Gravity Waves in Shear and Implications for Organized Convection

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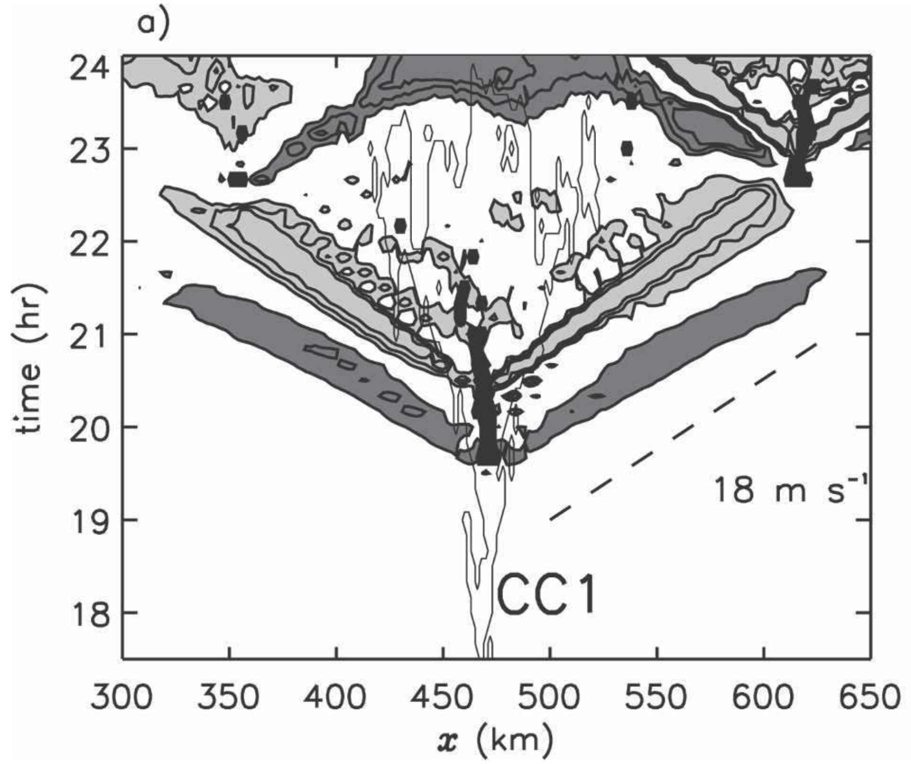
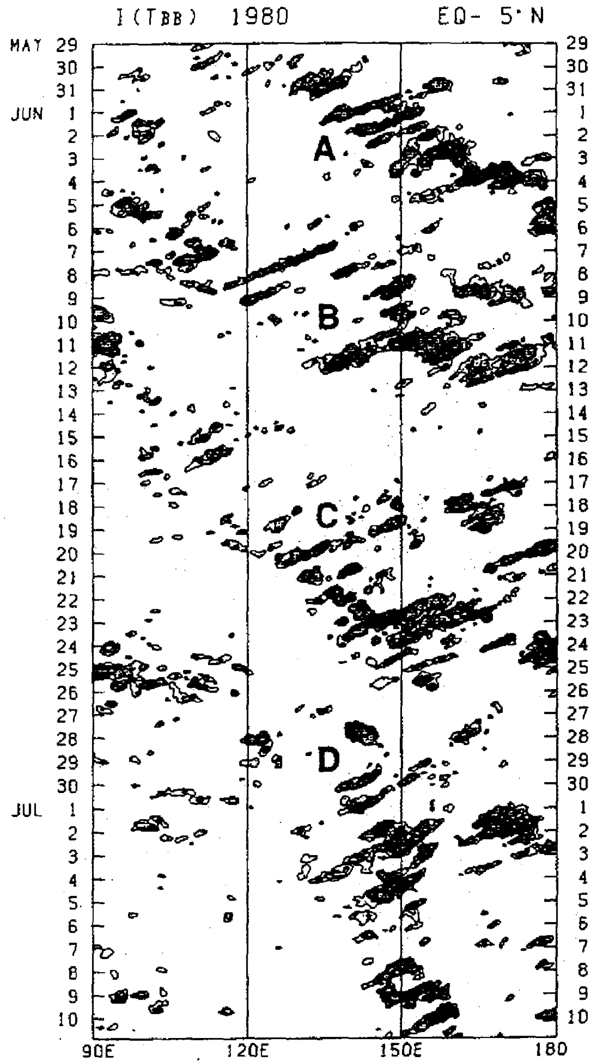
Gravity waves and organized convection

- Convection can excite gravity waves
- Gravity waves can suppress or excite new convection



from Tulich and Mapes (2008)

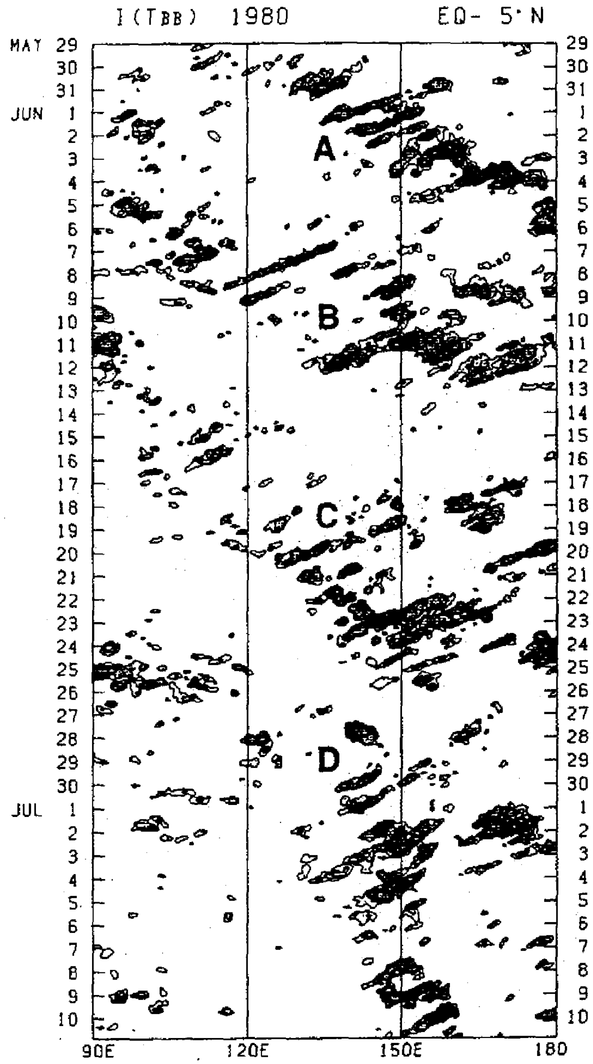
Convectively coupled waves:
Envelopes of mesoscale convective systems



from Nakazawa (1988)

- Embedded cloud systems propagate in opposite direction of wave envelope
- New cloud systems tend to form on a preferred side of preexisting cloud systems

Convectively coupled waves: Envelopes of mesoscale convective systems



from Nakazawa (1988)

- What causes wave trains to form preferentially (rather than scattered convection)?
- What determines the preferred propagation direction of the convectively coupled wave?
- Hypothesis: interactions of gravity waves with wind shear

Outline

- Designing a simple model with waves in shear
- Testing a simple model with waves in shear
- Application: preferred propagation direction of convectively coupled waves
- Application: formation of new convective cells in front of convective system

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Gravity waves in the tropical atmosphere

Hydrostatic Boussinesq equations:

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + W\frac{\partial U}{\partial z} + \frac{\partial P}{\partial x} = 0$$

$$\frac{\partial P}{\partial z} = g\frac{\Theta}{\theta_{ref}}$$

$$\frac{\partial \Theta}{\partial t} + U\frac{\partial \Theta}{\partial x} + W\frac{\partial \Theta}{\partial z} + W\frac{d\theta_{bg}}{dz} = 0$$

$$\frac{\partial U}{\partial x} + \frac{\partial W}{\partial z} = 0$$

$U =$ horizontal velocity $P =$ pressure

$W =$ vertical velocity $\Theta =$ temperature

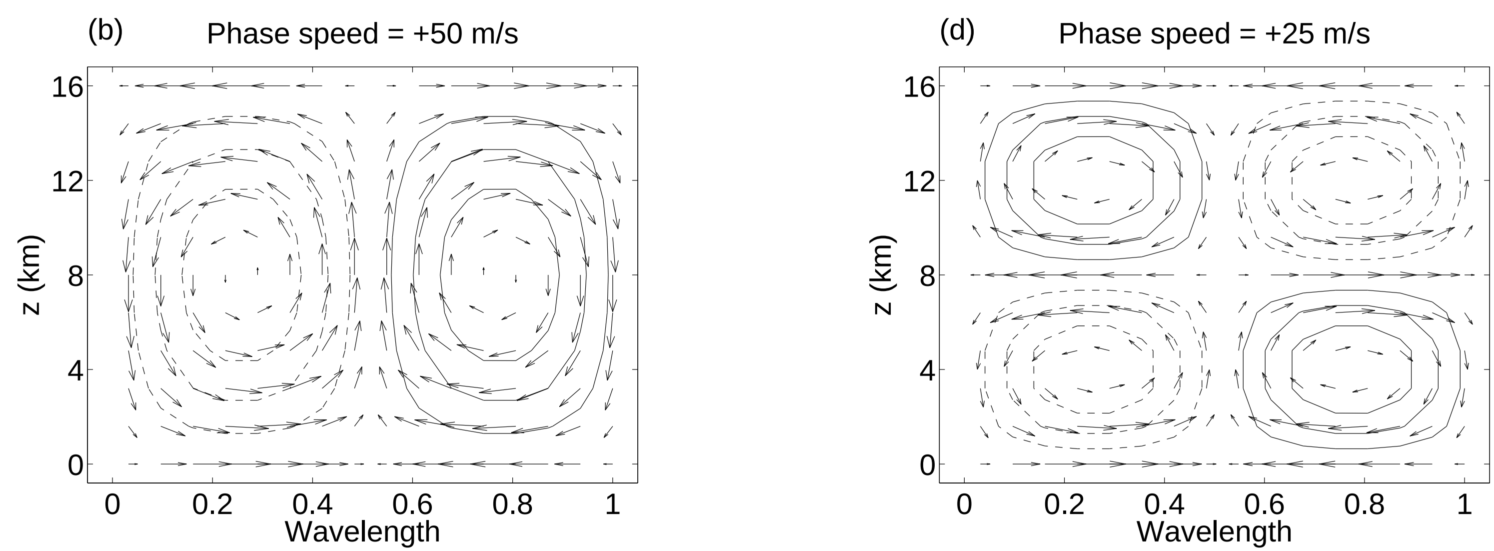
Gravity waves in the tropical atmosphere

Linearized equations with trivial background state $\bar{U}(z) = 0$:

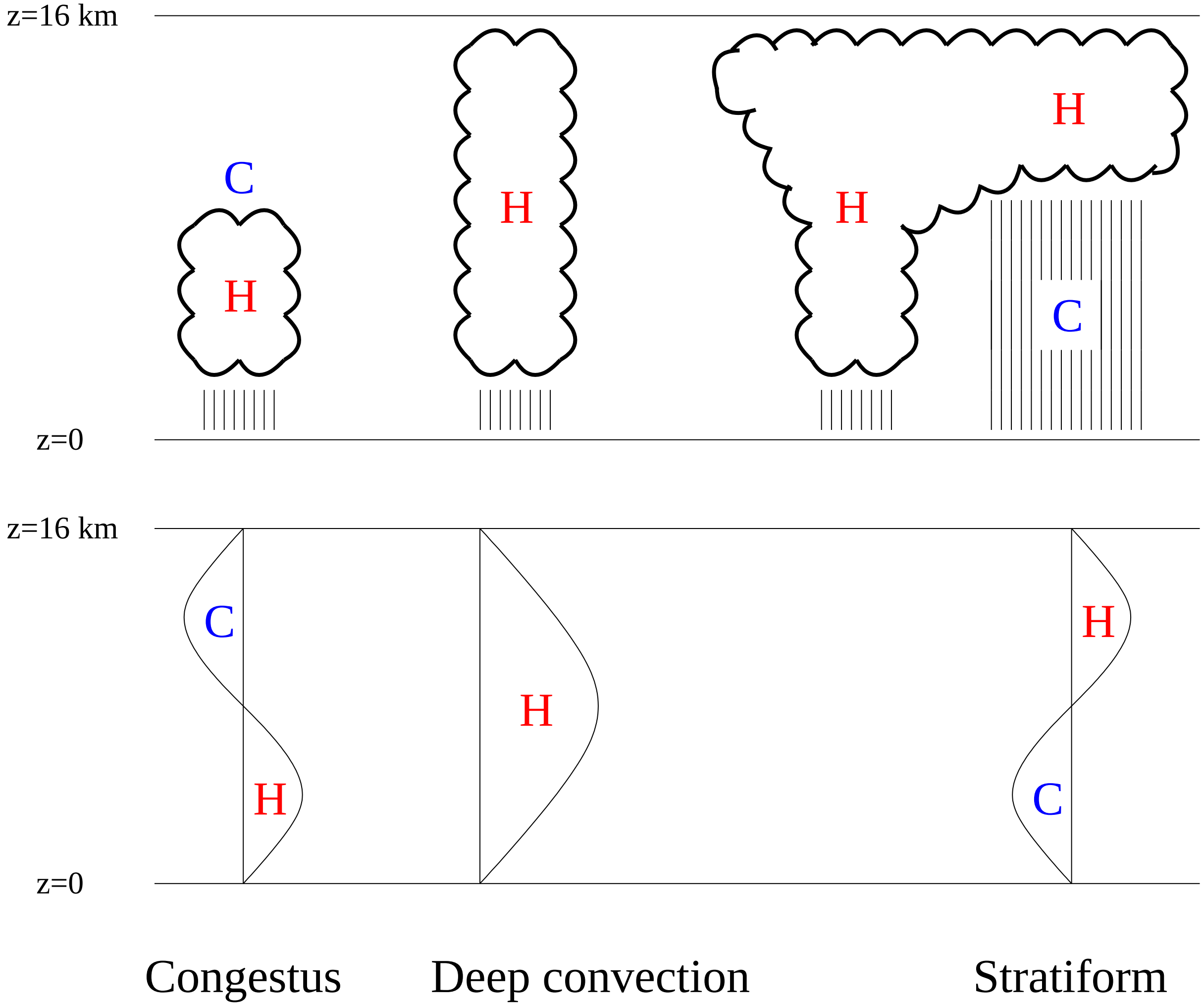
- Independent vertical modes: $U'(x,z,t) = \sum_j u'_j(x,t) \cos jz$, etc.
- Shallow water system for each vertical mode j:

$$\frac{\partial u'_j}{\partial t} - \frac{\partial \theta'_j}{\partial x} = 0$$

$$\frac{\partial \theta'_j}{\partial t} - \frac{1}{j^2}\frac{\partial u'_j}{\partial x} = 0$$

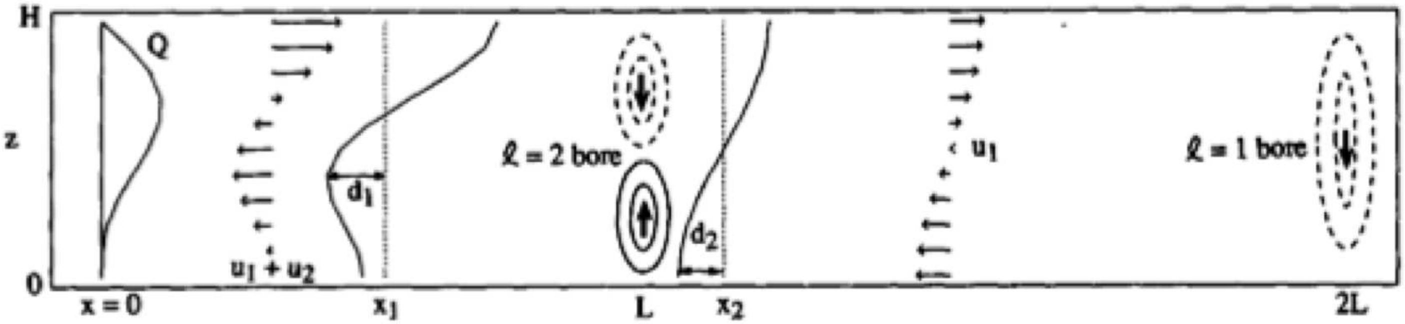


Cloud types and vertical modes

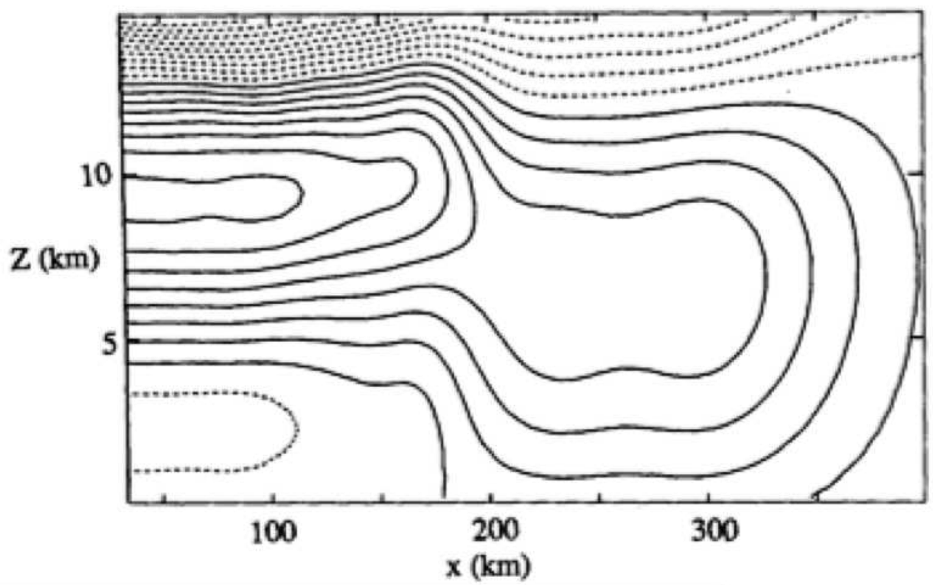


Gravity waves and organized convection

Theory: the role of (1) deep convection and (2) stratiform heating



Buoyancy contours



from Mapes (1993)

Gravity waves and organized convection

Previous work:

Simplified models without wind shear

- Nicholls et al (1991), Pandya et al (1993), Mapes (1993), Liu and Moncrieff (2004)

Cloud resolving models with/without shear

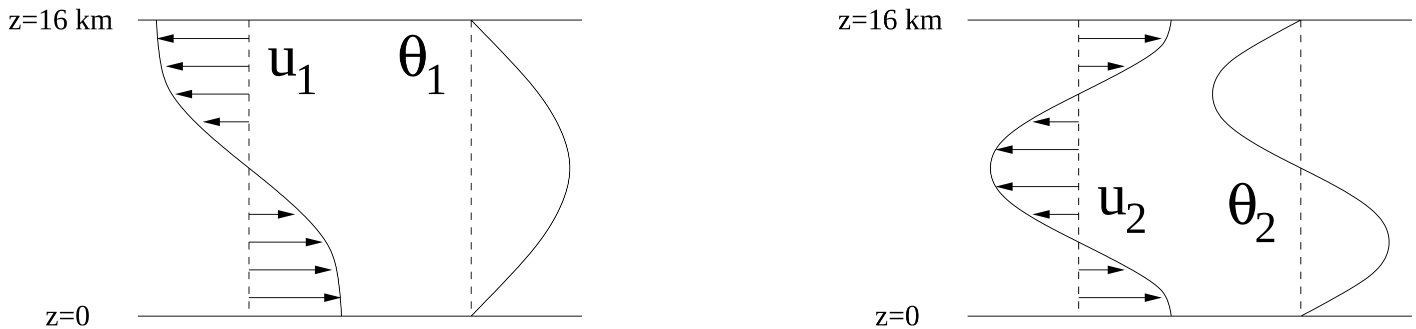
- Bretherton and Smolarkiewicz (1989), Oouchi (1999), Lane and Reeder (2001), Shige and Satomura (2001), Lac et al (2002), Tulich and Mapes (2008)

Present work:

Simplified nonlinear model with wind shear

- Stechmann and Majda (2009), Stechmann, Majda, Khouider (2008)

A simple model with waves in shear



Project nonlinear equations

$$\partial_t U + U\partial_x U + W\partial_z U + \partial_x P = 0$$

onto vertical modes

$$U(x,z,t) = u_1(x,t)\sqrt{2}\cos\frac{\pi z}{H} + u_2(x,t)\sqrt{2}\cos\frac{2\pi z}{H}$$

The result is ...

2-Mode Shallow Water Equations

$$\begin{cases} \dfrac{\partial u_1}{\partial t} - \dfrac{\partial \theta_1}{\partial x} & = & -\dfrac{3}{\sqrt{2}}\left[u_2\dfrac{\partial u_1}{\partial x} + \dfrac{1}{2}u_1\dfrac{\partial u_2}{\partial x}\right] \\ \dfrac{\partial \theta_1}{\partial t} - \dfrac{\partial u_1}{\partial x} & = & -\dfrac{1}{\sqrt{2}}\left[2u_1\dfrac{\partial \theta_2}{\partial x} + 4\theta_2\dfrac{\partial u_1}{\partial x} - u_2\dfrac{\partial \theta_1}{\partial x} - \dfrac{1}{2}\theta_1\dfrac{\partial u_2}{\partial x}\right] \end{cases}$$

$$\begin{cases} \dfrac{\partial u_2}{\partial t} - \dfrac{\partial \theta_2}{\partial x} & = & 0 \\ \dfrac{\partial \theta_2}{\partial t} - \dfrac{1}{4}\dfrac{\partial u_2}{\partial x} & = & -\dfrac{1}{2\sqrt{2}}\left[u_1\dfrac{\partial \theta_1}{\partial x} - \theta_1\dfrac{\partial u_1}{\partial x}\right] \end{cases}$$

- Nonlinear, hydrostatic internal gravity waves with effect of background shear

Outline

- Designing a simple model with waves in shear
- Testing a simple model with waves in shear
- Application: preferred propagation direction of convectively coupled waves
- Application: formation of new convective cells in front of convective system

Interesting properties of the 2MSWE

- Non-conservative

$$\mathbf{u}_t + A(\mathbf{u})\mathbf{u}_x = 0, \qquad A(\mathbf{u}) \neq \frac{\partial \mathbf{f}}{\partial \mathbf{u}}, \qquad \mathbf{u} = (u_1, \theta_1, u_2, \theta_2)$$

- Eigenstructure is not analytically accessible
- Energy is conserved: $(u_1^2 + u_2^2 + \theta_1^2 + 4\theta_2^2)/2$
- Conditionally hyperbolic
- Neither genuinely nonlinear nor linearly degenerate
- Background shear can affect propagating waves

Several similarities, some differences from properties of

2-phase fluid equations and 2-layer shallow water equations

Numerical Methods

Numerical methods are a challenge for non-conservative PDE

$$\frac{\partial \mathbf{u}}{\partial t} + A(\mathbf{u})\frac{\partial \mathbf{u}}{\partial x} = 0$$

Our approach: split A into conservative and non-conservative parts:

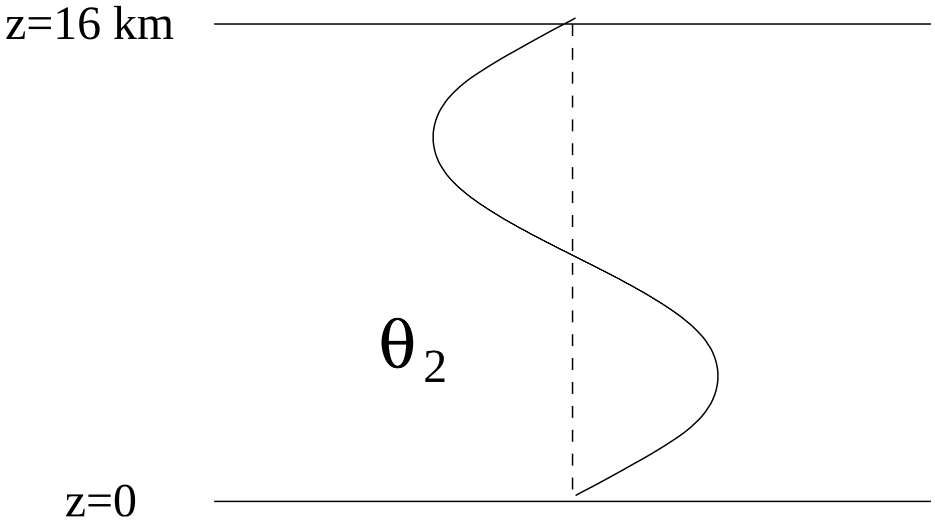
$$\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial}{\partial x}\mathbf{F}(\mathbf{u}) = -A_{nc}(\mathbf{u})\frac{\partial \mathbf{u}}{\partial x}, \qquad \text{where } \frac{\partial \mathbf{F}}{\partial \mathbf{u}} = A_c$$

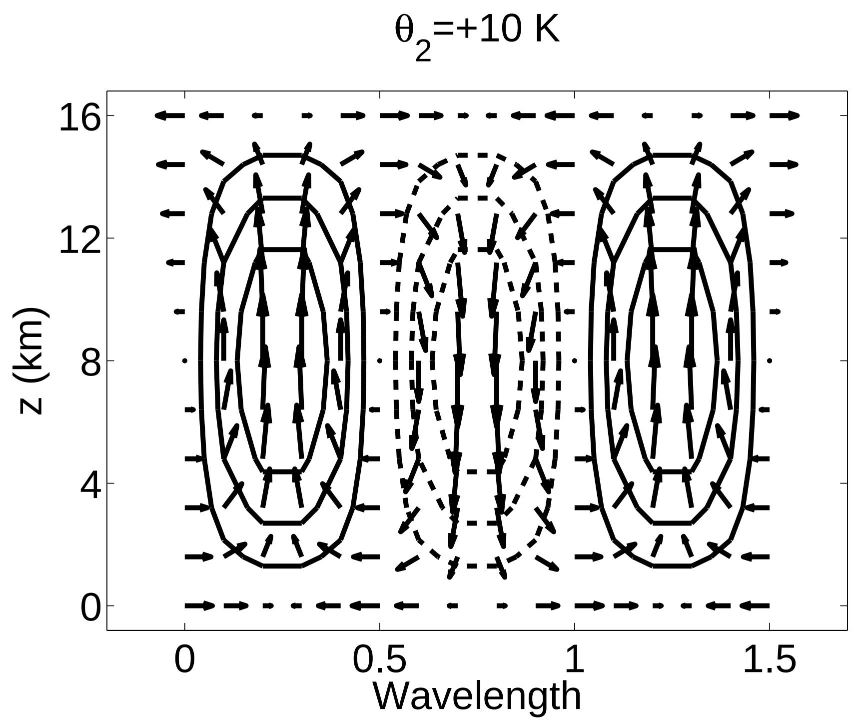
Operator splitting:

1. Non-oscillatory central scheme of Nessyahu and Tadmor (1990) for $\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial}{\partial x}\mathbf{F}(\mathbf{u}) = 0$
2. Centered spatial differences with 2nd order Runge–Kutta for $\frac{\partial \mathbf{u}}{\partial t} = -A_{nc}(\mathbf{u})\frac{\partial \mathbf{u}}{\partial x}$
   (Note: eigenvalues of A_{nc} are all zero)

2MSWE are conditionally hyperbolic

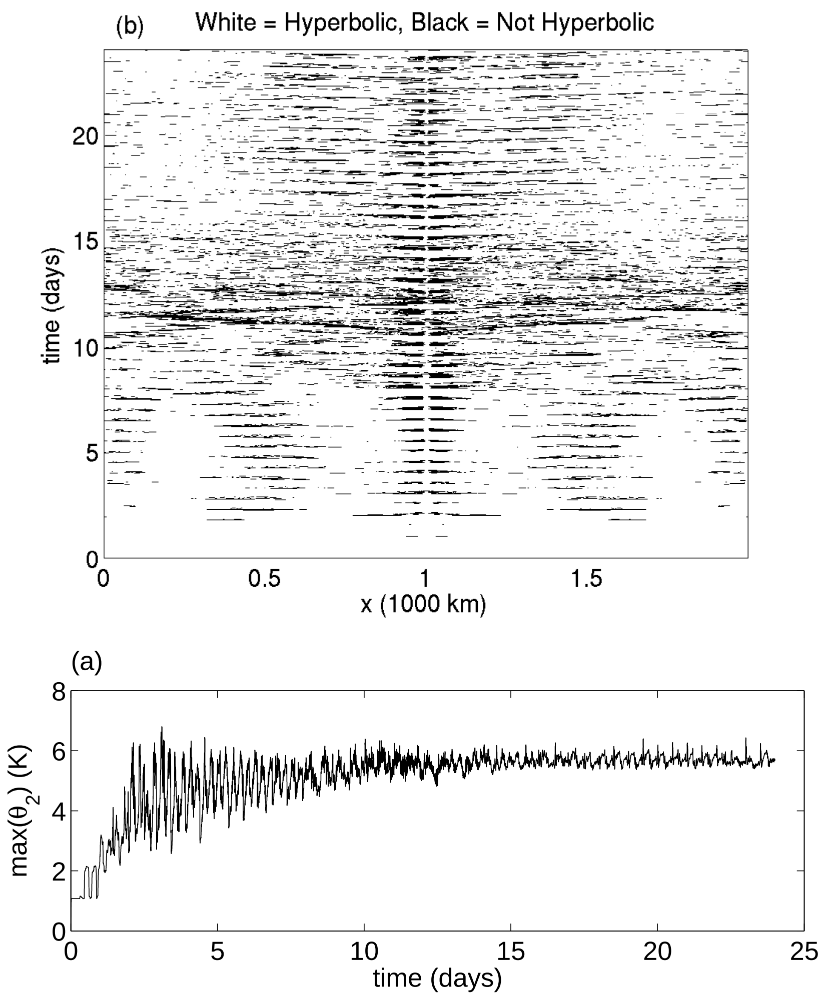
- Hyperbolic for lower values of u and θ
- Not hyperbolic for larger shears or temperatures
  - Richardson number-like criterion for instability
  - Unstable waves have overturning circulation to stabilize





Numerical Test:
Forcing the system into non-hyperbolic states

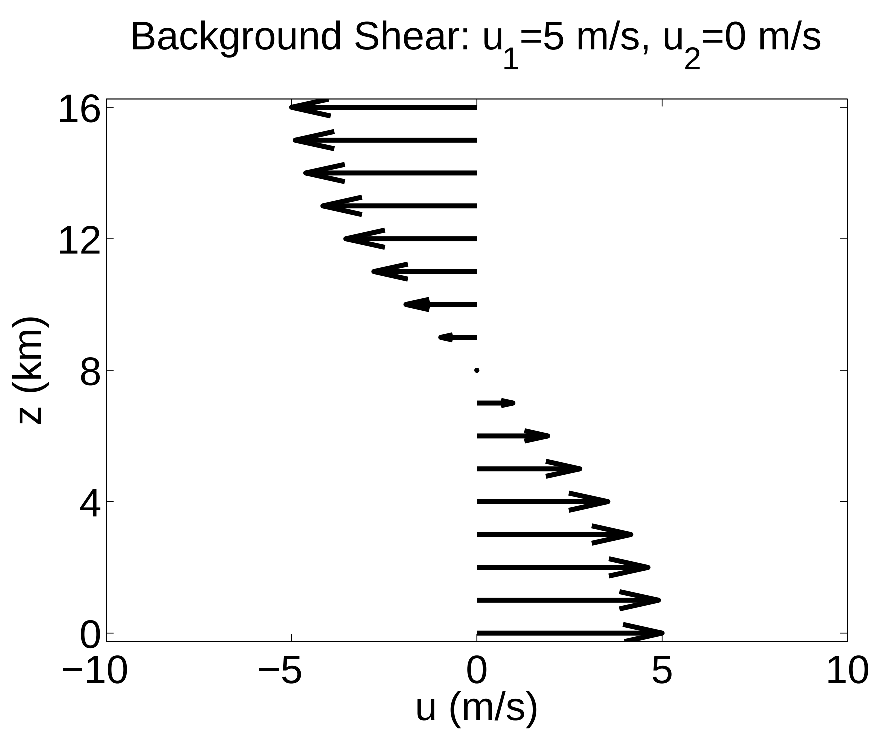
Add an imposed source term: $\mathbf{u}_t + A(\mathbf{u})\mathbf{u}_x = \mathbf{S}(x,t)$

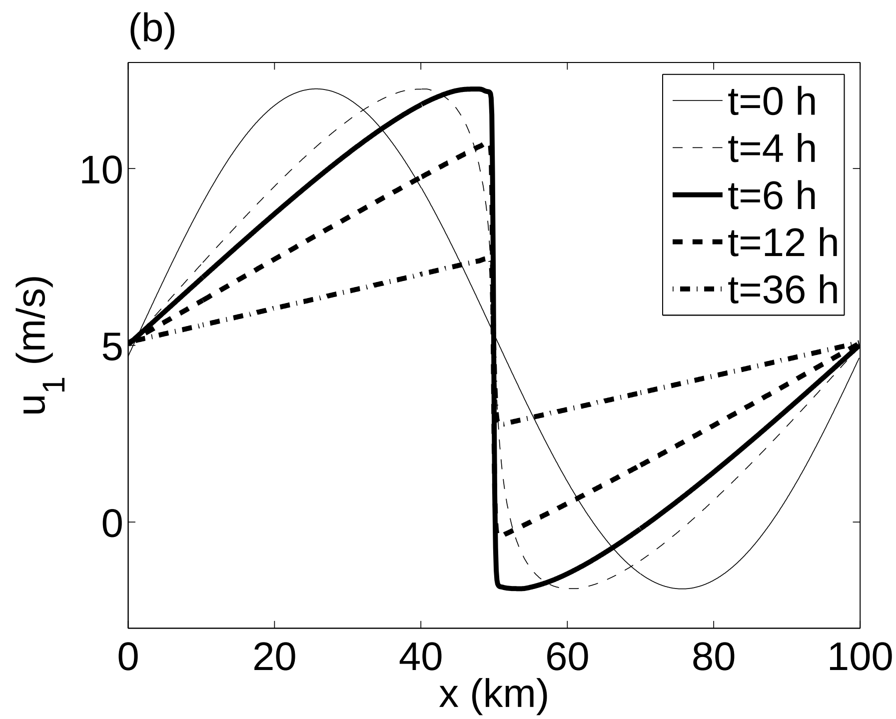


- Non-hyp. states accessed often
- But no catastrophic effects from non-hyperbolicity
- Energy also levels off with time

Smooth waves can break sometimes (but not always)

- With u_1 "background shear," smooth waves break in finite time





Summary of 2-mode shallow water equations

- Simple model for nonlinear interaction of waves and shear
- Several interesting mathematical properties
- Simple numerical method passes several difficult tests

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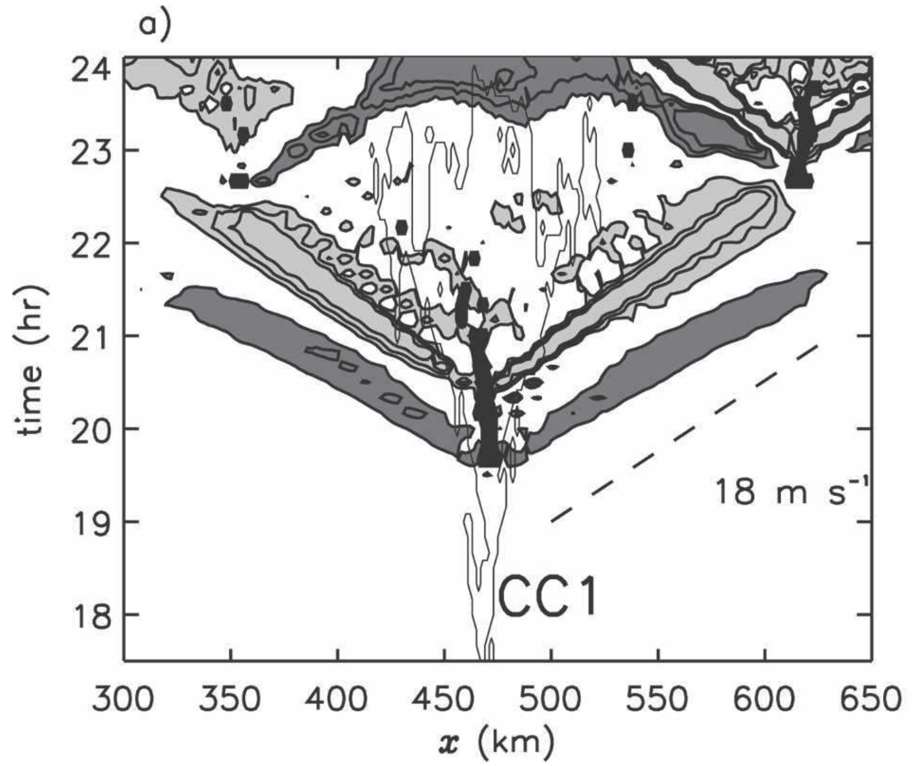
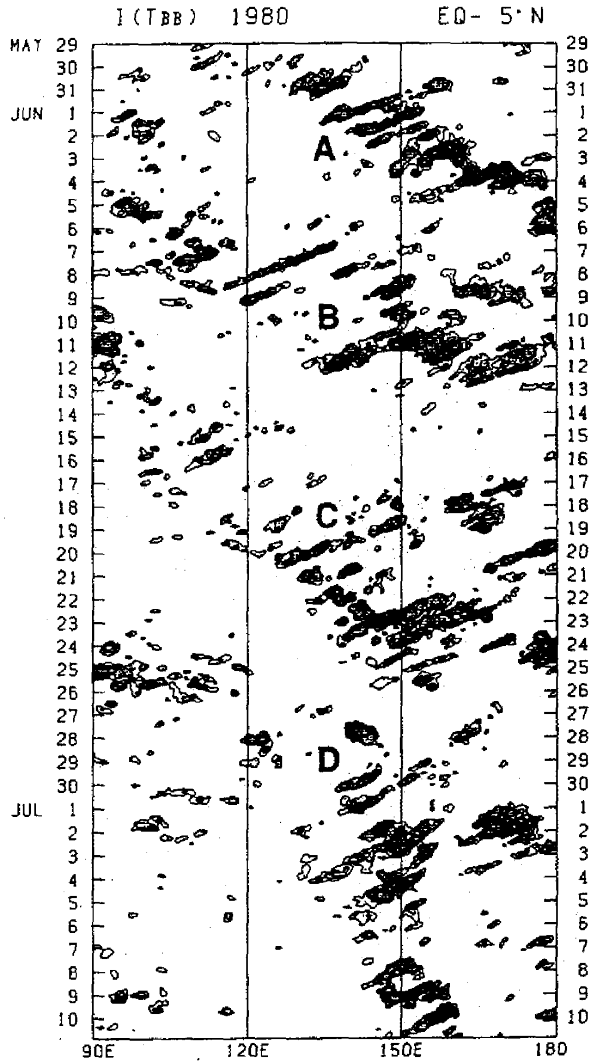


from Tulich and Mapes (2008)

Convectively coupled waves:
Envelopes of mesoscale convective systems



from Nakazawa (1988)

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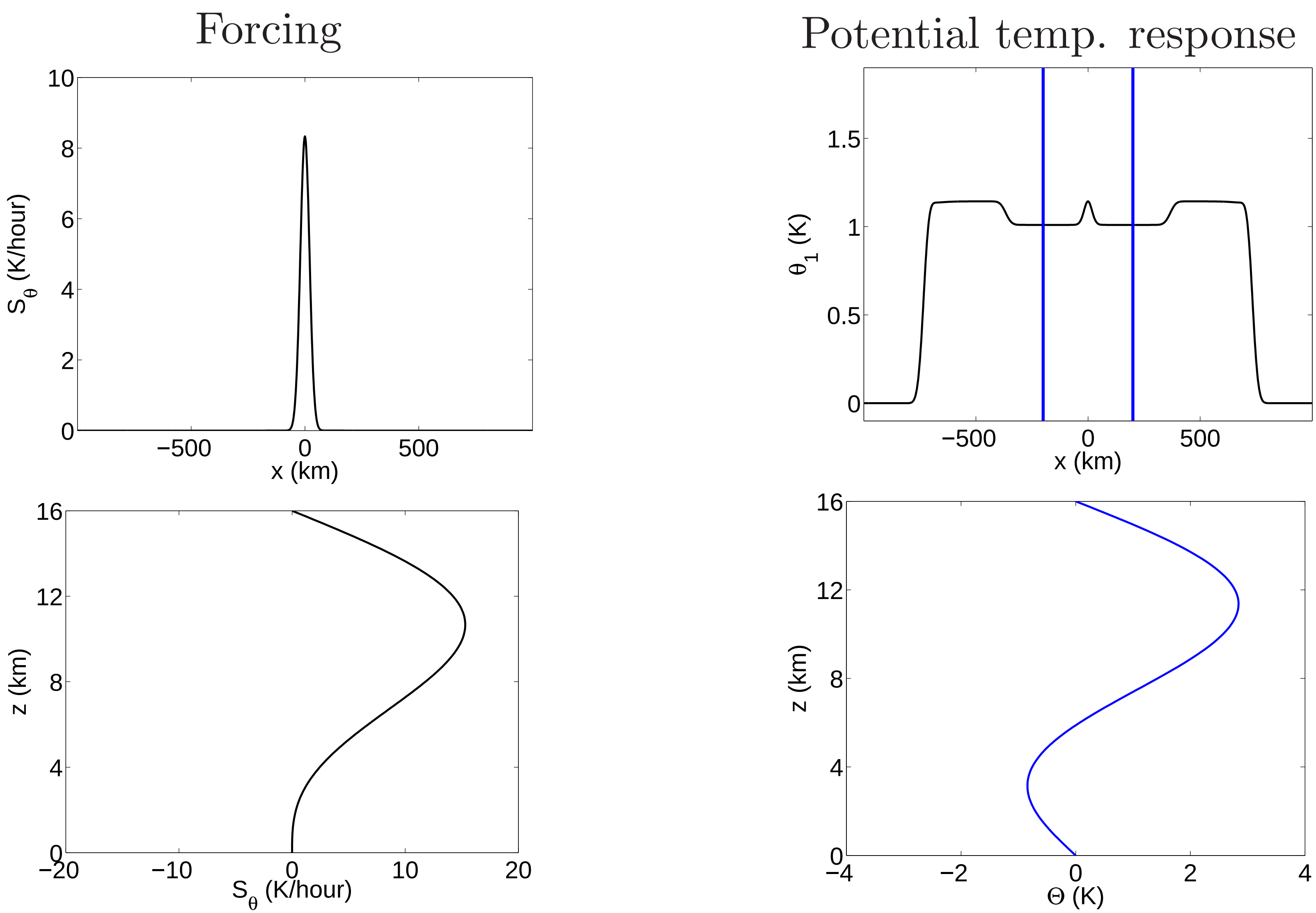
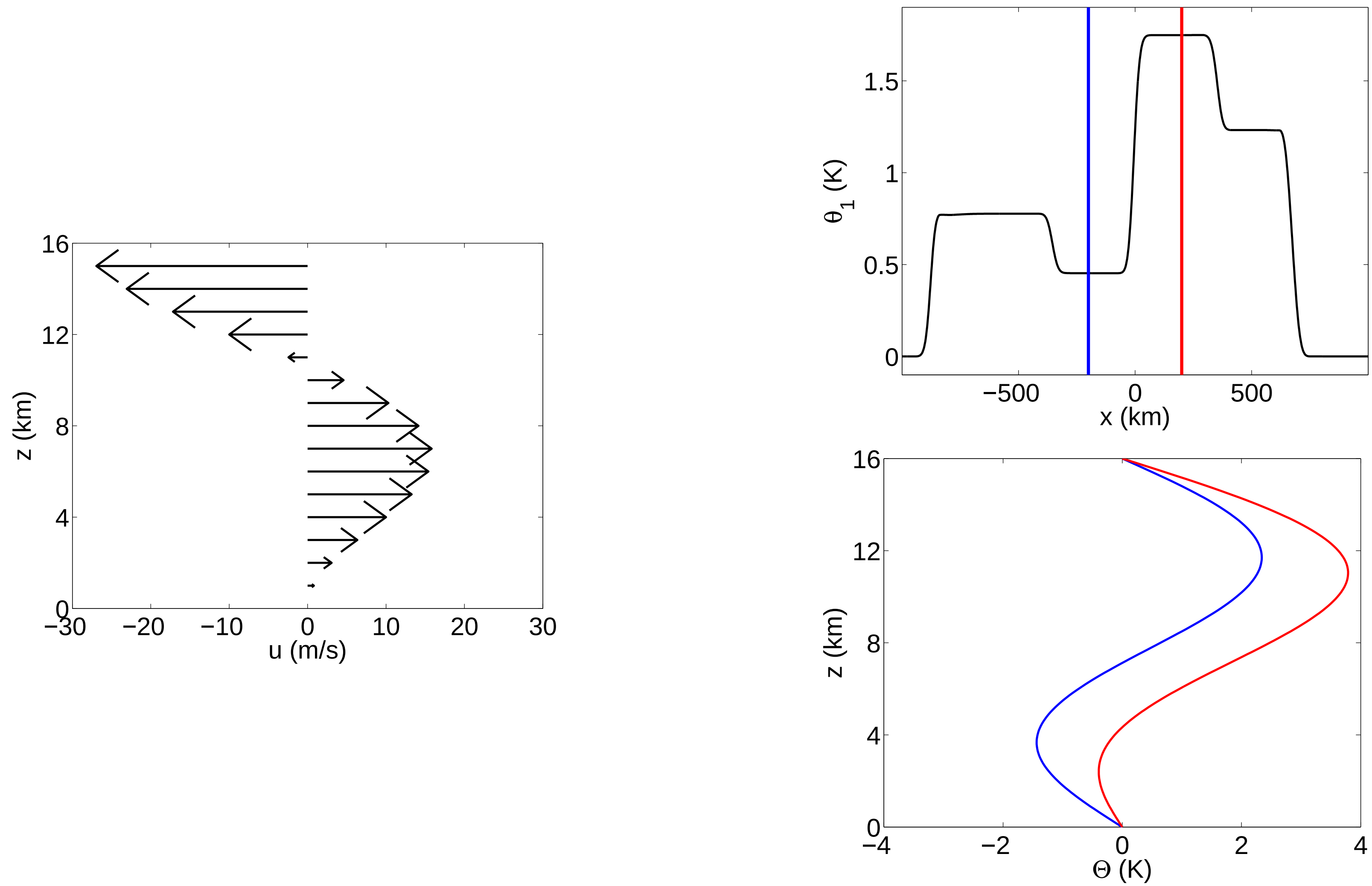
Numerical experiment WITHOUT wind shear



Results symmetric to east and west of forcing

Numerical experiment WITH wind shear



- West of forcing is more favorable for new convection than east
- In agreement with observations for this wind shear (Wu and LeMone, 1999)
- Consistent with features of CCW envelope and embedded cloud systems

Linear theory

A measure of the east–west asymmetry due to wind shear:

- the jump in θ across the source, $[\theta] = \theta^+ - \theta^-$

Linearized equations with singular source term:

$$\partial_t \mathbf{u} + A(\bar{\mathbf{u}})\partial_x \mathbf{u} = \mathbf{S}^* \delta(x)$$

Rankine–Hugoniot jump conditions at location of source:

$$A(\bar{\mathbf{u}})[\mathbf{u}] = \mathbf{S}^*$$

Results: linear theory agrees with nonlinear simulations to within 10 %

Optimal shears for east–west asymmetry

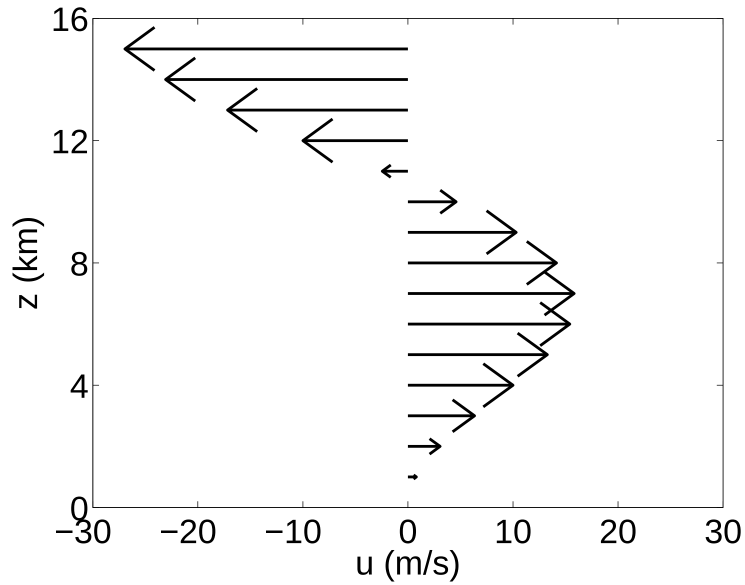
Which shear profiles $\bar{U}(z)$ maximize $[\theta_1]$?

Which shear profiles $\bar{U}(z)$ lead to $[\theta_1] = 0$

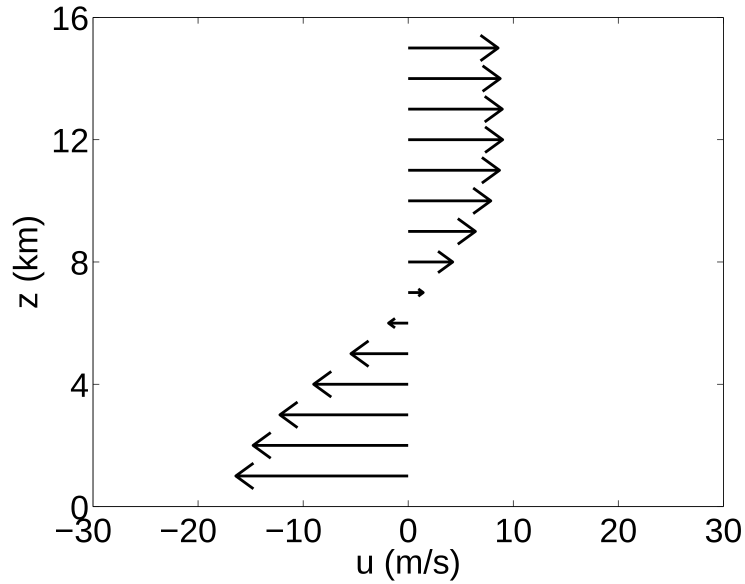
Use linear theory solutions: $A[\mathbf{u}] = \mathbf{S}^*$

Results:

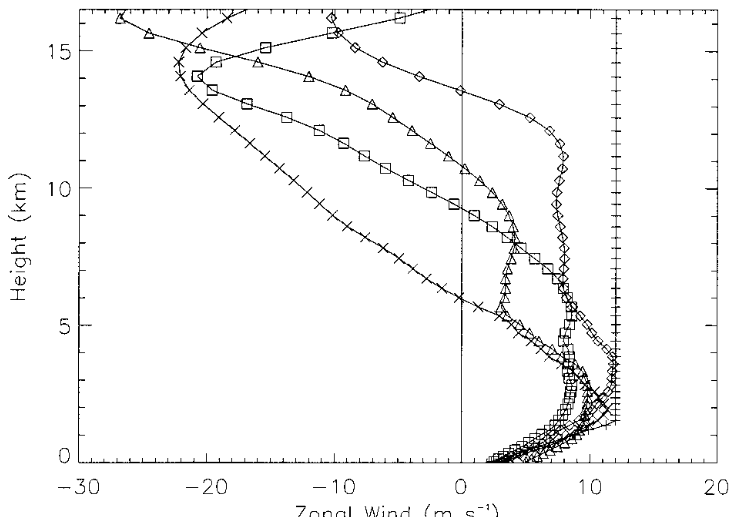
Jet shears maximize $[\theta_1]$

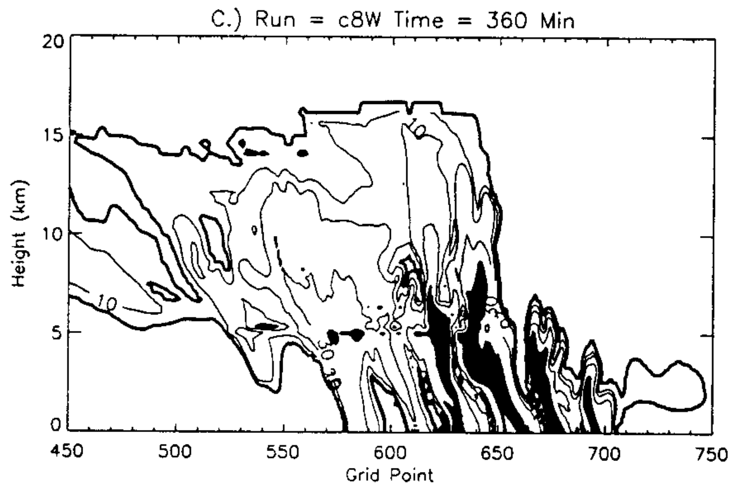


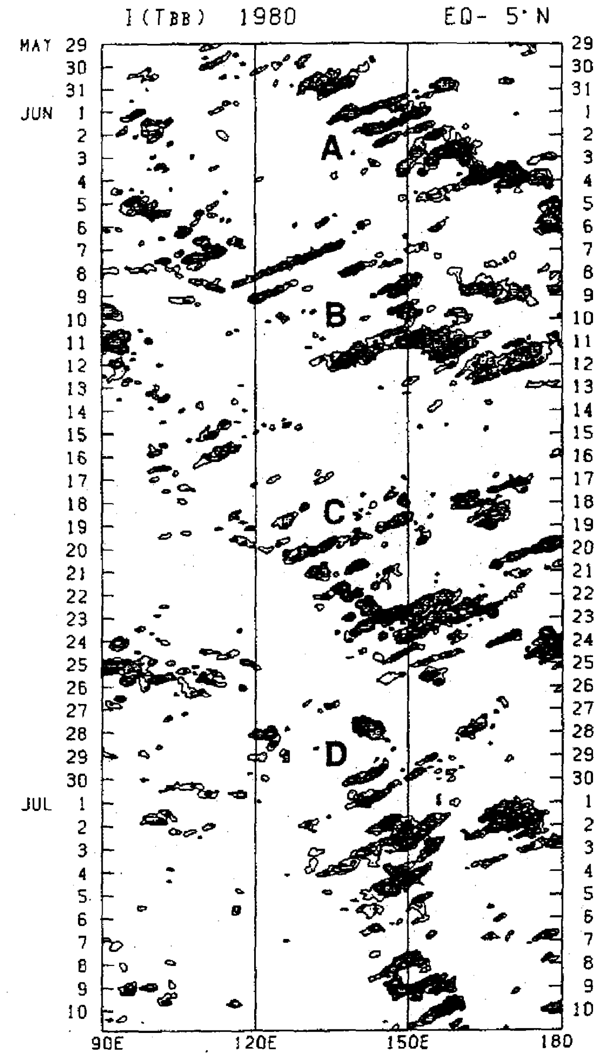
Profiles with zero shear at upper levels lead to $[\theta_1] = 0$



Wave trains of cloud systems







- Individual cloud systems propagate eastward
- Convectively coupled wave propagates westward

Summary of Application #1

- Wind shear can lead to east–west asymmetry
  - jet shears lead to largest east–west asymmetry
  - linear theory is accurate to within 10 % (usually)
  - *predictions of preferred propagation direction for convectively coupled gravity waves in a background wind shear*

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- Designing a simple model with waves in shear
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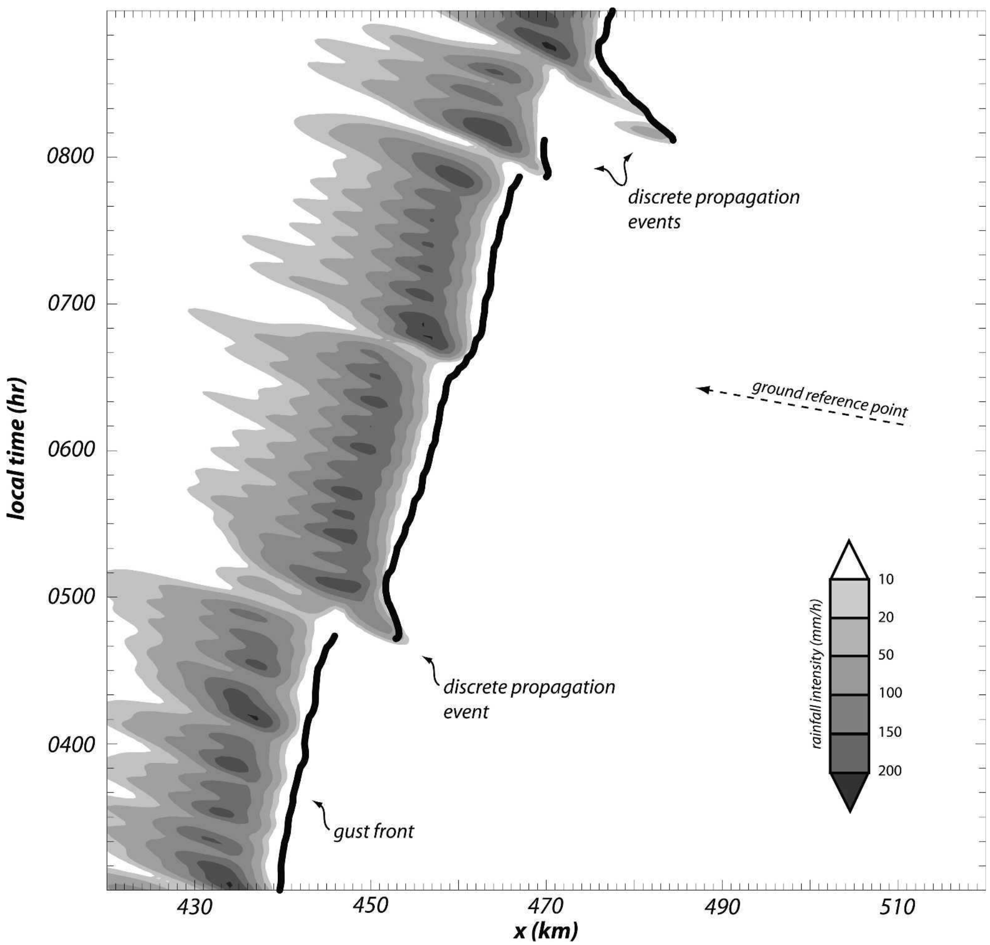
The role of gravity waves within an individual convective system

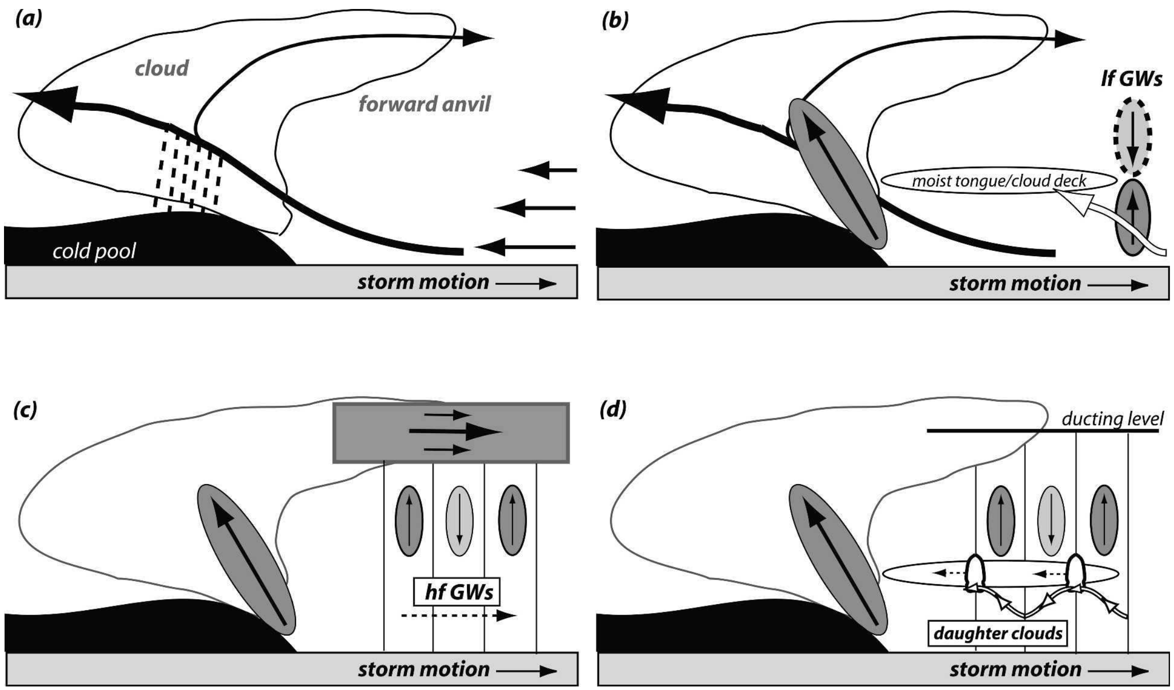


- New convective cells initiated ahead of existing squall line
- New cells merge with existing squall line

from "Discrete propagation in numerically simulated nocturnal squall lines"

by Fovel et al. (2006)

The role of gravity waves within an individual convective system

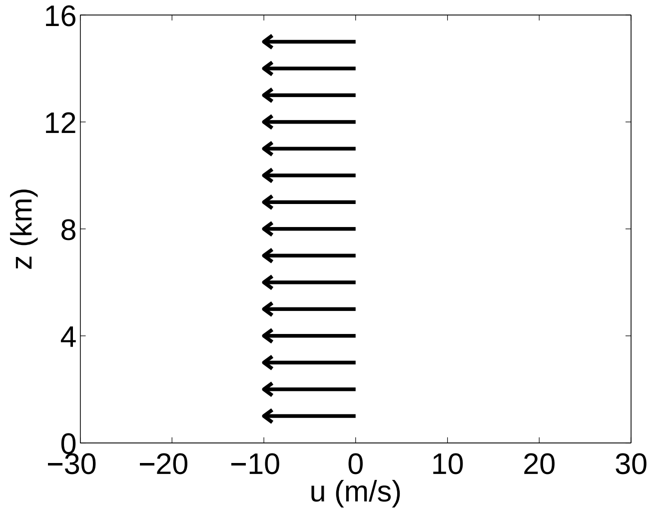


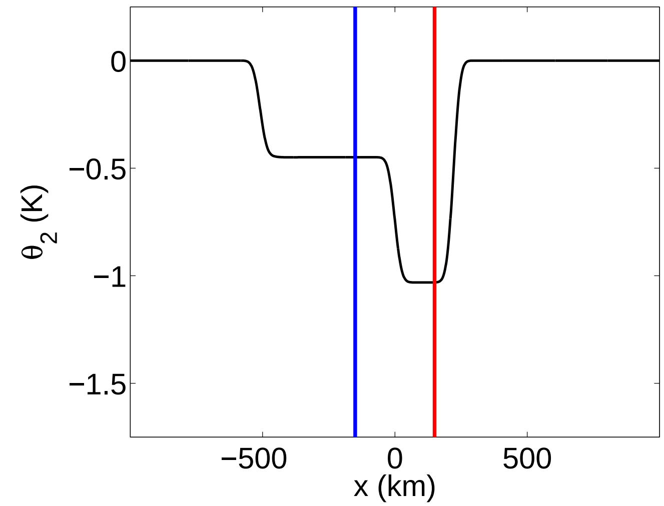
- Gravity waves initiate new convective cells
- What are the physical mechanisms involved?

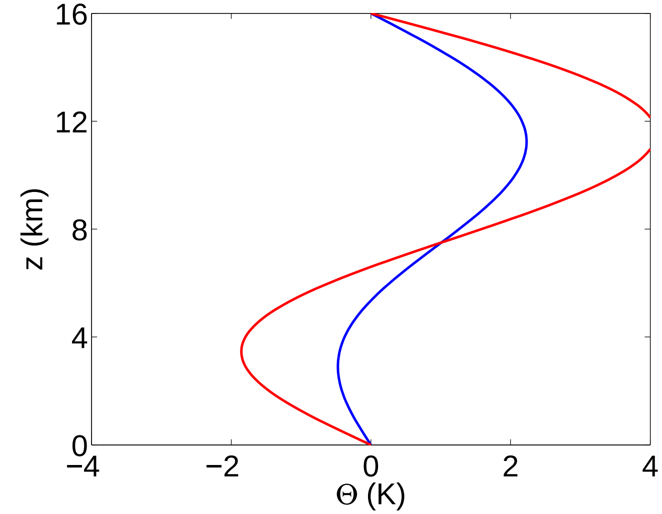
from "Discrete propagation in numerically simulated nocturnal squall lines" by Fovel et al. (2006)

Results with
simplified model for waves in shear

Numerical experiment with headwind



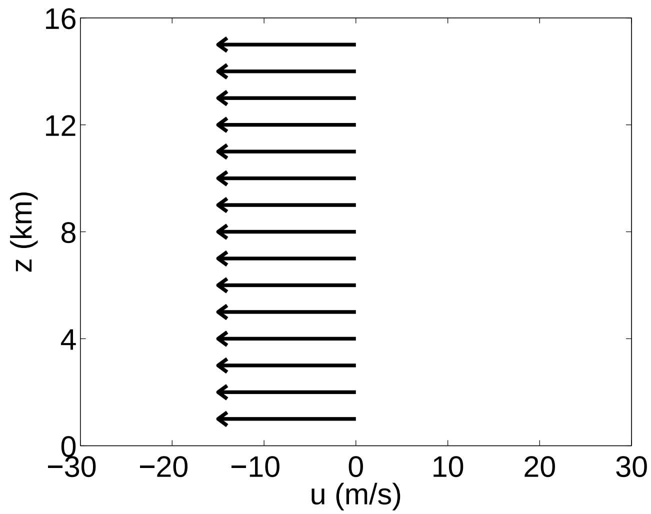


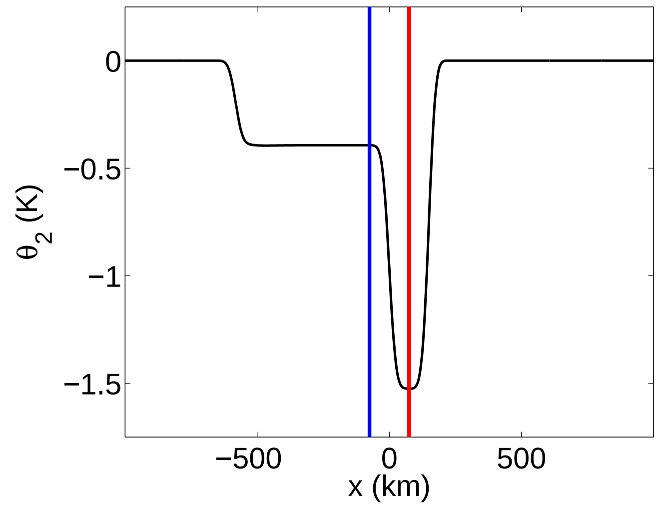


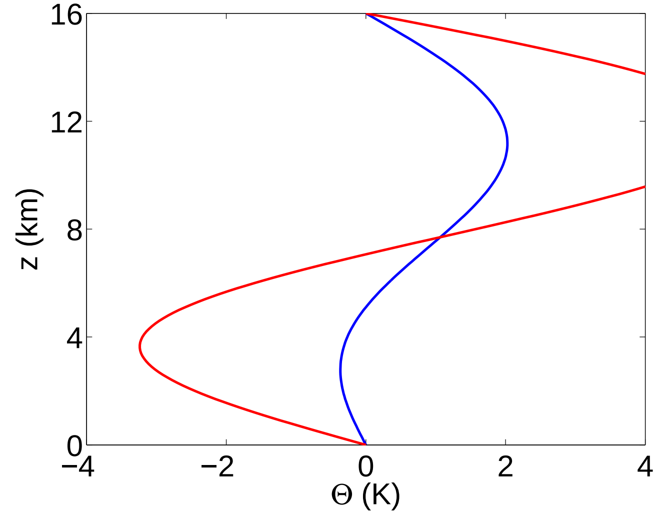
- Repeat earlier jet-shear experiment with headwind added

- Headwind is equivalent to propagating source (i.e., propagating squall line)
- Headwind confines upstream wave to vicinity of source
- East is more favorable for new convection than west *at low levels*

Numerical experiment with stronger headwind







Source propagation speed = 15 m/s

2nd baroclinic wave speed = 25 m/s

$\Rightarrow$ near resonant forcing

- Faster propagation leads to more favorable environment
- If squall line propagation speed ≈ gravity wave speed, then *wave amplitude is large due to near-resonance*

Summary

- 2-mode shallow water equations:
  - simplified nonlinear model for waves interacting with wind shear
  - interesting mathematical properties
  - numerical method passes several difficult tests
- Predictions of preferred propagation direction of convectively coupled waves in a background wind shear
  - wind shear can lead to east–west asymmetries
  - jet shears lead to largest east–west asymmetries
  - linear theory is accurate to within 10 % (usually)
- Initiation of new convective cells ahead of individual convective system
  - Propagation of source leads to near-resonant forcing and amplification of upstream waves

Stechmann and Majda (2009), *J. Atmos. Sci*

Stechmann, Majda, Khouider (2008), *Theoretical and Computational Fluid Dynamics*