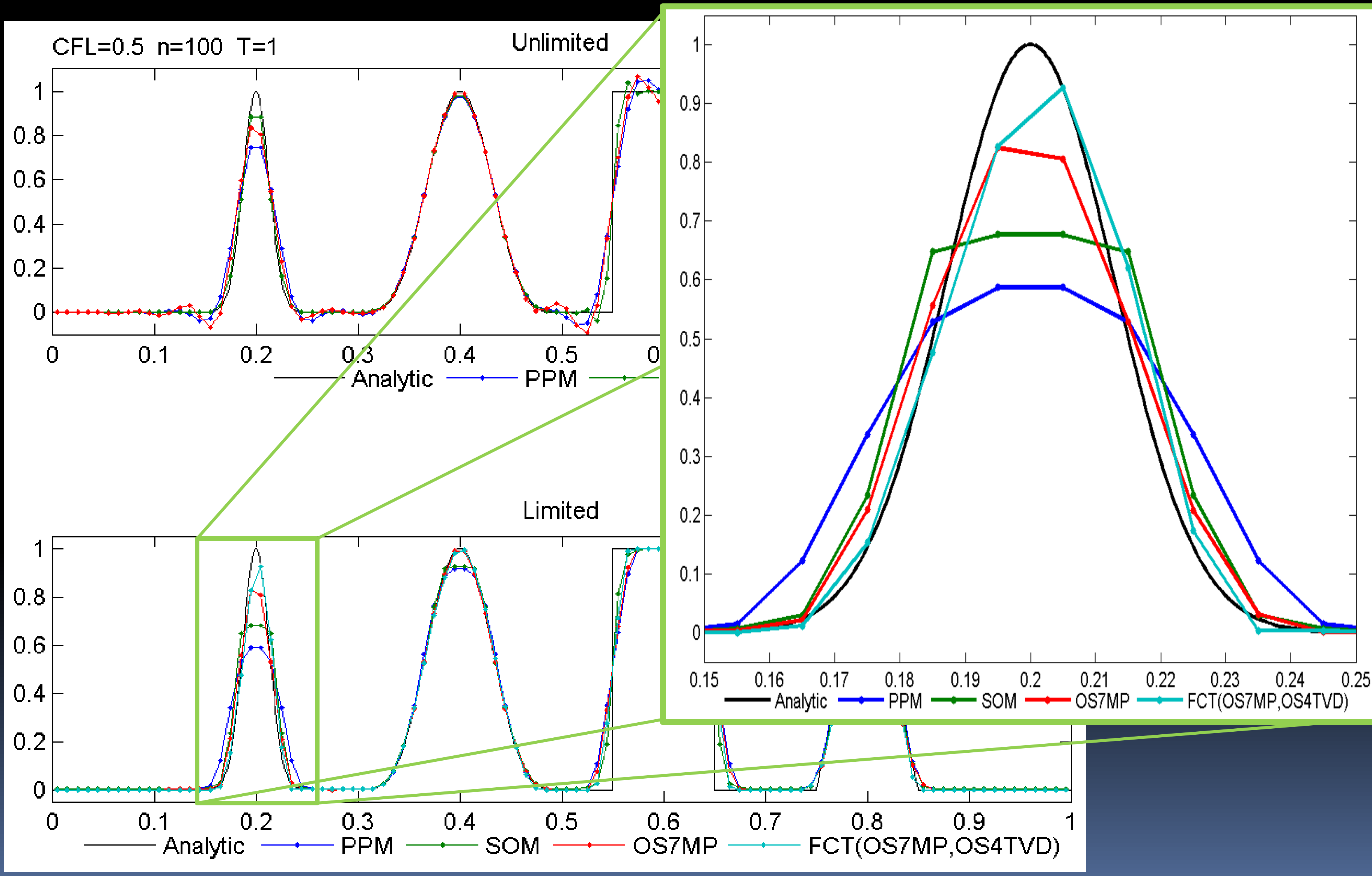
- Accuracy often thought of i.t.o. convergence
- Significant improvements for given resolution

- High order edge values often used with PPM
  - but PPM is only ever $O(\Delta^3)$



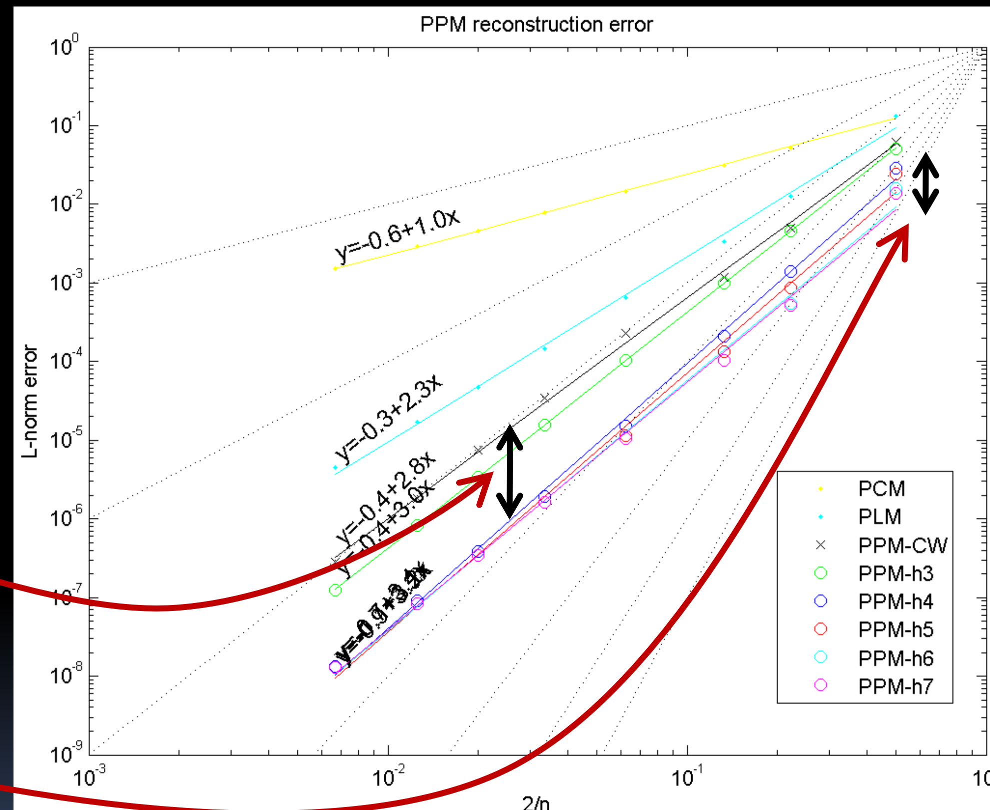
OSnMP schemes (n=1..7)
Daru & Tenaud, JCP 2004

On SOM (a side comment)



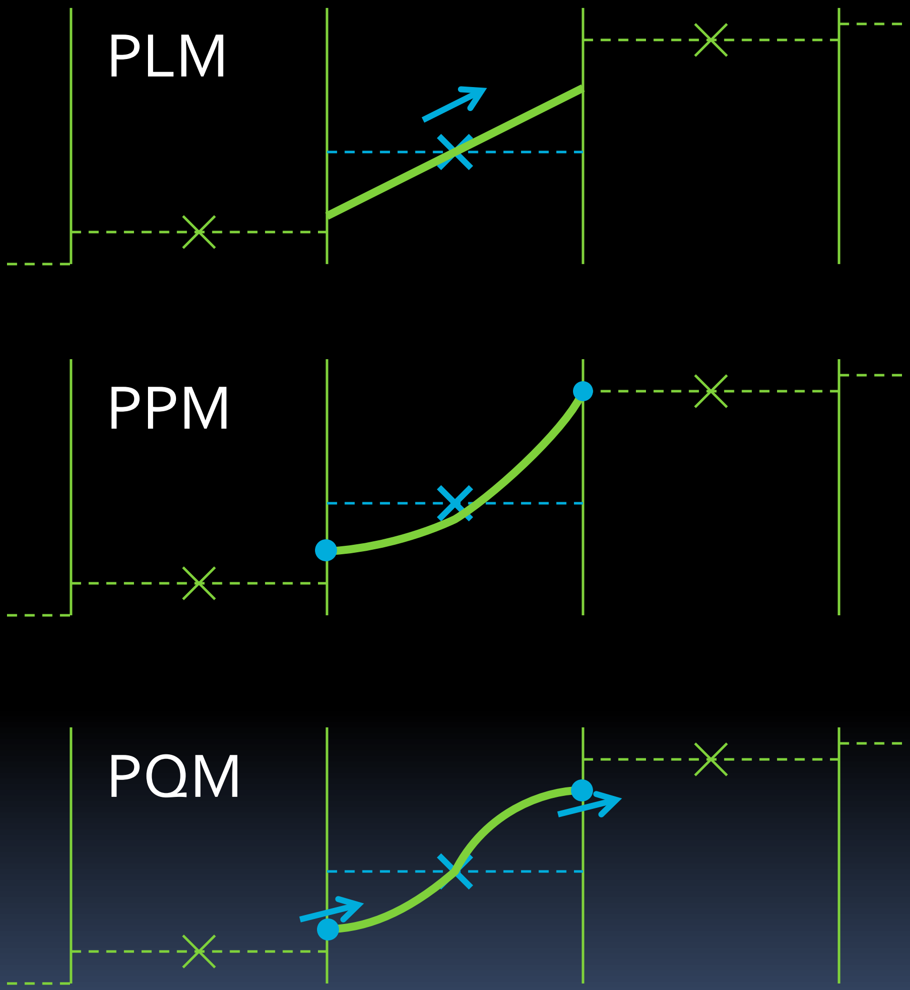
Impact of PPM edge values

- PCM, PLM, PPM scale as $O(\Delta)$, $O(\Delta^2)$ and $O(\Delta^3)$
- More accurate edge values can scale error down (shift in log-log plot)
- Most notably at low resolutions



Piecewise * Method (* = C,L,P or Q)

- PLM: two degrees of freedom
  - Cell mean + slope
- PPM: three degrees of freedom
  - Very widely used
  - Cell mean + two edge values
- PQM: five degrees of freedom
  - Cell mean + two edge values + two edge slopes



Successive schemes provide more flexibility to represent structures → more accurate

White & Adcroft, JCP 2008

PQM reconstruction

Algorithm

1. Estimate edge values/slopes

2. Bound edge values/slopes

3. Limit (monotonize) reconstruction

- This was the first (simple) algorithm we thought of (inspired by CW)
- Like for PPM, there are clearly plenty of other choices
- Already can think of better

PQM reconstruction 1

Algorithm

1. Estimate edge values/slopes
2. Bound edge values/slopes
3. Limit (monotonize) reconstruction

- F.V. curve fits to N-cell means
- To capture fifth order accuracy:
  - Values must be at least $O(\Delta^5)$
  - Slopes must be at least $O(\Delta^4)$
- Implicit schemes are viable in vertical

We are targeting the vertical direction

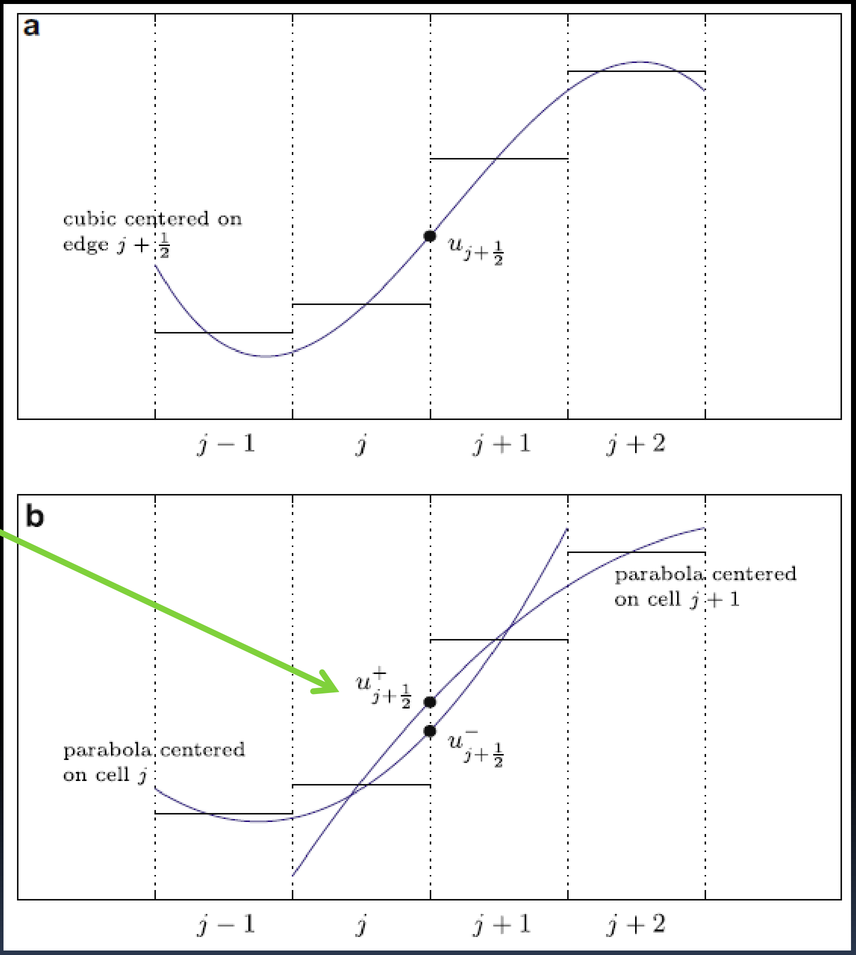
F.V. curve fitting

- Explicit interpolation
  - P_{n-1} fit to n cells
  - Yields $O(\Delta^n)$
- Odd orders are shifted
  - Discontinuous edge estimate
- Implicit interpolation
  - F.V. version of compact differencing

$$\alpha u_{j-\frac{1}{2}} + u_{j+\frac{1}{2}} + \beta u_{j+\frac{3}{2}} = a\bar{u}_{j-1} + b\bar{u}_j + c\bar{u}_{j+1} + d\bar{u}_{j+2}$$

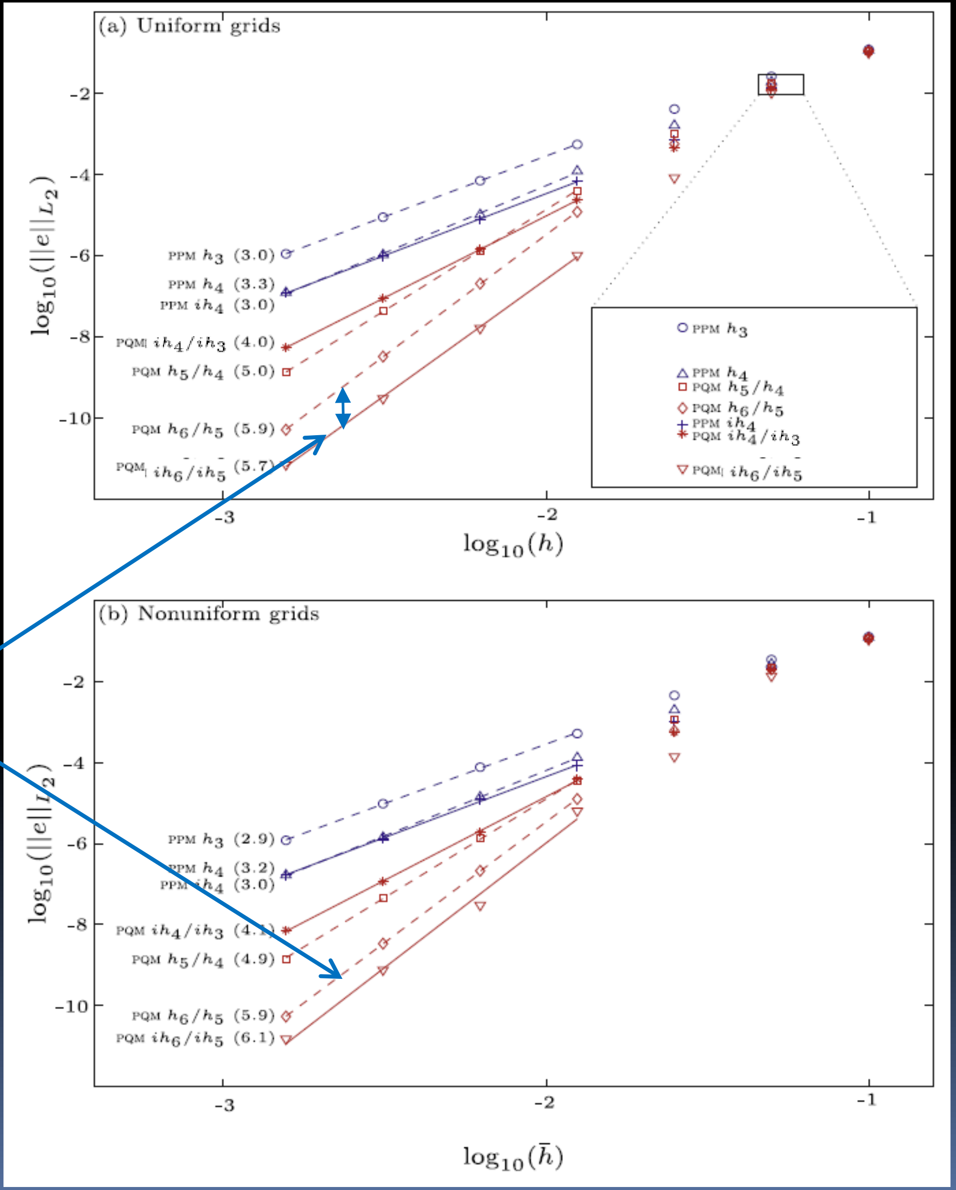
$$\alpha u'_{j-\frac{1}{2}} + u'_{j+\frac{1}{2}} + \beta u'_{j+\frac{3}{2}} = a\bar{u}_{j-1} + b\bar{u}_j + c\bar{u}_{j+1} + d\bar{u}_{j+2}$$

  - Tri-diagonal matrices ✓
  - Yields $O(\Delta^4)$ and $O(\Delta^6)$



Convergence analysis

- O (Δ^n) convergence for PQM-h_n/h_{n-1}
- Even for n=6
  - (should max at out at 5)
  - I don't understand this!
- Implicit interpolation significantly more accurate
  - even PPM benefits (at low resolution)



PQM reconstruction 2

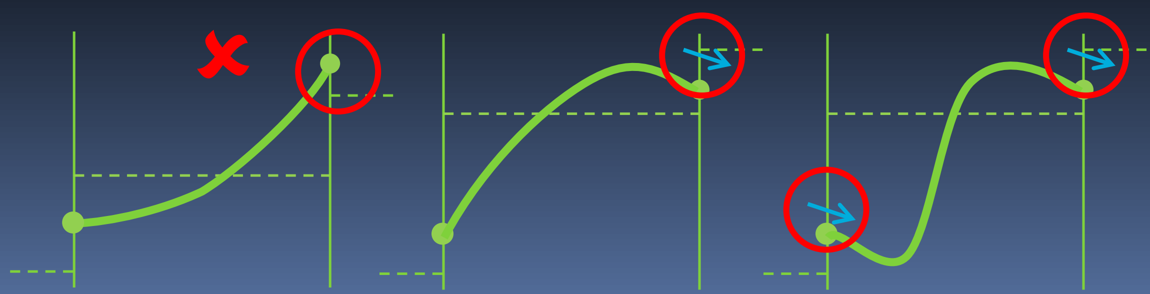
Algorithm

1. Estimate edge values/slopes
2. Bound edge values/slopes
3. Limit (monotonize) reconstruction

Require:

- Edge values to be bounded by neighbours
- Edge values are monotonic
- Edge slopes to be consistent with PLM

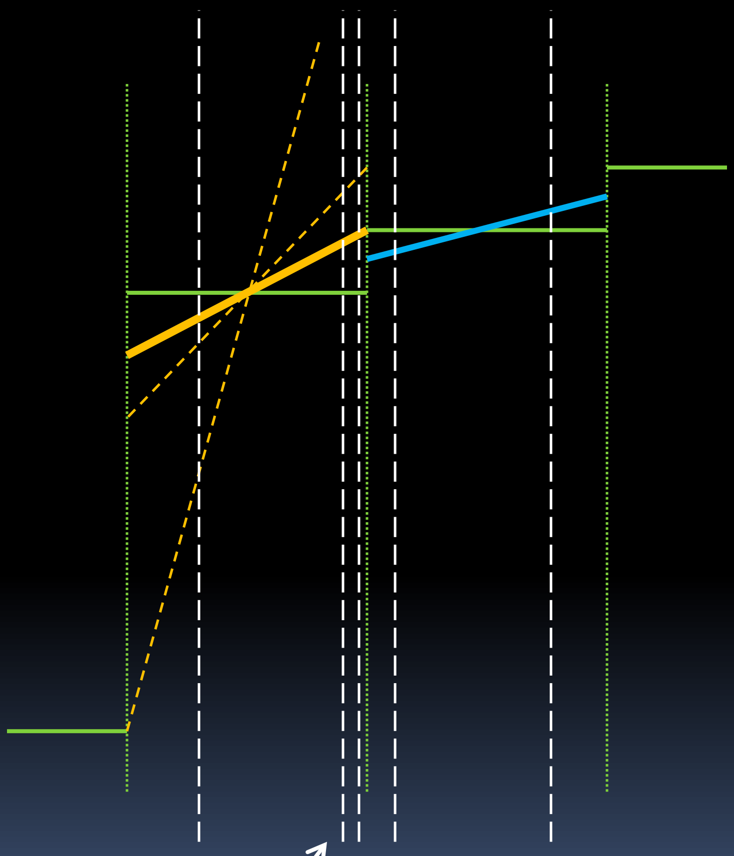
  (set equal to PLM if inconsistent)



Non-monotonic edge values

- In the 1-D non-divergent advection problem, monotonicity requires:
  - reconstruction is bounded by cell means
  - reconstruction is monotonic within cell
- For arbitrary remapping (i.e. to any grid)
  - edge values must be ordered

PLM example



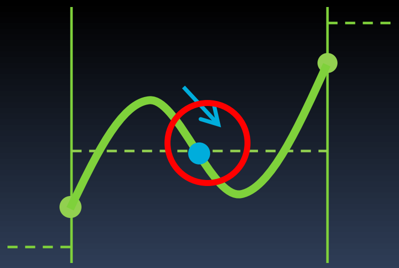
New target grid might sample non-monotonic profiles

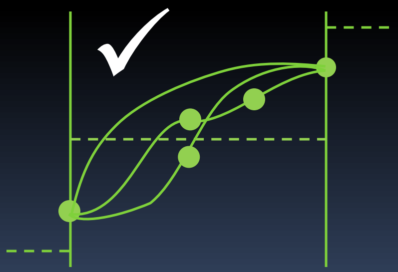
PQM reconstruction 3

Algorithm

1. Estimate edge values/slopes
2. Bound edge values/slopes
3. Limit (monotonize) reconstruction

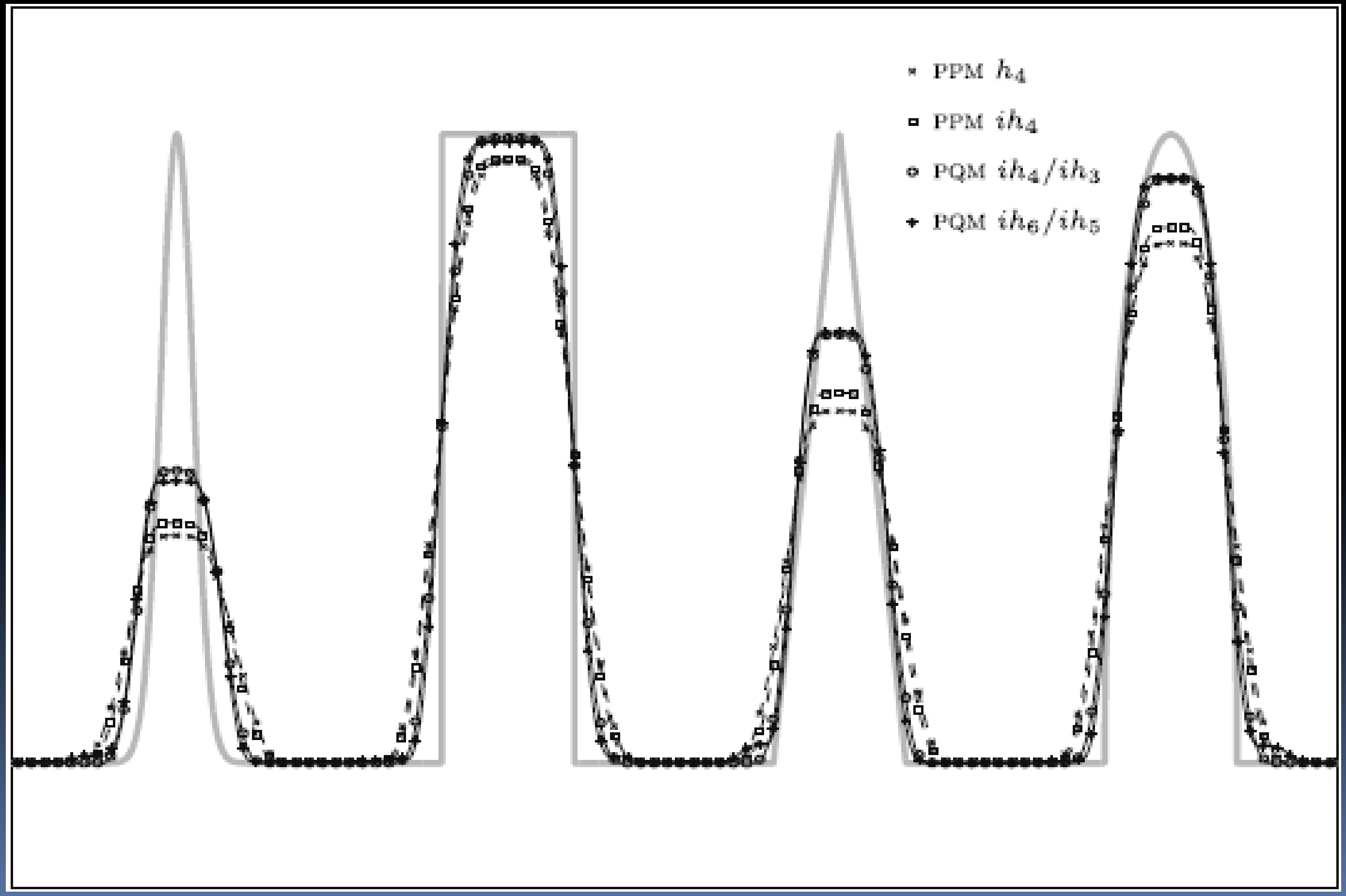
- Examine inflexions inside cell
  - Slope at inflexion should be same sign as PLM
  - Otherwise expel inflexions to edge with smaller slope





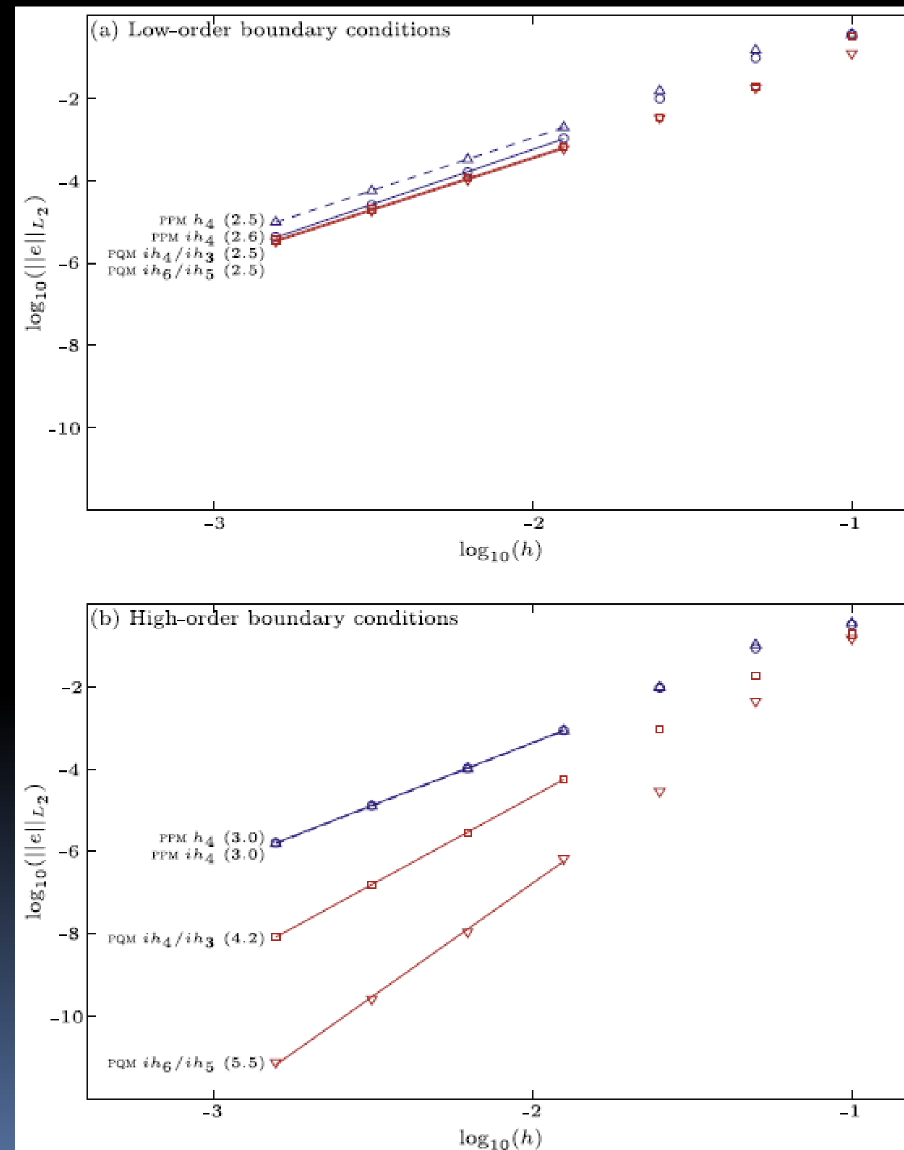
Remapping results: PPM & PQM

- Remap between uniform (100 cells) and <u>random</u> non-uniform grid (90 cells)

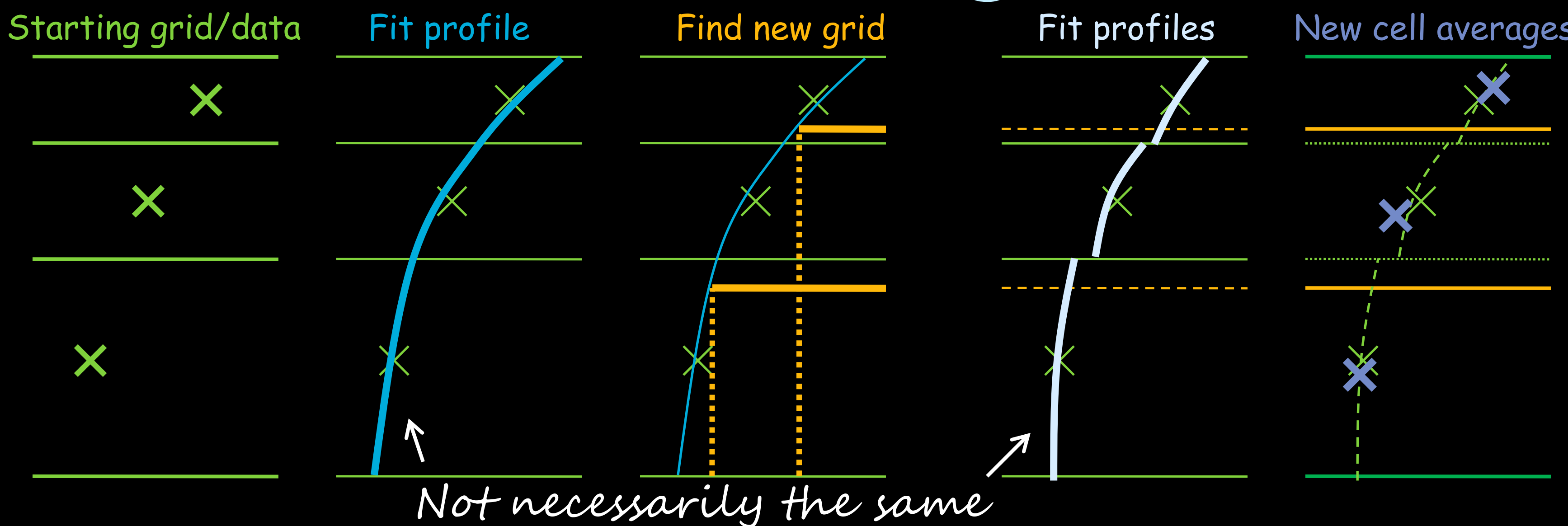


Boundaries (top/bottom)

- Boundaries are extrema…
- …and should not be limited
  - should any? Blossey & Durran 2008
- Here we use extended polynomial
  - Later we use rational functions
- Error due to a low order extrapolation on boundary dominates the L2-norm



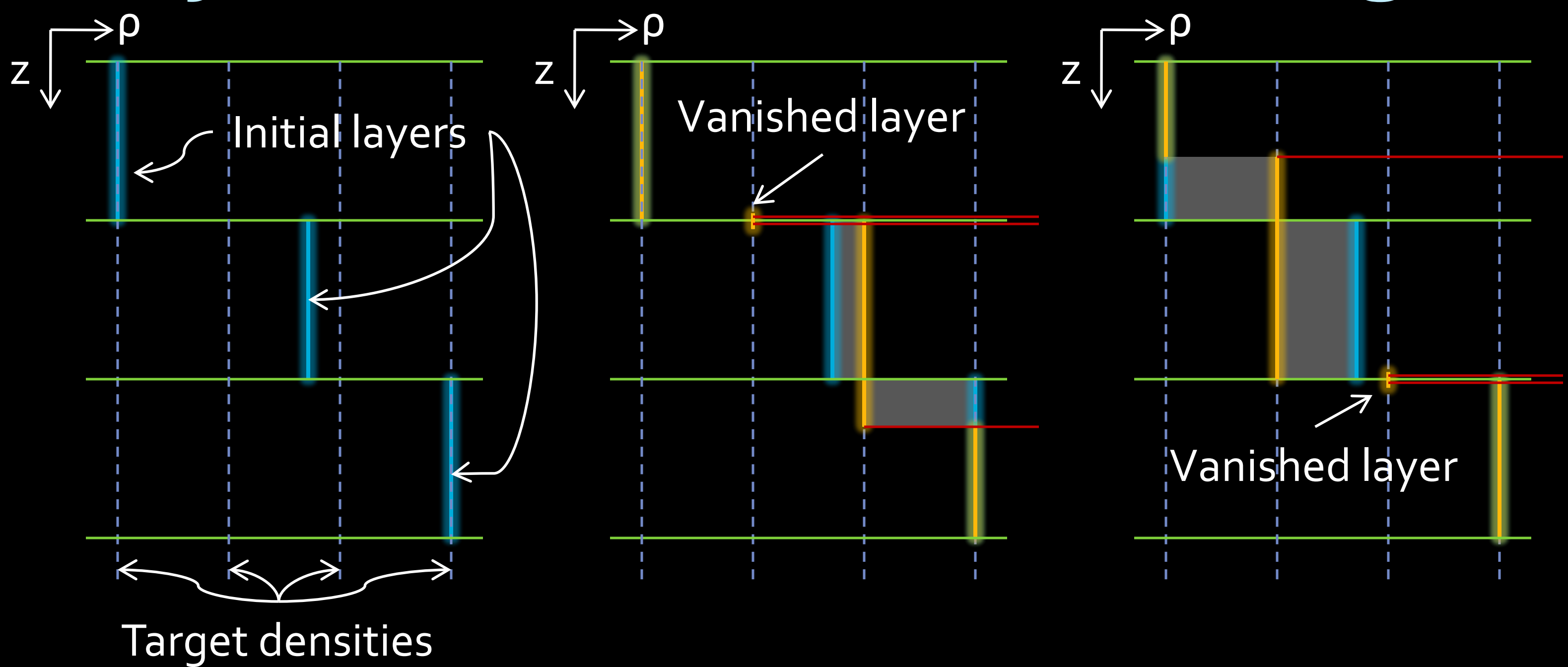
Coordinate free algorithm



- Re-gridding
  - Re-construct **global** profile
    - **Single valued (monotonic)**
    - (continuous or not)
    - (conservative or not)
  - Find position of new grid
- Re-mapping
  - Re-construct **local** profiles
    - **Conservative**
    - Limited (monotonic) or not
    - Discontinuous (exclusive!) or not
  - Integrate for new cell averages

•*Can iterate on procedure: high order converge faster*

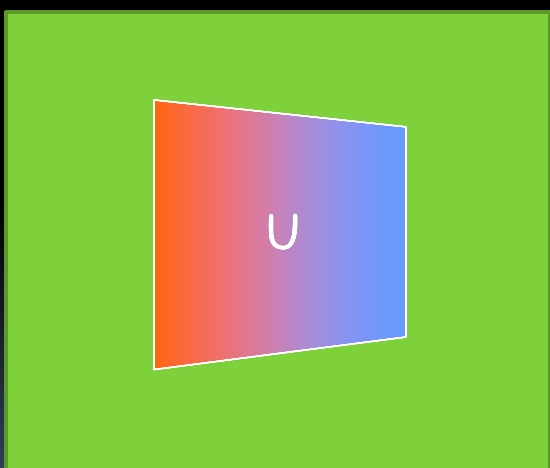
Layer v's interface targets



- Regridding with layer model mindset of "target" densities
  - Multiple or no solutions for some configurations
- Instead, specify density of bounding interfaces
  - No longer treats layers as constant density (a.k.a layer models)
  - Reconstruct variations in vertical
  - Refer to as "continuum isopycnal" (although can be discontinuous)

PGF error

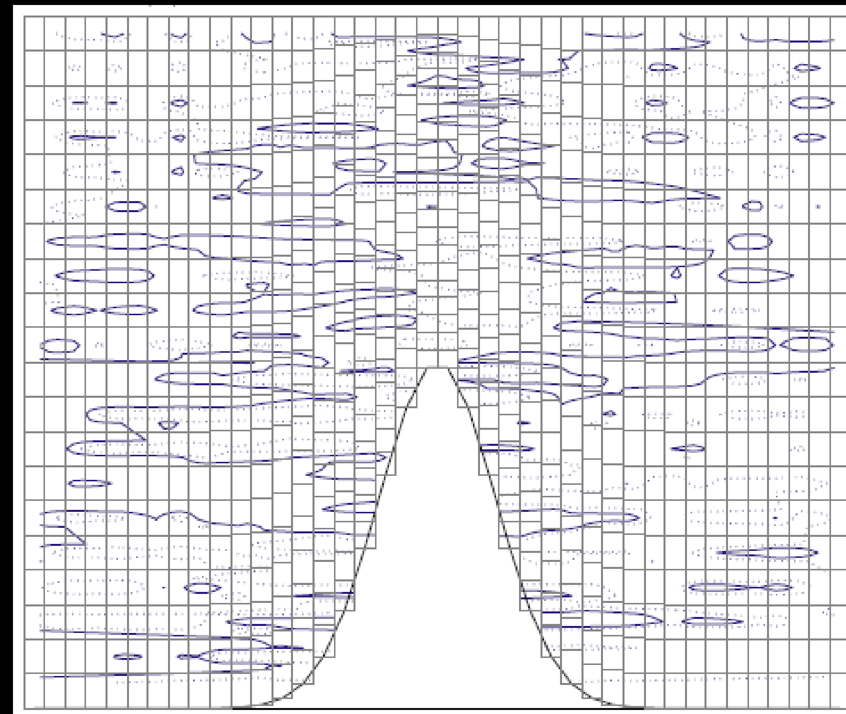
- Analytically integrate FV PGF
  - Necessary in isopycnal ocean model to avoid thermobaric instability



$$\boxed{\alpha = A + \frac{\lambda}{(P+p)}}$$

$$A = A(\theta, S)$$
$$P = P(\theta, S)$$
$$\lambda = \lambda(\theta, S)$$

Seamount resting ocean test



C.I. = 10^{-13} m/s, max $|u| \sim 10^{-11}$ m/s

$$\Phi_{tX}(x) = \Phi_{bX}(x) + A(x)\left(p_{bX}(x) - p_{tX}(x)\right) - \lambda(x)\ln\left|\frac{P(x)+p_{tX}(x)}{P(x)+p_{bX}(x)}\right|$$

$$\int_{p_{bX}}^{p_{tX}} \Phi_X \, dp = (p_{tX}-p_{bX})\left(\Phi_{bX} + \lambda_X - \tfrac{1}{2}A_X(p_{tX}-p_{bX})\right) - \lambda_X(P_X+p_{tX})\ln\left|\frac{P_X+p_{tX}}{P_X+p_{bX}}\right|$$

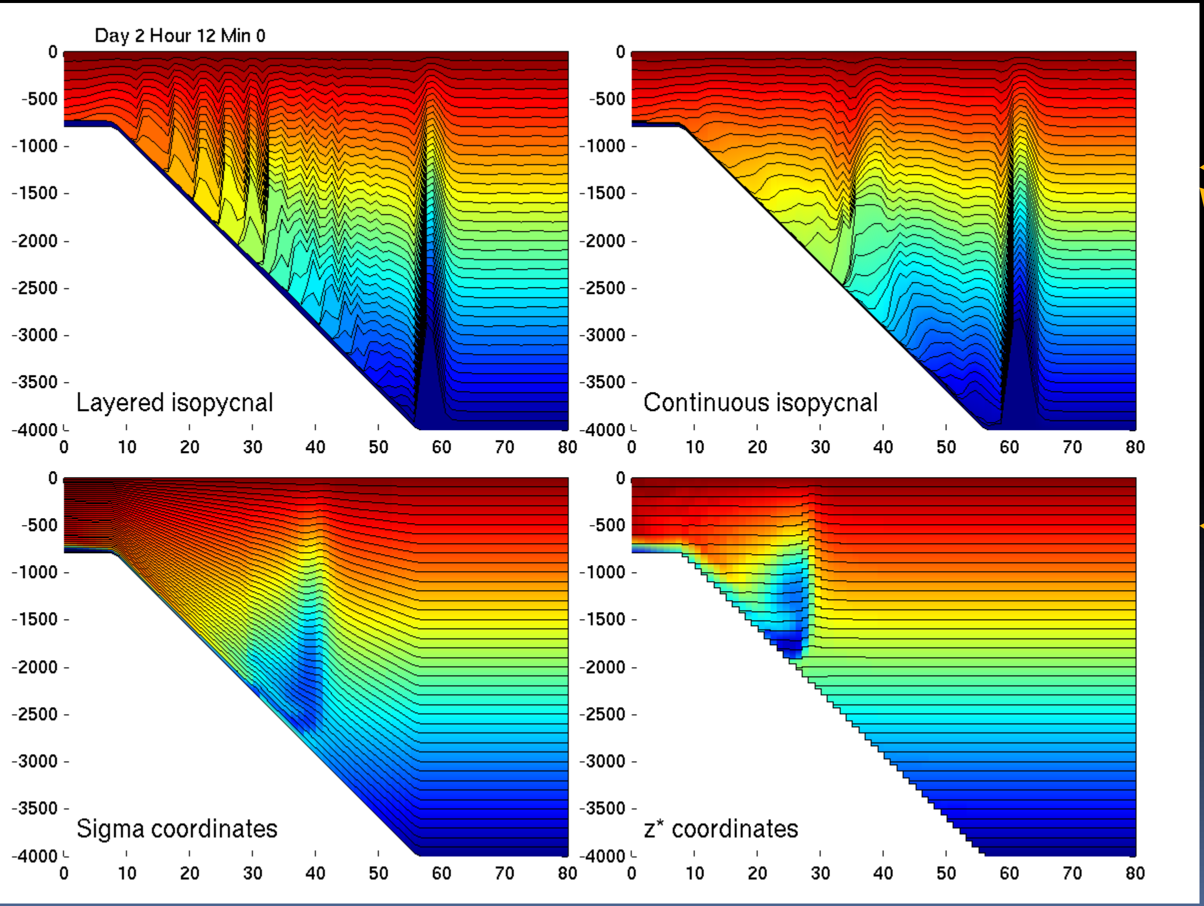
Adcroft et al., 2008

Gravity current (2D)

- Spurious diffusion significantly dilutes gravity current
- Continuous isopycnals do as well (look better) than layered
- Re-mapping to non-isopycnal clearly diffusive

"True" sol^n (adiabatic)

Better sol^n :-)



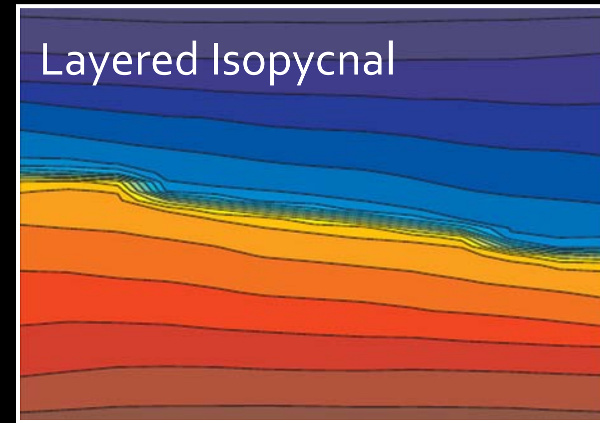
Z* and σ dillute buoyancy anomaly

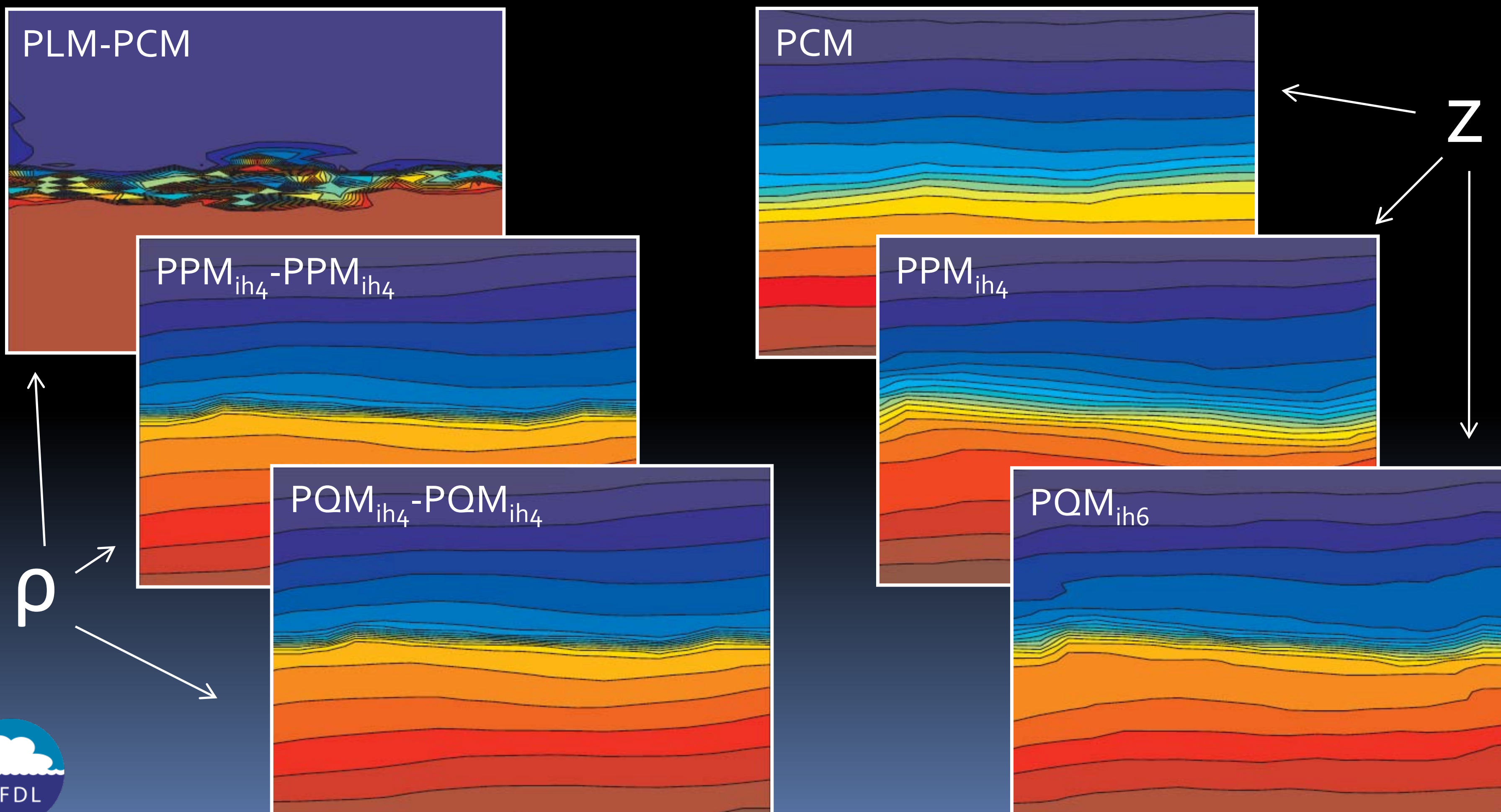
Same numerics for non-layered models

White, Adcroft & Hallberg, JCP 2009

Sloshing test case

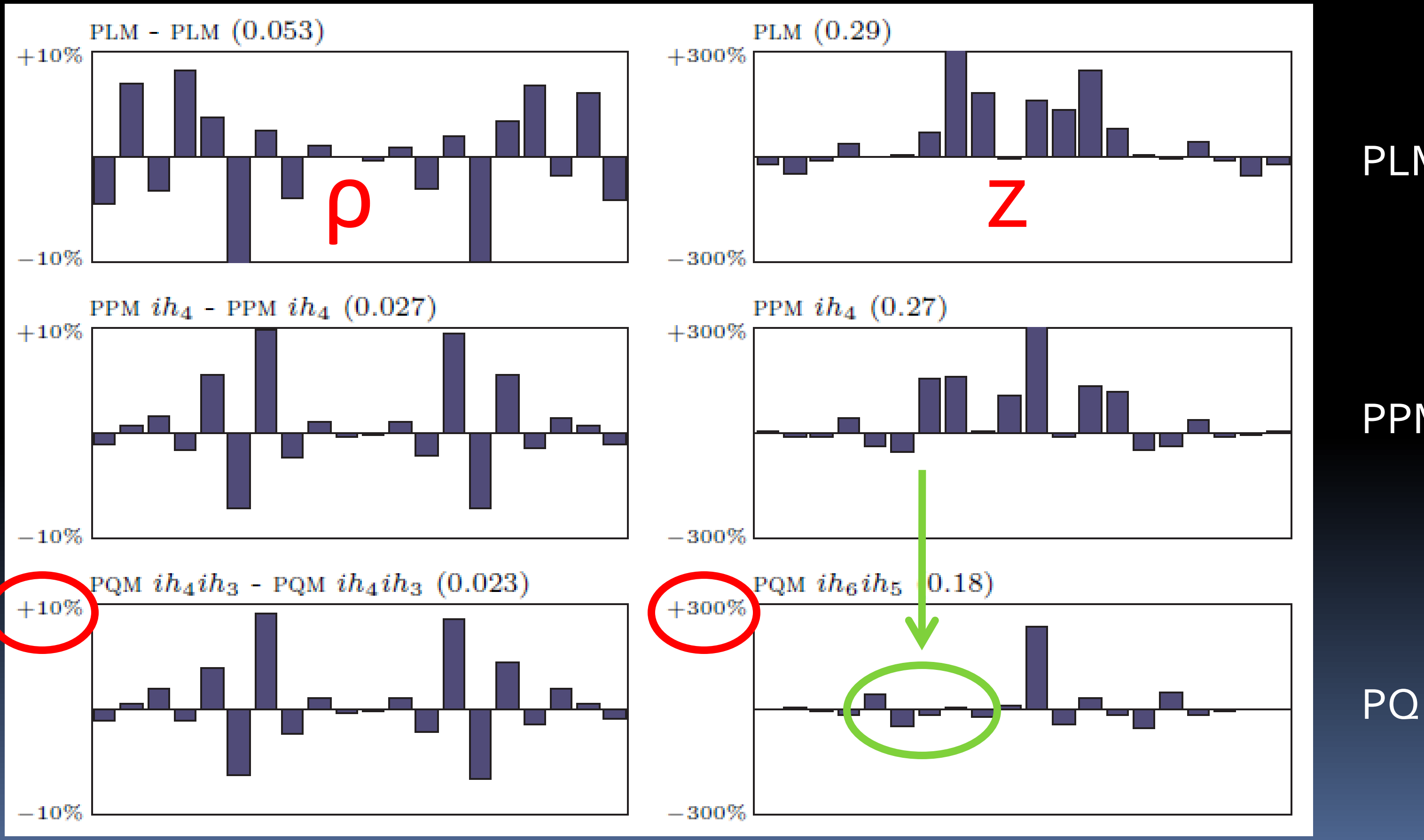
- Remapping to ρ works
  - PPM visibly diffusive in z-coordinates
  - PQM-PQM as good as layered





Sloshing test case

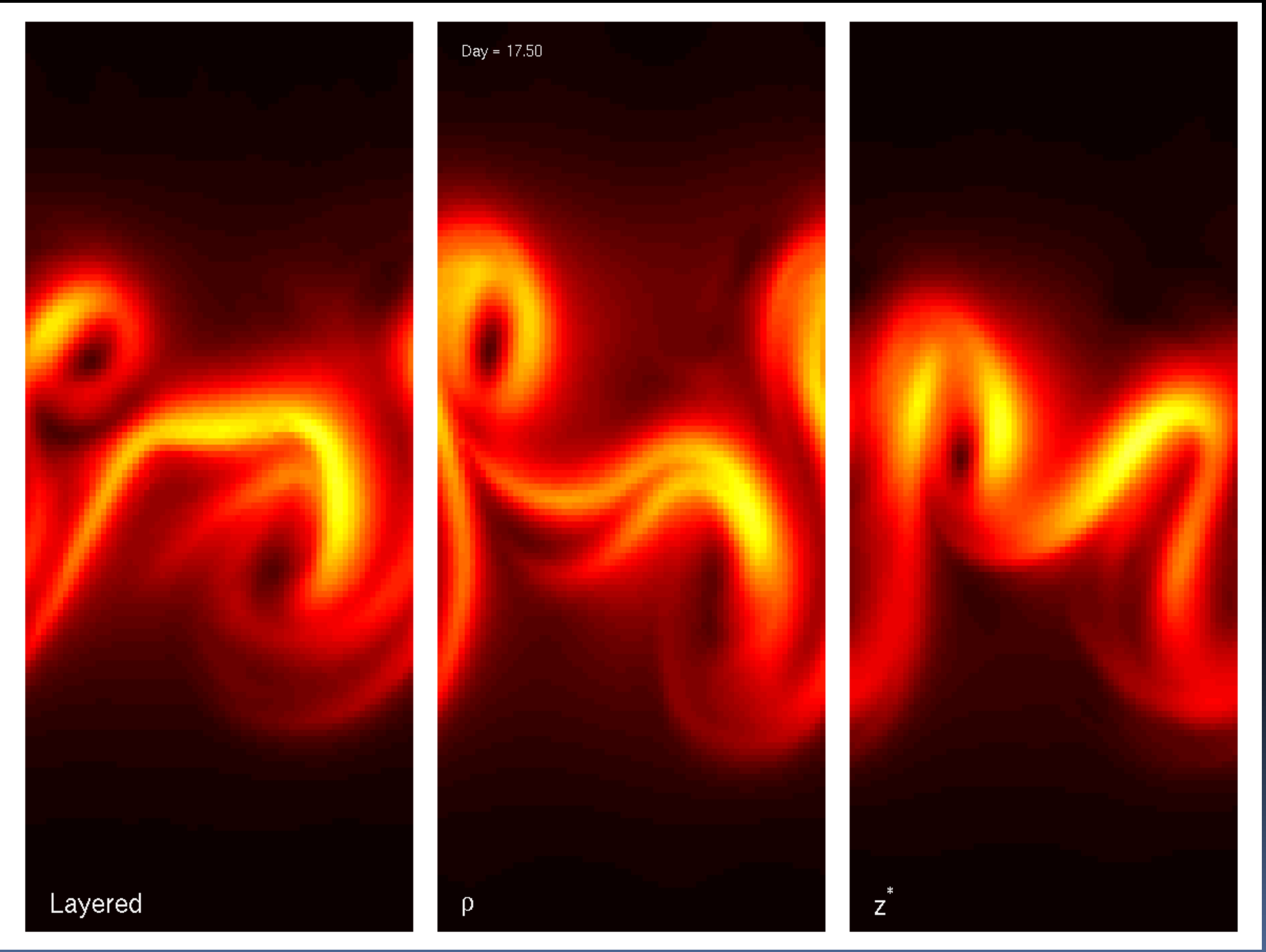
- Internal wave displacing a thermocline (tanh)
  - Note that numerical mixing is not simple diffusion



% volume change in each density class

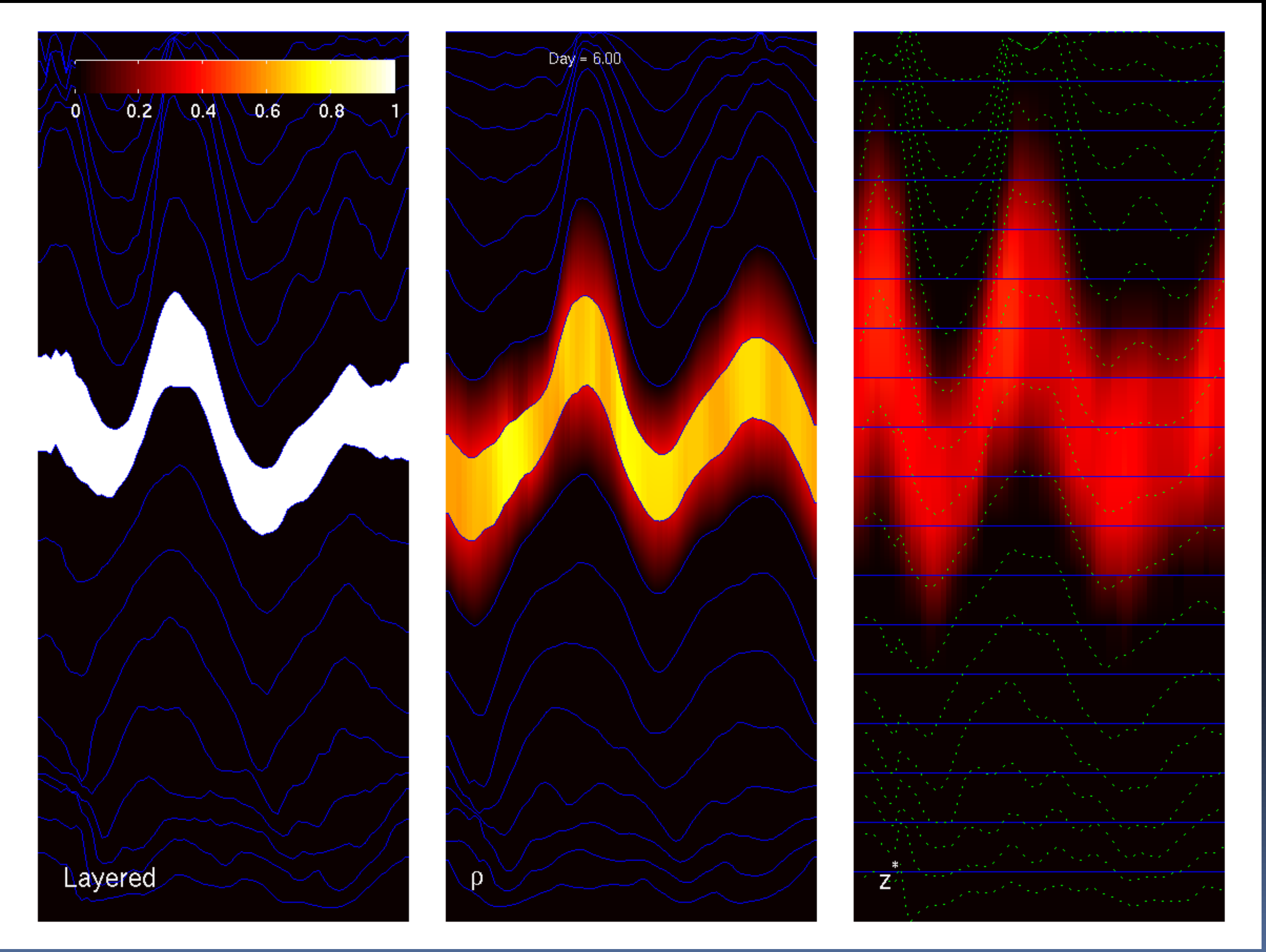
Eddying problems

- Expectation is that more energy nearer grid scale will lead to more spurious mixing



Tracer release: how not to measure buoyancy mixing

… at least when using limiters on extrema



Final thoughts

- GOLD can use same method throughout water column whether isopycnal or not
  - **Continuous isopycnal approach works (as well as layered)**
  - Not tied to pot. density, more flexible than layered isopycnal
- Spurious diffusion is minimized when <u>remapping to isopycnals</u>
  - … using PQM
  - PLM is too diffusive; don't yet know about PPM

Need to quantify in context of global application (measure κ)

- Verdict on non-isopycnal coordinates
  - Jury is out … but not looking good
  - High order approaches don't seem to be enough
  - Quantifying the spurious mixing is challenging
- Ready to explore new [hybrid] coordinates
- Consolidate "physics", e.g. bulk mixed layer vs. KPP

White & Adcroft, JCP 2008 White, Adcroft & Hallberg, JCP 2009